The endogenous choice of delegation in a duopoly with input outsourcing to the rival

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Abstract

In a market in which a vertically integrated producer (VIP) supplies an essential input to a retail rival, we explore the role of managerial delegation when it shapes downstream firms' incentives and determine the endogenous choice of delegation under both Cournot and Bertrand competition. The equilibrium choice of acting as a managerial firm, which is a standard result in literature of strategic delegation, is shown to be robust to the presence of a vertical supply relationship in both the quantity competition and the price competition framework, regardless of the degree of product differentiation. The paper, however, highlights the different motives pushing the integrated firm and the independent retailer towards delegation, which also revert the standard result that delegation causes a prisoner's dilemma-type equilibrium under Cournot and a more profitable outcome under Bertrand.

Jel Classification: D43, L13, L21.

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1 Introduction

Starting from the pioneering works by Vickers (1985), Fersthman (1985), Fersthman and Judd (1987) and Sklivas (1987) (VFJS hereafter), the need for managerial compensation on the basis of sales' maximization, stemming from separation of ownership from management in large companies (Berle and Means, 1932) and widely recognized in corporate governance literature (e.g., Baumol, 1958), has been reconsidered in a strategic context. The VFJS approach draws on the use of incentive contracts designed by firms' owners to manipulate their managers' behavior on the product market and attain a strategic advantage. Their main findings are that owners find optimal to distort their managers from profit maximization in order to commit to a more (less) aggressive behavior respectively under quantity (price) competition. Literature on strategic delegation has also extended the above basic models to explicitly model the endogenous choice of delegation in a duopoly, showing that the choice of hiring a manager emerges as the Nash equilibrium solution of both the Cournot game and the Bertrand game (e.g., Basu, 1995, Lambertini, 2017).[1] While in the former game firms face a prisoner-dilemma situation due to higher competition induced by the equilibrium quantities exceeding the profit-maximizing level,[2] in the latter the cooperative effect of delegation on equilibrium prices leads them to enjoy higher profits with respect to no-delegation. Such a result is also sustained by the fact that unilateral delegation induces a Stackelberg outcome, which raises the delegating firm's profits and reduces those of the rival under Cournot, while it results in higher profits for both firms under Bertrand.

The above works hinge on the assumption that delegation to managers occurs within an integrated firm sourcing inputs and selling output, while it has never been assumed in a vertical structure in which an independent retailer buys an input rather than makes it. More generally, strategic delegation has received less attention in the literature on firms' vertical relationships, with the exception of Park (2002) and Moner-Colonques et al. (2004).[3]

The objective of this paper is to revisit strategic delegation of market decisions to managers in both a Cournot and a Bertrand duopoly in which one

[1]Most recent literature on strategic delegation has extended the VFJS framework to allow for R&D investments (Zhang and Zhang, 1997, Kopel and Riegler, 2006), firms' unionization (Fanti and Meccheri, 2013), endogenous timing (Lambertini, 2000; Fanti, 2017), mergers (Gonzalez-Maestre and Lopez-Cunat, 2001), to quote a few examples. See Lambertini (2017) for a comprehensive overview of strategic delegation in oligopoly games.

[2]This is clearly resumed in the words of Berr (2011, p. 251): "Unfortunately, this [i.e. the use of managerial delegation] results in lower payoffs for both owners than in a standard Cournot game, and a prisoner's dilemma situation emerges, i.e. although both owners would benefit by abstaining from the use of incentives and, hence, play a normal Cournot game, they will not".

[3]In a vertical structure with an upstream monopoly and two managerial firms, Park (2002) assumes that managerial delegation affects both the monopolist's decision regarding the wholesale price and retail competition. In this framework, he finds that managers are instructed to maximize profits at equilibrium, regardless of whether firms compete downstream in quantities or prices. Conversely, in Moner-Colonques et al. (2004) delegation of sales to independent retailers in a multi-product context is interpreted as strategic delegation, but is not associated with the use of incentive schemes.

retailer is integrated with a manufacturer and the latter provides a key input to the downstream competitor. This assumption is common in the Industrial Organization literature which focuses on the profit and welfare consequences of vertical integration (Riordan, 1998; Kuhn and Vives, 1999; Chen, 2001; Arya, 2008; Moresi and Schwartz, 2017).[4] Outsourcing to a vertically integrated rival also fits many real-world cases. For example, in the telecommunications and in the railway industries, vertically integrated incumbent operators routinely supply key inputs (e.g., telephone loops and fixed railway network) to retail competitors. Also the electronic industry is a typical real-world example of "giant" firms purchasing key inputs from direct competitor: as reported by Chen (2010, p. 302) "in 1980s, IBM outsourced the micro-processor for its PC to Intel and the operating system to Microsoft". Moreover, as also noted by Arya et al. (2008, pp. 1-2), other firms such as soft-drink producers, cereal manufacturers, and gasoline refiners have long supplied key inputs to both their downstream affiliates and retail competitors.

Within the above framework, this paper examines the strategic choice to hire a sales-interested manager or not. This choice is made by firms' owners at a preplay stage of a Cournot and a Bertrand game. At the second stage, the vertically integrated producer (VIP) charges the independent retailer a wholesale linear price. At the third stage, each owner decides upon the degree of discretion to include in a managerial contract (if any), while quantity or price competition with managers acting as decision-makers takes place at the last stage of the game.[5] The results we obtain are as follows. We find that a Nash equilibrium with symmetric delegation is the equilibrium choice of each game. The presence of a supply relationship in a duopoly, however, is shown to reverse the payoff sequence popularized by the established literature addressing the case of independent firms. In contrast to the result achieved in previous literature, indeed, we get that unilateral delegation by each firm benefits the rival in the Cournot model and harms the rival in the Bertrand model, which causes a Pareto-improving equilibrium arising in the former and a prisoner-dilemma in the latter. Such a result is driven by the VIP's incentive to exploit through delegation its monopoly power on the upstream input market by inducing a greater demand of inputs from the rival, regardless of the mode of competition. For the independent firm, conversely, it is driven by the advantages it gains from an output expansion and from higher retail prices induced by delegation respectively in Cournot and Bertrand. Such unilateral incentives are shown to hold and yield higher profits than no-delegation, irrespective of the rival's choice to delegate or not market decisions to managers. This leads symmetric delegation to arise as an equilibrium regardless of the mode of competition. Moreover, the effects of such unilateral incentives towards delegation are found to benefit

[4] See Wu (1992) for an analysis concerning the strategic impact of vertical integration in oligopolistic markets. See also Lafontaine and Slade (2007) for empirical evidence on the consequences of vertical mergers, and Abiru (1988) for some implications for anti-trust laws.

[5] In this work we assume that managerial contracts are publicly observable. For a discussion on the observability and commitment problems in delegation models, see Kockesen and Ok (2004) and literature therein.

one firm's rival under Cournot competition, since they result in independent firm's output expansion which is profit-enhancing for both firms, while they harm one firm's rival under Bertrand competition, the VIP being interested in an output expansion by the independent firm, thus its higher demand of inputs, and the latter in softening retail price competition associated with its output contraction. This argument sustains the fact that symmetric delegation arising at equilibrium in Cournot is more profitable for both firms with respect to no-delegation, due to the coincidence of firms' objectives achieved in this setting through delegation, while the equilibrium with symmetric delegation in Bertrand entails a prisoner dilemma, which is due to the underlying conflict between firms' objectives caused by delegation.

This paper contributes to the existing literature in several ways. First, it captures the implications of outsourcing to an integrated producer on firms' delegation choices in a duopoly, so far not explored in literature on vertical relationships. Second, it highlights some circumstances in which symmetric delegation has a profit-enhancing role under quantity competition, which contributes to the established literature on strategic delegation.[6]

The reminder of the paper is organized as follows. Section 2 develops the model under both Cournot and Bertrand competition. Section 3 draws some conclusions.

2 The model

We assume that firm 1 is a vertically integrated producer (VIP) that is the sole producer of an essential input supplied to its downstream unit and to its downstream independent competitor, firm 2. Assumptions on firms' technology are that each retailer can convert one unit of input into one unit of output at no cost, and sustains output production costs implying constant marginal costs, equal to c_1 and c_2 respectively for firm 1 and firm 2 (with $c_1 \leq c_2$),[7] and zero fixed costs. Without loss of generality, firm 1's cost of input production is normalized to zero.

We introduce the following demand for differentiated product faced by firm i ($i = 1, 2$), as in Dixit (1979) and Singh and Vives (1984):

[6] One exception is given in Berr (2011) who shows that the inherent prisoner's dilemma in a Cournot duopoly with managerial firms can be overcome by assigning managers the appropriate incentive contract, i.e., a multiplicative incentive. One partial exception is the result by Fanti and Meccheri (2017), according to which at the equilibrium of the standard Cournot game "the more efficient firm may obtain higher profits provided that the degree of cost asymmetry between firms is sufficiently large" (p. 279). However such a result concerns only one firm and holds only under appropriate conditions, while the present one always applies to both rival firms. In this regard, it should be also considered the result of Fanti and Meccheri (2017) that, at the Cournot equilibrium with two managerial firms, "the more efficient firm may obtain higher profits provided that the degree of cost asymmetry between firms is sufficiently large" (p. 279), despite such a result concerns only one firm and holds only under appropriate conditions.

[7] Indeed, as standard in this literature (see also Arya et al., 2008), we assume that the VIP is at least as efficient as its downstream rival.

$$p_i = a - \gamma q_j - q_i$$

where q_i and p_i are respectively firm i's retail output and retail price, while the parameter $\gamma \in (0, 1)$ captures imperfect substitutability.

Given the above assumptions, the profits of the two firms are given by:

$$\pi_1 = zq_2 + (p_1 - c_1)\, q_1 \tag{1}$$

$$\pi_2 = (p_2 - z - c_2)\, q_2 \tag{2}$$

Indeed, the VIP's profit π_1 is the sum of the wholesale profit gained from supplying the input to firm 2 at a unit input price z and the profit accruing from its own retail sales, while π_2 reflects the firm 2's retail profits.

The model is described by a multi-stage game solved in the usual backward fashion. We assume that at stage 0 (i.e., a pre-play stage) firm i's owner chooses whether to maximize profits, thus acting as an entrepreneurial firm, or to hire a sales-interested manager who is offered the following compensation contract:

$$u_i = \pi_i + \lambda_i q_i \tag{3}$$

where parameter λ_i $(i = 1, 2)$ represents the weight attached to the volume of sales and is optimally chosen by firm i's owner on a profit-maximizing basis (Vickers, 1985).[8] The higher (lower) the assigned λ_i, the more (less) aggressive is the manager's behavior on the product market, with $\lambda_i = 0$ capturing pure profit maximization and $\lambda_i > 0$ $(\lambda_i < 0)$ implying more (less) aggressiveness than under profit maximization, i.e., managers are allowed to care about (penalized for) sales. At stage 1, firm 1's owner sets the input price charged to firm 2, while at the stage 2 she chooses the compensation scheme, i.e., the optimal λ_i to assign her manager, if any. The last stage of the game identifies product market competition in which managers or owners, depending on the choice made at stage 0, simultaneously decide upon the optimal level of market variable, price or quantity.

According to the firms' choice made at the first stage, the following configurations may arise, where 'M' stands for 'managerial': and 'E' stands for 'entrepreneurial':

- (MM):both firms delegate retail choices (output or prices) to a manager,
- (EE): both firms are profit-maximizers,
- (ME): firm 1 hires a manager and firm 2 does not,
- (EM): firm 2 hires a manager and firm 1 does not,

which lead to the four market subgames described in the following subsections. More precisely, the solutions of such subgames in the hypothesis of quantity competition are derived in Subsection 2.1, which also investigates the optimal choice of one firm's structure, managerial or entrepreneurial, determined at the first stage of the game. Moreover, Subsection 2.2 solves the game for the subgame Nash equilibrium under price competition.

[8] In our model, using the managerial objective function introduced by Vickers (1985) is formally equivalent to using that defined by Fershtman and Judd (1987) as a linear combination of firm's profits and revenues.

2.1 The quantity competition case

2.1.1 Symmetric behavior

We consider the two subgames in which both firms delegate or not output decisions to managers, solving them for the equilibrium outcome.

Symmetric delegation (MM)

Given the above-mentioned timing of the game and given the objective function in (3), under symmetric delegation the two managers solve the following maximization problems:

$$\begin{aligned}\max_{q_1} u_1 &= \pi_1 + \lambda_1 q_1 \\ \max_{q_2} u_2 &= \pi_2 + \lambda_2 q_2\end{aligned}$$

yielding the following reaction functions, respectively for firm 1 and firm 2:

$$\begin{aligned}q_1 &= \frac{a - c_1 - \gamma q_2 + \lambda_1}{2} \\ q_2 &= \frac{a - c_2 - \gamma q_1 + \lambda_2 - z}{2}\end{aligned}$$

which exhibit strategic substitutability. Solving the system of the two reaction functions, we obtain the optimal quantities as functions of the input price z and the incentive parameters λ_1 and λ_2:

$$q_1 = \frac{2(a - c_1) - \gamma(a - c_2) + \gamma(z - \lambda_2) + 2\lambda_1}{4 - \gamma^2} \tag{4}$$

$$q_2 = \frac{2(a - c_2) - \gamma(a - c_1) - \gamma\lambda_1 - 2(z - \lambda_2)}{4 - \gamma^2} \tag{5}$$

At the second stage of the game, i.e., the delegation stage, owner i $(i = 1, 2)$ maximizes with respect to λ_i her own profits obtained after substituting (4) and (5) respectively in (1) and (2), thus choosing the optimal extent of delegation to assign each manager. We get the following reaction functions:

$$\begin{aligned}\lambda_1 &= \frac{\gamma\left(2\gamma(a - c_1) - \gamma^2(a - c_2) - 2z\left(2 - \gamma^2\right) - \gamma^2\lambda_2\right)}{4(2 - \gamma^2)} \\ \lambda_2 &= \frac{\gamma^2\left(2(a - c_2) - \gamma(a - c_1) - 2z - \gamma\lambda_1\right)}{4(2 - \gamma^2)}\end{aligned}$$

Notice that $\frac{\partial\lambda_1}{\partial\lambda_2} < 0$ and $\frac{\partial\lambda_2}{\partial\lambda_1} < 0$, which implies strategic substitutability of delegation.

We get the following solutions of the delegation stage:

$$\lambda_1 = \frac{\gamma\left(\gamma\left(4-\gamma^2\right)(a-c_1)-2\gamma^2(a-c_2)-2z\left(4-3\gamma^2\right)\right)}{\left(\gamma^4-12\gamma^2+16\right)} \tag{6}$$

$$\lambda_2 = \frac{\gamma^2\left(\left(4-\gamma^2\right)(a-c_2)-2\gamma(a-c_1)-2z\left(2-\gamma^2\right)\right)}{\left(\gamma^4-12\gamma^2+16\right)} \tag{7}$$

It is easy to check that $\frac{\partial\lambda_1}{\partial z}<0$ and $\frac{\partial\lambda_2}{\partial z}<0$.

At the second stage of the game, by maximizing with respect to z firm 1's profits calculated at the optimal quantities and the optimal delegation parameters, we solve for the equilibrium wholesale price:

$$z^{MM} = \frac{16\left(2-\gamma^2\right)\left(4-3\gamma^2\right)(a-c_2)+\gamma^7(a-c_1)}{2\left(128+48\gamma^4+\gamma^6-160\gamma^2\right)}$$

The equilibrium incentive parameters are:

$$\lambda_1^{MM} = -\frac{2\gamma\left(4-\gamma^2+2\gamma\right)\left(4-\gamma^2-2\gamma\right)\left(a-c_2-(a-c_1)\gamma\right)}{128+48\gamma^4+\gamma^6-160\gamma^2}$$

$$\lambda_2^{MM} = \frac{\gamma^2\left(16-\gamma^4-8\gamma^2\right)\left(a-c_2-(a-c_1)\gamma\right)}{128+48\gamma^4+\gamma^6-160\gamma^2}$$

The equilibrium quantities are therefore:

$$q_1^{MM} = \frac{\left(128+\gamma^6+8\gamma^4-96\gamma^2\right)(a-c_1)-8\gamma\left(8-5\gamma^2\right)(a-c_2)}{2\left(48\gamma^4+128+\gamma^6-160\gamma^2\right)}$$

$$q_2^{MM} = \frac{2\left(16-\gamma^4-8\gamma^2\right)\left(a-c_2-\gamma(a-c_1)\right)}{128+48\gamma^4+\gamma^6-160\gamma^2}$$

We find that a non-foreclosure condition for firm 2 (i.e., $q_2 \geq 0$) applies when $\gamma \leq \frac{a-c_2}{a-c_1}$.[9] Under such a condition, we find $\lambda_1^{MM} \leq 0$ and $\lambda_2^{MM} \geq 0$, which reveals that at equilibrium firm 1's manager is penalized for sales (Fershtman and Judd, 1987, p. 937-8),[10] while firm 2's manager is allowed to consider sales to some extent.

We include the equilibrium profits of this subgame in Appendix A (eqts A1-A2).

[9] Such a condition will have to be met in all other settings throughout the paper. Therefore, by allowing for cost heterogeneity, we will always consider the interval of the product substitutability parameter that ensures the non-foreclosure condition for firm 2 , i.e., $\gamma \in \left(0, \frac{a-c_2}{a-c_1}\right)$ with $\frac{a-c_2}{a-c_1} \leq 1$, such an interval coinciding with the unit-interval of imperfect product substitutability in the absence of firms' cost differences.

[10] In the words of Fershtman and Judd (1987, pp. 937-8), such an overcompensation for profits (i.e., $\lambda_1^{MM} \leq 0$) can be interpreted as an owner's tax imposed on sales.

Symmetric no-delegation (EE)

We solve the game under the (EE) configuration in which, at the market stage, the profit-maximizer owners directly make their output choices. We can place $\lambda_1 = 0$ and $\lambda_2 = 0$ in (4) and (5) and run accordingly the above model, thus recovering the following solutions also identified by Arya et al. (2008):

$$z^{EE} = \frac{4\left(2-\gamma^2\right)(a-c_2)+\gamma^3(a-c_1)}{2\left(8-3\gamma^2\right)}$$

$$\begin{aligned} q_1^{EE} &= \frac{\left(8-\gamma^2\right)(a-c_1)-2\gamma(a-c_2)}{2\left(8-3\gamma^2\right)} \\ q_2^{EE} &= \frac{2\left(a-c_2-\gamma(a-c_1)\right)}{8-3\gamma^2} \end{aligned}$$

The equilibrium profits of this (EE) framework are included in Appendix A (eqts A3-A4).

The comparison between the two settings of symmetric delegation and no-delegation under quantity competition allows us to introduce the following remark.

Remark 1 Symmetric delegation under quantity competition implies that, at the retail market stage, the VIP behaves less aggressively and the independent retailer more aggressively than under no-delegation. This result relies on the one hand on the higher VIP's incentive to exploit upstream monopoly power by encouraging a higher demand of inputs from the rival rather than to compete aggressively downstream. On the other hand, it mirrors the incentive of the independent firm to achieve a competitive advantage by committing to a more aggressive manager.[11]

2.1.2 Unilateral delegation

Following the standard procedure above described, we derive the solutions of the two subgames in which only one of the two firms delegates output decisions to a manager.

Unilateral delegation by firm 1 (ME)

Let us consider that only firm 1 delegates the output choice to a manager and firm 2 is a non-delegating (profit-maximizing) firm. We can find the solutions

[11] See Fershtman and Judd (1987, Section II).

of the quantity stage by posing $\lambda_2 = 0$ in (4) and (5) and running the model accordingly. We thus obtain the following equilibrium market variables:

$$z^{ME} = \frac{a - c_2}{2}$$

$$\lambda_1^{ME} = -\gamma \frac{a - c_2 - (a - c_1)\gamma}{2(2 - \gamma^2)}$$

$$q_1^{ME} = \frac{a(2 - \gamma) - 2c_1 + \gamma c_2}{2(2 - \gamma^2)}$$

$$q_2^{ME} = \frac{a(1 - \gamma) - c_2 + \gamma c_1}{2(2 - \gamma^2)}$$

Notice that $\lambda_1^{ME} \leq 0$ under the non-foreclosure condition, i.e., when $\gamma \leq \frac{a-c_2}{a-c_1}$,[12] which implies that firm 1's manager receives an overcompensation for profits, as under symmetric delegation.

The equilibrium profits of this (ME) framework are included in Appendix A (eqts A5-A6).

Remark 2 Unilateral delegation under quantity competition alters the trade-off between the VIP's incentive to behave aggressively downstream and the incentive to exploit a higher demand of inputs upstream as follows. We find $\lambda_1^{ME} \leq 0$, regardless of γ in the given interval, which reveals that firm 1 assigns lower discretion to its manager under unilateral delegation than under no-delegation, thus competing less aggressively downstream and exploiting to a greater extent the upstream profit margin. Firm 1's lower aggressiveness on the retail market results in its lower output (i.e., $q_1^{ME} \leq q_1^{EE}$), which contributes to enhance profitability of the upstream channel by inducing a higher demand of inputs from the rival (i.e., $q_2^{ME} \geq q_2^{EE}$) due to strategic substitutability of quantities. Lower aggressiveness of the VIP, moreover, turns out to be consistent with its greater commitment to set a higher wholesale price (i.e., $\partial\lambda_1/\partial z < 0$, which causes $z^{ME} \geq z^{EE}$), which further positively affects the upstream market's profitability.

It is worth considering that, with respect to this unilateral delegation framework, interactions with a managerial independent firm in the symmetric delegation case cause a further reduction of firm 1's aggressiveness (i.e., $\lambda_1^{MM} \leq \lambda_1^{ME} \leq 0$), which is consistent with increased rival's aggressiveness compared to no-delegation and with strategic substitutability of delegation in the symmetric setting. According to the above argument, lower aggressiveness of the VIP under symmetric delegation amplifies the effect of delegation in further reducing

[12] It can be easily checked that the solution of the delegation stage of the game, which yields firm 1's incentive parameter as a function of the wholesale price z, is $\lambda_1 = \gamma\left(2\gamma(a - c_1) - \gamma^2(a - c_2) - 2z\left(2 - \gamma^2\right)\right) / \left(4\left(2 - \gamma^2\right)\right)$ and that $\partial\lambda_1/\partial z < 0$ for any γ in the considered interval.

its own output in favour of the rival's one (i.e., $q_1^{MM} \leq q_1^{ME}$ and $q_2^{MM} \geq q_2^{ME}$), while higher rival's greater aggressiveness limits the VIP's ability to set a high input price with respect to unilateral delegation (i.e., $\partial\lambda_2/\partial z < 0$, which causes $z^{MM} \leq z^{ME}$).

Unilateral delegation by firm 2 (EM)

When firm 2 is assumed to delegate market discretion to a manager and firm 1 is a profit-maximizing firm, the solutions of the last stage of the game are obtained by assessing (4) and (5) at $\lambda_1 = 0$. Given that, the equilibrium outcome of this subgame is as follows:

$$z^{EM} = \frac{2\left(4-3\gamma^2\right)(a-c_2)+\gamma^3(a-c_1)}{2\left(8-5\gamma^2\right)}$$

$$\lambda_2^{EM} = \frac{\gamma^2\left(a-c_2-\gamma(a-c_1)\right)}{8-5\gamma^2}$$

$$\begin{aligned} q_1^{EM} &= \frac{a(2+\gamma)(4-3\gamma)+2\gamma c_2-c_1\left(8-3\gamma^2\right)}{2\left(8-5\gamma^2\right)} \\ q_2^{EM} &= \frac{2\left(a-c_2-\gamma(a-c_1)\right)}{8-5\gamma^2} \end{aligned}$$

Notice that $\lambda_2^{EM} \geq 0$, under the non-foreclosure condition, i.e., when $\gamma \leq \frac{a-c_2}{a-c_1}$.[13]

The equilibrium profits of this EM framework are included in Appendix A (eqts A7-A8).

Remark 3 According to the standard result in the literature on managerial incentives under quantity competition, unilateral delegation allows the independent retailer to gain a competitive advantage by committing to a more aggressive behavior than under no-delegation. That is, firm 2 instructs its manager to put a positive bonus on sales (i.e., $\lambda_2^{EM} \geq 0$ regardless of γ in the given interval), which results in a higher output (i.e., $q_2^{EM} \geq q_2^{EE}$). Accordingly, the non-delegating VIP, acting as a Stackelberg follower on the retail market, will produce a lower final output than under no-delegation ($q_1^{EM} \leq q_1^{EE}$); moreover, being unable to alter downstream interactions, it will induce a higher demand of inputs from the rival by charging it with a lower input price ($z^{EM} \leq z^{EE}$).

With respect to this unilateral delegation framework, interactions with a managerial VIP in the symmetric delegation case cause a further increase of firm 2's aggressiveness, namely $\lambda_2^{EM} \leq \lambda_2^{MM}$, which is consistent with the

[13] It can be easily checked that the solution of the delegation stage of the game, which yields firm 2's incentive parameter as a function of the wholesale price z, is $\lambda_2 = \gamma^2\left(2(a-c_2)-(a-c_1)\gamma-2z\right)/\left(4\left(2-\gamma^2\right)\right)$ and that $\partial\lambda_2/\partial z < 0$ for any γ.

rival's lower aggressiveness compared to no-delegation and with strategic substitutability of delegation in the symmetric framework. According to the above argument, higher aggressiveness of firm 2 under symmetric delegation amplifies the effect of delegation, thus further enhancing its own output and reducing the rival's one (i.e., $q_2^{MM} \geq q_2^{EM}$ and $q_1^{MM} \leq q_1^{EM}$). This also weakens the VIP's incentive to induce a higher rival's demand of inputs by reducing z, which turns out to be higher at equilibrium ($z^{MM} \geq z^{EM}$).

2.1.3 The Cournot delegation game

The following lemma puts together the market variables discussed in Remarks 1, 2 and 3 and highlights the forces driving one firm' choice to act as a managerial firm or not.

Lemma 1 *The following rankings regarding the equilibrium market variables (i.e., the wholesale input price and individual output) of the above subgames apply as long* $\gamma \in \left(0, \frac{a-c_2}{a-c_1}\right)$*:*

- $z^{EM} \leq z^{EE} \leq z^{MM} \leq z^{ME}$
- $q_1^{MM} \leq q_1^{ME} \leq q_1^{EM} \leq q_1^{EE}$
- $q_2^{EE} \leq q_2^{ME} \leq q_2^{EM} \leq q_2^{MM}$

Proof.
It follows from direct comparisons across the equilibrium market variables of the subgames in Subsections 2.1.1 and 2.1.2.

The endogenous choice of whether to act as a managerial firm or not is identified with the subgame perfect Nash equilibrium of the delegation game described in the following matrix.

Figure 1
The pay-off matrix of the Cournot delegation game

1/2	M	E
M	${}_{c\vert}\pi_1^{MM};\ {}_{c\vert}\pi_2^{MM}$	${}_{c\vert}\pi_1^{ME};\ {}_{c\vert}\pi_2^{ME}$
E	${}_{c\vert}\pi_1^{EM};\ {}_{c\vert}\pi_2^{EM}$	${}_{c\vert}\pi_1^{EE};\ {}_{c\vert}\pi_2^{EE}$

where the pay-offs are the equilibrium profits of the above-described subgames which have been included in Appendix A (eqts. A1–A8).

We can state the following proposition:

Proposition 1. *Managerial delegation emerges as the endogenous choice made by both firms in the Cournot game of delegation, regardless of* γ *in the given* *interval. Indeed, it represents a dominant strategy for each firm, which leads the symmetric choice (MM) to arise as the unique subgame perfect Nash equilibrium and to Pareto-dominate the symmetric outcome with no-delegation.*

Proof.

Let us consider the following profit differentials:

- $_{c|}\pi_1^{ME} - _{c|}\pi_1^{EE} = \frac{\gamma^2(a(1-\gamma)+\gamma c_1-c_2)^2}{4(2-\gamma^2)(8-3\gamma^2)} \geq 0$
- $_{c|}\pi_1^{MM} - _{c|}\pi_1^{EM} = \frac{\gamma^2\left(16-\gamma^4-8\gamma^2\right)(a(1-\gamma)+\gamma c_1-c_2)^2}{(128+48\gamma^4+\gamma^6-160\gamma^2)(8-5\gamma^2)} \geq 0$
- $_{c|}\pi_2^{EM} - _{c|}\pi_2^{EE} = \frac{2\gamma^4(4-3\gamma)(4+3\gamma)(a(1-\gamma)+\gamma c_1-c_2)^2}{(8-5\gamma^2)^2(8-3\gamma^2)^2} \geq 0$
- $_{c|}\pi_2^{MM} - _{c|}\pi_2^{ME} = \frac{\gamma^4\left(1024+128\gamma^4-1280\gamma^2+320\gamma^6-81\gamma^8-8\gamma^{10}\right)(a(1-\gamma)-c_2+c_1\gamma)^2}{4(128+48\gamma^4+\gamma^6-160\gamma^2)^2(2-\gamma^2)^2} \geq 0$

The above inequalities prove that M is a dominant strategy for both firms. Also, consider the following inequalities:

- $_{c|}\pi_1^{MM} - _{c|}\pi_1^{EE} = \frac{\gamma^2\left(48-24\gamma^2-\gamma^4\right)(a(1-\gamma)-c_2+c_1\gamma)^2}{(128+48\gamma^4+\gamma^6-160\gamma^2)(8-3\gamma^2)} \geq 0$
- $_{c|}\pi_2^{MM} - _{c|}\pi_2^{EE} = \frac{2\gamma^2(a(1-\gamma)-c_2+c_1\gamma)^2\left(8192-13\,824\gamma^2+7936\gamma^4-1984\gamma^6+416\gamma^8-80\gamma^{10}-9\gamma^{12}\right)}{(128+48\gamma^4+\gamma^6-160\gamma^2)^2(8-3\gamma^2)^2} \geq 0$

which demonstrate that the equilibrium (MM) Pareto-dominates (EE).

The effects highlighted in Remarks 1-3 explain the reasons for (MM) to arise as an equilibrium under quantity competition. Indeed, delegating control to a manager, thus appropriately orienting incentives on the retail market and inducing a higher demand of inputs from the rival, is the optimal strategy for firm 1 irrespective of the rival's strategy, namely the strategic device through which the integrated firm enjoys its monopoly power on the input channel. Monopoly power is further exploited by credibly committing to a relatively high input wholesale price at the upstream stage of the game. Hiring a manager is a dominant strategy also for the independent firm, which gains a competitive advantage on the retail market through an output expansion induced by delegation. Moreover, in such a framework of quantity competition unilateral delegation, not only represents a mechanism through which firm 1 (firm 2) gains higher profits on the upstream (downstream) market, but also a strategy allowing the rival to benefit from a competitive advantage downstream (a higher profit margin upstream). This leads (MM) to Pareto-dominate (EE).[14]

[14] Consistently with higher firms' profits in (MM), we find that both consumer surplus and social welfare are higher in (EE) than in (MM), whatever γ in the given interval.

2.2 The price competition case

We consider the following direct demand function faced by firm i ($i = 1, 2$):

$$q_i = \frac{a(1-\gamma) - p_i + \gamma p_j}{(1-\gamma^2)}$$

We keep the assumptions on technology and managerial contracts of the previous section and solve the subgames under the same market configurations as above.

2.2.1 Symmetric behavior

Symmetric delegation (MM)

Given the manager's objective function in (3), symmetric delegation under price competition implies the following maximization problems:

$$\begin{aligned}\max_{p_1} u_1 &= \pi_1 + \lambda_1 q_1 \\ \max_{p_2} u_2 &= \pi_2 + \lambda_2 q_2\end{aligned}$$

We obtain the following reaction functions:

$$p_1 = \frac{a(1-\gamma) + c_1 + \gamma p_2 - \lambda_1 + z\gamma}{2} \tag{8}$$

$$p_2 = \frac{a(1-\gamma) + c_2 + \gamma p_1 - \lambda_2 + z}{2} \tag{9}$$

which exhibit strategic complementarity, and the following solutions of the price stage:

$$\begin{aligned}p_1 &= \frac{a(2+\gamma)(1-\gamma) + \gamma(c_2 - \lambda_2) + 2(c_1 - \lambda_1) + 3z\gamma}{4-\gamma^2} \\ p_2 &= \frac{a(2+\gamma)(1-\gamma) + \gamma(c_1 - \lambda_1) + 2(c_2 - \lambda_2) + z\left(2+\gamma^2\right)}{4-\gamma^2}\end{aligned}$$

At the delegation stage, profit maximization by each owner gives the following reaction functions:

$$\begin{aligned}\lambda_1 &= \frac{\gamma\left(\gamma^2(a-c_2) - \gamma\left(2-\gamma^2\right)(a-c_1) + \gamma^2\lambda_2 + 4z\left(1-\gamma^2\right)\right)}{4(2-\gamma^2)} \\ \lambda_2 &= \frac{\gamma^2\left(\gamma(a-c_1) - \left(2-\gamma^2\right)(a-c_2) + \gamma\lambda_1 + 2z\left(1-\gamma^2\right)\right)}{4(2-\gamma^2)}\end{aligned}$$

Notice that $\frac{\partial\lambda_1}{\partial\lambda_2} > 0$ and $\frac{\partial\lambda_2}{\partial\lambda_1} > 0$, namely the reaction functions exhibit strategic complementarity.

The solutions of the delegation stage are the following:

$$\begin{aligned}\lambda_1 &= -\frac{\gamma\left(a\gamma(1-\gamma)\left(4-\gamma^2+2\gamma\right)-c_1\gamma\left(4-3\gamma^2\right)+c_2\gamma^2\left(2-\gamma^2\right)-2z\left(1-\gamma^2\right)(2-\gamma)(2+\gamma)\right)}{(4-\gamma^2+2\gamma)(4-\gamma^2-2\gamma)}\\ \lambda_2 &= -\frac{\gamma^2\left(a(1-\gamma)\left(4-\gamma^2+2\gamma\right)+c_2\left(3\gamma^2-4\right)+c_1\gamma\left(2-\gamma^2\right)-4z\left(1-\gamma^2\right)\right)}{(4-\gamma^2+2\gamma)(4-\gamma^2-2\gamma)}\end{aligned}$$

It can be easily checked that $\frac{\partial\lambda_1}{\partial z} > 0$ and $\frac{\partial\lambda_2}{\partial z} > 0$.

Finally, at the second stage of the game, by maximizing with respect to z firm 1's profits evaluated at the optimal prices and the optimal delegation parameters, we solve for the equilibrium wholesale price:

$$z^{MM} = \frac{\gamma^7\left(a-c_1\right)+8\left(4-\gamma^2\right)\left(2-\gamma^2\right)^2\left(a-c_2\right)}{2\left(128+64\gamma^4-7\gamma^6-160\gamma^2\right)}$$

The equilibrium delegation parameters are:

$$\begin{aligned}\lambda_1^{MM} &= \frac{\gamma\left(2-\gamma^2\right)\left(4-\gamma^2+2\gamma\right)\left(4-\gamma^2-2\gamma\right)\left(a-c_2-\left(a-c_1\right)\gamma\right)}{128+64\gamma^4-7\gamma^6-160\gamma^2}\\ \lambda_2^{MM} &= -\frac{\gamma^2\left(5\gamma^4+16\left(1-\gamma^2\right)\right)\left(a-c_2-\left(a-c_1\right)\gamma\right)}{128+64\gamma^4-7\gamma^6-160\gamma^2}\end{aligned}$$

while equilibrium prices are:

$$\begin{aligned}p_1^{MM} &= \frac{a\left(4-\gamma^2+2\gamma\right)\left(3\gamma^4+10\gamma^3-24\gamma^2-16\gamma+32\right)-4c_2\gamma^3\left(2-\gamma^2\right)+c_1\left(128-11\gamma^6+72\gamma^4-160\gamma^2\right)}{2(128+64\gamma^4-7\gamma^6-160\gamma^2)}\\ p_2^{MM} &= \frac{4a\left(48-2\gamma^6+21\gamma^4-56\gamma^2\right)+2c_2\left(4-\gamma^2\right)\left(2-\gamma^2\right)\left(4-3\gamma^2\right)-\gamma\left(64\left(1-\gamma^2\right)+\gamma^4\left(20-\gamma^2\right)\right)(a-c_1)}{2(128+64\gamma^4-7\gamma^6-160\gamma^2)}\end{aligned}$$

A non-foreclosure condition $\gamma \leq \frac{a-c_2}{a-c_1}$, implying $q_2^{MM} \geq 0$, also applies under price competition. Under such a condition, we find $\lambda_1^{MM} \geq 0$ and $\lambda_2^{MM} \leq 0$, for any γ, which shows that at equilibrium firm 1's manager is instructed to care to some extent about sales, while firm 2's manager is penalized for sales.

We include the equilibrium profits of this setting in Appendix B (eqts B1-B2).

Symmetric no-delegation (EE)

We solve the game under the (EE) configuration in which, at the market stage, the profit-maximizer owners directly choose their price levels. By solving the second stage after placing $\lambda_1 = 0$ and $\lambda_2 = 0$ in (8) and (9), we obtain the following equilibrium market variables:

$$z^{EE} = \frac{8\left(a-c_2\right)+\gamma^3\left(a-c_1\right)}{2\left(8+\gamma^2\right)}$$

$$p_1^{EE} = \frac{8(a+c_1)+2\gamma(a-c_2)-\gamma^2(a-3c_1)}{2(8+\gamma^2)}$$

$$p_2^{EE} = \frac{2a\gamma^2+4(3a+c_2)-\gamma\left(4+\gamma^2\right)(a-c_1)}{2(8+\gamma^2)}$$

We include the equilibrium profits of the (EE) subgame in Appendix B (eqts B3-B4).

The comparison between the two settings of symmetric delegation and no-delegation under price competition allows us to introduce the following remark.

Remark 4 Symmetric delegation under price competition implies that the VIP behaves at the market stage more aggressively and the independent retailer less aggressively than under no-delegation. This result relies on the one hand on the higher incentive of the VIP to exploit its upstream monopoly power by letting a higher demand of inputs accrue from the rival upstream rather than to gain a competitive advantage downstream. On the other hand, it relies on the incentive of firm 2 to relax downstream competition through delegation to a less aggressive manager, which is a standard result in literature of strategic delegation.[15]

2.2.2 Unilateral delegation

Unilateral delegation by firm 1 (ME)

When only firm 1 delegates the output choice to a manager and firm 2 is a non-delegating (profit-maximizing) firm, the solutions of the quantity stage of the game are obtained posing $\lambda_2 = 0$ in (8) and (9). Running the model under this assumption, we obtain the following equilibrium market variables:

$$z^{ME} = \frac{a-c_2}{2}$$

$$\lambda_1^{ME} = \frac{\gamma(a-c_2-\gamma(a-c_1))}{4}$$

$$p_1^{ME} = \frac{a+c_1}{2}$$

$$p_2^{ME} = \frac{a(3-\gamma)+\gamma c_1+c_2}{4}$$

[15] See Fershtman and Judd (1987, Section V).

Notice that $\lambda_1^{ME} \geq 0$ under the non-foreclosure condition, i.e., when $\gamma \leq \frac{a-c_2}{a-c_1}$.[16]

We include the equilibrium profits of this setting in Appendix B (eqts B5-B6).

Remark 5 Unilateral delegation under price competition alters the trade-off between the VIP's incentive to behave aggressively downstream and the incentive to exploit a higher demand of inputs upstream as follows. We find $\lambda_1^{ME} \geq 0$, regardless of γ in the given interval, which reveals that firm 1 assigns greater discretion to its manager under unilateral delegation than under no-delegation, namely its manager is instructed to put a positive weight on the volume of sales, thus competing more aggressively downstream and exploiting to a greater extent the upstream profit margin. Indeed, firm 1's higher aggressiveness on the downstream market results in its lower retail price (i.e., $p_1^{ME} \leq p_1^{EE}$), which lets a higher demand of inputs accrue from the rival due to a reduction of the rival's price (i.e., $p_2^{ME} \leq p_2^{EE}$) caused by strategic complementarity. Firm 1's profitability on the upstream channel rises accordingly. Higher aggressiveness of the VIP, moreover, is consistent with a greater commitment to set a higher wholesale price (i.e., $\partial\lambda_1/\partial z > 0$, which causes $z^{ME} \geq z^{EE}$), which further positively affects the upstream market's profitability.

Symmetric delegation, however, implies a reduction of firm 1's aggressiveness with respect to unilateral delegation (i.e., $0 \leq \lambda_1^{MM} \leq \lambda_1^{ME}$), due to lower aggressiveness of the managerial independent firm and strategic complementarity of delegation. The above argument also suggests that lower aggressiveness of the VIP under symmetric delegation raises its own retail price as well as the rival's one (i.e., $p_1^{MM} \geq p_1^{ME}$ and $p_2^{MM} \geq p_2^{ME}$) with respect to unilateral delegation, while lower firm 2's aggressiveness limits the VIP's ability to set a high input price (i.e., $\partial\lambda_2/\partial z > 0$, which causes $z^{MM} \leq z^{ME}$).

Unilateral delegation by firm 2 (EM)

When firm 2 is assumed to delegate market discretion to a manager and firm 1 is a profit-maximizing firm, the solutions of the last stage of the game are obtained by assessing (8) and (9) at $\lambda_1 = 0$. By solving the subsequent stages, we obtain the following market variables at the subgame perfect equilibrium:

$$z^{EM} = \frac{\left(4-\gamma^2\right)\left(2-\gamma^2\right)(a-c_2)+(a-c_1)\gamma^3}{2\left(\gamma^4-5\gamma^2+8\right)}$$

$$\lambda_2^{EM} = -\frac{\gamma^2\left(a-c_2-\gamma\left(a-c_1\right)\right)}{\gamma^4-5\gamma^2+8}$$

[16] It can be easily checked that the solution of the delegation stage of the game, yielding firm 1's incentive parameter as a function of the wholesale price z, is $\lambda_1 = \gamma\left((1-\gamma)\left(4z(1+\gamma)-a\gamma(2+\gamma)\right)+\gamma c_1\left(2-\gamma^2\right)-\gamma^2 c_2\right)/\left(4\left(2-\gamma^2\right)\right)$ and that $\partial\lambda_1/\partial z > 0$.

$$p_1^{EM} = \frac{2a\left(4+\gamma^4+\gamma\right)-\gamma^2 a\left(7+\gamma\right)-\gamma c_2\left(2-\gamma^2\right)+c_1\left(8-3\gamma^2\right)}{2\left(\gamma^4-5\gamma^2+8\right)}$$
$$p_1^{EM} = \frac{2a\left(4+\gamma^4+\gamma\right)-\gamma^2 a\left(7+\gamma\right)-\gamma c_2\left(2-\gamma^2\right)+c_1\left(8-3\gamma^2\right)}{2\left(\gamma^4-5\gamma^2+8\right)}$$

Notice that $\lambda_2^{EM} \leq 0$, under the non-foreclosure condition, i.e., when $\gamma \leq \frac{a-c_2}{a-c_1}$.[17]

We include the equilibrium profits of this setting in Appendix B (eqts B7-B8).

Remark 6 According to the standard result in the literature on managerial incentives under price competition, unilateral delegation allows the independent retailer to commit to a less aggressive behavior than under no-delegation, namely to instruct its manager to put a negative weight on sales (i.e., $\lambda_2^{EM} \leq 0$, regardless of γ in the given interval). This results in a higher price (i.e., $p_2^{EM} \geq p_2^{EE}$). Accordingly, the non-delegating VIP, acting as a Stackelberg follower on the retail market and thus being unable to orient the rival's behavior, sets a lower retail price than under no-delegation ($p_1^{EM} \geq p_1^{EE}$) and induces a higher demand of inputs from the rival by charging it with a lower input price ($z^{EM} \leq z^{EE}$).

Symmetric delegation, moreover, leads firm 2 to compete more aggressively than under unilateral delegation (i.e., $\lambda_2^{EM} \leq \lambda_2^{MM} \leq 0$), due to higher aggressiveness of the managerial VIP and strategic complementarity of delegation. Higher aggressiveness of firm 2 under symmetric delegation further reduces its own retail price as well as the rival's one (i.e., $p_2^{MM} \leq p_2^{EM}$ and $p_1^{MM} \leq p_1^{EM}$). This also weakens the VIP's incentive to induce higher rival's demand of inputs by reducing z, which turns out to be higher at equilibrium ($z^{MM} \geq z^{EM}$).

2.2.3 The Bertrand delegation game

We first introduce the following lemma to highlight the pattern of the market variables discussed in Remarks 4, 5 and 6. This will allow us to capture the forces driving one firm' choice to act as a managerial firm or not.

Lemma 2 *The following rankings regarding the equilibrium market variables (i.e., the wholesale input price and the retail prices) of the above subgames apply as long* $\gamma \in \left(0, \frac{a-c_2}{a-c_1}\right)$*:*

- $z^{EM} \leq z^{EE} \leq z^{MM} \leq z^{ME}$
- $p_1^{ME} \leq p_1^{MM} \leq p_1^{EE} \leq p_1^{EM}$
- $p_2^{ME} \leq p_2^{EE} \leq p_2^{MM} \leq p_2^{EM}$

[17] It can be easily checked that the solution of the delegation stage of the game, which yields firm 2's incentive parameter as a function of the wholesale price z, is $\lambda_2 = \gamma^2\left(a\left(2+\gamma\right)\left(\gamma-1\right)+c_2\left(2-\gamma^2\right)-\gamma c_1+2z\left(1-\gamma^2\right)\right)/\left(4\left(2-\gamma^2\right)\right)$ and that $\partial\lambda_2/\partial z > 0$.

Proof.
It follows from direct comparisons across the equilibrium market variables of the subgames in Subsections 2.2.1 and 2.2.2.

Now, let us turn to search for the subgame perfect Nash equilibrium of the following delegation game.

Figure 2
The pay-off matrix of the Bertrand delegation game

1/2	M	E
M	${}_{b\|}\pi_1^{MM};\ {}_{b\|}\pi_2^{MM}$	${}_{b\|}\pi_1^{ME};\ {}_{b\|}\pi_2^{ME}$
E	${}_{b\|}\pi_1^{EM};\ {}_{b\|}\pi_2^{EM}$	${}_{b\|}\pi_1^{EE};\ {}_{b\|}\pi_2^{EE}$

where the pay-offs are the equilibrium profits of the above-described subgames which have been included in Appendix B (eqts. B1–B8).

We can state the following proposition:

Proposition 2. *Managerial delegation emerges as the endogenous choice made by both firms in the Bertrand game of delegation, regardless of γ in the given interval. Therefore, (MM) is the unique subgame perfect Nash equilibrium in dominant strategies, which turns out to be a prisoner-dilemma-type equilibrium.*

*Proof***.**

Let us consider the following profit differentials:

- ${}_{b|}\pi_1^{ME} -{}_{b|}\ \pi_1^{EE} = \frac{\gamma^2(a-c_2-\gamma(a-c_1))^2}{8(1-\gamma^2)(8+\gamma^2)} \geq 0$
- ${}_{b|}\pi_1^{MM} -{}_{b|}\ \pi_1^{EM} = \frac{\gamma^2\left(2-\gamma^2\right)^2\left(16-\gamma^4-8\gamma^2\right)(a-c_2-\gamma(a-c_1))^2}{4(1-\gamma^2)(\gamma^4-5\gamma^2+8)(128+64\gamma^4-7\gamma^6-160\gamma^2)} \geq 0$
- ${}_{b|}\pi_2^{EM} -{}_{b|}\ \pi_2^{EE} = \frac{\gamma^4\left(64-46\gamma^2-5\gamma^4+6\gamma^6-\gamma^8\right)(a-c_2-\gamma(a-c_1))^2}{(1-\gamma^2)(\gamma^4-5\gamma^2+8)^2(8+\gamma^2)^2} \geq 0$
- ${}_{b|}\pi_2^{MM} -{}_{b|}\ \pi_2^{ME} = \frac{\gamma^4\left(1024-49\gamma^8+96\gamma^6+384\gamma^4-1280\gamma^2\right)(a(1-\gamma)-c_2+c_1\gamma)^2}{16(1-\gamma^2)(128+64\gamma^4-7\gamma^6-160\gamma^2)^2} \geq 0$

The above inequalities prove that M is a dominant strategy for both firms. Also, consider the following inequalities:

- ${}_{b|}\pi_1^{MM} - \pi\,{}_{b|}\pi_1^{EE} = -\frac{\gamma^2\left(16-8\gamma^2-3\gamma^4+2\gamma^6\right)(a(1-\gamma)+\gamma c_1-c_2)^2}{(1-\gamma^2)(8+\gamma^2)(128+64\gamma^4-7\gamma^6-160\gamma^2)} \leq 0$
- ${}_{b|}\pi_2^{MM} -{}_{b|}\ \pi_2^{EE} =$
$\frac{-\gamma^2(a(1-\gamma)+\gamma c_1-c_2)^2\left(49\gamma^{14}-650\gamma^{12}+3328\gamma^{10}-2560\gamma^8-27776\gamma^6+91\,648\gamma^4-111616\gamma^2+49152\right)}{(1-\gamma^2)(8+\gamma^2)^2(128+64\gamma^4-7\gamma^6-160\gamma^2)^2}$
≤ 0

which prove that the equilibrium (MM) is a prisoner dilemma.

The effects highlighted in Remarks 4-6 explain the reasons for (MM) to arise as an equilibrium under price competition. Indeed, delegating to a manager is the optimal VIP's reaction, whatever the rival's strategy is. As in quantity competition case, it allows the VIP to gain a higher profit margin upstream by inducing a higher rival's demand of inputs and successfully committing to a relatively high input price, rather than that obtained competing downstream. Delegation also enables firm 2 to gain higher profits by relaxing downstream competition, regardless of the rival's strategy. Despite unilateral delegation represents a profit-enhancing mechanism through which firm 1 and firm 2 exploit higher profitability respectively on the input market and the retail market, it hurts the rival by weakening firm 2's competitive position downstream and reducing firm 1's upstream profit margin. This leads (MM) to emerge as a prisoner dilemma.[18]

3 Concluding remarks

In this paper we have pointed out how the presence of a vertical supply relationship in a duopoly with an integrated and an independent firm affects the strategic choice of delegation to managers. Indeed, literature on managerial incentives has focused on the profit-enhancing character of delegation as a means for independent firms to credibly commit to a more or less aggressive conduct respectively under Cournot and Bertrand, which weakens the rival leading to a more competitive market outcome in the former, while it benefits the rival softening overall market competition in the latter. As a result, a prisoner-dilemma-type equilibrium arises under Cournot and a Pareto-improving equilibrium arises under Bertrand. Conversely, in our scenario we have shown that delegation represents the equilibrium choice made by both the independent firm and the integrated firm since it works as a mechanism through which the former exploits the competitive advantage on the product market, while the latter orients downstream interactions to fully exploit its market power on the upstream market. In such a context, managerial delegation turns out to be beneficial to the rival under strategic substitutability of quantities and to be detrimental under strategic complementarity of prices, which results in higher profits for both firms under Cournot and causes a prisoner dilemma under Bertrand. Our

[18] By performing welfare analysis, we find that symmetric delegation, despite always less profitable for both firms, turns out to be welfare-reducing al long as γ is sufficiently high (with the interval of γ in which this occurs enlarging as the cost differential between the two firms increases). This result reflects the fact that consumer surplus is negatively affected by the firm 2's incentive to keep its retail price higher (or its final output lower) under delegation. When product substitutability is high enough, and firm 2's foreclosure becomes more likely, symmetric no-delegation yields welfare improvements with respect to symmetric delegation because of the greater positive impact of firm 2's output expansion on social welfare than the negative impact of firm 1's output reduction. Calculations are available from the authors upon request. Notice that the circumstances in which a prisoner-dilemma-type equilibrium is beneficial to consumers and society have been not highlighted in previous literature so far, to the best of our knowledge, with the exception of Burr et al. (2013) in a game of R&D and Cournot competition.

findings rely on the assumption that the integrated firm sets the wholesale input price prior to the optimal choice of the managerial contract(s). The analysis of the decision concerning the optimal firm structure when delegation also affects the upstream monopolist's choice of the wholesale price, as well as the analysis under different assumptions on the vertical structure of the industry (e.g., assuming competition on the upstream market or vertical separation between the upstream and downstream units) or under the hypothesis that more than one input is required for final production,[19] are left to future research.

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[19] This assumption is introduced in Matsushima and Mizuno (2013) who investigate vertical separation decisions when one or multiple manufacturers supply complementary inputs to a downstream monopolist and the wholesale input price is determined through Nash bargaining.

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Appendix A

$$
\begin{aligned}
{}_{c|}\pi_1^{MM} &= \frac{a^2\left(4-\gamma^2-2\gamma\right)\left(48-\gamma^4+2\gamma^3-24\gamma^2-8\gamma\right)+c_1^2\left(128+\gamma^6+16\gamma^4-96\gamma^2\right)}{4(128+48\gamma^4+\gamma^6-160\gamma^2)} - \\
&\quad \frac{2c_1 a\left(128+32\gamma^3+\gamma^6-96\gamma^2+16\gamma^4-64\gamma\right)+32c_2\left(2-\gamma^2\right)(2a(1-\gamma)-c_2+2c_1\gamma)}{4(128+48\gamma^4+\gamma^6-160\gamma^2)} &\text{(A1)}\\
{}_{c|}\pi_2^{MM} &= \frac{2\left(2-\gamma^2\right)\left(16-\gamma^4-8\gamma^2\right)^2(a(1-\gamma)-c_2+c_1\gamma)^2}{(128+48\gamma^4+\gamma^6-160\gamma^2)^2} &\text{(A2)}
\end{aligned}
$$

$$
\begin{aligned}
{}_{c|}\pi_1^{EE} &= \frac{\left(8+\gamma^2\right)(a-c_1)^2+4(a-c_2)^2-8\gamma(a-c_1)(a-c_2)}{4(8-3\gamma^2)} &\text{(A3)}\\
{}_{c|}\pi_2^{EE} &= \frac{4((a-c_2)-\gamma(a-c_1))^2}{(8-3\gamma^2)^2} &\text{(A4)}
\end{aligned}
$$

$$
\begin{aligned}
{}_{c|}\pi_1^{ME} &= \frac{a^2(3-2\gamma)-2c_2 a(1-\gamma)-2ac_1(2-\gamma)+2c_1^2+c_2^2-2c_2\gamma c_1}{4(2-\gamma^2)} &\text{(A5)}\\
{}_{c|}\pi_2^{ME} &= \frac{(a(1-\gamma)+\gamma c_1-c_2)^2}{4(2-\gamma^2)^2} &\text{(A6)}
\end{aligned}
$$

$$
\begin{aligned}
{}_{c|}\pi_1^{EM} &= \frac{4\left(3a^2+c_2^2+2c_1^2\right)-8c_2 a(1-\gamma)-8c_1 a(2-\gamma)-\gamma^2(a-c_1)^2-8\gamma\left(a^2+c_2c_1\right)}{4(8-5\gamma^2)} &\text{(A7)}\\
{}_{c|}\pi_2^{EM} &= \frac{2\left(2-\gamma^2\right)(a(1-\gamma)+\gamma c_1-c_2)^2}{(8-5\gamma^2)^2} &\text{(A8)}
\end{aligned}
$$

Appendix B

$$
\begin{aligned}
{}_{b|}\pi_1^{MM} &= \frac{a^2\left(192-320\gamma^2-31\gamma^6+176\gamma^4\right)-(a-c_1)^2\gamma^8-8c_2\left(2-\gamma^2\right)^3(2a-c_2)}{4(1-\gamma)(1+\gamma)(128+64\gamma^4-7\gamma^6-160\gamma^2)} - \\
&\quad \frac{16\gamma\left(2-\gamma^2\right)^3(a-c_2)(a-c_1)+c_1(2a-c_1)\left(128-23\gamma^6+128\gamma^4-224\gamma^2\right)}{4(1-\gamma)(1+\gamma)(128+64\gamma^4-7\gamma^6-160\gamma^2)} && \text{(B1)}\\
{}_{b|}\pi_2^{MM} &= \frac{2\left(2-\gamma^2\right)\left(16+5\gamma^4-16\gamma^2\right)^2(a(1-\gamma)-c_2+c_1\gamma)^2}{(1-\gamma)(1+\gamma)(128+64\gamma^4-7\gamma^6-160\gamma^2)^2} && \text{(B2)}
\end{aligned}
$$

$$
\begin{aligned}
{}_{b|}\pi_1^{EE} &= \frac{a^2(1-\gamma)(2+\gamma)\left(\gamma^2-\gamma+6\right)+4c_2(c_2-2a(1-\gamma))}{4(1-\gamma^2)(8+\gamma^2)} + && \text{(B3)}\\
&\quad \frac{2c_1a(\gamma-1)\left(\gamma^3+\gamma^2+4\gamma+8\right)-8c_2\gamma c_1+c_1^2\left(-\gamma^4-3\gamma^2+8\right)}{4(1-\gamma^2)(8+\gamma^2)}\\
{}_{b|}\pi_2^{EE} &= \frac{(a(1-\gamma)+\gamma c_1-c_2)^2\left(2+\gamma^2\right)^2}{(1-\gamma^2)(8+\gamma^2)^2} && \text{(B4)}
\end{aligned}
$$

$$
\begin{aligned}
{}_{b|}\pi_1^{ME} &= \frac{a^2(3+\gamma)(1-\gamma)+c_2^2-2c_2(a-\gamma(a-c_1))+c_1^2\left(2-\gamma^2\right)-2c_1a(2+\gamma)(1-\gamma)}{8(1-\gamma^2)} && \text{(B5)}\\
{}_{b|}\pi_2^{ME} &= \frac{(a(1-\gamma)+\gamma c_1-c_2)^2}{16(1-\gamma^2)} && \text{(B6)}
\end{aligned}
$$

$$
\begin{aligned}
{}_{b|}\pi_1^{EM} &= \frac{a^2(1-\gamma)\left(2\gamma^4-\gamma^3-9\gamma^2+4\gamma+12\right)+c_1^2\left(8+2\gamma^4-9\gamma^2\right)}{4(1-\gamma^2)(8-5\gamma^2+\gamma^4)} + && \text{(B7)}\\
&\quad \frac{2c_1a(\gamma-1)\left(\gamma^4-\gamma^3-5\gamma^2+4\gamma+8\right)+c_2\left(2-\gamma^2\right)^2(2\gamma(a-c_1)+c_2-2a)}{4(1-\gamma^2)(8-5\gamma^2+\gamma^4)}\\
{}_{b|}\pi_2^{EM} &= \frac{2\left(2-\gamma^2\right)(a(1-\gamma)+\gamma c_1-c_2)^2}{(1-\gamma^2)(8-5\gamma^2+\gamma^4)^2} && \text{(B8)}
\end{aligned}
$$