R&D Policy with Heterogeneous Innovators

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Abstract: This paper presents a second-generation semi-endogenous Schumpeterian growth model to investigate the existence of possible links between firm's heterogeneity, R&D investment and adjustment dynamics. In the model, R&D is affected by two types of uncertainty: (i) uncertainty in innovation success, which is governed by a Poisson arrival rate; (b) uncertainty in the stepsize of the innovation, which is in turn governed by a Pareto distribution. The paper finds that there exists a direct link between R&D realization, conditional convergence speed and effectiveness of R&D policy. More specifically, the model predicts that economies characterized by stiffer competition in the manufacturing displays a longer adjustment path than economies with loose competition, with the result that the former tend to keep the beneficial effects of R&D policy more in circle than the latter.

JEL classification E10, L16, O31, O38.

Keywords Schumpeterian Growth, Random Quality Jumps, R&D policy, Transitional Dynamics, Market Structure

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1 Introduction

This paper is about R&D policy and economic growth. It is widely known that the first generation of R&D-based endogenous growth models - Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992) - predicts that permanent changes in variables that are potentially affected by government policy can induce permanent changes in equilibrium growth rates. In all these models, the long-run growth rate of the economy is proportional to the total amount of research undertaken in the economy, so that an increase in the size of the resources devoted to R&D leads to a permanent rise in the equilibrium rate of economic growth. This is the so called *growth effects* or "strong" *scale effects*.

Jones (1995a) has criticized this prediction by showing that it is strongly at odds with 20th century empirical evidence of many industrialized countries. Since then, a second wave of R&D-based growth models have been developed to eliminate the scale-effect prediction. The final outcome of such a new wave has been the creation of a new class of models called *semi-endogenous* growth models[1] - Jones (1995b), Kortum (1997) and Segerstrom (1998) - according to which (*i*) R&D investment is still endogenously driven by self-interested firms; (*ii*) long-run growth is exogenously pinned down by the growth rate of the population/workforce; (*iii*) permanent changes in R&D policy do not have permanent effects on the long-run growth rate of the economy. Such a new wave of models, hence, exhibit "weak" *scale effects*, in the sense that it is the steady-state level of per capita income that is an increasing function of the size of the economy, not its growth rate.

Although semi-endogenous growth models have no policy implications for long-run growth, permanent changes in government policies can potentially have significative effects on growth during adjustment dynamics. As Temple (2003) points out, if the length of the transition path were very long the growth effects generated by government interventions might persist very long, and hence have very important implications for economic agents. And that is the case of semi-endogenous R&D-based growth models, in which the pace of transitional dynamics is quite slow.

Indeed, Jones (1995b) finds that adjustment dynamics is quite slow, and crucially dependant upon the size of R&D spillover. According to his calibration, the time required to go half the distance to the steady state - the so-called "half-life" - ranges from 35 years when the size of R&D spillover is low, to 347 years when the size of R&D spillover is close to one, with speeds of convergence equating 2% and 0.2% per year respectively.[2] As discussed by Eicher and

[1] For the sake of truth, the second wave of R&D-based models can be divided up into two different blocks of studies depending on whether the steady-state innovation rate of the economy is endogenously determined by the model, or whether it is the private investment in R&D to be determined endogenously. Belonging to the first block of papers - also referred to as the *fully-endogenous* models - are the contributions by Peretto (1998), Howitt (1999), and Dinopoulos and Syropoulos (2007).

[2] Jones's numerical results about adjustment speed has been confirmed by Steger (2003), who finds a very low convergence rate also for the case of Segerstrom's (1998) quality-ladder model with growing R&D complexities. In his simulation exercise, Steger finds that it takes almost 40 years to go half the distance to the steady-state, with an implied conditional rate of convergence of about 1.7% per year.

Turnovsky (1999) though, the excessively slow speed of convergence predicted by Jones (1995b) is due to the underlying assumption that the sectorial allocation of factors does not change during the transition. By correcting for this rigidity, Eicher and Turnovsky demonstrate that a two-scale variant of Jones' model of semi-endogenous growth can predict speeds of convergence close to the 2-3% prescribed by the empirical literature.

However, recent estimates of conditional speed of convergence have been found to range between 2% - Barro, 1991; Mankiw, Romer and Weil (1992) - and 10% - Islam, 1995; Caselli, Esquivel and Lefort (1996) -, implying that the semi-endogenous approach to growth theory seems to be not able to cover those studies finding half-lives of 6/7 years.[3] In this paper, I try to reconcile the dynamic properties of the semi-endogenous growth models with this recent findings of the growth econometrics literature. In doing so, I allow for firms' heterogeneity in terms of employment, sales and profits, by developing a formal model in the spirit of Minniti, Parello and Segerstrom (2013) in which out-pricing industry leaders coexist with monopolistic leaders, and where the industrial environment characterizing the economy is made up of a mix of unconstrained monopolies and Bertrand-like oligopolies. In the model, firms address two types of uncertainty when investing in innovation: (1) a traditional uncertainty related to the outcome of R&D races; (2) an additional uncertainty related to the size of innovation, where the probability distribution of quality improvements is Pareto.[4]

Neither Jones' nor Eicher and Turnovsky's model pay attention to the market environment that characterizes the manufacturing sector. While allowing for growth in set of intermediate varieties available for final good production, in Jones' (1995) model the quality of the existing

[3] The empirical growth literature on convergence is a very hot, an still on motion, literature. It may be split up into two different streams of studies, in which the demarking line is the econometric method applied for the estimates. In the first stream of the literature, known as Growth Regressions, the estimated rate of conditional convergence rotates around 2%-3% per year (Barro, 1991; Barro and Sala-I-Martin, 1992, 2004; Sala-I-Martin, 1994; Mankiw, Romer and Weil, 1992). However, these works have been heavily criticized for ignoring several econometric issues (mainly omitted variables, country specific effects, the endogeneity of the dependent variables, and measurement errors), and for delivering downwardly biased estimates. In the second stream of the literature that followed, such econometric issues have been tackled by either using a panel data approach taking account of fixed effects (Islam, 1995), or using GMM estimator to correct for sources of inconsistency due to correlated country-specific effects and endogenous explanatory variables, (Caselli, Esquivel, and Lefort, 1996), or using Bayesian techniques (Sala-I-Martin, Dopplerhofer and Miller, 2008). In this literature, the estimated convergence speed is found to range between 4.7 percent (Islam, 1995) and 10 percent (Caselli, Esquivel, and Lefort, 1996).

[4] As the size of innovation, i.e. the size of each quality improvement, can differ by chance only, in our model industry leaders can differ one another in terms of pricing, sales and profits, implying that the industry structure of the model is no longer symmetric as in the bulk of the Schumpeterian literature. Recently, Chu et al. (2017) and Iwaisako and Ohki (2017) have developed two different variants of Minniti et (2013) asymmetric framework to study the long-run implications of R&D investment on monetary policy and inflation (Chu et al., 2017), and the effects of industrial policy on the research propensity of market leaders and followers. In contrast to our model, in their models innovation is always non drastic, with the consequence that the unique market form surviving in the equilibrium is Bertrand oligopoly (Iwaisako and Ohki, 2017).

brands is fixed by construction (Romer, 1990; Grossman and Helpman, 1991). In contrast, in Steger (2003) it is the set of brands to be fixed, while the quality level of each brand can improve over time by a fixed proportion because of R&D. Consequently, in Jones's set-up successful R&D firms are always ever-lasting monopolists, while in Steger's paper industry leaders are always price-setting oligopolists because innovation are always non-drastic (or incremental). Our model differs from Jones (1995) and Steger (2003) because I allow for a mix of market regimes, and for a time-varying quality jump.

The ultimate goals of the paper are to (*i*) demonstrate that firms' size and competition stiffness can affect the length of the transition path, and thus the effectiveness of R&D policy; (*ii*) revisit the policy result of the semi-endogenous growth literature, according to which R&D policy might have very important implications for economic agents if the length of the transition path is very long; (*iii*) show that the short-run effects of R&D policy are always negative, and highly dependant on the industrial composition of the manufacturing sector.

We find that the length of the transition path crucially depends on the industrial composition characterizing the economy, i.e. on what market form - oligopoly Vs. monopoly - tends to prevail in the long-run equilibrium. More specifically, I find that the higher is the share of oligopolies in total industries, the longer is the adjustment path to the new steady state and *vice versa*. Our simulations show that it takes almost 53 years to go half the distance to the steady state if the share of oligopolies in total economy is about 99.9 percent, 31.6 years if it drops down to 75 percent, and about 7 years if it falls short to 50 percent.

The policy implications of the model are also interesting because I find that R&D subsidy - although they do not last forever - are more effective when the slice of industries led by price-setting oligopolists is as largest as possible. That means that the effectiveness of R&D policy depends the market environment characterizing each industry, and hence on the average degree of market competition of the whole economy.

The outline of the paper is the following. Section 2 introduces the model. Section 3 characterizes the dynamic equilibrium of the laissez-faire economy and describes the property of the steady-state equilibrium. Section 4 analyzes the relationship between the length of the transition path and the effectiveness of R&D policy through several simulation exercises. Finally, Section 5 concludes.

2 The model economy

2.1 Households

There is a representative household that is modeled as a dynastic family that grows over time at an exogenous rate n. Normalizing the initial number of members of this family to equal one, the economy's population size at time t amounts to $L(t) = e^{nt}$. Households own firms in equal shares and provide labor services in exchange for wages. They also choose from the continuum of products $\omega \in [0, 1]$, where each product ω can potentially be supplied in a countably-infinite number of qualities. Quality vintage j of product ω provides quality level $q_j(\omega, t)$. By the definition of quality improvement, new generations are better than the old ones; i.e., $q_j(\omega,t) > q_{j-1}(\omega,t)$. At time $t=0$, the state-of-the-art quality q for each product ω equals one.

The representative household maximizes lifetime utility

$$U_t \equiv \int_t^\infty e^{-(\rho-n)(s-t)} \ln u(s)\, \mathrm{d}s, \tag{1}$$

subject to

$$u(t) = \left\{ \int_0^1 \left[\Sigma_j q_j(\omega,t)^{1/(\sigma-1)} d_j(\omega,t) \right]^{(\sigma-1)/\sigma} \mathrm{d}\omega \right\}^{\sigma/(\sigma-1)}, \tag{2}$$

$$\dot{a}(t) = [r(t)-n]\, a(t) + w(t) - c(t), \tag{3}$$

$$c(t) = \int_0^1 \left[\Sigma_j p_j(\omega,t)\, d_j(\omega,t) \right] \mathrm{d}\omega. \tag{4}$$

Eq. (1) is the discounted utility of the representative household, where $\rho > n$ is the subjective discount rate. Eq.(2) is the static utility function that takes the traditional Dixit-Stiglitz form, where $d_j(\omega,t)$ denotes consumption by an individual of a product ω with quality vintage j at time t and $\sigma > 1$ is the elasticity of substitution between products. Eq. (3) is the intertemporal budget constraint of the household, where $c(t)$ denotes individual consumer spending, $r(t)$ is the rate of returns on financial assets, $a(t)$ denotes the real financial asset holdings of the representative household, and $w(t)$ is the wage rate earned by each household member. Finally, Eq. (4) is the static budget constraint of the households, according to which, al all $t \geq 0$, per capita consumption expenditure $c(t)$ must equate to the value of all final goods consumed, where $p_j(\omega,t)$ is the price of quality j of product ω at time t.

The household maximization problem can be solved in three steps. In the first step, the household allocates expenditure for each product across available quality levels. This is a static maximization problem, whose solution leads to the indifferent condition between quality vintages, $p_j(\omega,t) = \lambda(\omega,t)^{\frac{1}{\sigma-1}} p_{j-1}(\omega,t)$, where $\lambda(\omega,t) \equiv q_j(\omega,t)/q_{j-1}(\omega,t)$ is the stochastic quality jump separating two adjacent vintages of product ω at time t. I assume that when consumers are indifferent between two vintages, they only buy the higher quality product, with the result that only the highest quality level available is sold in equilibrium.

Once consumers decided how to choose among several qualities, in the next step the household chooses how to allocates expenditure across all the continuum $[0,1]$ of existing product varieties. The result of this further static maximization problem is the individual consumer demand function

$$d_j(\omega,t) = \frac{p_j(\omega,t)^{-\sigma} q_j(\omega,t)\, c(t)}{P(t)^{1-\sigma}}. \tag{5}$$

where

$$P(t) \equiv \left[\int_0^1 q_j(\omega',t)\, p_j(\omega',t)^{1-\sigma} \mathrm{d}\omega' \right]^{1/(1-\sigma)} \tag{6}$$

is the relevant quality-adjusted price index.

According to (5), the representative household member demands the amount $d_j(\omega, t)$ of product ω with a quality level $q_j(\omega, t)$, and no units of lower quality versions of that product. Substituting the individual demand function (5) into (2) and simplifying using (6), I obtain the indirect utility index

$$u(t) = \left[\int_0^1 q_j(\omega, t)^{1/\sigma} d_j(\omega, t)^{(\sigma-1)/\sigma} \, \mathrm{d}\omega \right]^{\sigma/(\sigma-1)} = \frac{c(t)}{P(t)}. \tag{7}$$

As indirect utility index $u(t)$ is proportional to consumer expenditure $c(t)$, (7) is a good proxy for measuring real consumption, and it is appropriate to use the growth rate of static utility as a measure for economic growth.

Finally, in the last step the household allocates life-time expenditure, $c(t)$, across time. Plugging (7) into (1), the discounted utility of the representative household can be written as

$$U_t = \int_t^\infty e^{-(\rho-n)(s-t)} \left[\ln c(t) - \ln P(t) \right] \mathrm{d}s, \tag{8}$$

Given an initial value for the real financial holdings of the household $a(0) > 0$, the optimal control problem consists of maximizing discounted utility (8), subject to (3), and (6). The optimal path of consumption and the transversality condition are

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho, \tag{9}$$

$$\lim_{t \to \infty} e^{-(\rho-n)t} \frac{a(t)}{c(t)} = 0. \tag{10}$$

2.2 Product Markets

In each industry, firms compete in prices. Labor is the only input used in production and there are constant returns to scale. One unit of labor is required to produce one unit of output, regardless of quality. As labor market is perfectly competitive and labor is the numéraire of the model, each firm has a constant marginal cost of production equal to one

Because preferences are homothetic, aggregate demand equals $D_j(\omega, t) = d_j(\omega, t) L(t)$ in each industry ω. The pricing decision of each state-of-the-art good producer (henceforth, industry leader) depends on the size of innovation, $\lambda(\omega, t)$. If $\lambda(\omega, t) \geq [\sigma/(\sigma-1)]^{\sigma-1}$, the innovation is said to be *drastic*, and the innovating firm can capture all of the market by setting its price at the unconstrained monopoly price $p = \sigma/(\sigma-1)$. On the other hand, if $\lambda(\omega, t) < [\sigma/(\sigma-1)]^{\sigma-1}$, the innovation is said to be *non drastic* or *minor* or *incremental*, and the innovating firm is compelled to limit-pricing competitors at $p = \lambda(\omega, t)^{1/(\sigma-1)}$ in order to capture all of the market.

Let $\tilde{\lambda} \equiv [\sigma/(\sigma-1)]^{\sigma-1}$ denote the threshold size of λ that determines whether an innovation is either incremental or drastic, and let superscript "D" and "I" denote "drastic" and "Incremental" type of profit-maximizing prices. Firms' pricing decision can thus be summarized by the following pair of pricing

$$\begin{aligned} p_j^D(\omega, t) &= \sigma/(\sigma-1) && \text{if } \lambda(\omega, t) \geq \tilde{\lambda} \\ p_j^I(\omega, t) &= \lambda(\omega, t)^{1/(\sigma-1)} && \text{if } \lambda(\omega, t) < \tilde{\lambda} \end{aligned} \tag{11}$$

Using pricing rule (11) to substitute for $p_j(\omega,t)$ in (5), sales of each type of industry leader are respectively equal to

$$d_j^D(\omega,t) = \frac{q_j(\omega,t)\,c(t)\,L(t)}{[\sigma/(\sigma-1)]^{\sigma}\,P(t)^{1-\sigma}} \tag{12}$$

$$d_j^I(\omega,t) = \frac{q_j(\omega,t)\,c(t)\,L(t)}{\lambda(\omega,t)^{\sigma/(\sigma-1)}\,P(t)^{1-\sigma}}. \tag{13}$$

From (11)-(13), the instantaneous flow of profits earned by each industry leader will depend on the size of the quality increment. Using (5) and (11), the flow of profits when innovation is drastic is

$$\pi_j^D(\omega,t) = \frac{\sigma^{-\sigma}(\sigma-1)^{\sigma-1}q_j(\omega,t)\,c(t)\,L(t)}{P(t)^{1-\sigma}},$$

whereas the flow of profits when innovation is non-drastic is

$$\pi_j^I(\omega,t) = \frac{\left[\lambda(\omega,t)^{\frac{1}{\sigma-1}}-1\right]\lambda(\omega,t)^{-\frac{\sigma}{\sigma-1}}\,q_j(\omega,t)\,c(t)\,L(t)}{P(t)^{1-\sigma}}.$$

2.3 Research and development

2.3.1 R&D technology and Pareto distribution

There is free entry into each R&D race, so firms may target their research effort at any of the continuum of state-of-the-art quality products. Labor is the only input used in R&D and all firms have the same R&D technology. Any firm i that hires $\ell_i(\omega,t)$ units of R&D labor in industry ω at time t is able to discover the next higher quality product $j+1$ with instantaneous probability (or Poisson arrival rate)

$$\iota_i(\omega,t) = \frac{Q(t)^{\phi}\,\ell_i(\omega,t)}{\eta q_j(\omega,t)}, \tag{14}$$

where $Q(t) \equiv \int_0^1 q_j(\omega,t)\,d\omega$ is the average quality across industries at time t, $\phi > 0$ is an across-industry R&D spillover parameter and $\eta > 0$ is a R&D productivity parameter.

Eq. (14) is similar to that used by Minniti, Parello and Segerstrom (2013). It captures one reason why innovating can become more difficult over time, and one reason why innovating can become less difficult over time. First, innovating can become more difficult over time because $q_j(\omega,t)$ increases. Since $q_j(\omega,t)$ only increases when innovation occurs, this term highlights a source of increasing R&D difficulty, namely research successes. As products improve in quality and become more complex, the creation of the next vintage quality becomes more difficult. Second, innovating can become less difficult over time because $Q(t)^{\phi}$ increases. This term captures the possibility of positive across-industry knowledge spillover. As other industries experience R&D successes and $Q(t)$ increases over time, this contributes to increasing the likelihood of research success by individual firms. These positive R&D spillover have been found to be significant in many empirical studies [see Griliches (1992) and Sveikauskas (2007)].

Once a firm wins a R&D race, it observes its realized quality jump λ and decides whether to charge the unconstrained monopoly price or the limit price. We assume that the size of the quality jump is drawn from a Pareto distribution with a shape parameter $1/k$ and a scale parameter equal to 1. The probability density function for this Pareto distribution is given by

$$g(\lambda) = \frac{1}{k}\lambda^{-(1+k)/k}, \qquad \lambda \in [1, \infty). \tag{15}$$

The law of large numbers implies that the fraction of firms observing a certain realization is exactly equal to the probability of this realization. Consequently, by letting f^I and f^D denote, respectively, the probability that a successful firm can get either an incremental or a drastic innovation, with $f^I + f^D = 1$, we can use (15) to obtain

$$f^I \equiv \int_1^{\tilde{\lambda}} g(\lambda)\mathrm{d}\lambda = 1 - [\sigma/(\sigma-1)]^{-(\sigma-1)/k}. \tag{16}$$

$$f^D \equiv \int_{\tilde{\lambda}}^{\infty} g(\lambda)\mathrm{d}\lambda = [\sigma/(\sigma-1)]^{-(\sigma-1)/k}. \tag{17}$$

At each moment of time, a fraction of firms equal to (16) are limit-pricing oligopolists, and a fraction equal to (17) are unconstrained monopolists. When k approaches 0, f^I approaches 1 and f^D approaches 0. In this case, innovation is always incremental, and the manufacturing sector is populated by a continuum of heterogeneous oligopolists whose individual flow of sales is give by (13). Conversely, when k approaches 1, f^I approaches $1 - 1/\tilde{\lambda} = 1 - [\sigma/(\sigma-1)]^{-(\sigma-1)}$ and f^D approaches $1/\tilde{\lambda} = [\sigma/(\sigma-1)]^{-(\sigma-1)}$. In this case, the manufacturing sector can be split into a fraction of $1 - 1/\tilde{\lambda}$ oligopolistic industries with individual sales equal to (13), and a fraction of $1/\tilde{\lambda}$ monopolistic industries selling an individual flow of sales equal to (12).

In this vein, hence, the parameter k can be interpreted as a measure of dispersion in R&D realizations. A lower value of k corresponds to a thinner upper tail of the distribution of quality jumps, which corresponds to a higher share of oligopolies in total economy. A higher value of k corresponds to a fatter upper tail of the distribution of quality jumps, which corresponds to a lower share of oligopolies in total industries.

2.3.2 R&D optimization

The R&D sector consists of a large number of firms engaged in innovation races. The prize for winning an innovation race is given by the stream of profits the successful firm will gain until a new innovation will be introduced in the same industry. At the beginning of each race, R&D firms face two different types of uncertainty. The first one is related to the outcome of the race, because the firm may fail to win the R&D race. The second one is related to the size of the quality jump and determines whether the firm will practice limit-pricing or pure monopoly pricing.

Let $v_{j+1}^e(\omega, t)$ denote the expected value of the uncertain profit stream for winning a R&D race and discovering the next higher quality product $j+1$ in industry ω at time t, and let s_R denote the fraction of firm's R&D cost subsidized by the government.[5] By hiring $\ell_i(\omega, t)$ units of R&D labor for a time interval of length dt, firm i expects to realize $v_{j+1}^e(\omega, t)$ with

[5]Throughout the paper we will assume that the chosen R&D subsidy s_R is fully financed through lump-sum taxation.

probability $\iota_i(\omega, t)\mathrm{d}t$. Thus, at each point in time t, firm i will choose its R&D employment ℓ_i in order to solve

$$\max_{\ell_i}\left\{v^e_{j+1}(\omega,t)\frac{Q(t)^{\phi}\ell_i(\omega,t)}{\eta q_j(\omega,t)}-\ell_i(\omega,t)(1-s_R)\right\}.$$

Perfect competition in the research sector implies

$$v^e_{j+1}(\omega,t)=\eta q_j(\omega,t)\,Q(t)^{-\phi}(1-s_R)\,. \tag{18}$$

If $v^e_{j+1}(\omega,t) < \eta q_j(\omega,t)\,Q(t)^{-\phi}(1-s_R)$, then the marginal cost of R&D exceeds the marginal benefit, and it is profit-maximizing for firms to devote no labor to R&D. In contrast, if $v^e_{j+1}(\omega,t) > \eta q_j(\omega,t)\,Q(t)^{-\phi}(1-s_R)$, then the marginal benefit of R&D exceeds the marginal cost, and it is profit-maximizing for firms to devote infinite resources to R&D. Only if (18) holds for all ω can a symmetric equilibrium exist where the innovation rate $\iota(t)$ is positive, finite and the same in all industries.[6]

2.4 Quality dynamics

Consider now how average quality $Q(t)$ evolves over time. In industry ω, the quality index $q_j(\omega,t)$ jumps to $q_{j+1}(\omega,t)=\lambda q_j(\omega,t)$ at the rate $\iota(t)$ when an innovation occurs. Since this process of quality improvement is common to all industries in the economy, the time derivative of $Q(t)$ can be written as

$$\dot{Q}(t)=\int_0^1(\lambda-1)q_j(\omega,t)\,\iota(t)\ \mathrm{d}\omega.$$

Using the law of large numbers, the previous equation becomes

$$\dot{Q}(t)=\iota(t)\int_0^1 q_j(\omega,t)\left[\int_1^{\infty}\lambda g(\lambda)\ \mathrm{d}\lambda-1\right]\ \mathrm{d}\omega,$$

which can be rewritten as

$$\frac{\dot{Q}(t)}{Q(t)}=\frac{k}{1-k}\iota(t)\,. \tag{19}$$

The growth rate of the average quality is an increasing function of the Pareto parameter k (which measures the mean of the innovation size distribution) and the industry-level innovation rate $\iota(t)$.[7]

[6] In fact, when (18) holds with equality, firms are indifferent to R&D projects across industries and labor can be chosen such that each industry has the same flow rate.

[7] By defining the average quality jump by $\lambda^e\equiv 1/(1-k)$, it is easy to verify that the $k/(1-k)$ ratio on the right-hand side of [19] can be written as λ^e-1. Thus, an increase in k positively affects the growth rate of Q through an increase in λ^e.

2.5 The stock market

There is a stock market that channels consumer savings to R&D projects and helps consumers to diversify the risks of holding stocks issued by firms. The stock market valuation of each innovation is the expected discounted profits that the innovation generates. We now solve for these expected discounted profits.

Regardless of whether an innovation is drastic or incremental, over a time interval $\mathrm{d}t$ the shareholder receives an expected dividend $\pi^e_{j+1}(\omega,t)\mathrm{d}t$, and the value of the quality leader appreciates by $\dot{v}^e_{j+1}(\omega,t)\mathrm{d}t$. Because each quality leader is targeted by other firms that conduct R&D to discover the next higher quality product, the shareholder suffers an expected loss of $v^e_{j+1}(\omega,t)$ if further innovation occurs. This event occurs with probability $\iota(t)\mathrm{d}t$, whereas no innovation occurs with probability $1-\iota(t)\mathrm{d}t$. Efficiency in the stock market requires that the expected rate of return from holding a stock of a quality leader is equal to the risk-less rate of return $r(t)$ that can be obtained through complete diversification. Taking limits as $\mathrm{d}t$ approaches zero, I get the following no-arbitrage condition for the stock market

$$\frac{\pi^e_{j+1}(\omega,t)}{v^e_{j+1}(\omega,t)}+\frac{\dot{v}^e_{j+1}(\omega,t)}{v^e_{j+1}(\omega,t)}=r\left(t\right)+\iota(t).$$

In equilibrium, the dividend rate, $\pi^e_{j+1}(\omega,t)/v^e_{j+1}(\omega,t)$, plus the expected rate of capital gains, $\dot{v}^e_{j+1}(\omega,t)/v^e_{j+1}(\omega,t)$, equals the risk-less interest rate, $r\left(t\right)$ plus a risk premium, equal to $\iota(t)$, capturing the risk that leaders can be driven out of business by further innovation. Using free-entry condition (18) and taking into account that $q_j\left(\omega,t\right)$ is fixed during the R&D race, the appreciation rate of the innovation value is given by $\dot{v}^e_{j+1}(\omega,t)/v^e_{j+1}(\omega,t)=-\phi\dot{Q}\left(t\right)/Q\left(t\right)$. Consequently, the expected dividend rate is

$$\frac{\pi^e_{j+1}(\omega,t)}{v^e_{j+1}(\omega,t)}=r\left(t\right)+\iota(t)+\phi\frac{\dot{Q}\left(t\right)}{Q\left(t\right)}.$$

Solving for the expected profit flow of the firm that wins the R&D race and produces the top quality $q_{j+1}\left(\omega,t\right)$, I get[8]

$$v^e_{j+1}(\omega,t)=\frac{\dfrac{q_j\left(\omega,t\right)c\left(t\right)L\left(t\right)}{Q\left(t\right)}\dfrac{k\left(1+k\right)\theta}{\sigma+k-1}}{r\left(t\right)+\iota(t)+\phi\dot{Q}\left(t\right)/Q\left(t\right)},\tag{20}$$

where θ is completely determined by parameter values.[9]

Thus, combining (20) with (18) and using (19) to get rid of $\dot{Q}\left(t\right)/Q\left(t\right)$, I obtain

$$\frac{\dfrac{k\left(1+k\right)\theta}{\sigma+k-1}c\left(t\right)}{r\left(t\right)+\iota\left(t\right)/\left(1-k\right)}=\eta x\left(t\right)\left(1-s_R\right),\tag{21}$$

where $x\left(t\right)\equiv Q\left(t\right)^{1-\phi}/L\left(t\right)$ is a new endogenous variable that measures relative R&D difficulty in the economy.

[8] See Appendix A.1 for further details.

[9] For the esact composition of the constant θ, see Appendix A.1.

Equation (21) is the research equation of the model, equating the discounted stream profits for winning an R&D race - left-hand side of (21) - to the cost of innovating - the right-hand side of (21). The benefit from innovating increases when $c(t)$ increases (the representative consumer buys more), when $r(t)$ decreases (future profits are discounted less) and when $\iota(t)$ decreases (the industry leader is less threatened by further innovation). The cost of innovating increases when η increases (R&D workers become less productive at generating innovation), when $x(t)$ increases (innovating becomes relatively more difficult), and when s_R decreases (the government subsidizes R&D less).

2.6 The labor market

Labor is perfectly mobile across industries, and between production and R&D activities. In each industry ω, consumers only buy from the current quality leader. Employment in the manufacturing sector is given by[10]

$$L_M(t) = \int_0^1 D_j(\omega, t)\ \mathrm{d}\omega = \frac{(\sigma-1)(1+k)}{\sigma(k+1)-1} c(t) L(t).$$

Total employment in the R&D sector is given by

$$L_I(t) = \eta\iota(t) Q(t)^{-\phi} \int_0^1 q_j(\omega, t)\ \mathrm{d}\omega = \eta\iota(t) Q(t)^{1-\phi}.$$

Using the clearing condition for the labor market $L(t) = L_M(t) + L_I(t)$, I obtain

$$1 = \frac{(\sigma-1)(1+k)}{\sigma(1+k)-1} c(t) + \eta\iota(t) x(t). \tag{22}$$

The two terms on the right-hand-side of (22) are the shares of labor in production and R&D activities, respectively. The production employment share increases when $c(t)$ increases (the representative consumer buys more). The R&D employment share increases when η increases (more R&D labor is needed to generate any given innovation rate), when $\iota(t)$ increases (firms innovate at a faster rate), and $x(t)$ increases (innovating is relatively more difficult).

3 Macroeconomic equilibrium

3.1 Characterization of the dynamic equilibrium

The dynamic equilibrium is defined by the paths $\{c(t), x(t), \iota(t)\}_{t=0}^{\infty}$ and prices $\{p(\omega, t),\ \omega \in [0,1]\}_{t=0}^{\infty}$, such that: (i) households and firms maximizes, respectively, intertemporal utility and profits; (ii) free entry (18) condition is met; (iii) all markets clear.

To characterize the dynamic equilibrium, I begin from the R&D difficulty index, $x(t) \equiv Q(t)^{1-\phi}/L(t)$. Differentiating $x(t)$ with respect to time and then using (19) and (22) to get rid of $\dot{Q}(t)$ and $\iota(t)$ in the resulting expression, I obtain the first dynamic equation of the model governing the time evolution of relative R&D difficulty

$$\frac{\dot{x}(t)}{x(t)} = \frac{B}{\eta(1-k)x(t)}\left[1 - \frac{(\sigma-1)(1+k)}{\sigma(k+1)-1}c(t)\right] - n, \tag{23}$$

[10] See Appendix A.2 for further details.

where $B \equiv (1-\phi)k$ is a positive constant provided that $\phi < 1$.

We now turn to consider the time evolution of per capita expenditure, $c(t)$. Substituting $\iota(t)$ and $r(t)$ from (22) and (21) in the Euler equation (9) yields

$$\frac{\dot{c}(t)}{c(t)} = \frac{k(1+k)\theta c(t)}{(\sigma+k-1)\eta x(t)(1-s_R)} - \frac{1}{(1-k)\eta x(t)}\left[1 - \frac{(\sigma-1)(1+k)c(t)}{\sigma(k+1)-1}\right] - \rho. \tag{24}$$

Equation (24) is the second dynamic equation of the model governing the time behavior of consumption expenditure, $c(t)$. Relative R&D difficulty x is predetermined at time $t = 0$, while consumption expenditure is not predetermined. Given the initial conditions $x(0) > 0$, the pair of non linear differential equations (23) and (24), along with the transversality condition[11]

$$\lim_{t\to\infty} e^{-(\rho-n)} c(t)^{-1} \eta x(t) = 0, \tag{25}$$

completely characterizes the time evolution of the economy.

3.2 The stationary state

In the steady state, per capita consumption $c(t)$ and relative-R&D difficulty $x(t)$ are not growing variables. Setting $\dot{c}(t) = \dot{x}(t) = 0$ in equations (23) and (24), I can solve the resulting equations to obtain

$$\hat{c} = \frac{\left(B + \frac{n}{\rho}\right)\frac{\sigma(1+k)-1}{(1+k)(\sigma-1)}}{B + \frac{n}{\rho} + \frac{k(1-k)\theta\left[\sigma(1+k)-1\right]n}{\rho(\sigma-1)(k+\sigma-1)(1-s_R)}} \tag{26}$$

$$\hat{x} = \frac{\frac{B}{\eta(1-k)n}\frac{k(1-k)\theta\left[\sigma(1+k)-1\right]n}{\rho(\sigma-1)(k+\sigma-1)(1-s_R)}}{B + \frac{n}{\rho} + \frac{k(1-k)\theta\left[\sigma(1+k)-1\right]n}{\rho(\sigma-1)(k+\sigma-1)(1-s_R)}}, \tag{27}$$

where the hat "$\hat{}$" indicates steady-state values for c and x.

Plugging (26) and (27) into (22), and then solving for ι gives the unique steady-state innovation rate

$$\hat{\iota} = \frac{1-k}{(1-\phi)k}n. \tag{28}$$

As in Minniti, Parello and Segerstrom (2013), the steady-state innovation rate $\hat{\iota}$ is an increasing function of the population growth rate n, an increasing function of the strength of R&D spillover parameter ϕ, and a decreasing function of the Pareto distribution parameter governing the expected size of innovation, k.

Finally, I solve for the steady-state rate of economic growth $\hat{g}$, I log-differentiate (7) with respect to time to obtain[12]

$$\hat{g} \equiv \frac{\dot{u}(t)}{u(t)} = -\frac{\dot{P}(t)}{P(t)} = \frac{n}{(\sigma-1)(1-\phi)}, \tag{29}$$

[11] The aggregate real value of financial wealth held by all households is $a(t)L(t) = \int_0^1 v_{j^e}(\omega,t)d\omega$, which, from free-entry condition (18), yields $a(t) = \eta Q(t)^{1-\phi}/L(t) = \eta x(t)$. Using this result to substitute for $a(t)$ in (10), we obtain the transversality condition (25).

[12] For the analytical details of the derivation of the growth rate of the price index $P(t)$, see Appendix A.1.

The steady-state rate of economic growth $\hat{g}$ is increasing in the population growth rate n, is increasing in the strength of R&D spillover ϕ and is decreasing in the elasticity of substitution between products σ.

3.3 Stability analysis

Taylor-expanding system (23)-(24) about the steady-state vector $\langle \hat{x}, \hat{c} \rangle^T$, I obtain the following linearized system

$$\begin{pmatrix} \dot{x}(t) \\ \dot{c}(t) \end{pmatrix} = \begin{pmatrix} -n & -\dfrac{B(1+k)(\sigma-1)}{(1-k)\eta\left[\sigma(1+k)-1\right]} \\ -\dfrac{\eta\rho\left[n+B(\rho-n)\right](k+\sigma-1)(1-s_R)}{Bk(1+k)\theta} & J_{22} \end{pmatrix} \begin{pmatrix} x(t)-\hat{x} \\ c(t)-\hat{c} \end{pmatrix}, \tag{30}$$

where

$$J_{22} = \left(\rho + \frac{n}{B}\right) \frac{k^2\theta(1-k\sigma) + (\sigma-1)\left[k(\theta+1-s_R) + (\sigma-1)(1-s_R)\right]}{k(1-k)\theta\left[\sigma(1+k)-1\right]}.$$

Letting J denote the matrix of coefficients in (30), straightforward computations lead to

$$\det J = -\left\{ nJ_{22} + \frac{\rho(\sigma-1)\left[n+B(\rho-n)\right](k+\sigma-1)(1-s_R)}{k(1-k)\theta\left[\sigma(1+k)-1\right]} \right\}.$$

If σ is sufficiently low, $J_{22} > 0$ and the determinant of the matrix of coefficient J is always negative, meaning that the steady-state equilibrium (26)-(27) is saddle-path stable. Given an initial level for the relative R&D difficulty index, $x(0) > 0$, the transversality condition (25) allows households to choose the initial value of their consumption expenditure so as to allow the economy to jump onto the unique converging path. That implies that convergence to the steady state occurs smoothly over time.

Proposition 1 *The steady-state equilibrium given by (26) and (27) is saddle-path stable.*

Though both eigenvalues can be determined analytically, the resulting expressions are too unwieldy to yield useful insights. To get around, in the next section I calibrate the model parameters to US data to determine the magnitude of the rate of convergence and assess the persistence of R&D policy.

4 Quantitative analysis

4.1 Empirical considerations

In the simulation I set $\rho = 0.07$, $n = 0.01$, $\sigma = 2$, $\eta = 2.5$ and $\phi = 0.5$. The subjective discount rate ρ is chosen to match the long-run average interest rate in the data. Following Mehra (2008), in the calibration I set $\rho = 0.07$ consistent with the 7.6 percent average real return on the US stock market over the past 116 years. The population growth rate n is chosen to match the US population growth rate. According to the World Development Indicators (World Bank, 2007), the average annual rate of population growth in the US between 1975 and 2007 was around 1.0 percent. Therefore I set $n = 0.01$.

The elasticity parameter σ is one of the key parameters because it affects simultaneously the threshold size of innovation, $\tilde{\lambda} \equiv [\sigma/(\sigma-1)]^{\sigma-1}$,and the level of the markup charged by monopolists, $p^D - 1 = 1/(\sigma-1)$. We set $\sigma = 2$, so that to get a threshold size of innovation, $\tilde{\lambda}$, equal to 2, and the highest markup over marginal cost of 100 percent, such that $p^D - 1 = 1$.

Finally, both the R&D productivity parameter η and the R&D spillover ϕ play no particular role in determining the equilibrium share of labor in R&D. Thus, in the simulation I set $\eta = 2.5$ in order to avoid some endogenous variables - notably per capita consumption, c - to become negative because of policy interventions, and $\phi = 0.5$ so as to guarantee that the steady-state rate of economic growth is 2 percent per year, which is consistent with the average US GDP per capita growth rate from 1950 to 1994 reported in Jones and Williams (2000) and Jones (2005). Indeed, given $n = 0.01$, $\phi = 0.5$ and $\sigma = 2$, Eq. (29) delivers a value for g of 0.02.

4.2 Transitional dynamics and speed of convergence

In this section I assess how changes in the parameter governing the shape of the Pareto distribution can affect the length of the adjustment process of the economy. In doing so, I will simulate three different economies, each characterized by three different values of the Pareto parameter: $k = 0.1$, giving a percentage of oligopolies out of total industries, f^I, close of 100 per cent - specifically $f^I = 0.999$ and $f^D = 0.001$; $k = 0.5$, giving a percentage of oligopolies out of total industries f^I of 75 percent, and a percentage of monopolies out of total industries f^D of 25 percent; $k \approx 1$, giving a percentage of oligopolies equal to the percentage of unconstrained monopolies - i.e. $f^I \approx 0.5$ and and $f^D \approx 0.5$.[13]

In the first economy, my choices of $k = 0.1$ and $\sigma = 2$ implies that the average price level characterizing such an economy - henceforth denoted by $E[p]$ - is very close to the marginal cost of production and is equal to 1.10, while the top price charged by unconstrained monopolists is $p_{max} = 2.0$.[14] Consequently, in the remainder of the paper I will refer to this economy as the *stiff competitive economy.*

In the second economy, $k = 0.5$ and $\sigma = 2$ imply that the share of leaders charging an unconstrained-monopoly price adds to 25 percent of total industries, while the share of leaders underpricing competitors shrinks to 75 percent. Differently from the previous case, thus, in such an economy the average price level of products is not so close to the marginal cost of production and equals $E[p] = 1.50$. We will refer to this economy as the *intermediate economy.*

[13] As the model delivers indeterminate values for $k \to 1$, in simulating the third economy we set $k =$ 0.99999999375.

[14] Basu (1996) and Norrbin (1993) estimates that the average markup of price over marginal cost ranges between 1.05 and 1.4 - see . Consequently, for the first scenario the average markup is close to the bottom of this range, $Ep = 1.10$ (i.e. 10 percent).

Finally, in the third economy a value for k very close to 1 determines a fifty-fifty distribution of oligopolies and unconstrained monopolies. Again, the average price products increases compared to the previous two economies because of loose competition, and it reaches $E[p] = 1.69$. We will refer to this last economy as the *limp competitive economy.*

Table 1 displays the dynamic properties for each of the three economies. The first column of the table specify the type of the economy figures refer to, while Columns 2-3 indicate the length of the transition path predicted by the model expressed in terms of both the conditional rate of convergence and the time required to close half of the gap to the steady state - the so-called *half-life*. Finally, column 4-6 describe the industrial composition of each economy, along with the strength of product competition measured by the average price of products $E[p]$.

	Convergence rate	Half-life	f^D	f^I	$E[p]$
Stiff competitive economy	-0.013	52.6	0.001	0.999	1.111
Intermediate economy	-0.022	31.6	0.250	0.750	1.500
Limp competitive economy	-0.090	7.7	0.500	0.500	1.693

Table 1: Convergence speed and industry composition. The convergence rate displayed in the table refers to the nonexplosive eigenvalue of the matrix of coefficients (30).

Glancing at the reported figures, the table clearly indicates the existence of a negative relationship between the length of the adjustment process and the industrial composition of the manufacturing sector of each economy. The rates of conditional convergence reported by the table range from -0.013 when the percentage of monopolies is only 0.1 percent of total industries and the average price of products is not too far from the marginal cost of production, to -0.09 when half of total industries are led by unconstrained monopolists and the average price of products is far higher than the marginal cost of production. Hence, the higher the average markup over the marginal cost, the shorter the length of the convergence path to the steady state, implying that economies characterized by *limp* competition in manufacturing are usually very near to their steady states, while this is not the case for economies where competition is far more tough. The implied time lapse required to go half the distance to the steady-state is thus 52.6 years for the *stiff* competitive economy, 31.6 for the *intermediate* economy, and 7.7 years for the *limp* competitive economy.

Armed with these results, in the next section I will explore the policy implications of the model. We will begin by assessing the steady-state impact of a 10 percent subsidy on research on the three economies under considerations, by focusing on four key variables of the model, namely relative R&D difficulty $\hat{x}$, per capita consumption $\hat{c}$, innovation rate $\hat{\imath}$ and growth rate $\hat{g}$, plus a consumer welfare index $\hat{u}$, whose composition will be spelled out later on in the section. Then I will move to analyze the short-run impact of such a growth policy, by paying particular attention to the welfare implications of R&D subsidy.

4.3 The steady-state effects of R&D policy

Consider the three economies described in the previous section, and suppose that each of them is in its long-run equilibrium. At $t = 0$, each government introduces a 10 percent R&D subsidy, $s_R = 0.1$.[15] Table 2 compares the pre- and post-policy steady-state values of the following endogenous variables of the model: consumer welfare $\hat{u}^*$, relative R&D difficulty index $\hat{x}$, per capita consumption $\hat{c}$, the innovation rate $\hat{\imath}$ and the growth rate $\hat{g}$.[16]

	Stiff competitive economy $f^I = 0.999$, $f^D = 0.999$ Half-time: 52.6 years			*Intermediate* economy $f^I = 0.750$, $f^D = 0.250$ Half-time: 31.6 years			*Limp competitive* economy $f^I = 0.500$, $f^D = 0.500$ Half-time: 7.7 years		
	Initial s.s.	Final s.s.	$\Delta\%$	Initial s.s.	Final s.s.	$\Delta\%$	Initial s.s.	Final s.s.	$\Delta\%$
s_R	–	0.100	–	–	0.100	–	–	0.100	–
$\widehat{u}$	0.023	0.028	12.2	7.271	8.613	18.5	$1.88 \cdot 10^{17}$	$2.23 \cdot 10^{17}$	18.2
$\hat{x}$	0.151	0.166	9.9	2.496	2.736	9.6	$3.76 \cdot 10^{8}$	$4.13 \cdot 10^{8}$	9.7
$\hat{c}$	1.017	1.009	−0.8	1.167	1.151	−1.4	1.324	1.306	−1.4
$\hat{\imath}$	0.180	0.180	–	0.020	0.020	–	0.00	0.00	–
$\hat{g}$	0.020	0.020	–	0.020	0.020	–	0.02	0.02	–

Table 2: The long-run effects of R&D Policy

The welfare index used in the simulation is a de-trended version of the indirect utility index (7). More specifically, in the simulation I use the following index[17]

$$\hat{u} = \hat{x}^{1/[(1-\phi)(\sigma-1)]}\hat{c}.$$

In all economies, introducing a 10 percent increase in R&D subsidy causes consumer welfare $\hat{u}$ and R&D difficulty $\hat{x}$ to increase permanently, and capita consumption $\hat{c}$ to decrease permanently. Not surprisingly, supporting research through public funds has no steady-state effects on neither the innovation rate $\hat{\imath}$, nor the rate of economic growth $\hat{g}$.

The increase in consumer welfare is higher in the *limp* and the *intermediate* economy (around 19 percent) than in the *stiff* competitive economy (about 12 percent), due to the combined effects of the increase in $\hat{x}$, and the fall in $\hat{c}$. Differentiating $\hat{u}$ with respect to s_R, I can manipulate terms to obtain

$$\frac{\partial \hat{u}^*}{\partial s_R} > 0 \quad \Longrightarrow \quad \frac{\hat{c}}{(1-\phi)(\sigma-1)\,\hat{x}}\frac{\partial \hat{x}}{\partial s_R} > -\frac{\partial \hat{c}}{\partial s_R}.$$

[15] Our choice of the size of the R&D subsidy is not arbitrary and follows an OECD study - OECD (2000) - according to which the average share of business enterprise R&D expenditure financed by government amounts 10 percent.

[16] For convenience, Table 1 also reports the speed of convergence predicted by the model, expressed in terms of the time required to cover half the gap to the steady state, for each of the three economies under considerations.

[17] See Appendix A.3 for the construction of $\hat{u}$.

where $\frac{\partial \hat{x}}{\partial s_R}$ captures the change in the steady-state level of R&D difficulty, and $\frac{\partial \hat{c}}{\partial s_R}$ captures the change in the steady-state value of per capita consumption. As Table 2 illustrates, the fall in per capita consumption ranges from the -0.8 percent of the *stiff competitive* economy to the -1.4 percent of the *intermediate* and *limp competitive* economy, and can be explained by the need of each local labor market to relocate workers out of the manufacturing sector and towards R&D because of the subsidy. As regards the effects on in state of technology captured by the level variable x, figures in Table 2 show that the long-run impact induced by the increase in s_R is not very sensitive to changes in the industrial composition of the economy, and that it can be quantified in about 9/10 percent. Consequently, the overall improvement in index $\hat{u}$ can be explained by the positive effects induced by the permanent increase in the R&D difficulty index $\hat{x}$, which are able to more than offset the negative effect induced by the fall in $\hat{c}$. As f^D gets larger and larger, though, the positive welfare effects generated by the increase in $\hat{x}$ become stronger and stronger, with the consequence that the long-run impact on welfare turns out to be larger in the economy with the largest share of monopolies out of total industries.

Summing up, though R&D policy fails to affect the long-run economic growth, my model predicts that research subsidy definitely improves consumer welfare and the state of technology. However, revisiting this result form the perspective of the expected size of innovation and firms composition, my model also predicts that the effectiveness of R&D subsidy to improve living standards and welfare depends crucially on whether innovations are on average drastic or incremental.

Such a result seems to give theoretical support to the conclusion that R&D policies are more persistent in time when innovation gives firms a high probability of becoming unconstrained monopolists. However, in the next section I will show that the short-run effects generated by R&D policy are far from being consumer-friendly, and that the magnitude of the short-run deviations from the steady-state induced by subsidy tends to depend on the industry composition of the economy.

4.4 Assessing the short-run impact on welfare

In line with most R&D-based endogenous growth models, in the previous section I saw that R&D subsidy always improves consumer welfare. This section investigates how the economy reacts to the introduction of the subsidy, and whether the size of the short-run impact can be dependant upon the industry composition of manufacturing.

Figures 1 depicts the impulse response functions (IRFs) of each of the three economies considered in the paper. As is easy to verify, the introduction of a positive R&D subsidy makes consumer welfare to fall initially. However, the fall is less dramatic in the *stiff competitive* and *intermediate* economy than in the *Limp competitive* economy, reflecting the different combined effects that the R&D policy generates on both c and x in the short term. In fact, by making innovation cheaper, R&D subsidy encourages firms to hire R&D workers. However, because the economic system was already in full employment before government intervention, the increase

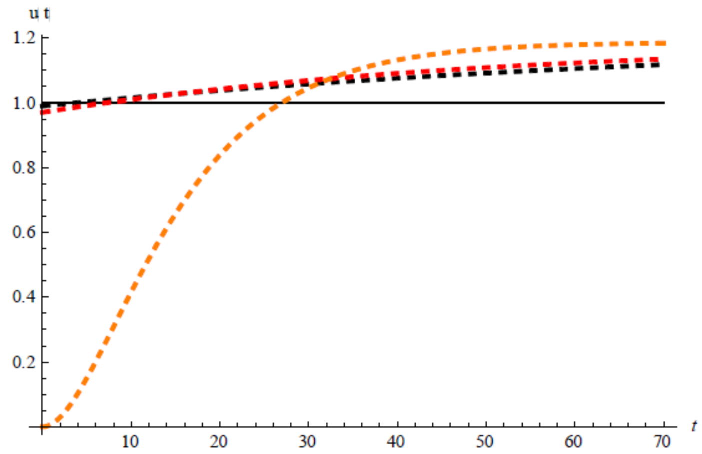

Figure 1: IRFs for the welfare index: Stiff competitive economy (black) Vs. Intermediate economy (red) Vs. Limp competive economy (orange).

in the demand for R&D workers can be matched only at the expenses of manufacturing employment. That generates a permanent reduction in the production of goods and services, implying that that consumption shrinks down in the after-policy equilibrium because of the burst in the research activity. The increase in the demand for R&D workers is stronger in the case of the *Limp competitive* economy because higher values of k makes the expected size of innovation higher, and thus expected profits larger.[18] That gives firms a strong incentive to engage more in R&D, and hence to hire a larger number of R&D workers in order to be successful in improving the state of the art of technology.

Even though R&D policy is beneficial for long-run consumer welfare, hence, in the short run its implementation is not harmless for consumers and tends to depend on the market composition characterizing the implementing economy. And this for at least two reasons. First, the absence of transitional dynamics characterizing many R&D-based endogenous growth models makes these models useless to study the short-run effect of R&D policy. For instance, In fully-endogenous Schumpeterian models, particularly those belonging to the first wave of R&D-based endogenous growth models, if the system is in its own balanced-growth path and a research subsidy is introduced by the government, then the economy's response will be to attain the level of per capita consumption that make the economy jump instantaneously onto the new balanced-growth path equilibrium. In such a situation, the conditional convergence rate predicted by this literature approaches infinity, implying that the time lapse required to recoup the negative impact of R&D policy approaches zero.

Second, the use of an infinitely-lived representative consumer to describe the demand-side of the economy makes all short-run considerations regarding growth and welfare totally uninteresting from a theoretical point of view. In semi-endogenous Schumpeterian models, for example, even though the economic system tends to adjust smoothly in case of a change in R&D policy,

[18] Notice that the percentage increase in the rate of innovation amounts to 557 for the high-markup case, and to 127 for the low-markup case.

the presence of the infinitely-lived representative consumer makes the temporary fall in welfare only temporary and thus totally uninteresting for the discussion of the model. However, as shown in Table 3, the time required to recoup the welfare lost because of R&D subsidy can be quite long and exceed 26 years.

	Stiff economy	Intermediate	Limp economy
t	3.7 years	7.2 years	26.9 years

Table 3: Time required to recoup the pre-policy welfare level

In a more sophisticated model in which individuals are allowed to live for only a finite period of time, some cohorts of consumers, e.g. the oldest ones, could find R&D policy not so appealing in terms of welfare when the odds of getting drastic innovation are high and life-expectancy low. Intuitively, rather than passively accept the policy, older cohorts could work against the implementation of any R&D-based growth policies, and in favor of either the status quo or alternative policies not necessarily growth-friendly. In the case of my model, for instance, those individuals expecting to live no more than 26 years could simply vote against the introduction of a research subsidy in a majority voting, with the result that economies characterized by low growth, old population and loose competition in product markets could find it difficult to get R&D-based growth policy passed.

5 Final remarks

In this paper I have studied the possible links between firm's heterogeneity, innovation size and adjustment dynamics in a second-generation R&D-based semi-endogenous growth model. In the model, innovation is affected by two types of uncertainty. The first type relates to the eventuality of being successful in R&D, which is governed by a Poisson arrival rate. The second type relates to the eventuality that the size of the innovation is either higher or lower than a certain threshold, which is instead governed by a Pareto distribution.

The outcomes of the paper are the following. Firstly, I find that the length of the transition path turns out to depend upon the industrial composition of the manufacturing sector. From the policy point of view, this result implies that the persistence in time of R&D policy is dependant upon the expected size of innovation, and hence on how many industries are run by oligopolistic leaders and how many are run by monopolistic leaders. Specifically, I find that the larger the share of oligopolies out of total industries, the lower the convergence rate to the steady state, and thus the more persistent are the affects of R&D subsidy on economic growth.

Secondly, I find that the short-run impacts of R&D policy on consumption, consumer welfare and R&D spending tend to depend also on firms' heterogeneity in manufacturing. In particular, I find that introducing a 10 percent subsidy in the R&D sector causes per capita consumption and welfare to fall temporary, and R&D difficulty to rise. The fall in consumption and welfare is found to be negatively related to the length of the transition path. The larger the share of monopolies in the economy, the longer the time required to close the temporary welfare gap generated by R&D policy.

Appendix

A.1 Calculation of the expected profit flow earned by an industry leader

In this section, I determine the expected value of the uncertain profit stream earned by an industry leader that produces the quality product $j+1$ in industry ω at time t, that is

$$\begin{aligned}\pi^e_{j+1}(\omega,t) &= \int_1^{[\sigma/(\sigma-1)]^{\sigma-1}} \pi^I_{j+1}(\omega,t) g(\lambda)\ \mathrm{d}\lambda + \\ &+ \int_{[\sigma/(\sigma-1)]^{\sigma-1}}^{\infty} \pi^D_{j+1}(\omega,t) g(\lambda)\ \mathrm{d}\lambda.\end{aligned}$$

By substituting the flow of profits for incremental and drastic innovation, the expected profit flow $\pi^e_{j+1}(\omega,t)$ can be written as

$$\pi^e_{j+1}(\omega,t) = \frac{q_j(\omega,t)c(t)L(t)}{P(t)^{1-\sigma}} \left[\underbrace{\int_1^{[\sigma/(\sigma-1)]^{\sigma-1}} \lambda \left(\lambda^{\frac{1}{\sigma-1}} - 1\right) \lambda^{-\frac{\sigma}{\sigma-1}} g(\lambda)\ \mathrm{d}\lambda +}_{(a)} \right.$$

$$\left. + \underbrace{\int_{[\sigma/(\sigma-1)]^{\sigma-1}}^{\infty} \sigma^{-\sigma} (\sigma-1)^{\sigma-1} \lambda g(\lambda)\ \mathrm{d}\lambda}_{(b)} \right]. \qquad \text{(A1.1)}$$

First, I calculate the two integrals (a) and (b) of Eq. (A1.1). As concerns the first integral, I notice that the term $\lambda \left(\lambda^{\frac{1}{\sigma-1}} - 1\right) \lambda^{-\frac{\sigma}{\sigma-1}}$ can be easily written as $\left(1-\lambda^{\frac{1}{1-\sigma}}\right)$. As the probability density function $g(\lambda)$ is Pareto and is equal to $\frac{1}{k}\lambda^{-\frac{1+k}{k}}$, integral (a) can be rearranged as

$$\frac{1}{k}\int_1^{[\sigma/(\sigma-1)]^{\sigma-1}} \left(1-\lambda^{\frac{1}{1-\sigma}}\right) \lambda^{-\frac{1+k}{k}}\ \mathrm{d}\lambda = \frac{k\sigma - [\sigma/(\sigma-1)]^{\frac{1-\sigma}{k}} [\sigma(k+1)-1]}{(k+\sigma-1)\sigma}. \qquad \text{(A1.2)}$$

Let us now focus on integral (b). Using the Pareto density function $g(\lambda)$ and solving, I get

$$\frac{\sigma^{-\sigma}(\sigma-1)^{\sigma-1}}{k} \int_{[\sigma/(\sigma-1)]^{\sigma-1}}^{\infty} \lambda^{-\frac{1}{k}}\ \mathrm{d}\lambda = \frac{(\sigma-1)^{\sigma-1}[\sigma/(\sigma-1)]^{\frac{(k-1)(\sigma-1)}{k}}}{\sigma^\sigma(1-k)}. \qquad \text{(A1.3)}$$

Summing the two expressions in equations (A1.2) and (A1.3), it yields

$$\frac{k}{(\sigma+k-1)} \left[1 + \frac{k\left(\frac{\sigma}{\sigma-1}\right)^{-\frac{\sigma-1}{k}}}{1-k} \right]. \qquad \text{(A1.4)}$$

Next, I calculate $P(t)^{1-\sigma}$ which is at the denominator of the ratio that is outside the square brackets of Eq. (A1.1). By using Eq. (6), this term can be written as

$$\begin{aligned}P(t)^{1-\sigma} &\equiv \int_0^1 \left[\int_1^{[\sigma/(\sigma-1)]^{\sigma-1}} \frac{q_j(\omega',t)}{\lambda} g(\lambda)\ \mathrm{d}\lambda + \right. \\ &\left. + \int_{[\sigma/(\sigma-1)]^{\sigma-1}}^{\infty} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} q_j(\omega',t)\, g(\lambda)\ \mathrm{d}\lambda \right] \mathrm{d}\omega',\end{aligned}$$

which can be further rearranged as

$$P(t)^{1-\sigma} \equiv \int_0^1 q_j\left(\omega', t\right) \left[\underbrace{\int_1^{[\sigma/(\sigma-1)]^{\sigma-1}} \frac{g(\lambda)}{\lambda} \ \mathrm{d}\lambda}_{(c)} \right] \mathrm{d}\omega' +$$

$$+ \left(\tfrac{\sigma}{\sigma-1}\right)^{1-\sigma} \int_0^1 q_j\left(\omega', t\right) \left[\underbrace{\int_{[\sigma/(\sigma-1)]^{\sigma-1}}^{\infty} g\left(\lambda\right) \ \mathrm{d}\lambda}_{(d)} \right] \mathrm{d}\omega'. \tag{A1.5}$$

Firstly, I consider integral (c). As before, replacing the Pareto density function $g(\lambda)$ and solving, I get

$$\frac{1}{k} \int_1^{[\sigma/(\sigma-1)]^{\sigma-1}} \lambda^{-1-\frac{1+k}{k}} \ \mathrm{d}\lambda = \frac{1}{(1+k)} \left[1 - \left(\frac{\sigma}{\sigma-1} \right)^{\frac{(k+1)(1-\sigma)}{k}} \right]. \tag{A1.6}$$

Secondly, I calculate integral (d). Once again, using the Pareto density function $g(\lambda)$, this integral boils down to

$$\frac{1}{k} \int_{[\sigma/(\sigma-1)]^{\sigma-1}}^{\infty} \lambda^{-\frac{1+k}{k}} \ \mathrm{d}\lambda = \left(\frac{\sigma}{\sigma-1} \right)^{\frac{1-\sigma}{k}}. \tag{A1.7}$$

Plugging equations (A1.6) and (A1.7) into (A1.5), the term $P(t)^{1-\sigma}$ can be rewritten as

$$\begin{aligned} P(t)^{1-\sigma} \ &\equiv \ \int_0^1 q_j\left(\omega', t\right) \left\{ \frac{1}{(1+k)} \left[1 - \left(\frac{\sigma}{\sigma-1} \right)^{\frac{(k+1)(1-\sigma)}{k}} \right] \right\} \mathrm{d}\omega' + \\ &+ \left(\tfrac{\sigma}{\sigma-1}\right)^{1-\sigma} \int_0^1 q_j\left(\omega', t\right) \left[\left(\frac{\sigma}{\sigma-1} \right)^{\frac{1-\sigma}{k}} \right] \mathrm{d}\omega'. \end{aligned}$$

The latter can be rearranged as

$$\begin{aligned} P(t)^{1-\sigma} \ &\equiv \ \frac{1}{(1+k)} \left[1 - \left(\tfrac{\sigma}{\sigma-1}\right)^{\frac{(1+k)(1-\sigma)}{k}} \right] \underbrace{\int_0^1 q_j\left(\omega', t\right) \ \mathrm{d}\omega'}_{=Q(t)} + \\ &+ \left(\frac{\sigma}{\sigma-1} \right)^{\frac{(k+1)(1-\sigma)}{k}} \underbrace{\int_0^1 q_j\left(\omega', t\right) \ \mathrm{d}\omega'}_{=Q(t)}. \end{aligned}$$

Collecting $Q\left(t\right)$ and simplifying terms, I get

$$P(t)^{1-\sigma} \equiv Q\left(t\right) \left[\frac{1+k\left(\frac{\sigma}{\sigma-1}\right)^{-\frac{(\sigma-1)(1+k)}{k}}}{1+k} \right]. \tag{A1.8}$$

Finally, using equations (A1.4) and (A1.8), I can write the expected profit stream as

$$\pi_{j+1}^{e}(\omega, t) = \frac{q_j(\omega, t) c\left(t\right) L\left(t\right)}{Q\left(t\right)} \cdot \frac{k\left(1+k\right)\theta}{\left(\sigma+k-1\right)}.$$

with θ being equal to $\left(1+\frac{k\left(\frac{\sigma}{\sigma-1}\right)^{-\frac{(\sigma-1)}{k}}}{1-k}\right) / \left[1+k\left(\frac{\sigma}{\sigma-1}\right)^{-\frac{(\sigma-1)(1+k)}{k}}\right]$.

A.2 Labour in the manufacturing sector

In this section, I determine the amount of employment in the manufacturing sector, that is

$$\begin{aligned} L_M(t) &= \int_0^1 D(\omega,t)\ \mathrm{d}\omega \\ &= \int_0^1 \left[\int_1^{[\sigma/(\sigma-1)]^{\sigma-1}} D(\omega,t)\ g(\lambda)\ \mathrm{d}\lambda + \int_{[\sigma/(\sigma-1)]^{\sigma-1}}^{\infty} D(\omega,t)\ g(\lambda)\ \mathrm{d}\lambda\right] \mathrm{d}\omega. \end{aligned}$$

By using equations (5) and (11) to substitute for $D(\omega,t)$, the previous expression becomes

$$L_M(t) = \underbrace{\frac{c(t)L(t)\int_0^1 q_j(\omega,t)\ \mathrm{d}\omega}{\int_0^1 q_j(\omega',t)p_j(\omega',t)^{1-\sigma}\ \mathrm{d}\omega'}}_{(e)} \left[\underbrace{\int_1^{[\sigma/(\sigma-1)]^{\sigma-1}} \lambda^{-\frac{\sigma}{\sigma-1}}\ g(\lambda)\ \mathrm{d}\lambda +}_{(f)}\right.$$

$$\left. + \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \underbrace{\int_{[\sigma/(\sigma-1)]^{\sigma-1}}^{\infty} g(\lambda)\ \mathrm{d}\lambda}_{(g)}\right]. \tag{A2.1}$$

We first consider the ratio (e) outside the square brackets in Eq. (A2.1). The denominator is $P(t)^{1-\sigma} \equiv \int_0^1 q_j(\omega',t)\, p_j(\omega',t)^{1-\sigma}\ \mathrm{d}\omega'$, that is equal to (A1.8); in the numerator, $\int_0^1 q_j(\omega,t)\ \mathrm{d}\omega$ can be replaced by $Q(t)$. Simplifying terms, the ratio (e) boils down to

$$\frac{c(t)L(t)\int_0^1 q_j(\omega,t)\ \mathrm{d}\omega}{\int_0^1 q_j(\omega',t)p_j(\omega',t)^{1-\sigma}\ \mathrm{d}\omega'} = \frac{(1+k)\,c(t)\,L(t)}{1+k\left(\frac{\sigma}{\sigma-1}\right)^{-\frac{(1+k)(\sigma-1)}{k}}}. \tag{A2.2}$$

Then, I focus on the two integrals inside the square brackets of Eq. (A2.1). Integral (g) is the same of integral (d) of formula (A1.5); the solution is given by Eq. (A1.7). Integral (f) reads as

$$\int_1^{[\sigma/(\sigma-1)]^{\sigma-1}} \lambda^{-\frac{\sigma}{\sigma-1}}\ g(\lambda)\ \mathrm{d}\lambda.$$

Using the Pareto density function $g(\lambda)$ and solving the integral, I get

$$\frac{1}{k}\int_1^{[\sigma/(\sigma-1)]^{\sigma-1}} \lambda^{-\frac{\sigma}{\sigma-1}-\frac{k+1}{k}}\ \mathrm{d}\lambda = \frac{\sigma-1}{\sigma(k+1)-1}\left[1-\left(\frac{\sigma}{\sigma-1}\right)^{-\frac{\sigma(k+1)-1}{k}}\right]. \tag{A2.3}$$

By using Equations. (A2.2), (A1.7) and (A2.3) into Eq. (A2.1), it yields

$$\begin{aligned} L_M(t) &= \frac{(1+k)\,c(t)\,L(t)}{1+k\left(\frac{\sigma}{\sigma-1}\right)^{-\frac{(1+k)(\sigma-1)}{k}}}\left\{\frac{\sigma-1}{\sigma(k+1)-1}\left[1-\left(\frac{\sigma}{\sigma-1}\right)^{-\frac{\sigma(k+1)-1}{k}}\right]+\right. \\ &\quad \left. + \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{\frac{1-\sigma}{k}}\right\}. \end{aligned}$$

which, after a bit of algebra, can be simplified as

$$L_M(t) = \frac{(\sigma-1)(1+k)}{\sigma(k+1)-1} c(t) L(t).$$

A.3 The consumer welfare index

This appendix presents some results used in the simulations. First, I calculate the expected price in the economy $E[p]$. Since marginal cost equals one for each firm, the markup of price over marginal cost for each firm is the firm's price, so the average markup is the expected price $E[p]$ charged by industry leaders. Using (11) and (15), the average markup in the economy is

$$E[p] \equiv \int_1^\infty p(\lambda) g(\lambda) \, \mathrm{d}\lambda = \frac{\left(\frac{\sigma-1}{\sigma}\right)^{-(\sigma-1)\left(\sigma-1-\frac{1}{k}\right)} - 1}{k(\sigma-1)-1} + \left(\frac{\sigma-1}{\sigma}\right)^{\frac{\sigma-1}{k}-1}$$

and the highest markup in the economy is

$$p_{max} \equiv \frac{\sigma}{\sigma-1}.$$

In the quantitative analysis I set k so $E[p] = 1.25$ and $p_{max} = 2.0$.

Second, I calculate what static utility equals for the representative consumer in steady-state equilibrium. From before $u(t) = c(t)/P(t)$ and in steady-state equilibrium, consumer expenditure $c(t)$ takes on the constant value $\hat{c}$. The price index satisfies $P(t) = \left[Q(t) \cdot E[p]^{1-\sigma}\right]^{1/(1-\sigma)}$. Since $x(t) \equiv Q(t)^{1-\phi}/L(t)$, it follows that

$$Q(t) = x(t)^{1/(1-\phi)} e^{[n/(1-\phi)]t}$$

and

$$P(t) = \left\{x(t)^{1/(1-\phi)} e^{[n/(1-\phi)]t} \cdot E[p]^{1-\sigma}\right\}^{1/(1-\sigma)}.$$

Thus, indirect utility is given by

$$u(t) = c(t) x(t)^{1/[(1-\phi)(\sigma-1)]} \cdot E[p] e^{\hat{g}t}.$$

As the $E[p] e^{\hat{g}t}$ is exogenously given, in calibrating consumer welfare I will use the population-adjusted sub-utility index

$$\hat{u} \equiv \frac{u(t)}{E[p] e^{\hat{g}t}} = c(t) x(t)^{1/[(1-\phi)(\sigma-1)]}.$$

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