Innovation and International Fragmentation of Production: complements or substitutes?

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Preliminary version. Comments most welcome

Abstract

In this paper we investigate the relationship between international fragmentation of production (IFP) and innovation performance at the country level. The sign of this relationship is not obvious a priori, as with IFP a country might generate more innovation thanks to positive spillovers, or it might reduce its innovative output because it relays on innovations produced elsewhere. We measure a country’s participation to IFP or to global value chains by using the recently released World Input-Output Database (WIOD), computing the share of foreign value added in a country’s gross export proposed by Koopman et al. (2014), and taking into account the weighted average of the R&D stock of a country’s production partners. We use these indicators to test empirically the relationship between a country’s innovation outcome, proxied by patent per capita, and the characteristics of its participation in the global value chains. Our preliminary results show that the involvement in IFP is positively related with a country’s innovation outcome.

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when the R&D stock level of the partners in IFP linkages is taken into account. In particular, the analysis by sub-samples shows that the results are driven by developing countries, suggesting that IFP can be a channel of technology spillovers allowing for international technology transfer from developed to developing countries.

**KeyWords:** global value chains, international fragmentation of production, innovation, R&D.

**JEL Classification:** F1(4), F6(0), O3(0), O4(0).
1 Introduction

Since the phenomenon of international fragmentation of production (IFP) has been highlighted in the economic literature (Hummels et al., 2001; Yi, 2003) much discussion occurred about its role and its effects on trade patterns and specialization. More recently, many efforts have been devoted to finding the appropriate measures of countries’ involvement in this international organization of production or in global value chains (GVC) (Timmer et al., 2014; Koopman et al., 2014). The importance of measuring correctly this phenomenon is due to the fact that IFP, by spreading the different phases of the production chain of a good across many countries, is in many ways different than trade in final goods, and straight trade measures can overestimate or underestimate its relevance. The creation of production linkages across countries, whether through different firms or between units of the same multinational firm, besides international trade, can affect many other economic features of a country. IFP will generally change the domestic organization of production not only between industries, as in traditional trade models, but also within industries, and it can be seen as a specific technological change (Feenstra and Hanson, 1996; Deardorff, 2001; Grossman and Rossi-Hansberg, 2008).

IFP can be not only a channel of domestic technological change, but also a vehicle of diffusion of technological progress. Several contributions have shown that research is carried out in only a few countries and a large part of technological change is due to international technology diffusion (Keller, 2002, 2004), that can be fostered by international trade in inputs and production sharing. It is therefore interesting to investigate how the participation to IFP can affect the technological frontier of a country, both directly and indirectly, as this paper wants to do.

In the paper, we compute two indicators of a country’s participation in global value chains, by using recently released World Input-Output Database (WIOD) www.wiod.org (Stehrer, 2012; Timmer et al., 2014). A first indicator is the share of foreign value added in a country’s gross export proposed by Koopman et al. (2014). This index should convey the overall weight of international production linkages in a country’s export and it is used to capture the involvement of a country in GVCs (De Backer and Miroudot, 2013). The second indicator is the weighted average of a country’s production partners’ R&D stock, where the weights are the foreign value added share by partner. This index should measure the exposure of a country to R&D spillovers through production linkages.

We then employ an empirical specification of a knowledge production function (Bottazzi and Peri, 2007; Malerba et al., 2013; Bloom et al., 2013)
in order to explore the relationship between a country’s involvement in IFP, proxied by the two indicators mentioned above, and its innovation outcome, measured by patent per capita. Our preliminary results show that the involvement of a country in IFP is positively related with a country’s innovation outcome when the R&D stock level of the partners is taken into account, this suggesting that IFP can indeed be a channel of technology spillovers. In particular, the analysis by sub-samples shows that the results are driven by developing countries, international technology transfer from developed to developing countries being potentially at work.

The structure of the paper is as follows: Section 2 discusses the possible relationship between IFP and innovation in a country, Section 3 reports our empirical estimation of this relationship between IFP and innovation; Section 4 concludes.

2 The relationship between innovation and participation in the global value chain

In principle, a country involvement in a vertical international relationship may affect its innovation performance both positively or negatively. First of all, involvement in a global value chain can generate relevant technology spillovers. Knowledge spillovers associated to trade, both through the interaction with foreign researchers and knowledge and through the exchange of technology incorporated in goods, has been highlighted by the literature since the seminal theoretical contributions of the endogenous growth theory (Grossman and Helpman, 1991; Rivera-Batiz and Romer, 1991), and the rich empirical stream of literature on international knowledge flows (Malerba et al., 2013; Keller, 2002, 2004; Gong and Keller, 2003; Bottazzi and Peri, 2003, 2007; Eaton and Kortum, 1999; Coe and Helpmann, 1995; Coe et al., 2009; Acharya and Keller, 2009; Bloom et al., 2013). The relevance of these spillovers can be enhanced when some features of the production process need to be shared between the production units that are exchanging intermediate goods. Therefore, compared to the traditional measures of knowledge spillovers through trade (Coe and Helpmann, 1995; Coe et al., 2009; Acharya and Keller, 2009), in the case of production sharing we can expect a stronger impact (see Piermartini and Rubnov (2014)).

Moreover, the international fragmentation of production (IFP) may increase the incentives to invest in R&D activities by increasing the market size allowing access to foreign markets and therefore to foreign demand for domestic input (Eaton and Kortum, 2001), at the same time increasing the
amount of resources available for R&D activities thanks to cost reduction induced by access to low cost inputs (Glass and Saggi, 2001).

But the involvement in a global value chain also changes the incentive system of a country, and the optimal allocation of resources to different sectors and to innovation activities. Participation into the global value chain may negatively affect the incentives to innovate and patent new goods through a ‘substitution’ effect, since cost reduction may represent an alternative strategy to compete in the international markets. At the same time, incorporating new foreign technologies by importing new foreign intermediate inputs may decrease the incentive to introduce innovation in intermediate goods (Eaton and Kortum, 2001), allowing for higher quality of the final goods without introducing (patenting) new-to-the-world inputs. This potential ‘substitution’ effect operates as a negative ‘spillover’ effect (as opposed to the positive ‘technology spillover’ mentioned above). Differently from the negative ‘spillover’ due to the product market rivalry effect of R&D, recently highlighted by Bloom et al. (2013), this negative spillover may be at work along ‘vertical’ relationship.

Following Coe et al. (2009), we exploit bilateral import relationship as a source of knowledge transfer, but we move a step further in the measures of participation into the international fragmentation of production, by considering the foreign content of imported inputs incorporated in goods then re-exported, corresponding to the the vertical specialization index (VS1) introduced by Hummels et al. (2001). Moreover, by using a measure of partners’ R&D weighted by the foreign value added index by partner, we combine production relationships with their potential R&D content. A positive relationship between a country’s participation in GVCs and its innovation performance would suggest that IFP is a channel of international technology transfer, therefore potentially positively affecting the world income distribution, at the country level, and the world income growth. Instead, a negative relationship would imply that through IFP innovative activities are shifted across countries, with a potential positive effect at the world level due to greater efficiency, but with possible negative effects for some countries and positive effects for others.

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1See also, Coe and Helpmann (1995); Keller (2002, 2004); Acharya and Keller (2009).
3 An empirical investigation on the relationship between innovation and participation in the global value chain.

3.1 The empirical framework

We consider the underlying empirical framework:

$$\text{PAT}_{it} = a_0 + a_1 \text{RDstock}_{it} + a_2 \text{IFP}_{it} + a_3 X_{it} + u_t + u_i + \epsilon_{it}$$  \hspace{1cm} (1)

where the dependent variable is country $i$’s patent applications per capita at time $t$, and $\text{RDstock}_{it}$ is country $i$’s R&D stock at time $t$, standard measures of innovation output and input, respectively. $\text{IFP}_{it}$ are our measures of international fragmentation of production and spillovers described below, $X_{it}$ is a vector of other potential explanatory variables, $u_t$ and $u_i$ are time and country fixed effects, respectively. We follow the knowledge production function approach by considering the R&D stock accumulated at time $t$ as the main determinant of a country’s patent applications at time $t$.

As a first step, we introduce in the above model of knowledge production function an indicator of participation in the global value chain following Koopman’s methodology (Koopman et al., 2014), the foreign value added content of a country’s gross export, which we interpret as a potential channel of both international knowledge spillovers, since it implies production relationship across countries, but at the same time a channel of ‘substitution’ of innovation activities as mentioned before.

As a second step, in line with the main contributions in the literature of international knowledge spillovers (Coe and Helpmann, 1995; Coe et al., 2009), we introduce another index in order to measure specifically the R&D content of these international production relationships, by weighting the aggregate R&D stock of the ‘offshoring’ partner with the foreign value added flows imported from the same partner by the reporting country:

$$\text{INTTECH}_{it} = \sum_s fvs_{st}^i * \text{R&D}_{st}$$  \hspace{1cm} (2)

where $i$ is the reporting country, $t$ is time, $s$ is the partner from which a country imports intermediate goods. This measure should better capture the potential effect of the international vertical relationships on innovation, showing the average net effect.

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3See, for instance, Malerba et al. (2013); Coe and Helpmann (1995); Coe et al. (2009).
4See Appendix 5.2.
We study the above relationship by initially considering the whole sample and then sub-samples of countries which should show a different position both in the global value chain and with respect to the domestic knowledge generation.

Our measures of international fragmentation of production are based on the recently released World Input-Output Database (WIOD) [www.wiod.org](http://www.wiod.org) (Stehrer, 2012, Timmer et al. 2014). The database indicates the transactions of intermediate products and final goods within and between each country at the industry level. It is built on official statistics: national Input-Output tables and national Supply-Use tables for 40 countries, for the period 1995-2011, extended with National Accounts time-series, linked using bilateral international trade statistics on goods and services. It provides domestic and international input-output flows at two digit level (40 sectors, 60 products), and factor income flows broken down by skill. Our measures of innovation are based on the World Bank database. We base therefore our analysis on an unbalanced panel of 39 countries plus the rest of the world (see the appendix) for the period 1999-2011.  

### 3.2 Results

In Column 1 of Table 1, we show the estimates of model (1), where we include, as a potential determinant of a country’s innovation performance, the foreign value added share of a country’s export, our first measure of participation in the global value chain. Our results show that, as expected, in line with previous literature, the R&D stock is significantly and positively related to countries’ innovation performance, while the overall measure of foreign value added is not significant. In Column 2, going a step further and in line with previous contributions (Coe et al., 2009) we include the second measure of potential international technological transfer, as in equation 2. Our measure of international technological transfer, where the R&D stock of all partners is weighted by the foreign value added flow is significantly and positively related with the innovation performance at the country level.

According to the first two channels mentioned in section 1, we should observe a positive relationship between IFP and patenting due to higher incentivises in investing in R&D. Since we control for this channel by controlling for own R&D stock, the coefficient of our IFP’s indicators should not be capturing the incentive channel. We are therefore capturing an effect which goes over and above what affects investments in R&D. As discussed above, since our indicators represent bilateral production relationships, they may convey

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5Taiwan is not included in the World Bank database.
both a positive effect (‘technological spillover’) and a negative effect (‘substitution effect’) on a country’s innovation performance. The overall positive sign shows that the ‘technological spillover’ prevails in the whole sample of countries.

In Table 2 we estimate model (1) in sub-samples of countries, namely the High Income countries and the Upper-Middle-Low Income countries, according to the World Bank classification. Our results show that the positive sign emerging in the whole sample is driven by the Upper-Middle-Low Income countries. In all the other sub-samples the coefficient is not significant.

4 Conclusion

In this paper we have employed an empirical model of Knowledge Production in order to explore the relationship between a country’s involvement in IFP, proxied by two indicators based on the recently released International Input-Output tables (WIOD), and its innovation outcome, measured by patent per capita. Our preliminary results show that the involvement of a country in IFP is positively related with a country’s innovation outcome when the R&D stock level of the partners is taken into account, this suggesting that IFP can be a channel of technology spillovers. In particular, the analysis by sub-samples shows that the results are driven by developing countries, allowing to think that a potential international technology transfer from developed to developing countries is at work. The empirical strategy does not allow us to identify a causal link, only suggesting the existence of a positive association which needs to be further investigated by improving the identification strategy. Furthermore, at the aggregate level, the index used does not allow to disentangle the potential positive effect due to the knowledge flows from the negative one, i.e. the ‘substitution’ effect. Therefore, we plan to also extend the analysis in order to take into account the industry level, which is important to identify the (positive) ‘technology spillover’ from the (negative) ‘substitution’ effect. Moreover, in future work we may consider more sophisticated indicators of a country’s involvement in the global value chain in order to better characterize some potential channels of the knowledge transfer and of the substitution effect.

6See Appendix 5.1.
**Table 1:** Models of international knowledge spillovers. All countries.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R&amp;D stock</td>
<td>2.911***</td>
<td>2.921***</td>
</tr>
<tr>
<td></td>
<td>(0.375)</td>
<td>(0.370)</td>
</tr>
<tr>
<td>gdp per capita</td>
<td>-0.041</td>
<td>-0.047</td>
</tr>
<tr>
<td></td>
<td>(0.221)</td>
<td>(0.251)</td>
</tr>
<tr>
<td>fvs</td>
<td>-0.005</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.176)</td>
<td></td>
</tr>
<tr>
<td>inttech</td>
<td></td>
<td>0.113**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.054)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.4573</td>
<td>0.4595</td>
</tr>
<tr>
<td>N</td>
<td>385</td>
<td>385</td>
</tr>
</tbody>
</table>

* p<0.10, ** p<0.05, *** p<0.01

Note: Dependent variable: Patent (resident) per capita. All models include year and country FE. All variables are in log and lagged one year. Driscoll-Kraay standard errors, robust to general forms of spatial correlation, are reported. (a): within R-squared.

**Table 2:** Models of international knowledge spillovers. Developed and developing countries.

<table>
<thead>
<tr>
<th></th>
<th>HIGH INCOME</th>
<th>MIDDLE-LOW INC.</th>
</tr>
</thead>
<tbody>
<tr>
<td>R&amp;D stock</td>
<td>2.319***</td>
<td>4.286***</td>
</tr>
<tr>
<td></td>
<td>(0.338)</td>
<td>(0.345)</td>
</tr>
<tr>
<td>gdp per capita</td>
<td>0.084</td>
<td>-1.903***</td>
</tr>
<tr>
<td></td>
<td>(0.249)</td>
<td>(0.392)</td>
</tr>
<tr>
<td>inttech</td>
<td>-0.040</td>
<td>0.577**</td>
</tr>
<tr>
<td></td>
<td>(0.126)</td>
<td>(0.175)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.2699</td>
<td>0.7737</td>
</tr>
<tr>
<td>N</td>
<td>294</td>
<td>91</td>
</tr>
</tbody>
</table>

* p<0.10, ** p<0.05, *** p<0.01

Note: Dependent variable: Patent (resident) per capita. All models include year and country FE. All variables are in log and lagged one year. Driscoll-Kraay standard errors, robust to general forms of spatial correlation, are reported. (a): within R-squared.
5 Appendix

5.1 Countries’ classification

List of countries: Austria, Australia, Belgium, Bulgaria, Brazil, Canada, China, Cyprus, Czech Rep., Germany, Denmark, Estonia, Greece, Spain, Finland, France, Hungary, Indonesia, Ireland, India, Italy, Japan, Korea, Lithuania, Luxembourg, Latvia, Malta, Mexico, Netherlands, Poland, Portugal, Romania, Russia, Sweden, Slovenia, Slovakia, Turkey, United Kingdom, United States and Rest of the World (40, 39 plus RoW; 27 EU).

5.1.1 World Bank countries’ classification

- High income includes: Austria, Australia, Belgium, Canada, Cyprus, Czech Rep., Germany, Denmark, Estonia, Greece, Spain, Finland, France, Ireland, Italy, Japan, Korea, Lithuania, Luxembourg, Latvia, Malta, Netherlands, Poland, Portugal, Russia, Sweden, Slovenia, Slovakia, United Kingdom, United States.

- Upper-Middle-Low income includes: Bulgaria, Brazil, China, Hungary, Mexico, Romania, Turkey, India, Indonesia.

5.2 The Foreign Value Added in a country’s export

Here we describe the parts of the Inter-Country Input-Output model of (?) that we have used to compute the foreign value-added embodied in a country’s exports.

Assume a $G$-country world, in which each country produces goods in $N$ differentiated sectors. Goods in each sector might be consumed directly or used as intermediate input. Each country can also export both intermediate and final goods.

All gross output produced by country $s$ must be used as either an intermediate good or a final good at home or in other countries,

$$X_s = A_{ss}X_s + A_{sr}X_r + Y_{ss} + Y_{rs}, r, s = 1, ..., G r \neq s \quad (3)$$

where $X_s$ is the $N \times 1$ gross output vector of country $s$, $Y_{sr}$ is the $N \times 1$ final demand vector that represent demand in country $r$ for final goods produced in $s$ and $A_{sr}$ is the $N \times N$ Input-Output coefficient matrix, showing the use in $r$ of intermediate goods produced in $s$. The $G$-country production
and trade system can be written as Inter-Country Input-Output model in block matrix notation

\[
\begin{bmatrix}
X_1 \\
X_2 \\
\vdots \\
X_G
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & \cdots & A_{1G} \\
A_{21} & A_{22} & \cdots & A_{2G} \\
\vdots & \vdots & \ddots & \vdots \\
A_{G1} & A_{G2} & \cdots & A_{GG}
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2 \\
\vdots \\
X_G
\end{bmatrix} +
\begin{bmatrix}
Y_{11} + Y_{12} + \ldots + Y_{1G} \\
Y_{21} + Y_{22} + \ldots + Y_{2G} \\
\vdots \\
Y_{G1} + Y_{G2} + \ldots + Y_{GG}
\end{bmatrix}
\] (4)

and rearranging

\[
\begin{bmatrix}
X_1 \\
X_2 \\
\vdots \\
X_G
\end{bmatrix} =
\begin{bmatrix}
1 - A_{11} & -A_{12} & \cdots & -A_{1G} \\
-A_{21} & 1 - A_{22} & \cdots & -A_{2G} \\
\vdots & \vdots & \ddots & \vdots \\
-A_{G1} & -A_{G2} & \cdots & 1 - A_{GG}
\end{bmatrix}^{-1}
\begin{bmatrix}
\sum_r^G Y_{1r} \\
\sum_r^G Y_{2r} \\
\vdots \\
\sum_r^G Y_{Gr}
\end{bmatrix}
= 
\begin{bmatrix}
B_{11} & B_{12} & \cdots & B_{1G} \\
B_{21} & B_{22} & \cdots & B_{2G} \\
\vdots & \vdots & \ddots & \vdots \\
B_{G1} & B_{G2} & \cdots & B_{GG}
\end{bmatrix}
\begin{bmatrix}
Y_1 \\
Y_2 \\
\vdots \\
Y_G
\end{bmatrix}
\] (5)

where \( B_{sr} \) denotes the \( N \times N \) block Leontief inverse matrix, which is the total requirement matrix that gives the amount of gross output produced in country \( s \) for one-unit increase in final demand in country \( r \), \( Y_s = \sum_r^G Y_{sr} \) is the \( N \times 1 \) vector that gives the global use of \( s \)'s final products. This system can be also expressed as:

\[
X = (I - A)^{-1}Y = BY
\] (6)

where \( X \) and \( Y \) are \( GN \times 1 \) vectors, and \( A \) and \( B \) as \( GN \times GN \) matrices.

Having defined the Leontief inverse matrix, we turn to show how domestic and foreign contents of gross exports are computed. Let \( V_s \) be the \( 1 \times N \) direct value-added coefficient vector. Each element of \( V_s \) gives the share of direct domestic value added in total output. This is equal to one minus the intermediate input share from all countries (including domestically produced intermediates):

\[
V_s \equiv u(I - \sum_r A_{rs})
\] (7)

where \( u \) is a \( 1 \times N \) unity vector. To be consistent with the Inter-Country model, we define \( V \) the \( G \times GN \) matrix of direct domestic value added for all countries,
\[
V \equiv \begin{bmatrix}
V_1 & 0 & 0 & 0 \\
0 & V_2 & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & V_G
\end{bmatrix}
\]  \hspace{1cm} \text{(8)}

As in Koopman et al. (2014), combining \(V\) with Leontief inverse matrix \(B\) produces the \(G \times GN\) value-added share (\(VB\)) matrix, \(VB\) is our basic measure of value-added shares by source of production:

\[
VB = \begin{bmatrix}
V_1B_{11} & V_1B_{12} & \ldots & V_1B_{1G} \\
V_2B_{21} & V_2B_{22} & \ldots & V_2B_{2G} \\
\vdots & \vdots & \ddots & \vdots \\
V_GB_{G1} & V_GB_{G2} & \ldots & V_GB_{GG}
\end{bmatrix}
\]  \hspace{1cm} \text{(9)}

Within \(VB\), each element \(V_sB_{sr}\) is a \(1 \times N\) vector. Vectors on the diagonal denote domestic value-added share of domestically produced \(N\) products. The out-diagonal vectors along columns denote instead the foreign country’s value-added shares in the same domestically produced \(N\) products. Each of the first \(N\) columns in the \(VB\) matrix includes all value added components, domestic and foreign, needed to produce one additional unit of domestic product at home.

Because all value added must be either domestic or foreign, the sum along each column is unity.

The \(VB\) matrix contains all the information to separate domestic and imported content shares in each country’s gross exports at the sectoral level.

Let \(E_{sr}\) be the \(N \times 1\) vector of gross exports from \(s\) to \(r\). For consistency with the Inter-Country Input-Output model we also define

\[
E_{ss} = \sum_{r \neq s}^{G} E_{sr} = \sum_{r \neq s}^{G} (A_{sr}X_r + Y_{sr}) \quad r, s = 1 \ldots G
\]  \hspace{1cm} \text{(10)}

\[
E = \begin{bmatrix}
uE_{1s} & 0 & \ldots & 0 \\
0 & uE_{2s} & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & uE_{Gs}
\end{bmatrix} = \begin{bmatrix}
E_{1s} & 0 & \ldots & 0 \\
0 & E_{2s} & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & E_{Gs}
\end{bmatrix}
\]  \hspace{1cm} \text{(11)}

where \(E\) is a \(GN \times G\) export matrix and each element \(E_{ss} = uE_{ss}\) is a \(N \times 1\) vector given by the product of the unity \(N \times 1\) vector \(u\) and the scalar \(E_{ss}\).
The combination of value added share matrix $VB$ and the export matrix $E$ produces a $G \times G$ matrix ($VBE$) that represents the aggregate measures of value-added by origin in countries gross exports

$$VBE = \begin{bmatrix} V_1B_{11}E_{1*} & V_1B_{12}E_{2*} & \ldots & V_1B_{1G}E_{G*} \\ V_2B_{21}E_{1*} & V_2B_{22}E_{2*} & \ldots & V_2B_{2G}E_{G*} \\ \vdots & \vdots & \ddots & \vdots \\ V_GB_{G1}E_{1*} & V_GB_{G2}E_{2*} & \ldots & V_GB_{GG}E_{G*} \end{bmatrix}$$  \hspace{1cm} (12)

Diagonal elements of $VBE$ define the domestic value-added in each countrys gross exports. Off-diagonal elements along each column give the foreign value-added embodied in each countrys exports by origin. Therefore, gross exports of country $s$ can be decomposed into two components: domestic value-added content of gross exports ($DV_s$) and foreign value-added content of gross exports ($FV_s$) as follows

$$DV = \begin{bmatrix} DV_1 \\ DV_2 \\ \vdots \\ DV_G \end{bmatrix} = \begin{bmatrix} V_1B_{11}E_{1*} \\ V_2B_{22}E_{2*} \\ \vdots \\ V_GB_{GG}E_{G*} \end{bmatrix}$$  \hspace{1cm} (13)

$$FV = \begin{bmatrix} FV_1 \\ FV_2 \\ \vdots \\ FV_G \end{bmatrix} = \begin{bmatrix} \sum_{r \neq 1} V_rB_{r1}E_{1*} \\ \sum_{r \neq 2} V_rB_{r2}E_{2*} \\ \vdots \\ \sum_{r \neq G} V_rB_{rG}E_{G*} \end{bmatrix}$$  \hspace{1cm} (14)

$FV$ and $DV$ are both $G \times 1$ vectors. Elements of $FV$ are the result of the sum of out-diagonal elements along each column of $VBE$.

It holds that for the generic country $s$

$$E_{ss} = V_sB_{ss}E_{ss} + \sum_{r \neq s} V_rB_{rs}E_{ss}$$  \hspace{1cm} (15)

Therefore we can easily derive the aggregate measures of domestic and foreign shares of value-added incorporated in country $s$ gross exports as

$$DVA_s = DV_s/E_{ss}$$  \hspace{1cm} (16)

$$FVA_s = FV_s/E_{ss}$$  \hspace{1cm} (17)

Note that measures indicated here as $DV$ are instead denoted in (Koopman et al., 2014) as DC standing for domestic content of gross export; measures indicated here as $FV$ are instead labeled in (Koopman et al., 2014) as
VS standing for the foreign content of gross exports. VS indeed corresponds to the index proposed by (Hummels et al., 2001) for measuring vertical specialization.

References


