

Teaching an old dog a new trick:
reserve price and unverifiable quality
in repeated procurement*

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April 5, 2017

Abstract

In procurement markets, unverifiable quality provision may be obtained either by direct negotiation or by competitive processes which discriminate firms on the basis of their past performance. However, discrimination is not allowed in many institutional contexts. We show that a non-discriminatory competitive process with a reserve price may allow the buyer to yield an efficient allocation of the contract and to implement the level of quality desired by the buyer. Quality enforcement arises out of a relational contract whereby the buyer threatens to set a ‘low’ reserve price in future competitive tendering processes if any contractor fails to provide the required quality. We study an infinitely repeated procurement model with many firms and one buyer imperfectly informed on the firms’ cost, in which, in each period, the buyer runs a standard low-price auction with reserve price. We study the cases of players using grim trigger strategies, analysing both the case of a committed and uncommitted buyer. We find that a competitive process with reserve price is able to elicit the desired level of unverifiable quality provided that the buyer’s valuation of the project is not too high and the value of quality is not too low; under these conditions, the buyer can credibly threaten the firms to set, in case a contractor fails to deliver the required quality level, a reserve price so low that no firm is willing to participate to the tender. A committed buyer can elicit the desired quality level for a wider range of preference parameters.

Keywords: public procurement, relational contracts, unverifiable quality, reserve price.

*We are grateful to Armin Schmutzler and seminar participants at Lisbon (EARIE 2016) for useful comments.

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1 Introduction

During the last two decades public procurement has undergone profound changes. Policy makers, academics and practitioners alike share the broad view that public procurement has evolved from a clerical signoff-ridden set of activities to a strategic tool to pursue a wide array of socially relevant objectives. Among the different procurement systems, there is a wide consensus on the most important objectives being the *efficiency* in the acquisition of required goods, works or services and in the procurement process; *integrity*, that is, avoiding corruption and conflicts of interest; *equality and fairness of treatment* for providers.

In many circumstances, open competition in procurement processes is an effective procedure to select the most efficient contractor. This is true when the quality of the procured object is verifiable by a third party at a reasonable cost. The procurement contract can indeed be designed so as to deter the contractor from breaching contract clauses, and from renegeing on promised quality levels.¹ There exist cases, though, where procurement contracts are characterised by performance dimensions that are observable by contracting parties, but cannot be objectively measured. Examples comprise Information Technology or management consulting services, where it is virtually impossible to measure a consultant's pro-activeness or his/her ability to provide innovative solutions. Lack of verifiability may also affect quality dimensions such as a software's degree of user-friendliness or the palatability of catering services.

Although competitive procedures are deemed to work ineffectively when quality is non-contractible², a buyer may enhance a standard competitive process so as to provide the necessary incentives for non-verifiable quality provision. The main objective of the current paper is to prove that a public buyer can strategically use the reserve price in a standard low-price auction to select the most efficient supplier and implement the desired level of non-verifiable quality. In a context of repeated interaction, the relationship between the buyer and the firms takes the nature of a relational procurement contract (RPC, henceforth). Under this RPC, the buyer selects the most efficient supplier by means of an auction with reserve price and obtains the desired level of quality by setting a high reserve price as the selected contractor delivers the quality level the buyer desires, and a low reserve price otherwise. On the other hand, firms, when awarded the project,

¹This conclusion would hold under the assumptions of effective contract management and law enforcement.

²Manelli and Vincent (1995) show that negotiation-like procedures with a restricted set of highly reputable suppliers perform better than competitive procedures. Bajari et al. (2009) provide empirical evidence in the case of the non-resident private sector building contracts in Northern California from 1995 to 2000.

provide the required quality as long as the buyer has set a sufficiently large reserve price, and a lower quality otherwise. This relational contract is non-discriminatory (in the language of the auction literature) or multilateral (in the language of the relational contract literature, see Levin, 2002), in that a deviation by any player results in a breakdown of all existing relationships.

In a model of infinitely repeated procurement, we show that a non-discriminatory competitive process with a reserve price may yield an efficient allocation of the contract and induce the contractor to deliver the buyer's desired quality. We study a model with $N \geq 2$ firms and one buyer imperfectly informed on the firms' cost. In each period, the buyer awards the contract by means of a standard low-price auction with reserve price. We study the cases of players using grim trigger strategies and analyse both the case of a committed and uncommitted buyer; we also analyse the case of stick-and-carrot strategies. Quality enforcement is carried out through a RPC whereby the buyer may apply a *cooperative* (that is, high) reserve price to reward the contractor when the required quality is delivered and threaten to set a *punitive* (that is, low) reserve price in future competitive tendering processes if any contractor fails to provide the required quality.

When the buyer is *uncommitted*, we find two types of equilibria, which are determined by the level of the *cooperative* reserve price. In first type of equilibrium, the cooperative reserve price is 'high'; along the equilibrium path, the buyer's desired quality is always delivered and a positive rent is always left to the contractor even if the latter bears the highest possible (fixed) cost. In the second type of equilibrium, the cooperative reserve price is 'low'; along the equilibrium path, the buyers runs the risk of not awarding the contract when all competing firms draw the highest possible (fixed) cost. Both types of equilibria entail a *punishment* reserve price equal to the lowest (fixed) cost. Given the multilateral nature of the relational contract, when setting the *punishment* reserve price, the buyer anticipates that no quality would be delivered during the punishment phase, therefore she sets the reserve price so as to minimise the cost of the project.

We also characterise the optimal contract(s), that is, the contract(s) yielding the buyer the highest utility. We find two optimal contracts according to the value the buyer attaches to quality. When the buyer cares sufficiently enough about quality, the optimal contract entails a high *cooperative* reserve price, which never constraints firms' bids under any cost configuration. The project is always awarded to the most efficient firm and quality always delivered. When, instead, the buyer cares less about quality, the project is not always delivered as the optimal contact entails a lower *cooperative* reserve price, which only makes the more efficient firms able to participate in the auction. These

results suggest that an optimal contract induces the delivery of the socially optimal quality only when its level is sufficiently high. When, instead, the optimal level is not too high, only sub-optimal levels of quality are enforceable.

Under the assumption of *commitment*, the buyer can enforce the provision of quality by threatening the competing firms to use a *punishment* reserve price higher than the cost of the efficient firm. The main difference with the *uncommitted* case is that the committed hypothesis makes it credible for the buyer to threaten a cheating firm with a softer *punishment* reserve price. Since this softer reserve price leaves a strictly positive rent to an efficient cheating firm, the lowest *cooperative* reserve price inducing the delivery of quality is higher than the one under the *uncommitted* assumption. The new higher *punishment* reserve prices makes the conditions on the discount factor more stringent with respect to the *uncommitted* case. In other words, all other things being equal, when the buyer is *committed* the delivery of quality needs more patient firms. Unlike the *uncommitted* case, since the buyer is now unable to deviate from her cooperative strategy, an equilibrium exists even for low values of quality. In terms of optimal contract, similarly to the *uncommitted* case, any sufficiently high *cooperative* reserve price is optimal. However, unlike the *uncommitted* case, a *committed* buyer is always able to enforce her desired level, even when her evaluation is low.

We finally check for the robustness of our results by looking at the case of players using stick-and-carrot strategies, which entails a time-limited punishment implying that players are not willing to give up forever the value of their cooperative interaction in case of a deviation. We find that, when firms are close to be infinitely patient, it is possible to induce the delivery of quality even with a punishment of finite length, provided that this length is above a minimal level.

The paper is organised as follows. After reviewing the related literature in Section 2, we describe the model in Section 3. Section 4 analyses the static game, while Section 5 looks at dynamic game with grim trigger strategies, under the different assumptions of an *uncommitted* and a *committed* buyer. Section 6 looks at the case of players using stick-and-carrot strategies. Section 7 concludes. All proofs are relegated to the Appendix.

2 Related literature

Our paper contributes to a growing literature studying the enforcement of unverifiable quality in procurement, first analysed by Lewis and Sappington (1991), Laffont and Tirole (1993) and Che (1993). While these papers and the ensuing literature look at

this issue in the static setup of incentive and auction theory,³ we set up our analysis in the context of a repeated interaction between our players, thus investigating relational contracts and the incentives that they create to provide quality. Relational contracts are informal agreements and unwritten codes of conduct that are sustained by the value of future relationships, and are applicable in cases where the outcome of a repeated relationship is based on some unverifiable variables.⁴

Several papers have conducted the analysis of opportunistic behaviour in repeated procurement in the context of a long-term relationship. Klein and Leffler (1981) show that an optimal strategy for the buyer is to promise a rent to the contractor under the threat of terminating the relationship in case of opportunistic behaviour. When the buyer faces more than one potential supplier, following the approach in Levin (2002) relational contracts can be classified as either *bilateral* or *multilateral* contracts: in the former case any deviation by the buyer triggers a reaction only by the firm that has been hurt; in the latter case all firms react to a deviation by the buyer. While the Albano et al. (2017) and Doni (2006) study bilateral RPCs, Calzolari and Spagnolo (2013) analyse RPCs under both assumptions

Che (2008) provides and reviews solutions to the problem of unverifiable quality, other than in the context of relational contracts. Among others, Taylor (1993) and Che and Hausch (1999) introduce an option contract whereby the supplier pays a fee to the buyer, who then may accept or reject (at no penalty) the provision at a price equal to its desired level of quality. Che and Gale (2003) show that a buyer may be better off by allowing suppliers to bid on their reward, as in a standard auction. Other papers, in line with Manelli and Vincent (1995), study the enforcing power of competitive procedures versus negotiations when quality is unverifiable; Tunca and Zenios (2006), among the others, study the interaction between a competitive auction and a relational contract. Buyers procure low-quality products by running a competitive price auction and high-quality products by means of a relational contract with a single supplier. They find that, for some values of quality, the use of competitive auctions for low-quality products may ease the enforcement of relational contracts for high-quality products.⁵

³See, for instance, Hanazono et al. (2013) and Giebe and Schweinzer (2015).

⁴Relational contracts have been pioneered by Bull (1987) and MacLeod and Malcomson (1989) and applied in several fields: labour market (MacLeod, 2003; Levin, 2003; and Li and Matouschek, 2013), interaction between/within firms (Baker et al., 2002; and Rayo, 2007), regulation (Cesi et al., 2012) and experimental economics (Fehr and Schmidt, 2007; and Bigoni et al., 2014).

⁵See, also, Branco (1997), Wang (2000), Kessler and Lulfesmann (2004) and Milgrom (2004). Empirical analyses provide mixed evidence. Bajari et al. (2009), studying the private-sector building contracts in Northern California from 1995 to 2000, find that those awarded by negotiation perform better. On the other hand, analysing the procurement of regional railway services in Germany, Lalive et al. (2015) find that auctioned lines provide a higher frequency of service (seen as a proxy for quality) compared to

In our model, the buyer is able to enforce *multilateral* RPC by strategically using the *reserve price*. The reserve price plays a major role in the auction literature, although its role in enforcing unverifiable quality has not yet been explored. In selling auctions with private values under incomplete information, Riley and Samuelson (1981) show that a profit-maximising seller always sets a level of the reserve price strictly higher than her valuation for the object, thus running the risk of not trading with any of the competing buyers. Hence, a trade-off between revenue and efficiency arises.⁶ When bidders' values are affiliated – which is a special form of correlation –, the seller may use the reserve price to signal her private information about the value of the object which can be used by competing bidders to fine-tune their bids. This is explored, for instance, by Cai, Riley and Ye (2007). If a cartel may rig the bidding process, the seller faces also the dilemma of whether or not to publicly announce the reserve price. On the one hand, announcing the reserve price does provide a clear focal point to a(n) (all-inclusive) cartel, but maximises the chances of trade taking place; on the other hand, keeping the reserve price secret makes the cartel's choice of a focal point more difficult and thus raises the chances that the object remains unsold (see McAfee and McMillan, 1992).

3 The model

The players. We assume a buyer who wants to procure a project of fixed size. The project is procured repeatedly in each period of time t , with $t = 0, \dots, \infty$. The quality of the project, denoted by q , may vary, so that $q \in [0, \infty)$. The gross utility the buyer derives from the project is equal to $v + \nu(q)$, where v denotes the value of the project, regardless its quality, and $\nu(q)$ is the value of the quality of the project. Setting $\nu(q) = q$ allows to write the buyer's net utility as $U(v, q, p) = v + q - p$, where p is the price the buyer pays for the project.

A group of n firms are able to provide the project.⁷ At each t , firm i 's cost (with $i = 1, \dots, n$) is given by $C_t(\theta_{it}, q_t) = \theta_{it} + \psi(q_t)$ (we will drop the subscript t whenever possible). The fixed firm-specific cost component θ_i is a discrete random variable whose possible realisations are θ_L and θ_H , with $\theta_L < \theta_H$. At all t , we let $Prob(\theta_{it} = \theta_L) = \beta$, where $\beta \in [0, 1]$. The other cost component $\psi(q)$ is the cost of quality; it is identical across firms when the quality they provide is also identical. We assume that $\psi(q)$ is increasing and convex in q , and satisfies $\psi(0) = 0$; Thus, firm i 's profits are given by

lines awarded by direct negotiations.

⁶For a comprehensive survey on the role of the reserve price in auctions under different assumptions on the nature of bidders' information see Krishna (2009).

⁷The buyer is referred to as *she*, while firms are referred to as *it*.

$\pi_i \equiv \pi(p, q) = p - \theta_i - \psi(q)$ when it provides the project, and $\pi(p, q) = 0$ otherwise. We assume that firms have no access to credit, so that they face a single-period non-negativity constraint on profits.⁸

To ensure that the procurement activity is socially beneficial and that the provision of quality is desirable, we focus on the cases in which $v \geq \theta_L$ and $q > \psi(q)$. All players have a common discount factor equal to δ , where $\delta \in [0, 1)$.

The competitive tendering. In each period of time, the buyer awards the project running a low-price auction with reserve price. More specifically, at all t , the competitive procedure is such that the buyer first publicly announces a reserve price r and then firms make their bids. If a bid is above the reserve price, it is meant to be equivalent to $+\infty$. The project is awarded to the lowest bid firm; in case of a tie, the project is awarded randomly to one firm with equal probabilities.

Bids are mono-dimensional. This is because, despite the project has varying quality, quality is assumed to be not verifiable (see below). Therefore, quality is not contractible and cannot be made part of the bid. Firms face no bidding costs.⁹

Informational structure. We look at a game of incomplete information. The buyer has incomplete information on the firm's cost; on the other hand, the rivals' and its own cost parameters are perfectly known by each firm when they are realised. Everyone observes the reserve price the buyer sets and the bids made by all firms. On the other hand, the quality offered by a contractor is observed by the buyer only. Despite quality is perfectly observable, the lack of verifiability makes it non-contractible and non-enforceable by a court of law. This simple informational structure allows us to ensure the tractability of a game in which the main ingredients of our economic problems are present: the buyer faces the dual problem of selecting the (unknown) more efficient supplier and, at the same time, of inducing the required level of the quality of the project.

The game. We analyse an infinitely repeated game resulting from an infinite repetition of the following sequential stage game:

stage 1: the buyer sets and announces a reserve price r ;

stage 2: (the *bidding stage*) firms learn their cost parameters and make their bids; the project is awarded by means of a low-price auction with reserve price;

⁸See Arve (2014) for an analysis of a repeated procurement market with financially constrained firms.

⁹This assumption would apply, for instance, when participation costs are of an order of magnitude much lower than the cost of the project itself.

stage 3: (the *execution stage*) the contractor chooses the quality level and delivers the project. The buyer pays the contractor a price equal to its bid, quality is observed and all payoffs are collected.

4 The static game

In this section, we analyse the static constituent game; we show that, in equilibrium, the buyer cannot induce any quality different from 0. For future reference, it is first useful to concentrate on the bidding stage and characterise the firms' bids.

Lemma 1. *Let $\mathbf{q} \equiv (q_1, \dots, q_N)$ be a vector of exogeneously given quality levels to be delivered by firms at the execution stage. Let $\underline{C} \equiv \min_i \{C(\theta_i, q_i)\}$ and $\underline{C}_{-i} \equiv \min_{j \neq i} \{C(\theta_j, q_j)\}$, for all i and j , be the lowest cost among all firms and among all firms but firm i , respectively. Then, for any \mathbf{q} , the bidding stage of the static game admits a Nash equilibrium (in undominated strategies) whereby firm i bids $b_i = C(\theta_i, q_i)$ for all i , unless $\underline{C} < r$ and $C(\theta_i, q_i) = \underline{C}$ holds for firm i only, in which case $b_i = \min\{r, \underline{C}_{-i}\}$ and $b_j = C(\theta_j, q_j)$ for all $j \neq i$.*

The Lemma is a simple application of standard results on low-price auctions with a reserve price, and the intuition is very simple. Assume that the anticipated quality levels are given and the random components of the firms' costs are realised, so that costs are given (and known) to all firms. When the reserve price is (weakly) lower than the lowest cost, all firms find it optimal to bid their cost and be excluded from the competitive tendering.¹⁰ When, instead, the reserve price is higher than the lowest cost, only the lowest cost(s) and either the reserve price or the next to lowest cost matter for characterising the awarding price. If only one firm has a cost advantage over all the rivals, this firm bids the lowest between the reserve price and the lowest of the rivals' costs, and is awarded the contract; if, instead, more than one firm enjoy the cost advantage, these more efficient firms bid their costs and the contract is awarded randomly between them. In both cases, all rival firms would not gain from placing a bid different from their cost.

The following Proposition characterises the equilibrium of the static game.

Proposition 1. *Let*

$$\bar{v} \equiv \frac{N\beta}{1-\beta} \Delta\theta + \theta_H. \quad (1)$$

¹⁰In fact, because of the zero bidding cost assumption firms are indifferent between bidding their costs and not bidding at all. We assume that firms bid and that, as firms' bids are above the reserve price, the buyer rejects firms' bids.

The following strategy profile forms a subgame perfect equilibrium of the static game:
stage 1: if $v \leq \bar{v}$, the buyer sets a reserve price $r = \theta_L$; otherwise, if $v \geq \bar{v}$, the buyer sets a reserve price $r \geq \theta_H$;
stage 2: firms bid as described in Lemma 1, anticipating to deliver quality equal to 0;
stage 3: the winning firm, if any, delivers the project, with quality equal to 0.

In the final stage of the game, irrespective of the bid and the awarding price, the selected contractor always finds it optimal to deliver quality equal to 0. Hence, in the second stage, *all* firms bid anticipating to deliver no quality. In the first stage of the game, the buyer anticipates that no quality will be provided. When setting the reserve price, the buyer faces the ‘standard’ trade-off in a single-object low-price auction: she may set a ‘high’ reserve price so that all firms are willing to participate in the auction, irrespective to their cost, or she may charge a ‘low’ reserve price, so that only an efficient firm will actively bid. In the first case, clearly, the buyer benefits from the project for any possible realisation of the firms’ fixed cost parameter; this comes, however, at the cost of a possibly higher final price. Conversely, a ‘low’ reserve price lowers the final price, but makes a firm unwilling to participate to the auction when inefficient; in case *all* firms are inefficient, the buyer does not award the contract and does not benefit from the project.

The resolution of this trade-off hinges on the relationship between v and \bar{v} ; when the value of the project is large, the buyer prefers to set a high reserve price to make sure that she will reap the utility she derives from the project under all cost realisations. The threshold, in turn, depends on the two fundamental parameters of the model: It is higher the larger the number of firms in the market and the larger their individual probability of having a low fixed cost. Both parameters indeed concur in making less likely the event of all firms being high cost, reducing the probability that the project is not completed. Also, \bar{v} increases with θ_H and decreases with θ_L , and the larger the cost differential the larger the saving accruing to the buyer when the latter sets a low reserve price.

Finally, notice that the buyer’s utility does not depend on the reserve price when this is sufficiently large not to constrain the firm’s bids. On the other hand, when the reserve price is ‘low’ and binding, the buyer’s utility is decreasing in the reserve price and the buyer clearly prefers to set the lowest admissible reserve price compatible with at least one firm participating to the auction, i.e. θ_L .

5 The dynamic game with trigger strategies

We turn our attention to the dynamic game given by an infinite repetition of the constituent game analysed in the previous section. We focus on a relational procurement contract (RPC) which describes, for any history of the game, the reserve price the buyer sets, the bids the firms make and the quality the contractor chooses. This RPC is self-enforcing if it describes a subgame perfect equilibrium of the repeated game.

We carry out our analysis with players adopting the following grim trigger strategies:¹¹

Buyer: in each repetition of the game, the buyer sets r_B^C if each firm i (with $i = 1, \dots, n$) delivered quality equal to $q_{B,i}^C$ in all previous periods in which it was awarded the project (if any). Otherwise, the buyer sets r_B^P .

Firm i ($i = 1, \dots, N$): in each repetition of the game, firm i bids as in Lemma 1 with $q_i = q_i^C$ and, if awarded the project, offers quality q_i^C if the buyer sets a reserve price r_i^C in the current and in all previous periods (if any). Otherwise, the firm bids as in Lemma 1 with $q_i = q_i^P$ and, if awarded the project, delivers quality q_i^P .

These strategies can be defined as $\sigma_B(r_B^C, r_B^P, q_{B,1}^C, \dots, q_{B,N}^C)$ for the buyer, and $\sigma_i(q_i^C, q_i^P, r_i^C)$, for firm i , with $i = 1, \dots, N$. In the rest of the paper, we will however focus on buyer's 'symmetric' strategies, that is, on strategies in which the quality the buyer requires from the winning firms is identical across firms; in other words, we set $q_{B,i}^C = q_{B,j}^C = q_B^C$, for $i, j = 1, \dots, N$ and $i \neq j$. This restriction has several justifications: First, the buyer should ensure an 'equality of treatment' across the firms. Since firms are *ex-ante* identical, there are no grounds for differential quality requirements. Secondly, we will look if and under what conditions the buyer is able to elicit the provision of optimal quality, that is of the quality which maximises her utility. As we will discuss later, given the nature of the firms' costs, optimal quality is independent from the identity of the contractor and from its realised cost. Notice also that quality has no value to the firms other than its strategic long-term value in the relationship with the buyer and results only in an additional cost. Hence, in case of a deviation by the buyer, the best punishment a firm may put in place is to choose $q_i^P = 0$. We can then unambiguously

¹¹Clearly, many alternative strategies could be possible and, thus, our game clearly has multiple equilibria.

denote the strategy of the buyer and of firm i as

$$\sigma_B(r_B^C, r_B^P, q_B^C) \text{ and } \sigma_i(q_i^C, r_i^C). \quad (2)$$

Before turning to the equilibrium analysis, it is useful to introduce the following definition,

Definition 1. For any quality level q , let

$$\rho_{low} \equiv \{r | r \in [\theta_L + \psi(q), \theta_H + \psi(q)]\}; \quad (3)$$

$$\rho_{high} \equiv \{r | r \in [\theta_H + \psi(q), +\infty)\}; \quad (4)$$

$$\rho_{low}^0 \equiv \{r | r \in [\theta_L, \theta_H]\}; \quad (5)$$

$$\rho_{high}^0 \equiv \{r | r \in [\theta_H, +\infty)\}. \quad (6)$$

This Definition introduces two pairs of interval for the reserve price which will be relevant in the rest of the analysis; the first pair (i.e. ρ_{low} and ρ_{high}) is of relevance in the case of a firm providing quality, while the second pair matters in the case of a firm not providing quality. When a reserve price is contained in the interval ρ_{low} , only efficient firms are able to cover their cost when providing quality; on the other hand, when the reserve price is contained in ρ_{high} , all firms, irrespective to their efficiency level, are able to cover their cost when providing quality. A similar argument applies to ρ_{low}^0 and ρ_{high}^0 , but in the case of firms not providing quality.

5.1 The case of an uncommitted buyer

In this section, we characterise a self-enforcing RPC whereby the players' strategies form a subgame perfect equilibrium of the repeated game. We characterise the equilibrium of the game by checking the conditions for the absence of profitable one-shot deviations (POSDs, henceforth) for each player.¹²

Our players have several possible deviations from their candidate equilibrium strategies. A firm may deviate at the *execution stage* once awarded the contract, when no previous deviation has occurred, by providing a quality different from the cooperative one. A firm may also deviate at the *bidding stage*, by making a bid that does not anticipate the full cost of quality; this allows a better chance to win the contract, but implies that, also in this case, no quality is provided at the *execution stage*. Finally, a firm may also deviate by not punishing a previous deviation by the buyer (i.e. off the

¹²See Mailath and Samuelson (2006).

equilibrium path); in this case it may provide a quality different from the one prescribed by its strategy in case of a reserve price different from the ‘cooperative’ one. A buyer, too, has several possible deviations from her candidate equilibrium strategy. First, the buyer may prefer to set a reserve price different from the ‘cooperative’ one in the absence of a previous deviation by a firm. Also, off the equilibrium path, it may prefer to forgive a previous deviation by a firm and set a reserve price different from the ‘punitive’ one.

Our equilibrium analysis is illustrated in the following Proposition.

Proposition 2. *Let*

$$\bar{\delta}(q) \equiv \frac{\psi(q)}{\psi(q) + \beta(1 - \beta)^{N-1} [\min\{r^C - \psi(q), \theta_H\} - r^P]}; \quad (7)$$

$$\bar{q}_1 \equiv \psi(q) + (1 - \beta^N)\Delta\theta - (1 - \beta)^N(v - \theta_L); \quad (8)$$

$$\bar{q}_2 \equiv r^C - \theta_L - \frac{\beta^N}{1 - (1 - \beta)^N}(r^C - \psi(q) - \theta_L). \quad (9)$$

If $v > \bar{v}$, no self-enforcing RPC exists. Assume, instead, $v \leq \bar{v}$; when $\delta \geq \bar{\delta}(q)$, the strategy profile $\sigma_B(r^C, r^P, q)$ and $\sigma_i(q, r^C)$ (with $i = 1, \dots, N$) defines a self-enforcing RPC under which the project is awarded to the most efficient firm, which delivers quality q , under the following conditions:

- i) $r^P = \theta_L$ and $r^C \in \rho_{high}$, provided that $q \geq \bar{q}_1$;
- ii) $r^P = \theta_L$ and $r^C \in \rho_{low}$, provided that $q \geq \bar{q}_2$.

The Proposition illustrates the conditions for the players’ strategies to define a RPC under which the buyer is able to select the most efficient firm and the firm awarded the contract is induced to deliver the required quality level.¹³ The role of the different choice variables or market parameters is discussed below.

‘Cooperative’ and ‘punitive’ reserve prices. Depending on the value of r^C , a self-enforcing RPC induces two different types of equilibria. In an equilibrium of the first type, the project is always delivered; in the other type of equilibrium, the project is delivered only under some conditions on the realisation of the firms’ cost parameters. More specifically, when r^C is large (i.e. $r^C \in \rho_{high}$), the reserve price allows even an inefficient firm to submit a bid covering the cost of quality. Thus at the bidding stage the highest possible bid, along the equilibrium path, will be at most equal to the reserve price. When, instead, r^C is small (i.e. $r^C \in \rho_{low}$), an inefficient firm would not be able to recover its cost when delivering quality. Therefore, in order to avoid being awarded the project and

¹³These conditions are necessary and sufficient, given the players’ strategies.

being forced to renege on quality – something which would trigger a punishment by the buyer –, a firm willing to stick to its ‘cooperative’ strategy has to bid above the reserve price. In the event of *all* firms being inefficient, all firms bid above the reserve price and the buyer is unable to have the project delivered.

Both types of equilibria are sustained by the threat of setting a ‘punitive’ reserve price equal to the cost of an efficient firm, when not delivering quality. Because of the multilateral nature of the relational contract, a deviation by any firm triggers a punishment which affects all firms; the buyer then sets a reserve price which is based on the firms’ fixed costs only (that is, not including the cost of quality), anticipating that no firm would be willing to deliver quality. While a reserve price equal or above θ_H is too mild a threat to induce a firm to cooperate, a reserve price above θ_L would induce the buyer to renege on her strategy and further lower the reserve price in order to reduce the contractor’s rent.

Critical discount factor. A self-enforcing RPC exists provided that firms are sufficiently patient, that is, provided that they have a discount factor above the critical level in (7). Notice that the discount factor does not play a role in the buyer’s choices; because of the sequential nature of the stage game, the buyer is punished immediately in case of a deviation and does not, therefore, face a trade-off between short- and long-term effects of her choice. As to the firms, when the ‘cooperative’ reserve price is high (i.e. $r^C \in \rho_{high}$), a firm’s reward from a RPC does not depend on the level of the reserve price; thus, the critical discount factor depends on market parameters only. When instead the ‘cooperative’ reserve price is tighter (i.e. $r^C \in \rho_{low}$), the firm’s ‘cooperative’ profits do depend on the level of the reserve price, which then affects the level of the critical discount factor: clearly, the lower is the ‘cooperative’ reserve price, the more patient a firm must be to stick to its strategy $\sigma_i(\cdot)$. These features of the threshold level of the discount factor are illustrated in Figure 1, where the critical value of δ is plotted against r^C .

The critical discount factor is decreasing in the term $\beta(1 - \beta)^{N-1}$; this term gives the probability that a firm will be the only efficient firm in the market, thus being able to make positive profits along the ‘cooperative’ phase. Clearly, the higher is this ‘cooperative’ profit, the lower is the critical discount factor ensuring cooperation. Notice however that the term $\beta(1 - \beta)^{N-1}$ also affects the (expected) profits during the ‘punishment’ path; however, since the ‘cooperative’ profits are larger than the ‘punishment’ profits, the effect of the ‘cooperative’ profits dominates. It is then immediate that the higher β the higher the probability that each individual firm will be the only efficient firm in the market, which reduces the critical level of the discount factor. Finally, the critical

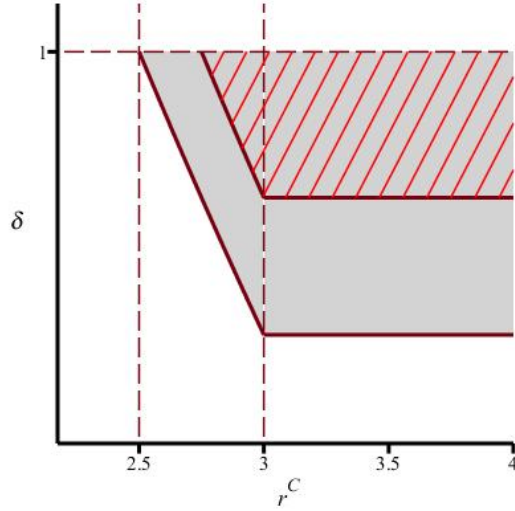


Figure 1: Equilibrium combinations of δ and r^C (when $\beta = \frac{1}{2}$, $N = 2$, $\psi(q) = 2$, $\theta_H = 1$ and $\theta_L = \frac{1}{2}$); grey shaded area relates to the case of an *uncommitted* buyer setting $r^P = \theta_L$, while the red-striped area to the case of *committed* buyer setting $r^P \in \rho_{low}^0$.

discount factor increases with $\psi(q)$, which measures the cost of quality, but captures also a firm's highest single-period gain when deviating from its cooperative strategy. When the short-run gain from a deviation is large, only a patient firm will find it optimal to stick to its strategy $\sigma_i(\cdot)$.

Other market parameters. The existence of a self-enforcing RPC requires additional conditions on other market parameters. The combinations of values of v and q ensuring the existence of a self-enforcing RPC are illustrated in Figure 2 (for given values of the other market parameters).

First, an equilibrium exists only provided that the buyer's evaluation of the project is not too high. The reason is simple: when the buyer has a 'too' high valuation of the project, she prefers not to punish a deviating firm since the punishment implies giving up the project under some cost conditions. Additionally, a high enough quality level is also needed; this ensures that the buyer is willing to set a high reserve price that induces firms to cooperate, rather than setting a tight reserve price which limits the firms' rent, but which does not induce the provision of the desired quality.

The number of firms does affect the conditions for the existence of a self-enforcing RPC through different channels. First, the higher N the lower the probability that each single firm is the only efficient firm in the market, which in turns raises the critical discount factor. Second, the number of firms also affects \bar{v} and the thresholds values

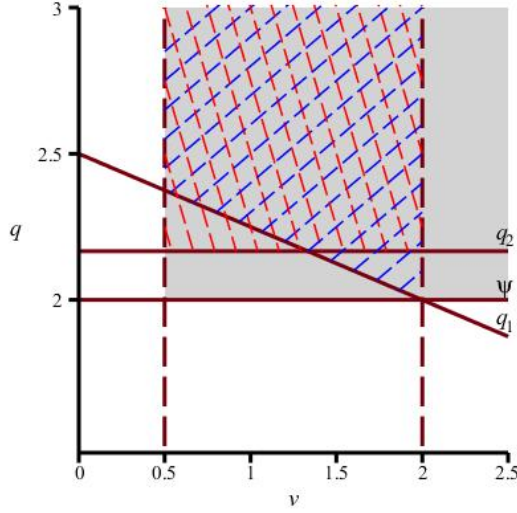


Figure 2: Combinations of v and q for which an equilibrium exists (when $\beta = \frac{1}{2}$, $N = 2$, $\psi(q) = 2$, $\theta_H = 1$ and $\theta_L = r^P = \frac{1}{2}$). In the case of an *uncommitted* buyer, an equilibrium entails $r^C \in \rho_{low}$ in the red-striped area and $r^C \in \rho_{high}$ in the blue-striped area. In the case of a *committed* buyer, an equilibrium entails any r^P and r^C such that $r^P \in \rho_{low}^0$ and $r^P + \psi(q) \leq r^C$ in the grey area.

for q , that is q_1 and q_2 . The reason is simple: a large N reduces the probability of all firms being inefficient, making less costly for the buyer the tough punishment necessary to induce cooperation by the firms. Also, \bar{q}_1 and \bar{q}_2 both increase with the number of firms for a similar reason. When N is high, for the reasons described above, the utility the buyer obtains in case of a deviation is high and, for the buyer to find it optimal to stick to her cooperative strategy, the value of the quality she derives from the project must be high too.

5.1.1 Optimal contract and optimal quality with an uncommitted buyer

The previous section illustrates that there exist many RPCs which implement the the buyer's desired quality. We now turn our attention to characterising the *optimal* RPC for the buyer. This issue may be interpreted in two different ways. First, one may ask which RPC ensures that the buyer obtains the highest utility for a *given quality level*. Alternatively, one may ask if and under what conditions a RPC may induce the *optimal quality level*, that is, her most preferred quality level; in case of a public benevolent buyer, this is also the socially optimal level.

We will address these two questions in the rest of this section. A preliminary result,

which is instrumental to formulate our answers, is illustrated in the following Lemma.

Lemma 2. *Let*

$$\hat{r}^C \equiv \psi(q) \left(1 + \frac{1-\delta}{\delta} \frac{1}{\beta(1-\beta)^{N-1}} \right) + r^P. \quad (10)$$

Then,

- i) when $r^C \in \rho_{high}$, all self-enforcing RPCs yield the buyer the same utility;*
- ii) when $r^C \in \rho_{low}$, the self-enforcing RPC yielding the buyer the highest utility, entails $r^C = \hat{r}^C$.*

This Lemma illustrates a very simple result. When the buyer sets a high reserve cooperative price (i.e. $r^C \in \rho_{high}$), the reserve price does not constraint the firms' bids and, therefore, does not affect the buyer's utility; this implies that any sufficiently large reserve price gives the buyer the same utility level. On the other hand, when the buyer sets a low cooperative reserve price (i.e. $r^C \in \rho_{low}$), this caps the firms' bids and, therefore, the price the buyer ends up paying for the project. In this case, given the firms' discount factor, the buyer finds it optimal to set the lowest reserve price consistent with the firm having an incentive to deliver the required quality. In other words, the reserve price is chosen as to make the firms' incentive compatibility constraint just binding.

The following Proposition illustrates the nature of the RPC which ensures that the buyer obtains the highest utility for a *given quality level*; in the rest of the discussion, we define this RPC the *optimal contract*.

Proposition 3. *Let*

$$\bar{q}_3 \equiv \frac{1-\beta^N}{(1-\beta)^N} (\psi(q) + \theta_H - \hat{r}_{low}^C) - (v - \hat{r}^C). \quad (11)$$

Then, the self-enforcing RPC which gives the buyer the highest utility entails $r^P = \theta_L$ and

- i) when $q \in [\bar{q}_2, \bar{q}_3)$, $r^C = \hat{r}^C$;*
- ii) when $q \in [\max\{\bar{q}_1, \bar{q}_3\}, \infty)$, $r^C \in \rho_{high}$*

When the buyer sufficiently cares about quality, an optimal contract entails a 'high' (and never binding) 'cooperative' reserve price: *all firms*, irrespective to their efficiency levels, always bid to be awarded the project, having their bids constrained by competition only, and the project is awarded to the most efficient firm. Firms are induced to deliver

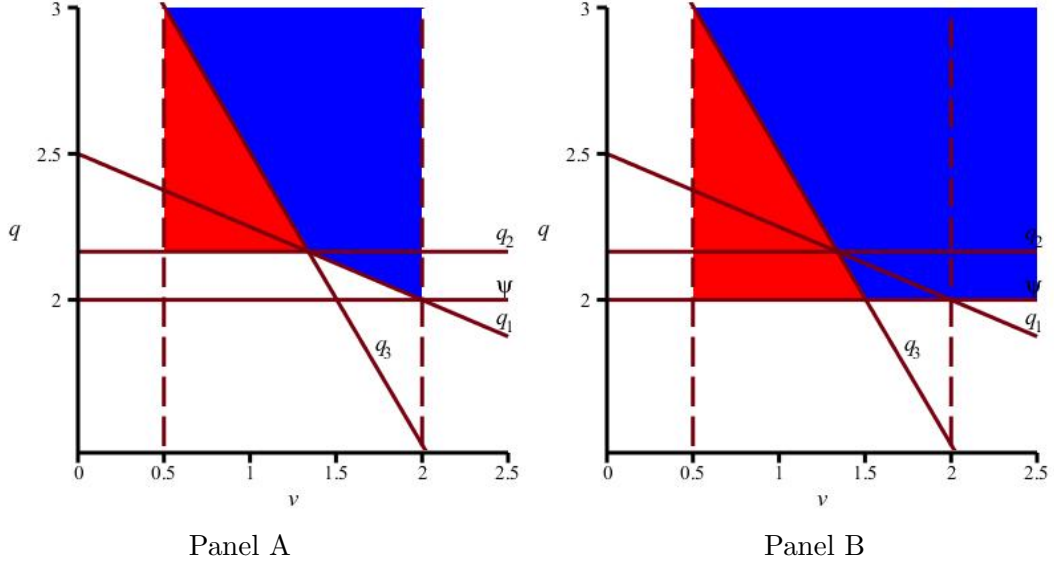


Figure 3: Optimal contracts for combinations of parameters v and q (when $\beta = \frac{1}{2}$, $N = 2$, $\psi(q) = 2$, $\theta_H = 1$ and $\theta_L = r^P = \frac{1}{2}$). In the case of an *uncommitted* buyer (Panel A), an optimal contract entails $r^P = \theta_L$ always and $r^C = \hat{r}^C$ in the red area and $r^C \in \rho_{high}$ in the blue area. In the case of a *committed* buyer (Panel B), an optimal contract entails $r^P \in \rho_{low}^0$ always and $r^C = \hat{r}^C$ in the red area and $r^P + \psi(q) \leq r^C$ in the blue area.

quality by the threat of a stricter reserve price only. When the buyer cares less about quality, the optimal contract induces the buyer to apply a ‘low’ reservation price during the cooperative phase; its level is just high enough to make *only the efficient firms* willing to (actively) participate to the auction and win the project. When all firms are inefficient, they are not willing to (actively) participate to the auction and the buyer is not able to procure the project.

In other words, when the buyer cares a lot about the quality of the project, it sets the reserve price in such a way that she always has the project delivered. The ‘cooperative’ reserve price is not meant to limit the winning firm’s rent, which is in any case limited by competition. When instead the buyer does not care much about the quality of the project, she sets the reserve price primarily to reduce the firm’s rent, even if this makes her not having the project delivered under some realisation of the firms’ costs.

The other issue introduced at the beginning of this Section regards the ability of a RPC to induce the optimal quality. This is simply defined as the one that maximises the buyer’s utility $v + q - p$, subject to a non-negativity constraint on the profits of the firms which delivers the project, say, firm i . Denoting this optimal quality level by q^* , it is

implicitly defined by the standard optimality condition for quality, $\frac{d\psi(q^*)}{dq^*} = 1$, provided that $\psi(q^*) < q^*$. Notice that the optimal quality level does not depend on the realisation of the contractor's cost parameter, since the cost function is additive separable in quality.

An immediate corollary of Proposition 2 is that it may be not possible to enforce the optimal quality level when this is too low. This is because a RPC, as illustrated in the previous section, does exist only for sufficiently high levels of quality. Under some market conditions, such as the ones illustrated in Figure 2, the optimal quality level could be enforced provided that also v is sufficiently large, but this is not a general result.¹⁴ In general, it is possible to conclude that, when q^* is too low, only sub-optimally high levels of quality are enforceable.

5.2 The value of commitment

What if the buyer could commit to a certain level of the reserve price? This is more than a theoretical question. When the procuring entity carrying out the procurement procedure is also the final beneficiary/user of the project, the assumption of lack of commitment seems quite appropriate as the procurer-user is, at least in principle, in a position of fully exploiting the strategic value of the reserve price, thus responding at any point in time to firms' strategies. When awarding a contract on behalf of another public organisation, instead, the procuring entity is less likely to strategically manipulate the reserve price as some of the rules of the procurement game might be directly determined by the final user(s).¹⁵ The highest price that the procuring entity is willing to accept might then be closely linked to (if not coincide with) the budget made available by the final user(s), which, in turn, may be related to the observed quality of the projet. Hence, the reserve price might inherit the budget's commitment feature stemming from national public finance rules or by the financial resources being transferred by international donors.

In this section, we analyse our dynamic model under the hypothesis that our buyer is able to commit to her strategy $\sigma_B(\cdot)$. In other words, for any given choice of r^C and r^P , we assume that she will set the reserve prices exactly as instructed by $\sigma_B(\cdot)$, even if a different choice were preferable. The assumption of a committed buyer implies that we need to characterise a self-enforcing RPC whereby the firm' strategies form a subgame perfect equilibrium of the repeated game, taking the buyer's strategy as given. Our equilibrium analysis is illustrated in the following Proposition.

¹⁴Additional details are available from the authors upon request.

¹⁵This is the case, for instance, of centralised procurement agencies awarding public contracts on behalf of other public bodies or procuring organisations specifically created for awarding big infrastructure projects.

Proposition 4. *Assume the buyer is committed to her strategy. Provided that $\delta \geq \bar{\delta}(q)$, for any admissible q and v , and for any $r^P \in \rho_{low}^0$, and r^C such that $r^P + \psi(q) \leq r^C$, the strategy profile $s_B(r^C, r^P, q)$ and $s_i(q, r^C)$ (with $i = 1, \dots, N$) defines a self-enforcing RPC under which the project is awarded to the most efficient firm, which delivers quality q .*

The Proposition illustrates the nature of the self-enforcing RPCs in the case of a *committed* buyer. These equilibrium RPCs share many features with the ones in case of an *uncommitted* buyer; thus, the following discussion focuses only on the differences relatively to that case. Focus first on the punitive preserve price. Unlike the *uncommitted* case, any reserve price below the cost of an inefficient firm (not providing quality) is a sufficient threat to induce the provision of quality. While an *uncommitted* buyer would have reneged on any punitive reserve price above the minimal level (i.e. θ_L), the commitment hypothesis makes it credible for her to threaten using a punitive reserve price which leaves a rent to the contractor. The (possible) rent given to an (efficient) contractor in case of a deviation, however, changes the nature of the 'cooperative' reserve price. Only a 'cooperative' reserve price sufficiently high – relatively to the punitive one – ensures that the firm's benefits from cooperation are large enough for the firms to cooperate.

In terms of the critical discount factor, the combinations of δ and r^C consistent with the existence of a self-enforcing RPC are the same as in the *uncommitted* case, provided that r^P is set to its minimal value (i.e. θ_L). Higher values of r^P , instead, make the condition on δ more stringent; this is illustrated in Figure 1, where the diagonally striped region illustrates the combinations of δ and r^C under which a self-enforcing RPC exists, when r^P is higher than θ_L .

Another important difference relative to the case of an *uncommitted* buyer is now that a much wider set of combinations of the buyer's utility parameters are consistent with the existence of a self-enforcing RPC. Indeed, an equilibrium exists even for values of v above \bar{v} . The reason is simply that now the buyer, in order to ensure that a high-valued project is always delivered, is not able to renege on the punishment of a deviating firm. Additionally, an equilibrium exists even when the level of quality is low (i.e., below \bar{q}_1 and \bar{q}_2). Contrary to what happens in the case of a *uncommitted* buyer, she cannot deviate from the cooperative strategy even if the value of quality is not high enough.

5.2.1 Optimal contract and optimal quality with a committed buyer

Mirroring the analysis in Section 5.1.1, we now characterise the contract which gives the highest utility to a *committed* buyer among the many self-enforcing ones characterised in Proposition 4. This is done in the following Proposition.

Proposition 5. *Assume the buyer is committed to her strategy. Then, the self-enforcing RPC which gives the buyer the highest utility entails $r^P \in \rho_{low}^0$, and*

i) when $q \in [\psi(q), \bar{q}_3)$, $r^C = \hat{r}^C$, where \hat{r}^C is as in (10);

ii) when $q \in [\bar{q}_3, \infty)$, $r^C \in \rho_{high}$.

Optimal contracts are similar to the ones described in the case of an *uncommitted* buyer. Some differences are, however, to be noted, first, when quality is sufficiently large, a wider set of contract giving the highest utility level are available to the buyer. As in the *uncommitted* case, any large enough cooperative reserve price is optimal; however, unlike with an *uncommitted* buyer, many punitive reserve prices are optimal, as long as they keep out of the market an inefficient firm. All these contracts give the buyer the same utility; the 'cooperative' reserve price is so high that it is not binding, while the punitive reserve price is not observed on the equilibrium path.

The second difference relates to the fact that a self-enforcing RPC is available also for combinations of the buyer's utility parameters for which such a RPC would not exist for an *uncommitted* buyer. In other words, an equilibrium is now possible also when the intrinsic value of the project is sufficiently large and the value of quality is sufficiently low. The possibility of enforcing a sufficiently low level of quality has a noticeable effect on the existence of a self-enforcing RPC able to enforce the optimal quality level. Differently from the case of an *uncommitted* buyer, the immediate corollary of Proposition 5 is that commitment makes it possible for the buyer to always enforce the optimal level of quality, even when this is low.

6 The dynamic game with stick-and-carrot strategies

In this section, we check to robustness of our result to the change of the hypothesis regarding the players' strategies. We assume now that players use stick-and-carrot strategies: any player deviating from the cooperative path is punished for a limited number of periods only. The punishment ends if, during the punishment phase, the punished players maintain an appropriate behaviour. The everlasting punishment typical of trigger strategies may indeed be not very realistic, since it implies that players give up

forever the value of their cooperative interaction in case of a deviation. Thus, especially in public procurement, the analysis of more forgiving strategies seems also appropriate.

We concentrate on players adopting the following stick-and-carrot strategies:

Buyer: the buyer sets a reserve price equal to r_B^C as long as the firm awarded the project delivered quality q_B^C in the previous period. Otherwise, the reserve price is set equal to r_B^P for T_B periods; if during these T_B periods, the quality delivered is q_B^P , in period $T + 1$ the buyer reverts to reserve price equal to r_B^C . Otherwise, if during any of these T_B periods, a quality different from q_B^P is delivered, the buyer imposes a reserve price equal to r_B^P again for the next T_B periods.

Firm i ($i = 1, \dots, N$): firm i bids as in Lemma 1 with $q_i = q_i^C$ and, if awarded the project, offers quality q_i^C as long as the buyer sets a reserve price r_i^C in the current period. Otherwise, the firm bids as in Lemma 1 with $q_i = q_i^P$ and, if awarded the project, delivers quality q_i^P in the same and in the following $T_i - 1$ periods; if during these $T_i - 1$ periods, the reserve price is set equal to r_i^P , in period T_i the firm reverts to the initial choice of $q_i = q_i^C$. Otherwise, if during any of these $T_i - 1$ periods, a reserve price different from r_i^P is set, the firm bids as in Lemma 1 with $q_i = q_i^P$ and, if awarded the project, delivers quality q_i^P in the same and in the following $T_i - 1$ periods.

When referring to these strategies, we will denote the buyer's strategy as $\sigma_B(r_B^C, r_B^P, q_B^C, q_B^P, T_B)$; differently from the previous section, the strategy depends also on the variables characterising the time-limited punishment phase. As to firm i 's strategy, we write it as $\sigma_i(r_i^C, r_i^P, q_i^C, q_i^P, T_i)$.

In the rest of this section, we restrict our analysis to the case in which, during the punishment phase, the buyer's stick-and-carrot strategy entails a quality level equal to 0 (i.e., $q_B^P = 0$), so that the quality level required after a deviation is lower than in equilibrium. This restriction implies that we need not worry about a deviation by firm i during the punishment phase. Note that a firm optimally set equal to zero the punishment quality in case of a buyer's deviation; this is because, in this case, it has no strategic or market value and ensure the firms avoid bearing unnecessary costs. Finally, to avoid messy notation, we restrict our analysis to buyer's and firms' strategies such that $T_B = T_i = T$.

We concentrate on the case of the discount factor arbitrarily close to 1. This simplifies the technical analysis and is less restrictive than it appears: if an equilibrium exists when

$\delta \rightarrow 1$, by continuity, there always exists a critical discount factor sufficiently close to 1 but strictly smaller than 1, such that a self-enforcing RPC exists.

We can now state the main result of this section.

Proposition 6. *Assume $\delta \rightarrow 1$. Let*

$$\bar{T} \equiv \max \left\{ \frac{\psi(q)}{\beta(1-\beta)^{N-1} [\min \{r^C - \psi(q), \theta_H\} - r^P]}, 1 \right\}$$

Then, provided that $v \leq \bar{v}$ and $\bar{T} \leq T$, the strategy profile $\sigma_B(r^C, r^P, q, 0, T)$ and $\sigma_i(r^C, r^P, q, 0, T)$ (with $i = 1, \dots, N$) defines a self-enforcing RPC whereby the project is awarded in each period to the most efficient firm, which delivers quality q , under the following conditions:

- i) $r^P = \theta_L$ and $r^C \in \rho_{high}$, provided that $q \geq \bar{q}_1$;*
- ii) $r^P = \theta_L$ and $r^C \in \rho_{low}$, provided that $q \geq \bar{q}_2$*

The Proposition illustrates the sufficient conditions such that, when players are close to be infinitely patient, it is possible to induce the delivery of quality even with a punishment of finite length, provided that this length is above a minimal level.

Together with conditions on v , q and the punishment price r^P , by now familiar from the analysis with trigger strategies, the Proposition provides the sufficient conditions for an equilibrium to exist in terms of a lower bound, \bar{T} , for the punishment length. This lower bound is strictly increasing in r^C when this is low and does not depend on r^C when this is large. When the reserve price is low, the punishment length and the reserve price are substitutes to each other; a low r^C reduces the incentive to cooperate that, in turn, must be restored by a higher minimum punishment length. When, instead, r^C is large, it does not affect the firms bids, whose behaviour is only determined by the competitive pressure of rivals. The minimum punishment length is then only a function of the expected profits a firm may make along the equilibrium path, which in turn depends on the possible competitive advantage.

7 Conclusions

Unverifiable quality is known to cause severe problems in procurement, especially if the buyer is constrained to use competitive procedures. When the overarching regulatory allows the buyer to evaluate firms' past performance, such as the federal public procurement regulation in the U.S., a *discriminatory* scoring auction is instrumental to enforce the socially optimal level of quality. When past performance cannot be taken into account in tenders evaluation, such as in the case of the EU public procurement regulation

and of the UNCITRAL Model Law, a public buyer can still exploit one dimension of the tender design, namely the reserve price, to enforce unverifiable quality.

In this paper, we have shown that a public buyer can credibly threaten to punish competing firms with a low reserve price if quality is not delivered. Such a punishment reserve price affects equally the expected profit of *all* firms, thus it represents a *non-discriminatory* enforcement mechanism.

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Appendix

We provide here the proofs of all Lemmata and Propositions in the paper.

Proof of Lemma 1. Trivial, then omitted. ■

Proof of Proposition 1. In stage 3, since quality is costly and does not affect the firms' revenues, the firm awarded the contract chooses to deliver quality equal to 0. In stage 2, the N firms, anticipating that the one winning the contract will deliver quality equal to 0, bid as in Lemma 1 anticipating to deliver qualities $q_i = 0$, for $i = 1, \dots, N$. In stage 1, the buyer's utility depends on the level of the reserve price r . Before turning to analyse the buyer's expected utility, it is useful to remind that i) β^N is the probability that all firms draw θ_L ; ii) $(1 - \beta)^N$ is the probability that all firms draw θ_H ; iii) $N\beta(1 - \beta)^{N-1}$ is the probability that only one firm draws θ_L ; iv) $1 - N\beta(1 - \beta)^{N-1} - (1 - \beta)^N$, is the probability that at least two firms draw θ_L .

Consider a reserve price $r' \in \rho_{low}^0$. Depending on the realisations of the firms' cost parameters, the buyer's utility is as follows:

$$U(r') = \begin{cases} v - r' & \text{with probability } N\beta(1 - \beta)^{N-1}; \\ 0 & \text{with probability } (1 - \beta)^N; \\ v - \theta_L & \text{with probability } 1 - (1 - \beta)^N - N\beta(1 - \beta)^{N-1}. \end{cases} \quad (12)$$

The buyer's expected utility is then $EU(r') = (v - r')N\beta(1 - \beta)^{N-1} + (v - \theta_L)(1 - (1 - \beta)^N - N\beta(1 - \beta)^{N-1})$. Since $\frac{\partial EU(r')}{\partial r'} = -\beta(1 - \beta)^{N-1} < 0$, it is dominant for the buyer to set r' equal to its lower bound, θ_L ; hence, at the buyer's optimal choice, $EU(r') = (v - \theta_L)(1 - (1 - \beta)^N)$.

Let ρ_{high}^0 and ρ_{low}^0 be defined as in (6) and (5), respectively. Consider now a reserve price $r'' \in \rho_{high}^0$. Depending on the realisations of the firms' cost parameters, the buyer's utility is as follows:

$$U(r'') = \begin{cases} v - \theta_H & \text{with probability } N\beta(1 - \beta)^{N-1}; \\ v - \theta_H & \text{with probability } (1 - \beta)^N; \\ v - \theta_L & \text{with probability } 1 - (1 - \beta)^N - N\beta(1 - \beta)^{N-1}. \end{cases} \quad (13)$$

The buyer's expected utility is then $EU(r'') = (v - \theta_H)(N\beta(1 - \beta)^{N-1} + (1 - \beta)^N) + (v - \theta_L)(1 - (1 - \beta)^N - N\beta(1 - \beta)^{N-1})$.

Comparing $EU(r'')$ and $EU(r')$ gives $\bar{v} = \frac{N\beta}{1-\beta}\Delta\theta + \theta_H$. ■

Proof of Proposition 2. We begin by establishing a Lemma which describes a firm's static optimal deviation, given the rivals' strategies.

Lemma 3. *Consider the static game of Section 4. Assume that all firms, but firm i , bid as in Lemma 1, anticipating to offer a common quality \tilde{q} if awarded the project. Then, for any r and any realisation of the firms' cost parameters, bidding its best reply to its rivals' bids yields firm i a gain bounded from above by $\psi(\tilde{q})$ relatively to Lemma 1 where firm i anticipates to deliver quality \tilde{q} as all other competitors.*

Proof. We have to consider three cases:

i) $C_i(\theta_i, \tilde{q}) \leq \min\{r, \underline{C}_{-i}\}$. Firm i bids as in Lemma 1 and delivers $\tilde{q} = 0$. In this case firm i is the most efficient even when anticipating to deliver quality $\tilde{q} > 0$. It then bids as in Lemma 1 and optimally deviates at the execution stage, thus reaping an additional profit of $\psi(\tilde{q})$.

ii) $C_i(\theta_i, 0) \leq \min\{r, \underline{C}_{-i}\} < C_i(\theta_i, \tilde{q})$. Firm i would lose the auction if bidding according to Lemma 1, but would win it if it were to anticipate to deliver zero quality. Thus firm i 's optimal deviation consists in bidding $b'_i = \min\{r, \underline{C}_{-i}\}$ - thus winning the auction - and delivering $\tilde{q} = 0$. Firm i 's additional profit is equal to $\psi(\tilde{q})$ if $\min\{r, \underline{C}_{-i}\} = \underline{C}_{-i}$, and strictly lower than $\psi(\tilde{q})$ if $\min\{r, \underline{C}_{-i}\} = r$.

iii) $\min\{r, \underline{C}_{-i}\} < C_i(\theta_i, 0) < C_i(\theta_i, \tilde{q})$. In this case, any deviation at the bidding stage would not alter the outcome of the stage game, thus firm i makes zero additional profit. \square

Next, we check whether firm i ($i = 1, \dots, N$) or the buyer have a POSD from the strategies described in the Proposition, on and off the equilibrium path.

i) conditions for no POSDs for a firm off the equilibrium path: when facing a reserve price $\theta_L \leq r^P$ because of a previous deviation from the equilibrium path, quality provision has no strategic value and it is only a cost; hence, no POSD exists whenever a firm bids as in Lemma 1, anticipating to deliver a quality level equal to 0;

ii) conditions for no POSDs for a firm on the equilibrium path: when no previous deviation has occurred, no POSD exists for firm i if

$$\pi_i^C + \frac{\delta}{1-\delta} E\pi_i^C \geq \pi_i^D + \frac{\delta}{1-\delta} E\pi_i^P \quad (14)$$

where π_i^C denotes the firm's profits when it stick to its cooperative strategy $\sigma_i(\cdot)$, π_i^D denotes the firm's profits when it (optimally) deviates from its cooperative strategy $\sigma_i(\cdot)$ and π_i^P denotes the firm's profits when the buyer sets a 'punitive' reserve price; E denotes

the expectation operator, before the cost parameters are realised. This constraint can be re-expressed as

$$\delta \geq \frac{\pi_i^D - \pi_i^C}{(\pi_i^D - \pi_i^C) + (E\pi_i^C - E\pi_i^P)} \quad (15)$$

(provided that the denominator is positive). Since the RHS of this inequality is increasing in the numerator, the constraint is verified to always hold when it holds for the largest value of the difference $\pi_i^D - \pi_i^C$ which, from Lemma 3, is equal to $\psi(q)$. As to the expressions for expected profits, they may take different values depending on the values of r^C and r^P , as described in the subcases below. In the analysis of all these subcases below, following the previous argument, we set $\pi_i^D - \pi_i^C = \psi(q)$ and derive the values of $E\pi_i^C$ and $E\pi_i^P$ making use of standard results on Bertrand games (the derivation is trivial and it is omitted):

ii.i) $r^C \in \rho_{high}$ and $r^P \in \rho_{high}^0$: we have that $E\pi_i^C = E\pi_i^P = \beta(1 - \beta)^{N-1}\Delta\theta$. Hence, (15) becomes $\delta \geq 1$, which, clearly, can be satisfied only in the limit case of $\delta = 1$.

ii.ii) $r^C \in \rho_{high}$ and $r^P \in \rho_{low}^0$: we have that $E\pi_i^C = \beta(1 - \beta)^{N-1}\Delta\theta$ and $E\pi_i^P = \beta(1 - \beta)^{N-1}(r^P - \theta_L)$. These imply that (15) becomes

$$\delta \geq \frac{\psi(q)}{\psi(q) + \beta(1 - \beta)^{N-1}(\theta_H - r^P)} \equiv \delta_a. \quad (16)$$

ii.iii) $r^C \in \rho_{low}$ and $r^P \in \rho_{low}^0$: we have that $E\pi_i^C = \beta(1 - \beta)^{N-1}(r^C - \theta_L - \psi(q))$ and $E\pi_i^P = \beta(1 - \beta)^{N-1}(r^P - \theta_L)$. These imply that (15) becomes

$$\delta \geq \frac{\psi(q)}{\psi(q) + \beta(1 - \beta)^{N-1}(r^C - \psi(q) - r^P)} \equiv \delta_b. \quad (17)$$

Notice that for δ_b to be smaller than 1, it must be that $r^C \geq \psi(q) + r^P$.

ii.iv) $r^C \in \rho_{low}$ and $r^P \in \rho_{high}^0$: we have that $E\pi_i^C = \beta(1 - \beta)^{N-1}(r^C - \theta_L - \psi(q))$ and $E\pi_i^P = \beta(1 - \beta)^{N-1}\Delta\theta$. These imply that (15) becomes

$$\delta \geq \frac{\psi(q)}{\psi(q) + \beta(1 - \beta)^{N-1}(r^C - \psi(q) - \theta_H)} \equiv \delta_c. \quad (18)$$

Since $\delta_c > 1$, this is, clearly, never satisfied.

Notice that δ_a and δ_b in (16) and (17) differ only for the term in the denominator which multiplies $\beta(1 - \beta)^{N-1}$. Also, δ_a is derived under the assumption that $r^C \geq \theta_H + \psi(q)$, which implies $r^C - \theta_H < \psi(q)$; on the other hand, δ_b is derived under the

assumption that $r^C < \theta_H + \psi(q)$, which implies $r^C - \theta_H \geq \psi(q)$. Hence, we can write

$$\begin{aligned}\bar{\delta}(\psi(q)) &\equiv \max\{\delta_a, \delta_b\} \\ &= \frac{\psi(q)}{\psi(q) + \beta(1 - \beta)^{N-1}[\min\{r^C - \psi(q), \theta_H\} - r^P]}.\end{aligned}\tag{19}$$

iii) conditions for no POSDs for the buyer off the equilibrium path: when facing a previous deviation by a firm from the equilibrium path, the reserve price has no strategic value to the buyer and only affects her single-period utility; hence, from Proposition 1, no POSD exists provided that the buyer sets a reserve price $r^P = \theta_L$ if $v \leq \bar{v}$ and $r^P \geq \theta_H$ if $v \geq \bar{v}$;

The analysis in *i)*, *ii)* and *iii)* restricts the candidate equilibria only to those strategy profiles in which $r^C \leq \theta_H + \psi(q)$ and $r^P = \theta_L$, for any level of q and provided that $v \leq \bar{v}$ and $\delta \geq \bar{\delta}(\psi(q))$. In the rest of the proof, we restrict our attention to these candidate equilibria.

iv) conditions for no POSDs for the buyer on the equilibrium path: when no previous deviation has occurred, no POSD exists for the buyer i if

$$EU^C + \frac{\delta}{1 - \delta}EU^C \geq EU^D + \frac{\delta}{1 - \delta}EU^P,\tag{20}$$

where U^C denotes the buyer's utility when she sticks to her cooperative strategy $\sigma_B(\cdot)$, U^D denotes the buyer's utility when she (optimally) deviates from her cooperative strategy $\sigma_B(\cdot)$ and U^P denotes the buyer's utility when she sets a 'punitive' reserve price; E denotes the expectation operator, before the cost parameters are realised. Notice that, since the buyer chooses her action before the realisation of the firms' cost parameters, all utilities are expected.

From Proposition 1, when $v \leq \bar{v}$, the optimal deviation is setting a reserve price equal to θ_L . Therefore, $EU^D = EU^P$. This allows to rewrite (20) simply as

$$EU^C \geq EU^P,\tag{21}$$

where EU^C may take different values depending on the value of r^C , as described in the subcases below. In the analysis of all these subcases below, we derive the values of EU^C and EU^P making use of standard results on Bertrand auctions (the derivation is trivial and it is omitted); since $r^P = \theta_L$ always, EU^P is constant and such that $EU^P = (1 - (1 - \beta)^N)(v - \theta_L)$.

iv.i) $r^C \in \rho_{high}$: we have that $EU^C = v + q - \psi(q) - \theta_H + \beta^N \Delta\theta$. Simple algebra shows that $EU^C \geq EU^P$ when $q \geq \bar{q}_1$. [Notice that \bar{q}_1 is monotonically decreasing in v - i.e. $\frac{d\bar{q}_1}{dv} < 0$. When evaluated at the minimum value of v (i.e. $v = \theta_L$), then $\bar{q}_1 = \psi(q) + (1 - \beta^N)\Delta\theta$, when instead evaluated at the maximum value of v (i.e. $v = \bar{v}$), then $\bar{q}_1 = \psi(q)$ when $N = 2$ and $\bar{q}_1 = \psi(q) + \Delta\theta$ when $N \rightarrow \infty$.]

iv.ii) $r^C \in \rho_{low}$: we have that $EU^C = (1 - \beta^N - (1 - \beta)^N)(v + q - r^C) + \beta^N(v + q - \psi(q) - \theta_L)$. Simple algebra shows that $EU^C \geq EU^P$ when $q \geq \bar{q}_2$.

Proof of Lemma 2. In equilibrium, when $r^C = r'$ with $r' \in \rho_{low}$, depending on the realisations of the firms' cost parameters, the single period buyer's utility is as follows:

$$U(r') = \begin{cases} v + q - r' & \text{with probability } N\beta(1 - \beta)^{N-1}; \\ 0 & \text{with probability } (1 - \beta)^N; \\ v + q - \psi(q) - \theta_L & \text{with probability } 1 - (1 - \beta)^N - N\beta(1 - \beta)^{N-1}. \end{cases} \quad (22)$$

The buyer's expected utility is then $EU(r') = (v + q - r')N\beta(1 - \beta)^{N-1} + (v + q - \psi(q) - \theta_L)(1 - (1 - \beta)^N - N\beta(1 - \beta)^{N-1})$. Since $\frac{\partial EU(r')}{\partial r'} = -N\beta(1 - \beta)^{N-1} < 0$, r' is set to its lowest level compatible with the firms' incentive to play their cooperative strategies. This is found by solving $\delta = \bar{\delta}(q)$, from (7) w.r.t. r^C , which gives $r' = \hat{r}^C$, as in (10). Notice that $\psi(q) + r^P \leq r'$ always holds, as it requested to ensure that $\delta \leq 1$.

In equilibrium, when $r^C = r''$ with $r'' \in \rho_{high}$, depending on the realisations of the firms' cost parameters, the single period buyer's utility is as follows:

$$U(r'') = \begin{cases} v + q - \psi(q) - \theta_H & \text{with probability } N\beta(1 - \beta)^{N-1}; \\ v + q - \psi(q) - \theta_H & \text{with probability } (1 - \beta)^N; \\ v + q - \psi(q) - \theta_L & \text{with probability } 1 - (1 - \beta)^N - N\beta(1 - \beta)^{N-1}. \end{cases} \quad (23)$$

The buyer's expected utility is then $EU(r'') = (v + q - \psi(q) - \theta_H)(N\beta(1 - \beta)^{N-1} + (1 - \beta)^N) + (v + q - \psi(q) - \theta_L)(1 - (1 - \beta)^N - N\beta(1 - \beta)^{N-1})$, which does not depend on r'' . ■

Proof of Proposition 3. Trivial, by comparing $EU(\hat{r}^C)$, with \hat{r}^C evaluated at $r^P = \theta_L$, and $EU(r'')$ from Lemma 2. ■

Proof of Proposition 4. The result follows directly from points *i*) and *ii*) in the proof of Proposition 2 where we have shown that the cases in which $\theta_H < r^P$ induce a deviation by the firm. ■

Proof of Proposition 5. Trivial and, therefore, omitted. ■

Proof of Proposition 6. Next, we check whether firm i ($i = 1, \dots, N$) or the buyer have a POSD from the strategies described in the Proposition, on and off the equilibrium path.

Notice that a deviation in terms of length of the punishment T , is actually a deviation on the reserve price (for the buyer) or from the quality (bid or actual, for the firm) prescribed by the equilibrium strategy during the punishment (in case of a punishment shorter than T) or during the cooperative path (in case of a punishment longer than T). Hence, checking for a POSD in terms of length of the punishment is equivalent to a check for a POSD in terms of those actions; hence, no specific attention is devoted in the rest of this proof to deviation in terms of length of the punishment T .

i) conditions for no POSDs for a firm off the equilibrium path: when facing a reserve price $\theta_L \leq r^P$ because of a previous deviation from the equilibrium path, quality provision has no strategic value and it is only a cost; hence, no POSD exists whenever a firm bids as in Lemma 1, anticipating quality equal to 0;

ii) conditions for no POSDs for a firm on the equilibrium path: when all rival firms sticks to their ‘cooperative’ strategies, when no previous deviation has occurred, no POSD exists for firm i if:

$$\pi_i^C + \sum_{t=1}^{\infty} \delta^t E\pi_i^C \geq \pi_i^D + \delta\Pi_i^P, \quad (24)$$

where π_i^C , π_i^D and π_i^D are defined as in the proof of Proposition 2; also, Π_i^P , which gives the expected future profits after a deviation (taking as given a punishment of T periods), is given by

$$\Pi_i^P = \sum_{t=0}^{T-1} E\pi_i^P + \sum_{t=T}^{\infty} E\pi_i^C \quad (25)$$

where π_i^D is defined as in the proof of Proposition 2.

When $\delta \rightarrow 1$, equation (24) becomes

$$T \geq \frac{\pi_i^D - \pi_i^C}{E\pi_i^C - E\pi_i^P} \equiv \bar{T} \quad (26)$$

Since \bar{T} is increasing in the numerator, (24) is verified to always hold when it holds for the largest value of the difference $\pi_i^D - \pi_i^C$ which, from Lemma 3, is equal to $\psi(q)$. As to the expressions for expected profits, they may take different values depending on the value of r^C and r^P . For all the relevant combinations of r^C and r^P , we will derive the threshold value for \bar{T} in (26), by setting $\pi_1^D - \pi_1^C = \psi(q)$ and borrowing the values of

$E\pi_i^C$ and $E\pi_i^P$ from the four cases *i*)-*iv*) in the proof of the Proposition 2.

Case i): $r^C \in \rho_{high}$ and $r^P \in \rho_{high}^0$. Since $E\pi_i^C = E\pi_i^P = \beta(1 - \beta)^{N-1}\Delta\theta$, then the value of \bar{T} is not defined.

Case ii): $r^C \in \rho_{high}$ and $r^P \in \rho_{low}^0$. Since $E\pi_i^C = \beta(1 - \beta)^{N-1}\Delta\theta$ and $E\pi_i^P = \beta(1 - \beta)^{N-1}(r^P - \theta_L)$, then

$$T \geq \frac{\psi}{\beta(1 - \beta)^{N-1}(\theta_H - r^P)} \equiv T_a; \quad (27)$$

Case iii): $r^C \in \rho_{low}$ and $r^P \in \rho_{low}^0$. Since $E\pi_i^C = \beta(1 - \beta)^{N-1}(r^C - \theta_L - \psi)$ and $E\pi_i^P = \beta(1 - \beta)^{N-1}(r^P - \theta_L)$, then

$$T \geq \frac{\psi}{\beta(1 - \beta)^{N-1}(r^C - \psi(q) - r^P)} \equiv T_b. \quad (28)$$

Notice that for the denominator to be positive it must be that $\psi(q) + r^P \leq r^C$.

Case iv): $r^C \in \rho_{low}$ and $r^P \in \rho_{high}^0$. Since $E\pi_i^C = \beta(1 - \beta)^{N-1}(r^C - \theta_L - \psi)$ and $E\pi_i^P = \beta(1 - \beta)^{N-1}\Delta\theta$, then

$$T \geq \frac{\psi(q)}{\beta(1 - \beta)^{N-1}(r^C - \theta_H - \psi)} \quad (29)$$

which clearly gives a negative \bar{T} .

Since we only consider punishment length at least longer than one period ($\bar{T} \geq 1$), note that $T_a \geq 1$ if $\psi(q) \geq \beta(1 - \beta)^{n-1}(\theta_H - r^P)$ and $T_b \geq 1$ if $\psi(q) \geq \beta(1 - \beta)^{n-1}(r^C - \psi(q) - r^P)$.

iii) conditions for no POSDs for the buyer on the equilibrium path: when no previous deviation has occurred, no POSD exists for the buyer if

$$EU^C + \frac{\delta}{1 - \delta}EU^C \geq EU^D + \sum_{t=1}^{T-1} \delta^t EU^P + \sum_{t=T}^{\infty} \delta^t EU^C \quad (30)$$

where U^C , U^P and U^D are defined as in the proof of Proposition 2. This inequality reduces to (21) and the analysis carried out in the proof of Proposition 2 applies

iv) conditions for no POSDs for the buyer off the equilibrium path: as shown in the proof of the Proposition 2 no POSD from the punishment reserve price exists provided that the buyer sets a reserve price $r^P = \theta_L$ if $v \leq \bar{v}$ and $r^P \geq \theta_H$ if $v \geq \bar{v}$. ■