How the Representativeness Heuristic Shapes Decisions

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The rational expectations equilibrium takes into account the information carved out of market prices. This equilibrium (mostly) hinges on the information and beliefs that agents bring to the market. In general, let p be a market price, w a utility-relevant state and s^a be the a's non-price signal with $a = 1, \ldots, A$. Define the market model π as a family of the probability distribution $\pi(w \mid s^a, p)$. The equilibrium price vector will be a measurable function (the so called *forecast function*) of the joint non-price signal $p = \psi(s)$ with $s = s_1, \ldots, s_A$. Given a price function ψ and a probability distribution F_a of w and s, the rational market condition for trader a is

$$\pi(w' \mid s^{a\prime}, p') \equiv \mathrm{Pr}_{F_a}[w = w' \mid s^a = s^{a\prime} \text{ and } \psi(s) = p']. \tag{1}$$

Define the market excess supply as $\Delta(s, p, \psi)$. It is plain that $\Delta(\cdot)$, for any p and s hinges also on the price function ψ, since it is used to calculate the rational condition. The REE is thus a price function ψ^* such that, for almost every s, the excess supply is zero at the price vector $\psi^*(s)$

$$\Delta(\psi^*(s), s, \psi^*) = 0 \quad \text{for a.e. } s. \tag{2}$$

The forecast function ψ describes the statistical relationship between market prices and their signals, and it is justified by an assumed common knowledge about the structure of the market. An equilibrium arises when each agent believes that the other agents use their equilibrium strategies. Assuming common knowledge and full rationality, then no-trade theorems (Milgrom and Stokey 1982) and no bubble conditions (Tirole 1982) prevent rational market from being distorted by any non-fundamental shocks. In any case, by the efficient market hypotheses (see Fama 1970), temporary effects of nonrational players are absorbed by rational agents who exploit arbitrage bringing back prices on their equilibrium path. However, arbitrage may be quite limited (see Shleifer and Vishny 1997). There are limits in the availability of capital which is needed for exploiting arbitrage opportunities, and there are limits in assuming risky positions (De Long et al. 1990). When limitations arise, the presence of a sufficient amount of non-rational players may endanger the market efficiency which relies on the fulfillment of the rational expectations equilibrium. It is thus crucial to understand what do we mean with "non-rationality". On the one hand, rationality is well defined (for example see Muth 1961 or Radner 1972), on the other hand nonrationality is still not well defined, it can be expressed by several definitions. In so far as market distortions are concerned, a promising one is that of the *representativeness heuristic*. People rely on representativeness heuristic whenever they have to assess the probability that an object A belongs to a class $\mathcal{A}$ or that A originates from a process $\mathcal{A}$. Agents thus rely on a shortcut from which it is high probable that A is belonged and generated from $\mathcal{A}$ in order of A resembles $\mathcal{A}$, that is by the degree to which A is representative of $\mathcal{A}$. It follows from the fact that perception (and intuitive evaluation) is reference-dependent: the perceived attributes of a focal stimulus reflect the contrast between that stimulus and a context of prior and concurrent stimuli (Kahneman 2003). It means that agents suffer from systematic and common errors in processing information. This finding, along with the presence of limitations in exploiting distortions, paves the way for the necessity to understand which models better describes cognitive limits in economics, what are the possible effects to markets and how to prevent these distortionary effects to come up as a surprise (see Gennaioli, Shleifer, and Vishny 2012 for the effects of a "surprise effect"in securitized markets).

1 Deciphering the intuition

From Debreu 1959, commodities are not just defined by their physical characteristics, availability locations and dates. Commodities do rely also on the *environmental event* in which they are available and used. A fresh ale of beer in a hot day is different with respect to the same ale but available in a cold night. A *state* of the environment is the complete specification of the environmental variable. Moreover, the *event* is a set of states; a set of all possible histories of the environment with a common characteristic. In this framework, future utilities of the commodities, depend on the state of the environment. In terms of the rational condition defined in equation (1), the market model π defines the inferences of the agents by a family of conditional probability distributions of the environment given the market prices p and the agent's nonprice information s^a. When agents' expectations are homogeneous and information is common knowledge, then it follows that nonprice information and signals embodied in price movements will coincide: prices will reflect the whole information. In this case distortions cannot arise, or in a weaker form, they cannot be long-lasting.

However, when homogeneous expectations are biased, information of s and p still coincide, but their understanding is biased which in turn distort how conditional probabilities are computed. The biased expected environment could thus differ from that expected by rational agents giving rise to distortionary effects in real and financial markets even if the total excess supply is zero (see Strati 2017b). The problem is that $\Delta(\cdot) = 0$ for a biased forecast function which detects exaggerate relations between s and p. By denoting this biased clearing excess supply by Δ^θ and the rational one by just Δ, then the distortion will be $\Delta^\theta - \Delta > 0$. The intertemporal equilibrium will be unfeasible ending up with a sharp reverting process. For example in Strati 2017a, is showed that this unfeasible path can trigger a severe fall in the aggergate demand.

2 A model of representativeness

We shall describe a general model of representativeness heuristic which can be applied to several situations. This heuristic has been studied in Kahneman and Tversky 1972; Kahneman and Tversky 1983 and recentely formalized in Gennaioli and Shleifer 2010; Bordalo et al. 2016. In Kahneman and Tversky 1983 the representativeness can be defined as follows

> *A person who follows this heuristic evaluates the probability of an uncertain event, or a sample, by the degree to which it is:* (*i*) *similar in essential properties to its parent population; and* (*ii*) *reflects the salient features of the process by which it is generated.*

As Gennaioli and Shleifer 2010, we consider limited cognitive capabilities, that is to say representativeness is an intuitive and fast thinking, and it is based on how scenarios become accessible from agents' memory. In particular, what comes to mind is a *limited* and *selective* retrieval from memory. Scenarios come to mind in order of their representativeness of a specific group rather than their probability, statistical theory of prediction is not fully considered: agents mix up likelihood and representativeness fostering a bias in the formation of expectations which do not fully rely on scientific probabilistic reasoning.

A decision maker judges the distribution of a trait A in a group G, $f(A = a \mid G)$. Because of the ease of recall, the decision maker will weight this distribution relatively to a distribution which describes the same trait A but with respect to a comparison group $-G$ for which $-G \cap G = \emptyset$, that is $f(A = a \mid -G)$. The representativeness can be measured by the following

$$\mathcal{R} \equiv \frac{f(A = a \mid G)}{f(A = a \mid -G)}. \tag{3}$$

A trait is more representative if it is *relatively* more frequent in G than in $-G$. In few words if $\mathcal{R} > 1$. In order to understand this heuristic, let us take an illuminating example from Bordalo, Gennaioli, and Shleifer 2017[1]: consider that someone has to guess the hair color of an irishman. Red irishmen are the 10% among the Irish, while that percentage in the rest of the world is only the 1%. However blond and light brown irishmen are the 40% among the Irish and the 14% in the rest of the world. Finally dark hair irishmen are the 50% of the Irish, while in the rest of the world the percentage is 85%. The representativeness works in this way:

$$\frac{\Pr(\text{red hair}|\text{Irish})}{\Pr(\text{red hair}|\text{World})} = 10 \quad \frac{\Pr(\text{blonde}|\text{Irish})}{\Pr(\text{blonde}|\text{World})} \approx 2,9 \quad \frac{\Pr(\text{dark}|\text{Irish})}{\Pr(\text{dark}|\text{World})} \approx 0,6$$

where the highest likelihood ratio gives 10, for which there is a disproportionate belief in favour of the frequency of red hair irishmen. Notice that half of them

[1]See data from `http://www.eupedia.com/genetics/origins_of_red_hair.shtml`

are dark haired and only a 10% are red haired. Agents are thus highly focused just on the differences between the two groups Irish and rest of the world. If this heuristic is applied to issues of finance and economics, then definitions of equilibria will change. For example in Gennaioli, Shleifer, and Vishny 2012, albeit they take into account a total neglection, this heuristic explains the overissue of backed securities which caused fire sales in the 2007-2008 crisis. Also in Strati 2017b, the representativeness heuristic is the cause of intertemporal mispricings in the collateral market triggering agents to deleverage.
As in Bordalo et al. 2016 and getting a more formal introduction to Equation (3), consider a set of (ordered) reference traits $A = \{a_1, \ldots, a_A\}$ and an overall population Ω for which $G \subset \Omega$. There is a probability distribution $\pi : A \times \Omega \to [0, 1]$ that sets off a conditional distribution $\pi_{a,G} = f(A = a \mid G)$, that is a probability of a type a in group G. Denote the vector $\pi_G = (\pi_{a,G})_{a \in A}$ which contains the conditional distribution. Define the comparison group as $-G \subset \Omega \setminus A$, then the repesentativeness of Equation (3) can be re-written as

$$\mathcal{R}(a, G, -G) \equiv \frac{\pi_{a,G}}{\pi_{a,-G}}.$$

The extent of representativeness is captured by a function $h : \mathcal{R}_+ \to \mathbb{R}_+$ called weighing function (see Bordalo et al. 2016). It is a symmetric function of representativeness of types $a \neq k$

$$h \equiv h\left(\frac{\pi_{a,\Gamma}}{\pi_{a,\Gamma^c}}; \left(\frac{\pi_{k,\Gamma}}{\pi_{k,\Gamma^c}}\right)_{k \in \Omega \setminus \{A\}}\right)$$

in which h is weakly increasing in its first argument, while it is weakly decreasing in the other arguments $k \in \Omega \setminus \{A\}$ (see Bordalo et al. 2016 for details).

In contonuous models it is convenient to use the so called "representativness based discounting", a specific form of functional weight of representativeness, defined as $h = (\mathcal{R})^{\theta}_{\mathbb{R}_+}$. Notice that h increases in the representativeness of the event considered, while decreases in the representativeness of the others (see Bordalo et al. 2016). It can be defined as a sensitivity of agents with respect to representative news. In particular, the parameter $\theta \geq 0$ captures the extent to which representativeness distorts beliefs (see Bordalo et al. 2016). Whenever $\mathcal{R} > 1$ and $\theta > 1$, agents are *over*-sensitive (also called local thinkers) exaggerating the extent of rational expectations albeit following the same direction: representativeness gives rise to biased expectations that contain a "kernel of truth", that is they differentiate groups along existing and highly diagnostic characteristics (Bordalo et al. 2016).[2]

Assuming a discrete time $t = 1, \dots,$ this model is considered as normal and triggers a linear distortion as (see Bordalo, Gennaioli, and Shleifer 2017 and Strati 2017b for further details)

$$\mathbb{E}^{\theta}_{t}(p_{t+1}) = \mathbb{E}_{t}(p_{t+1}) + \theta[\mathbb{E}_{t}(p_{t+1}) - \mathbb{E}_{t-1}(p_{t+1})]. \tag{4}$$

in which rational expectations $\mathbb{E}_{t}(p_{t+1})$ are distorted by $\theta[\mathbb{E}_{t}(p_{t+1}) - \mathbb{E}_{t-1}(p_{t+1})]$.

3 The main result

Since $p = \psi(s)$, then $\mathbb{E}^{\theta}_{t}(p_{t+1})$ describes this relation in a biased fashion. The biased forecast function is thus defined as $p^{\theta} = \psi^{\theta}(s)$ for which the biased excess supply function is $\Delta^{\theta}(p^{\theta}, s, \psi^{\theta})$. Now, the biased expected equilibrium is thus a price function $\psi^{*\theta}$ such that, for almos every s, the excess supply is zero at

[2] "This suggests that many (although not necessarily all) stereotypes are based on some empirical reality, although they may exaggerate the extent to which a particular group can be characterized in a certain way..." (Schneider 2004).

the price vector $\psi^{*\theta}(s)$

$$\Delta^\theta(\psi^{*\theta}(s), s, \psi^{*\theta}) = 0 \quad \text{a.e. } s \tag{5}$$

However Δ^θ is based on a forecast function which exaggerates the extent of the signal s making $\Delta^\theta \neq \Delta$. This distortions should be further studied since neglecting these possibilities can prevent institutions from hedging psychological risk factors that plainly depart from fundamentals.

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