

Endogenous labour supply, endogenous lifetime and economic growth: local and global indeterminacy

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Motivations

- Demographic and macroeconomic outcomes are recognised to be dramatically related to each other in the process of economic growth and development of nations
- The well-known phenomenon of population ageing experienced in several developed countries has called attention of governments to reform labour markets and pension systems to overcome this concern, especially because of the demographic shift due to the steadily reducing number of young workers and the steadily increasing number of old and healthy pensioners

Motivations (continued)

- **First step: Endogenous fertility.** The idea that fertility can be viewed as a result of a rational choice of individuals that compare benefits and costs of having children, has opened the route to causes for reflection that have given birth to several theoretical works for explaining the behaviour of fertility and income (Becker, 1960; Becker and Barro, 1988; Barro and Becker, 1989; Benhabib and Nishimura, 1989; Cigno, 1992; Raut and Srinivasan, 1994; Galor and Weil, 1996). Overlapping generations models provide a natural basis where studying this issue
→ Models of economic growth and development

Motivations (continued)

- **Second step: Endogenous lifetime.** A subsequent step in the literature has been the analysis of the effects of health of individuals and life expectancy on long-term demo-economic outcomes. As pointed out by Weil (2007, p. 1265), in fact, “People in poor countries are, on average, much less healthy than their counterparts in rich countries. How much of the gap in income between rich and poor countries is accounted for by this difference in health?” (see also Cervellati and Sunde, 2011, 2013)
- Public health expenditure, private health expenditure or both. Effects on health status and labour productivity

Motivations (continued)

- **Third step: Endogenous fertility and endogenous lifetime.** Amongst the contributions on this issue are the seminal paper by Blackburn and Cipriani (2002) and the papers by Varvarigos and Zakaria (2013) and Fanti and Gori (2014). They try to explain the Demographic Transition phenomenon (see also Galor and Weil, 1999, 2000)

Related literature

- Chakraborty (2004) JET. **Public health investments** are directly provided by the government. Underinvestment in health may determine poverty traps because high mortality discourages saving
- Chakraborty and Das (2005) J Econ Growth. **Private health investments** affect labour productivity, education and bequests. OLG economy with rich and poor agents. Initial differences between economic agents of the two different types may perpetuate across generations when annuities markets are imperfect

Related literature (continued)

- Bhattacharya and Qiao (2007) JEDC. **Public and private health investments.** The paper shows that the existence of public health expenditures in an economy where individuals privately invest in health, may cause endogenous fluctuations in GDP and life expectancy. There is a unique steady state and labour supply is exogenous
- Our paper extends BQ (2007) with endogenous labour supply. **It shows the existence of multiple (determinate or indeterminate) fixed points, endogenous fluctuations and local and global indeterminacy**

The model

- General equilibrium OLG closed economy comprised of a continuum of (two-period lived) rational and identical individuals of measure one per generation. Each generation overlaps for one period with the previous generation and then overlaps for one period with one
- An individual survives at the onset of old age with certainty, and he is alive only for a fraction $\theta \in (0, 1]$ of the second period of his lifetime. The probability of surviving when old is endogenous and determined by the individual state of health
- Time is discrete and indexed by $t=0,1,2,\dots$

The model (continued)

- Economic agents: individuals, firms, government
- Longevity, θ_t , and longevity production function, $\theta(x_t, \eta_t) \rightarrow \theta_t = \theta(x_t, \eta_t)$

x_t private input

η_t public input

- Different from BQ (2007), we assume that the private input and the public input are complements for every x_t

that is, the public expenditure on health always increases the marginal productivity of the private one: $\theta''_{x_t, \eta_t}(x_t, \eta_t) > 0$

The model (continued)

- Cobb-Douglas longevity p.f.

$$\theta(x_t, \eta_t) := \frac{x_t^\rho \eta_t^{1-\rho}}{Z}$$

where $\rho \in (0, 1)$ is the (constant and independent from the public input) elasticity of longevity with respect to private investments in health and Z is a positive parameter that will appropriately be fixed to get well defined economic dynamics, that is

$$\theta(x_t, \eta_t) \in (0, 1]$$

- The relationship between public input in the longevity p.f. and public health expenditure is the following: $\eta(p_t) := \underline{\eta} + p_t^\delta$

The model (continued)

- **Government b.c.:** $p_t = \tau w_t l_t$ $0 < \tau < 1$
- **The relationship $\eta(p_t)$ satisfies the following properties:**

if $\tau = 0$, $\eta(0) = \underline{\eta} > 0$

if $\tau > 0$, $\eta(p_t) > \underline{\eta}$

$\delta > 0$ is a parameter that weights the intensity of the effects of public health expenditure as a means of higher longevity

if $\delta \leq 1$, η is a concave function

if $\delta > 1$, η is a convex function

The model (continued)

- Preferences: consumption (C), leisure (2-l), private health expenditure (x) \rightarrow Expected utility

$$U(2 - l_t, x_t, C_{t+1}) = \frac{(2 - l_t)^{1-\gamma}}{1 - \gamma} + \theta(x_t, \eta_t) \frac{(C_{t+1}/B)^{1-\mu}}{1 - \mu}$$

where $\gamma > 0$ ($\gamma \neq 1$) is a measure of the constant elasticity of utility with respect to leisure time and $\mu \in (0, 1)$, which contributes to determine the elasticity of utility with respect to consumption is fixed between zero and one to avoid paradoxical effects of longevity on utility (Hall and Jones, 2007)

The model (continued)

- The utility function is concave if and only if $\rho < \mu$
- Lifetime b.c.

$$C_{t+1} = R_{t+1}^e [w_t l_t (1 - \tau) - x_t]$$

where R_{t+1}^e is the expected interest factor, w_t is the wage rate

- Consumer's FOC's imply:

$$\frac{x_t}{l_t} = \frac{\rho w_t (1 - \tau)}{\rho + 1 - \mu}$$

The model (continued)

- **Result.** When individuals experience an increase in labour income (wage), they increase the proportion of private health spending with respect to labour, while also increasing the proportion of private health spending with respect to leisure. This means that private health investments play an important role in countries where wages are sufficiently high, that is when the wage increases agents increase private health spending more than leisure in their optimal consumption bundles

The model (continued)

- **Firms. Production function and marginal productivities**

P.f. $Y_t = AF(K_t, L_t) = AK_t^\alpha L_t^{1-\alpha}$

M.p. labour $w_t = (1 - \alpha)AK_t^\alpha l^{-\alpha}$

M.p. capital $R_t = \alpha AK_t^{\alpha-1} l^{1-\alpha}$

$A > 0$ and $0 < \alpha < 1$

are a production scale parameter and the capital share

The model (continued)

- **Equilibrium.** In the case agents are perfect foresight the forward equilibrium dynamics in the capital stock and labour supply are given by the following two-dimensional map:

$$K' = \frac{AK^\alpha l^{1-\alpha}(1-\tau)(1-\mu)(1-\alpha)}{\rho+1-\mu}$$

$$l' = \left(\frac{\left(\frac{(\rho+1-\mu)Bl^{\alpha-1}K^{-\alpha}}{\alpha(1-\mu)(1-\tau)(1-\alpha)A^2} \right)^{1-\mu} \left(\frac{l(1-\mu)Z}{(\rho+1-\mu)\eta^{1-\rho}} \right) (K')^{(1-\alpha)(1-\mu)}}{(2-l)\gamma \left(\frac{\rho AK^\alpha l^{1-\alpha}(1-\tau)(1-\alpha)}{\rho+1-\mu} \right)^\rho} \right)^{\frac{1}{(1-\alpha)(1-\mu)}}$$

Results

By assuming some restrictions on parameters such that the fixed point $(1,1)$ exists, we get the following proposition

Proposition. (a) The map generically admits an odd number of fixed points (at most three). (b) In particular, if

$$[(1 - \tau)^\delta (1 - \mu)^\delta + \tau^\delta (\rho + 1 - \mu)^\delta] (-\rho + \mu + \gamma) + \tau^\delta (\rho + 1 - \mu)^\delta (-1 + \rho) \delta < 0$$

there exist three fixed points with $(1,1)$ being the intermediate one. *Ceteris paribus*, the latter inequality is verified when δ is sufficiently high

Results (continued)

The proposition characterises the number of fixed points of the map (and then of long-term behaviours of the economy) depending of the relative interactions of the main parameters of the problem (that includes policy variables).

In particular, if the relative weight of the effects of public health expenditure as a means of higher longevity is high, three fixed points can exist. This opens several questions with regard to how an economy may behave in the long term depending on initial conditions, and stability properties of a fixed point at both local and global perspectives.

Results (continued)

Difference between local and global indeterminacy

Local indeterminacy (LI). A fixed point is locally indeterminate if for every arbitrarily small neighbourhood of it and for a given value of the state variable close enough to its coordinate values at the stationary state, there exists a continuum of values of the control variable (the labour supply) for which equilibrium trajectories converge towards the fixed point.

Global indeterminacy (GI). A system is globally indeterminate when there exist values of the state variable such that different choices on the control variable lead to different invariant sets. In this case, the initial condition of the state variable is not sufficient to define the long-term dynamics of the system.

Global analysis

Lemma. The map is invertible on set W_3

where

$$W_3 = \{(K, l) \in R^2 : M^n(K, l) \in W_2, \forall n > 0\}$$

and $M^n(K, l)$ is the n th iterate of the map applied to point (K, l)

- **Case 1.** Existence of quasiperiodic or chaotic long-term dynamics for the system.
- **Case 2.** Coexistence of attractors or other feasible trajectories for the system (saddle path stability).
- **Case 3.** GI (expectations driven).

Global analysis (continued: **Case 1**)

- **Case 1.** Existence of quasiperiodic or chaotic long-term dynamics for the system. It is possible to find evidence of the existence of flip bifurcations or NS bifurcations around $(1,1)$ that create an attracting two-period cycle (which represents the first step towards the birth of a chaotic attractor, (Figures 2.a and 2.b) and an attracting closed invariant curve (Figure 3.a), respectively
- The birth of a closed invariant curve through a NS bifurcation can occur around a non-normalised fixed point (Figure 3.b)

Global analysis (continued: **Case 1**)

Parameter set: $\alpha = 0.12$, $\delta = 4$, $\rho = 0.15$, $\mu = 0.3$ and $\tau = 0.6$.

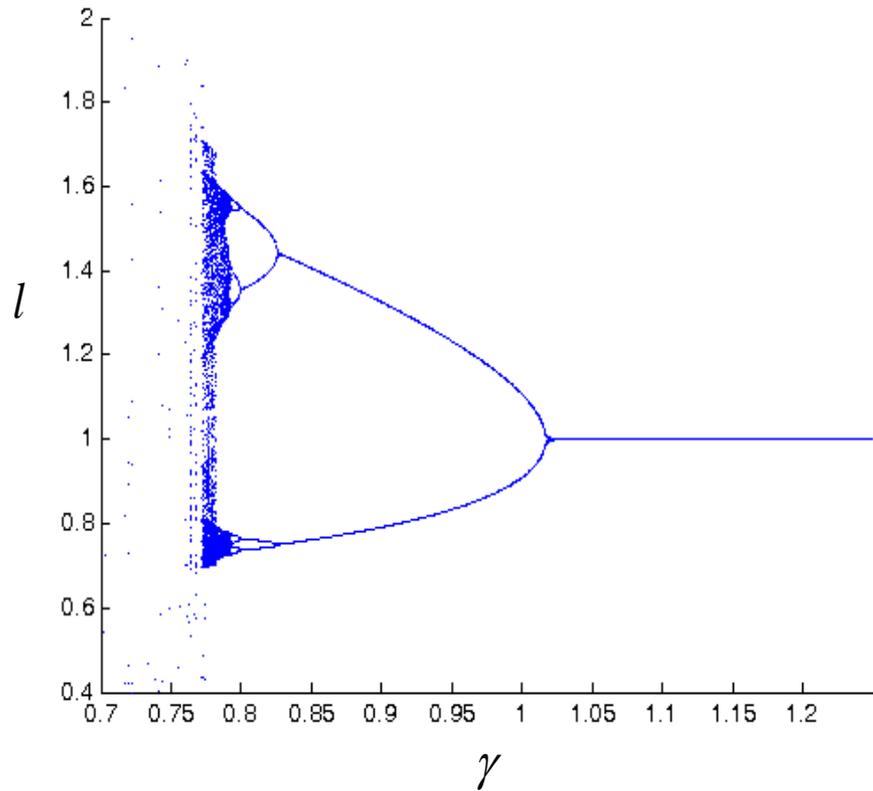


Figure 2.a
 $\gamma \in [0.7, 1.25]$

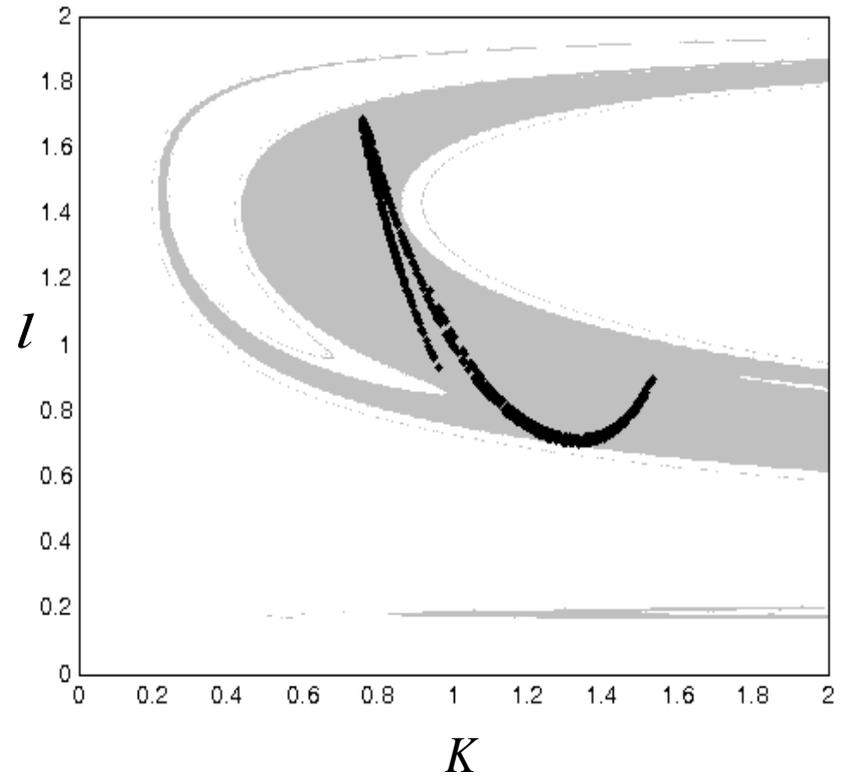


Figure 2.b
 $\gamma = 0.78$

Global analysis (continued: **Case 1**)

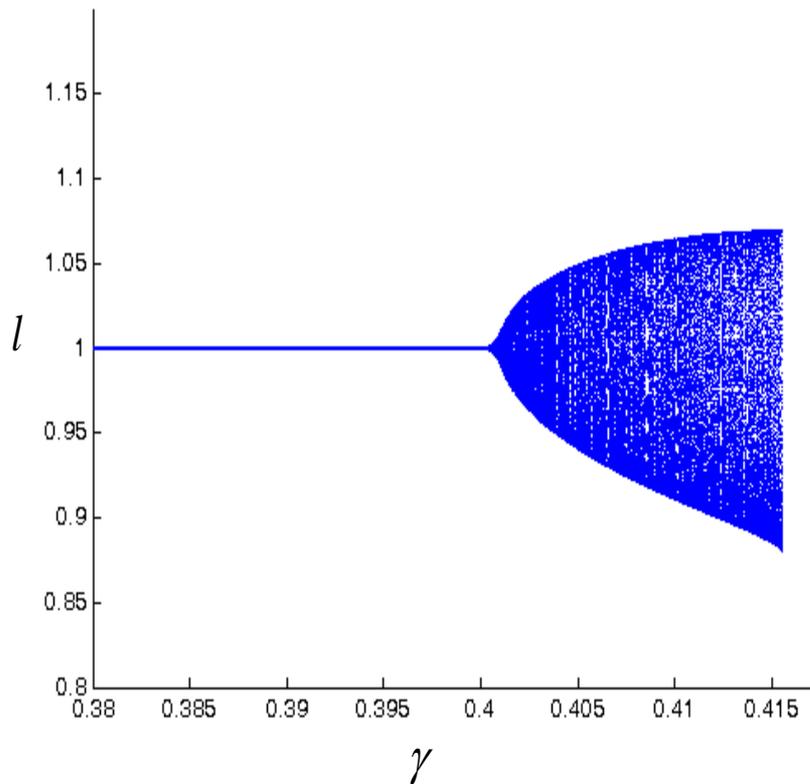


Figure 3.a

Parameter set:

$$\alpha = 0.33, \delta = 7, \rho = 0.27$$

$$\mu = 0.31 \text{ and } \tau = 0.35.$$

$$\gamma \in [0.38, 0.416]$$

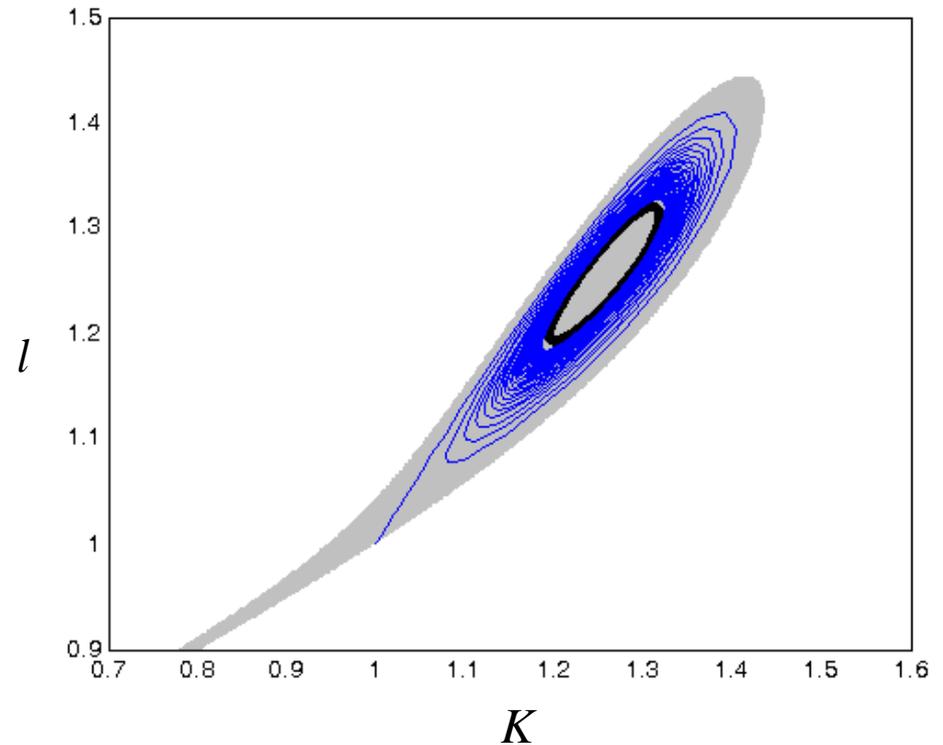


Figure 3.b

Parameter set:

$$\alpha = 0.33, \delta = 4.2, \rho = 0.26$$

$$\mu = 0.27, \tau = 0.29 \text{ and } \gamma = 0.285$$

Global analysis (continued: Case 2)

- **Case 2.** Depending on the initial condition, it is possible to converge towards different fixed points (path dependence). Figure 4.a shows coexistence of two attractors: the normalised fixed point and a three-period cycle born through a saddle node bifurcation. Since labour supply is a control variable, agents may coordinate either on stable manifolds that define the basins of attraction of the saddles or on the saddle that lies in the south-west region of the phase plane. It is important to note that saddle node bifurcations can also be observed together with a sequence of flip bifurcations (around $(1,1)$). In this case attractors of high periodicity coexist (Figure 4.b).

Global analysis (continued: **Case 2**)

Parameter set:

$\alpha = 0.127$, $\delta = 4$, $\rho = 0.25$, $\mu = 0.35$ and $\tau = 0.55$

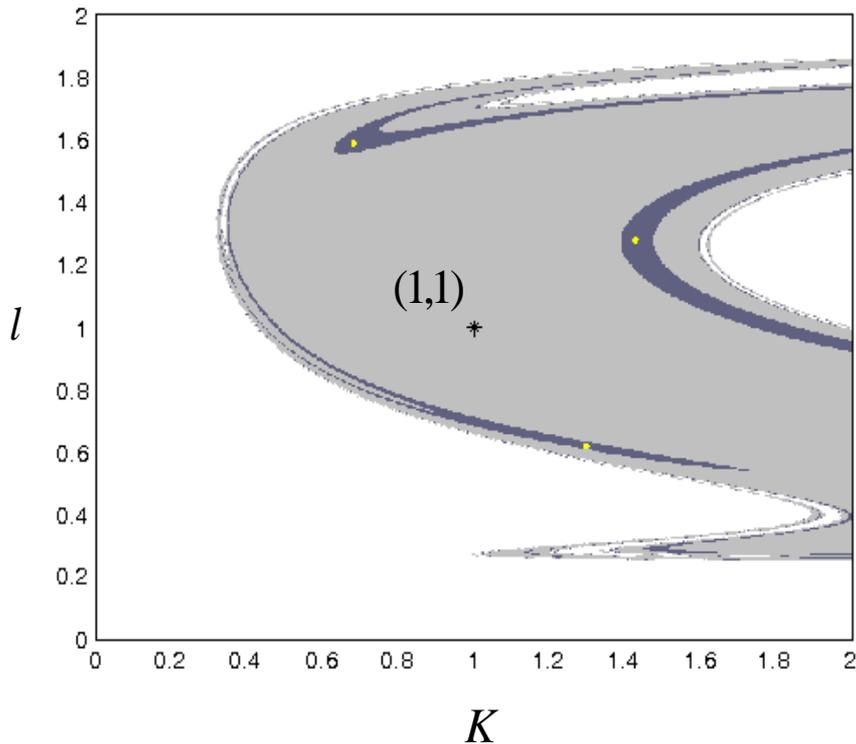


Figure 4.a

$\gamma = 0.9$.

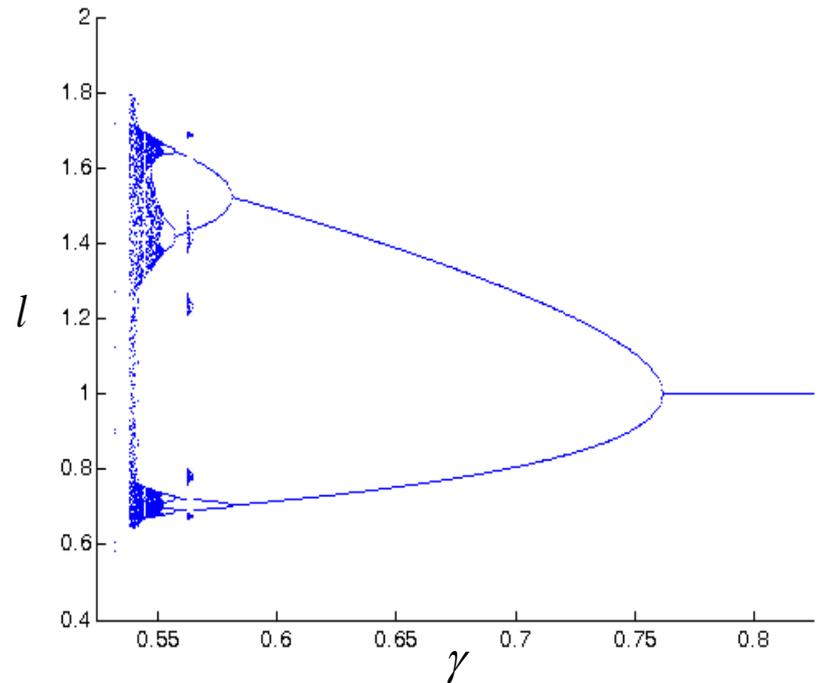


Figure 4.b

$\gamma \in [0.5, 0.85]$

Global analysis (continued: Case 3)

- **Case 3.** GI arises when there exist values of the state variable for which different choices on the control variable lead to different invariant sets. Figure 5 shows that GI can be observed also when the longevity production function is concave in all its arguments, that is when the external effect created by public health investments is sufficiently low ($\delta < 1$)

Global analysis (continued: Case 3)

Parameter set:

$$\alpha = 0.3, \delta = 0.8, \rho = 0.49, \mu = 0.501, \tau = 0.4 \text{ and } \gamma = 0.4$$

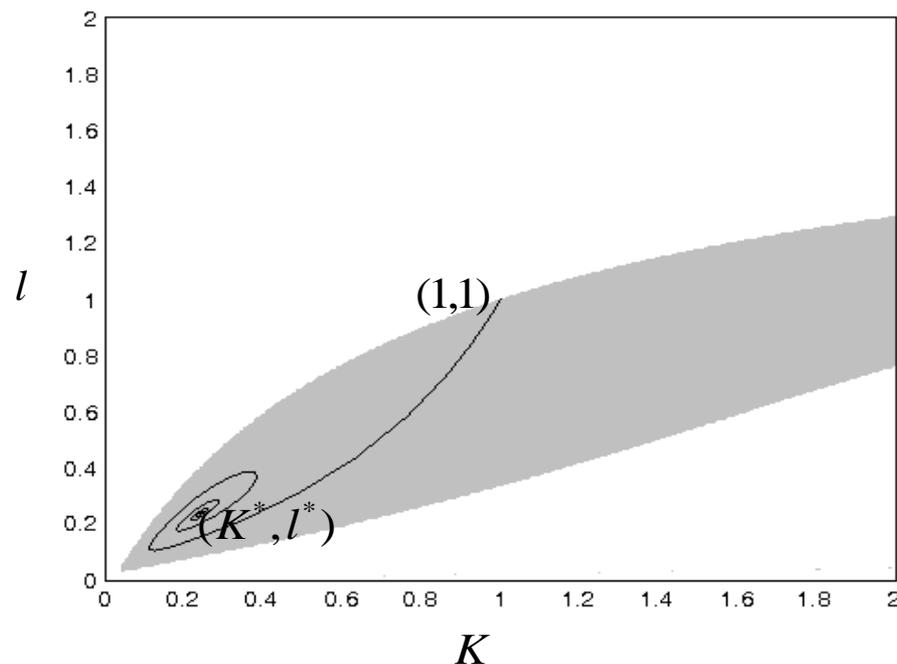


Figure 5

Global indeterminacy with a concave utility function and concave Cobb-Douglas technologies. A heteroclinic connection exists between $(1, 1)$ and (K^*, l^*)

Conclusions

- General equilibrium OLG economy with endogenous longevity and endogenous labour supply
- Multiple fixed points (different development paths) and local and global indeterminacy (GI) can exist
- Due to the externality of health technology on the macro-economy, GI becomes a plausible result with respect to several configurations of parameters

Conclusions (continued)

- GI arises when utility function, production function and production of longevity are concave functions
- GI may occur when all fixed points of the system are determinate (saddles and sources). This implies that the long-term behaviour of the economy can be strongly related to decisions regarding the magnitude of the public health policy.
- Welfare?... Research agenda

Thank you!