DSGE vs CGE Models: Modelling Sustainable Development in a Computable General Equilibrium Context

Federico Perali and Pasquale Lucio Scandizzo

Abstract

This paper offers an overview of a family of general equilibrium models, which extends the results already obtained in several earlier and recent contributions of the authors, with the main objective to evaluate the effects of policy interventions by taking into account of the micro-macro structural characteristics of the economy. The models described are based on a theoretical approach that considers demand and supply systems as the main drivers of the economy both directly and indirectly, through the different "upstream" and "downstream" links that characterize the production structure and the multiplicative effects that occur through the movement of prices and consumption. The methodology proposed combines theoretically micro-based macroeconomic modelling with bottom up aggregation to simulate and test response to unanticipated policy interventions and other exogenous shocks.

Even though so far they have been used separately to address problems of different scale and purpose, Stochastic Dynamic General Equilibrium (DSGE) and Computable General Equilibrium Models (CGEs), provided that they are appropriately integrated through multilevel procedures, appear a promising methodology to explore the implications of sustainable development for several reasons. First, SDGEs are a new variety of macroeconomic models reaching for micro-foundations on the basis of standard economic theory assumptions, such as utility and profit maximization of decentralized agents. CGEs, on the other hand, are constructed by complementing national account data and constraints, in a disaggregated form (extended SAMs) with behavioral and technical relations also consistent with the same basic economic theory of rational behavior. Both models, therefore, at different levels of aggregation, appear to be useful complements to address the issue of extending the classical and neoclassical notions of capital to the stocks and services provided by the ecosystem. Second, as counterfactual machines, SDGEs and CGEs can simulate alternative states of the world to measure the interdependent causes and effects involving economic and environmental variables. Third, while SDGEs may be effective tools to explore capital accumulation over time, they are generally too aggregate to address problems of natural resources and natural services. CGEs, on the other hand, once backed by a solid representation of the dynamics of capital formation, can easily explore the impact of alternative policies of physical and natural capital accumulation and destruction. Fourth, both models can thus provide specific indications on trade-offs between the economy and the ecology through direct substitution and transformation effects. They can also both be used to yield indicators of present and future resource scarcity through the computation of shadow prices for different forms of capital (physical, human and natural), as well as factors of production and ecosystem services.

1. Introduction

General equilibrium (GE) modelling as a methodology to analyze broad policy issues, has been around for many years, at least since the pioneering efforts of Wassily Leontief, Hollis Chenery and Leif Johansen in the 60's. The revival which we are witnessing today, however, is based on several new facts and advancements of both theory and practice. First, the extensive experimentation with computable general equilibrium (CGE) models in the past 50 years has been instrumental in generating a greater degree of understanding of both the potential and the limitations of both GE ideas and CGE models. Second, the advancement of computational techniques and the power of modern computers have made possible to construct more transparent models, more easily penetrable by numerical techniques and, as a consequence, much less "black boxes" that their earlier progenitors. Third, the combination of CGEs and Social Accounting Matrices (SAMs) has become a standard that allows to treat the GE model as an extension, however complex, of national accounting. Fourth, the greater availability of microdata has widened the horizon of the SAM-CGE possible coverage, extending their reach to seemingly elusive phenomena, such as income distribution and employment, trade and migration flows, factor markets and their spatial mobility, the environment and climate change. For example, labor and workforce accounts measuring labor force in terms of hours, occupations, full versus part time, type of household, gender, skills, wages within the frame of Industry-Occupation matrices are increasingly available for a high level of sectoral detail and for the smallest administrative territorial units. The evaluation of the impact of policy programs or environmental shocks on well-beings is now enriched by satellite accounts, statistically consistent with national accounts, that collect and order information about human, social, cultural and political dimensions of economic and social life. Common examples are satellite accounts for the environment, or tourism/migration/commuting, unpaid household work or related to different forms of capital besides the traditional financial and physical capital such as human capital, natural capital in the form of amenity indices, social capital, cultural and political capital. This information adds value to a modern analysis of an economy not simply because it allows representing an "augmented" reality where the effective productivity, for example, of a unit of physical capital accounts for the fact that it is invested within a community that is also endowed with a high or low level of human and social capital. Finally, new techniques, based on sophisticated statistical and mathematical algorithms, have become available to estimate and calibrate model parameters, by incorporating and integrating information from macro and micro data, using time series, surveys as well other model estimates. This advanced computing capacity makes it easier to handle highly detailed information sets and large-scale models thus opening new prospects for inferential and causal analysis within a general equilibrium context.

As we learn more about their potential and hidden messages, CGEs have become the tool of selection for economists and policy makers to perform evaluative simulations within a context of coherent and transparent hypotheses on the technology, the behavior of the economic agents and the status and the evolution of the external environment and the representative exogenous variables. They have become the only point of encounter of macroeconomic policies with project evaluation, where they promise to perform a critical function to connect two frameworks that typically don't mingle and often risk contradicting each other.

In this study, we look at the basic design of modern CGEs and some of their more interesting variants, with a special focus on the emerging connection between the policy and the project level. We present and discuss several different attempts to operationalize the CGE context to analyze the connection between policies and projects and, in some cases, the corresponding macro-micro nexus. For this, we develop model structures that correspond to a common framework and aim to both clarify and simplify the intricacies of the CGE procedures.

2. Project evaluation as a new frontier for modelling

The individual assessment of investment projects, as developed by economic theory, is based on the consideration of quantitative traits related to financial and economic "profitability" of the project. These are measured by the difference between the so-called “benefits” and “costs” of the project with and without the project under consideration. The costs are generally concentrated in the investment or construction phase of the project, while the benefits are almost exclusively part of the subsequent operational phase. Since the individual assessment is based on the characteristics of the project, it regards as "given" the external conditions of the overall economic system, which are synthetically represented by so-called shadow prices used. For this reason, the costs and benefits that depend on the interaction between the project and its economic environment are typically neglected in whole or in part in the cost-benefit analysis, particularly regarding the effects of the stimulation of economic activity prevailing during the construction phase. It also neglects – as each project is evaluated independently of the other – any interdependencies with other projects, thus creating the risk of making a mistake that will tend to be greater, the greater will be the size and degree of complexity of the group of projects selected for funding. Finally, none at the so-called "external effects", i.e. the provision of public goods and environmental impact of the project are considered. These effects are particularly relevant in the case of public projects, which themselves, ultimately, are a vehicle for improving the physical and economic environment of the country.

As we said, the quantitative measurement of the costs and benefits of investment projects is based on the dichotomy: construction – operational phase. This dichotomy is part of an approach that does not consider the multiplicative effects of investment on factor employment. In fact, the benefits of traditional investment analysis arise especially during operations through increasing production, driven in turn by an increase of fixed assets. The costs are concentrated in the construction phase, because it is at this stage that fixed assets are built by committing productive resources in the hope of future benefits. The very concept of productive investment is therefore defined by the dichotomy between anticipation of costs and of realization of benefits according to a time profile that constitutes one of the fundamental determinants of the profitability of the project.

The ability to calculate the values of equilibrium prices, quantities, household incomes and other variables of interest in complex multi-sectoral models is on the other hand a recent achievement of applied economics. It is based on the specification of mathematical structures which reflect the rigorous definitions of economic equilibrium, developed by Kenneth Arrow, Gerard Debreu, Michio Morishima and others, and other simplifications and approximations necessary to allow the calculation of the equilibrium values.

In a series of important research attempts, in large part conducted at the World Bank, several generations of computable general equilibrium models (CGE) since the late 70’s were developed and gradually became important and useful tools for policy analysis. In these models, social accounting matrices (SAM) became the core of the representation of general equilibrium as a circular flow of production, consumption and incomes, with prices in all markets as the equilibrating variables. Solving algorithm started with fixed point (Scarf and Hansen, 1973) and mathematical programming procedures (Norton and Scandizzo, 1981, Walbroeck and Ginsburg, 1981) and gradually developed into nonlinear equation systems and local or global search solution methods (Devarajan et al.,1997). At present, while the macro-econometric models prevailing in the 1970s have all but disappeared from the economic practice, CGEs are increasingly used around the world, both in their static and dynamic versions, as tools to analyze economic policy options.

3. Some general equilibrium concepts

The main concepts of the category that goes by the name of general economic equilibrium can be found only with considerable effort in the economic literature. This is because the notion of equilibrium depends on the historical context in which it is used, and the model of the economy to which it refers.

Classical economists such as Adam Smith, David Ricardo, J.S. Mill, and Karl Marx believed that the value was determined by the cost of production and the absence of profits. Both conditions can be regarded as characteristics of an equilibrium condition: the equality of prices to the cost of production, in fact, ensures that individual producers do not wish to change their plans, while the absence of profits implies the absence of competitive pressure from new companies trying to enter in the markets. This equilibrium can be described as "general" if it extends to all markets.

Although at first sight satisfactory, especially for its simplicity, this classical view of equilibrium reveals two weaknesses. First, equality between prices and unit costs of production, if acceptable as a condition of balance for goods and services produced, says nothing about the value of primary production factors and especially labour. The state of equilibrium is not then "general" because it does not extend to factor markets. Secondly, because consumers are not involved either in the equality of prices and costs, or in the absence of profits, they also appear to be excluded from the equilibrium described that turns out, therefore, to be wholly partial.

A general equilibrium model in the modern sense of the word must have some essential requirements, both in terms of the equilibrium condition than that of “being general”. Equilibrium should, in fact, result from supply and demand equality, but it must also assume that consumers and producers are, individually, where they want to be, that is, on their individual curves of supply and demand. This means in practice that every solution must depend parametrically on taste, technology and the initial distribution of goods. Secondly, the equilibrium should be "general". This implies that no price (except the numeraire) can be considered a purely exogenous variable. If this were the case, in fact, the corresponding market could not be in balance except by chance and the description of the model would be incomplete. In addition, equilibrium between demand and supply should cover not only the goods produced, but also the primary factors of production such as land, capital, labour and other resources that characterize the initial endowment.

Both in the classical description, and in the newer ones, general equilibrium is finally typically characterized as a set of conditions of real balance of a closed economy. This implies that the demand functions are homogeneous of degree zero in prices (no money illusion) and that therefore it is possible to apply an appropriate normalization rule, such as the choice of a simple or composite commodity (a numeraire) whose price is conventionally equal to unity.

Given these characteristics, the concept of equilibrium is also associated with efficiency and to the question of its existence, which was taken up by Arrow and Debreu (1954), as well as McKenzie's (1959) in a series of celebrated contributions. Their work, although focused on a rather narrow sub-problem, essentially proved that under certain conditions, given a set of demand and supply equations of individual agents, aggregate demand and supply could be equated by a set of non- negative prices. This was a non-trivial result, that was contingent on a series of rather restrictive assumptions, but was obtained through a mathematical powerful and unifying instrument (the fixed point theorem) that was in itself shining for originality and simplicity. The result had two drawbacks, however. First, it did not cover nor it proved to be a feasible base for finding circumstances under which the equilibrium was unique. Second, as proved in a series of important and somewhat astounding later contributions by Sonnenschein (1972, 1973), Debreu (1974) himself, and Mantel (1974), the base of the existence proof was an aggregate excess demand function, which, although resulting from the aggregation of individual demand and supply, was not bound by the limitations deriving from the postulates of rationality. In what has been called "the everything goes" conclusion, in fact, it was proved that such a function, even though the result of individual rational behaviour, is not characterized by any special mathematical property. Thus, the existence of general equilibrium seemed to be quite independent of its "micro-foundations", as a consequence of an essential weakness of the microeconomic "rationality" assumptions, which were proved to be not sufficiently discriminating to impose anything resembling rationality on aggregate behaviour.

A further point arises from the consideration of the causal chains contained, or implied by the process of reaching the equilibrium, or re-establishing it after a perturbation (the comparative static problem). From the point of view of the underlying causal chain, a general equilibrium model, from the original Walrasian formulations to the latest computable forms, does not in itself indicate any direction of causality, as relations between its variables are fully simultaneous. If full employment is considered to be the crucial element of discrimination between the classical and Keynesian approach, it is clear that this condition does not characterize necessarily a solution that meets the conditions of general equilibrium. If it is true, indeed, that such a solution cannot contain involuntary unemployment, given that the supply of labour and other resources depend entirely on household preferences and prices, it is also true that the level of employment in the solution found is not necessarily the maximum possible, given the fact that there may be multiple equilibria. Even when uniqueness of equilibrium is guaranteed by ad hoc conditions, a higher employment level could be achieved by changing the attitudes of consumers (and among them we can mention the expectations) technology or deployment of resources. If the change in autonomous expenditure that sets in motion the Keynesian causal chain is interpreted, as it seems legitimate to do, as an exogenous change in preferences, technologies or distribution, the general equilibrium model is therefore fully compatible with income stabilizing fiscal policies.

More generally, the level of employment of a solution of a specific model depends both on the characteristics of the solution (if it is not unique), and the characteristics of the model. The latter consist of the structure (number of equations, functional forms, variables included and excluded etc.) as well as parameters, that is, variables whose value depends on the model but is set exogenously. A causal chain of Keynesian type, then, is the sequence of changes caused by an exogenous variation of one or more parameters. For example, if the level of domestic demand depends on the percentage of wealth held by the richest 5% of the population and that percentage changes after the imposition of a 1% tax, the consequent change in demand will result in a new parametric balance that may result in less than full employment. Stabilizing fiscal policy will then consist in determining the value of another parameter: an exogenous variable in the model, but subject to political control, such as government spending, to reconstruct a situation which is as close as possible to that which preceded the distributive variation.

4. The "closures"

The incompleteness of the classical general equilibrium model has been overcome in modern models since the days of Walras, through the introduction of both labor and leisure in the household behavioral function and the inclusion of all factors, including labor, in the original allocation of resources. However, several problems remain for a realistic representation of the economic system. First, the model does not consider the formation of savings and investment. Secondly, it only considers real quantities and prices and therefore does not include the supply and demand of money and other financial resources. Finally, it describes a closed economy and thus ignores the possibilities for international trade as well as domestic and foreign currency and relations between domestic and international prices.

These three areas are all theoretical “holes” of general economic equilibrium, in the sense that their "closing" forces us to deal with the problem of reconciling the micro with the macro-economy, making choices that may be justified by personal beliefs and ideological reasons, as well as by empirical evidence. In some sense, this is equivalent to choose between Keynesian and monetarist theories in one of their many meanings.

Consider the simple case of the introduction of money. If we accept the classical scheme, we can also introduce money through a quantitative equation. This equation says that the monetary value of income is proportional to the amount of money exogenously supplied. Substituting this equation into a normalizing equation that assigns to an arbitrary good a unit price, we get a system with two characteristics. (a) There is a commodity called money, the price of which is fixed to unity, and whose application is due only to the fact that it is necessary to carry out transactions. (b) The general price level (understood as the arithmetic average of the weighted prices with quantity quotas) is proportional to the amount of money supplied. The system resulting from the introduction of money through the quantitative equation is then "dichotomous" in the sense that its real part continues to determine the general price level.

Once money is introduced, however, the issue of savings and investment arises: a funding activity becomes possible based on the availability of some traders to surrender their temporary surpluses of money to other operators that are characterized by temporary deficits. In the neoclassical story, aggregate surplus represents excess savings, which are matched to excess investment to achieve equilibrium. The Bank money, which is only a particular type of numeraire, promises in addition to pay investors who save. The "price" of these promises is greater the smaller the interest owed by debtors to creditors. The interest rate thus becomes the variable balancing savings and investment. The introduction of the savings-investment balance at the aggregate level allows to give more substance to the activities of the banking sector, which is not limited to distribute a "currency, but acts as an intermediary between families and businesses.

Finally, we consider the introduction of the external sector. As for the other two "closures" model, the inclusion of international trade in the aggregate is not difficult, because it is, in fact, the simple addition of a macro-enterprise: the "external sector", that transforms exports into imports (and vice versa) with a given technology (transactions at world prices).

Considering for simplicity only imports that are finished products, household budgets will be divided between spending on the domestic market and in foreign markets, while the balance sheets of firms will in turn be fed either by domestic sales, and/or by those in the foreign market. Equilibrium conditions will not be changed, except for the addition of condition of balance in value between imports and exports (balance-of-payments constraint).

The extension of this model through Keynesian assumptions does not pose any special problem. The liquidity preference can be incorporated either at the aggregate level or directly in the demand functions that describe the behaviour of households. The dependence of the demand for money on the interest rate, however, forces us to introduce at the same time, both an investment demand schedule (through an appropriate function of corporate behaviour) as a function of the interest rate, and a saving function. The latter, barring multi temporal complications, can be introduced directly into the household budget constraints by assuming, in the Keynesian tradition, that saving is a function of income, but not of the interest rate. The extension to international trade can now be performed in a not dissimilar way from the one already described above for the neoclassical model.

The model obtained differs from previous one in that it reflects the basic differences between the neoclassical and Keynesian macro-economic structure, since the interest rate has to perform the task to balance money demand and supply in the Keynesian model. These differences, however, are not such as to affect the simultaneity characterizing both models, which are both the combination of hypotheses of aggregate type (e.g. quantitative equation, investment function) on a disaggregated, Walrasian type structure. Much more important are the differences relating to the causal chains that can be associated to the exogenous variables or parameter changes. It is these changes that have profound consequences on the use of two models for the valuation of investments.

5. The New Frontier of the CGE Models

5.1 **General Equilibrium as a model foundation**

What is "general equilibrium"? One is tempted to reply that general equilibrium describes a condition where all markets are in equilibrium, both in the sense that all markets are cleared (demand equals supply) and all agents fulfil their plans. However, while this definition certainly appears simple and direct, it is neither complete, nor satisfactory. It is intrinsically incomplete, since general equilibrium, unlike partial equilibrium, in addition to material balances and subjective fulfilment, requires that the distribution of wealth is consistent with resource allocation. It is not satisfactory, because market clearance depends on a flow condition, i.e. it can be satisfied only for one particular interval of time. If we take the year as the reference time frame, for example, there may be several markets that require more or less than a year to be cleared. Inventories and other capital goods bridge the gap, both as flows and accumulating stocks, between the production and consumption timelines and play a special role in both static and dynamic CGEs. The discriminant between general and partial equilibrium, however, does not depend only on the number of markets, prices and quantities that simultaneously interact, but also by a radically different perspective between the two different equilibrium mechanisms respectively studied by Marshall and Walras, and their followers. In addition to confine itself to a limited number of markets, Marshall-style economics focused in fact on short-run quantity adjustments to external shocks, for given price expectations, while Walrasian economics aimed to reproduce all mechanisms of maintenance and adjustment to equilibrium for quantities and prices, typically in a long run perspective (Costas-Aziaradis, p. 1539, 2018). Because of these differences, more than claiming a different interpretation of the same phenomena, the two approaches can be considered complementary, at least to the extent that adopting one of them does not exclude pursuing the other.

In order to address the time matching and the stock-flow problem, general equilibrium modelling must address four different circles of causation: (i) between demand and supply of goods and services on one hand, and prices and incomes on the other; (ii) between the formation of incomes from demand and supply of factors of production and their prices, (iii) between the initial resource endowment and the redistribution caused by productive choices and institutional transfers, (iv) between investment and savings and the rates of return to all forms of capital. The precise way in which these four circles interact is still not clear, especially for what concerns the link between flow and stock variables although R. Stone and A. Brown (1962) formalized the main flow balance relations in a form essentially consistent with the Keynesian model in the so called Social Accounting Matrix (SAM). As Taylor (2010) persuasively argues, computable general equilibrium models (CGE), mainly developed because of research efforts at the World Bank in the ‘70s, are a spinoff of the application of Input-Output matrices and SAMs, more than any attempt to compute Walrasian equilibria. Even in their advanced, present day form, they tend to reflect a basic indeterminacy of capital accounting, deriving both from lack of consensus on capital theories and on best practices of accounting. They also evoke an intrinsic dualism between a core set of social accounts and a complementary, highly variable set of behavioural and technical equations.

Figure 1 summarizes the fundamental variables of general equilibrium, as conceived in most CGE models. The figure also shows the causal links, according to the two extreme versions of the classical and Keynesian theory. Following the arrows connected by solid lines, and starting from the top, we can follow the classical chain, in which the productive capacity determines the level of employment (which is the one that maximizes the profits of the entrepreneur). This in turn determines the level of production, and prices of these factors determine in turn the level of prices of goods, income and consequently the level of consumption. In the Keynesian version, on the other hand, it is the level of consumption that determines monetary income and then, through employment, the level of production, by establishing a series of effects on product and factor prices with feedbacks on incomes and consumption. The Keynesian causal sequence differs from the neoclassical one, in both the origin and the direction of change, but eventually recovers parts of the neoclassical relationships through income and price feedbacks on consumption.

Although the representation of equilibrium just described contains all the essential ingredients of a “general” equilibrium, it is not sufficiently analytic, because it is limited to considering only the "final" variables. This form of the model can be called "reduced", because it is constituted by a set of relationships among key variables, (i.e. variables that cannot be suppressed without depriving the model from its “generality”) and cannot be reduced to a form with fewer variables.

In order to formulate a structural model, one has to explicitly introduce some relations and variables that carry the assumptions on the causal chain moving the model itself from disequilibrium to equilibrium. The easiest way to introduce these structural elements is shown in Figure 2 where supply and demand for factors and products are included as four additional structural variables. A variable "changes in the stock of capital" includes new capital accumulated through investment, including inventories, and allows supply and demand flows to differ from production flows.

The supply-demand equilibrium is achieved through four causal relations according to which, in particular: (a) factor supply and demand are determined by households, "given" price and income levels, (b) the supply of goods and the demand factors is determined by the firms, given price levels. Assume further that (c) the price level of goods and services is determined by the firms to cover the costs of production (or to maximize profits).

Figure 1: The Basic Economic Model

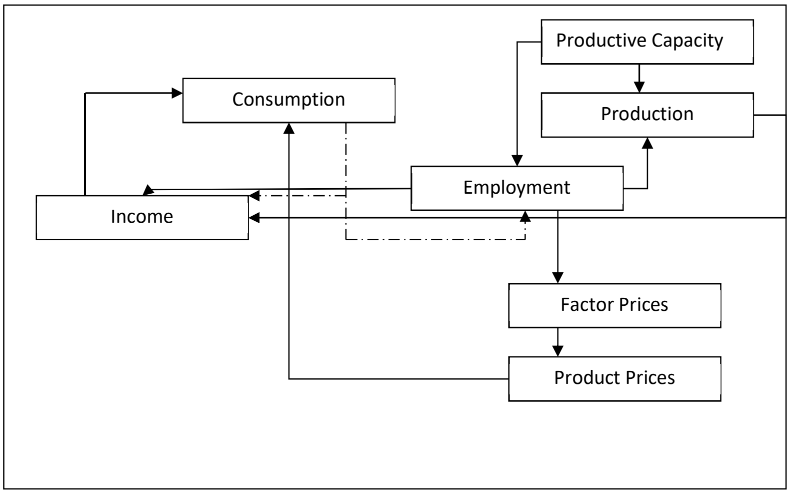
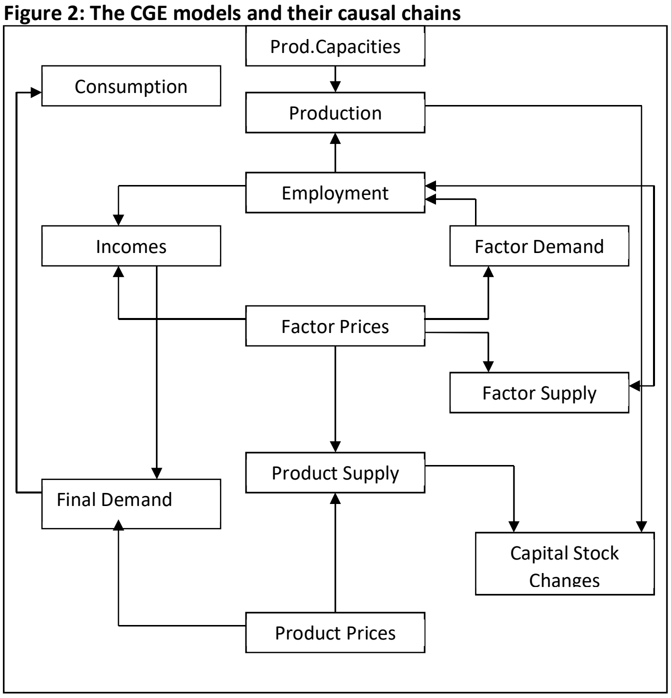


Figure 2: The CGE models and their causal chains



These relationships, together with those already present in the reduced form, complete the model whose equilibrium depends on a series of simultaneous relations. In these relations, the “causal links” (the level of the prices "cause" the level of demand, of the offer, etc.) only describe subjective relationships of the type: each consumer determines the quantities requested of the goods assuming that the prices and its income are given. There are, however, no objective causal links, except for the productive capacity-to-production link, which is objective because if it is true that prices determine the level of supply and demand, it is also true that in equilibrium prices are themselves to be determined.

A comparative static exercise consists in disrupting the balance described in Figures 1 and 2 by introducing an exogenous shock in one of the variables subject to simultaneous determination or "given" assumptions in the equilibrium situation. Depending on the variables perturbed, a sequence of different reactions will be generated, that can represent a different theory of achieving the general economic equilibrium. For example, an increase in production capacity due to technical progress will tend, through the reaction of firms that maximize profits, to increase demand for factors and therefore employment, supply of goods and production. This will put in motion a causal chain of classic type, which moves from production to consumption. Conversely, an increase in demand for goods, due to an exogenous variation in consumer preferences (or producers as regards investment goods) will tend to result in a causal chain of Keynesian type in which the increase in global demand ultimately causes an increase in production.

In both cases, however, the initial cause-effect sequence, which moves in a different direction depending on the original exogenous impulse, follows an adjustment phase based on the reciprocal interaction between the variables. Thus, for example, given an increase in production capacity, the first reaction of producers will be to modify their production plans, increasing demand for factors and employment. However, the supply of factors constrains the possibilities of expanding employment and, through the increase in income of the factors themselves and therefore of families, to expand the demand for goods. After the first impact on the variables directly linked to it, the exogenous shock and the causal chain connected to it will then be "absorbed" by the mechanism of general economic equilibrium that would restore the simultaneousness of the interactions between variables and, ultimately, the equilibrium.

5.2 The aggregable micro-macro link and the DSGE experience

The micro-macro connection in the CGE construction has an interesting intersection with the attempt of micro-founding macroeconomics, pursued through a series of models developed in the past 30 years. These models can be ascribed to three basic theoretical families: the neo-classical (NC), the new business cycle (NBC) and the new Keynesian (NK) approach. They are also often loosely identified as dynamic stochastic general equilibrium (DSGE) models, with reference to their aim to represent the economy as the result of an “aggregable” microeconomic general equilibrium of the Walrasian type.

The first example of a DSGE model aiming at reconciling macroeconomic phenomena such as the business cycle and money illusion with a Walrasian general equilibrium was the exercise on “Expectations and the Neutrality of Money,” by Lucas (1972). This was a very simple model, based on a two period-two generation world with a single commodity and fiat money, but incorporated equilibrium in all markets under flexible prices and rational expectations, combined with two distinct market failures: limited information on the nature of the shock analysed (technological or monetary) and dynamic inefficiency. The model displayed many characteristics of a Walrasian equilibrium, but achieved its results by reducing all agent heterogeneity to the difference between two types of differently motivated traders (the young and the old). As such, as most of its followers and more recent epigones, Lucas' search for micro-foundations was more based on minimal disaggregation of a macro-economy, rather than from scaling up a heterogeneous micro-economy.

Since Lucas seminal contribution, DSGEs have further evolved into a very broad family of aggregate models with tight theoretical structures, built around the idea of constructing a micro-founded macro-economy through "representative" utility maximizing agents, in a context of rational /Ricardian expectations and possibly of downward rigid prices and monopolistic behavior. Within these models, the neoclassical (NC) and New Business Cycle (NBC) varieties incorporate neoclassical optimizing assumptions, but these assumptions are directly applied to aggregate behavior through the use of representative agents, so that they occupy a rather peculiar position in the realm of Walrasian economics. On one hand, in fact they may be described as consistent with the mechanisms of adjustment to general equilibrium under fully flexible prices and quantities (possibly modified to reflect market distortions). On the other hand, the fact that they are not the fruit of bottom up aggregation from individual decision units, in the absence of meaningful restrictions from economic theory (see the "everything goes" theorem) reduces behavioral responses to "calibrated" parameters acting through macro-variables such as interest rates and money supply. A case in point is the impact of government fiscal shocks, which is the consequence of government crowding out private investment, which appears more the fruit of an assumption (e.g. lack of complementarity between private and public goods), than the result of the working of the model, with determining effects on private wealth reduction and households' decision to employ a higher proportion of their time in work activities rather than in leisure. Furthermore, regardless of the destination of government expenditure, the desired level of capital increases and the more so the higher is the perceived persistence of the public expenditure increase (Ramey, 2011, 2019).

Both these effects are hard to reproduce in more disaggregated CGE models, where agents' heterogeneity and multiple price effects tend to absorb much of the wealth adjustment, while reallocation of consumption and production and redistribution across income groups bear the bulk of the response to the external shocks. In typical CGEs, in particular, market competition for resources increases the costs of expanding supply in response to the demand increases, with a mechanism that may include, according to the different "closures" that can be chosen, the savings-investment equilibrium mechanism of the neoclassical models or the compensating nature of foreign balance and international trade. The disaggregated nature of this adjustment, depending on agents' heterogeneity, sector interdependencies and externalities, by redistributing wealth and income effects across households, reduces both crowding out and its impact from fiscal or other policy shocks. Furthermore, most CGE formulations of the CGE of new generation (Perali and Scandizzo, 2018), even under closures that reflect the Keynesian paradigm, exhibit non monotonic aggregate behavior that appears more sensitive to resource limitations, complementarities between private and public goods, and includes both crowding in and crowding out effects.

For example, as shown by Scandizzo, 2019, one of the key features exhibited by CGE models that incorporate both private and public goods is a non-monotonic behavior of the relationship between phenomena such as income distribution, environmental changes and aggregate growth. This is reminiscent of the so called Kutznetz curves, which suggest that in one phase growth is negatively and in one positively related to public goods provision and income distribution depending on two important effects. First, the relationship between aggregate economic performance and disaggregated distribution and environmental characteristics in the CGE models also tends to be highly non-linear (Scandizzo, 2019). This is the consequence of both static and dynamic non convexities, that arise from the externalities and the economies of scale generated by income redistribution across heterogeneous agents, and public goods such as natural resource services in spite of the decreasing return to scale built in the production function assumptions. Second, variations of aggregate value added and contributions from income redistribution policies and eco-services tend to display substitution properties in a region of development where there appears to be a trade-off between growth and broader public good performance and, once they have reached a turning point, revert to a complementary behavior where private and public goods provision tends to be a mutually reinforcing activity. One implication of this non monotonicity is that a growth depressing feedback may prolong indefinitely the negative relationship between development and a deteriorating environment, but also that a virtuous cycle can be created by an increase in growth and/or an investment in conservation, depending on whether the economy is on the decreasing (trade-off) or increasing (complementary) branch of the EKC.

Going back to the DSGE models, outside the context of the neoclassical framework, (Woodford, 2011), the New Keynesian (NK) approach provides a dynamic version departing somewhat from neoclassical assumption, and based on three critical blocks: a dynamic IS curve, an expectations-augmented Phillips curve, and a modified Taylor rule (Azariadis, 2019). In this case, however, in spite of the crucial role assigned by NK to prices, extreme aggregation and representative agents have the effect to make monetary policy the only arbiter of effective equilibrium adjustments. In the "benchmark" case illustrated by Woodward, moreover, crowding out of private investments and negative wealth effects of government expenditure fail to occur only in the special circumstances determined by a combination of recession and extremely accommodating monetary policies. In this case, the neoclassical closure of the model is neutralized by reaching the monetary trap of the zero nominal interest rate, beyond which responses to expenditure shocks become independent on how slack resources are i.e. it is independent of the cost of supply, even though such a cost does matter in determining the cost of the monetary policy measures needed to maintain the interest rate constant (cost of present goods in terms of future goods or indexed bonds). Aggregation also brings to the conclusion that monetary policy is the best instrument to maintain price stability and, at the same time to ensure a first best (full information) resource allocation. While monetary policy acts through the market in a neutral way, in fact, government expenditure, being necessarily non neutral in the sense that it consists of a specific pattern of purchases, is likely to determine a suboptimal allocation of resources, thus reducing the efficiency of the economy.
DSGE-RBC models also rely on utility-maximizing representative households for whom Ricardian equivalence holds with fully competitive labor and goods markets. Furthermore, these models, as in the NC and NK varieties of DSGEs, RBC models' claim to micro-foundations entirely depends on the optimizing assumptions that are adopted for the aggregate representative agents. These assumptions ultimately imply that the public and private sector compete for resources, with no possibility of using redistribution and heterogeneity to mold the response pattern. As a consequence, expansionary fiscal policies and any autonomous increase in spending, including investment, is unable to increase GDP via a Keynesian demand effect, but can only act via neoclassical wealth or substitution effects that force increased aggregate labor supply (Baxter and King, 1993) in the form of a larger number of working hours. Because government expenditure subtracts resources to the private sector, crowding

out is generally massive and results in a negative impact of fiscal expansion (and a positive impact of fiscal repression), with the precise value of the effect depending on the elasticities of demand for labor and the elasticity of substitution of consumption and leisure (Woodford, 2011). Even though these models are characterized by extreme aggregation, some of their variants do suggest that their results may be substantially modified by considering agents' heterogeneity, for example by factoring in complementarity effects across different households' utility functions, (Linnemann, 2006; Mazraani, 2010), but also by crowding out and distortionary effects (Ardagna, 2001; Fatàs and Mihov, 2001).

But what is the relationship between SDGEs and CGEs? It is not only a question of aggregation, even though the two classes of models tend to reflect the different importance given to agents' heterogeneity in their overall structure. Rather, SDGEs aim at analyzing macroeconomic policies, especially of the fiscal variety, by designing top down model structures anchored to a few microeconomic, and typically neoclassical tenets, such as utility maximization and price formation mechanisms. In the process of achieving these goals, they make some heroic assumptions about agents' behavior, expectations, uncertainty and aggregation. CGEs, on their part, are essentially a compromise between micro and macro representation, since they model behavior on the part of aggregate, but heterogeneous economic agents and, at the same time, design systems of equilibrium in the research tradition of macroeconomic accounting with "no black holes" (Taylor, 2010). While a complete consistency is assured to national accounting, on the other hand, based on the basic Keynesian identity between expenditure and income, the macroeconomics of CGEs is often confined to the so called "closure", that is a set of simple hypothesis on whether the anchor of the system is a "sticky" monetary wage (the "Keynesian closure") with equilibrium determined by shocks in aggregate demand, or a Walrasian price system, with more complex interplay between demand and supply. CGEs also typically embody the hypothesis of Keynesian expectations, as in Chapter 18 of the General Theory (1937). Keynesian agents are thus assumed to hold rational expectations, but only in the sense that they align expenditure to incomes on the basis of a continuous attempt to estimate expectations of others in an endless iterative process (Davidson, 1991).

An integrated DSGE-CGE Approach

For a more detailed analysis of the potential that DSGEs offer to CGE modelling to address the micro-macro challenge, it is useful to examine the comparative structures of two typical SDGE and CGE models, aiming to reproduce key macro "trends" such as economic growth or business cycles and to anticipate the effects of economic policy. SDGE models of this type combine firm and households' dynamic behaviour affected by stochastic shocks, by adopting a simple notion of applied general equilibrium across complete and efficient markets, while the corresponding CGE models aim also at presenting a consistent "meso and macro-economic" picture of the possible outcomes of alternative policy shocks. These models are both implemented using either calibrated parameters or parameters estimated using econometric models based on time-series data or a combination of both. The so called New-Keynesian (NK) DSGE models, in particular, build on a structure similar to Real Business Cycle models while striving to provide microeconomic foundations by introducing rational expectations and monopolistic competitions.

Table 1 below shows the comparative features of a typical NK- DSGE (see the appendix for details) and an equally typical CGE (Perali and Scandizzo, 2018). As a detailed analysis of the equations show, the two classes of model share several similar characteristics and assumptions, including: (i) a set of neoclassical assumptions on optimizing agents' behaviour, (ii) aggregation rules that avoid specialization, based on CES-like aggregation, (iii) price formation mechanisms differentiating tradable from nontradables, (iv) structure of wealth and property rights, based on the distribution of value added to factor owners (households), government, and rest of the world. This latter property seems to suggest that the social accounting principles are satisfied by both model baseline data and solutions, as well as macro-micro consistency. Ricardian equivalence, however, is a condition enforced by DSGEs, but in more recent NK versions, tempered by the introduction of Keynesian agents. Depending on the proportion of the two agents, the mixture of Ricardian and Keynesian consumers may impose a much more rigid form of intertemporal consistency which is not satisfied by CGEs and may cause strong divergence between the solutions of the two types of models.

As shown in the last two columns of Table 1, in addition to agents' expectations, Government and Central Bank behaviour constitute the main area where the two classes of models present the more substantive theoretical differences. These differences correspond to the treatment of the monetary sector, which is subject to a Taylor rule or equivalent automatic adjustments in SDGEs, and by the government budget constraint, which is typically supposed to be enforced through strict equality between tax revenues and expenditure. The monetary sector is generally neglected by CGEs, which consider money essentially neutral and financial instruments only implicit components of capital formation, both on the investment and the savings side. On the other hand, CGEs tend to be more flexible and may contemplate different closures on government budget rules. In sum, it appears that SDGEs and CGEs could be integrated to take advantage of theoretical differences and possible complementarities. CGEs could provide a compatible dimension of disaggregated results to DSGEs solutions. Vice versa DSGEs could strengthen the different CGEs' macroeconomic closures, by providing a consistent framework for both monetary and fiscal policy rules.

Table 1. Differences and similarities between DSGEs and CGEs

Households		Firms		Government and Central Bank	
DSGE	CGE	DSGE	CGE	DSGE	CGE
Aggregate Representative household $U(i)\ , i = A, B$	Representative households disaggregated by income class $U(i,j)\ , j = 1,2 \dots H$	Aggregate (of a continuum of firms) representative firm operating in a monopolistic non tradable goods sector, according to a neoclassical production function to yield a single	Several sector specific representative firms operating in competitive (according to different levels of competition) non tradable goods sectors, according to neoclassical production functions to yield a plurality	The central bank sets nominal gross interest rates and adjusts them in response to inflation according to a Taylor rule	No monetary sector. Money is assumed to be neutral.

		variety of non-tradable goods	of non-tradable goods		
Aggregate Consumption as constant elasticity index of traded and non traded goods	Disaggregated consumption, according to NACE-CLIO or other international statistic classification for non traded and traded goods	A single tradable good is produced both at home (in each region) and abroad in a perfectly competitive setting. Domestic producers combine imported intermediate goods and domestic value-added goods according to a CES technology	A plurality of tradable goods is produced both at home (in each region) and abroad in a perfectly competitive setting. Domestic producers combine imported intermediate goods and domestic value-added goods according to a CES technology	The fiscal authority balances the budget in each period by ensuring that government expenditures and transfers to the households are fully financed by lump sum taxes. Ricardian equivalence is embedded in the model.	The fiscal authority balances the budget in each period by ensuring that government expenditures and transfers to the households are fully financed by lump sum taxes or by increases in the internal or external deficit. No Ricardian equivalence is assumed.
The tradable good is a CES composite index of home and foreign tradable goods	Each tradable good is a CES composite index of home and foreign tradable goods	Labor and capital in the North are more productive than in the South	Labor and capital in the North are more productive than in the South	In equilibrium the model simultaneously determines regionally differentiated output and CPIs, consumptions, investment, employment and interest rate.	In equilibrium the model simultaneously determines regionally differentiated output by product, consumption, investment, employment and interest rate.
Home consumption is a CES composite index of the domestic consumption produced in the North (H_A) and	Each home consumption good k is a CES composite index of the domestic consumption produced in the North (H_{kA}) and	Value added produce in the North and in the South aggregate to total domestic value added with the CES-like aggregation	Value added produce in the North and in the South aggregate to total domestic value added		

in the South (H_B)	in the South (H_{kB})				
Price of total consumption is aggregated using CES composite indices	Price of each consumption good is obtained by solving the CGE model				
The price of aggregate tradable good is determined abroad following the law of one price.	Price of each tradable good is determined abroad following the law of one price (international price multiplied by the exchange rate).				
The price of domestic tradables $P_{H,t}$ are regionally differentiated and are aggregated according to the CES weighting procedure	Prices of (disaggregate) domestic tradable goods are aggregated according to the CES weighting procedure				
(Aggregate) households hold physical capital that rent to firms of each specific sector. Movements of capital between tradable and non-tradable sectors, differently from labor, is not admitted.	(Disaggregate) households hold physical capital that rent to firms of each specific sector. Movements of capital between tradable and non-tradable sectors, differently from labor, is not admitted.				
Households' wealth is defined as	Households' wealth is defined as				

remuneration to labour and capital supplied to the firms, from interest on bonds, and from profits derived from ownership of firms.	remuneration to labour and capital supplied to the firms, from savings (capital formation) and from profits derived from ownership of firms.				

The model so designed is supposed to be suitable to run simulations that assume an increasing reduction of the factor productivity gap between the North and the South to gauge the impact on price parities differential. The present model, after appropriate validation for out of sample behaviour, would serve also the purpose of anticipating the macro consequences for growth CUM environmental equity across regions and generations to come associated with a widening productivity divide and/or different solutions to manage environmental externalities.

There are several general *caveats* that should be kept in mind when evaluating the usefulness of SDGE macro models for policy analysis and design. The implicit assumption underlying the DSGE model-based policy analysis is that the parameters describing the preferences of a representative agent and the technologies of a representative firm including exogenous structural shocks are policy invariant and therefore subject to Lucas critique. Further, while it should be acknowledged that the treatment of rational expectations and dynamic behaviour is nicely developed in SDGE models, the problems due to aggregation are normally understated (Chang, Kim, and Schorfheide 2012). For example, the discount factor should be treated as endogenous so that it can vary through time as the characteristics of agents, that can be more or less myopic or forward looking, change through time and to different extents in the various regions of a country. This source of heterogeneity, though relevant for policy analysis, can hardly be accommodated in a SDGE model because of the instabilities introduced in the dynamic system. In analogy, also problems of omission of relevant information are a potential source of bias. An example is the treatment of technical change as a possible motion laws of the economy neglecting the role of institutional change and the associated endogenous process of social class formations.

Solow (2010) makes this point abundantly clear in the 2010 United States congress hearings on macroeconomic modelling methods asking why macroeconomists failed to foresee the 2008 financial crisis referring explicitly to DSGE models contending that "they take it for granted that the whole economy can be thought about as if it were a single, consistent person or dynasty carrying out a rationally designed, long-term plan, occasionally disturbed by unexpected shocks, but adapting to them in a rational, consistent way." Solow is also very critical with the idea of macro models being micro funded in a statistically consistent way.

These feeble relations with microeconomic reality are judged to be one of the main causes for the lack of relevant predictions from DSGE models that were also validated by history. We think, as it will be explained in the next Section, that large-size disaggregated computable general equilibrium model perform much more realistically because not necessarily imprisoned to representative behaviour so that heterogenous preferences and technologies can exactly aggregate the micro behaviour and data up to the macro level and vice versa without loss of relevant information (Jorgenson, Lau and Stoker 1980). This micro-macro consistency can also be extended to microeconomic shocks as recently shown by Baqaee and Farhi (2019).

5.3 An exactly aggregable micro-macro link

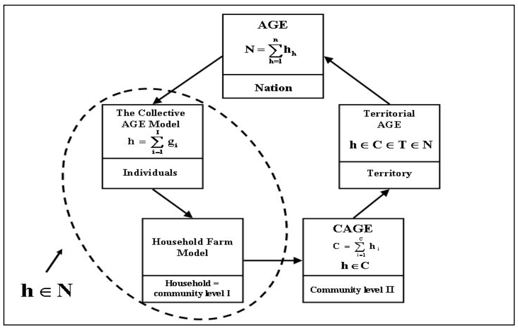
In the traditional simulation literature linking the micro and macro level of policy analysis the micro level of analysis refers to the household while the macro level of refers to society as a whole. This representation neglects several layers of aggregation of high policy relevance. Each household is a collection of individuals with their own preferences and levels of well-being that are employed in both marketable and non-marketable household production activities. The household enterprise is *per se* a miniature economy that can be studied within an equilibrium framework. The recent advances in the collective theory of the household (Chiappori 1992, Chiappori and Ekeland 2011, Chiappori and Lewbel 2015) makes it possible to identify preferences of each member of the household and distributive effects within the household so that, for example, adults and children can be regarded as social classes of the family micro-society. It is then natural to describe "input-output" transactions within the household using a social accounting framework (Matteazzi, Menon and Perali 2017).

Until the recent past, the micro-macro link was missing also at the community or village level (Taylor and Adelman 2006, Taylor 2012, and Taylor and Filipski 2014). This limitation was mainly due to lack of complete statistical data at low administrative levels. Non-survey methods working top-down from macro input-output tables produce approximation errors that increase the larger is the zooming at the micro level. Trade information is especially exposed to this imprecision. One of the main problems in the assembly of local economy tables consistent with the system of regional input-output tables is obtaining inter-community commodity flows, along with their respective zones of influence. Moreover, it is especially difficult to account for the possibility of existing simultaneous import and export of the same product (cross hauling).

When the level of disaggregation or the local economy of interest does not coincide with administrative units, as for example in the case of a natural park, an industrial district or a large project, it is preferable to supplement the published statistics available at the local level with representative business surveys about the input-output structure of local industries and the associated trade flows with the surrounding zone of influence and the rest of the world (Taylor 2012). To save survey costs, household consumption information may be inferred from sufficiently representative national income, consumption or living standard surveys.

Figure 3 describes the micro-macro link between the general equilibrium model at the macro level of the economy and the general equilibrium model at the micro level of the household economy that accounts for individual preferences and well-beings. The dashed set diagram emphasizes the fact that the primitive macro-micro link is the one aggregating all household individuals into the family seen as a mini-society. The *h* household farm, or enterprise, level can be interpreted as a first community level aggregating each *g* member of the household using the collective theory of the household (Chavas, Menon, Pagani and Perali 2017). Then, households at the micro level aggregate up to the macro-level of the whole economy. As shown in the right panel of the graph, households can aggregate also at the intermediate level of a community, such as a village, or of a territory such as a natural park, an industrial district or a region.

Figure 3: A "general" Micro-Macro link



The micro dimension of the "general" representation of the micro-macro link described in Figure 3 is specular to the general equilibrium macro dimension. In the traditional microsimulation literature, the approach is partial in the sense that the focus is limited either to labor supply or consumption, health, housing, education, marketable production, home production or other issues that are analyzed in separate modules.

To implement this approach, data must be general too. The multi-topic approach implemented by the World Bank Living Standard Measurement Study (LSMS) integrates information about the household, marketable and non-marketable household production (when the time use module is included) and the service and business community is an appropriate example. In less developed economies, where agriculture still contributes significantly to domestic production, it is relevant to record who does what in the family, both in agricultural and household related activities to construct a reliable input-output matrix of agriculture (or other marketable family business) and of home activities. This aspect is recommended in the Wye Group Handbook (2007) that also stresses the relevance to record individually disaggregated data and the consumption of goods assignable to specific members of the household, if the interest is to implement the micro-macro link starting from the individual to the family, community and society level.

Using surveys with a design that integrates consumption, production, and time use information as in the LSMS (Grosh and Glewwe 2000) or the ISMEA case (Finizia, Magnani, Perali 2004) and reporting individual specific information, it is possible to implement collective household enterprise models within an equilibrium framework (Caiumi and Perali 1997, Matteazzi, Menon, Perali 2017) representing the base micro level depicted in Figure 3. These studies extend the traditional farm household model (Singh, Squire and Strauss 1986, De Janvry, Fafchamps, Saudolet 1991) to encompass recent advances in collective theory. The model represents production and consumption-leisure choices along with the rule governing intra-household resource allocation to analyse the income and wage responses of each family member and recover their level of well-being. The household is treated as an equilibrium model whose accounts are based on a collective household accounting matrix, with the social dimension being the wife/husband classes. The micro data permit the joint estimation of behavioral parameters characterizing consumption, farm production and household production choices of farm-households. These estimates are used to construct the micro farm-household model. The household enterprise, be it a farm or a firm, is by analogy the micro-level mirror image of the macro-economy. The appendix describes the collective household farm or enterprise model in detail.

At the household level, production and consumption decisions are non-separable. The collective approach permits deducing the welfare levels of individual household members thus making it possible to account for gender and inter-generational differences in the evaluation of policy impacts and individual responses to policy changes in the labor or capital markets.

Because of non-separability, farm production and household consumption are estimated jointly. The econometric methodology consists first in estimating household production and deriving the price of the aggregate non-marketable domestic product, and, secondly, in estimating the production and consumption side of the household economy conditional on the estimated instrumented domestic price.

The specification of the micro econometric model takes the following behavioral aspects into account: non-separability of the farm, household and home activities, time allocation and associated labor supply between on-farm, off-farm and on-home production, corner solutions related to the choice of input use such as capital and labor and optimal portfolio of production activities, rule governing the intra-household distribution of resources permitting the recovering of individual preferences and welfare levels of the husband, the wife and, possibly, the children, estimation of shadow wages considering family labor as a quasi-fixed input.

The design of this estimation strategy is clearly general in the sense that it estimates consumption, leisure and both marketable non-marketable production jointly so that the econometric model can be transferred as such within the structure of a general equilibrium model without the need of a traditional calibration (Matteazzi, Menon and Perali 2017). Interestingly, the modelling of zero consumption, labor or other factor choices at the micro-level can also be easily mirrored at the macro-level adopting Löfgren and Robinson’s (1997, 1999) mixed complementarity approach.

When integrated surveys are not available, as in the case of many developed societies that prefer to maintain the higher level of detail of separate consumption, income, wealth, labor and time use surveys, statistical matching techniques should be applied (Wolff *et al.* 2012, Dalla Chiara, Menon, Perali 2016). Such integrated data bases are suited for the analysis of standard of living and for the estimation of complete collective demand systems describing consumption, domestic production and leisure/labor choices (Caiumi and Perali 2015). If the available micro data-base is not integrated, then the micro level of analysis is by force partial.

Clearly, the production side of the economy is not represented and should be econometrically summarized using micro-data from business surveys possibly incorporating a design that records input-output and trade transactions or macro-data recording input use per sector as in the KLEM-style data bases (Jorgenson 2007, Jorgenson and Samuel 2014). Both data sources would be consistent with the aggregate account data of the corresponding input-output data.

Farm or non-agricultural enterprise household models can be aggregated into a local general-equilibrium framework representing a village, larger communities or counties linked together spatially through trade (Taylor and Adelman 2006, Taylor 2012, Taylor and Filipski 2014) as illustrated in Figure 3. The aggregation process may continue bottom-up and stop at the desired level of policy analysis.

In this section, we stressed the importance of a "general" and integrated data design with an input-output structure and sufficient individual level information so that a single source of information may feed both the micro and macro behavioural model. Micro data would then exactly aggregate to the macro level (Jorgenson, Lau and Stoker 1980, Savard 2003, Magnani and Mercenier 2009) so that it would be possible to consistently zoom in from the macro to the micro level or to implement a micro-macro zooming out statistically consistent across levels of aggregation. It would also be possible to estimate exactly aggregable micro-econometric models of consumption, labour supply and production and the associated set of parameters along with their standard errors that would permit a more flexible representation of economic behaviour as compared to the standard CES forms used in traditional applied general equilibrium analyses. In addition, there would be no need for a traditional calibration procedure, aiming at deriving the set of parameters from a given SAM or borrowing them from other external econometric studies, because the set of parameters of interest would be produced "in-house" along with their confidence sets (Jorgenson *et al*. 2013, Taylor 2012, Taylor and Filipski 2014, Matteazzi, Menon, Perali 2017).

Another virtue of integrating micro-econometric modelling with applied general equilibrium is the possibility to obtain confidence intervals for the equilibrium outcomes of changes in economic policies. Jorgenson *et al*. (2013) show how to apply the Delta method for policy evaluation given the knowledge of the asymptotic covariance matrix of the parameters. It becomes also practicable to run project impact simulations using Monte Carlo methods (Taylor 2012, Taylor and Filipski 2014). Such simulation methods in an applied general equilibrium context may become a complement to randomized control trials (RCTs) and an effective tool for impact evaluation where RCTs are not feasible. When randomized control trials are not appropriate or cannot be implemented, impact evaluation at the local level can be performed comparing the same local economy before and after the program or with adjacent local economies "not treated" with the program.

This inferential feature of the micro-macro approach should become a standard feature of policy and project evaluation in an equilibrium framework, though it should be recognized that it is constrained more by data availability rather than modelling capabilities. It also opens the doors to the implementation of causal analysis with observational data, represented in our context by the micro integrated data base, along the lines traced by Heckman (2010), striving to reconcile structural and program evaluation policies by using LATE techniques or by studying causal models through Directed Acyclic Graphs (DAGs) or Bayesian Networks (Pearl, 2013). Clearly, causal inference can be pursued provided that the applied framework is an econometrically estimated and calibrated general equilibrium model and is an important opportunity for policy analysis, because, as stressed by Imbens (2010: 401), questions concerning the causal effects of macroeconomic policies or involving general equilibrium effects can rarely be settled by randomized experiments.

So far, we have examined a circular micro-macro link that can be made operational either from the bottom up or indifferently top down exactly aggregating individuals to families, communities or societies or disaggregating society into their individual members. However, as it will be apparent in the next section, the micro-macro modelling approach is not fully integrated because the tax-impact component of the policy analysis is mainly omitted.

5.4 A fully integrated micro-macro modelling approach

A fully integrated micro-macro modelling approach refers to a modelling strategy that builds a formal communication flow between the macro equilibrium analysis, the micro behavioural analysis and the non-behavioural tax-benefit simulator. A tax-benefit microsimulation model calculates the effects of direct and indirect taxes and benefits on household incomes and work incentives for the population of a country or, in a comparable manner, for a set of countries. It executes a highly detailed and exhaustive set of policy rules, that must be updated on a yearly basis, using representative expenditure, income or standard of living surveys whose reported incomes are checked for consistency with available administrative data. The simulator is usually too large to be hosted in a general equilibrium model that normally incorporates a gross representation of a country tax system. This explains why it is rare to observe applications that integrate an analytical tax-benefit simulator with a less systematic and accurate macro representation of the fiscal system.[1] Previous discussions of micro-macro modelling integration do not explicitly include a tax-benefit simulator (Savard 2003, Davies 2009, Ahmed and O'Donoughe 2007, Cockburn, Corong and Cororaton 2010, Cockburn, Savard and Tiberti 2014, Peichl 2016) in the communication flow.

The macro level is normally represented by a CGE with a representative agent or more agents mirroring the social class differentiation of the underlying SAM. To establish an exactly aggregable link, it is recommendable that the SAM information about household classes come from the same micro dataset used for the behavioural micro-simulation model. This feature would help retain the heterogeneity that CGE models alone normally do not account for and the micro-macro consistency of income and poverty simulations at the micro level by linking intra-group heterogeneity to a statistical relationship between averages and a measure of entropy dispersion.

Two main approaches can be identifiedto build a formal communication between the macro and micro layer of the analysis. The approach proposed by Savard (2003), Cockburn, Corong and Cororaton (2010) and Cockburn, Savard and Tiberti (2014) is an integrated method that incorporates all the "real" households in the micro data set directly in the general equilibrium model. It does so by simply extending the set of households treated in the model while ensuring the coherence between income and expenditure accounts in the household survey and in the SAM. This method uses traditional calibration techniques to recover demand parameters rather than importing directly more flexible and sophisticated

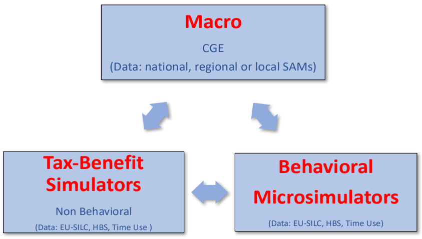
[1] Bourguignon and Spadaro (2006) explain that performing a microsimulation entails three basic inputs. The policy rules to be evaluated describing, for example, a tax reform, an appropriate behavioural model of individual response to policy and an informative micro dataset.

consumption and labour supply models estimated before the execution of the micro-macro exercise. This evolution, along with the theoretical result in Magnani and Mercenier (2009), would eliminate both the need to reconcile incomes with expenditures and savings and the model aggregate account and would respond to the concern raised by Savard (2003) and Bourguignon, Robilliard and Robinson (2005) about the difficulties investigating policies involving discrete choices or regime switching behaviour.

The second main approach establishes a sequential top-down link, whereby the macro CGE model generates equilibrium prices that are passed on to a partial-equilibrium behavioural household micro-simulation model. This approach is usually performed with an iterative feed-back "top-down/bottom-up" process that ensures consistency between the behaviour of the aggregate classes in the CGE and that of individual households (or individuals if a collective approach is pursued) in the micro databases (Savard 2003, Bourguignon, Robilliard and Robinson 2005). As noticed also by Cockburn, Savard and Tiberti (2014), the importance of the feedback effect critically depends on the aggregation error due to the lack of exact aggregation of the micro behavioural functions. If the exactness property is maintained, then the micro and macro models will be consistent and the iterative feedback process will be minimal.

The micro-macro dialogue depicted in Figure 4 traditionally occurs between the macro models passing on information about equilibrium prices and incomes of the new post-reform economic situation, to the micro behavioural models. It simulates how consumers or workers respond to the change and the associated impact on poverty and income distribution, or between the tax-benefit non-behavioural simulators, therebytransferring information about post-reform incomes to the behavioural micro-simulation models. This clarifies what we mean by a fully integrated micro-macro modelling system that takes advantage from a circular communication flow between the macro and both the behavioural and non-behavioural micro models, as illustrated in Figure 4.

Figure 4. A fully integrated micro-macro modelling approach



We now provide a formal representation of a fully integrated approach.[2] In the following expression, a tax-benefit simulator gets the records about households' gross income $\mathrm{y_h}$ from microdata obtained from information about the household h supply of working hours l at aggregate wage w and non-labour income as the rent r of capital k

$$\mathrm{y_h = rk_h + wl_h}$$

and derives households' net income $\mathrm{y_h^{net}}$ by applying a household specific tax-benefit rule τ to gross incomes

$$\mathrm{y_h^{net} = (1-\tau_h\,)y_h + transf_h = \big((1-\tau_h\,)rk_h + transf_h\big) + (1-\tau_h\,)wl_h =}$$
$$\mathrm{= y_h^{exo} + (1-\tau_h\,)wl_h,}$$

where net incomes are decomposed into an exogenous component $\mathrm{y_h^{exo}}$ and endogenous labour income.

At the micro level, each member j of household h chooses her/his optimal bundle of consumption goods c and labour l by maximizing her/his own utility U subject to the individual budget constraint corresponding to a proportion $0 < \mu^j < 1$ of household net income

$$Max_{c_h^j\ ,l_h^j}\ \ \{U_h^j\,(c^j, 1-l^j)\ \left|pc^j = \mu^j \cdot ((1-\tau_h\,)\,\mathrm{l}_h\,\mathrm{w}_h + \mathrm{y}_h^{exo}\,)\right.\}$$

whose solution gives optimal collective consumption for each consumption aggregate (or sector) s such

[2] We thankfully acknowledge Jean Mercenier for this integrated representation and for many fruitful discussions on frontier topics of general equilibrium.

as food, housing, education, health, transportation, recreation, and others, and collective labour supply

$$\begin{cases} c^{j}_{(h|s)} = c^{j}_{(h|s)}\,(\mathrm{w}, \mathrm{p}, \mathrm{y}^{exo}_{h}, \tau) \\ l^{j}_{h} = l^{j}_{h}\,(\mathrm{w}, \mathrm{p}, \mathrm{y}^{exo}_{h}, \tau) \end{cases} \quad \forall \mathrm{j} = \text{male, female, children and } \forall \mathrm{s} \text{ sectors}$$

that feed the macro CGE level.

At the macro level, to exemplify we suppose that the economy reaches a Walrasian equilibrium that we summarize as follows

$$\begin{cases} \sum_{h} \mathrm{l_h} = \mathrm{L^{dem}}(\frac{\mathrm{w}}{\mathrm{p}}, \mathrm{Q}) \\ \sum_{h} \mathrm{k_h} = \mathrm{K^{dem}}(\frac{\mathrm{w}}{\mathrm{p}}, \mathrm{Q}) \\ \sum_{h} \mathrm{c_h} = \mathrm{Q} \end{cases} \Rightarrow \quad (\mathrm{w}^*, \mathrm{p}^*, \mathrm{r_{numeraire}};\ \mathrm{Q}^*)$$

where individual factor demands aggregate to the total factor endowments in the economy (L^{dem}, K^{dem}) and aggregate demand equals aggregate supply Q. The unique solution of the system gives equilibrium prices and output $(\mathrm{w}^*, \mathrm{p}^*, \mathrm{r_{numeraire}};\ \mathrm{Q}^*)$ that may feed and feedback both the behavioural microsimulation models and, for a complete integration, the tax-benefit simulator

$$\mathrm{y_h^{net}} = \mathrm{y_h^{exo}} + (1 - \tau_\mathrm{h})\mathrm{w}^* \mathrm{l}^*_h\,.$$

Then, to maintain an exact micro-macro consistency, researchers would need to iterate until convergence, where two adjacent iterations give wages and prices differing for a small error of the $1 \mathrm{x} 10^{-8}$ size. Incorporating labour supply and consumption schemes consists in replacing the first order conditions in the CGE by the estimated consumption and labour (collective) supply

$$\begin{cases} c^{j}_{h} = \hat{c}_{h}\,(\mathrm{w}, \mathrm{p}, \mathrm{y}^{exo}_{h}, \tau) \\ l^{j}_{h} = \hat{l}_{h}(\mathrm{w}, \mathrm{p}, \mathrm{y}^{exo}_{h}, \tau) \end{cases}.$$

It is relevant to note that failing to close the model at the macro level would be "distortionary partial". For example, focusing on labour supply alone, would break the utility optimization and $\hat{c}_h$ would be determined residually to satisfy the budget constraint $\mathrm{p}c_h = (1 - \tau_h)\,\mathrm{l}_h\,\mathrm{w}_h + \mathrm{y}^{exo}_h$.

The importance of a direct integration between CGEs and tax-benefit models should not be overlooked. For example, consider an aggregate shock, such an economic crisis or the exit of a European country from the Community. In this case the iterative use of the tax-benefit simulator and the CGE model would allow determining the endogenous policy rule $\hat{\tau}$ that neutralizes the shock.

This step is critical to place tax-benefit simulators as an integral component of the micro-macro behavioural modelling approach that is going to make more and more extensive use of big data warehouses. A relevant example is the Rhomolo model (Mercenier *et al*. 2016) that is a spatial computable general equilibrium model for EU regions and sectors that is calibrated using the SAMs of all regions in Europe, uses inter-regional trade information of both products and factors, and incorporates systematic information of the fiscal and legal systems specific to each region.

Similarly, there is a large investment in creating "big micro data" warehouses that are uniformed across European regions of interest so that it is practical to run the same behavioural (consumption and labour supply) models that would allow to support micro-econometric based inter-regional general equilibrium models that will be able to perform causal policy analysis. Such EU-level matched data bases, would allow a more thorough understanding of the policy impact at the micro level on the well-being of policy-relevant types of families, such as fragile and migrant families, and a more complete cross-country comparison of standards of living, possibly accounting for differences in prices and access to public services.

6. Conclusions

General equilibrium (GE) modelling includes both Dynamic Stochastic General Equilibrium (DSGE) and Computable General Equilibrium (CGE) models as an evolving methodology of applied economics, that is fast becoming the main tool of policy analysis capable to link the aggregate level of the economy, to different disaggregate levels, such as, in particular, specific regions, population aggregates and investment projects. Although some of their theoretical foundations can be traced to the historical debate on the theory of market equilibria from Walras to Debreu and beyond, modern GE models are pragmatic constructions based on available statistics and relatively simple estimates. SDGEs are essentially macroeconomic structures built around micro-foundations such as rational economic behaviour on the part of decentralized and rational agents. On the basis of similar postulates, CGEs try to capture the essentials of the economic system using the mathematical and statistical structure of the Social Accounting Matrix, as a core element of their description of a transaction based economy. This implies not only that they are not an attempt to quantify the complicate relations described by the classical economists under the invisible hand hypothesis, but also that they do not address some of the big unresolved issues concerning equilibrium among stocks and flows and, in particular, by different capital theories.

Because of their reliance on SAMs, CGEs also present the peculiarity of being a consistent and complete set of accounts that represent a disaggregation of the national accounts and provide monetary indicators of value consistent with the UN methodologies and the gross domestic product (GDP) concept. They also allow to easily extend the same accounting principles to represent and measure values of non-market goods, externalities, environmental damages, and natural resources. As such, they present themselves as useful complements to the compressed view of the economy provided by the more sophisticated, but less differentiating dynamics of DSGEs.
While in CGEs the linear homogeneity of the price system in the model makes equilibrium price variations and prices specified only in relative terms, SDGEs contain a monetary module based on a simple rule for money supply, which makes money non neutral and inflation meaningful.

By virtue of these characteristics, and in spite of its many limitations, both the theory and the application of SDGEs and CGE modelling has been growing fast, in part as the consequence of advancing knowledge in the fields of social accounting, mathematical programming and development of computational power. Even though the SAM transactions do reflect a fully circular economy and the CGE computations register the interdependencies across a plurality of autonomous agents, the results of the simulations tend to reveal well defined adjustment mechanisms and typically can be interpreted as the consequence of adding flexibility and price effects to a basic input-output structure. Model closures based on explicit SDGE structures may be very useful to avoid the "black box" effect characteristic of large models, by imposing explicit directions to the causal chains of the policy experiments and project evaluations. If we further consider that SAMs are the aggregate accounting structure of microdata about firms and households' transactions that can be used by micro econometric models that supply behavioural parameters and associated standard errors to the macro model, then policy and project evaluations techniques can be implemented in a causal framework.

Yet some important questions remain and suggest that many dimensions of a new frontier of SDGE-CGE , as "GE" structures, modelling are ready for exploration. Among these, the most important appears to be their reliance on aggregate agents, even though aggregation is less extreme in CGEs than in SDGEs. While the assumption of agents' homogeneity is somewhat released in CGEs, in fact, GEs behavioural functions are generally assumed to be equivalent or similar to those of the individual, decentralized agents acting in the economic system. We know that this is not only a gross simplification, but because of the "everything goes" result, it is not consistent with economic theory. Any set of excess demand functions, in fact, may be capable of being solved for non-negative equilibrium prices, regardless of the properties of the underlying disaggregated structures that may have been used to generate them. This means that using well behaved demand and supply functions at aggregate level does not per se add any validity to the general equilibrium solutions found. In other words, more specific micro-foundations, such as those derived by the collective theory of the household, are not only useful, but necessary to fully legitimize the aggregate model and ensure the exact aggregation of the micro-macro link within a modelling approach that fully integrates the macro CGE environment with the tax-benefit analysis of fiscal reforms and the micro-behavioural impacts.

The above considerations also bring about the question of the causal nature of GE models and their use in inference. As causal models, GEs have to deal with three sets of tasks in this respect:

- Defining the set of hypotheticals or counterfactuals (a scientific theory),
- Identifying parameters (causal or otherwise) from hypothetical population data (mathematical analysis of point or set identification),
- Identifying parameters from real data (estimation and testing theory).

From the point of view of inference this implies that counterfactuals, even in dynamic settings, may face two different problems: (i) identifying causal models from idealized data of population distributions (infinite samples without any sampling variation), (ii) identifying causal models from actual data, where sampling variability is an issue. In case (ii), the hypothetical populations may be subject to selection bias, attrition and the like, but all issues of sampling variability are irrelevant for this problem. In case (ii), the analysis must recognize the difference between empirical distributions based on sampled data and population distributions generating the data.
The tension that has been driving recent developments in the theory and applications of general equilibrium towards greater integration between policy and project analysis led to major achievements in moving the frontier of knowledge forward. As this new stock of knowledge is increasingly

transferred to institutions and practitioners, the quality of the policy process and project evaluation will be greatly enhanced.

References

Ahmed, V. and C. O'Donoghue (2007): "CGE-Microsimulation Modelling: A Survey," MPRA Paper No. 9307.

AmbriŠko, R. (2015). "A Small Open Economy with the Balassa-Samuelson Effect," CERGE-EI, Prague.

Arrow, K. J. (1951): "An Extension of the Basic Theorems of Classical Welfare Economics," Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, ed. J. Neyman.

Arrow, K. J. and G. Debreu (1954): "Existence of an equilibrium for a competitive economy". Econometrica 22 (3): 265–290.

Aziaradis, Costas (2018): "Riddles and Models: A Review Essay on Michel De Vroey's A History of Macroeconomics from Keynes to Lucas and Beyond", Journal of Economic Literature, 56(4), 1538–1576.

Balassa, B., (1964). "The purchasing power parity doctrine: a reappraisal," Journal of Political Economy, 72: 684-596.

Baqaee, D.R. and Farhi, E. (2019). "The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem," Econometrica, 87(4):1155-1204.

Bourguignon, F., A.S. Robilliard and S. Robinson (2005): "Representative Versus Real Households in the Macroeconomic Modelling of Inequality," in T. J. Kehoe, T.N. Srinivasan and J. Whalley (eds.) "Frontiers in applied general equilibrium modelling", Cambridge, Cambridge University Press.

Bourguignon, F. and A. Spadaro (2006): "Microsimulation as a Tool for Evaluating Redistribution Policies," Journal of Economic Inequality, 4: 77-106.

Calvo, G. (1983). "Staggered prices in a utility maximising framework", Journal of Monetary Economics.

Chang, Y., Kim, S-B., and Schorfheide, F. (2012). "Labor-Market Heterogeneity, Aggregation, and the Policy-(In)variance of DSGE Model Parameters," Working papers 2012rwp-51, Yonsei University, Yonsei Economics Research Institute.

Caiumi A. and F. Perali (1997): "Female Labor Force Participation: A Comparison between Urban and Rural Families," American Journal of Agricultural Economics 79: 595-601.

Caiumi, A. and F. Perali (2015): "Who Bears the Full Cost of Children? Evidence from a Collective Demand System," Empirical Economics.

Chavas, J.P., M. Menon, E. Pagani and F. Perali (2018): "Collective Household Welfare and Intra-Household Inequality," Theoretical Economics.

Chiappori P.A. (1992): "Collective Labor Supply and Welfare," Journal of Political Economy, 100(3): 437-467.

Chiappori P.A. and Ekeland I. (2011): "New Developments in Aggregation Economics," Annual Review of Economics, 3: 631-668.

Chiappori P.A. and Lewbel A. (2015): "Gary Becker's A Theory of the Allocation of Time," Economic Journal, 125(583): 410–442.

Cockburn, J., E. Corong and C. Cororaton (2010): "Integrated Computable General Equilibrium (CGE) Micro-Simulation Approach," International Journal of Microsimulation, 3(1): 60-71.

Cockburn, J., L. Savard and L. Tiberti (2014): "Macro-Micro Models," Handbook of Microsimulation Modelling (Contributions to Economic Analysis, Volume 293), Ed. C. O'Donoghue, Emerald Group Publishing Limited: 275–304.

Colander, D. (1995): "Marshallian General Equilibrium Analysis", Eastern Economic Journal, Vol.21, No. 3.

Damania R. and Scandizzo, P.L. (2016): "The Serengeti ecosystem—Burden or bounty?" Journal of Policy Modeling.

Dalla Chiara, E., M. Menon, and F. Perali (2019): "An Integrated Database to Measure Living Standards," Journal of Official Statistics.

Davies, J. (2009): "Combining Microsimulation with CGE and Macro Modelling for Distributional Analysis in Developing and Transition Countries," International Journal of Microsimulation: 49-65.

Debreu, G. (1974): "Excess-demand functions". Journal of Mathematical Economics 1: 15–21.

Debreu, G. (1951): "The Coefficient of Resource Utilization," Econometrica.

De Janvry, A., M. Fafchamps, and E. Sadoulet (1991): "Peasant Household Behavior with Missing Markets: Some Paradoxes Explained," Economic Journal, 101:1400-17.

Devarajan, S., D. Go, J. Lewis, S. Robinson, and P. Sinko (1997): Simple General Equilibrium Modeling, in J. Francois and K. Reinert, eds., Applied Methods for Trade Policy Analysis, Cambridge University Press.

Commentato [CFP1]: Not referenced in the text

Dixit, A. K., and J. E. Stiglitz “Monopolistic Competition and Optimum Product Diversity,” American Economic Review, 67(3), 297–308, 1977.

Finizia, A., R. Magnani and F. Perali (2004): “A General Equilibrium Analysis of the Mid Term Review of the CAP on the Italian Economy,” Franco Angeli, Rome.

Grosh, M. and P. Glewwe (2000): “Designing Household Survey Questionnaires for Developing Countries: Lessons from 15 Years of the Living Standards Measurement Study,” World Bank.

Heckman, J. (2010): “Building Bridges between Structural and Program Evaluation Approaches to Evaluating Policy,” Journal of Economic Literature, 48(2): 356-398.

Imbens, G. (2010): “Better LATE Than Nothing: Some Comments on Deaton (2009) and Heckman and Urzua (2009),” Journal of Economic Literature, 48(2): 399-423.

Jorgenson D.W., Lau L.J. and Stoker T.M. (1980): “Welfare Comparisons and Exact Aggregation,” American Economic Review, 70(2): 268-272.

Jorgenson, D. W. (2007): ‘35 Sector KLEM’. http://hdl.handle.net/1902.1/10684.

Jorgenson, D. W. and J. Samuels (2014): “Long-term Estimates of U.S. Productivity and Growth,” Prepared for Presentation at Third World KLEMS Conference Growth and Stagnation in the World Economy, Tokyo, May 19-20, 2014.

Jorgenson, D., R. Goettle, M. Ho, and P. Wilcoxen (2013): “Double Dividend. Environmental Taxes and Fiscal Reform in the United States,” The MIT Press, MA.

Löfgren, H. and S. Robinson (1997): “The Mixed-Complementarity Approach to Specifying Agricultural Supply in Computable General Equilibrium Models,” No 197058, 1997 Conference, August 10-16, 1997, Sacramento, California, International Association of Agricultural Economists.

Löfgren, H., and S. Robinson (1999): “To Trade or Not to Trade: Non-Separable Farm Household Models in Partial and General Equilibrium,” TMD Discussion Paper No. 37, IFPRI, Washington, D.C.

Magnani, R., and J. Mercenier (2009): “On linking microsimulation and computable general equilibrium models using exact aggregation of heterogeneous discrete-choice making agents,” Economic Modelling, 26: 560–570.

Mantel, R. (1974): "On the characterization of aggregate excess-demand". Journal of Economic Theory 7: 348–353.

Matteazzi, E., M. Menon and F. Perali (2017): “The Collective Farm-household Model: Policy and Welfare Simulations,” Applied Economic Perspectives and Policy, 39 (1): 111–153.

Mercenier, J., M. Álvarez-Martínez, A. Brandsma, F. Di Comite, O. Diukanova, d’A. Kancs, P. Lecca, M. López-Cobo, Ph. Monfort, D. Persyn, A. Rillaers, M. Thissen and W. Torfs (2016): “Rhomolo v-2

Model Description. A Spatial Computable General Equilibrium Model for EU Regions and Sectors," JRC Technical Reports.

McKenzie, L. W. (1959): "On the Existence of General Equilibrium for a Competitive Economy," Econometrica 27 (1): 54–71.

Menon, M., F. Perali, R. Ray, and Tommasi, N. (2019). "The Tale of the Two Italies: Regional Price Parities Accounting for Differences in the Quality of Services," Working Paper Series, Department of Economics, University of Verona, Italy.

Nash, J. F. Jr. (1950): Equilibrium Points in n-Person Games, Proc. Nat. Acad. Sci. U.S.A., 36, 48-49.

Nash, J. F. Jr. (1951): Non-Cooperative Games, Ann. of Math., 54, 286-295.

Norton, R. and P. L. Scandizzo (1981): "General Equilibrium Computations in Activity Analysis Models", Operations Research, 29 (2): 243-262.

Obstfeld, M. and Rogoff, K., (2000). "New directions for stochastic open economy models," Journal of International Economics, 50: 117-153.

Pearl, J. (2013): "Reflections on Heckman and Pinto's `Causal Analysis After Haavelmo," UCLA Cognitive Systems Laboratory, Technical Report (R-420), Working Paper.

Peichl, A. (2016): "Linking Microsimulation and CGE Models," International Journal of Microsimulation: 9(1):161-75.

Ravenna, F. and F. M. Natalucci (2008). "Monetary Policy Choices in Emerging Market Economies: The Case of High Productivity Growth." Journal of Money, Credit and Banking 40.

Rogoff, K. (1992). "Traded Goods Consumption Smoothing and the Random Walk Behavior of the Real Exchange Rate," Bank of Japan Monetary and Econ. Stud., Nov. 1992, 10(2):1-29.

Samuelson, P. A. (1964). Theoretical Notes on Trade Problems. Review of Economics and Statistics 46, 145-154.

Savard, L. (2003): "Poverty and Income Distribution in a CGE-Household Micro- Simulation Model: Top Down/Bottom Up Approach," Laval University, CIRPEE Working Paper 03-43, CIRPEE, Université Laval: Quebec.

Scandizzo, P.L. and Ferrarese, C. (2015): "Social Accounting Matrix, a New Estimation Methodology", Journal of Policy Modeling.

Scandizzo, P.L. (2014), "Equilibrium Theory": Encyclopaedia of Law and Economics, Springer.

Scandizzo, P.L. (1995): A general equilibrium analysis of ownership, property rights and control in a transition economy,   ESA Working Paper (FAO), no. 02-01 / FAO, Rome (Italy).

Scandizzo, P.L. (2019): Natural Capital, the Lesson of CGE Models. World Bank Technical Paper.

Scarf, H.E. and T. Hansen (1973): The Computation of Economic Equilibria, Cowles Foundation for Research in economics at Yale University, Monograph No. 24, New Haven, CT and London, UK: Yale University Press.

Singh, I., L. Squire, and J. Strauss, eds. (1986): Agricultural Household Models, Extensions, Applications and Policy. Washington DC: The World Bank and The Johns Hopkins University Press.

Smets, F., and Wouters, R. (2002). An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. International Seminar on Macroeconomics. European Central Bank, Working Paper No. 171.

Solow, R. (2010). "Building a Science of Economics for the Real World" Prepared Statement to the House Committee on Science and Technology, Subcommittee on Investigations and Oversight.

Sonnenschein, H. (1973): "Do Walras' identity and continuity characterize the class of community excess-demand functions?". Journal of Economic Theory 6: 345–354.

Sonnenschein, H. (1972): "Market excess-demand functions". Econometrica (The Econometric Society), 40 (3): 549–563.

Stone, R. and A. Brown (1962): A computable model for economic growth, Cambridge, UK: Cambridge Growth Project.

Taylor, Ed. and I. Adelman (2006): “Village Economies: The Design, Estimation, and Use of Villagewide Economic Models,” Cambridge University Press.

Taylor, Ed. (2012): “A Methodology for Local Economy-wide Impact Evaluation (LEWIE) of Cash Transfers,” Working Paper, 99, International Policy Centre for Inclusive Growth.

Taylor, Ed. and M. Filipski (2014): “Beyond Experiments in Development Economics. Local Economy-wide Impact Evaluation,” Oxford University Press.

The Wye Group (2007): “Rural Households’ Livelihood and Well-Being. Statistics on Rural Development and Agriculture,” United Nations, NY.

Walbroeck, J.L. and Ginsburg, V.A. (1986): Activity analysis and general equilibrium modelling. Amsterdam, North Holland Publishing Co. Webley, O.

Wolff, E. N., Zacharias, A., Masterson, T., Eren, S., Sharpe, A., and E. Hazell (2012): “A comparison of Inequality and Living Standards in Canada and the United States Using an Expanded Measure of Economic Well-Being”. Working paper n. 703, Levy Economics Institute.

Appendix 1
A reference DSGE structure for a North –South representation for Italy

A goal of modern macro research is to provide a model that is consistent with macro "trends" such as economic growth or business cycles and is capable to anticipate the effects of economic policy. The model combines firm and households' dynamic behavior affected by stochastic shocks and includes a simple notion of applied general equilibrium across complete and efficient markets. SDGE models are implemented using either calibrated parameters or parameters estimated using econometric models based on time-series data or a combination of both. The New-Keynesian DSGE models build on a structure similar to Real Business Cycle models while striving to provide microeconomic foundations by introducing rational expectations and monopolistic competitions.

The objective of the structural representation of a SDGE model is to capture the essential features of a country such as Italy with a large technological and productivity divide between the North and the South of the nation that, according to the Balassa (1964) and Samuelson (1964) hypothesis, is one of the major determinants of the large gap in purchasing power parity (Menon et al. 2019). We also investigate the future environmental consequences of such a dualistic North-South development. A side objective is to use this prototype model to discuss the robustness of the microeconomic foundations of SDGE models in general and their potential to deliver useful ex-ante policy indications and/or ex-post policy evaluations.

The model reproduces a stylized small open economy aiming at understanding the welfare impact of changing terms of trade (ToT) between the richer region in the North of a country and the poorer South. In line with the Balassa (1964) and Samuelson (1964) hypothesis, richer countries (or regions) tend to have higher price levels than poor countries, because richer regions are relatively more productive in the tradable goods sector. Non-tradable goods are traditionally produced in the more labor intensive service sector (retail, banks, education, health, insurance, public administration, supply of public utilities, and others) where technological change has been traditionally adopted at a lower pace.

In a setting were the exchange rate is exogenously determined, as is the case for a ToT change within a country, a rise in productivity in the tradable goods sector translates into a rise in wages in the tradable goods sectors. Considering also that low-skilled labor is relatively more abundant and relatively cheaper in poor countries producing mainly more labor intensive non-tradable goods, it is reasonable to expect higher price levels in richer countries. In the present context, where the rich and poor "countries" of interest are two regions of the same country, the sector of non-tradable goods is characterized by wage rigidities due to the fact that wages are fixed through centralized negotiations setting the same wages for services for the whole country disregarding efficient mechanisms. This stickiness is also part of the explanations for the wide north-south price level gap.

We also intend to investigate what are the consequences of the regional differences in PPP on the regional capacity to reinvest resources on activities that prevent or cure climate change effects. The more productive region has a higher polluting potential and at the same time has larger control over resources to be diverted to improve environmental quality. Therefore, the price gap is expected to translate in a widening environmental gap depending on the relative costs and effectiveness of the quality of the cleaning technology adopted in each region.

The small open economy produces a non-tradable good (N) and a domestic tradable good (H) that is also produced abroad, and its price is exogenously determined in the world market. There is a continuum of goods (and varieties) in each category. Each region specializes in the production of a subset of tradable goods that are no longer perfect substitutes. Changes in productivity lead to an adjustment of the ToT and relative prices of the non-tradable goods. Consumers are employed in both production sectors and choose a basket of tradable (T) goods, composed of an imported foreign good (F) and a domestically produced good (H), and non-tradable goods (N). Consumers own their sector-specific labor and capital. Investments are a combination of tradable and non-tradable goods so that an increase in capital in one sector must be met by an increase in production in all sectors. The inputs of the domestic tradable sector come from a domestic value-added function, specified as a Cobb-Douglas aggregator of labor and capital, and an imported intermediate input. Similarly, output in the non-tradable sector is obtained as a combination of labor and capital. Graph 1 of this Appendix describes the stylized features of the North-South economy proposed here in a SDGE environment.

The general structure of the SDGE model follows Rogoff (1992), Obstfeld and Rogoff (2000), Smets and Wouters (2002), Ravenna and Natalucci (2008), and AmbriŠko (2015). Our model departs from the traditional representation because the effective quality of the inputs producing both the tradable and non-tradable goods varies by region, thus generating regionally differentiated goods and contributing to the regional separation in price levels. While wages for non-tradable goods are sticky, the wages of the tradable goods sector reflect productivity differentials. The other main distinctive feature is the inclusion of regional environmental taxes that are in toto reinvested for the prevention and management of climate change. These modelling features differentiate the behaviour of the representative producer, though maintaining strong limitations in terms of the exact micro-macro aggregation of the model.

Household preferences

In the economy there exists a representative household that maximizes its lifetime expected utility by choosing c_t and hours of work L_t

$$U_J = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ D_{t_J} log\left(C_{t_J}\right) - \lambda \frac{\left[L_{t_J}\right]^{1+\eta_L}}{1+\eta_L} \right\}$$

where $J = \aleph, S$ denotes the consumer of the North and the South, β is an exogenous discount factor, λ is the parameter capturing the disutility of labor supply, η_L is the inverse of the labor supply elasticity, $L_t = L_t^N + L_t^H$ is labor hours and D_{tJ} is an exogenous preference shock that is specific to each region. Total consumption is a Constant Elasticity of Substitution (CES) composite index of tradable T and non-tradable N goods

$$C_{t_J} = \left[\gamma_n^{\frac{1}{\rho_n}} C_{N_J}^{\frac{(\rho_n - 1)}{\rho_n}} + (1-\gamma_n)^{\frac{1}{\rho_n}} C_{T_J}^{\frac{(\rho_n - 1)}{\rho_n}} \right]^{\frac{\rho_n}{(\rho_n - 1)}}$$

where $0 \leq \gamma_n \leq 1$ is the share of non-tradable goods and $\rho_n > 0$ is the elasticity of substitution between tradable and non-tradable goods. Similarly, the tradable good is a CES composite index of home and foreign tradable goods

$$C_{T_J} = \left[\gamma_h^{\frac{1}{\rho_h}} C_{H_J}^{\frac{(\rho_h - 1)}{\rho_h}} + (1 - \gamma_h)^{\frac{1}{\rho_h}} C_{F_J}^{\frac{(\rho_h - 1)}{\rho_h}} \right]^{\frac{\rho_h}{(\rho_h - 1)}}$$

where $0 \leq \gamma_h \leq 1$ is the share of home tradable goods and $\rho_n > 0$ is the elasticity of substitution between home (H) and foreign (F) tradable goods. We also introduce north-south interregional trade that presumes that home consumption is a CES composite index of the domestic consumption produced in the North ($H_{\aleph}$) and in the South (H_S)

$$C_H = \left[\gamma_r^{\frac{1}{\rho_r}} C_{H_{\aleph},t}^{\frac{(\rho_r - 1)}{\rho_r}} + (1 - \gamma_r)^{\frac{1}{\rho_r}} C_{H_S,t}^{\frac{(\rho_r - 1)}{\rho_r}} \right]^{\frac{\rho_r}{(\rho_r - 1)}}$$

where $0 \leq \gamma_r \leq 1$ is the share of home tradable goods and $\rho_r > 0$ is the elasticity of substitution between the domestic goods ($H_{\aleph}$) from the North and domestic (H_S) goods from the South.

Given the above preference structure, also the P_t price of total consumption is aggregated using CES composite indices

$$P_t = \left[\gamma_n \; P_{N,t}^{(1-\rho_n)} + (1 - \gamma_n) \;\; P_{H,t}^{(1-\rho_n)} \right]^{\frac{1}{(1-\rho_n)}}$$

where P_t is the consumer price index (CPI) and P_N is the price index for non-tradable goods.

The price of tradable goods is determined abroad following the law of one price. Therefore, the price for the domestic tradable good P_H is

$$P_{H,t} = ER_t P_t^*$$

where P_t^* is the exogenous foreign currency price of tradable goods and ER_t is the nominal exchange rate assuming full pass-through of the exchange rate. The price of the domestic tradable good $P_{H,t}$ are regionally differentiated and are aggregated according to the CES weighting procedure

$$P_{H,t} = \left[\gamma_r \; P_{H_{\aleph},t}^{(1-\rho_r)} + (1 - \gamma_r) \;\; P_{H_S,t}^{(1-\rho_r)} \right]^{\frac{1}{(1-\rho_r)}}.$$

Prices (and wages) of the non-tradable sector are centrally determined and have no regional variation. Investments aggregates I_t^J, $I_{T,t}^J$ and $I_{N,t}$ for J=N,H are CES-like combination similar to the consumption aggregates.

Households hold physical capital that rent to firms of each specific sector. Movements of capital between tradable and non-tradable sectors, differently from labor, is not admitted. Capital accumulation is subject to convex adjustment costs Φ(·) to limit investment volatility. Capital K_{t-1}^J depreciates at a common constant rate δ>0. The law of accumulation of the capital stock takes the form

$$K_t^J = \Phi\left(\frac{I_t^J}{K_{t-1}^J}\right) \; K_{t-1}^J + (1-\delta)\, K_{t-1}^J, \qquad J = N, H.$$

The household budget constraint is

$$P_t C_t + B_t + ER_t B_t^* + P_t(I_t^N + I_t^H) = W_t^H L_t^H + W_t^N L_t^N + R_{t-1} B_{t-1} + R_{t-1}^* \, ER_t B_{t-1}^* + P_{N,t} R_t^N K_{t-1}^N + P_{H,t} R_t^H K_{t-1}^H + \Pi_t + \Im_t$$

where W_t^N and W_t^H are the nominal wages in the non-tradable and domestic tradable sector, B_t and B_t^* denote holdings of bonds denominated in domestic and foreign currency earning R_t and R_t^* interest rates, R_t^N and R_t^H are the real returns to capital in the non-tradable and domestic tradable sector and Π_t are nominal profits from monopolistically competitive firms in the non-tradable sector. The right-hand side of the constraints represents wealth as remuneration to labor and capital supplied to the firms, from interest on bonds, and from profits derived from ownership of firms. The left-hand side describes how wealth inclusive of lump sum government transfers net of lump sum taxes $\Im_t$, is used to purchase consumption and investment goods or saving bonds.

Firms

Non-tradable sector. The production technology of the monopolistically competitive non-tradable goods sector, where a continuum of firms $z \in [0,1]$ for $J = \aleph, S$ operates, is of the Cobb-Douglas form. It aggregates labor and capital to yield a single variety of non-tradable goods

$$Y_{N,t}(z) = A_t^N [L_t^N(z)]^{(1-\alpha_N)} [K_{t-1}^N(z)]^{\alpha_N}$$

where A_t^N is a factor-neutral technical change shift in the non-tradable sector that is not subject to random shocks.

Tradable sector. The tradable good H is produced both at home and abroad in a perfectly competitive setting. Domestic producers combine imported intermediate goods $X_{M,t}$ and domestic value-added goods $V_{H,t}$ within a CES technology

$$Y_{H,t} = \left[\gamma_v^{\frac{1}{\rho_V}} V_{H,t}^{\frac{(\rho_V-1)}{\rho_V}} + (1-\gamma_v)^{\frac{1}{\rho_V}} X_{M,t}^{\frac{(\rho_V-1)}{\rho_V}}\right]^{\frac{\rho_V}{(\rho_V-1)}}$$

where $0 \leq \gamma_v \leq 1$ is the share of domestic tradable goods in tradable output, and $\rho_V > 0$ is the elasticity of substitution between imported intermediate goods $X_{M,t}$ and domestic value added goods $V_{H,t}$ produced with labor and tradable (mobile) capital whose productivity varies regionally in relation to the factor specific efficiency parameters θ_L and θ_K and output elasticities $\alpha_{h_\aleph}$ and α_{h_S}

$$V_{H_\aleph,t} = A_t^{H_\aleph}\left[\frac{L_t^{H_\aleph}}{\theta_{L_\aleph}}\right]^{(1-\alpha_{h_\aleph})}\left[\frac{K_{t-1}^{H_\aleph}}{\theta_{K_\aleph}}\right]^{\alpha_{h_\aleph}} - \psi_{H_\aleph} V_{H_\aleph,t-1}$$

$$V_{H_S,t} = A_t^{H_S}\left[\frac{L_t^{H_S}}{\theta_{L_S}}\right]^{(1-\alpha_{h_S})}\left[\frac{K_{t-1}^{H_S}}{\theta_{K_S}}\right].^{\alpha_{h_S}} - \psi_{H_S} V_{H_S,t-1}$$

Effective labor and capital in the North are more productive $0 < \theta_{L_\aleph}, \theta_{K_\aleph} \leq 1$ than in the South where $1 \leq \theta_{L_S}, \theta_{K_S} \leq 2$. The upper limit of the efficiency parameter in the South guarantees that factor productivity in the South cannot plausibly be less than half with respect to the North. A_t^H is a factor neutral exogenous technology shock. The fixed cost term $\psi_{H_J} V_{H_J,t-1}$ for $J = \aleph, S$ and $0 < \psi_{H_J} < 1$ is a production cut due to the reinvestment of part of the revenues obtained in the previous period in environmentally friendly technologies. This is a means, proportional to the volume of potentially polluting production, to enforce the internalization, though sub-optimal, of environmental externalities produced by the firms. The technology process can be represented as

$$\frac{A_t^J}{A_{t-1}^J} = e^{\mu_{J,t}};\ J = \aleph, S$$

with μ being the technology growth rate following an AR(1) process that is supposed to be higher in the North than in the South $\mu_{H_\aleph} > \mu_{H_S}$. This stochastic specification of technical change occurring in the domestic tradable sector introduces a permanent productivity increase at the expenses of a certain degree of non-stationarity in the model. Value added produce in the North and in the South aggregate to total domestic value added with the CES-like aggregation

$$V_{H,t} = \left[\gamma_\delta^{\frac{1}{\rho_D}} V_{H_\aleph,t}^{\frac{(\rho_D-1)}{\rho_D}} + (1-\gamma_\delta)^{\frac{1}{\rho_D}} V_{H_S,t}^{\frac{(\rho_D-1)}{\rho_D}}\right]^{\frac{\rho_D}{(\rho_D-1)}}$$

where 0≤ γ_δ ≤1 is the share of domestic tradable goods produced in the North in domestic value added, and ρ_D >0 is the elasticity of substitution between value added produced in the North and value added produced in the South. Cost minimization implies a regionally differentiated capital-labor ratio equal to the ratio of the corresponding region-specific output elasticities.

Monetary and Fiscal Policy

The central bank sets nominal gross interest rates and adjusts them in response to inflation according to the following general Taylor rule

$$R_t = (R_{t-1})^{\sigma}\left[R\left(\frac{\Pi_t}{\Pi}\right)^{\varphi_p}\right]^{1-\sigma} exp(MS_t - NS_t^{MS})$$

where R is the steady state nominal gross interest rate, $\varphi_p \geq 0$ is the feedback coefficient to CPI inflation, Π is the central bank's inflation target, Π_t is the CPI inflation rate, $0 \leq \sigma < 1$ is the interest rate smoothing parameter, MS_t is an exogenous monetary policy shock, and NS_t^{MS} is the news shock, whose strength depends on the length of the announcement and its persistence (AmbriŠko, 2015). The fiscal authority balances the budget in each period by ensuring that government expenditures and transfers to the households are fully financed by lump sum taxes. Because of the representative agent assumption, Ricardian equivalence is embedded in the model.

Market Clearing

Output of each firm producing non-tradable goods is either consumed domestically or spent on investment. Similarly, all tradable goods are consumed or invested domestically or abroad. The resource constraints for the tradable and non-tradable sectors therefore are

$$Y_{N,t} = C_{N,t} + I_{N,t}^{N} + I_{N,t}^{H}$$

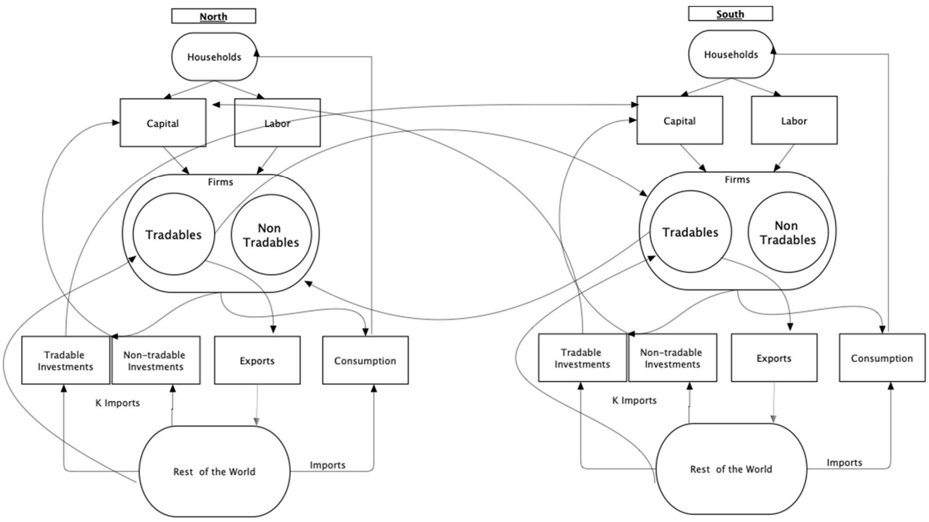
$$Y_{H,t} = C_{H,t} + I_{H,t}^{N} + I_{H,t}^{H} + X_t.$$

The value of aggregate output equals the value of tradable and non-tradable output that can also be expressed in deflated terms using their respective CPI

$$Y_t = \frac{P_{N,t}}{P_t} Y_{N,t} + \frac{P_{H,t}}{P_t} Y_{H,t}$$

In equilibrium the model simultaneously determines regionally differentiated output and CPIs, consumptions, investment, employment and interest rate. Because the model does not have a closed-form solution, the empirical implementation of the model requires the derivation of the first order conditions of the model in order to describe the non-stochastic steady state at which it is possible to log-linearize the model.

Appendix. Graph 1. A Stylized North-South Economy in a DSGE Environment



Appendix 2.
An Integrable CGE Model for Italy

PRICE EQUATIONS

$PMi = pwmi(1+tmi) \cdot ER$ (1)

$PEi = PWEi(1-tei) \cdot ER$ (2)

Where:

PMi = domestic price of imports

PEi = domestic price of exports

Pwmi=world prices of imports

PWEi=world prices of exports (fixed)

Tmi=import tariffs

Tei=export subsidies

ER=exchange rate

These two equations represent the domestic price of imports PM and exports PE, where pwm and pwe are the world prices and set exogenously, incorporating the assumption of small country. These prices are multiplied for import tariff for imports and export subsidies for exports times the exogenous exchange rate ER.

$$P_{ij} = \frac{(PD_{ij} * D_{ij}) + (PM_{ij} * M_{ij})}{X_{ij}} \quad (3)$$

Where:

Pij=composite price of domestic goods in region j; j=A,B

PDij = domestic price of domestic goods in region j;

PMij = domestic price of imports in region j

Dij=domestic produced goods in region j

Mij=imported goods in region j

Xij=quantity of the composite good in region j

This equation reflects the composite price paid by domestic demanders, which is composed by domestic products and imports, where D and M are respectively domestic produced and imported good, with the respective prices PD and PM, and Xi is the quantity of the composite good. The price, however, is different in region A from region B, since the product mix in the two regions is different.

$$PX_{ij} = \frac{(PD_{ij} * D_{ij}) + (PE_{ij} * E_{ij})}{Q_{ij}} \qquad (4)$$

Where:

PXij=Sales price of output in region j

PDij = domestic price of domestic goods in region j;

PEij = domestic price of exports in region j

Dij=domestic produced goods in region j

Eij=exported goods from region j

Qij=domestic output in region j

Equation (4) above reflects the homogeneity of the export transformation function and represents the price of the domestic output of region j Xij. PXij is the sale Price, composed by the price of domestic regional good PDij and the price of exports PEij times their respective quantities D and E, divided by the sectorial output Q.

$$PV_{ij} = (PX_{ij} * (1 - tx_i)) - (\Sigma_k\, io_{ki} * P_{ki}) \qquad (5)$$

Where:

PVij=Value added price of ith good in region j

PXij = Sales price of output in region j

txi=indirect tax rate

ioik=input-output coefficients

Pj in region j =composite price of domestic goods in region j

Equation (5) defines the sectoral value added, which is given by the output price, minus indirect taxes and the cost of intermediate input with fixed input-output coefficients. The equation defines the value added at factor costs paid to primary factors labor and capital.

$$PK_{ij} = (\Sigma_j \, imat_{ki} * P_k) \quad (6)$$

Where:

PKi=unit price of capital

imatik = capital composition matrix

Pk=composite price of domestic goods

This equation states the price of a unit of capital installed in each sector, which differs among sectors and regions due to the heterogeneity in use of capital.

PINDEX = GNPVA/ RGNP (7)

Where:

PINDEX = aggregate price index (GNP deflator)

GNPVA = Nominal GNP

RGNP = real GNP

This equation defines the numeraire price index, which is fixed and defines the absolute price level against which all relative prices are measured.

PRODUCTION EQUATIONS

$$\boldsymbol{Q_i = ad_i * \Pi_f(FDSC_{if}^{\alpha_{if}})} \quad (8)$$

where:

Qi=domestic output

adi = Production function shift parameter

FDSCif = factor demand

αif = share parameter in production function

This equation defines the Cobb-Douglas value-added production function. FDSC represents primary factors demand (labor and capital). First order condition for profit maximization bring to the next equation:

$$WF_f * wfdist_{if} = \frac{PV_i * \alpha_{if} * Q_i}{FDSC_{if}} \quad (9)$$

Where:

WFf = average factor price

wfdistif = factor market distortion parameters

FDSCif = factor demand

αif = share parameter in production function

Qi=domestic output

PVi=Value added price

The value added price PV is used for profit maximization wfdist is distortion in factor markets, WF is the remuneration of the factors and α is the shift parameter.

$$INT_i = \Sigma_j (io_{ji} * Q_i) \quad (10)$$

where:

INTi = intermediate input demand

ioij=input-output coefficients

Qi=domestic output

This equation reflects the demand for intermediate inputs, which assumes fixed input-output coefficients and are function of domestic output Qi. Intermediate inputs are function of domestic production (X)

Sectoral output is defined by a CET function combining domestic and export sales:

$$\boldsymbol{Q_{ie} = at_{ie} * (\gamma_{ie} * E_{ie}{}^{\rho_{ie}} + (1-\gamma_{ie})D_{ie}{}^{\rho_{ie}})^{\frac{1}{\rho_{ie}}}} \quad (11)$$

where:

Qie=domestic output for sector with exports

at_{ie}= CET function shift parameter

γ_{ie}= CET function share parameter

ρ_{ie}= CET function exponent

E_{ie}= Exports

D_{ie}= Domestic sales for sector with exports (ie)

This equation reflects the export transformation function and the functional form is a constant elasticity of transformation (CET) between domestic sales and exports, where revenue maximization from sales is subject to the CET transformation function.

$$\boldsymbol{E_{ie} = D_{ie} * (\frac{PE_{ie}}{PD_{ie}} * \frac{(1-\gamma_{ie})}{\gamma_{ie}})^{\frac{1}{\rho_{ie}-1}}} \quad (12)$$

where:

γ_{ie}= CET function share parameter

ρ_{ie}= CET function exponent

E_{ie}= Exports

D_{ie}= Domestic sales for sector with exports (ie)

PDie = domestic price of domestic goods ;

PEie = domestic price of exports

The equation above represents sectorial export supply function, which depends on relative price (PE/PD). It represents the first order condition for the CET equation and is function on relative export price to domestic price, the elasticity of transformation and the share parameter in the CET function.

The world export demand function for exports is expressed as:

$$\boldsymbol{E_{ied}} = \boldsymbol{\varphi_{ied}} * ((\frac{\boldsymbol{PWE_{ied}}}{\boldsymbol{pwse_{ied}}})^{-\psi_{ied}} \qquad (13)$$

where:

E_{ie}= Exports for the sector that have export demand from the rest of the world (ied)

φ_{ied}= Export demand function shift parameter

PWEied= world prices of exports

pwseied= world prices of export substitutes

ψ_{ied}= function exponent by sector

Next equation describes composite good supply, determining, how imports and domestic products are demanded.

$$\boldsymbol{X_{im}} = \boldsymbol{ac_{im}} * (\boldsymbol{\delta_{im}} * \boldsymbol{M_{im}}^{-\eta_{im}} + (\boldsymbol{1} - \boldsymbol{\delta_{im}})\boldsymbol{D_{im}}^{-\eta_{im}})^{\frac{-1}{\eta_{im}}} \qquad (14)$$

where:

Xim=quantity of the composite good for sector with imports (im)

ac_{im}= CES function shift parameter

δ_{im}= CES function share parameter

η_{im}= CES function exponent

M_{im}= imports

D_{im}= Domestic sales for sector with imports (im)

The equation represents the import aggregation function, which is a Constant elasticity of substitution function (CES) aggregate of imported and domestic goods, where consumers demand a composite good. The first order condition for minimizing the cost of buying a given amount of composite good is given by the next equation:

$$M_{im} = D_{im} * (\frac{PD_{im}}{PM_{im}} * \frac{(1-\delta_{im})}{\delta_{im}})^{\frac{1}{\eta_{im}+1}} \quad (15)$$

where:

δ_{im}= CES function share parameter

η_{im}= CES function exponent

M_{im}= imports

D_{im}= Domestic sales for sector with imports (im)

PDim = domestic price of domestic goods ;

PMim = domestic price of imports

INSTITUTION EQUATIONS

$$Y_f^F = \Sigma_i(WF_f * wfdist_{if} * FDSC_{if}) \quad (16)$$

Where:

Y_f^F = incom of primary factors (labor and capital)

WFf = average factor price

wfdistif = factor market distortion parameters

FDSCif = factor demand

This equation reflects the factor income equation Y_f^F defined as the sum of factor income times the sectoral capital stock FDSC times the specific sectoral distortion wfdist

$$Y^{ent} = Y_f^F + GENT - ENTSAV - ENTTAX \quad (17)$$

where:

Y^{ent}= enterprise income

GENT= transfer payment from Government to enterprises

ENTSAV= enterprise savings

ENTTAX= enterprises taxes

Last equation defines enterprise income, which is equal to factor income plus transfers from government to enterprises minus enterprise saving and taxes.

Similarly, household income is defined as:

$$\boldsymbol{Y_{hh} = Y^{ent} + HHT + REMIT * EX} \quad (18)$$

where:

Y_{hh}= households' income

HHT= transfer payment from Government to households

REMIT= Remittances from abroad

EX= Exchange rate

Tariff, indirect and direct taxes are expressed as follow:

$$\boldsymbol{TARIFF = \Sigma_{im}(tm_{im} * pwm_{im} * M_{im} * EX)} \quad (19)$$

where:

$TARIFF$= import Tariff

tm_{im}=Tariff rate on imports

Pwmim=world prices of imports

Mim= imports

EX= Exchange rate

$$\boldsymbol{INDTAX = \Sigma_{i}(PX_{i} * Q_{i} * tx_{i})} \quad (20)$$

where:

$INDTAX$= Indirect taxes (by sector i)

PX_i= Output price

Qi=Domestic Output

txi= indirect tax share (from I-O coefficient table)

$$\boldsymbol{HHTAX} = \boldsymbol{\Sigma_i}(\boldsymbol{Y_{hh}} * \boldsymbol{tx_{hh}}) \tag{21}$$

where:

$HHTAX$= Households' taxes

Y_{hh}= houesolds' income

txhh= direct household tax share (from I-O coefficient table)

The sum of these variables gives the revenues for the government:

$$\boldsymbol{GR} = \boldsymbol{TARIFF} + \boldsymbol{INDTAX} + \boldsymbol{HHTAX} + \boldsymbol{ENTTAX} + \boldsymbol{nc_g} \tag{22}$$

where:

GR= Government revenues

$INDTAX$= Indirect taxes (by sector i)

$TARIFF$= import Tariff

ENTTAX= enterprises taxes

$HHTAX$= Households' taxes

$\mathrm{nc_h}$= Restoration of natural capital by government expenditure

$$\boldsymbol{HHSAV} = \boldsymbol{\Sigma_i}(\boldsymbol{Y_{hh}} * (\boldsymbol{1} - \boldsymbol{tx_{hh}}) * \boldsymbol{mps}) \tag{23}$$

Where:

$HHSAV$= Households' saving

Y_{hh}= Houesolds' income

txhh= direct household tax share (from I-O coefficient table) mps=marginal propensity to save

Last equation represents household saving, which is function of marginal propensity to save (mps) and household income Yh.

Total saving includes government, household and enterprise saving, as well as foreign saving in domestic currency (FSAV*ER):

SAVING = HHSAV + GOVSAV + ENTSAV + FSAV*EX (24)

Where:

$SAVING$= total saving

$HHSAV$= Households' saving

ENTSAV= enterprise savings

GOVSAV=Government savings

FSAV= Foreign savings

EX=exchange rate

Household consumption is defined by the following equation:

$$\boldsymbol{CD_i} = \frac{\boldsymbol{\Sigma_{hh}(\beta_{ihh} * (1 - mps_{hh}) * Y_{hh} * (1 - tx_{hh}) * (1 - nc_h))}}{\boldsymbol{P_i}} \quad (25)$$

Where:

CD_i = consumer expenditure

β_{ihh}= household expenditure share

mps=marginal propensity to save

txhh= direct household tax share (from I-O coefficient table)

Y_{hh}= Households' income

Pi=composite price of domestic goods

$\mathrm{nc_h}$= Restoration of natural capital by households expenditure (shares)

This equation reflects the consumer expenditure function, which is function of prices and income, according to a simplified version of a Linear Expenditure System (LES). This function in its simple form, reduces to fixed expenditure shares and a Cobb-Douglass utility function.

Government consumption is defined by next equation:

$$\boldsymbol{GD = \vartheta_i{}^G * GDTOT} \quad (26)$$

Where:

GD= Government Consumption

$\vartheta_i{}^G$= government expenditure shares

GDTOT = real aggregate government consumption

This equation is defined in terms of fixed shares of real aggregate government spending on goods and services.

Next equations determine the demand for capital goods:

$$\boldsymbol{DK_i = \tau_i * \frac{FXDINV}{PK_i}} \quad (27)$$

$$\boldsymbol{ID_i = \Sigma_j(imat_{ij} * DK_j)} \quad (28)$$

Where:

FXDINV=aggregate nominal fixed investment

DK_i=Fixed investment by sector of destination

τ_i=share of investment by sector of destination (from I-O coefficient matrix)

ID_i= final demand for investment good

$imat_{ij}$=capital composition matrix

FXDINV represents the aggregate nominal fixed investment and is determined by total investments. DK is the sector in investment destination, calculated as fixed investment times the investment shares (τ_i). Multiplying DK for capital composition matrix, we obtain the demand for capital goods by sector of origin ID. This basic CGE model is static with the capital stock as exogenous variable. The model generates savings, investments and capital goods that are assumed not to be installed during the period.

$$\boldsymbol{GNPVA = \Sigma_i(PV_i * Q_i) + INDTAX + TARIFF} \quad (31)$$

$$\boldsymbol{RGNP} = \boldsymbol{\Sigma_i}(\boldsymbol{CD_i} + \boldsymbol{ID_i} + \boldsymbol{GD_i}) + (\mathbf{E}_{ie} + \boldsymbol{CDTOUR}) - \boldsymbol{M_{im}} \quad (32)$$

Where:

$GNPVA$=GNP generated from Value added

$RGNP$=Real GNP

PVi=Value added price

Qi=Domestic Output

$INDTAX$= Indirect taxes (by sector i)

$TARIFF$= import Tariff

CD_i = consumer expenditure

ID_i= final demand for investment good

GD= Government Consumption

Mim= imports

E_{ie}= Exports

These two last equations define real and nominal GNP. These are used to define GNP deflator in the price equation block.

System of Constraints

$$\boldsymbol{X_i} = \mathbf{INT}_i + \boldsymbol{nc_i} + \boldsymbol{CD_i} + \boldsymbol{ID_i} + \boldsymbol{GD_i} \quad (33)$$

Where:

Xi=quantity of the composite good

INTi = intermediate input demand

CD_i = consumer expenditure

ID_i= final demand for investment good

GD= Government Consumption

$\mathrm{nc_i}$= depletion of natural capital by government expenditure

$$\boldsymbol{\Sigma_i}(\boldsymbol{FDSC_{if}}) = \boldsymbol{fs_f} \quad (34)$$

Where

FDSCif = factor demand

fs_f= factor supply

$$\boldsymbol{GR = \Sigma_i(GD_i * P_i) + GOVSAV + GENT + HHT} \quad (35)$$

Where:

GR= Government revenues

GD= Government Consumption

GOVSAV=Government savings

HHT= transfer payment from Government to households

GENT= transfer payment from Government to enterprises

Pi=composite price of domestic goods

$$\boldsymbol{pwm_i * M_i = pwe_i * E_i + FSAV} \quad (36)$$

Where

M_{im}= imports

E_{ie}= Exports

Pwmi=world prices of imports

PWEi=world prices of exports

FSAV= Foreign savings

SAVING = INVEST (37)

The first equation of the block (33) states that sector supply of composite commodities must equal demand, defining market-clearing condition for product market. A separate equation for domestically goods sold on domestic market is not needed, since the market for domestic goods will clear when the market for composite goods is in equilibrium.

The second equation (34) defines equilibrium in factor markets, where the supply of primary factors equals the factor demand and the equilibrating variable is the factor wage. The supply of primary factors fs is assumed exogenous and given as a parameter. WF (f), which represents the average factor prices, is the equilibrating variable into the factor market. In the model, all factors are freely mobile.

The last three equations (35-37) define the three major macroeconomic closures: Government deficit, saving-investments and balance of trade. According to Walras' law, these three macro-balances satisfy the identity: private savings + government savings + foreign savings = aggregate investment.

All financing items of the balance of trade equation are fixed, (e.g. net foreign savings and remittances), hence the balance of trade in the model is set exogenously with the nominal exchange rate as the equilibrating variable. When the nominal exchange rate changes, the relative prices of non-tradables (PD) as well as tradables (PE and PM) also change. The model determines the exchange rate that brings about equilibrium in the balance of trade.

Government revenues are determined by constant tax parameters and Government savings is determined residually by equation 35. Constant coefficient based institutional and household savings determine private savings, while foreign saving is set exogenously.

The net effect of these hypotheses in the model is the so-called "neoclassical closure", where aggregate investment is determined by aggregate savings in a "saving-driven" model.