Simultaneous Hedging of Upgrading Costs and Optimal Public Policies with Many Orphan Diseases

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Abstract

In a General Equilibrium framework, we consider a large number of countries and international agencies simultaneously fighting an arbitrary number of orphan diseases. Those bodies seek to insure against the upgrading costs associated with the introduction of innovative treatments. We exhibit a large class of economies where markets for insurance against those upgrading costs are incomplete, and where the introduction of an insurance against one particular disease makes a large number of countries strictly worse off because of substitution effects. We show that there exists a sequence of assets with more complex payoff patterns whose sequential introduction makes every country better off after every introduction. Finally, we argue and recommend that the simultaneous introduction of every original insurance for every disease under treatment avoids this pitfall, and leads to Pareto optimal outcomes.

Keywords: Orphan diseases, Optimal public policies,
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1 Introduction

The main health problem in many developing countries is the simultaneous occurrence of many widespread diseases, which take an enormous toll both at human and economic level. For instance, the simultaneous incidence of HIV/AIDS and malaria make the medical situation nearly unmanageable in most Sub-Saharan countries already facing severe budget shortfalls (see Cleary et al. (2) and Leoni (13) Ch. 2). Every disease requires significant, specific and mostly irreversible investments in situations of severe public budget shortfalls, leading to crowding-out effects in some of the poorest countries. The problem has become so striking that the eradication of those diseases is one of the main targets set by the United Nations to eradicate poverty worldwide (see Millennium Goal (15), or Lampsey et al. (11)).

Given the severe strain on national expenditures in developing countries that simultaneously fighting those diseases take, the identification of optimal public policies is critical for the relevant developing countries and international agencies dealing with those health problems. One important aspect of those optimal policies, which has somewhat been overshadowed, is the need to hedge against upgrading costs resulting from the introduction of innovative treatment technologies. Leoni and Luchini (14) shows that those upgrading costs are mostly irreversible and they annually amounts to billion of dollars worldwide. Moreover, hedging against costs resulting from the introduction of innovative and more effective drugs (see Webber and Kremer (16) for orphan diseases in general, and Klausner et al. (10) for a therapeutic vaccine against
HIV/AIDS), or also an innovative treatment strategy (Lasserre et al. (12)), is shown to foster investments in available treatments, and to reach Pareto optimal allocation of national resources.

On the other hand, failure to hedge against those upgrading costs may result in optimal underinvestment and/or delay in investment in current treatment therapies (see Dixit and Pyndick (4) Ch. 7-11). The basic insight is that the possibly rapid obsolescence of current and mostly irreversible investments may trigger severe opportunity costs and losses in some of the poorest countries, making those investments sub-optimal because of their too-short lifespan. Leoni and Luchini (14) introduces a particular set of derivatives called Arrow securities, targeted at HIV/AIDS but extendable to any other orphan disease, whose objective is to efficiently insure any subscriber against any such upgrading cost.

The main potential problem when introducing an Arrow security against the upgrading costs resulting from a particular disease, whereas many others upgrading costs are also to be simultaneously hedged, is that it may trigger negative substitution effects across health expenditures. The intuition is that the introduction of one particular Arrow security may change, depending on preferences, the price of already existing similar assets against other diseases to the point where previous hedging needs are no longer affordable for a large number of countries. Given so, new hedging plans are to be implemented at the new equilibrium prices, and they may lead to sub-optimal insurance level. This large impact on insurance prices typically make
them prohibitive for some other countries in needs of those hedges, leading to over-
investment in some diseases and under-investment in others. This issue is critical
when dealing with orphan diseases, since a surge in demand from some countries
particularly affected by a particular disease (Sub-Saharan countries with HIV/AIDS
for instance) is likely to occur given high national incidences.

The question addressed here from a General Equilibrium standpoint is two-fold;
first, we identify the determinants under which many developing countries simulta-
neously fighting many diseases become strictly worse off after introducing one Arrow
security against a particular disease; second, we identify which sequence of insurance
schemes always leads to Pareto optimal allocations for every country and every in-
ternational agency involved in this fight. We find that most economies will display
sub-optimal substitution effects leading a large number of countries to be worse off
after introducing one particular Arrow security, and we show that the simultaneous
introduction of every Arrow security against every disease cancel out those effects
and leads to Pareto-optimal outcomes.

We consider an economy composed of a large number of countries and interna-
tional agencies (a continuum) simultaneously fighting an arbitrary number of orphan
diseases, in a General Equilibrium framework. This continuum assumption is solely
meant to rule out the possibility that one country only affects too dramatically equi-
librium prices of Arrow securities, and it is not essential to our results. Under fairly
general conditions, we give a large class of economies where markets for insurance
against upgrading costs for many diseases are incomplete, and where the introduction of one non-redundant Arrow security makes a strictly positive measure of countries strictly worse off as a result. The basic intuition is that the introduction of the new Arrow security change prices so dramatically, because of substitutions effects, that previous hedging plans are no longer affordable and under-insurance obtain in equilibrium for this large measure of countries (Proposition 3).

To avoid this issue, we show that there exists a sequence of assets, whose payoff pattern is different from that of Arrow securities, and whose sequential introduction makes every country better off after every introduction (this result is a consequence of Th. 1 in Hara (7)). The basic intuition for the construction of those assets is that they must pay off for the simultaneous upgrades of many diseases, instead of one disease only as for Arrow securities, so that a change in equilibrium holdings after an asset introduction does not trigger overly large substitution effects as previously observed. Finally, we argue and recommend that the simultaneous introduction of every Arrow security for every orphan disease under treatments avoids all of the above pitfalls, and leads to Pareto optimal outcomes.

The idea that the introduction of a new asset may have significant impact on welfare dates back to Hart (9), and Hakansson (8) later gave some conditions leading to welfare changes. The importance of substitution effects was first pointed in Cass and Citanna (3) and Elul (5; 6); however, the whole point of this literature is that the too-small number of traders trigger those substitution effects. In sharp
contrast, we show that those substitution effects also occur in large economies, and they entirely drive the affordability and optimality of previous hedging plans after an asset introduction. Hara (7) shows the existence of a welfare-improving sequence of assets, without showing that the multi-span payoff pattern of those securities allows to smooth out substitution effects.

The paper is organized as follows. In Section 2, we describe the economy; in Section 3, we construct the class of economies for which a welfare impairment occurs after the introduction of one Arrow securities; in Section 4, we show existence of a welfare improving sequence and the Pareto optimality of all of the Arrow securities; Section 5 contains some concluding remarks, and the technical proofs are given in the Appendix.

2 The model

We next develop our formal analysis. In a first step, we define the model. We then show that, for any given asset structure resulting in incomplete markets, the arbitrary addition of a non-redundant Arrow security may make at least one country strictly worse off depending on substitution effect across needs for treatments. We finally prove that there exists a sequence of securities, possibly different from Arrow securities, such that their sequential introduction makes every country better off after every introduction. Finally, we argue that the simultaneous introduction of every Arrow securities leads the economy to an equilibrium outcome that is Pareto-optimal.
There is a large number of countries and/or international agencies, which are involved in actively providing treatments to cure an arbitrary but finite number of orphan diseases. Formally, the set of countries and international agencies are modeled a continuum normalized to $[0, 1]$ without any loss in generality; a country is represented by $i \in [0, 1]$. We could have assumed instead that there is a finite but large number of those bodies without changing of our results, but the analysis turns out to be more complicated in this case. There are 2 periods; the first period corresponds to certainty where decisions are taken, and in period 2 there are $S > 1$ states that may occur. A state corresponds to the appearance or not of possibly many innovative treatments against these diseases treated by the countries. The state $S$ will then correspond to the case where the innovative treatments for every possible disease under treatment become available, with the convention that no appearance occurs in the first period. To fix ideas, consider 2 diseases such as AIDS and malaria and an innovative treatment potentially available against every of them. There are three states of nature: $s_1 = (1, 0)$, $s_2 = (0, 1)$ and $S = s_3 = (1, 1)$; the state $s_1$ corresponds to the appearance of an innovative treatment against AIDS but not against malaria, state $s_2$ corresponds to the appearance of an innovative treatment against malaria but not against AIDS, and state $s_3$ corresponds to the appearance of an innovative treatment both against AIDS and against malaria. Every state $s \in \{1, ..., S\}$ occurs in period 2 with probability $\alpha_s > 0$, and the odds are common knowledge among countries (without any loss of generality).
There is one consumption good in every period, which can be used to purchase public goods and specific treatments for every disease. Our analysis readily extends to the case of finitely many consumption goods, we omit this issue to simplify the exposition. In period 1, every country \( i \in [0, 1] \) has a certain integrable endowment \( w^i_0 > 0 \). In period 2, this same country receives a state-contingent integrable endowment \( w^i_s > 0 \) for every state \( s \). Endowments’ fluctuations can be justified as a function of tax revenues and reduction in international subsidies resulting from the appearance of a successful treatment against one or many diseases.\(^1\)

The problem that we analyze is that of the optimal management of public budgets across various diseases, as a function of the success/failure of the R&D for innovative treatments, and it does not address the optimal purchase and local production of drugs as in Leoni and Luchini (14). Those issues are equivalent though, since the funds allocated to a specific state can be use for the purchase of public goods and various available therapies. We avoid this last issue to emphasize the role of the introduction of our hedging tools on welfare enhancement and also possible reduction.

We next describe the asset markets. We identify a security by its payoff pattern in period 2; i.e., security \( a \in \mathbb{R}^S_+ \) is a vector \((a_1, ..., a_S)\). An Arrow security designed against a particular disease, as described in the Introduction, thus payoffs one unit in every state where the directed disease is present, and 0 otherwise. Going back to the previous example with solely AIDS and malaria, an Arrow security against AIDS

\(^1\)This issue is discussed in length in Leoni and Luchini (14)
for instance thus pays off one unit of consumption good in states $s_1$ and $s_3$, and 0 in state $s_2$.

Consider a sequence of securities $(a_1, ..., a^p)$ for some $p > 1$, which we will call an asset structure. Its span is defined as

$$sp(a_1, ..., a^p) = \{ x \in \mathbb{R}^S | \exists \theta \in \mathbb{R}^p : \sum_{r \leq p} \theta_r a^r_s = x_s \forall s \}$$

We say that a market is complete for a sequence $(a_1, ..., a^q)$ (for some $q > 1$) when $sp(a_1, ..., a^q) = \mathbb{R}^S$. This means that every desirable hedging plan can be achieved through trading the appropriate securities, even if this may not be affordable. It is clear that one must have at least $S$ linearly independent securities for a market to be complete, that the introduction of every Arrow security allowing to hedge against the appearance of every innovative treatment leads to a complete market.

We next describe the budget of every country. Assume that the asset market consists of an arbitrary span $sp(a_1, ..., a^p)$ for some $p > 1$, and that every country has full access to it. The budget constraint of country $i \in [0, 1]$ in period 0 is given by

$$c_0 + \sum_r \theta_r \cdot q_r \leq w^i_0, \quad (1)$$

and for every $s$ by

$$c_s \leq w^i_s + \sum_r \theta_r a^r_s, \quad (2)$$

where $(c_0, c_1, ..., c_S)$ is the vector of consumption of the country, the vector $(q_1, ..., q_p)$ corresponds the current market prices of available securities in the asset span, and $(\theta_1, ..., \theta_p)$ is the vector of holdings of corresponding securities. In words, the budget
in period 0 is allocated between current budget needs, and the purchase of available securities to be redeemed next period as an addition to future state-contingent endowments.

A country $i$ seeks to maximize the expected sum of its discounted consumption, which specifically takes the form

$$U^i(c_0, c) = u^i(c_0) + \sum_{s=1}^{S} \alpha s u^i_s(c_s). \tag{3}$$

We assume that, for every $s$ and $i$, the functions $u^i_s$ and $u^i$ are increasing, concave and differentiable, and that $\beta > 0$ is an intertemporal discount factor. We next define our key concept of competitive equilibrium.

**Definition 1** Consider an arbitrary span $sp(a^1, ..., a^p)$ for some $p > 1$. A competitive equilibrium for this economy is a sequence of prices $(q_1, ..., q_p)$, and for every country $i \in [0, 1]$ an integrable sequence of consumption $(c^i_0, c^i_1, ..., c^i_S)$ and asset holdings $(\theta^i_1, ..., \theta^i_p)$ such that

- for every $i$, and taking asset prices as given the sequences $(c^i_0, c^i_1, ..., c^i_S)$ and $(\theta^i_1, ..., \theta^i_p)$ maximize Eq. (3) subject to Eq. (1) and (2), and

- markets clear; i.e., for every $s \in \{0, 1, ..., S\}$ we have that $\int_{[0,1]} c^i_s di = \int_{[0,1]} w^i_s di$

  and $\int_{[0,1]} \theta^i_r di = 0$ for every $r = 1, ..., p$.

The above definition is a fairly standard concept of competitive equilibrium, and in particular it requires that there is no outstanding shares of securities in the economy. Budgets components can be freely traded across countries, and then transformed into
public goods and therapies; this issue extends the analysis in Leoni and Luchini (14) without any loss in generality.

3 Welfare deterioration and asset structure

We now show that, for a given set of diseases to treat, the sequential introduction of Arrow securities may make one country strictly worse off during the process.

We first define the notion of price preservation.

**Definition 2** Consider a sequence of linearly independent assets \((a_1, ..., a_S)\), and for every \(j \leq S\), let \((q_1, ..., q_j)\) denote the sequence of equilibrium price for \(sp(a_1, ..., a_j)\). We say that \((a_1, ..., a_S)\) is price-preserving if, for every \(j \leq S\), the sequence \((q_1, ..., q_j)\) is a sequence of equilibrium prices for \(sp(a_1, ..., a_j)\).

Intuitively, the notion of price-preservation requires that the addition of a new security along the original sequence, and in this order of introduction, does not affect the equilibrium prices of the previously introduced securities. With this notion, we can state our intermediary result which extends to broader classes of utility functions. We first assume, for the following result only, that the utility function for every country \(i\) takes the form

\[
U^i(c) = u(c_0) + \sum_s \alpha_s \left[ u(c_s) \cdot \gamma^i_s \right],
\]

where \((\gamma^i_1, ..., \gamma^i_S) \in [0, 1]^S\) are positive constants that characterize country \(i\), and capture in a simple manner substitution effects across states.
**Proposition 3** Assume that the utility functions are of the form Eq. (4). If the sequential introduction of assets \((a_1, \ldots, a_S)\) is not price preserving then a strictly positive measure of countries will be strictly worse off after the introduction of one of those assets.

The proof of Proposition 3 is given in the Appendix. The result readily extends to a broader of utility functions, but our choice strongly emphasizes the importance of substitution effects. The basic insight is that the introduction of one particular Arrow security modifies equilibrium prices of traded assets in a manner that hedging prices become prohibitive for many countries, as follows. This phenomenon occurs when some countries exhibit strong substitution effects across states, as emphasized by the choice of our utility functions, leading to significantly increase their hedges when the appropriate Arrow security is introduced. When the demand for this security increases and to the point where all hedging prices are dramatically changed, as is typically the case in a general equilibrium framework, one can find a large set of countries for whom the resulting equilibrium prices make the hedging prohibitive to the point where they become strictly worse off after this last introduction. Those strong substitution effects are typically found in countries with a very high infection rate in a particular disease (such as HIV/AIDS in Sub-Saharan countries for instance), and lower with other diseases, and whose needs to treat this dominating disease are of paramount political and medical importance.
With Proposition 3 above, it is straightforward to construct economies where the sequential introduction of Arrow securities will violate the price-preservation property, and in turn leads to welfare-impairment after the arbitrary introduction of Arrow securities. This issue is elementary, and it is avoided here for sake of brevity. Actually, the class of economies that violate this price-preserving property is fairly large, showing that in most cases the sequential introduction of Arrow securities eventually results in welfare deterioration. The identification of asset introduction that avoids this pitfall is discussed next.

4 Sequential introduction of assets and Pareto optimality

We now show that there exists a sequence of assets such that their sequential introduction makes every country better off, even if the economy described in the previous section.

The basic problem when sequentially introducing Arrow securities only is that substitution effects may affect too dramatically equilibrium prices of hedging tools, to the point where some countries become strictly worse off. One natural way of smoothing out those substitution effects is to make sure that any newly introduced asset should pay off in many states, so that some previous hedging needs can still be satisfied with the purchase of this security without reducing too much the holdings
of previous assets, so that the price structure is not overly modify. This idea is at
the heart of the result of this section, where we exhibit a sequence of assets with this
pattern that do not affect welfare.

We first define a notion of welfare improvement associated with the introduction
of a sequence of securities.

**Definition 4** We say that a sequence of linearly independent assets \((a_1, ..., a_S)\) is
welfare improving if the sequential introduction of every asset makes every country
better off in equilibrium after every new introduction, and also leads to a Pareto-
optimal allocations after the last introduction.

We next state the main result of this section. It is a consequence of Theorem 1 in
Hara (7).

**Proposition 5** Assume that every \(U^i\) is quasi-concave. For every economy, there
exists a sequence of securities \((a_1, ..., a_S)\) that is welfare-improving.

The point of assuming quasi-concave utility functions is solely to make sure that equi-
librium allocations are unique (see Hakansson (8) for issues arising without unique-
ness). It should be clear now, from the proof of Proposition 3 and the discussion
above, that the welfare-improving sequence described in Proposition 5 is different
from the sequential introduction of Arrow securities against every disease in general.
Identifying this sequence may prove difficult in practice, since it would require to
know in particular the various substitution effects across diseases for many countries.
However, every security in a welfare-improving sequence can replicated by a linear combination of Arrow securities, when such Arrow securities against every disease are available. Therefore, when it is possible to simultaneously hedge against the upgrading costs linked with every orphan disease under care, then we must reach a Pareto-optimal outcome. This remark is formalized in the following proposition.

**Proposition 6** Consider any economy as above, and assume that all of the Arrow security against every disease are simultaneously introduced. Then the resulting equilibrium allocations are Pareto-optimal.

The above result states that, in practice, one simple way to reach a Pareto-optimal allocation in situations where many orphan diseases are to be treated is to simultaneously introduce all of the Arrow securities to hedge against every related upgrading cost.

**5 conclusion**

In a standard general equilibrium framework, we have analyzed the equilibrium behavior of many countries hedging against the cost of upgrading to innovative treatments when dealing with many orphan diseases. When markets are incomplete, we have first presented a fairly standard economy where the introduction of a non-redundant hedging product against one disease only makes a large number of countries strictly worse off in equilibrium. The basic insight of this result is that, for some countries,
substitutions effects across diseases is so strong that, when presented with a new hedging tool, the prices of every financial asset would be negatively affected as the result of those countries’ trading. Some hedging prices would then become so high that many other countries are prohibited from implementing hedging strategies, making them worse off. This idea is of practical importance, since those substitution effects are likely to be found when one disease such as HIV/AIDS in some areas drains a considerable amount of national resources to be fought.

We have also seen that it is always possible to reduce those substitution effects by sequentially introducing non-redundant assets whose payoff pattern simultaneously encompasses the upgrading cost of many diseases. With such well-chosen assets, no country would be worse off after the introduction of a new financial product.

In practice, the identification of those last assets is difficult because of the large diversity of hedging needs and idiosyncratic substitution effects worldwide. It turns out that the simultaneous introduction of all of the Arrow securities, allowing to hedge against the upgrading cost of every disease under care, avoids welfare losses and leads to Pareto optimal allocations in equilibrium.

A Proof of Proposition 3

Define, for every country $i$, the utility function

$$U^i(c) = u(c_0) + \sum_s \alpha_s \left[ u(c_s) \cdot \gamma^i_s \right],$$

(5)
where $(\gamma_{i_1}^i, \ldots, \gamma_{i_S}^i) \in [0, 1]^S$ are positive constants. Assume the sequential introduction of the sequence $a = (a_1, \ldots, a_S)$ that is not price preserving. Therefore, there exists $m \leq n \leq S$ and $q$ (resp. $q'$) equilibrium prices under $sp(a_1, \ldots, a_m)$ (resp. $sp(a_1, \ldots, a_n)$), and one individual holding $\theta$ such that $q\theta < q'\theta$ (without any loss in generality). We find a country with utility of the form Eq. (5) that is strictly worse off in equilibrium under $sp(a_1, \ldots, a_n)$. This leads to a contradiction with the assumed welfare-preserving property the sequence of assets. We then show that we can extend this property to a set of strictly positive measure of countries with close enough characteristics.

Consider country $i_0$ whose $\gamma_{i_0}^{i_s}$'s are defined as solution to

$$q'_{s} = \gamma_{i_0}^{i_s} \frac{\alpha_s u'(w_{i_0} + \theta a)}{w'(w_{i_0}^0 - q'\theta)} \quad \text{for every } s. \quad (6)$$

We next show that country $i_0$ is strictly worse off under $sp(a_1, \ldots, a_m)$. By construction, the holding $\theta$ is optimal for country $i_0$ under complete markets at equilibrium prices $q'$. Moreover, and also by construction we have that $\theta \in sp(a_1, \ldots, a_m)$, the holding $\theta$ is also optimal subject to standard budget constraints under the asset span $\in sp(a_1, \ldots, a_m)$. Since $q\theta < q'\theta$, it follows that

$$U^{i_0}(w_{i_0}^0 - q\theta, w_{i_0}^0 + q\theta) > U^{i_0}(w_{i_0}^0 - q'\theta, w_{i_0}^0 + q'\theta). \quad (7)$$

We also know that the consumption sequence $w_{i_0}^0 - q\theta, w_{i_0}^0 + q\theta$ belongs to the budget set of country $i_0$ when markets prices are $q$. Therefore, the holding $\theta$ is an equilibrium holding for both price system, and country $i_0$ is strictly worse off under $sp(a_1, \ldots, a_m)$. 

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Since the utility functions depend continuously on the $\gamma$'s, one can easily show that Eq. (7) extends to a set of $\gamma \in [0,1]^S$ of strictly positive measure containing $(\gamma_1^{i_0}, ..., \gamma_S^{i_0})$. This completes the proof.

B Proof of Proposition 5

The result is a consequence of Theorem 1 in Hara (7), which is stated next.

**Theorem 7** For every $j \in \{1, ..., S\}$ and every asset span $G$ of dimension $j$, there exists a price-preserving sequence $(a_1, ..., a_j)$ such that $sp(a_1, ..., a_j) = G$.

With the above result, we now prove Proposition 5. Consider the asset span of dimension $S$, by Theorem 7 there exists a price-preserving sequence $(a_1, ..., a_S)$ that generates this span, with corresponding equilibrium prices $(q_1, ..., q_S)$. Since markets are complete after the introduction of the last asset, the last resulting equilibrium allocations must be Pareto optimal. We next show that, after every introduction of asset along this last sequence, every country is better off.

Fix any $j < S$, and let $(\theta^{i,j})_{i \in [0,1]}$ be the unique (by quasi-concavity) equilibrium asset holdings under $sp(a_1, ..., a_j)$ at equilibrium prices $(q_1, ..., q_j)$. Since $sp(a_1, ..., a_j) \subset sp(a_1, ..., a_{j+1})$, we have that $\theta^{i,j} \in sp(a_1, ..., a_{j+1})$ for every $i \in [0, 1]$. Moreover, by the price-preserving property we have that $\theta^{i,j}$ is part of the budget constraint of country $i$ under $sp(a_1, ..., a_{j+1})$ at equilibrium prices $(q_1, ..., q_{j+1})$. Therefore, the same equilibrium utility level under $sp(a_1, ..., a_j)$ can attained under
\(sp(a_1, ..., a_{j+1})\), and the equilibrium allocation under \(sp(a_1, ..., a_{j+1})\) leaves country \(i\) at least as well off as that under \(sp(a_1, ..., a_{j+1})\) for every \(i\). This implies that the sequence \((a_1, ..., a_S)\) is welfare-improving, and the proof is now complete.
References


