

Climate Change, Catastrophes and Dismal Theorem: a critical Review.

How to analyse the extreme scenarios related to the climate.*

Preliminary

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Abstract

The aim of this paper is to provide an overview of the principal studies about catastrophic scenarios and environmental policy in presence of natural disaster risk related to climate change. We investigate how the literature has analysed these topics designing policy recommendation using modelling entailing also the extreme case scenarios.

Throughout this review we want to investigate how the related literature has evolved during the recent years and how the historical evolution of climate change issue has influenced the Academic studies (and reverse). In doing so, we analyse the most important works and report entailing the future possible catastrophic effect. This, related to the conferences of parties made during the last thirty years (from Kyoto to Paris).

We focus on two main criticalities: i) the intergenerational equity between present and future generations (Stern, 2006), with the huge debate around the fact; ii) the time rate preference linked to the intergenerational equity, from the use of fat-tail and the consequent debate around the Dismal Theorem (Weitzman, 2009) to the debate around the usefulness of EU theory.

Keywords: Catastrophe, Uncertainty, Environmental Policy, Risk, Natural Disaster.

JEL Classification: Q54

1 Introduction

In the last decades the literature on climate change risk, and overall on environmental policy, has significantly increased. It is sure that the failure of Kyoto agreement, the general consensus about the climate change issue (which is not always granted) along with the rising of policymakers' effort, focused the attention on this topic and to the studies on extreme events.

The main issues that the literature is engaging are both theoretical and technical: on the theoretical side it is not straightforward to justify the politics cost due to an environmental policy; and on the other side it is controversial any probability structure used in modelling future catastrophic scenario.

Regarding the politics cost there are two sort of debatable points: the inter-generational equity, and so

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the opportunity to engage policy increasing the well-being of future generations (Stern, 2006), (Weitzman, 2009); the social equity, meaning the opportunity to suffer the cost of policy only for the rich countries or also to the poorer ones (Dasgupta, 2007).

Thus, every paper or report investigating catastrophic scenario linked to the climate change has to provide analysis assuming or recommending an intergenerational policy and it becomes necessary to make choices able to gain political consideration (Stern, 2006), (Tol and Yohe, 2006), (Dietz et al., 2007), (Tol and Yohe, 2007), (Ackerman et al., 2009), (Sterner and Persson, 2008), (Weitzman, 2009).

Consequently, the more controversial debate arising in the literature regards the value of discount factor between present and future utility (pretty always present and future generation), the assumption of a stochastic discount factor (Weitzman, 2009) or others different kind of this.

These modelling unresolved problems, are given by the reality that there are more than one source of uncertainty: the future world (future utility), the effect of probable environmental damage due to climate change, the probability of extreme events and the severity of the extreme events.

The pure time rate preference, and so the discount factor, is the most debated issue in this decade. This is so because the discount factor is the aspect containing the different theoretical disputable points.

According to this, before going deeper to the debate about the Dismal Theorem, (Weitzman, 2009) we will investigate how the approach on modelling the climate change in literature has been modified after the Stern Review (Stern, 2006).

We made this choice because the critique moved on the document defined a “political one” from Weitzman (2007) and the consequent debate on the discount factor has been precursor to the current debate on Dismal Theorem.

The Dismal theorem has been provided by Weitzman (2009), who introduce a not so innovative framework of modelling, entailing any source of uncertainty and so overestimating any state of world. Thus Weitzman (2009), uses a fat-tailed function in order to estimate the risk of a climate catastrophe, this leading to the result that it is necessary to conduct extremely expensive environmental policy in order to be safety. The effect in literature has been shattering, the paper attracted many critiques, mainly technical but also regarding the conclusion and policy recommendation (too much politics costs) provided.

The politics cost, was and still is the principal obstacle to mitigation effort. Every study in '90s, during the 2000 and in the recent years arrive (and in some papers start) to this conclusion or throws to this wall.

The rest of the paper is organized as follows, any section has a paper like landmark and discuss the entire debate exploited around the crucial work: in section 2, we discuss the Stern Review (Stern, 2006), considered from many the handbook of economics of climate change; in section 3 we will present the Dismal Theorem (Weitzman, 2009) commenting the technical choices of the author and the main cri-

tiques received; the section 4 is focused only on the critiques against the Dismal Theorem and the debate on this; in section 5 we will present the most recent work paying attention to the recent (and future) debate stressing the EU theory and the assumption of rationality; section 6 concludes giving remark and suggestion for future works.

2 Stern Review and debate

At the beginning of the current century, there has been an increasing interest on the field of catastrophic scenarios with a particular focus on climate catastrophes. The breakdown of Kyoto agreement and the general consensus about the climate change issue from populations, academia and governments has guided the research on this topic.

Thus, many studies providing interesting policy recommendations has been made along these years. The most important work about climate change catastrophic scenario and natural disaster studies, overall about environmental policies and analysis, is the one made by Stern and published by U.K. government: *The Economics of Climate Change: The Stern Review* (Stern, 2006).

The reason why a government decides to finance a similar study is to avoid the same mistakes done through the Kyoto agreement. They should think: “This time, maybe, it is better to ask to scientists before taking choices”.

It is interesting, indeed, to remember the *DICE-Model* made by Nordhaus (1993) and the title of the paper that follows the same assumption framework *Requiem for Kyoto: An Economic Analysis of the Kyoto Protocol*¹, one of not many examples in which the economists were right.

The Stern Review is a huge study on the risk and policies around the climate change issue. The aim of the Review is to become the handbook for every model about climate policy and to discuss the divergences in the different models implemented in the economics analysis.

The academic debate regarding the climate change effect is focused on the uncertainty about the future scenario and the policy expenditure in order to avoid the worst-case scenarios. The Stern Review is focused exactly on these aspects. Indeed, Stern (2006) provides simulations, attending different assumptions frameworks, about the standard inter-temporal welfare function $W = U(C_0) + \beta E[U(C_i)]$. Thus he asserts that, modelling the catastrophic climate change, it is useful to impose a discount rate (β represents the discount factor) extremely low due to the large inter-temporal horizon (C_i regards the future generations), to use a different way entails to underestimate the catastrophic scenario. Straightforward, this assumption will face some scepticism, first of all from Weitzman (2007) and also some others authors like Henderson (2007), Carter et al. (2007) and Tol and Yohe (2007).

¹ Nordhaus and Boyer (1999).

The other controversial point of the academic debate regards the expectation $E(\cdot)$ of future utility. The analysis about the right assumption framework to use in a climate policy model, few years after, has conducted to the Dismal Theorem (Weitzman, 2009) and it is still like this nowadays.

Thus, how to treat β ? which time preference to use? an ecological one or a rate more close to the intergenerational egoistic individual time-preference? which expectation rules to use? the worst-case scenarios are on the tail of the random variable, is it licit to cut these tails? even though the tails show the scenario you are analysing? one solution could be to use fat tails, but in doing so the conclusions are given by the assumptions framework. Should it be a good strategy?

The Stern review furnishes clear and reasoned answers entailing a policy prospective. A debatable point of the assumption's framework utilized, is the discount factor giving a great importance to the future generations' utility.

As argued by the author, this choice entails a policy recommendation: *“future generations should have a right to a standard of living no lower than the current one”*², which follows an idea about sustainability. Thus, in order to modelling it, Stern uses a very low discount rate r defined as $r = \delta + \eta g$, where δ is the pure discount rate, η is the elasticity of the growth rate path $g = \frac{\dot{c}}{c}$ and, straightforward, the discount factor is given by $\beta = e^{-rt}$.

This particular assumption has faced many objections in the Academia. Weitzman (2007) argues that, according to the model provided by Stern, it should be necessary to save the entire present endowment in order to mitigate the pure losses. Weitzman, in fact, plugging the values of r , δ and η used by Stern (2006) in a balance growth path having a rate $g = sr$, underlines the result regarding the saving rate: $s = \frac{r-\delta}{\eta r}$. Actually, using the values of δ and r assumed by Stern (2006) the simulation gives a $s \approx 100$ per cent of saving and through this Weitzman (2007) asserts that to use $\delta \approx 0$, $\eta \approx 1$ is a non-conventional assumption too strong with respect to the mainstream economics and involves a “unconvincing” super-strong policy recommendation. Dietz et al. (2007), defending the Stern Review, argue that introducing the technological progress the saving rate will change.

Dasgupta (2007) highlights another important point of discussion, analysing the distinction between δ and η , that is “crucial”, according to the author, because if the first shows the intergenerational distribution, the second entails the distribution between agents in the same time. Thus, Dasgupta (2007) makes a note of the evidence that, Stern (2006) uses an extremely egalitarian view of the intergenerational distribution but an extremely non-egalitarian view between rich and poorer countries.

An accurate analysis on parameters values is made by Sterner and Persson (2008) that conduct a deep observation on every Stern's choice:

δ Here, Sterner and Persson (2008) note that those using a high value for the pure rate of time

² Stern (2006) page 42.

preference, are suggesting policies making extinction more likely to occur. Thus, also according to Sterner and Persson, Stern uses a “*very egalitarian view of intergenerational distribution*” but this does not seem to be a mistake.

η Regarding the utility of growth, Sterner and Persson (2008) argue that “*it is ironic that Stern has been accused of using too low value for η when he has used a value of one*”. In fact, in many others real business models, η is treated close to zero and it is straightforward to think of a concave utility function.

g The authors ask on the possibility that Stern uses a growth rate too high and, moreover, if it is licit or not to use an eternal growth in a similar analysis³.

Controversies regarding the parameter’s values aside, Weitzman (2007) critiques how Stern (2006) faces the uncertainty regarding the future value of r : $E(r)$. Stern uses a linear combination between a high value and a low value of r , and then he solves $\frac{1}{2}e^{r_1} + \frac{1}{2}e^{r_2}$. Even though, the value obtained is acceptable, Weitzman critiques the expectation, proposing to use a set of subjective probabilities $\sum p_i$ of the value of the discount rate r_i . Thus, he obtains $r(t) = -\frac{\ln \sum p_i e^{-r_i t}}{t}$, that declines monotonically over time to an asymptotic limit for $r(\infty) = \min_i \{r_i\}$. Hence, the Weitzman’s conclusion is: “*Stern is right for the wrong reasons*”.

Indeed, regarding the Ramsey formula used by Stern, Weitzman suggests to treat the growth rate g as a normal random variable: $g \sim N(\mu, \sigma^2)$, obtaining $r^f = \delta + \eta\mu - \frac{1}{2}\eta^2\sigma^2$. Actually, to use a random variable in order to represents the growth rate gives the fair importance of the volatility with respect to the interest rate.

Overall, Weitzman (2007) discards some aspects of the “political document” known as Stern Review⁴ because these are too strong and given by strong theoretically assumptions, that are not easily testable empirically. Maybe this is true but on the other side, it is possible to move the same objection also to the modification proposed by Weitzman.

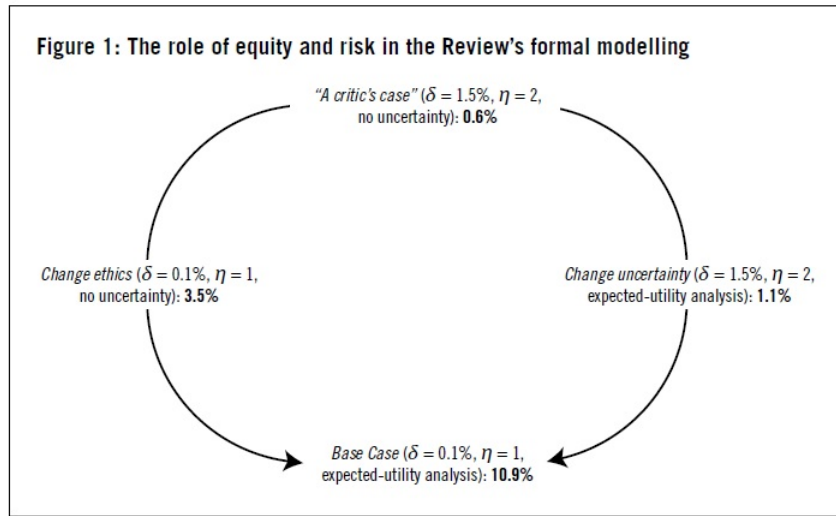
In fact the answer to this, is based on the ethical discussion behind the choice of the values; indeed, the author gives technical answer to “political” questions. On this ground, the analysis proposed by Dietz is extremely interesting, Dietz et al. (2007) argue that the ethic consideration regarding the responsibilities between generations, necessarily, “boil down” the model in a very simplistic one (the paper is entitled “*Right for the Right Reasons*”). The point is the same for any value chosen: “*how do we, as a generation, value benefits to collective action to protect the climate for generations a hundred or more years from now?*” and taking into consideration the fact, that the damage due to the climate change could affect the future growth, therefore, also the values of g is not straightforward.

³ Regarding this point, Sterner and Persson lament the Academic discredit on the limit to growth theory (Meadows et al., 1972), and his Malthusian “absurd” idea that if the planet is finite an eternal exponential growth could be a problem (Malthus, 1798).

⁴ Weitzman (2007) page 723.

Thus, it is unambiguous that the evaluation of risk and of ethic's intra-generation is not unilateral.

Figure 1: Consumption Choice along two time periods, [Dietz et al. \(2007\)](#).



A critic's case: [Nordhaus \(2006\)](#)
Base Case: [Stern \(2006\)](#)

Figure 1 shows two extremes present in the literature: [Nordhaus \(2006\)](#) and [Stern \(2006\)](#) and it is added to this two intermediate simulations. In the underside part of the figure, indeed, it is represented the estimation made by the Stern Review asserting that the climate change will involve more than the ten per cent of loss in global mean consumption per-capita.

At the top of the illustration there is an example of study in which the effects of climate change are taken into consideration in an optimistic way (considering the risk) and using a smaller coefficient concerning the utility of the future generations: [Nordhaus \(2006\)](#) in which $\delta = 1.5\%$ and $\eta = 2$. Moreover, in that paper there is not (or very little) uncertainty. [Dietz et al. \(2007\)](#) argues that within a similar simulation it is not possible to justify a policy action because, according to the parameter chosen the damage estimated is only the 0.6% of the current consumption.

The key point of the Stern Review is the following: it is necessary a precise ethic view about the utility of future utility and the model has to entail an interaction between ethics values and risk. This is so because, straightforward, climate risks fall in the future (for almost any kind of assumption) and to give a significant weight of future utility is possible only if this is evaluated according to an ethic' view, similar to the one proposed by the Stern Review.

[Dietz et al. \(2007\)](#) focus the attention on the aspect more sensitive of the Stern's framework of analysis, making two additional simulations. The first, entailing "more ethical weight" but omitting the uncertainty, and the second one introducing the uncertainty, but using the same parameters used by [Nordhaus \(2006\)](#). In that case, the cost of climate change increases with respect to [Nordhaus \(2006\)](#) but

not significantly. This, because “*the interaction between risk and ethics ... is crucial*”⁵.

Nevertheless it is licit to assert that some assumptions imposed by [Stern \(2006\)](#) are given by a “political” decision, is also true that, is possible to make this critique for any assumption framework chosen. In fact, it is curious and funny to note that the authors of this critique will receive the same kind of comments after a couple of years (It is the case of comments of the Weitzman’s Dismal Theorem).

Notwithstanding the doubts, the critiques and the controversies, the Stern Review is the environmental policy work that obtained a widespread attention in the Academia, in the supranational and national institutions. This, more than any other study.

Moreover, even the idea that Stern overestimates the climate risk ([Tol and Yohe, 2006](#)) is contradicted by some papers, an interesting example of this is [Ackerman et al. \(2009\)](#). They use the same assumptions of [Stern \(2006\)](#) changing some parameters considered to be too “optimistic”, concluding that the climate damage for the future is bigger than the one computed by the Review in terms of future loss of GDP. According to their paper, the climate damage is estimated as a loss of 10.8% with respect to the current GDP in 2100, whereas Stern has estimated this loss as the 2.2%.

The authors treats three important parameters differently: the threshold of a climate catastrophe⁶, the probability of it⁷ and the damage function⁸. Thus, ([Ackerman et al., 2009](#)) conclude that [Stern \(2006\)](#) does not inflate the damage (according to [Ackerman et al. \(2009\)](#) is the opposite) and estimate correctly the cost of climate change.

The Academic debate regarding the technical aspects of the Stern Review modelling covers, more or less, every part of the assumption framework built. It is so unequivocal that the Stern Review represents a break point of the environmental policy study. “*(...) has come to symbolize something of a dividing line in the evolution of the common appreciation of the climate problem. (...) the Stern Review is the first major, official economic report to give climate change a really prominent place among global problems*”. ([Stern and Persson, 2008](#))

The controversy around the conclusions and estimations has been widespread, the kind of integrated assessment model used (called PAGE2002) has been involved in many other papers also covering different environmental fields (an example could be [Alberth and Hope \(2007\)](#)).

The PAGE2002 is able to follow all the damages caused by climate change as it has been described by the third IPCC Report ([UNFCCC, 2001](#)). This, reminding that the most recent UN reports was not

⁵ [Dietz et al. \(2007\)](#) page 245.

⁶ Stern uses a Monte Carlo simulation in order to establish a threshold temperature (degree above the present one) that must be reached before catastrophic events becoming possible. [Stern \(2006\)](#): < 2, 5, 8 >; [Ackerman et al. \(2009\)](#): < 2, 3, 4 >.

⁷ Meaning that there is a rate at which the probability of having a catastrophe increases along with the increasing in the temperature. Monte Carlo simulation, [Ackerman et al. \(2009\)](#): < 1, 10, 20 >; [Ackerman et al. \(2009\)](#): < 10, 20, 30 >.

⁸ In [Stern \(2006\)](#) the damages function is given by aT^N , where is an uncertain parameter that is allowed to vary in the Monte Carlo analysis. He set the values as < 1, 1.3, 3 >, whereas [Ackerman et al. \(2009\)](#) as < 1.5, 2.25, 3 >.

yet produced in 2006⁹ and the ones produced after, followed the Stern's framework¹⁰.

Hope (2006) summarizes that the PAGE2002 is an update of the PAGE95 integrated assessment model (Plambeck and Hope, 1996) able to investigate all the following aspects: *Emissions of the primary greenhouse gases, CO₂ and methane, including changes in natural emissions stimulated by the changing climate; The greenhouse effect; Cooling from sulphate aerosols; Regional temperature effects; Non-linearity and transience in the damage caused by global warming; Regional economic growth; Adaptation to climate change; The possibility of a future large-scale discontinuity.*

In order to deeper analyse the modelling of PAGE2002, the appendix of Hope (2006) furnish a great synthesis.

One aspect that will be deeply stressed during the same years, regards the probabilistic features of the model nevertheless Stern spreads the level of uncertainty in the his model, according to Weitzman and many others authors (...) it is not enough. The point is an endemic one: every study on catastrophic scenario has to analyse also situations with a small probability of realization. Moreover, the different sources of uncertainty and the limits to information justify the using of "fat-tailed" probability.

Thus, three years after Weitzman will write his Dismal Theorem (Weitzman, 2009) and the debate that will rise will be widespread.

3 Dismal Theorem and recent literature

As argued above, in the recent years the literature production about environmental economics, catastrophic scenarios and policy against natural disaster opportunity has increased. This, thanks to a general awareness about the issue, for the predicted failure¹¹ of Kyoto agreement and also thank to the Stern Review (2006) already shown.

The many sources of uncertainty have been, and still are in nowadays Academic debate the most controversial point. Which mathematical and empirical tools and values are more useful to use is unequivocally an arguable choice; Barrett (2013) work is one of not many example of a paper that investigates the uncertainty behind the level of pollution (or lack of pollution control) that causes a natural disaster. It is an interesting study where the author analyses the uncertainty by using a uniform and a Gaussian random variable in order to analyse the level of pollution that could determine a catastrophe or not, according to this assumption any additional emission unit provides a higher risk to cause a disaster.

His paper, concerns the climate treaties among countries and the theoretical model giving different results entailing a different probability to experiment a natural disaster for any equilibrium obtained. An interesting conclusion made by Barrett (2013) is that the severity of the possible disaster does not affect

⁹ UNFCCC (2007) and UNFCCC (2014).

¹⁰ http://unfccc.int/adaptation/nairobi_work_programme/knowledge_resources_and_publications/items/5447.php

¹¹ Nordhaus (1993).

the Nash equilibria of the model.

Another unresolved debate present in the literature at this time regards the appropriate welfare framework in order to evaluate the environmental policy, especially about the climate change. One stimulating work about this, is made by [Heal \(2009\)](#) that conducts an analysis about the different tools used in the literature in order to investigate climate change policy.

A key point of academic discussion, as investigated in the previous section, is the discount rate. As we show, [Stern \(2006\)](#) argues that the pure rate of time preference (PRTP) is extremely low, and there exists a widespread discussion about this issue.

[Heal \(2009\)](#) agrees to the prospective that it is also important to study the elasticity between goods, moreover he stresses also the role of uncertainty investigating the solutions proposed by [Stern \(2006\)](#) and [Nordhaus \(2007\)](#). According to [Heal \(2009\)](#), in fact, “*Uncertainty is one of the dominant facts in any analysis of climate change*”. The point is that in the field there are different sources of uncertainty, these regard the magnitude of possible catastrophe, which kind of pollution could cause this, etcetera...

Straightforward, any study entailing the possibility of having natural disaster has to face the uncertainty about this possibility and about the information disposal to the agents facing the issue.

Hence, entailing the possibility of having a terrible catastrophe, required a deep analysis of events having a negligible probability to occur. This stresses the normal tools of *Expected Utility theory* that usually is insensitive to small probability events.

The point carried out from some environmental problems, is exactly what is insensitive in the EU theory. For instance, the aim of economic issue posed by climate change is to study, under uncertainty, the best policy for low-probability events within high-impact catastrophic consequence. Indeed, [Weitzman \(2009\)](#) focuses his work on this economic uniqueness studying the structural uncertainty of this framework of analysis.

The main point of [Weitzman \(2009\)](#), is to show “*that the economic consequences of fat-tailed structural uncertainty (...) can readily outweigh the effects of discounting in climate-change policy analysis*”. This is so, criticizing models of uncertain catastrophes in which the uncertainty has form of a know thin-tailed PDF. Indeed, the author provides a model with fat-tailed PDF, which are the consequence of the ratio between the thick-tailed probability, the one entailing the evidence, and the thick-tailed probability entailing the likelihood function.

The particular feature of [Weitzman \(2009\)](#) work, is the discount factor. Since he used a two time periods theoretical model, he has chosen to use a stochastic discount factor instead of a fixed rate. The stochastic discount factor used by Weitzman is defined as follows: $M := \beta \frac{E[U'(C)]}{U'(1)}$.

According to the [Weitzman \(2009\)](#) framework of analysis, through this stochastic factor M , it is possible to show the marginal rate of substitution between present and future consumption of a certain

infinitesimal unit δ . The conclusion is that this stochastic discount factor M tends to infinity.

The author obtains this, starting from three main assumptions¹²:

- i) The coefficient of relative risk aversion $\eta(c) := -c \frac{U''(C)}{U'(C)}$ is strictly greater than zero as $c \rightarrow 0$.
- ii) The consumption growth rate $y := \log C$ is distributed according to a probability density function $h(y|s) = \frac{1}{s} f(\frac{y-\mu}{s})$, where f is a normalizable function, μ is a known location parameter, and s is an uncertain scale parameter. Weitzman interprets s as a “generalized climate sensitivity parameter” (...).
- iii) The prior on s is of the form $p(s) \propto s^{-k}$, where $k > 0$, and you are given a finite number n of independent observations y_n of the random variable y .

Thus, the author concludes that:

- a) The posterior distribution for y , $q(y|y_n) \propto \int_0^\infty h(y|s) \Pi_n h(y_n|s) p(s) ds$ scales like $|y|^{-(n+k)}$ as $y \rightarrow -\infty$.
- b) The stochastic discount factor $M \rightarrow \infty$.

As argued, this work is a big innovation in modelling the climate change underlying the risk of a catastrophe and the reaction in the Academia has been a crowd. The idea of a CBA entailing a fat-tailed risk is not accepted from the entire literature, also because this involves a too much expensive policy at the present time. This is so, even though it is commonly accepted that the policy should be oriented to reduce the probability to experiment a catastrophe in the future.

Moreover, the author (due to the strong critiques made against the assumptions of the Dismal Theorem), provides interesting reasoning to the use of framework here analysed, observing the probability to assign of the climate change response to GHG change, namely the equilibrium climate sensitivity (“a good example of a known unknown”¹³). Thus, Weitzman (2011) argues that results can depend on seemingly casual decisions about how to model tail probabilities; specifically, if use a Pareto PDF or a normal one. It is true, indeed, that using the same probability of UNFCCC (2007), there is a great difference in upper tail behaviour between the Pareto and the Normal distribution. This, in addition to the fact that, only on this point there are at least two sources of uncertainty: the change in temperature (Table 1) and the sensitivity to GHG emissions (Table 2).

Table 1: $Prob[S \geq \hat{S}]$ for fat-tailed Pareto and thin-tailed Normal distributions

$\hat{S} =$	$3^\circ C$	$4,5^\circ C$	$6^\circ C$	$8^\circ C$	$10^\circ C$	$12^\circ C$
$Prob_P[S \geq \hat{S}]$	0,5	0,15	0,06	0,027	0,014	0,008
$Prob_N[S \geq \hat{S}]$	0,5	0,15	0,02	0,003	$7 * 10^{-7}$	$3 * 10^{-10}$

Weitzman (2011) p. 279

¹² The Dismal theorem is provided by Weitzman (2009) work, this summary of the assumptions and conclusions is given by Millner (2013).

¹³ Weitzman (2011) page 277.

The following table represent the link between emissions and temperature increase. According to the different probability structure assumed, Pareto = $Prob_P(\cdot)$ = fat-tails and Normal distribution = $Prob_N(\cdot)$ = thin-tails. So, the author is able to show how (and why), using fat-tailed distribution the outcome is more worrisome (Byrnes, 2015).

Table 2: Probabilities of exceeding $T = 5^\circ C$ and $T = 10^\circ C$ for given $G = ppm$ of CO_2e

G	400	500	600	700	800	900
<i>MedianT</i>	1,5°	2,5°	3,3°	4,0°	4,5°	5,1°
$Prob_P[T \geq 5^\circ C]$	1,5%	6,5%	15%	25%	38%	52%
$Prob_N[T \geq 5^\circ C]$	10^{-6}	2,0%	14%	29%	42%	51%
$Prob_P[T \geq 10^\circ C]$	0,20%	0,83%	1,9%	3,2%	4,8%	6,6%
$Prob_N[T \geq 10^\circ C]$	10^{-30}	10^{-10}	10^{-5}	0,1%	0.64%	2,1%

Weitzman (2011) p. 279

Generally, the debate affects, mainly, the mathematical tools and assumptions used in the Dismal Theorem, not his principles. This, nevertheless, on the issue here treated, the tail events are not statistical curiosities, they dominate the way we think about our options and our strategies (Nordhaus, 2011). Thus, in presence of fat-tailed outcomes and strong risk aversion, it is not possible to use the standard EU theory, because the probability of catastrophic events does not decline sufficiently rapidly to compensate the risk aversion. On the other hand, it remains crucial to include the deep structural uncertainties concerning catastrophic outcomes, influencing plausible applications of any Cost-Benefit analysis to the economics of climate change for the foreseeable future.

4 Critiques to Dismal theorem

The interpretations and the assumptions of Weitzman (2009) paper had faced a hotly debate (it is possible to count more than one thousand citations) and many authors argued against few mathematical assumptions¹⁴. In fact, according to a relevant branch of the environmental literature, the dismal theorem has not great relevance regarding the climate problem and the modelling proposed by Weitzman is not useful in order to conduct a cost-benefit analysis.

We can summarize the assertions against the DT assumptions' framework in three kinds of it: *i*) The

¹⁴ Horowitz and Lange (2014), Karp (2009), Nordhaus (2011) and others.

assumption that does not permit to transfer endowment in the future (Horowitz and Lange, 2009); *ii*) The fact that infinite marginal willingness to pay does not imply total willingness to pay is infinite (Karp, 2009); *iii*) The use of a CRRA utility function (Arrow and Priebsch, 2014).

Horowitz and Lange (2009) show that it is possible to apply the Weitzman's model only in the cases in which it is not possible to shift the wealth to the future time period. This is a stronger limit of the Weitzman's modelling, and even though many papers give different kinds of critiques, this remains a central point of the uselessness of the Dismal theorem.

In fact, the impossibility to do investment during the first time period affects the expected value of the stochastic discount factor. The authors, providing the same model but considering the explicit investment decision, obtain quite different results. According to this, they have that (...) *the optimal expected marginal rate of substitution between current and future consumption is finite* (...). This is the main difference within the authors' methodology and the one's by Weitzman (2009), because Horowitz and Lange (2009) do not consider that consumption include all adaptation and mitigation options (the investment, for example).

As argued above, the difference from Weitzman (2009) is that the investment decision is explicit. Thus, in the second time period there will be $C_1(S) + R(p, s)$, where $R(p, s)$ is the return to p^{15} investment decided during the first time period. Thus, Horowitz and Lange (2009) do not obtain $\beta \frac{E[U'(C_1(S))]}{U'(1)} = \infty$ as optimal investment choice if it exists a technology which is able to transfer an asset into the future¹⁶. In fact, even though we assume optimal choice $p = 0$ we have that $\beta \frac{E[U'(C_1(S))]}{U'(1)} = \frac{1}{\bar{R}(0)}$, contradicting Weitzman (2009) straightforward.

The point is that the Weitzman (2009) work proceeds in the Bayesian paradigm¹⁷ and so, he uses a likelihood function in order to investigate the behaviour of the random variable. According to this framework of analysis the conclusion regarding an infinite marginal rate of substitution is obvious.

The critiques made by Karp (2009) work, follows the same idea. In fact, also Karp highlights this point of analysis concluding that the likelihood function entailing the intra-generational conflict arising from climate change policy could be exaggerated under two circumstances: (a.) if it is not possible to change the composition of investment between man-made input and natural capital; or (b.) if the correct investment decisions are not able to internalize neither future nor present damages¹⁸.

Karp (2009) conducts an analysis of consumption along two time periods, studying any possible Nash equilibrium (figure Figure 2). According to Karp, the internalization of present damage and the opportunity to abate during the future time period (and so, the possibility to consume abatement), shifts to right the consumption frontier possibility to the dotted curve.

¹⁵ $0 < p < 1$ where the entire consumption is normalized to 1.

¹⁶ Horowitz and Lange (2009): Proposition 1, page 4.

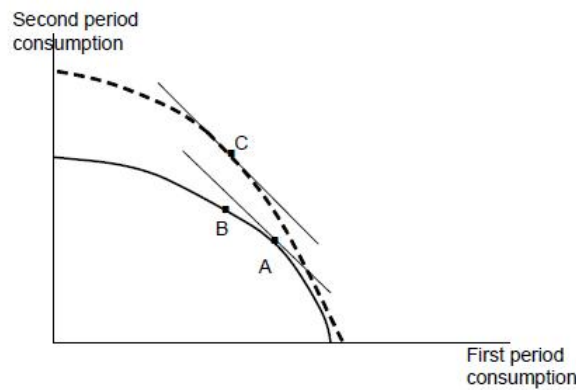
¹⁷ Millner (2013)

¹⁸ Karp (2009), page 27.

Hence, the discount rate has a key role in order to take the decision to abate or not. According to the Nash equilibrium C, there is a positive amount of abatement during the both time periods. The point is that in this first best the investment decision during the first time period claim emissions during the second one, thus the emissions decision is not exogenous. In fact, if the agent takes the emission during the second time period as exogenous, the equilibrium is given by point B.

Moreover, both [Karp \(2009\)](#) and [Horowitz and Lange \(2009\)](#) have criticized also the fact that Weitz-

Figure 2: Consumption Choice along two time period, [Karp \(2009\)](#).



- Point A: Constrained optimum with abatement set equal to 0.
- Point B: BAU equilibrium
- Point C: First Best with abatement

man shows his result in terms of marginal willingness to pay instead the total.

[Horowitz and Lange \(2009\)](#) criticize the Dismal Theorem by using the same CRRA function allied in the [Weitzman \(2009\)](#) work, in order to investigate the temporal substitution of consumption along two time periods: $U(C) = \frac{C^{1-\eta}}{1-\eta}$ where the future consumption is a random variable that captures uncertainty about her value.

The [Nordhaus \(2011\)](#) paper argues against the DT in a different way. In fact, the main point of debate regards the absence of policy in the Dismal Theorem. According to the author, in the Weitzman modelling this lack influences the conclusion that is a pessimistic one because there is not the opportunity for the policymaker to do something in order to avoid the catastrophic scenario: “*Weitzman discusses many implications of the dismal theorem, but one of the most striking is its destructive effect on benefit cost analysis, particularly for climate change*”.

Indeed, the author recognizes the importance of Dismal Theorem in terms of helping to identify when tail events are significant; but, it is doubtless that Weitzman assumes a complete (and strong) inability of society to learn and act in time.

Regarding this, with the aim of supporting his theory, [Weitzman \(2011\)](#) argues that the huge time

required by the system to naturally remove extra atmospheric CO_2 (Solomon et al., 2009) represents the evidence of ineffectiveness of any policy.

5 What's Future in Climate Change Research and Policy underlying Catastrophic' Scenarios

Nowadays, the challenges previously described, have been important in order to design the new global strategy undertaken by the policy makers¹⁹. Undoubtedly, the scientific approach and the simulation regarding the future outcomes has been seriously considered.

This is an additional reason in order to face the fact the expected power utility is fragile with respect to heavy-tailed distributional assumptions²⁰. Specifically, Ikefuji et al. (2015) provide a general theoretical framework on the utility function (present and future) in EU model, leaving probability distribution unrestricted. The authors follow, at the same time, the idea that it is reasonable to use fat-tail to model future consumption and observe that there is a limit to present consumption effort in terms of present policy (or in other words, the insurance cannot have an infinite price).

The authors rise few objection to the mainstream EU theory axiomatization by VonNeumann and Morgenstern (1944) and Savage (1954), without discredit the dismal theorem and arguing that it is appropriate questioning if EU theory is an appropriate theory for normative analysis on catastrophic risk. Indeed, they use a pretty similar (to Stern and Weitzeman) inter-temporal welfare function $W(C_0, C_1) = U(C_0) + \frac{1}{1+\rho}E[U(C_1)]$, assuming that it is possible to transfer endowment from the present to the future. Thus, the future wealth depends on the state of nature of time $t = 1$: $W_1 = (W_0 - C_0)e^{(A\epsilon_1)}$, where ϵ_1 represents the risk.

Consequently, Ikefuji et al. (2015) assert that the representative agent faces catastrophic consumption risk at the future time $t = 1$ when ϵ_1 is heavy-tailed to left under the (objectively or subjectively) given probability measure \mathcal{P} . In fact, deriving the optimal consumption bundle, the inter-temporal marginal rate of substitution will be given by: $P_{C_1^*}(C_1^*) = \frac{U'(C_1^*)}{(1+\rho)U'(C_0^*)}$; so, it exists, at least the option to postpone current consumption because in the future could be a catastrophic consumption (a sort of self-insurance). Straightforward, if the rate of substitution is finite or not, depends on the assumption regarding the risk aversion²¹.

Specifically, if the relative risk aversion is non-negative, and non-decreasing for any positive consumption; and the absolute risk aversion is non-increasing for any positive consumption, then the expected

¹⁹Paris agreement: <https://unfccc.int/resource/docs/2015/cop21/eng/109r01.pdf>

²⁰(Ikefuji et al., 2015)

²¹Ikefuji et al. (2015): page 309, Proposition 5.2.

inter-temporal marginal rate of substitution is finite: $E(P) < \infty$. The authors object to [Weitzman \(2009\)](#) only the fact that the Dismal Theorem is based on ex-ante incompatible model specification; asserting that, it is possible to use EU theory to conduct similar modelling.

Thus, [Ikefuji et al. \(2015\)](#) provide an utility function of present and future consumption avoiding fragility to distributional assumptions. Note, the authors observe that “*only the Savage’s axiomatization of EU is valid*”; they did not say the same, regarding Von Neumann Morgenstern axiomatization.

Notwithstanding the recent evolution of the debate around the EU framework, it is becoming time in time more interesting the new framework of analysis stressing the rationality assumption.

Even on 2000, Chichilnisky has written against the Von Neumann Morgenstern. Indeed, [Chichilnisky \(2000\)](#) provides an axiom allowing extreme responses to extreme events (“*sensitivity to low-probability events*”).

[Chanel and Chichilnisky \(2013\)](#) conduct an interesting experiment asking (two times, the first on 1998 and the second one during the 2009): “Imagine that you are offered the opportunity to play a game in which you must choose and swallow one pill out of 1 billion identical pills. Only one contains a lethal poison that is sure to kill you, all the other pills being ineffective. If you survive (i.e. you swallow one of the 999.999.999 ineffective pills), you receive a tax-free amount of 152.450 €. Are you willing to choose one pill and to swallow it?”

The experiment has been repeated two times and the authors investigate also the reasoning of answer change (if the individual are rich or poor, if they are educated or not, if they have child or not, etc...). Straightforward, the results of experiment contradict the EU theory because the interviewed overestimate the risk. Substantially, this result follows the [Posner \(2004\)](#) idea: “*a very low probability of materializing, but that if does materialize will produce a harm so great and sudden as to seem discontinuous with the flow of events that preceded it*”. Thus, a possible and innovative solution proposed by [Chanel and Chichilnisky \(2013\)](#), is to use a discontinuous function to collect to any state of word in order to provide “The Topology of Fear” ([Chichilnisky, 2009](#)).

Of course, this is the future of any analysis regarding this field, even though the “normal” expected utility theory will remain a fundamental basis. The new frontier of economics science, that is breaking the wall of rationality, could provide interesting results regarding some human (strange) behaviour.

An emblematic real example, of this, is the fear of a terrorist attack, that is higher with respect to the fear of a car crash. In a rational view this is absurd, however, the reason why of this behaviour is common knowledge. In the first case, in fact, the victims and the perpetrators are easily identifiable, in the second case, not. The policy makers behaves consequently, after 9/11 air-traffic was banned in the entire US and the growth of victims of car crashes was higher than the victim of world trade center attack ([Sunstein, 2007](#)). Also for the climate change risk the perpetrator is not identifiable, so population are

oriented to underestimate this risk, and the economics science has to provide answers also regarding this point, it is not sufficient to say to the policy makers what to do, it is important also to explain why there is not people consensus regarding climate change policy.

Undoubtedly, the new approaches recently used in any economics field will be extremely useful on this ground; even during 1990s (but at time, it was only a niche of literature), somebody asserted that people behave not rationally in front of risk of catastrophe (Coursey et al., 1993). Sunstein (2007) investigating how people and governments manage the worst case scenarios, reports experimental work in which the individuals underestimate the risk (so they treat this as 0) when the probability of having a catastrophe is below a certain threshold, whereas when this probability exceeds this probability threshold the people are willing to pay a big amount in order to defend themselves from this eventuality. Thus, if the people do not ignore the huge risk, they overestimate it; this is exactly what Coursey et al. said in 1993.

6 Conclusions and Suggestions

The great importance of the literature related to the climate change and possibility to incur in catastrophic scenarios, is commonly recognized both in the Academia and in the Institutional environment. This, has lead to design any possible modelling feature and any possible interpretation of it; so far, the it has proposed extremely interesting and innovative studies considering different points of view.

Indeed, the catastrophic climate change and the long horizon to investigate (i.e. 2100) gives many responsibility to the researcher who is investigating the field, some work are able to provide answers and interesting policy recommendation like Stern (2006); instead, Weitzman (2009) rises a great Academic debate around his Dismal Theorem and, consequently, on its utility. The use of fat-tailed made by Weitzman (2009), attracted many critiques and, sometimes, hilarity, but also an important debate regarding the different sources of uncertainty and how to deal with them.

Finally, it is not possible to conclude any survey of catastrophic climate change without stressing the entire Expected Utility theory and his sixty years of existence. The human behaviour in front of the possibility to suffer a global disaster entails some feature that, undoubtedly, a stochastic discount factor is not able to capture. Thus, for the future, should be extremely useful a type of investigation which uses the common EU theory in order to provide policy recommendation, but experimental and behavioural tools will be important in order to understand the actions made by individual, insurance companies, lobbies and governments.

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