What Determines Volunteer Work? On the Employer's Choice of a Mission

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Abstract

A mission-oriented firm hires workers without knowing their skills and their intrinsic motivation, which is nonetheless driven by the firm's mission. The firm's optimal choice of its mission is studied and related to the optimal screening contracts that satisfy workers' limited liability. Volunteerism emerges when the firm sacrifices a high fraction of its revenues for socially worthwhile projects, inducing a high workers' motivation.

Jel classification: C61, D82, D86, J41, M55.

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1 Introduction

Intrinsic motivation is the worker's enjoyment of her personal contribution to the employer's mission or goals. As such, it should not be conceived of as an exogenously given characteristic of workers; intrinsic motivation is rather induced by the firm's choice of its mission.

This paper studies the endogenous mission choice of an employer, and its effects on workers' intrinsic motivation, extending the analysis carried out in Burani and Palestini (2016). In that work, we consider the screening problem of a firm willing to hire potential applicants, who have heterogeneous and unobservable skills, but the same observable level of intrinsic motivation. Optimal contracts consist in different effort-salary pairs offered by the firm; each worker selects the preferred pair thus revealing her private information about her skills. When workers are protected by limited liability and cannot accept negative salaries, volunteerism is the contractual outcome for low-ability workers, whose motivation is sufficiently high. These workers are asked to provide the same effort level independently of their skills in exchange for a null reward.

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This paper takes the screening contracts satisfying workers' limited liability as a starting point, and analyses the firm's choice of its mission. In the present context, selecting a mission means that the firms optimally decides how much to sacrifice of its profits (or revenues) in the social interest. The firm's level of engagement in socially worthwhile activities determines the degree of workers' intrinsic motivation (which is taken as given in Burani and Palestini, 2016). In turn, the higher the fraction of revenues that the firm sacrifices to pursue its mission, the higher workers' motivation. Moreover, highly motivated workers are ready to work harder for their employer while accepting lower wages (this reflects the so-called "donative-labour hypothesis", see Preston 1989). I consider a family of functions describing how the firm's choice of a mission drives workers' motivation, and derive the conditions under which a sufficiently high motivation is generated so as to induce the least able workers to become volunteers.

Optimal contracting with endogenous project mission has already been analyzed by Cassar (2016), who does not consider workers' skills and volunteerism. Her results are thus not directly comparable to mine.

2 The model

The model is borrowed from Burani and Palestini (2016) and slightly modified to take explicitly into account the impact of the employer's mission on screening contracts.

Consider a principal-agent model with adverse selection. The principal (he) is a firm willing to hire a worker (she) to perform a given task. Both the firm and the agent are risk neutral.

The firm produces output according to a linear technology with labour as the only input. Its production function is $q(e) = e$, where e is the observable effort that the worker is asked to exert. The firm's payoff, per-worker hired, is

$$\pi(e, w) = \alpha q(e) - w = \alpha e - w, \tag{1}$$

where the (exogenous) price of output is set equal to 1, w is the total salary paid to the worker, and $\alpha \in [0, 1]$. The firm is a mission-oriented organization that need not strictly maximize profits, but rather sacrifices a fraction $(1 - \alpha)$ of its revenues (or, more generally, some of its profits) for the social interest.[1] For example, consider a non-profit hospital whose mission consists in providing treatment to both insured and uninsured patients. The hospital earns positive revenues only for the fraction α of insured patients, whereas it receives a null compensation for the fraction $(1 - \alpha)$ of charity care provided.[2] For the time being, the firm's mission, represented by the level of α, is taken as given, but it will be endogenized in what follows (see Section 2.1).

[1] See Barigozzi and Burani (2016).

[2] Or else, consider a non-profit university offering special tuition wavers to some incoming students.

Workers differ in productive ability, which lowers the cost of effort provision θ. High realizations of θ represent workers with high cost of effort provision and low ability, whereas low realizations of θ correspond to high-skilled workers. For simplicity, assume that $\theta \sim U\,[0,1]$. Ability cannot be observed by the firm, which only knows its distribution. Workers are also characterized by intrinsic motivation, $\gamma \in [0,1]$, representing the enjoyment of one's personal contribution to the firm's mission. For the time being, let motivation γ be homogeneous across workers and observable. In the sequel (see Section 2.1), I'll assume that motivation, although being unobservable to the firm, can exactly be inferred because it is driven by the firm's choice of its mission.

For each type θ, worker's utility is

$$u\,(e,w) = w - \frac{1}{2}\,(\theta+1)\,e^2 + \gamma e. \tag{2}$$

If a worker is not hired by the firm, she receives zero utility.

The firm aims at maximizing expected profits: it chooses effort levels $e\,(\theta)$ and wages $w\,(\theta)$ based on the worker's truthful report of θ. Let

$$U\,(\theta) = w(\theta) - \frac{1}{2}\,(\theta+1)\,e\,(\theta)^2 + \gamma e\,(\theta) \tag{3}$$

denote the information rent of type θ accepting contract $\{e\,(\theta)\,, w\,(\theta)\}$. Solving (3) for $w(\theta)$ and taking into account limited liability, which prevents the worker from accepting negative transfers, one has

$$w\,(\theta) = \max\left\{0, U\,(\theta) + \frac{1}{2}\,(\theta+1)\,e\,(\theta)^2 - \gamma e\,(\theta)\right\}. \tag{4}$$

When both $e\,(\theta) < \frac{2\gamma}{\theta+1}$ and $U\,(\theta) < \gamma e\,(\theta) - \frac{1}{2}\,(\theta+1)\,e\,(\theta)^2$ hold, the liability constraint is binding and $w\,(\theta) = 0$.

The firm's problem becomes

$$\max E\,[\pi\,(e,w)] = \max_e \int_0^1 [\alpha e\,(\theta) - w\,(\theta)]\,d\theta \tag{5}$$

subject to $w\,(\theta)$ satisfying (4) and

$$\frac{\partial e\,(\theta)}{\partial \theta} \leq 0, \tag{$C.1$}$$

$$\frac{\partial U\,(\theta)}{\partial \theta} = -\frac{1}{2} e\,(\theta)^2\,, \tag{$C.2$}$$

$$U\,(\theta) \geq 0 \text{ for all } \theta \in [0,1]\,. \tag{$C.3$}$$

The solution to the above program yields the following optimal incentive schemes.[3]

[3] For a complete description of the features of the optimal contract and for the intuition of these results, the reader is referred to Burani and Palestini (2016).

Proposition 1 *The optimal contract is such that the firm asks workers to provide effort*

$$e^*(\theta) = \begin{cases} \frac{\alpha+\gamma}{2\theta+1} & \textit{for } 0 \leq \theta \leq \overline{\theta} = \frac{\alpha}{2\gamma} \\ \gamma & \textit{for } \overline{\theta} = \frac{\alpha}{2\gamma} \leq \theta \leq 1 \end{cases}$$

and offers the wage schedule

$$w^*(\theta) = \begin{cases} \frac{(3\alpha+4\theta\alpha-2\theta\gamma)(\alpha-2\theta\gamma)}{4(2\theta+1)^2} & \textit{for } 0 \leq \theta \leq \frac{\alpha}{2\gamma} \\ 0 & \textit{for } \frac{\alpha}{2\gamma} \leq \theta \leq 1 \end{cases}.$$

Proof. See Appendix A.1 ■

The optimal contract is fully separating as long as $\frac{\alpha}{2\gamma} \geq 1$, i.e. provided that workers' motivation is sufficiently low. Otherwise, the least able worker are offered the same pooling contract with a fixed positive level of effort and a null salary. These workers become volunteers to the firm.

2.1 The optimal mission choice

Given the optimal contract, and the constraints imposed by limited liability when workers' motivation is high, let me take a step back and address the firm's problem consisting in the choice of its mission, i.e. the value of α.[4] This induces a specific level of workers' motivation, which the firm cannot observe directly, but rather infers provided that there exists a causal relationship between the fraction of revenues that the firm devotes to socially worthwhile projects and workers' motivation.

Indeed, assume that workers' motivation behaves according to function $\gamma(\alpha)$ which is such that: (i) $\gamma : [0,1] \to [0,1]$; (ii) $\gamma'(\alpha) < 0$; and (iii) $\gamma''(\alpha) \leq 0$ for all $\alpha \in [0,1]$. Condition (i) requires that motivation be bounded above so that labor donations are not excessive; in particular it requires that $\gamma(\alpha) \to 1$ when $\alpha \to 0$ and $\gamma(\alpha) \to 0$ when $\alpha \to 1$. Condition (ii) guarantees that the lower the firm's commitment to the social interest, i.e. the higher α, the lower workers' motivation, whereas condition (iii) ensures that the problem of the firm is well-behaved: concavity of γ translates into profits being concave in α, so that there always exists an optimal interior mission choice.

Substituting the optimal contract $(e^*(\theta), w^*(\theta))$ into firm's profits (5) and taking into account that worker's motivation depends on α according to function $\gamma(\alpha)$, the problem of the firm becomes

$$\max_{\alpha} \pi^*(\alpha) = \begin{cases} \int_0^{\frac{\alpha}{2\gamma(\alpha)}} \left(\frac{(\alpha+\gamma(\alpha))^2(4\theta+1)}{4(2\theta+1)^2} + \frac{1}{4}\gamma(\alpha)(2\alpha-\gamma(\alpha)) \right) d\theta + \frac{\alpha}{2}(2\gamma(\alpha)-\alpha) & \text{if } 0 \leq \alpha \leq 2\gamma(\alpha) \\ \int_0^1 \left(\frac{(\alpha+\gamma(\alpha))^2(4\theta+1)}{4(2\theta+1)^2} + \frac{1}{4}\gamma(\alpha)(2\alpha-\gamma(\alpha)) \right) d\theta & \text{if } 2\gamma(\alpha) \leq \alpha \leq 1 \end{cases}. \quad (6)$$

[4] For instance, a non-profit hospital is concerned with setting its optimal level of compensated care.

The associated first-order conditions are as follows

$$\int_0^{\frac{\alpha}{2\gamma(\alpha)}} \left(\frac{(\alpha+\gamma(\alpha))\big(1+\gamma'(\alpha)\big)(4\theta+1)}{2(2\theta+1)^2} + \frac{\gamma'(\alpha)(2\alpha-\gamma(\alpha))}{4} + \frac{\gamma(\alpha)\big(2-\gamma'(\alpha)\big)}{4} \right) d\theta$$
$$+\frac{\alpha\big(\gamma(\alpha)-\alpha\gamma'(\alpha)\big)}{2\gamma(\alpha)} + (\gamma(\alpha)+\alpha(\gamma'(\alpha)-1)) = 0 \qquad \text{if } 0 \le \alpha \le 2\gamma(\alpha) \tag{7}$$

$$\int_0^1 \left(\frac{(\alpha+\gamma(\alpha))\big(1+\gamma'(\alpha)\big)(4\theta+1)}{2(2\theta+1)^2} + \frac{\gamma'(\alpha)(2\alpha-\gamma(\alpha))}{4} + \frac{\gamma(\alpha)\big(2-\gamma'(\alpha)\big)}{4} \right) d\theta = 0 \quad \text{if } 2\gamma(\alpha) \le \alpha \le 1 \tag{8}$$

There are several effects at work that first order conditions impose to balance. First of all, an increase in α yields higher profits to the firm directly in the form of higher revenues. Nonetheless, an increase in α triggers a decrease in motivation which in turn influences negatively the effort level provided by workers and their labor donations, so that the salary increases. Moreover, considering (7), an increase in α determines an increase in the upper bound of integration, which is beneficial for profits because the mass of workers who are offered the separating contract increases. On the other hand, an increase in α decreases profits because it decreases the mass of workers who are offered the pooling contract, and also because it reduces the fixed effort provided by volunteers.

Of course, a solution to the above problem can only be found once a specific function $\gamma(\alpha)$ is provided. So, in what follows, I'll focus attention on a family of functions $\gamma(\alpha)$ and consider different examples, computing the optimal α^* and the associated level of motivation $\gamma(\alpha^*)$.

2.2 Examples

Let motivation be given by function

$$\gamma(\alpha) = (1-\alpha^x)^y, \tag{9}$$

which is decreasing in α and concave, provided that $x \ge 1$ and $y \le 1$.[5]

As a first example, set $x = y = 1$ and consider

$$\gamma(\alpha) = 1 - \alpha.$$

What is peculiar to this specification is that, when α is sufficiently high that limited liability does not bind and separation of workers according to ability is possible, profits are decreasing in α: optimal effort does not depend on α, neither directly not indirectly through motivation, whereas optimal salaries are increasing in α. When, instead, α is sufficiently low that limited liability binds and a pooling contract is proposed to low-ability workers, profits are increasing in α : optimal effort decreases in α, while wage is set equal to zero irrespective of α. Both first-order conditions (7) and (8) solve for $\alpha^* = \frac{2}{3}$, which is the point at which $\pi(\alpha)$ has a kink, but which is nonetheless the global maximum. The induced level of motivation is $\gamma(\alpha^*) = \frac{1}{3}$. The optimal contract is fully separating because the threshold $\overline{\theta}$ is precisely

[5] The examples that follow are examined in more detail in Appendix A.2.1.

equal to one and there are no volunteers, except for the least-able type, $\theta = 1$, who earns $w^*(1) = 0$ while providing positive effort $e^*(1) = \frac{1}{3}$.

As a second example, set $x = 1$ and $y = \frac{1}{2}$ and consider the function

$$\gamma(\alpha) = (1-\alpha)^{\frac{1}{2}}.$$

Condition (7) is the relevant one and it can be solved numerically for $\alpha^* = 0.7867$. Motivation is equal to $\gamma(\alpha^*) = 0.46184$. Furthermore, the threshold value $\overline{\theta}$ is given by $\frac{\alpha}{2\gamma} = 0.8517$ so types with high skills $\theta \in [0, 0.8517]$ are separated whereas types with low skills $\theta \in [0.8517, 1]$ are offered a pooling contract and are volunteers.

Similar results are obtained when one sets $x = 2$ and $y = 1$, so

$$\gamma(\alpha) = 1 - \alpha^2.$$

In this case, again, (7) is the relevant first-order condition and its numerical solution is $\alpha^* = 0.62416$, leading to $\gamma(\alpha^*) = 0.61043$. Workers with high skills, i.e. whose effort cost is $\theta \in [0, 0.51125]$, are offered a fully separating contract, whereas workers with low skills, i.e. such that $\theta \in [0.51125, 1]$, become volunteers.

The proposition that follows generalizes the results obtained so far, providing sufficient conditions under which the firm has incentive to hire volunteer workers.

Proposition 2 *Consider the function $\gamma(\alpha) = (1-\alpha^x)^y$. When $x = 1$ and $y \leq 1$ or when $x \geq 1$ and $y = 1$, the optimal choice of the firm's mission is such that volunteerism emerges.*

Proof. See Appendix A.2.1. ■

When $\gamma(\alpha)$ satisfies the above requirements, it is always in the interest of the firm to sacrifice a sufficiently high fraction of its revenues in the social interest so as to induce a high motivation in its workforce. In turn, high motivation makes liability limitations relevant and calls for optimal contracts being such that: (*i*) high-skilled applicants are separated and asked to exert effort levels which are increasing in ability and distorted downward (with respect to the efficient level), except for the most able worker; and (*ii*) low-skilled applicants are pooled and asked to provide a fixed level of effort in exchange for a null salary.

The main driver of this result is the concavity of $\gamma(\alpha)$. When α is already high, a further increase in the fraction of revenues retained by the employer, α, generates a sharp drop in workers' motivation; this is harmful for the firm because it causes a sizeable decrease in workers' willingness to work and a corresponding increase in salary, leading to lower profits.

As a final remark, consider the case in which $\gamma(\alpha)$ is still decreasing, but convex in α. In particular, suppose that $x = \frac{1}{2}$ and $y = 1$ so that

$$\gamma(\alpha) = 1 - \alpha^{\frac{1}{2}}.$$

Profits attain a maximum for $\alpha^* = 0.82949$, inducing motivation $\gamma(\alpha^*) = 0.089237$. Condition (8) is now the relevant one and limited liability has no bite. Intrinsic motivation is so low that all workers' types are offered a different contracts and receive a strictly positive wage in exchange for their labor services.

3 Conclusion

I analyze the screening problem of a firm that hires motivated workers who have private information about their ability, taking into account workers' liability limitations. Given the optimal contract, it becomes natural for the firm to choose its mission-orientation in such a way as to the drive the desired level of motivation for its pool of applicants. It is shown that, for a whole family of functions representing workers' mission-induced motivation, the firm sets its mission so as to generate a high motivation in its workforce and to induce the least able workers to become volunteers.

References

[1] Barigozzi F. and N. Burani (2016), "Competition and Screening with Motivated Health Professionals", *Journal of Health Economics*, 50, 358-371.

[2] Burani and Palestini (2016), "What determines Volunteer Work? On the Effects of Adverse Selection and Intrinsic Motivation", *Economics Letters*, 144, 29-32.

[3] Cassar, L. (2016) "Optimal Contracting with Endogenous Project Mission", CESifo Working Paper Series No. 6181.

[4] Preston, A. (1989), "The Nonprofit Worker in a For-Profit World", *Journal of Labor Economics* 7, 438-463.

A Appendix

A.1 Optimal contracts

The derivation of the optimal contracts, with α and γ taken as exogenous, follows Burani and Palestini (2016).

Suppose first that limited liability has no bite and that

$$w(\theta) = U(\theta) + \frac{1}{2}(\theta + 1) e(\theta)^2 - \gamma e(\theta) > 0. \tag{10}$$

The firm's problem is thus

$$\max_{e} \int_0^1 \left[(\alpha + \gamma) e(\theta) - U(\theta) - \frac{1}{2}(\theta + 1) e(\theta)^2 \right] d\theta, \tag{11}$$

subject to $(C.1)-(C.3)$, for all $\theta \in [0,1]$. Condition $(C.3)$ is the individual rationality constraint, whereas the monotonicity condition $(C.1)$ and the envelope condition $(C.2)$ characterize incentive compatibility. Given the envelope condition, incentive compatibility implies that only the participation constraint of the least able type be binding, whereby the participation constraint $(C.3)$ reduces to the boundary condition $U(1) = 0$.

This is an optimal control problem, where e is the control variable and U is the state variable. In order to solve this problem, build the Hamiltonian

$$H = (\alpha + \gamma) e(\theta) - U(\theta) - \frac{1}{2}(\theta + 1) e(\theta)^2 + \lambda(\theta)\left(-\frac{1}{2} e(\theta)^2\right),$$

where multiplier λ is the co-state variable. The first order conditions are the following ones

$$\begin{aligned} \frac{\partial H}{\partial e} &= (\alpha+\gamma) - (\theta+1) e(\theta) - \lambda(\theta) e(\theta) = 0 && (a) \\ -\frac{\partial H}{\partial U} &= 1 = \lambda'(\theta) && (b) \\ \frac{\partial U(\theta)}{\partial \theta} &= -\frac{1}{2} e(\theta)^2 && (c) \\ \lambda(0) &= 0 && (d) \end{aligned}$$

where (d) is the transversality condition, since there is no constraint on $U(0)$. Integrating (b) over θ, one gets

$$\lambda(\theta) = \theta + c$$

and, using (d) to compute the value of the constant c, one obtains $c = 0$ and $\lambda(\theta) = \theta$. Replacing the latter expression into (a) yields the optimal effort

$$(\alpha+\gamma) - (\theta+1) e(\theta) - \theta e(\theta) = 0 \quad \Longrightarrow \quad e^*(\theta) = \frac{\alpha+\gamma}{2\theta+1}. \tag{13}$$

The optimal wage rate is obtained by the envelope condition

$$\frac{\partial U(\theta)}{\partial \theta} = -\frac{1}{2} e(\theta)^2 = -\frac{(\alpha+\gamma)^2}{2(2\theta+1)^2}.$$

Integrating it over θ, with the requirement that $U(1) = 0$, yields

$$U(\theta) = \frac{(\alpha+\gamma)^2}{4(2\theta+1)} - \frac{(\alpha+\gamma)^2}{12} = \frac{(\alpha+\gamma)^2(1-\theta)}{6(2\theta+1)}. \tag{14}$$

Substituting (14) and (13) into (10) one gets

$$w^*(\theta) = \frac{(\alpha+\gamma)((2\theta+1)(\alpha-2\gamma) + (1-\theta)(1+\theta)(\alpha+\gamma))}{3(2\theta+1)^2}.$$

Since the wage schedule is non-increasing in θ, the sufficient condition for $w(\theta) > 0$ becomes $w(1) > 0$ which amounts to $\gamma < \frac{\alpha}{2}$.

Suppose then that limited liability is binding and consider $w(\theta) = 0$. From Burani and Palestini (2016), the optimal contract is such that all types of workers are employed, even though some types (those with high effort cost) are not separated and are offered the same contract. Thus, there exists an optimal threshold $\overline{\theta}$ such that types below $\overline{\theta}$ are fully separated whereas types above $\overline{\theta}$ are pooled. By continuity of the optimal allocation, the solution is such that, for some threshold $\overline{\theta}$,

$$e(\theta) = \begin{cases} \frac{\alpha+\gamma}{2\theta+1} & \text{for } 0 \leq \theta \leq \overline{\theta} \\ \frac{\alpha+\gamma}{2\overline{\theta}+1} & \text{for } \overline{\theta} \leq \theta \leq 1 \end{cases}. \tag{15}$$

When workers' types belong to the range $\overline{\theta} \leq \theta \leq 1$, the schedule of information rents is given by (3) with $w = 0$ and $e = \frac{\alpha+\gamma}{2\overline{\theta}+1}$ and it is such that

$$U(\theta) = \frac{(\alpha+\gamma)\left(4\overline{\theta}\gamma - (\alpha-\gamma) - \theta(\alpha+\gamma)\right)}{2\left(2\overline{\theta}+1\right)^2}$$

Under full participation, it is optimal for the employer to leave the worst worker with zero rents, so it must be that $U(1) = 0$, which yields the optimal threshold

$$\overline{\theta} = \frac{\alpha}{2\gamma}$$

and the optimal constant level of effort, that is required for types in the range $\overline{\theta} \leq \theta \leq 1$,

$$e = \frac{\alpha+\gamma}{2\left(\frac{\alpha}{2\gamma}\right)+1} = \gamma.$$

Accordingly, the optimal allocation (15) can be fully specified as

$$e^*(\theta) = \begin{cases} \frac{\alpha+\gamma}{2\theta+1} & \text{for } 0 \leq \theta \leq \frac{\alpha}{2\gamma} \\ \gamma & \text{for } \frac{\alpha}{2\gamma} \leq \theta \leq 1 \end{cases}.$$

When workers' types belong to the interval $0 \leq \theta \leq \frac{\alpha}{2\gamma}$, the function $U(\theta)$ can be recovered from the envelope condition $(C.2)$ and it is equal to

$$U(\theta) = \frac{(\alpha+\gamma)^2}{4(2\theta+1)} + c,$$

where the constant c can be computed using the continuity of the surplus function at $\overline{\theta}$ and the fact that

$$U\left(\theta, \overline{\theta} = \frac{\alpha}{2\gamma}\right) = \frac{(1-\theta)\gamma^2}{2}.$$

This leads to

$$U^*(\theta) = \begin{cases} \frac{(\alpha+\gamma)^2}{4(2\theta+1)} - \frac{1}{4}\gamma(2\alpha-\gamma) & \text{for } 0 \leq \theta \leq \frac{\alpha}{2\gamma} \\ \frac{(1-\theta)\gamma^2}{2} & \text{for } \frac{\alpha}{2\gamma} \leq \theta \leq 1 \end{cases}$$

Finally, the wage rate as a function of θ is such that

$$w^*(\theta) = \begin{cases} \frac{(\alpha+\gamma)(4\theta(\alpha-\gamma)+3\alpha-\gamma)}{4(2\theta+1)^2} - \frac{1}{4}\gamma(2\alpha-\gamma) = \frac{(3\alpha+4\theta\alpha-2\theta\gamma)(\alpha-2\theta\gamma)}{4(2\theta+1)^2} & \text{for } 0 \leq \theta \leq \frac{\alpha}{2\gamma} \\ 0 & \text{for } \frac{\alpha}{2\gamma} \leq \theta \leq 1 \end{cases}.$$

A.2 Optimal mission choice

Consider now function $\gamma(\alpha)$ and substitute the optimal contract $\{e^*(\theta), w^*(\theta)\}$ into $\pi(e,w)$ yielding

$$\pi(\alpha) = \begin{cases} \frac{(\alpha+\gamma(\alpha))^2}{4}\left(\frac{\gamma(\alpha)}{2(\alpha+\gamma(\alpha))} + \ln\left(\frac{\alpha+\gamma(\alpha)}{2\gamma(\alpha)}\right) - \left(\frac{1}{2} + \ln\left(\frac{1}{2}\right)\right)\right) + \frac{\alpha(7\gamma(\alpha)-2\alpha)}{8} & \text{if } 0 \le \alpha \le 2\gamma(\alpha) \\ \frac{1}{12}\left((3\ln 3 - 1)(\alpha+\gamma(\alpha))^2 + 3\gamma(\alpha)(2\alpha - \gamma(\alpha))\right) & \text{if } 2\gamma(\alpha) \le \alpha \le 1 \end{cases} \tag{16}$$

This is the same expression as in (6), with the difference that integration with respect to θ has already been performed. First-order conditions are

$$\frac{(\alpha+\gamma(\alpha))\left(1+\gamma'(\alpha)\right)}{2}\left(\ln 2 - \frac{1}{2} + \frac{\gamma(\alpha)}{2(\alpha+\gamma(\alpha))} + \ln\left(\frac{(\alpha+\gamma(\alpha))}{2\gamma(\alpha)}\right)\right) + \frac{\left(3\alpha\gamma(\alpha)\gamma'(\alpha) - \alpha\gamma(\alpha) + 4\gamma(\alpha)^2 - \alpha^2\gamma'(\alpha)\right)}{4\gamma(\alpha)} = 0 \tag{17}$$

if $0 \le \alpha \le 2\gamma(\alpha)$, and

$$\frac{(\alpha+\gamma(\alpha))\left(1+\gamma'(\alpha)\right)}{2}\left(\ln 3 - \frac{1}{3}\right) + \frac{1}{2}\left(\gamma(\alpha) + \alpha\gamma'(\alpha) - \gamma(\alpha)\gamma'(\alpha)\right) = 0 \tag{18}$$

for $2\gamma(\alpha) \le \alpha \le 1$. These conditions, respectively, are equivalent to (7) and (8) in the main text, the difference being, again, that now integration with respect to θ has already been performed.

A.2.1 Examples

Consider the family of functions $\gamma(\alpha) = (1-\alpha^x)^y$ with $x \ge 1$ and $y \le 1$.

As a first example examine the instance in which $x = 1$ and $y = 1$ so that $\gamma(\alpha) = 1 - \alpha$. Profits (16) specify as

$$\pi(\alpha) = \begin{cases} \frac{1}{4}\ln 2 - \frac{1}{4}\alpha + \frac{3}{8}\alpha^2 + \frac{1}{4}\ln\left(\frac{1}{2} + \frac{\alpha}{2(1-\alpha)}\right) + \frac{\alpha}{2}(2-3\alpha) & \text{if } 0 \le \alpha \le \frac{2}{3} \\ \alpha + \frac{1}{4}\ln 3 - \frac{3}{4}\alpha^2 - \frac{1}{3} & \text{if } \frac{2}{3} \le \alpha \le 1 \end{cases}$$

and are depicted in Figure 1, where the solid curve represents profits when α is smaller than the critical value, i.e. to the left of $\alpha = \frac{2}{3}$, whereas the dashed curve corresponds to profits when α is higher than its critical value, i.e. to the right of $\alpha = \frac{2}{3}$.

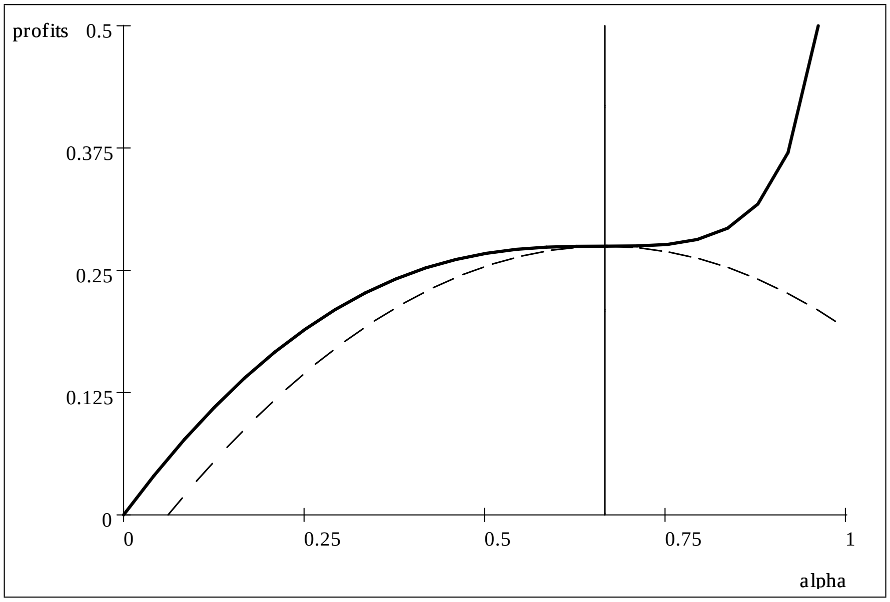

Figure 1

The first-order conditions (17) and (18) are given by

$$\begin{array}{ll} \frac{(3\alpha-2)^2}{4(1-\alpha)}=0 & \text{if } 0\leq\alpha\leq\frac{2}{3} \\ 1-\frac{3}{2}\alpha=0 & \text{if } \frac{2}{3}\leq\alpha\leq 1 \end{array},$$

respectively, and the unique solution is $\alpha^*=\frac{2}{3}$.

Consider now the second example in which $x=2$ and $y=1$ so that $\gamma(\alpha)=1-\alpha^2$. Now profits $\pi(\alpha)$ become

$$\frac{\left(\alpha+1-\alpha^2\right)^2}{4}\left(\frac{\left(1-\alpha^2\right)}{2(\alpha+1-\alpha^2)}+\ln\left(\frac{\alpha+1-\alpha^2}{2(1-\alpha^2)}\right)-\left(\frac{1}{2}+\ln\left(\frac{1}{2}\right)\right)\right) \quad +\frac{\alpha\left(7-7\alpha^2-2\alpha\right)}{8}$$
$$\text{if } 0\leq\alpha\leq\frac{\sqrt{17}-1}{4}=0.78078$$

and

$$\frac{1}{12}\left(\left(3\ln 3-1\right)\left(\alpha+1-\alpha^2\right)^2+3\left(1-\alpha^2\right)\left(2\alpha-1+\alpha^2\right)\right)$$
$$\text{if } \frac{\sqrt{17}-1}{4}\leq\alpha\leq 1$$

and they are represented in Figure 2 (the same rule concerning solid vs dashed curves is adopted).

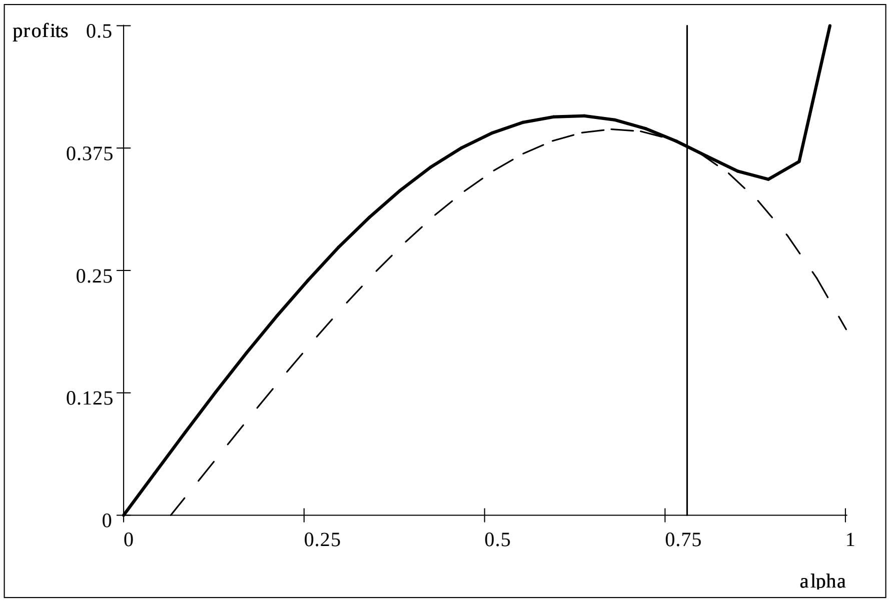

Figure 2

The first-order conditions specify as

$$\begin{aligned}&\frac{\left(\alpha+1-\alpha^2\right)(1-2\alpha)}{2}\left(\frac{1-\alpha^2}{2(\alpha+1-\alpha^2)}+\ln\left(\frac{\alpha}{2(1-\alpha^2)}+\frac{1}{2}\right)+\ln 2-\frac{1}{2}\right)+\frac{\left(3\alpha^3-14\alpha^2-\alpha+10\alpha^4+4\right)}{4(1-\alpha^2)} &= 0\\ &\text{if } 0\leq\alpha\leq\frac{\sqrt{17}-1}{4}\end{aligned} \tag{19}$$

and

$$\begin{aligned}&2\left(3\ln 3-1\right)\left(1-\alpha-3\alpha^2+2\alpha^3\right)+6\left(1-3\alpha^2+2\alpha-2\alpha^3\right) &= 0\\ &\text{if } \frac{\sqrt{17}-1}{4}\leq\alpha\leq 1.\end{aligned} \tag{20}$$

Condition (20) has solution $\alpha=0.68128$, which does not belong to the relevant range, so condition (20) can be discarded. Condition (19), instead, solves for $\alpha^*=0.62416$ which is the global maximum and yields $\gamma\left(\alpha^*\right)=1-\left(0.62416\right)^2=0.61043$.

One can generalize this example considering $x>1$ and $y=1$, so that $\gamma\left(\alpha\right)=1-\alpha^x$. First-order conditions are then given by

$$\begin{aligned}&\left(\alpha+1-\alpha^x\right)\left(1-x\alpha^{x-1}\right)\left(\frac{(1-\alpha^x)}{4(\alpha+1-\alpha^x)}+\frac{1}{2}\ln\left(\frac{\alpha}{2(1-\alpha^x)}+\frac{1}{2}\right)+\frac{1}{2}\ln 2-\frac{1}{4}\right)+\frac{4(1-\alpha^x)^2-3(1-\alpha^x)x\alpha^x-\alpha(1-\alpha^x)+x\alpha^{x+1}}{4(1-\alpha^x)} &= 0\\ &\text{if } 2-\alpha-2\alpha^x\geq 0\end{aligned} \tag{21}$$

and

$$\begin{aligned}&2\left(3\ln 3-1\right)\left(\alpha+1-\alpha^x-x\alpha^x-x\alpha^{x-1}\left(1-\alpha^x\right)\right)+6\left(1-\alpha^x-x\alpha^x+x\alpha^{x-1}\left(1-\alpha^x\right)\right) &= 0\\ &\text{if } 2-\alpha-2\alpha^x\leq 0.\end{aligned} \tag{22}$$

In Figure 3, the dotted curve is the locus of points (x,α) such that condition (21) holds whereas the solid curve is the locus of points satisfying condition (22). At the same time, Figure 3 depicts as a thick curve the locus of points (x,α) such that $2-\alpha-2\alpha^x=0$, i.e. such that the contract switches from separating to pooling. In particular, above the thick curve condition (22) is relevant whereas below the thick curve condition (21) matters. This picture shows that (22) can be discarded because it can only be satisfied when it is not relevant; condition (21) instead admits a solution in the relevant range.

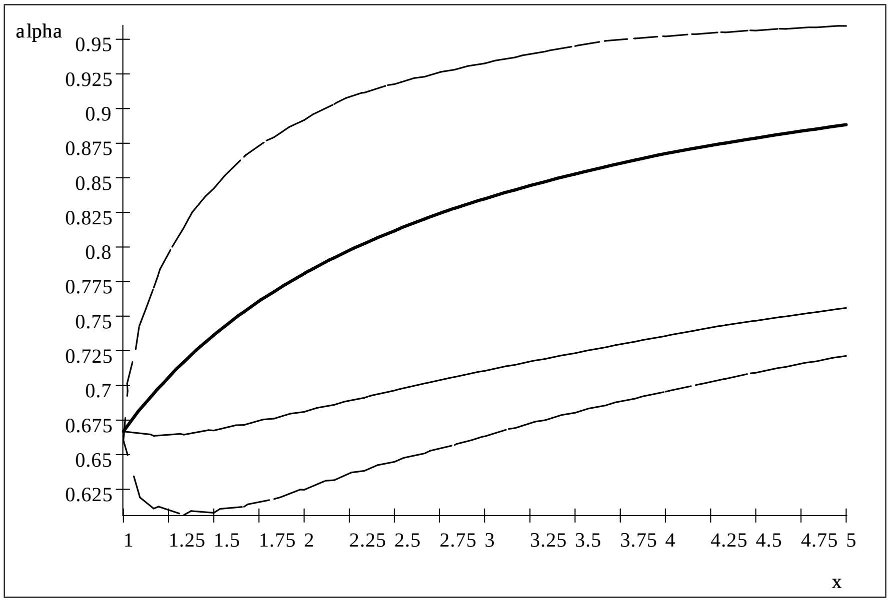

Figure 3

In the third example, one has $x=1$ and $y=\frac{1}{2}$ so that $\gamma(\alpha)=(1-\alpha)^{\frac{1}{2}}$. Profits to the firm $\pi(\alpha)$ are equal to

$$\frac{\left(\alpha+(1-\alpha)^{\frac{1}{2}}\right)^2}{4}\left(\frac{(1-\alpha)^{\frac{1}{2}}}{2\left(\alpha+(1-\alpha)^{\frac{1}{2}}\right)}+\ln\left(\frac{\alpha+(1-\alpha)^{\frac{1}{2}}}{2(1-\alpha)^{\frac{1}{2}}}\right)-\left(\frac{1}{2}+\ln\left(\frac{1}{2}\right)\right)\right)+\frac{\alpha\left(7(1-\alpha)^{\frac{1}{2}}-2\alpha\right)}{8}$$
$$\text{if } 0\le\alpha\le 2\left(\sqrt{2}-1\right)=0.828\,43$$

and

$$\frac{1}{12}\left((3\ln 3-1)\left(\alpha+(1-\alpha)^{\frac{1}{2}}\right)^2+3(1-\alpha)^{\frac{1}{2}}\left(2\alpha-(1-\alpha)^{\frac{1}{2}}\right)\right)$$
$$\text{if } 2\left(\sqrt{2}-1\right)\le\alpha\le 1$$

and they are represented in Figure 4.

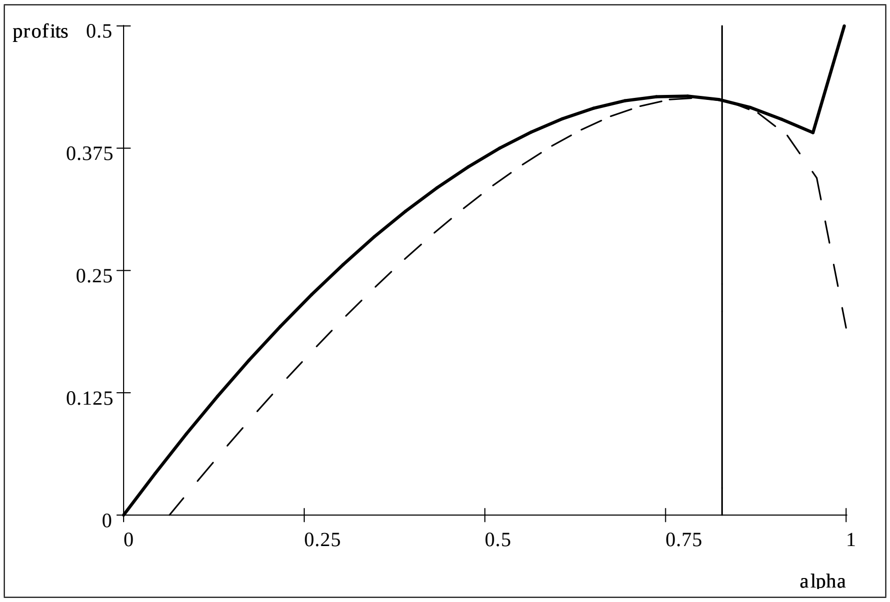

Figure 4

The first-order conditions specify as

$$\frac{\left(\alpha+(1-\alpha)^{\frac{1}{2}}\right)\left(1-\frac{1}{2}(1-\alpha)^{-\frac{1}{2}}\right)}{2}\left(\ln 2-\frac{1}{2}+\frac{(1-\alpha)^{\frac{1}{2}}}{2\left(\alpha+(1-\alpha)^{\frac{1}{2}}\right)}+\ln\left(\frac{\left(\alpha+(1-\alpha)^{\frac{1}{2}}\right)}{2(1-\alpha)^{\frac{1}{2}}}\right)\right)+\frac{\left(4(1-\alpha)-\frac{3}{2}\alpha-\alpha(1-\alpha)^{\frac{1}{2}}+\frac{\alpha^2}{2}(1-\alpha)^{-\frac{1}{2}}\right)}{4(1-\alpha)^{\frac{1}{2}}}=0$$
$$\text{if } 0\leq\alpha\leq 2\left(\sqrt{2}-1\right)=0.828\,43 \tag{23}$$

and

$$(2\,(3\ln 3-1)+6)\left((1-\alpha)^{\frac{1}{2}}-\frac{\alpha}{2(1-\alpha)^{\frac{1}{2}}}\right)+3+(2\alpha-1)\,(3\ln 3-1)=0$$
$$\text{if } 2\left(\sqrt{2}-1\right)\leq\alpha\leq 1, \tag{24}$$

where condition (24) solves for $\alpha=0.79138$ which does not belong to the relevant range, so condition (24) can be discarded; condition (23), instead, has solution $\alpha^*=0.76527$, so that $\gamma\left(\alpha^*\right)=0.48449$.

This example can be generalized considering considering $x=1$ and $y<1$, so that $\gamma\left(\alpha\right)=\left(1-\alpha\right)^y$. The associated first-order conditions are

$$\frac{(\alpha+(1-\alpha)^y)\left(1-y(1-\alpha)^{y-1}\right)}{2}\left(\ln 2-\frac{1}{2}+\frac{(1-\alpha)^y}{2(\alpha+(1-\alpha)^y)}+\ln\left(\frac{(\alpha+(1-\alpha)^y)}{2(1-\alpha)^y}\right)\right)+\frac{\left(4(1-\alpha)^y-3\alpha y(1-\alpha)^{y-1}-\alpha+\alpha^2y(1-\alpha)^{-1}\right)}{4}=0$$
$$\text{if } 2\left(1-\alpha\right)^y-\alpha\geq 0 \tag{25}$$

and

$$(2\,(3\ln 3-1)+6)\left((1-\alpha)^y-\alpha y\,(1-\alpha)^{y-1}\right)+(6-2\,(3\ln 3-1))\,y\,(1-\alpha)^{2y-1}+2\,(3\ln 3-1)\,\alpha=0$$
$$\text{if } 2\left(1-\alpha\right)^y-\alpha\leq 0. \tag{26}$$

In Figure 5, the dotted curve represents the locus of points (α, y) such that condition (25) holds whereas the solid curve is the locus of points satisfying condition (26). The thick curve is the locus of points (α, y) such that $2(1-\alpha)^y - \alpha = 0$, i.e. such that the contract switches from separating to pooling. In particular, above the thick curve condition (26) is relevant, whereas below the thick curve condition (25) matters. As before, this picture shows that (26) can be discarded because it can only be satisfied when it is not relevant; condition (25) instead admits a solution in the relevant range.

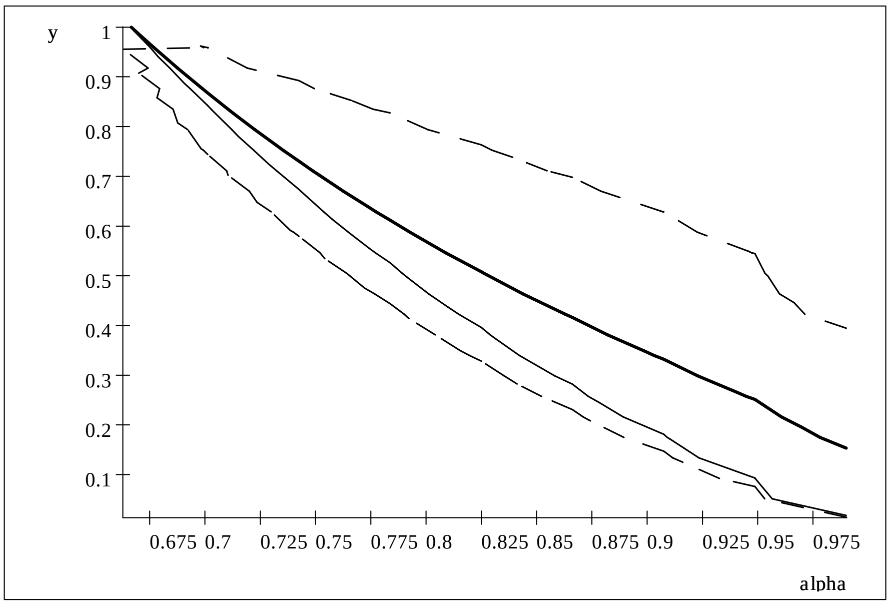

Figure 5

Finally, consider the case in which $\gamma(\alpha)$ is convex and examine $\gamma(\alpha) = 1 - \alpha^{\frac{1}{2}}$. Profits to the firm are still concave and are depicted in Figure 6. It is clear that a maximum is attained to the right of the critical α corresponding to $\overline{\theta} > 1$ and full separation, or else no volunteerism, is the outcome.

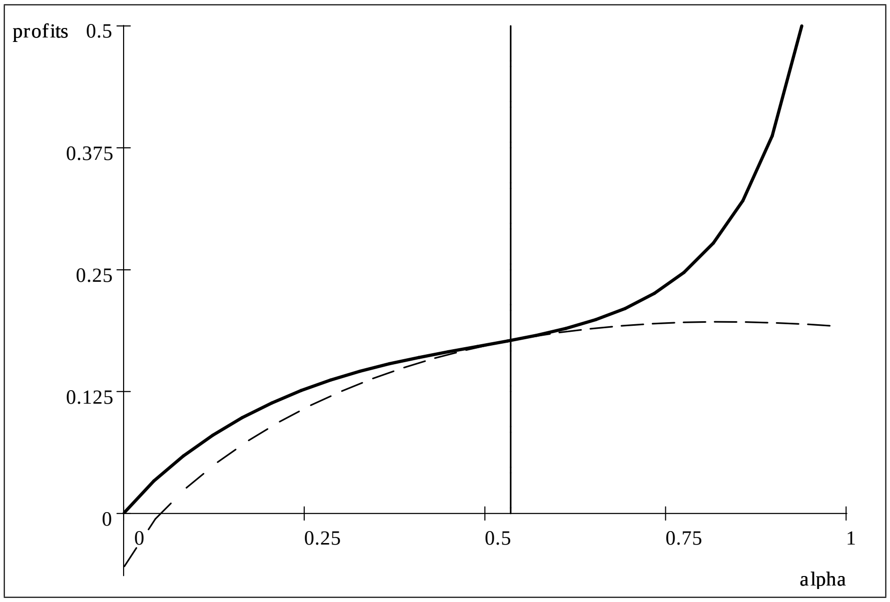

Figure 6