Inefficient rationing with post-contractual information

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Abstract

We study a contractual design problem where some information is publicly observed ex-post and the allocation, but not transfers, can be made contingent on it. In contrast with previous results, we find that, in the optimal contract, even with linear utility function and absent any limited liability constraint, efficiency can decrease with post-contractual information. Moreover, the allocation rule of the optimal contract displays inefficient rationing: there is an interval in the type space in which lower types are assigned the good more often than higher types.

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1 Introduction

In a long-run contractual relationship, it is not infrequent that some provisions are contingent on information that becomes available during the life of the contract. A famous result (Riordan and Sappington, 1988) states that the arrival of even a little amount of information that reduces the information asymmetry about the agent's type may restore allocation efficiency and enable the principal to extract all the surplus from the agent.[1] Essentially, the contract must include a premium if the new information is coherent in expectation with the contractual choice of the agent and a penalty in the opposite case.[2] This result, however, has the disturbing feature that premia and penalties must be larger and larger as the signal becomes less and less informative. Therefore, the literature that followed focused on the presence of liquidity constraints on payments. Demougin and Garvie (1991) prove that, under limited liability, both profit and efficiency increase with the informativeness of the signal. Kessler et al. (2005) show that this result crucially depends on the payoffs being linear in the outcome. They find that post-contractual information can be harmful for efficiency in the general case.

All the results just described assume that, in principle, every element of the contract can be contingent on the post-contractual information. However, in the optimal contract, what crucially depends on the ex-post information is the transfer function: by penalizing the agent when the information is incoherent with the agent's revealed type, transfers are used to induce truthtelling and to extract as much surplus as possible from the agent. Given truthtelling, the allocation rule is then used to maximize social surplus, that is eventually appropriated by the principal through the transfers themselves.

In this paper we consider a contractual design problem in which, although some information related to the agent's type is revealed after the contract has been signed, transfers cannot be contingent on it: only the allocation rule can depend on the ex-post signal. This situation is theoretically interesting as previous work highlighted that, when the mechanism can be contingent on future information, transfers play the major role in inducing

[1]The crucial technical assumption for the result is that the type of an agent and the conditional distribution over the signal be in a one to one relation.

[2]Cremer and McLean (1988), McAfee and Reny (1992) and Mezzetti (2007) extended this intuition to the multiple agents' case: in this case, the principal does not even need to observe a signal himself, but may exploit the correlation across agents' signals and/or the interdependence of their payoffs. See also Bose and Zhao (2007).

truthtelling and rent extraction.

Specifically, we consider a seller-buyer context, where the buyer's valuation for the good on sale is private information, but the seller observes a binary signal that is related to the agent's valuation. Our main finding is that the optimal contract has an allocation rule that is non-monotone in the agent's type. In particular, the allocation rule works as follows. For agents with low and high valuations, the signal is not used at all: the former are not assigned the good, the latter receive the good for sure. For agents with intermediate valuations, instead, the allocation depends on the signal, but in a seemingly counterintuitive fashion: the good is transferred only if the signal turns out to be bad, while it is retained by the seller if the signal is good. As a consequence, within this interval of valuations, agents are inefficiently rationed: those with lower valuations are assigned the good more often than those with higher valuations. This last fact generates per se a loss in efficiency. It is then not surprising that, as we show, the presence of post-contractual information can decrease social surplus. Hence, in contrast with previous results, we find that, even with linear utility function and absent any limited liability constraint, efficiency can decrease with post-contractual information.

Intuitively, inefficient rationing allows the principal to extract surplus from relatively low types, without leaving much information rent to high types. In fact, the mechanism assigns the good with high probability to relatively low types (because these types are very likely to have a bad signal): hence, the principal can extract a fair amount of surplus from them; at the same time, high types find it unattractive to underreport their types, because, if they did so, they would be assigned the good with very little probability (they are likely to have a good signal).

The rest of the paper is organized as follows. Section 2 outlines the model, that is solved and analyzed in Section 3. Section 4 provides intuitions behind the results and possible applications of the model to real world situations. Section 5 concludes.

2 The Model

Consider the following contractual relationship between a principal (e.g. the seller of a good) and an agent (e.g. the buyer). The contract has two elements: a decision or allocation $\pi \in [0, 1]$ (e.g. whether or not the good is transferred from the seller to the buyer or, in general, the probability that this occurs) and a transfer t (potentially negative) from the agent to the principal. If the decision π and the transfer t are implemented, the agent's payoff is $u_A = \pi \times v - t$, while the principal's payoff is simply $u_P = t$. The parameter v (e.g. the valuation of the agent for the good) is private information to the agent and represents the agent's type. The principal knows that v is the realization of a continuous random variable defined on the interval $V = [\underline{v}, \overline{v}]$, where $\overline{v} > \underline{v} \geq 0$, with cumulative distribution function G, and strictly positive density g.

After the agent selects a contract (if any), the principal receives a (publicly observable) signal s, that we assume to be binary ($s \in \{0, 1\}$), and that is correlated with the agent's type. Let $p_0(v)$ be the probability of observing signal $s = 0$ when the type of the agent is v. Clearly, for all v, $p_1(v) = 1 - p_0(v)$. We assume that $p_0(v)$ is a differentiable and decreasing function of v. We will say that the signal is informative when $p_0(v)$ is strictly decreasing[3], it is uninformative when $p_0(v)$ is constant. In the following, we will consider as a benchmark the case of no signal, that we assimilate to the case of an uninformative signal.

To better screen the agent, the principal can use the information provided by the signal. We posit that only the decision π can be made contingent on the realization of the signal, while the transfer cannot.[4]

Thanks to the revelation principle, the contract design problem can be framed in terms of the choice of a direct revelation mechanism; under our assumptions, a contract can then be summarized by a set of three functions $C(s, v) = \{\pi_0(v); \pi_1(v); t(v)\}$, with the following interpretation: (1) the principal commits to the mechanism $C(v)$; (2) the agent accepts or reject the mechanism; (3) in case of acceptance, the agent reports his type v; (4) depending on the report, the transfer $t(v)$ is determined and executed; (5) the signal s is publicly observed; (6) depending on the realization of the signal ($s = 0$ or $s = 1$) and on the report, the allocation $\pi_s(v)$ is implemented.

Lastly, we make the following regularity assumptions:

(A1) for $s = 0, 1$, $v - \frac{1-G(v)}{g(v)}(1 + \eta_s(v))$ is increasing in v, where $\eta_s(v) = \frac{dp_s(v)}{dv}\frac{v}{p_s(v)}$ is the type elasticity of $p_s(v)$;

(A2) for $s = 0, 1$, $\exists\, v_s \in (\underline{v}, \overline{v})$ such that $v_s - \frac{1-G(v_s)}{g(v_s)}(1 + \eta_s(v_s)) = 0$;

[3] This is exactly equailavent to saying that the likelihood ratio $p_0(v)/p_1(v)$ is strictly decreasing.

[4] This assumption, which crucially differentiates our work from previous ones, will be discussed in Section 4, along with its possible applications.

(A3) $p_0(v)v < p_0(v_0)v_0$ for $v < v_0$; $p_0(v)v \geq p_0(v_0)v_0$ for $v_0 \leq v \leq v_1$.

Assumption (A1) guarantees that if, in equilibrium, $\pi_s(v) > 0$ for some v, then it must be $\pi_s(v') > 0$ for all $v' > v$. Assumption (A2) reduces the number of cases to be examined, focusing on the most interesting one. Notice that (A1) and (A2) jointly imply that, if the signal is informative, then $v_0 < v_1$; if it is uninformative, clearly $v_0 = v_1 = \tilde{v}$. Assumption (A3) is necessary to satisfy incentive compatibility.

3 Analysis

To characterize the optimal menu of contracts, thanks to the revelation principle, we can restrict our attention to truth-telling equilibria of a direct revelation mechanism $C(v)$.

The expected payoff of the principal if the agent always reports truthfully is:

$$U_P = \int_V t(v)g(v)dv. \tag{1}$$

The expected payoff of the agent, type v, if he reports $\hat{v}$ is:

$$U_A(\hat{v}; v) = E[\pi_s(\hat{v}); v]v - t(\hat{v}),$$

where $E[\pi_s(\hat{v}); v] = p_0(v)\pi_0(\hat{v}) + p_1(v)\pi_1(\hat{v})$ is the expected allocation.[5]

The optimal contract is the one that maximizes the principal's expected payoff, conditional on the agent accepting the contract and reporting truthfully, i.e. it is the solution to the following program P:

$$\max_{C(v)} U_P,$$

subject to:

$$v \in \arg\max_{\hat{v}} U_A(\hat{v}; v), \quad \text{for all } v \in V, \tag{2}$$

and to:

$$U_A(v; v) \geq 0, \quad \text{for all } v \in V. \tag{3}$$

Constraint (2) is the incentive compatibility constraint, which requires that, for every agent's type, truth-telling is a best reporting strategy. Constraint (3) is the individual rationality or participation constraint, which requires that signing the contract is convenient for all types.

[5]Notice that the optimal mechanism is one for which the revenue equivalence theorem holds. To see this, note that payments enter linearly in the payoffs of both the principal and the agent (see, e.g., Krishna 2010, Chapter 3).

Notice that, by the envelope theorem, (2) implies that

$$\left.\frac{dU_A(\hat{v};v)}{dv}\right|_{\hat{v}=v} = \left.\frac{\partial U_A(\hat{v};v)}{dv}\right|_{\hat{v}=v} \tag{4}$$

where

$$\left.\frac{\partial U_A(\hat{v};v)}{dv}\right|_{\hat{v}=v} = \sum_{s=0}^{1} p_s(v)\pi_s(v)(1+\eta_s(v)).$$

To solve for the optimal mechanism, we relax program P, by neglecting (3) and by replacing constraint (2) with the necessary condition (4). We obtain a relaxed program P', with maximand (1) and constraint (4). We will then check ex-post that the solution to P' does indeed solve P as well.

Now, the expected payoff of the principal in any incentive compatible mechanism writes

$$\int_V \left[\sum_{s=0}^{1} p_s(v)\pi_s(v)v - U_A(v;v)\right] g(v)dv.$$

Using (4) it is possible to reduce the above maximand to[6]

$$\int_V \sum_{s=0}^{1} \left[v - \frac{1-G(v)}{g(v)}(1+\eta_s(v))\right] p_s(v)\pi_s(v)g(v)dv. \tag{5}$$

Now we are ready to state the main result.

Proposition 1. *If the signal is* informative, *the optimal mechanism is*

$$\pi_s^*(v) = \begin{cases} 0 & \text{if } \underline{v} \leq v < v_s \\ 1 & \text{if } v_s \leq v \leq \overline{v} \end{cases}, \qquad s = 0,1; \tag{6}$$

$$t^*(v) = \begin{cases} 0 & \text{if } \underline{v} \leq v < v_0 \\ p_0(v_0)\, v_0 & \text{if } v_0 \leq v < v_1 \\ p_0(v_0)\, v_0 + p_1(v_1)v_1 & \text{if } v_1 \leq v \leq \overline{v} \end{cases}.$$

If the signal is uninformative, *the optimal mechanism is*

$$\pi_s^*(v) = \begin{cases} 0 & \text{if } \underline{v} \leq v < \tilde{v} \\ 1 & \text{if } \tilde{v} \leq v \leq \overline{v} \end{cases}, \qquad s = 0,1;$$

[6] The result follows from integration by parts.

$$t^*(v) = \begin{cases} 0 & \text{if} \quad \underline{v} \leq v < \tilde{v} \\ \tilde{v} & \text{if} \quad \tilde{v} \leq v \leq \overline{v} \end{cases} .$$

Proof. We prove only the first part of the proposition, the one with informative signal (the proof of the second part is analogous). Under Assumptions (A1) and (A2), the expression within square brackets in (5) is strictly negative for $v < v_s$, strictly positive for $v > v_s$. Hence, (5) is maximized pointwise by the allocation stated in (6). To pin down $t(v)$, notice that, for all v:

$$t(v) = E[\pi_s(v); v]v - U_A(v; v),$$

where

$$U_A(v; v) = \int_{\underline{v}}^{v} \frac{\partial U_A(u; u)}{\partial u} du + U_A(\underline{v}; \underline{v}) = \int_{\underline{v}}^{v} \sum_{s=0}^{1} p_s(u)\pi_s(u)(1 + \eta_s(u))du + U_A(\underline{v}; \underline{v}).$$

It is immediate to verify that, given $\pi_s^*(v)$, $t^*(v)$ solves the above equations for all v, with $U_A(\underline{v}; \underline{v}) = 0$. In fact, for $v < v_0$, we have:

$$t^*(v) = E[\pi_s^*(v); v]v - \int_{\underline{v}}^{v} \sum_{s=0}^{1} p_s(u)\pi_s^*(u)(1 + \eta_s(u))du = 0;$$

for $v_0 \leq v < v_1$, we have:

$$\begin{aligned} t^*(v) &= E[\pi_s^*(v); v]v - \int_{\underline{v}}^{v} \sum_{s=0}^{1} p_s(u)\pi_s^*(u)(1 + \eta_s(u))du \\ &= p_0(v)v - \int_{v_0}^{v} p_0(u)(1 + \eta_0(u))du = p_0(v_0)v_0; \end{aligned}$$

for $v_1 \leq v \leq \overline{v}$, we have:

$$\begin{aligned} t^*(v) &= E[\pi_s^*(v); v]v - \int_{\underline{v}}^{v} \sum_{s=0}^{1} p_s(u)\pi_s^*(u)(1 + \eta_s(u))du \\ &= v - \int_{v_0}^{v} p_0(u)(1 + \eta_0(u))du - \int_{v_1}^{v} p_1(u)(1 + \eta_1(u))du = p_0(v_0)v_0 + p_1(v_1)v_1. \end{aligned}$$

This shows that $(\pi_s^*(v), t^*(v))$ solve program P'. To see that they also solve program P, we first have to check that (2) is satisfied. Now, under $(\pi_s^*(v), t^*(v))$, we have that:

$$\begin{aligned} U_A(\underline{v} \leq \hat{v} < v_0; v) &= 0, \\ U_A(v_0 \leq \hat{v} < v_1; v) &= p_0(v)v - p_0(v_0)v_0, \\ U_A(v_1 \leq \hat{v} \leq \overline{v}; v) &= v - p_0(v_0)v_0 - p_1(v_1)v_1. \end{aligned}$$

It is easy to see that, when $v < v_0$:

$$U_A(v;v) = U_A(\underline{v} \leq \hat{v} < v_0; v) > U_A(v_0 \leq \hat{v} < v_1; v) > U_A(v_1 \leq \hat{v} \leq \overline{v}; v),$$

where the first inequality follows from assumption (A3) and the second follows from (A3) and the fact that p_1 is strictly increasing; when $v_0 \leq v < v_1$:

$$U_A(v;v) = U_A(v_0 \leq \hat{v} < v_1; v) \geq U_A(\underline{v} \leq \hat{v} < v_0; v) > U_A(v_1 \leq \hat{v} \leq \overline{v}; v),$$

where the first inequality follows from assumption (A3) and the second follows from (A3) and the fact that p_1 is strictly increasing; and when $v \geq v_1$:

$$U_A(v;v) = U_A(v_1 \leq \hat{v} \leq \overline{v}; v) \geq U_A(v_0 \leq \hat{v} < v_1; v) > U_A(\underline{v} \leq \hat{v} < v_0; v),$$

where the first inequality follows from the fact that p_1 is strictly increasing and the second follows from (A3). Hence, incentive compatibility is satisfied for all v. Finally, notice that also (3) is satisfied and is binding for the lowest type. This completes the proof.

Notice that, with informative signal, the second best is achieved by a menu of three contracts: one in which the good is not transferred and no payment is required, intended for low types ($v < v_0$); one in which the good is transferred for sure and a relatively high payment is required, intended for high types ($v \geq v_1$); one, intended for intermediate types ($v_0 \leq v < v_1$), in which the required payment is low and the allocation depends on the signal, but in a counterintuitive fashion: the allocation is $\pi^* = 1$ in case of bad signal ($s = 0$), it is zero otherwise. As a result, within the interval $[v_0, v_1)$, the expected allocation is strictly decreasing in the agent's valuation, thereby producing an inefficient rationing: higher types are assigned the good less ofthen than lower types. Moreover, it turns out that the presence of the ex-post signal hurts some types, while benefits some lower types. These observations are collected in the following proposition.

Corollary 1. *In the case of informative signal, the expected allocation under the optimal mechanism is non-monotone over V. Moreover, relative to the case of no signal, the expected allocation is larger for types in $(v_0, \tilde{v})$, smaller for types in $[\tilde{v}, v_1)$.*

Proof. With informative signal, the expected allocation in the optimal mechanism is easily computed in:

$$E[\pi^*(v); v] = \begin{cases} 0 & \text{if } \underline{v} \leq v < v_0 \\ p_0(v) & \text{if } v_0 \leq v < v_1 \\ 1 & \text{if } v_1 \leq v \leq \overline{v} \end{cases}.$$

Notice that the above function is strictly decreasing in $v_0 \leq v < v_1$. With uninformative signal (or no signal), the expected allocation in the optimal mechanism is:

$$E[\pi^*(v); v] = \begin{cases} 0 & \text{if} \quad \underline{v} \leq v < \tilde{v} \\ 1 & \text{if} \quad \tilde{v} \leq v \leq \overline{v} \end{cases}.$$

From assumption (A1), we know that $v_0 < \tilde{v} < v_1$. Hence, for types in $[v_0, \tilde{v})$, the presence of an informative signal increases the expected allocation from 0 to $p_0(v)$; for types in $[\tilde{v}, v_1)$, the presence of an informative signal decreases the expected allocation from 1 to $p_0(v)$.

The next proposition show that, while the presence of the informative signal obviously increases the expected payoff of the principal (the principal could always decide not to use the signal), the effect on welfare is ambiguous.

Corollary 2. *The presence of an informative signal does not necessarily increase social surplus.*

Proof. We simply construct one example in which the social surplus is strictly larger when the signal is uninformative, and one example in which the opposite occurs.

(i) Let $v \in [0, 1]$, $G(v) = v$, and $p_0(v) = [1 + 4(1 - v)]/6$. In this case we have $v_0 = [9 - \sqrt{21}]/12$, $v_1 = [3 + \sqrt{21}]/12$, $\tilde{v} = 1/2$. The expected social surplus with informative is easily computed in 0.365; with uninformative signal, it is 0.375.[7] *(ii)* Let $v \in [0, 1]$, $G(v) = v$, and $p_0(v) = 1 + 4v(3v - 3 - v^2)/7$. In this case we have $v_0 \approx 0.36$, $v_1 \approx 0.57$, $\tilde{v} = 1/2$. The expected social surplus with informative is easily computed in 0.387; with uninformative signal, it is 0.375.

For what concerns the impact of signal informativeness on efficiency, we have been able to prove a general result, although of local nature.

Proposition 2. *When the elasticity of $p_1(v)$ is sufficiently small, inefficiency is certainly higher under informative signal than under uninformative signal.*

[7] This example is interesting because the loss in efficiency associated with the signal is entirely due to inefficient rationing: in fact, it can be shown that, in this case, the ex-ante (i.e. averaging over V) expected allocation is the same with informative and uninformative signal. Hence, the allocation rule under informative signal is more inefficient exclusively because, in the interval $[v_0, v_1]$, it favors lower types against higher types, whereas, with uninformative signal, rationing is efficient.

Proof. We approximate any function $p_1(v)$ around $\tilde{v}$ with the constant elasticity function $\tilde{p}_1(v) = \frac{p_1(\tilde{v})}{\tilde{v}} v^{\tilde{\eta}_1}$, where $\tilde{\eta}_1 = \eta_1(\tilde{v})$ is the (constant) elasticity. Clearly, $\tilde{p}_0(v) = 1 - \tilde{p}_1(v)$. We develop a comparative statics exercise on the elasticity $\tilde{\eta}_1$ around zero, which corresponds to the case of an uninformative signal. It is immediately seen that $\frac{d\tilde{p}_1}{d\tilde{\eta}_1} = \frac{d\tilde{p}_0}{d\tilde{\eta}_1} = 0$ for $\tilde{\eta}_1 = 0$. Furthermore, we have the following lemma.

Lemma 1. *If* $\frac{1-G(v)}{g(v)}$ *is decreasing, then* $\frac{dv_0}{d\tilde{\eta}_1} > -\frac{p_1(v_0)}{1-p_1(v_0)} \frac{1-G(v_0)}{g(v_0)}$.

Proof. Recall that, by assumption (A2), $v_s - \frac{1-G(v_s)}{g(v_s)}(1 + \eta_s(v_s)) = 0$, $s = 0, 1$. Let $\gamma(v) = \frac{1-G(v)}{g(v)}$. We have $\frac{dv_s}{d\tilde{\eta}_s} = \frac{\gamma(v_s)}{1-(1+\tilde{\eta}_s)\gamma'(v_s)}$. Since $d\tilde{\eta}_0 = -\frac{p_1(v)}{1-p_1(v)} d\tilde{\eta}_1$, then $\frac{dv_0}{d\tilde{\eta}_1} = -\frac{\frac{p_1(v_0)}{1-p_1(v_0)}}{1-\gamma'(v_0)\left(1-\tilde{\eta}_1 \frac{p_1(v_0)}{1-p_1(v_0)}\right)} \gamma(v_0)$. When $\tilde{\eta}_1 = 0$ and $\gamma'(v_0) < 0$, then $\frac{dv_0}{d\tilde{\eta}_1} > -\frac{p_1(v_0)}{1-p_1(v_0)} \frac{1-G(v_0)}{g(v_0)}$. This concludes the proof of the Lemma.

Next, consider the social surplus SS with informative signal under the optimal mechanism. It is given by

$$SS = \int_{v_0}^{v_1} v(1 - p_1(v))g(v)dv + \int_{v_1}^{\overline{v}} vg(v)dv.$$

Its derivative with respect to $\tilde{\eta}_1$ at $\tilde{\eta}_1 = 0$ is

$$\left. \frac{dSS}{d\tilde{\eta}_1} \right|_{\tilde{\eta}_1 = 0} = -v_0(1 - p_1(v_0))g(v_0)\frac{dv_0}{d\tilde{\eta}_1} - v_1 p_1(v_1)g(v_1)\frac{dv_1}{d\tilde{\eta}_1}.$$

Under the assumption of Lemma 1, $\left. \frac{dSS}{d\tilde{\eta}_1} \right|_{\tilde{\eta}_1 = 0} < \frac{1-G(v_0)}{g(v_0)} p_1(v_1) \left[v_0 g(v_0) - v_1 g(v_1) \right]$. Notice that the RHS is lower than or equal to zero; therefore $\frac{dSS}{d\eta_1}_{\eta_1 = 0} < 0$. The proof can then be generalized to any function $p_1(v)$ by considering the Gateaux derivative of SS with respect to the elasticity of $p_1(v)$, considered as a function of v. This concludes the proof of the proposition.

4 Discussion

The result that, in the optimal mechanism with informative signal, the allocation rule is non monotone is a remarkable one and deserves some comments.

The reason why, for the principal, it is optimal to impose inefficient rationing in the interval $[v_0, v_1)$ – lower types receive the good more often than higher types – is that this allows to extract surplus from relatively low types, and, at the same time, does not leave much information rent to high types. To see this, start considering an agent of type v_0. This agent knows that, if he reports truthfully, he is pretty likely to receive the good (because, for him, the signal is likely to be $s = 0$). If, instead, he underreports $\hat{v} < v_0$, he will certainly not receive the good. Hence, by reporting truthfully, this agent would enjoy a relatively large gross surplus, that is extracted by the principal through a proper transfer.[8] Consider, now, type v_1 (or any type above): this type knows that, if he reports truthfully, he will receive the good for sure; if, instead, he mimics type v_0 (or any type $v \in [v_0, v_1)$), he is very likely not to receive it, because, being v_1 relatively high, the probability that the signal will be $s = 0$ is pretty low. In other words, by mimicking v_0, type v_1 will face the same (low) transfer as v_0, but not the same probability of getting the good, because the allocation, depending on the signal, eventually depends (negatively) on the *true* type. Hence, the fact that the allocation rule decreases with the (true) type when the report is in the interval $[v_0, v_1)$, has the effect of making it less attractive for high types to underreport their type, thereby allowing the principal to extract a large amount of surplus from them.

Compared with the case of no signal, the optimal mechanism under informative signal assigns the good more often to types in the interval $[v_0, \tilde{v})$, and less often to types in the interval $[\tilde{v}, v_1)$. In terms of efficiency, these two facts operate in opposite directions. It turns out that, as a whole, the ranking is ambiguous: it is perfectly possible that social surplus is lower with informative signal than without it. In particular, when the informativeness of the signal is very small, the presence of ex-post information reduces social surplus.

Lastly, let us discuss how realistic it is our assumption that only the allocation rule, not the transfers, can depend on the ex-post signal. It is a common practice for sellers to offer, together with a basic product or service, an additional product or service for an extra-price. Sometimes, these additional products are not delivered immediately, but in the future, as soon as they are ready. For example, along with a basic software, the buyer can pay an extra-price to get add-ons or updates to the software as soon as they will be developed. In principle, the seller can make the actual delivery of these add-ons dependent on future information, and this information could be for example related to the actual usage of the basic software by the buyer (that can easily be monitored). Of course, in these cases, also the price paid by the customer could in principle be related to this ex-post information. However, the seller may have more than one reason to prefer

[8] Assumption (A3) guarantees all types $v \in (v_0, v_1)$ do not find it convenient to underreport $\hat{v} < v_0$ either.

not to ask for additional payments in the future: for example, the seller may fear that a buyer would simply reject a contract that calls for possible additional future payments; or, it may want to avoid the risk that the buyer has no liquidity in the future to make the additional payment. In a context like this, the optimal allocation rule will then have the following interpretation: low types purchase only the basic product; high types, along with the basic product, subscribe the right to get future updates in exchange for an additional (relatively high) price; intermediate types, beyond the basic product, pay a (relatively low) price to get the *possibility* of receiving future updates; whether or not the updates will be acknowledged to the agent depend on future information, which is somewhat related to the characteristics of the user.

A second, more intriguing application of this model is to employer-employee relations in institutions with an administered salary system. In these organizations, it is often the case that wages associated with different levels are exogenously fixed and an employee can climb the wage ladder only through promotions. Our model can be adapted to this context. Consider an institution with two job positions: a low-level job with fixed wage and a high-level one, with wage related to productivity. Applied to this context, the allocation rule in our model is the promotion decision. Transfers are not monetary transfer; rather, they are unpaid additional activies that the employee must comply with if he wants to be entitled for a promotion in the future. For example, it may be the case that it is the manager who is in charge for the low-level division that decides on promotions: his primary interest is to make his division work properly, not to promote productive workers; hence, he promotes only those workers that execute additional duties, even though they do not receive any additional compensation for them. The agent's type can be interpreted as a measure of productivity: this productivity is totally irrelevant in the low-level job, but becomes relevant for the worker if he is promoted (for example, in the higher level job, wages are related to output). The allocation rule in the optimal mechanism can then be given the following interpretation: low productivity workers are not required any additional activity but they will never be promoted in the future; high productivity workers are promoted for sure, conditional on the execution of (heavy) additional activities; types with intermediate productivity are required lighter additional activities, and their promotion will depend on future information which is somehow related to their productivity.

5 Conclusion

We studied a contractual design problem when some information is publicly observed ex-post and the allocation but not transfers can be made contingent on it. We derived the optimal mechanism, showing that the allocation rule produces inefficient rationing: there is an interval in the middle of the type space where lower types are assigned the good more often the higher types. in addition, we showed that, with respect to the case in which ex-post information is absent, the presence of an informative signal can increase inefficiency.

The model has been kept as simple as possible to focus on its main implications, but can be extended in several directions. The extension to an n-dimensional signal is straightforward but does not seem to offer new insights. Clearly, the number of contracts necessary to implement the optimal solution will increase with the number of outcomes. However, the main result that the allocation is non monotone in the type will remain valid. Relaxing assumption (A1) has very standard consequences on the equilibrium: types which choose different contracts in our equilibrium will bunch together.[9] Instead, the violation of assumption (A3) is fatal: no incentive-compatible contract can take advantage of the additional information and therefore the static lemon solution will emerge.

[9]See Krishna (2010) for a standard treatment of bunching in direct revelation models.

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