Macroeconomics with endogenous markups and optimal taxation

by Federico Etro

June 2016

Abstract

We extend the Real Business Cycle framework to any symmetric preferences over a variety of goods supplied under monopolistic, Bertrand or Cournot competition with a fixed or endogenous number of firms, and derive the implications for business cycles and optimal taxation. With an exogenous number of goods, the model delivers a modified Euler equation when preferences are non-homothetic: if the endogenous markups are countercyclical the impact of shocks on consumption and labor supply is magnified through new intertemporal substitution mechanisms, and the optimal fiscal policy requires a countercyclical labor income subsidy and a capital income tax that is positive along the growth path and converging to zero in the long run. With an endogenous number of goods, also entry affects markups and the optimal fiscal policy requires also a tax on profits.

Key words: Real Business Cycles, monopolistic competition, variable markups, optimal taxation.

JEL Codes: E1, E2, E3.

1I am extremely grateful to Fabio Ghironi, Mario Padula, Lorenza Rossi and especially Paolo Bertoletti and Lilia Cavallari for discussions, and to seminar participants at the University of Essex. Correspondence: Federico Etro: Ca’ Foscari University, Venice, Sestiere Cannaregio, 30121, Fond.ta S.Giobbe 873, Venice, Italy. Tel: +39-0412349172, email: federico.etro@unive.it.
In this work we develop a dynamic model with a richer microfoundation of preferences than usually assumed in standard macroeconomic theory, and we augment it with monopolistic and imperfect competition as well as with an exogenous or an endogenous number of firms. Such a framework nests the traditional Real Business Cycle (RBC) framework (as developed since Kydland and Prescott, 1982) and the flexible price version of modern New-Keynesian models (as developed since Blanchard and Kiyotaki, 1987), but it also generates new pervasive inefficiencies. Beside the inefficiency in labor supply typical of models with monopolistic competition based on CES preferences, our framework implies also inefficiency in the process of savings and capital accumulation and in the process of creation of new goods. After characterizing the optimal allocation of resources in various cases, we derive the optimal taxation system that restores this first best allocation. This requires a variable labor subsidy and a variable capital income taxation (which is zero in the long run). Moreover, when entry is endogenous, it also requires a tax on profits.

The motivation for this work is methodological and both positive and normative. The modern process of microfoundation of flexible price macroeconomics (started at least since Lucas and Rapping, 1969) has been mainly focused on the supply-side. Indeed, under either perfect competition or monopolistic competition over an exogenous number of goods with CES preferences, prices depend only on technological conditions (the markups are either zero or constant) and the demand-side has little chances to affect the business cycle. We depart from this tradition by modeling general symmetric preferences over the final goods for an infinitely living agent. The model has a standard perfectly competitive sector that produces an intermediate good with a Cobb-Douglas production function. However, under imperfect competition in the market for final goods, prices and markups depend on the consumption level because this affects the demand elasticity, and therefore markups change over time with changes in demand conditions. The variability of prices over time has two crucial implications when preferences are non-homothetic. The first positive implication is that the amplification of aggregate shocks can be amplified by monopolistic pricing through additional substitution effects: a temporary reduction in markups increases more current consumption and current labor supply. The normative implication is that savings decisions follow a modified Euler condition depending on relative price movements and are suboptimal unless preferences are homothetic: this requires variable labor subsidies and capital income taxes to restore optimality. Finally, Bertrand and Cournot competition increase the markups compared to monopolistic competition, affecting both the business cycle properties and the optimal tax rules.

As already noticed in the recent literature on dynamic entry (see for instance Etro and Colciago, 2010; Bilbiie, Ghironi and Melitz, 2008a, 2012), an additional

\[\text{As well known, homothetic and identical preferences in the consumption goods are essential for aggregation of the demand functions into a single demand function of a representative consumer with the average income. Since heterogeneity in preferences and the computational problem of aggregating demand functions are not our concern here, we assume that there is a single agent in the economy.}\]
limit of the modern process of microfoundation of flexible-price macroeconomics based on monopolistic competition is that the set of firms is usually assumed exogenous, and often normalized to unity (for a survey see, for instance, Woodford, 2003). This is in contrast with the basic principle of rational decision-making if one believes that the decision to enter in a market is the basic decision taken by firms. The literature has already analyzed business cycle in models with endogenous entry, but only under the assumption of CES or homothetic aggregators, and has analyzed optimal corrective taxation only in models without capital accumulation (see Bilbiie, Ghironi and Melitz, 2008b and Colciago, 2016). We extend the analysis to any symmetric intratemporal preferences and capital accumulation. In such a general case, the markups depend on the Morishima elasticity of substitution (see Blackorby and Russell, 1981, and Bertoletti and Etro, 2016) and through that on both the consumption level and the number of firms. The business cycle properties of the general model are much richer, and we propose examples based on different preference aggregators that can be used for quantitative investigations. The optimal taxation scheme inherits the labor subsidy and the capital income tax from the baseline framework with a fixed number of goods, and complements this with a profit tax needed to restore the optimal entry process.

The analysis builds on recent advances in microeconomic and macroeconomic theory. On the first front, the wide literature on dynamic consumption theory in partial equilibrium has already analyzed a variety of preference specifications (for an interesting treatment with direct additivity see, for instance, Browning and Crossley, 2000), but has usually neglected implications for pricing under imperfect competition and for its feedback on consumption. Our analysis builds mainly on the industrial organization literature which has recently provided more general microfoundations to the analysis of imperfect competition. In particular, while Dixit and Stiglitz (1977) introduced the formal analysis of monopolistic competition with directly additive preferences, Benassy (1996) has considered homothetic ones and Bertoletti and Etro (2017) have analyzed indirectly additive preferences. Bertoletti and Etro (2016) have put together these and more general symmetric preferences in a unique framework studying monopolistic, Bertrand and Cournot competition, and we employ such a framework in a dynamic model.

On the macroeconomic side, the introduction of monopolistic competition goes back to Blanchard and Kiyotaki (1987), but most of the subsequent applications have used their CES microfoundation. Notable exceptions include Kimball (1995), who has used a class of (implicitly additive) homothetic aggregators, and Bilbiie, Ghironi and Melitz (2008a), who have used translog preferences, but both these works (as well as a wide derived literature) have focused on sticky prices. The closest work in our spirit is the one by Bilbiie, Ghironi and Melitz (2008b, 2012) who have analyzed a dynamic entry model with flexible prices, monopolistic competition and general homothetic aggregators to study business cycle and optimal taxation. Besides differences in modeling entry and

\[3\text{In the literature on dynamic entry see also La Croce and Rossi (2014) and Poutineau and}\]
capital accumulation, our main contribution is to depart from homotheticity. As we have noticed, this has radical implications for the difference between perfect and imperfect competition when the number of goods is exogenous, for the propagation of the shocks and for the analysis of the optimal taxation. Lewis and Winkler (2015) have analyzed optimal taxation in a related but static environment. Colciago (2016) has analyzed optimal Ramsey taxation in a model based on CES preferences with Cournot and Bertrand competition and we will generalize some of his results. Finally, Etro (2016) has derived the modified Euler equation for a Ramsey model with monopolistic competition for a representative good without analyzing the business cycle implications of models with endogenous labor supply and endogenous entry and without considering labor and profit taxation to restore optimality.

The work is organized as follows. Section 1 presents the baseline model with a fixed number of goods and directly additive preferences with perfect and monopolistic competition, and extends it to strategic interactions and endogenous entry: the optimal taxation scheme is derived for each case. Section 2 extends the baseline model to general symmetric preferences characterizing the optimal taxation scheme for some relevant classes of preferences. Section 3 discusses the most general model with imperfect competition and endogenous entry under general preferences. Also in this case we derive informally the optimal tax system. Section 4 is the conclusion.

1 A dynamic model with additive preferences

Our analysis is based on a traditional Real Business Cycle model on the supply side. The main novelties are on the demand side. In this section we consider a single agent with the following intertemporal utility function:

\[ \tilde{U} = \mathbb{E} \sum_{t=1}^{\infty} \beta^{t-1} \left[ \sum_{j=1}^{n} u(C_{jt}) - \frac{v L_t^{1 + \frac{1}{\varphi}}}{1 + \frac{1}{\varphi}} \right] \]  

(1)

where \( \beta \in (0, 1) \) is the discount factor, \( \mathbb{E}[\cdot] \) is the expectations operator, \( n \in \mathbb{Z}^+ \) is the number of final goods \( j = 1, 2, ..., n \) (assumed exogenous for now), the subutility \( u(\cdot) \) is assumed to be the same for each good with \( u'(C) > 0 \) and \( u''(C) < 0 \), and the disutility from labor \( L_t \) is isoelastic with a Frish elasticity \( \varphi \geq 0 \) and \( v \geq 0 \).

Capital \( K_t \) and labor supply \( L_t \) are entirely employed by a perfectly competitive sector producing an intermediate good with a Cobb-Douglas production function:

\[ Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \]  

(2)

where \( A_t \) is total factor productivity. The intermediate good is the numeraire of the economy and can be used to invest in a standard process of capital

Vermandel (2015) for interesting related investigations with imperfections in the goods and financial markets.
accumulation with depreciation rate $\delta \in [0, 1]$, or to produce final goods with a linear technology.

Consider a decentralized organization where each variety $i$ is sold at price $p_{it}$ chosen in each period by a firm $i$ with profits:

$$ \pi_{it} = (p_{it} - 1) C_{it} $$

(3)

The consumer receives all the profits as dividends, $\Pi_t = \sum_{j=1}^{n} \pi_{jt}$, and the remuneration of the inputs.

The markets for the factors of production are perfectly competitive. The labor market implies the following wage in units of intermediate goods:

$$ w_t = (1 - \alpha)A_t K_t^{-\alpha}L_t^- $$

(4)

and the capital market implies the following rental rate:

$$ r_t = \alpha A_t K_t^{\alpha - 1}L_t^{-\alpha} $$

(5)

In each period, the consumer chooses spending on each variety $C_{jt}$ for $j = 1, 2, ..., n$, labor supply $L_t$ and the future stock of capital $K_{t+1}$ to maximize utility under the resource constraint:

$$ K_{t+1} = K_t(1 - \delta) + w_tL_t + r_tK_t + \Pi_t - \sum_{j=1}^{n} p_{jt}C_{jt} $$

(6)

where total profits $\Pi_t$ and prices $p_{jt}$ for any good $j$ are taken as given.

The FOCs for $C_{jt}$ are:

$$ u'(C_{jt}) = \lambda_t p_{jt} \quad \text{for } j = 1, ..., n $$

(7)

the FOC for $L_t$ is:

$$ uL_t^{\frac{1}{\beta}} = \lambda_tw_t $$

(8)

and the FOC for $K_{t+1}$ is:

$$ \lambda_t = \beta E[R_{t+1}\lambda_{t+1}] $$

(9)

where the Lagrange multiplier $\lambda_t$ corresponds to the marginal utility of income and $R_{t+1} = 1 + r_{t+1} - \delta$. The marginal utility of income can be computed as:

$$ \lambda_t = \frac{\sum_{j=1}^{n} C_{jt}u'(C_{jt})}{E_t} $$

(10)

where $E_t = \sum_{j=1}^{n} p_{jt}C_{jt}$ is total expenditure in period $t$.

Perfect competition in the production of the differentiated goods implies $p_{jt} = 1$ for each good, so that consumption is also symmetric, $C_{jt} = C_t$ for any $j$, and the equilibrium equations are the same of a standard RBC model:

$$ u'(C_t) = \beta E \left\{ [1 + \alpha A_{t+1} K_{t+1}^{\alpha - 1}L_{t+1}^- - \delta] u'(C_{t+1}) \right\} $$
\[ K_{t+1} = K_t(1 - \delta) + A_t K_t^\alpha L_t^{1-\alpha} - nC_t \]
\[ L_t = \left[ (1 - \alpha) A_t K_t^\alpha u'(C_t) \right]^{\frac{1}{\alpha + \gamma}} \]

for a given initial level of capital \( K_0 \). When \( n = 1 \) this is the usual RBC model. Otherwise, total consumption \( C_t = nC_t \) is equally divided between multiple goods. As well known, the concavity of the \( u(C) \) function and the transversality condition are necessary and sufficient to guarantee the existence of a unique steady state with a saddle-path stable equilibrium.

### 1.1 Monopolistic competition

We need additional conditions, compared to perfect competition, for the existence of a saddle-path stable equilibrium with monopolistic pricing for each good. In particular, we assume that the function \( u'(C)C \) is increasing and concave. This is equivalent to:

\[ u'(C) + Cu''(C) > 0 \quad \text{and} \quad 2u''(C) + u'''(C)C < 0 \quad (11) \]

Given this assumption, we can obtain the following characterization of the equilibrium:

**Proposition 1.** In a dynamic model with directly additive intratemporal preferences (1) satisfying (11) and monopolistic competition over an exogenous number of goods, the equilibrium satisfies:

\[
\begin{align*}
K_{t+1} &= K_t(1 - \delta) + A_t K_t^\alpha L_t^{1-\alpha} - n C_t \\
L_t &= \left[ (1 - \alpha) A_t K_t^\alpha u'(C_t) \right]^{\frac{1}{\alpha + \gamma}} \quad (12)
\end{align*}
\]

where \( \beta \equiv -\frac{u''(C)C}{u'(C)} \) is the index of relative risk aversion, and has a unique deterministic steady state which is saddle-path stable.

As before total consumption \( \bar{C}_t \equiv nC_t \) follows the same dynamic pattern of individual consumption \( C_t \) because the number of goods is given. The main difference due to market power is that the marginal utility of consumption \( u'(C_t) \) is replaced in both the Euler condition and the labor supply condition with \( u'(C_t) [1 - \epsilon(C_t)] \), which can be interpreted as the marginal revenue of each monopolistic firm (Etro, 2016). This is smaller than the marginal utility, but positive under (11), which creates a reduction in the production level due to the intratemporal distortion of labor supply. Moreover, the marginal revenue is decreasing under (11) and can be variable over time, which creates an intertemporal distortion affecting savings and capital accumulation.

\[ ^4 \text{A sufficient condition for this is } u'''(C) \leq 0. \]
To prove the Proposition, notice that each firm producing a variety $i$ maximizes its profits:

$$\pi_{it} = \left[ \frac{u'(C_{it})}{\lambda_t} - 1 \right] C_{it}$$

where we used $p_{it} = \frac{u'(C_{it})}{\lambda_t}$ and the marginal utility of income $\lambda_t$ is taken as given by each firm under monopolistic competition à la Dixit and Stiglitz (1977). The choice of the monopolistic price is equivalent to the optimal choice of $C_{it}$, and provides the FOC $u'(C_{it})/\lambda_t + u''(C_{it})C_{it}/\lambda_t = 1$ for any firm $i$. Notice that the SOC for profit maximization requires the marginal revenue $u'(C) + u''(C)C$ to be decreasing, or $u'(C)C$ to be concave, as assumed in (11). Rearranging, we obtain the same equilibrium price for each firm, with:

$$p_t = \frac{1}{1 - \epsilon(C_{it})}$$

(15)

The relative risk aversion $\epsilon(C)$, smaller than one under (11), is the key determinant of markups. As long as this measure of risk aversion is countercyclical, markups are countercyclical because the demand for each good becomes more elastic in a boom and firms reduce their prices. As we will see later on, under our assumptions on preferences, the relative risk aversion corresponds also to the reciprocal of the elasticity of substitution between goods, which is in general the relevant concept of demand elasticity for monopolistic pricing.

Replacing (15) in (7) and then in (9), we can rewrite the modified Euler equation as in Proposition 1 after using (5). This condition shows how the consumption behavior of the agent can change compared to perfect competition. More precisely, if markups are constant, their existence is not relevant _per se_ because higher markups generate higher profits that are entirely redistributed to the consumer. However, variations in the markups across periods are relevant. If a consumer faces high prices and expects them to decrease in the future, he or she may reduce its current consumption and save more to be able to consume more in the future. When consumption is increasing, this is exactly what happens when the relative risk aversion is decreasing: in such a case, consumption growth induces a reduction in prices which induces consumers to decrease initial consumption and increase future consumption. More important for business cycle analysis, a positive and temporary shock that increases consumption is also going to reduce markups under monopolistic competition, which induces an additional upward pressure on consumption on impact. In other words, the tendency toward excessive consumption smoothing of the neoclassical models is limited because changes in markup determine a new substitution effect. Of course, the opposite happens when the relative risk aversion is increasing in consumption.

The optimality condition for the labor supply is obtained replacing (15) and (7) in (8), and using the equilibrium wage from (4). The result is modified compared to the condition under perfect competition for the index of relative risk aversion. If markups are constant, labor supply is only shifted proportionally in each period. However, when the relative risk aversion is countercyclical
imperfect competition implies higher reactivity of labor supply. For instance, a shock that increases consumption reduces markups increasing the real wages, which promotes labor supply more compared to perfect competition. In other words, the tendency toward limited labor reactivity of the neoclassical models is limited because changes in markup determine a new substitution effect. The opposite happens when the relative risk aversion is increasing.

Finally, since total profits are $\Pi_t = n(p_t - 1)C_t$ and total expenditure is $E_t = np_tC_t$, replacing in the resource constraints we obtain:

$$K_{t+1} = K_t(1 - \delta) + w_tL_t + r_tK_t - nC_t \tag{16}$$

which, after replacing the equilibrium input prices (4) and (5) gives the equation of motion in Proposition 1. The uniqueness of the steady state and saddlepath stability follow from the fact that the dynamic system under monopolistic competition with utility $u(C)$ is the same of the dynamic system under perfect competition with utility $u'(C)C$, which has a unique steady state and is stable as long as the latter is concave as assumed in (11).

The model of this section may appear as a natural extension of the RBC framework with a general utility function to multiple goods. However, the fact that the shape of the same subutility function $u(C)$ governs both intratemporal and intertemporal substitutability is not without consequences under imperfect competition. The requirement of a relative risk aversion below unity imposes substantial restrictions on intertemporal substitutability. We will return on this limitation in a subsequent section.

**An example without capital** To relate the model with a well known macroeconomic framework, let us consider the simple case without capital. With $\alpha = 0$, all output is consumed, and total consumption $C_t = Y_t = A_tL_t$ must solve:

$$C_t = \frac{A_t^{1+\varphi}}{n} \left[ \frac{u'(C_t)[1 - \epsilon(C_t)]}{\nu} \right]^{\varphi}$$

Loglinearizing around the steady state we get:5

$$\dot{Y}_t = \frac{1 + \varphi}{1 + \epsilon(C)\varphi + \frac{\kappa(C)\epsilon(C)}{1 - \epsilon(C)}} \dot{A}_t \tag{17}$$

where $\kappa(C) \equiv \epsilon'(C)C/\epsilon(C)$ is the elasticity of the relative risk aversion. The multiplier of the technology shock is always larger than one under decreasing relative risk aversion: indeed $\epsilon(C) < 1$ and $\kappa(C) < 0$ imply $\dot{Y}_t > \dot{A}_t$. The mechanism of propagation relies on the variability of the prices of the final goods. More formally, if we loglinearize the price equation around the steady state and use obvious notation for expectations, we obtain:

$$\ddot{p} + \dot{p} = \frac{\kappa(C)\epsilon(C)}{1 - \epsilon(C)} \left( \dot{Y}_{t+1} - \dot{Y}_t \right) \tag{18}$$

which relates changes in output growth to relative price changes. With a decreasing relative risk aversion, markup countercyclicality magnifies the propagation of the shocks and the above relation can be rewritten in a more familiar form as:

$$\hat{p}_t = \hat{p}_{t+1} + \xi z_t^{e_t}$$

where $z_t^{e_t} \equiv \hat{Y}_{t+1} - \hat{Y}_t$ and $\xi \equiv \frac{\epsilon(C)\epsilon(C)'}{1 - \epsilon(C)} \geq 0$ if $\epsilon'(C) \leq 0$. This emphasises how markup inflation depends on a measure of the output gap growth when preferences are not homothetic.

1.2 Examples

We can now consider some examples of intratemporal preferences.

**CRRA/CES preferences**  Consider the standard case of isoelastic subutility:

$$u(C) = \frac{C^{1-\rho}}{1-\rho}$$  \hspace{1cm} (19)

where $\epsilon(C) = \rho > 0$ is the constant relative risk aversion, which here represents also the reciprocal of the intertemporal elasticity of substitution. These preferences are also homothetic and are the only homothetic preferences within the class of directly additive preferences under our consideration. They require the restriction $\rho < 1$ to guarantee the existence of the monopolistic price:

$$p_t = \frac{1}{1 - \rho}$$

Since this is constant, the Euler equation is the same as under perfect competition:

$$C_t^{1-\rho} = \beta R_{t+1} E[C_{t+1}^{1-\rho}]$$

This is due to the fact that consumers see through the veil of market power and realize that higher prices induce higher profits without affecting their real resources over time. It is now clear that the requirement of a relative risk aversion $\rho < 1$ imposes a substantial restriction on the intertemporal elasticity of substitution, which here corresponds to $1/\rho > 1$, and excludes unitary or less than unitary elasticity. We will overcome this limiting restriction in a later extension.

**Quadratic preferences**  Consider quadratic preferences as in Hall (1978), with subutility:

$$u(C) = \alpha C - \frac{C^2}{2}$$  \hspace{1cm} (20)

with $\alpha > 2\beta$. This satisfies all our assumptions and provides a procyclical risk aversion $\epsilon(C) = \frac{C}{\alpha - C}$. Monopolistic prices are:

$$p_t = \frac{\alpha - C_t}{\alpha - 2C_t}$$  \hspace{1cm} (21)
in period $t$ and are therefore increasing in consumption. The modified Euler equation becomes:

$$\alpha - 2C_t = \beta R_{t+1}E[\alpha - 2C_{t+1}]$$ (22)

which implies that consumption is a martingale, as under perfect competition (Hall, 1978).\(^6\)

**CARA preferences** Consider the case of constant absolute risk aversion with subutility:

$$u(C) = 1 - e^{-\chi C}$$ (23)

where $\chi > 0$. This provides a procyclical relative risk aversion $\epsilon(C) = \chi C$. Monopolistic prices (Bertoletti, 2006) are:

$$p_t = \frac{1}{1 - \chi C_t}$$ (24)

which shows a positive correlation with consumption. The modified Euler equation reads as:

$$e^{-\chi C_t} (1 - \chi C_t) = \beta R_{t+1}E \{e^{-\chi C_{t+1}} (1 - \chi C_{t+1})\}$$ (25)

generating more consumption smoothing compared to perfect competition.

**Stone-Geary preferences** Consider the following translated power subutility (see Bertoletti and Etro, 2014, for a discussion in a static context):

$$u(C) = \frac{\theta}{\theta - 1} (C - \bar{C})^{\frac{\theta - 1}{\theta}}$$ (26)

where $\theta > 1$ and consumption must be above the minimum level $\bar{C} > 0$. This provides a countercyclical relative risk aversion $\epsilon(C) = \frac{C}{u(C-C)}$. Monopolistic prices are:

$$p_t = \frac{\theta(C_t - \bar{C})}{(\theta - 1)C_t - \bar{C}\theta}$$ (27)

under the regularity condition $\bar{C} < \frac{(\theta - 1)C_t}{\theta}$ for any $t$, and are therefore decreasing in consumption for $\bar{C} > 0$. The modified Euler condition under monopolistic competition becomes:

$$\frac{(\theta - 1)C_t - \bar{C}\theta}{(C_t - \bar{C})^{1+\theta}} = \beta R_{t+1}E \left\{ \frac{(\theta - 1)C_{t+1} - \bar{C}\theta}{(C_{t+1} - \bar{C})^{1+\theta}} \right\}$$ (28)

which nests the CES case for $\bar{C} = 0$. In general, monopolistic competition generates higher reactivity of consumption compared to perfect competition. However, notice that the model is also consistent with a negative $\bar{C}$, generating the opposite effect.

\(^6\)In case of a constant interest rate, we have:

$$C_{t+1} = \beta_0 + \beta_1 C_t + \varepsilon_t$$

with $\beta_0 = \frac{\gamma}{\gamma-1} (1 - \frac{1}{\gamma})$, $\beta_1 = \frac{1}{\gamma}$ and $\varepsilon_t$ white noise. The only difference compared to perfect competition is that the constant $\beta_0$ is half as under perfect competition.
IES preferences For a similar case, consider a linear combination of isoelastic functions (defined as Increasing Elasticity of Substitution preferences by Bertoletti, Fumagalli and Poletti, 2008), as in:

\[ u(C) = C + \frac{\theta}{\theta - 1} C^{\frac{\theta - 1}{\theta}} \]  

(29)

This provides again a countercyclical risk aversion \( \epsilon(C) = \frac{1}{\theta(1 + C^{1/\theta})} \). Monopolistic prices are:

\[ p_t = \frac{\theta(1 + C_t^{1/\theta})}{\theta(1 + C_t^{1/\theta}) - 1} \]  

(30)

and decrease in consumption. The modified Euler condition under monopolistic competition reads as:

\[ 1 + \frac{(\theta - 1)C_t^{-1/\theta}}{\theta} = \beta R_{t+1} \mathbb{E} \left\{ 1 + \frac{(\theta - 1)C_{t+1}^{-1/\theta}}{\theta} \right\} \]  

(31)

and this is another case where monopolistic competition amplifies the reaction of consumption.

1.3 Optimal taxation with monopolistic competition

Monopolistic competition introduces an intratemporal and an intertemporal distortion in the economy. To characterize the optimal taxation that fixes this distortion we need first to solve the social planner problem for this economy. Assuming symmetry for all goods, this can be stated as follows:

\[
\max_{c_t, l_t, k_t} \mathbb{E} \sum_{t=1}^{\infty} \beta^{t-1} \left[ nu(C_t) - \frac{vL_t^{1+\frac{1}{\alpha}}}{1 + \frac{1}{\varphi}} \right] \\
\text{s.t.} \quad K_{t+1} - K_t(1 - \delta) = A_t K_t^\alpha L_t^{1-\alpha} - nC_t
\]  

(32)

or, after replacing the constraint in the objective function:

\[
\max_{k_{t+1}, l_t} \mathbb{E} \sum_{t=1}^{\infty} \beta^{t-1} \left[ nu \left( A_t K_t^\alpha L_t^{1-\alpha} - K_{t+1} + K_t(1 - \delta) \right) - \frac{vL_t^{1+\frac{1}{\alpha}}}{1 + \frac{1}{\varphi}} \right]
\]

This provides the following FOCs for \( K_{t+1} \) and \( L_t \):

\[ u'(C_t^*) = \beta \mathbb{E} \left\{ [1 + \alpha A_{t+1} K_t^{\alpha-1} L_{t+1}^{1-\alpha} - \delta] u'(C_{t+1}) \right\} \]  

(33)

\[ L_t^* = \left[ \frac{1 - \alpha}{v} A_t K_t^\alpha u'(C_t^*) \right]^{\frac{1}{1+\alpha}} \]  

(34)

These correspond to the equilibrium conditions when there is perfect competition in the market for the final goods. As well known (see also Bilbiie, Ghironi
and Melitz, 2008,a,b), the intuition for the difference between equilibrium and optimality goes back to the insights of Lerner (1934). Monopolistic competition introduces a wedge between intermediate and final goods prices. The size of the wedge determines the extent of the suboptimality of labor supply (since leisure is the only good provided without a monopolistic wedge), but it is the variability over time of the wedge that determines the inefficiency in the savings decision and, consequently, in capital accumulation.

The decentralized equilibrium can be easily augmented with taxation. In case of lump sum taxation available, Ricardian Equivalence holds because the capital market is perfectly competitive. Therefore, we can assume budget balance in each period without loss of generality and find the tax rates on capital income and labor income that restore the first best allocation of resources obtained above. Let us introduce a subsidy on labor income $L_t$ and a tax rate on gross capital income $K_t$ at time $t$. The equilibrium conditions change as follows:

$$u'(C_t) = \beta \mathbb{E} \left\{ (1 - \tau^K_{t+1} R_{t+1} u'(C_{t+1}) \frac{1 - \epsilon(C_{t+1})}{1 - \epsilon(C_t)} \right\}$$

$$L_t = \left[ \frac{1 - \alpha}{\nu} (1 + \tau^L_t) A_t K^\alpha_t u'(C_t) [1 - \epsilon(C_t)] \right]^{\frac{1}{1 + \gamma}}$$

where $R_{t+1} = 1 + \alpha A_{t+1} K_t^{\alpha - 1} L_t^{1 - \alpha} - \delta$. We only need to find the tax rules $\{\tau^L_t, \tau^K_t\}$ which equalize these conditions to the first best conditions. Immediate computation requires the optimal taxation summarized as follows:

**Proposition 2.** In a dynamic model with directly additive intratemporal preferences and monopolistic competition over an exogenous number of goods 1) the optimal labor income subsidy is:

$$\tau^L_t = \frac{\epsilon(C_t)}{1 - \epsilon(C_t)} > 0$$

and it is decreasing (increasing) with consumption if the relative risk aversion is decreasing (increasing); 2) the optimal capital income tax rate is:

$$\tau^K_t = \frac{\epsilon(C_{t-1}) - \epsilon(C_t)}{1 - \epsilon(C_t)} < 1$$

and it is positive (negative) if the relative risk aversion is decreasing (increasing) and consumption is growing, but it is zero in steady state.

This shows that in the traditional case of CES preferences a constant and negative labor income tax in each period is sufficient to establish optimality even in the presence of monopolistic distortions in the goods market. In other words, tax smoothing à la Barro (1979) occurs if and only if the relative risk aversion is constant. However, this is not the case in general: the result on a countercyclical labor subsidy reminds of the traditional fiscal policy requirement of countercyclical tax rates, but here it emerges due to monopolistic distortions, and not due to tax distortions. Most important, countercyclical tax rates on
labor income emerge when the markups are countercyclical, because a boom reduces prices increasing the real wages. Instead, a procyclical labor taxation becomes optimal when preferences exhibit a procyclical relative risk aversion.

The fact that the optimal capital income tax rate is zero in the steady state is reminiscent of the famous Chamley (1986) result on zero capital income taxation in the long run, but again it emerges here due to monopolistic distortions, and not due to tax distortions. Actually, in the short run when consumption is growing, the optimal capital income tax rate is positive (negative) if the relative risk aversion is decreasing (increasing). This result implies that along the growth process, as long as markups are decreasing over time, capital income should be taxed to promote consumption and slow down capital accumulation. Finally, zero taxes on capital income are always optimal with CES preferences and a negative capital income tax becomes optimal when preferences exhibit a procyclical relative risk aversion.

1.4 Bertrand and Cournot competition

An important step to study imperfect competition is the introduction of Cournot and Bertrand competition between firms, which requires us to consider the exact inverse and direct demand system for each good. This is possible using (10) to obtain the inverse demand system:

\[ p_{it} = \frac{u'(C_{it})E_t}{\sum_{j=1}^{n} C_{jt}u'(C_{jt})}, \quad i = 1, ..., n \]

that can be inverted to obtain also the direct demand system (see Bertoletti and Etro, 2016, for the derivation of Bertrand and Cournot equilibrium prices from this demand system).

Under competition in quantities, the profit of each firm \( i \) in each period \( t \) reads as:

\[ \pi_{it} = \frac{u'(C_{it})C_{it}E_t}{\sum_{j=1}^{n} C_{jt}u'(C_{jt})} - C_{it} \]

which is maximized with respect to the quantity \( C_{it} \) taking as given total expenditure \( E_t \). This delivers the symmetric Cournot equilibrium price can be derived easily as:

\[ p_t^C = \frac{n}{(n-1)[1 - \epsilon(C_t)]} \]

(37)

This is proportionally higher than the one under monopolistic competition and converges to the latter only for an infinity of firms.\(^7\) For this reason, the modified Euler equation remains the same as under monopolistic competition:

\[ u'(C_t) [1 - \epsilon(C_t)] = \beta \mathbb{E} \{ R_{t+1}u'(C_{t+1}) [1 - \epsilon(C_{t+1})] \} \]

\(^7\) Indeed, even when the demand becomes perfectly elastic (\( \epsilon(C_t) \to 0 \)), the markup remains positive.
Instead, the labor supply is reduced to:

\[ L_t = \left(1 - \frac{1}{n}\right) \left(1 - \alpha \right) A_t K_t^\alpha u'(C_t) \left[1 - \epsilon(C_t) \right] \left(1 + \epsilon(C_t) \right)^\gamma \]

which is proportionally smaller than the one under monopolistic competition. This implies that the dynamic properties of the equilibrium with Cournot competition are identical to those of the model with monopolistic competition when the number of goods is fixed.

The optimal taxation restoring the first best allocation is however dependent on the number of goods, and the next result, which can be easily verified matching optimality and equilibrium conditions, derives it precisely:

**Proposition 3.** In a dynamic model with directly additive intratemporal preferences and Cournot competition over an exogenous number of goods the optimal labor and capital income taxes are:

\[ \tau_t^{LC} = \frac{1}{n - 1} + \epsilon(C_t) \quad \text{and} \quad \tau_t^{KC} = \frac{\epsilon(C_t - 1) - \epsilon(C_t)}{1 - \epsilon(C_t)} \]

This taxation requires a larger subsidy compared to monopolistic competition to counteract the increase in markup due to Cournot competition. However the capital income tax is the same as in the baseline model because the intertemporal distortion is unchanged. Notice that when goods become homogenous \( \epsilon(C) \to 0 \), as considered in Colciago and Etro (2010) extending the baseline RBC model to Cournot competition, the optimal capital income tax is zero, but the labor subsidy is positive and constant, \( \tau^{LC} = 1/(n-1) \): of course, when the number of goods increases indefinitely also this subsidy tends to zero because the economy approximates perfect competition.

Competition in prices creates some complications. As shown in Bertoletti and Etro (2016), Bertrand competition delivers the following price:

\[ p_t^B = \frac{\epsilon(C_t) + n - 1}{(n - 1)[1 - \epsilon(C_t)]} \]

The markup is intermediate between those of monopolistic and Cournot competition. Replacing in the FOCs for consumer behavior, we obtain the modified Euler equation:

\[ u'(C_t) \left[ 1 - \epsilon(C_t) \right] = \beta R_{t+1} \mathbb{E} \left[ u'(C_{t+1}) \left[ \frac{1 - \epsilon(C_{t+1})}{\epsilon(C_{t+1}) + n - 1} \right] \right] \]

and the labor supply:

\[ L_t = \left[ \left(1 - \alpha \right) A_t K_t^\alpha u'(C_t) \left[1 - \epsilon(C_t) \right] \left(1 + \epsilon(C_t) \right)^\gamma \frac{1}{v} \right] \]

which provide additional mechanisms of intertemporal substitution when the relative risk aversion is decreasing. Indeed, a boom generates a stronger relative
change in markups compared to the other forms of competition, which promotes labor supply and consumption more.

In this case, the optimal taxation restoring the first best allocation is amended as follows:

**Proposition 4.** In a dynamic model with directly additive intratemporal preferences and Bertrand competition over an exogenous number of goods the optimal labor and capital income taxes are:

\[
\tau_{t}^{LB} = \frac{\epsilon(C_t)}{(1 - \frac{1}{n})(1 - \epsilon(C_t))}
\quad \text{and} \quad
\tau_{t}^{KB} = \frac{\epsilon(C_{t-1}) - \epsilon(C_t)}{[1 - \epsilon(C_t)] \left(1 - \frac{1 - \epsilon(C_{t-1})}{n}\right)}
\]

Both these fiscal wedges are higher than under monopolistic competition, because competition in prices increases markups and both the intratemporal and intertemporal distortions. In this case, homogenous goods \((\epsilon(C) \rightarrow 0)\) generate optimality without taxes if there are at least two firms.

It is clear that, as long as the number of goods is exogenous, strategic interactions have a minor impact on macroeconomic dynamics: this is the case when the number of firms changes endogenously.

### 1.5 Endogenous business creation

Our next step is to endogenize the number of varieties provided in the market under monopolistic competition when there is a fixed cost of creating new varieties in the market à la Romer (1990). Notice that this precludes perfect competition in the market for final goods, and leads to imperfect competition with free entry as the natural and endogenous structure of this market. Nevertheless, even the outcome of such a structure is not efficient in general and will be later compared to the optimal allocation of resources.

Retaining the directly additive preferences, we now assume that \(n_t\) is the endogenous number of varieties consumed in period \(t\), each one produced by a different firm. The number of firms/goods follows the simplest law of motion:

\[
n_{t+1} = n_t + n_t^e
\]  

(39)

where \(n_t^e \in \mathbb{Z}^+\) is the endogenous number of entrants in period \(t\) (which was assumed zero in the previous section).

The consumer chooses how much to spend in final goods, how much to invest in stocks of existing and new firms (as in Ghironi and Melitz, 2005) and how much to invest in physical capital, as already examined in entry models by Etro and Colciago (2010), Bilbiie, Ghironi and Melitz (2012) and others. All this must match the sum of labor and capital income, the value of the riskless asset, profits of the firms, and the value of the current firms (holdings of stocks of the firms through a mutual fund). The budget constraint of the agent (expressed in terms of the intermediate good) is:

\[
K_{t+1} + x_{t+1} \sum_{j=1}^{n_{t+1}} V_{jt} = K_t(1 - \delta) + w_tL_t + r_tK_t + \sum_{j=1}^{n_t} \left[ x_t(\pi_{jt} + V_{jt}) - p_{jt}C_{jt} \right]
\]
where $x_t$ is the share of mutual fund investing in risky stocks of the firms, $\pi_{jt}$ are the dividends and $V_{jt}$ the value of firm $j$ at time $t$ and the other variables are the same as before.

Augmenting the Lagrangian with this constraint leads to the same FOCs as before for $C_{jt}$, $L_t$ and $K_{t+1}$. The equilibrium of monopolistic competition follows the same price rule as before, $p_t = 1/(1-\epsilon(C_t))$, generating the following profits:

$$\pi_t(C_t) = \frac{\epsilon(C_t)C_t}{1-\epsilon(C_t)}$$

which are always redistributed as dividends at time $t$. This allows us to impose symmetry on the stock market value $V_t$ for any firm in what follows. Our assumption (11) implies that these profits are increasing in individual consumption, $\pi'_t(C_t) > 0$.

Entry requires a sunk cost of entry $F_t$ in units of intermediate good at time $t$. This could follow an exogenous process, possibly subject to shocks. The simplest case is the deterministic case where the entry cost is a constant $F_t$. The most natural case is the one in which an increase in total factor productivity makes the business creation sector more productive, as with $F_t = f/A_t$ for a constant parameter $f > 0$, so that a productivity shock affects the economy through a double channel, leading to higher productivity for the production of both intermediate goods and firms. However, we will leave unspecified the dynamics of the fixed cost in what follows.

Free entry requires that in every moment the number of entrants is such that the value of firms equates the fixed entry cost, $V_t = F_t$, or zero if the fixed costs are higher than the value of firms. To investigate how many firms enter in the market we first need to derive the value of the firms, which is the present discounted value of their expected profits. Notice that the new FOC for the consumer is the one for $x_{t+1}$, which is:

$$\lambda_t V_t(n_t + n^e_t) = \beta E \{ \lambda_{t+1} (\pi_{t+1} + V_{t+1}) n_{t+1} \}$$

Using $n_{t+1} = n_t + n^e_t$, the modified Euler equation and the equilibrium profits, this becomes a recursive asset pricing formula:

$$V_t = \beta E \left\{ \frac{\lambda_{t+1}}{\lambda_t} [\pi_{t+1}(C_{t+1}) + V_{t+1}] \right\} = \beta E \left\{ \frac{u'(C_{t+1})}{u'(C_t)} \left[ \frac{1}{1-\epsilon(C_{t+1})} - \frac{1}{1-\epsilon(C_t)} \right] \right\}$$

which expresses the value of a firm as the present discounted value of its future profit flows. If this is below the fixed cost there is no entry in the economy. In

\[\text{Indeed, } \pi(C) = \frac{u'(C)C}{u'(C)+u''(C)C} \text{ implies:}\]

$$\pi'(C) \propto [u'(C) + u''(C)C]^2 - u'(C)C \left[ 2u''(C) + u'''(C)C \right]$$

which must be positive under (11). I am thankful to Paolo Bertoletti for noticing this. See also Bertoletti and Epifani (2014).
such a case the number of firms is fixed and the evolution of the economy is the same as in the previous section: this “Ramsey regime” occurs when there is not much capital, the real interest rate is high and the present discounted value of future profits from a new good is therefore low compared to the fixed cost of creating a new firm. However, while capital accumulates, the real interest rate declines and individual consumption increases, which increases the present discounted value of profits until business creation becomes profitable. In such a “Romer regime” the endogenous entry condition:

\[ V_t = F_t \]  

is binding and provides implicitly the number of entrants in each period.\(^9\) In this entry regime the Euler and free entry conditions imply a no-arbitrage condition for investment in capital and firms. The number of new firms derives from the resource constraint. In equilibrium with \( x_t = 1 \) and \( V_t = F_t \) this becomes:

\[ K_{t+1} + (n_t + n_t^e)F_t = K_t(1 - \delta) + w_tL_t + r_tK_t + n_t(\pi_t + F_t) - n_t\rho_tC_t \]

which states the equality of total investment (in capital and new firms) to total savings:

\[ K_{t+1} - K_t(1 - \delta) + n_t^eF_t = A_tK_t^{\alpha}L_t^{1-\alpha} - n_tC_t \]

This can be solved for the number of new firms:\(^10\)

\[ n_t^e = \frac{A_tK_t^{\alpha}L_t^{1-\alpha} - n_tC_t - K_{t+1} + K_t(1 - \delta)}{F_t} \]

and replaced in the equation of motion for the number of firms. Therefore, the equilibrium in the entry regime is summarized as follows:

**Proposition 5.** In a dynamic model with directly additive intratemporal preferences satisfying (11) with monopolistic competition over an endogenous number of goods, the equilibrium with entry satisfies:

\[ u'(C_t) [1 - \epsilon(C_t)] = \beta \mathbb{E} \left\{ \left[ 1 + \alpha A_{t+1} K_t^{\alpha-1} L_{t+1}^{1-\alpha} - \delta \right] u'(C_{t+1}) [1 - \epsilon(C_{t+1})] \right\} \]  

\[ n_{t+1} = n_t + \frac{A_tK_t^{\alpha}L_t^{1-\alpha} - n_tC_t - K_{t+1} + K_t(1 - \delta)}{F_t} \]

\(^9\)The two regimes are vaguely reminiscent of Matsuyama (1999), where however market power is temporary, leading to cycles of capital accumulation and innovation.

\(^10\)We are neglecting the non-negativity constraint on \( n_t^e \), which is not binding in the transition path of a deterministic environment. Adding an exogenous exit probability, as we will do later on, the non-negativity constraint is not binding for any shock small enough, as usually assumed in the literature. Therefore we will neglect this constraint in the subsequent discussion.
\[ L_t = \left[ \frac{(1-\alpha)A_t K_t^\alpha u'(C_t) [1 - \epsilon(C_t)]}{\nu} \right]^{\frac{1}{1+\beta}} \]  
and has a unique deterministic steady state.

The system provides the dynamics for individual consumption \( C_t \), capital \( K_t \), labor supply \( L_t \) and number of firms \( n_t \) given the initial values \( K_0 \) and \( n_0 \). Condition (46) derives from replacing (40) and (43) in (42). Condition (47) derives from replacing (44) in (39). The remaining conditions are the same as before.\(^{11}\)

In a deterministic environment with constant productivity \( A \) and fixed cost \( F \), the steady state consumption level of each good \( \bar{C} \) must satisfy:

\[ \frac{\beta \pi(\bar{C})}{1 - \beta} = F \]  
whose left hand side is the expected discounted profit of a firm in steady state. Under our assumptions the profit is monotonic increasing in consumption and therefore, the consumption in the deterministic steady state is unique for any fixed cost. Notice that consumption depends on the entry cost and on preference factors, but not on the production side: this neutrality is inherited from the static Dixit-Stiglitz model where individual consumption is neutral in the income endowment (see Bertoletti and Etro, 2014). Of course the steady state stock of capital does depend on technological factors, namely on productivity:

\[ \bar{K} = \left( \frac{\alpha}{\beta - 1 + \delta} \right)^{\frac{1+\alpha}{1+\beta}} A^{\frac{1+\alpha}{1+\beta}} \left[ \frac{(1-\alpha)u'(\bar{C}) [1 - \epsilon(\bar{C})]}{\nu} \right]^{\frac{1}{1+\beta}} \]  
Therefore also the steady state number of firms \( \bar{n} \) and total consumption must depend on both technological and preference factors.

The equilibrium and the steady state are characterize by multiple inefficiencies, due to market power in final goods, undersupply of labor and also inefficient incentives to create new goods. As we will see below, appropriate tax tools can restore efficiency in all these dimensions. Before moving to that, even if it is beyond the scope of this paper to quantify the reaction of the economy to shocks, we should notice that in this model a temporary change in entry costs or productivity does affect the incentives to invest in business creation. In this environment, it is important to remark that, even if total consumption \( \bar{C}_t = n_tC_t \) grows over time, the individual consumption of each good may remain constant or change over time depending on whether the fixed entry cost is constant or variable. A simple case can be easily studied analytically.

\(^{11}\)Notice that, as long as entry occurs, the following no-arbitrage condition is satisfied:

\[ V_t = \mathbb{E} \left\{ \frac{\pi_{t+1}(C_{t+1}) + V_{t+1}}{R_{t+1}} \right\} \iff \mathbb{E} [r_{t+1}] - \delta = \mathbb{E} \left[ \frac{\pi_{t+1}(C_{t+1})}{F_t} + \frac{V_{t+1} - V_t}{V_t} \right] \]
An example without capital Consider the simple case without capital ($\alpha = 0$). For simplicity let us assume also a rigid labor supply ($\varphi = 0$). Then, all the output is used for consumption or to create new firms. The dynamics of consumption is summarized by the equation (46). It is easy to verify that this is unstable around the steady state ($\frac{dC_{t+1}}{dC_t}|_{C_t=C^*} > 1$), therefore the only equilibrium must have that the consumption of each good is constant at the steady state level $C$. If entry occurs. In this simple case, the equation of motion for the number of firms is unidimensional and extremely simple:

$$n_{t+1} = n_t + \frac{A - n_t \hat{C}}{F}$$

which is monotonically converging to the steady state $n = A/\hat{C}$ under the assumption that $\hat{C} < F$.

For instance, with a CES subutility (19) we can solve explicitly for the steady state values as:

$$\hat{n} = \frac{A \beta \rho}{F (1 - \rho) (1 - \beta)}$$

$$\hat{C} = \frac{F (1 - \beta) (1 - \rho)}{\beta \rho}$$

and monotonic convergence is always satisfied.

In conclusion, it can be useful to notice that when the productivity is increasing with the number of goods, this simple model can deliver an endogenous growth process à la Romer (1990).

1.5.1 Optimal taxation with monopolistic competition and entry

To identify the optimal tax system under endogenous entry, let us solve the deterministic social planner problem after imposing symmetry:

$$\max_{C_t, L_t, K_t, n_t, n_{t+1}} \sum_{t=1}^{\infty} \beta^{t-1} \left[ n_t u(C_t) - \frac{uL_t^{1+\frac{1}{\varphi}}}{1 + \frac{1}{\varphi}} \right]$$

s.v. : $K_{t+1} - K_t (1 - \delta) + n_t F_t = A_t K_t^{-\alpha} L_t^{1-\alpha} - n_t C_t$

$$n_{t+1} = n_t + n_t^*$$

Replacing the constraints in the objective function we have:

$$\max_{K_{t+1}, L_t, n_{t+1}} \mathbb{E} \sum_{t=1}^{\infty} \beta^{t-1} \left[ n_t u \left( A_t K_t^{-\alpha} L_t^{1-\alpha} - K_{t+1} + K_t (1 - \delta) - n_{t+1} F_t + F_t \right) - \frac{uL_t^{1+\frac{1}{\varphi}}}{1 + \frac{1}{\varphi}} \right]$$

The FOCs for $K_{t+1}, L_t$ and $n_{t+1}$ can be rearranged as follows:

$$u'(C_t^*) = \beta \left[ 1 + \alpha A_{t+1} K_{t+1}^{-\alpha - 1} L_{t+1}^{1-\alpha} - \delta \right] u'(C_{t+1})$$

\[12\] For instance, if $A_t = \hat{A} n_t$, it is immediate to obtain from (51) that output and number of goods grow at the constant rate $g = \frac{\hat{A}}{\varphi} \hat{C}$ if $\hat{A} > \hat{C}$. As well known, a technology using intermediate inputs sold by monopolist can generate such a framework.
The first two are standard and they can be reproduced in the decentralized equilibrium with the same subsidies on labor $L_t$ and total capital income $K_t$ as in the baseline model with a fixed number of firms. The third FOC equalizes the marginal cost of investing resources to create new goods (giving up to more consumption of existing goods) to its expected marginal benefit, which includes two aspects: a higher future utility from the consumption of the new goods and a reduction in the marginal utility from each good consumed in lower quantity. The last condition is the usual resource constraint needed to determine the number of firms.

The entire social planner allocation can be reproduced in the decentralized equilibrium adding a tax rate on profits or dividends $D_t$. The equilibrium free entry condition would be amended as follows in the presence of optimal tax rates:

$$u'(C_t)F_t = \beta \left[ u(C_{t+1}^*) - u'(C_{t+1}^*) (C_{t+1}^* - F_{t+1}) \right]$$

Therefore equating this to the optimality condition above we can solve for the optimal profit tax rate:

$$\tau_t^D = 1 - \frac{1 - \epsilon(C_t)}{\epsilon(C_t)} \left[ 1 - \psi(C_t) \right]$$

where $\psi(C) = u'(C)C/u(C) > 0$ is the elasticity of the sub-utility function. Notice that in case of CES preferences this tax is zero because $\psi(C) = 1 - \epsilon(C) = 1 - \rho$. This is just a dynamic extension of the principle established by Dixit and Stiglitz (1977) in a static context for which the number of firms is optimal under CES preferences. In such a case $\tau_t^D = \tau_t^K = 0$ and optimality can be reached with the labor subsidy only.

In the general case, using the fact that:

$$\psi'(C) = \psi(C) \frac{1 - \epsilon(C) - \psi(C)}{C}$$

we can also rearrange the optimal tax on profits and obtain:

**Proposition 6.** In a dynamic model with directly additive intratemporal preferences and monopolistic competition over an endogenous number of goods the optimal taxation requires:

$$\tau_t^L = \frac{\epsilon(C_t)}{1 - \epsilon(C_t)}, \quad \tau_t^K = \frac{\epsilon(C_{t-1}) - \epsilon(C_t)}{1 - \epsilon(C_t)} \text{ and } \tau_t^D = \frac{-C_t\psi'(C_t)}{\epsilon(C_t)\psi(C_t)^2}$$

and the optimal profit tax rate is positive (negative) if the elasticity of the utility function is decreasing (increasing).

One may notice that the model with endogenous entry allows for different reactions of the individual consumption of each good in front of shocks. For
instance, an expansionary productivity shock may affect the number of consumed goods while leaving unchanged the individual consumption, as when the fixed cost is constant over time (as in our example above): in such a case, the optimal capital income tax is zero and the optimal labor subsidy is constant, but the optimal profit tax is still depending on the shape of the utility function. With variable fixed costs and variable individual consumption, also the optimal labor and capital income taxes should change over the business cycle. It can be useful to mention that, when individual consumption grows, the examples with quadratic and CARA subutilities generate an increasing labor income taxation and a negative capital income and profit taxation, while the examples with Stone-geary and IES subutilities generate a decreasing labor income taxation together with positive optimal taxes on capital income and profits.

1.5.2 Strategic interactions with endogenous entry

Introducing strategic interactions in the endogenous entry model enriches its implications for markup variability. Bertrand competition delivers the equilibrium price:

\[ p_B^t(C_t, n_t) = \frac{\epsilon(C_t) + n_t - 1}{(n_t - 1)[1 - \epsilon(C_t)]} \]  

(55)

where entry induces a new reason for countercyclical markups: it strengthens competition and reduces equilibrium prices, which promotes intertemporal substitution. The equilibrium profits of each firm now read as:

\[ \pi_B^t(C_t, n_t) = \frac{n_t\epsilon(C_t)C_t}{(n_t - 1)[1 - \epsilon(C_t)]} \]

Strategic interactions have an important implication for the business cycle dynamics, already emphasized in Jaimovich and Floetotto (2008) and Etro and Colciago (2010) in frameworks based on CES preferences only. An expansionary shock can generate countercyclical markups not only when the relative risk aversion is countercyclical, but also when it is constant and even when it is procyclical: indeed, an increase in \( n_t \) tends to depress prices, and this competition effect can more than compensate an opposite effect of an increase in \( \epsilon(C_t) \).

The Euler and labor supply conditions are amended in obvious ways, and the free entry condition becomes:

\[
F_t = \beta E \left\{ A_{t+1,t}^B \left( \frac{n_{t+1}\epsilon(C_{t+1})C_{t+1}}{(n_{t+1} - 1)[1 - \epsilon(C_{t+1})]} + F_{t+1} \right) \right\} 
\]

with \( A_{t+1,t}^B = \frac{(\epsilon(C_t) + n_t - 1)u'(C_{t+1})(n_{t+1} - 1)[1 - \epsilon(C_{t+1})]}{(\epsilon(C_{t+1}) + n_{t+1} - 1)u'(C_t)(n_t - 1)[1 - \epsilon(C_t)]} \)

We omit straightforward computations to introduce taxation in the decentralized equilibrium. The optimal taxation restoring the first best allocation derived above can be obtained as follows:
Proposition 7. In a dynamic model with directly additive intratemporal preferences and Bertrand competition over an endogenous number of goods the optimal taxation requires:

\[ \tau_{t}^{LB} = \frac{\epsilon(C_t)}{(1 - \frac{1}{n_t}) [1 - \epsilon(C_t)]} \]
\[ \tau_{t}^{KB} = \frac{\epsilon(C_{t-1}) - \epsilon(C_t)}{[1 - \epsilon(C_t)] \left(1 - \frac{1 - \epsilon(C_{t-1})}{n_t} \right)} \]
\[ \tau_{t}^{DB} = 1 - \frac{(n_t - 1) [1 - \epsilon(C_t)] [1 - \psi(C_t)]}{n_t \epsilon(C_t) \psi(C_t)} \]

Notice that with CES preferences, using \( \epsilon(C) = 1 - \psi(C) = \rho \) we obtain a very simple optimal tax system, with zero capital income tax, a labor subsidy \( \tau_{t}^{LB} = \frac{n_t}{(n_t - 1)[1 - \epsilon(C_t)]} \), and a profit tax which is independent from substitutability and inversely proportional to the number of active firms, according to the simple tax rule \( \tau_{t}^{DB} = \frac{1}{n_t} \). The latter is consistent with a long run optimal tax obtained by Colciago (2016) under an equivalent assumption on the entry costs.\(^{13}\)

Let us move to Cournot competition. This delivers in each period \( t \) the equilibrium price:

\[ p_{t}^{C}(C_t, n_t) = \frac{n_t}{(n_t - 1)[1 - \epsilon(C_t)]} \tag{56} \]

where \( \epsilon(C_t) \) is the same elasticity defined above, and profits read as:

\[ \pi_{t}^{C}(C_t, n_t) = \frac{[1 + (n_t - 1)\epsilon(C_t)] C_t}{(n_t - 1)[1 - \epsilon(C_t)]} \]

Using this, the modified asset pricing equation in free entry equilibrium becomes:

\[ F_{t} = \beta E_{t+1} \left\{ \Lambda_{t+1,t}^{C} \left( \frac{[1 + (n_{t+1} - 1)\epsilon(C_{t+1})] C_{t+1}}{(n_{t+1} - 1)[1 - \epsilon(C_{t+1})]} + F_{t+1} \right) \right\} \]

with \( \Lambda_{t+1,t}^{C} = \frac{u'(C_{t+1}) (1 - \frac{1}{n_{t+1}})[1 - \epsilon(C_{t+1})]}{u'(C_t) \left(1 - \frac{1}{n_t} \right)[1 - \epsilon(C_t)]} \)

Again, strategic interactions contribute to generate countercyclical markups, which magnifies the propagation of shocks amplifying the effects on both labor supply and consumption. This happens even with homogenous goods (for \( \epsilon(C_t) \rightarrow 0 \)). We complete our analysis deriving the optimal taxation for this case:

Proposition 8. In a dynamic model with directly additive intratemporal preferences and Cournot competition over an endogenous number of goods the

\(^{13}\)In reality, Colciago (2016) assumes a fixed cost in units of labor, and has no capital. This is equivalent to assume a fixed cost in units of an intermediate good produced with labor only.
optimal taxation is:

\[ \tau^{LC}_t = \frac{1}{n_t - 1} + \epsilon(C_t) \]

\[ \tau^{KC}_t = \frac{\epsilon(C_{t-1}) - \epsilon(C_t)}{1 - \epsilon(C_t)} \]

\[ \tau^{DC}_t = 1 - \frac{(n_t - 1) [1 - \epsilon(C_t)] [1 - \psi(C_t)]}{[1 + (n_t - 1) \epsilon(C_t)] \psi(C_t)} \]

In the case of CES preferences, we obtain again a zero capital income tax, and countercyclical labor subsidy

\[ \tau^{LC}_t = 1 + \frac{(n_t - 1)}{(1 - \rho)(n_t - 1)} \]

which is again consistent with Colciago (2016). Notice that the optimal profit tax is always positive because imperfect competition attracts more entry than monopolistic competition, which was generating the efficient number of firms under CES preferences.

2 An RBC model with general preferences

After clarifying the role of market power in the RBC model with additive preferences, we can move to the case of general intratemporal preferences. The supply side is the same as in the baseline model. On the demand side, following the static analysis of Bertoletti and Etro (2016), let us consider the following generalized preferences:

\[ \bar{U} = E \sum_{t=1}^{\infty} \beta^{t-1} \left[ U(C_t, n) - \frac{\nu L_t^{1+\frac{1}{\eta}}}{1 + \frac{1}{\eta}} \right] \]  

where the period utility from consumption \( U(C_t, n) \) is now any symmetric function of the \( n \)-dimensional consumption vector \( C_t \equiv [C_{1t}, C_{2t}, ..., C_{nt}] \), assumed increasing and concave in the consumption of each good, but not necessarily additive. This allows us to separate intratemporal and intertemporal substitutability, which is crucial to investigate imperfect competition without imposing undue restrictions on the consumption dynamics. Nevertheless, notice that we retain intertemporal additivity, which is crucial for two-stage budgeting and time consistency of the consumption decisions.

For a given expenditure \( E_t = \sum_{j=1}^{n} p_j C_{jt} \) at time \( t \), the Marshallian demand vector in \( t \) is a function of the price vector \( p_t \equiv [p_{1t}, p_{2t}, ..., p_{nt}] \) and expenditure \( E_t \), and it is homogenous of degree zero, as in \( C_t \left( \frac{p_t}{E_t} \right) \). We can therefore define intratemporal preferences also through the intratemporal indirect utility function:

\[ V \left( \frac{p_t}{E_t}, n \right) = U \left( C_t \left( \frac{p_t}{E_t} \right), n \right) \]

where expenditure must be allocated across periods and the demand for each good can be derived through the Roy’s identity. In this perspective, the prefer-
ences in (57) can be expressed equivalently as:

\[ \tilde{U} = \mathbb{E} \sum_{t=1}^{\infty} \beta^{t-1} \left[ V \left( \frac{p_t}{E_t}, n \right) - \frac{uL_{it}^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] \]  

(58)

As shown by Bertoletti and Etro (2016) in a static analysis of monopolistic competition, the relevant elasticity under symmetry corresponds now to the elasticity:

\[ \epsilon(C, n) = -\frac{\partial(p_i/p_j)}{\partial C_i} \frac{C_i}{(p_i/p_j)} \]  

(59)

with \( p_i = p_j \). Direct computation allows one to obtain:

\[ \epsilon(C, n) = \frac{CU_{ij}(C, n)}{U_j(C, n)} - \frac{CU_{ii}(C, n)}{U_i(C, n)} \]

which is known as the Morishima elasticity of complementarity (see Blackorby and Russell, 1981) after defining utility and its first and second derivatives under symmetry as:

\[ U(C_t, n) \equiv U(C_t u, n), \quad U_i(C_t, n) \equiv \frac{\partial U(C_t u, n)}{\partial C_{it}} > 0 \quad \text{and} \quad U_{ij}(C_t, n) \equiv \frac{\partial^2 U(C_t u, n)}{\partial C_{it} \partial C_{jt}} \]  

where \( u \) is the \( n \)-dimensional unit vector.\(^{14}\)

If preferences are expressed in terms of the indirect intratemporal utility, Bertoletti and Etro (2016) show that the relevant elasticity can be also expressed as:

\[ \epsilon(C, n) = \frac{1}{\theta(s, n)} \]

where \( \theta(s, n) \) is the Morishima elasticity of substitution:

\[ \theta(s, n) \equiv \frac{sV_{ji}(s, n)}{V_j(s, n)} - \frac{sV_{ii}(s, n)}{V_i(s, n)} \]

under symmetry, and as before we defined \( V(s_t, n) \equiv V(s_t u, n), \quad V_i(s_t, n) \equiv \frac{\partial V(s_t u, n)}{\partial s_{it}} > 0 \quad \text{and} \quad V_{ij}(s_t, n) \equiv \frac{\partial^2 V(s_t u, n)}{\partial s_{it} \partial s_{jt}} \). Given any well-behaved symmetric preferences, we can directly compute these elasticities, and the equilibrium price under monopolistic competition becomes:

\[ p_t = \frac{1}{1-\epsilon(C_t, n)} \]

Notice that, as long as the number of goods is fixed, there are not important consequences on pricing. Nevertheless, the more general nature of preferences does affect the equilibrium dynamics. In particular, the modified Euler equation becomes:\(^{15}\)

\[ U_i(C_t, n) [1 - \epsilon(C_t, n)] = \beta R_{t+1} \mathbb{E} \{ U_i(C_{t+1}, n) [1 - \epsilon(C_{t+1}, n)] \} \]  

(60)

\(^{14}\)It is natural to notice that with directly additive preferences the cross effect \( U_{ij}(C, n) \) is null and the Morishima elasticity reduces to the relative risk aversion.

\(^{15}\)Here we assume that the second order conditions and the conditions for saddle-path stability are satisfied, which impose some restrictions on the shape of the utility function. In the examples below we can verify the conditions.
while labor supply is amended as:

\[ L_t = \left[ \frac{1 - \alpha}{\nu} A_t K_t^\alpha U_i(C_t, n) [1 - \epsilon(C_t, n)] \right]^{\frac{\alpha}{1 - \alpha}} \]  

(61)

and the model is always closed by the resource constraint:

\[ K_{t+1} = K_t(1 - \delta) + A_t K_t^\alpha L_t^{1-\alpha} - nC_t \]  

(62)

The business cycle properties of the model are qualitatively similar to the baseline case, but the general formulation is much more flexible in modeling intertemporal substitution, which governs savings, separately from intratemporal substitution, which governs markups and the impact of monopolistic competition.\(^{16}\)

The analysis could be extended also to public spending in the final goods to study the impact of an increase in public spending: while this reduces private consumption under perfect competition for a basic income effect, under imperfect competition it induces a substitution effect as long as it reduces markups. While a quantitative examination of this possibility is beyond the theoretical scope of this paper, it appears remarkable that market power in the goods market can generate an appealing response of consumption to demand shocks.

It can be easily verified as in the baseline model that the social planner optimum reproduces the equilibrium under perfect competition, which differs from the one above only for the presence of the markup. Accordingly, the optimal taxation is easily generalized as follows:

\[ \tau_t^L = \frac{\epsilon(C_t, n)}{1 - \epsilon(C_t, n)} \quad \text{and} \quad \tau_t^K = \frac{\epsilon(C_{t-1}, n) - \epsilon(C_t, n)}{1 - \epsilon(C_t, n)} \]  

(63)

Extensions to strategic interactions are also straightforward, therefore we will omit them here. Instead, some new examples will clarify how one can characterize and analyze equilibria in this generalized environment under monopolistic competition.

### 2.1 Quadratic direct utility

Let us consider the simplest example of non-separable preferences, those popularized by Melitz and Ottaviano (2008) in a quasilinear version applied to trade and modified by Ottaviano (2012) in a macroeconomic application (without capital accumulation or endogenous labor supply) as follows: \(^{17}\)

\[ U(C, n) = \alpha \sum_{j=1}^{n} C_j - \sum_{j=1}^{n} C_j^2 - \eta \left( \sum_{j=1}^{n} C_j \right)^2 \]  

(64)

\(^{16}\)Preliminary quantitative analysis with Lilia Cavallari shows that there is a wide potential for improving the ability of standard models in matching realistic impulse response functions to productivity shocks.

\(^{17}\)Bertoletti and Etro (2016) have proposed more general versions of preferences with quadratic direct and indirect utility functions, and these could be easily employed here.
where $\eta \geq 0$ parametrizes the cross-substitutability (for $\eta = 0$ we are back to the earlier case of quadratic additive preferences). In this case we have $U_i(C, n) = \alpha - (1 + \eta n) C$ and the Morishima elasticity is $\epsilon(C, n) = \frac{C}{\alpha - (1 + \eta n) C}$, which is increasing in individual consumption but also in the number of goods due to non-additivity. This delivers the monopolistic price:

$$p_t = \frac{\alpha - (1 + n\eta) C_t}{\alpha - (2 + n\eta) C_t}$$

that is increasing in consumption but also in the number of goods. Under monopolistic competition we have the modified Euler equation:

$$\alpha - (2 + n\eta) C_t = \beta R_{t+1} \mathbb{E} \{ \alpha - (2 + \eta n) C_{t+1} \}$$

which generalizes (22) and requires $C_t < \alpha/(2 + \eta n)$ for any $t$. In this case, the optimal taxation can be easily derived as:

$$\tau_t^L = \frac{C_t}{\alpha - (2 + n\eta) C_t} \quad \text{and} \quad \tau_t^K = \frac{\alpha(C_{t-1} - C_t)}{[\alpha - (2 + n\eta) C_t] [\alpha - (1 + n\eta) C_t]}$$

which provides a procyclical labor subsidy and a negative capital income taxation on the growth path.

### 2.2 Homothetic aggregators

Let us assume that intratemporal preferences for consumption can be expressed as:

$$U(C_t, n) = U[C(C_t, n)]$$

where, without loss of generality, $C(C_t, n)$ is a consumption index that is homogenous of degree one, and $U(\cdot)$ is an appropriate concave monotonic transformation. The traditional case in macroeconomic applications requires a CES aggregator and an isoelastic $U(\cdot)$ function, as in the logarithmic case:

$$U(C_t, n) = \log \left( \sum_{j=1}^{n} \frac{C_j^{\eta-1}}{C_t^{\eta-1}} \right)$$

Contrary to the baseline model, this specification allows one to distinguish the intertemporal elasticity of substitution, which here is unitary, from the intratemporal elasticity of substitution, which is $\theta > 1$.

Beyond the CES case, the class of preferences with homothetic aggregators includes other examples often used in macroeconomics, such as implicitly additive preferences (Kimball, 1995) or translog preferences (Feenstra, 2003, Bilbiie, Ghironi and Melitz, 2012). It is well known (see for instance Benassy, 1996)\footnote{This shows that an increase in the number of goods does not induce a markup reduction unless it forces lower consumption of each good, a point already noticed by Bertoletti and Epifani (2014).}
that the symmetric price of monopolistic competition for these preferences is a function \( \epsilon(n) \) of the number of goods \( n \). Since here the number of goods is exogenous, the price of the final goods:

\[
p_t = \frac{1}{1 - \epsilon(n)}
\]
is constant.\(^\text{19}\) Therefore the Euler equation remains identical as under perfect competition:

\[U'(C_t, n)) = \beta R_{t+1} \mathbb{E} \{U'(C_{t+1}, n))\} \] (69)

while the labor supply is distorted downward by a constant in every period. This has an important implication: under homothetic preferences, it is always optimal to adopt zero taxation on capital income and tax smoothing on the labor income tax:

\[
\tau^L_t = \frac{\epsilon(n)}{1 - \epsilon(n)} \quad \text{and} \quad \tau^K_t = 0
\] (70)

For instance, Feenstra (2003) has analyzed preferences based on a translog expenditure function. This implies the price \( p_t = 1 + \frac{\sigma}{\sigma n} \), where \( \sigma > 0 \) is a parameter related to substitutability between goods. In this case \( \epsilon(n) = \frac{1}{1 + \sigma n} \) and the Euler equation is identical under perfect and monopolistic competition as long as the number of goods is exogenous. See Etro (2016) for the case of Kimball preferences.

### 2.3 Directly additive aggregators

Let us consider the case where the intratemporal utility is a monotonic transformation of a directly additive aggregator:

\[
U(C_t, n) = U \left( \sum_{j=1}^{n} u(C_{jt}) \right)
\] (71)

Of course, the baseline model corresponds to this when \( U \) is a linear function and we have the traditional case (68) when \( U \) is a logarithmic function and \( u \) is a power function. In general, the intertemporal utility is not additive in the consumption of each good, while the intratemporal elasticity of substitution is the same as in the baseline model. This implies that in each period monopolistic pricing remains:

\[
p_t = \frac{1}{1 - \epsilon(C_t)}
\]

with \( \epsilon(C) = -u''(C)C/u'(C) \) that we keep labeling as relative risk aversion even if such a definition is not entirely appropriate due to the monotonic transformation applied to the intratemporal aggregator. The modified Euler condition

\(^{19}\text{We should remark that markups are variable in Kimball (1994) because prices are not flexible and consumption varies across goods, and in Bilbiie, Ghironi and Melitz (2012) because the number of firms is variable.}\)
can be expressed as:

\[ U'(nu(C_t))u'(C_t) [1 - \epsilon(C_t)] = \beta R_{t+1} \mathbb{E} \{ U'(nu(C_{t+1}))u'(C_{t+1}) [1 - \epsilon(C_{t+1})] \} \]

which differs from its perfectly competitive version only for the variable relative risk aversion. The same holds for the labor supply. Accordingly, the implications for optimal taxation are exactly the same as in the baseline model, with:

\[ \tau^L_t = \frac{\epsilon(C_t)}{1 - \epsilon(C_t)} \quad \text{and} \quad \tau^K_t = \frac{\epsilon(C_{t-1}) - \epsilon(C_t)}{1 - \epsilon(C_t)} \]

However, the mechanism of intertemporal substitution is now governed by the shape of the transformation function \( U(\cdot) \) and not just by the subutility \( u(\cdot) \). This allows us to separate the roles of intertemporal substitutability and market power in determining the reaction of aggregate variables to shocks. With the common logarithmic transformation, \( U(\cdot) = \log(\cdot) \) we would have \( U_i(C, n) = \frac{u'(C)}{nu(C)} \) and various examples can be examined on the basis of the common examples of directly additive aggregators.

### 2.4 Indirectly additive aggregators

The third general class of preferences recently analyzed in a static context of monopolistic competition is characterized by an indirect utility that is additive (Bertoletti and Etro, 2017). In particular, let us assume that the intratemporal indirect utility is additively separable as in:

\[ V \left( \frac{p_t}{E_t} \right) = U \left( \sum_{j=1}^{n} v \left( \frac{p_{jt}}{E_t} \right) \right) \]

where \( v(s) \) is decreasing and convex in the price-expenditure ratio and \( U(\cdot) \) is always a monotonic transformation that insures concavity in income.

In this case, we can reformulate easily the decentralized equilibrium. The demand for each good \( i \) in period \( t \) derives from the Roy identity, and the corresponding profits are:

\[ \pi_{it} = \frac{(p_{it} - 1) u' \left( \frac{p_{it}}{E_t} \right) E_t}{\sum_{j=1}^{n} v' \left( \frac{p_{jt}}{E_t} \right) \left( \frac{p_{jt}}{E_t} \right) \left( \frac{p_{jt}}{E_t} \right)} \]

whose denominator is directly related to the marginal utility of income\(^{20}\) and is taken as given under monopolistic competition. The monopolistic price satisfies:

\[ p_t = \frac{1}{1 - \frac{1}{v(p_t/E_t)}} \]

\(^{20}\) More precisely, \( \lambda_t E_t = - \sum_{j=1}^{n} v' \left( \frac{p_{jt}}{E_t} \right) \left( \frac{p_{jt}}{E_t} \right) \).
where $\theta(s) \equiv -v''(s)s/v'(s)$ is the demand elasticity in function of the price-expenditure ratio, and corresponds to what we defined above as the Morishima elasticity of substitution. However, symmetry implies $E_t = np_tC_t$, therefore the price can be seen as a function of $\theta(1/nC)$, and it is increasing in the consumption level if and only if $\theta' > 0$. The case of CES preferences emerges again if $\theta' = 0$.

In each period, since the direct demand of each good is already implicit in the specification of preferences, the consumer chooses only total spending $E_t$ and labor supply $L_t$ to maximize intertemporal utility (58) under the resource contraint. The problem:

$$\max_{E_{t+1},L_t} \hat{U} = E \sum_{t=1}^{\infty} \beta^{t-1} \left[ U \left( \sum_{j=1}^{n} v \left( \frac{p_j}{E_t} \right) \right) - vL_t^{1+\frac{1}{n}} \right]$$

leads to the Euler condition:21

$$U'[nv \left( \frac{p}{E_t} \right)]v' \left( \frac{p}{E_t} \right) p_t = \beta R_{t+1} E \left\{ U'[nv \left( \frac{p_{t+1}}{E_{t+1}} \right)]v' \left( \frac{p_{t+1}}{E_{t+1}} \right) p_{t+1} \right\}$$

This is a particular case of our general framework once we recognize that in a symmetric equilibrium $p_t/E_t = 1/nC_t$.

Since a well behaved indirect utility function is increasing in income but not necessarily concave, the transformation $U$ must insure concavity for the equilibrium to be well defined. For instance, if $U$ is linear, one can verify that this is the case if and only if $\theta(s) \in (1,2)$, which is quite restrictive for our analysis of monopolistic competition.22 Therefore, in what follows we focus again on the case of a logarithmic transformation, with $U(\cdot) = \log(\cdot)$.

Defining $\phi(s) = -v'(s)s/v(s)$ as the elasticity of the subutility, we have $\phi(p/E) = \phi(1/nC)$ and $\epsilon(C) = 1/\theta(1/nC)$. This allows us to rewrite the modified Euler condition as a particular case of the general model:

$$\phi \left( \frac{1}{nC_t} \right) \left[ 1 - \theta \left( \frac{1}{nC_t} \right)^{-1} \right] = \beta R_{t+1} E \left\{ \phi \left( \frac{1}{nC_{t+1}} \right) \left[ 1 - \theta \left( \frac{1}{nC_{t+1}} \right)^{-1} \right] \right\}$$

With CES preferences both elasticities are constant and we are back to the same results as with perfect competition. When the demand elasticity is variable, instead, market power has bite and affects the business cycle properties

21 A similar result is obtained in Etro (2016) to analyze a Ramsey model of consumption growth and used by Boucekkine et al. (2016) for an interesting analysis of endogenous growth. The FOC for the labor supply can be derived accordingly.

22 I am thankful to Paolo Bertoletti for insightful discussions on this condition and on the concept of intertemporal elasticity of substitution. On the latter, a useful reading is Browning and Crossley (2000).
of the model. Since indirectly additive utility functions have never been used in macroeconomics, it is useful to express also the general labor supply equation:

\[ L_t = \left\{ \frac{1 - \alpha}{vC_t} A_t R_t^\alpha \phi \left( \frac{1}{nC_t} \right) \left[ 1 - \theta \left( \frac{1}{nC_t} \right)^{-1} \right] \right\}^{\frac{\phi}{1 - \phi}} \]

The optimal taxation can be derived from the general principles stated above as follows:

\[ \tau_t^L = \frac{1}{1 - \theta (s_t)} \quad \text{and} \quad \tau_t^K = \frac{\theta (s_t) - \theta (s_{t-1})}{\theta (s_t) [\theta (s_t) - 1]} \quad (79) \]

where \( s_t = p_t / E_t = 1 / nC_t \).

In spite of apparent complication, this class of preferences can be easily analyzed in macroeconomic models. To show this, we will now introduce some new examples deriving the relevant equilibrium conditions.

**Addilog preferences** As a first example, let us consider the addilog preferences, already exploited by Bertoletti, Etro and Simonovska (2016) in a static context:

\[ v(s) = \frac{(a - s)^{1+\gamma}}{1 + \gamma} \quad (80) \]

where \( a > 0 \) represents the maximum willingness to pay (demand is zero for normalized prices above this) and \( \gamma > 0 \) parametrizes the demand elasticity. This parameter is estimated as unitary by Bertoletti, Etro and Simonovska (2016) in a multicountry context, which supports a linear demand function. In general, notice that \( \theta (s) = \gamma s / (a - s) \) which is increasing in \( s \) and therefore \( \epsilon (C) = \frac{anC - 1}{\gamma} \) is increasing in consumption. This implies the monopolistic price:

\[ p_t = \frac{\gamma}{1 + \gamma - anC_t} \quad (81) \]

which is procyclical under the regularity condition \( 1 < anC_t < 1 + \gamma \). The Euler equation under perfect competition can be derived as:

\[ \frac{1}{(anC_t - 1) C_t} = \beta R_{t+1} E \left\{ \frac{1}{(anC_{t+1} - 1) C_{t+1}} \right\} \]

The modified Euler equation under monopolistic competition is:

\[ \frac{1 + \gamma - anC_t}{(anC_t - 1) C_t} = \beta R_{t+1} E \left\{ \frac{1 + \gamma - anC_{t+1}}{(anC_{t+1} - 1) C_{t+1}} \right\} \]

which tends to enhance consumption smoothing compared to perfect competition as long as the number of goods is fixed.
Exponential preferences  Let us consider the exponential preferences with:

\[ v(s) = e^{-bs} \]  \hspace{1cm} (82)

with \( b > 0 \) parametrizing demand elasticity. Notice that \( \theta(s) = \phi(s) = bs \) which is increasing in \( s \), therefore \( \epsilon(C) = \frac{aC}{b} \) is instead increasing in individual consumption. Under monopolistic competition, the equilibrium price is:

\[ p_t = \frac{b}{b - nC_t} \]  \hspace{1cm} (83)

which is procyclical under the regularity condition \( C_t < b/n \). Under perfect competition we would have a very simple variation of the traditional Euler condition:

\[ \frac{1}{C_t^2} = \beta R_{t+1} E \left\{ \frac{1}{C_{t+1}^2} \right\} \]

while the modified Euler equation under monopolistic competition becomes:

\[ \frac{b - nC_t}{C_t^2} = \beta R_{t+1} E \left\{ \frac{b - nC_{t+1}}{C_{t+1}^2} \right\} \]

whose difference from the former crucially depends on the parameter \( b \) as well as on the number of goods.

Translated power preferences  A last example of indirect additivity emerges with:

\[ v(s) = (s - \bar{s})^{1-\vartheta} \]  \hspace{1cm} (84)

where \( \bar{s} > 0 \) and \( \vartheta > 1 \). Normalized prices must be above \( \bar{s} \) for demand to be finite. In this case \( \theta(s) = \vartheta s/(s - \bar{s}) \) is decreasing in \( s \) and \( \epsilon(C) = \frac{1 - \bar{s}nC}{\vartheta} \) is now decreasing in consumption. The monopolistic price can be computed as:

\[ p_t = \frac{\vartheta}{\vartheta - 1 + \bar{s}nC_t} \]  \hspace{1cm} (85)

which is countercyclical with \( C_t < 1/\bar{s}n \). The Euler condition with perfect competition reads as:

\[ \frac{1}{(1 - \bar{s}nC_t)C_t} = \beta R_{t+1} E \left\{ \frac{1}{(1 - \bar{s}nC_{t+1})C_{t+1}} \right\} \]

and the modified Euler equation under monopolistic competition becomes:

\[ \frac{\vartheta - 1 + \bar{s}nC_t}{(1 - \bar{s}nC_t)C_t} = \beta R_{t+1} E \left\{ \frac{\vartheta - 1 + \bar{s}nC_{t+1}}{(1 - \bar{s}nC_{t+1})C_{t+1}} \right\} \]

Both versions go back to the same classic Euler condition for CES preferences when \( \bar{s} \to 0 \). As long as \( \bar{s} \) is positive, however, monopolistic competition amplifies consumption fluctuations compared to perfect competition. While its properties are similar to the earlier example with Stone-Geary preferences, here the number of goods is an additional element which affects intertemporal substitutability.
3 Toward a general theory of dynamic endogenous market structures

The maximum level of generality proposed in this work should accounts for entry and exit of firms, different forms of competition and an intratemporal utility function $U(C_t, n_t)$ that depends in a general way on the endogenous number of consumed goods. Building on the microfoundation of symmetric preferences with variable number of goods of Bertoletti and Etro (2016) we can advance such a general framework.

First of all, we augment the equation of motion for the number of firms as:

$$n_{t+1} = (1 - \delta_n)(n_t + n^*_t)$$

where $\delta_n \in [0, 1)$ is an exogenous exit probability (Ghironi and Melitz, 2005) and $n^*_t$ is always the endogenous number of entrants in period $t$.

Monopolistic, Bertrand and Cournot competition deliver the respective prices:

$$p_t = \frac{1}{1 - \epsilon (C_t, n_t)} , p^B_t = \frac{\epsilon (C_t, n_t) + n_t - 1}{(n_t - 1)(1 - \epsilon (C_t, n_t))} , p^C_t = \frac{n_t}{(n_t - 1)(1 - \epsilon (C_t, n_t))}$$

Denoting the generic price as $p_t(C, n_t)$, profits read in general as $\pi_t (C_t, n_t) = p_t(C_t, n_t)C_t - C_t$.

The modified Euler equation and the labor supply equations are:

$$\frac{U_i(C_t, n_t)}{p_t(C_t, n_t)} = \beta E \left\{ \left[ 1 + \alpha A_t K_{t+1}^{1-\alpha} L_{t+1}^{1-\alpha} - \delta \right] \frac{U_i(C_{t+1}, n_{t+1})}{p_t(C_{t+1}, n_{t+1})} \right\}$$

and as long as entry takes place, which we will assume to be the case in what follows, the general free entry condition becomes:

$$\frac{U_i(C_t, n_t)}{p_t(C_t, n_t)} F_t = \beta (1 - \delta_n) E \left\{ U_i(C_{t+1}, n_{t+1}) \left[ C_{t+1} - \left( \frac{C_{t+1} - F_{t+1}}{p_t(C_{t+1}, n_{t+1})} \right) \right] \right\}$$

Using the resource constraint, the equation of motion for the number of firms becomes:

$$n_{t+1} = (1 - \delta_n) \left[ n_t + \frac{A_t K_t \alpha L_t^{1-\alpha} - n_t C_t - K_{t+1} + K_t (1 - \delta)}{F_t} \right]$$

Bilbiie, Ghironi and Melitz (2012) have studied a similar model with monopolistic competition and homothetic aggregators, where $\epsilon (C_t, n_t) = \epsilon (n_t)$ depends only on the number of goods provided in period $t$. The conceptual novelty of the above system is that it allows one to study the case of any symmetric intratemporal preferences. While a numerical simulation is beyond the scope of this theoretical work, it is clear that model is much more flexible than existing alternatives in reproducing cyclical behavior and moments emerging in the data.
Also in this case we can study the deterministic social planner problem as:

\[
\max_{K_{t+1}, L_t, n_{t+1}} \sum_{t=1}^{\infty} \beta^{t-1} \left[ U \left( \frac{A_t K_t^{1-\alpha} L_t^{1-\alpha} - K_{t+1} + K_t (1 - \delta) - \frac{n_{t+1}}{1 - \delta_n} + F_t, n_t} {n_t} \right) - \frac{\psi L_t^{1+\frac{1}{\psi}}}{1 + \frac{1}{\psi}} \right]
\]

The FOCs for \(K_{t+1}, L_t\) and \(n_{t+1}\) can be rearranged as:

\[
U_i(C_t^*, n_t^*) = \beta \left[ 1 + \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} - \delta \right] U_i(C_{t+1}^*, n_{t+1}^*)
\]

\[
L_t^* = \left[ \frac{1 - \alpha}{\psi} A_t K_t^{\alpha} U_i(C_t^*, n_t^*) \right]^{\frac{\psi}{\psi - \alpha}}
\]

\[
\frac{U_i(C_t^*, n_t^*) F_t}{n_t^*} = \beta (1 - \delta_n) \left[ U_n(C_{t+1}^*, n_{t+1}^*) - \frac{U_i(C_{t+1}^*, n_{t+1}^*) (C_{t+1}^* - F_{t+1})}{n_{t+1}^*} \right]
\]

which extend to a dynamic context the analysis of optimal market structures presented in Bertoletti and Etro (2016) for a static context. As there, the optimal number of firms derives from the trade-off between the costs of producing new varieties and the benefits of enjoying them (in the future) net of the reduction of consumption of each variety (needed to invest in replacing the lost varieties). In both the static and the dynamic context, this trade-off depends on the elasticities of the utility function with respect to consumption and to the number of goods:

\[
\psi^C(C, n) = \frac{U_i(C, n) C}{U(C, n)} \quad \text{and} \quad \psi^n(C, n) = \frac{U_n(C, n) n}{U(C, n)}
\]

Indeed, the last FOC can be rearranged as follows:

\[
U_i(C_t^*, n_t^*) F_t = \beta (1 - \delta_n) U_i(C_{t+1}^*, n_{t+1}^*) \left[ \frac{\psi^n(C_{t+1}^*, n_{t+1}^*) C_{t+1}^*}{\psi^C(C_{t+1}^*, n_{t+1}^*)} - (C_{t+1}^* - F_{t+1}) \right]
\]

to be compared to the decentralized free entry condition.

Let us consider optimal taxation under monopolistic competition. This requires simple extensions of our earlier tax rates on labor and capital income:

\[
\tau_t^L = \frac{\epsilon(C_t, n_t)}{1 - \epsilon(C_t, n_t)} \quad \text{and} \quad \tau_t^K = \frac{\epsilon(C_{t-1}, n_{t-1}) - \epsilon(C_t, n_t)}{1 - \epsilon(C_t, n_t)}
\]

Two remarks are in order. First, the traditional case of a constant labor subsidy and a zero capital income tax emerges only under intertemporal preferences that deliver a symmetric Morishima elasticity of substitution independent from consumption and number of goods. A well known case is the one of CES aggregators, but Bertoletti and Etro (2016) have shown that there are other possible cases. One of them is based on the generalized linear direct utility introduced by Diewert (1971): in special cases of this class of homothetic preferences the
equilibrium markup of monopolistic competition is constant and therefore capital income taxation is not needed to reach optimality and the optimal labor subsidy is constant.\textsuperscript{23}

Second, except for these special cases, the class of homothetic aggregators generates variable optimal tax rates over the business cycle. In particular, if the markups decrease with the number of firms, a shock that generates entry reduces the optimal labor subsidy and requires a negative capital income taxation. With more general preferences, the principle is that countercyclical markups require a countercyclical taxation. In the long run, however, the optimal capital income tax is zero for any preferences.

Given the optimal tax rates on labor and capital income, it is a matter of computation to show that the first best is fully replicated if the tax on profits satisfies:

$$
\tau^D_t = 1 - \frac{1 - \epsilon(C_t, n_t)}{\epsilon(C_t, n_t)} \left[ \frac{n_{t-1} \psi^n(C_{t-1}, n_t) - \psi^C(C_{t-1}, n_t)}{\psi^n(C_{t-1}, n_t)} - \frac{n_t - n_{t-1} \bar{E}_t}{n_t} \frac{C_t}{C_{t-1}} \right]
$$

To gain insights from these results, notice that in steady state the optimal taxation simplifies to:

$$
\tau^L = \frac{\epsilon(C, n)}{1 - \epsilon(C, n)} \quad \tau^K = 0 \quad \text{and} \quad \tau^D = \frac{1 - \epsilon(C, n) \psi^n(C, n)}{\epsilon(C, n) \psi^C(C, n)}
$$

which generalizes earlier results. Capital income taxation is always zero in the long run, but a labor subsidy and a profit tax are typically needed to insure the optimal production level and the optimal entry process.

Some examples can be useful. The traditional case of a CES aggregator with intratemporal preferences (68) implies $\epsilon(C, n) = 1/\theta$ and $\psi^n(C, n)/\psi^C(C, n) = \theta/(\theta - 1)$, therefore the optimal profit tax is zero in the long run. The mentioned case of translog preferences of Feenstra (2003) implies $\epsilon(C, n) = 1/(1 + \sigma n)$, $\psi^C(C, n) = 1$ and $\psi^n(C, n) = 1 + 1/2\sigma n$, which allows us to compute the long run optimal profit taxation $\tau^D = 1/2$. Both these results are consistent with what found by Biblič, Ghironi and Melitz (2008,b).\textsuperscript{24}

The business cycle properties of such a general model are much richer than in the standard RBC framework or in common models with dynamic entry based

\textsuperscript{23}For instance, this requires intratemporal utility $U(C, n) = U(\sum_{i=1}^n \sum_{j=1}^n \sqrt{C_i C_j})$ which delivers the monopolistic price $p_i = 2 > 1$. Strategic interactions imply $p_i^B = \frac{2m_i - 1}{m_i}$ and $p_i^C = \frac{2m_i}{m_i}$. See Bertoletti and Etro (2016) for details.

\textsuperscript{24}More in general, we can focus our interpretation of the optimal taxation scheme for our main classes of preference aggregators, for which the elasticities showing up above can be derived directly from preferences. With homothetic aggregators all these elasticities depend on the number of firms only, as can be derived from $U(C, n) = U[C(\text{C})C, n]$; this implies that the optimal labor subsidy is countercyclical and the capital income tax is positive and countercyclical if and only if $\epsilon'(n) < 0$, as for instance with translog preferences (but one can find examples of homothetic aggregators such that $\epsilon'(n) > 0$; see Bertoletti and Etro, 2016). With directly additive aggregators all the elasticities depend only on the consumption level, as can be derived from $U(C, n) = U[\text{C}(\text{C})C, n]$. Finally, with indirectly additive aggregators all the elasticities depend on total consumption $nC$, as can be derived from $U(C, n) = U[\text{C}(\text{C})C, n]$. 


on CES preferences and strategic interactions (since at least Etro and Colciago, 2010). Optimal taxation formulas can be derived as usual, in function of the same elasticities used before and of the number of goods.

4 Conclusions

Departing from perfect competition and CES preferences, flexible price DSGE models can offer new channels of propagation of the shocks that operate through changes in the endogenous markups and, possibly, endogenous business creation. Remarkably, these changes depend crucially on the properties of preferences, restoring a novel role for the demand side in determining the propagation of shocks along the business cycle. We have analyzed these dynamic models and evaluated the optimal taxation that restores the first best allocation of resources through taxes on labor income, capital income and firms’ profits.

Further work should evaluate the empirical properties of particular models after analyzing their impulse response functions and moments under standard calibrations. The framework could be applied to examine Ramsey policies of optimal distortionary taxation (in the absence of lump sum taxes) and to introduce price frictions for the analysis of monetary shocks in a New-Keynesian style.

Moreover, one could study shocks and policies in an open economy framework (see Ghironi and Melitz, 2005, and Cavallari, 2013, for open economy entry models with CES preferences). A flourishing literature in international trade has been recently departing from CES preferences to investigate non-homothetic ones, especially quadratic preferences (Melitz and Ottaviano, 2008), directly additive preferences (Arkolakis et al., 2015) and indirectly additive preferences (Bertoletti et al., 2016): a contamination between these literatures in the spirit of Ghironi and Melitz (2005) could be fruitful. An interesting tension emerges in an open economy framework: while we have seen that lower markups in boom periods can contribute to better explain the business cycle, the cross-country evidence suggests that markups are higher in richer countries. Apparently, one may think that matching both features is difficult within our framework: in reality, countercyclical markups due to competition effects on the supply side can be perfectly consistent with higher markups in richer countries due to lower demand elasticities for high income countries (as, for instance, with addilog preferences and strategic interactions). This is also consistent with the evidence of different reactivity of markups in front of supply and demand shocks (Nekarda and Ramey, 2013).

Our ultimate objective, however, is to provide a general microfoundation of RBC models to expand the ability of creating a framework which can replicate empirical findings, incorporate realistic imperfections in the labor and credit markets and be used for policy analysis. Many of the limits of the standard macroeconomic framework are deeply linked with the ubiquity of CES preferences in dynamic analysis.
References

Arkolakis, Costas, Arnaud Costinot, Dave Donaldson and Andres Rodríguez-Clare, 2015, The Elusive Pro-Competitive Effects of Trade, WP 21370, NBER
Bertoletti, Paolo and Federico Etro, 2014, Pricing to Market in the Krugman Model, Economics Bulletin, 34, 1, 459-68
Bertoletti, Paolo and Federico Etro, 2016, Preferences, Entry and Market Structure, RAND Journal of Economics, in press
Bertoletti, Paolo, Federico Etro and Ina Simonovska, 2016, International Trade with Indirect Additivity, NBER WP, 21983
Bertoletti, Paolo, Elena Fumagalli and Chiara Poletti, 2008, Price-Increasing Monopolistic Competition? The Case of IES Preferences, IEFE Working Paper No. 15, Bocconi University
Bilbiie, Florin, Fabio Ghironi and Marc Melitz, 2008b, Monopoly Power and Endogenous Variety: Distortions and Remedies, NBER WP 14383
Blanchard, Olivier and Nobuhiro Kiyotaki, 1987, Monopolistic Competition and the Effects of Aggregate Demand, American Economic Review, 77, 4, 647-66
Boucekkine, Raouf, Hélène Latzer and Mathieu Parenti, 2016, Variable Markups in the Long-Run: A Generalization of Preferences in Growth Models, mimeo, ECARES
Cavallari, Lilia, 2013, Firms’ entry, Monetary Policy and the International Business Cycle, Journal of Monetary Economics, 91, 2, 263-74
Kydland, Finn and Edward Prescott, 1982, Time to Build and Aggregate Fluctuations, *Econometrica*, 50, 6, 1345-70
La Croce, Carla and Lorenza Rossi, 2014, Endogenous Entry, Banking, and Business Cycle, DEM WP 072, University of Pavia
Lewis, Vivien and Roland Winkler, 2015, Product Diversity, Demand Structures and Optimal Taxation, *Economic Inquiry*, 53, 2, 979-1003
Nekarda, Chris and Valery Ramey, 2013, The Cyclical Behavior of the Price-Cost Markup, NBER 19099

Poutineau, Jean-Christophe and Gauthier Vermandel, 2015, Financial Frictions and the Extensive Margin of Activity, _Research in Economics_, 69, 4, 525-54


Woodford, Michael, 2003, _Interest and Prices_, Princeton University Press