Pass-Through Pricing on Production Chains*

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Abstract

We here want to analyze how the imperfect competition mark-up and pass-through are transmitted through the production chain and how they change, as a function of the number of firms existing at each production stage. In order to have an analytical closed form solution, we use the standard linear oligopoly framework. Without loss of generality and as first approximation, we assume an homogeneous good and constant returns to scale. The exercise, by using the well known standard results of Nash equilibrium with n firms and the monopoly double marginalization, obtains the mark-up solution parametric in the number of firms in S production stages.

Results are mainly the followings, where the first two are just a confirmation of the two-stage results, the third is original: (i) Mark-up vanishes in the increasing number of firms, at any production stage. (ii) Pass-through increases in the number of firms, existing at each production stage, because competition tranfers cost increases. (iii) Results are indifferent to the stage at which the larger number of firms happen. In other words, results do not discount the position in the sequence. Classical comparisons between integrated and decentralized chain have been computed with computed consequences on welfare results.

PRELIMINARY AND INCOMPLETE. DO NOT QUOTE.

KEYWORDS: pass-through, pass-on, mark-up, vertical control, production chain, bottleneck, Cournot Nash equilibrium, cartels, demand elasticity.

JEL: C63, D21, D40, D42, D43, D61, D85, E27, H22, L11, L12, L13, L41, L42, L43, L50, L51.

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1 Motivation

We here want to analyze how the imperfect competition mark-up is transmitted through the production chain and how it changes, as a function of the number of firms existing at each production stage. In order to have an analytical closed form solution, we use the standard linear oligopoly framework. Without loss of generality and as first approximation, we assume an homogeneous good and constant returns to scale. The exercise, by using the well known standard results of Nash-Cournot equilibrium with n firms, and the monopoly double marginalization, obtains the mark-up solution parametric in the number of firms in the (two) production stages.

In order to facilitate and to promote effective private enforcement of the EU antitrust rules, the European Commission has approved the Directive on Antitrust Damages Action (104/2014). Victims of the infringements to the EU Antitrust rules are entitled to the full compensation right. The Damages Directive specifically established that compensation for harm could be claimed by anyone who suffered it, irrespective of whether they are direct or indirect purchasers on an infringement. In fact, antitrust law infringements causing an overcharge can harm not only the direct purchasers of the affected good: an injured part may indeed reduce its actual loss by passing the overcharge on, entirely or in part, to its own purchasers at the next level of the supply chain. In this case, the total overcharge harm is spread among the direct purchaser and a number of indirect purchasers of the cartel, at different levels in the supply chain, including final consumers. Furthermore, in line with the compensatory principle, the Directive specifically grants to the defendant the right to invoke a "passing-on defence" against a claim concerning compensation for overcharge harm. This defence is based on the argument that the claimant has been able to pass (fully or partially) the overcharge on to his own customers, so the defendant can be able to reduce the amount due to the claimant: compensation for that amount is due to indirect customers, who in the end suffered from the price increase.

Here, in order to analyze how the cartel overcharge is transmitted through the production chain, we need to establish the elements creating it, parameterize them and describing the pass-on difference between a contestable oligopolistic market and a cartel.

The definition of the pass-through rate depends, at each stage of the production chain, on:

- the market structure, namely the distribution of firm market shares;
- the length of the chain;
- the cost structure and the economies of scales;
- the demand elasticities,
- the relative market power of the counterparts at each chain exchange.

All these parameters may of course change

- over time,
- depending on the duration of cartels,
- as soon as we consider product differentiation and whether the cartel includes or excludes differentiation.

The aim of this work is to present a first linear model, leading to simple analytic solutions, where parameters can be easily changed to understand the computational magnitudes of the setting changes.

In the analytic solution section we establish the simplest oligopoly production chain benchmark with a parametric number of production stages, a parametric number of firms at each stage, linear demand functions and linear variable cost functions to be added to fixed costs. In this setting we are able to get closed form results depending on parameters.

2 Notation and Assumptions

We consider a production chain with a number $s = 1, ..., S$ production stages, where at each stage there is a number n_s:of firms.

- Number of production stages $s = 1, ..., S$,
- Number of firms at each stage: $n_s = n_1, ..., n_S$, where the downstream stage is S.
- Each firm in the network is indexed by $\{i, s\}$, where $i = 1, ..., n_s$ is the position of the firm in that stage.
- Linear consumer aggregated demand $p\left[Q\left(n_s\right)\right] = a - b\left(q_1 + q_2 + ... + q_{n_S}\right).$
- Linear intermediate aggregated demand $w\left[Q\left(n_s\right)\right] = a - b\left(q_1 + q_2 + ... + q_{n_S}\right).$
- Cost functions: $C(q_{i,s}) = k_{i,s}q_{i,s} + F_{i,s}$, where we initially assume unit constant costs $k_{i,s} = k$ and fixed costs $F_{i,sj} = 0$.
- Each downstream firm will buy the single input from one of the upstream firms and has no other fixed costs.
- Production function: at each stage 1 unit of the intermediate product will produce 1 unit of subsequent product, therefore quantities are determined only at the final stage, when the final firms meet consumer demands.

3 Model I

3.1 Assumptions

We want to analyze how both, the imperfect competition mark-up and cost increases are transmitted through the production chain and how they change as a function of the number of firms existing at each production stage. In order to have an analytical closed form solution, we use the standard linear oligopoly framework. Without loss of generality and as first approximation, we assume an homogeneous good and constant returns to scale.

Many details can complicate the matter, so we start from the most parsimonious solution by assuming:

1. Goods at each stage of production are homogeneous, so that every firm can buy from any firm at the previous stage.
2. Price at each stage depends on the Nash equilibrium played at that stage. Since quantities are determined downstream, the only optimization is the price at that stage.
3. Equilibrium solution is by backward induction, as in standard vertical control theory.

The exercise, by using the well known standard results of Nash equilibria with n firms at each stage, obtains the mark-up solution parametric in the number of firms and in the number production stages. By changing the number of firms at each stage, we automatically control for the market regime at each bargaining stage.

In vertical chain control we have two elements to be considered in pricing:

- the mark-up added at each step,
- the possibility to “pass" downstream the cost increase, which is proportional to the derivative of the mark-up with respect to cost variations.

3.2 Definition of Mark-Up

In the following we like to precise what do we define as "mark-up" and "pass-through" in the general monopoly setting and in the linear model of it.

Given the most classic form of monopolistic profits

$$\pi(q) = p\,(q) \cdot q - c(q) - F$$

and the first order condition for a monopolistic firm:

$$\frac{d\pi(q)}{dq} : p\,(q) \left(1 - \frac{1}{\left|\varepsilon_{q/p}\right|}\right) - c'(q) = 0$$

the difference between the monopolistic price and the marginal cost is

$$p\,(q) - c'(q) = p\,(q)\, \frac{q}{p} \frac{dp}{dq}$$

where

Definition 1 *Mark-up*

$$mark - up :\equiv \frac{p}{\varepsilon_{q(p)/p}} = p\,(q) - c'(q)$$

or

$$\begin{aligned} mark - up \quad &: \quad \equiv \frac{1}{\left(1 - \frac{1}{\left|\varepsilon_{q/p}\right|}\right)} \\ &= \frac{1}{\left(1 - \left|\eta_{p(q)/q}\right|\right)} c'(q) \end{aligned}$$

where

$$\eta_{p(q)/q} \overset{def}{=} \frac{1}{\varepsilon_{q/p}}$$

3.3 Definition of Mark-Up and Pass-Through

Now, we want to know how much of the price increase is due to changes in the costs:

$$\frac{\partial p}{\partial c'(q)}$$

while costs may increase because of two reasons:

1. quantity increase;
2. marginal cost function shift, due to parameters change in the above cost function.

3.3.1 Price changes due to quantity increase.

Let's measure the change in price due to the quantity increase.

Given

$$p(q) = \frac{1}{\left(1 - \frac{1}{|\varepsilon_{q/p}|}\right)} c'(q)$$

$$\begin{aligned}
\frac{\partial p}{\partial q} &= \left(\frac{1}{\left(1 + \frac{1}{\varepsilon_{q/p}}\right)}\right)' c'(q) + \frac{1}{\left(1 + \frac{1}{\varepsilon_{q/p}}\right)} c''(q) \\
&= \frac{1}{\left(1 + \varepsilon_{q/p}\right)^2} c'(q) + \frac{1}{\left(1 + \frac{1}{\varepsilon_{q/p}}\right)} c''(q) = \\
&= \frac{1}{\left(1 + \varepsilon_{q/p}\right)^2} c'(q) + \frac{\varepsilon_{q/p}}{1 + \varepsilon_{q/p}} \cdot c''(q) \qquad (1)
\end{aligned}$$

The price variation due to the quantity increase is due to two effects:
(i) a decreasing price effect due to the downward sloping demand curve,
(ii) a price variation due to the behaviour of the cost function: increasing, constant, decreasing.

3.3.2 Example with linear demand and parametric cost functions

Let

$$\pi(q) = p^D(q)\, q - C(q)$$

where the inverse demand function is $p^D(q) = a - bq$ and the cost function is parametric in returns to scale $C(q) = kq^\gamma + F$. We have

$$\pi(q) = [a - bq]\, q - kq^\gamma - F$$

Retrieving the direct demand function $p(q) = a - bq, \Longrightarrow q(p) = \frac{a}{b} - \frac{1}{b}q$

$$\varepsilon_{q/p} = \frac{\partial\left(\frac{a}{b} - \frac{1}{b}p\right)}{\partial p} \cdot \frac{p}{\frac{a}{b} - \frac{1}{b}p} = -\frac{1}{b} \cdot \frac{p}{\frac{a}{b} - \frac{1}{b}p} = -\frac{p}{a - p} = -\frac{a - bq}{bq}$$

$$==> \frac{1}{\varepsilon_{q/p}} = -\frac{bq}{a - bq}$$

$$\eta_{p/q} = \frac{\partial(a - bq)}{\partial q} \cdot \frac{q}{a - bq} = -b \cdot \frac{q}{a - bq} = \frac{1}{\varepsilon_{q/p}}$$

$$\frac{d\pi(p)}{dp} : a - 2bq - \gamma kq^{\gamma - 1} = 0$$

The mark-up is represented by

$$\begin{aligned}
p(q) - c'(q) &= p\frac{1}{\varepsilon_{q/p}} \\
(a - bq) - \gamma kq^{\gamma - 1} &= (a - bq) \cdot \frac{bq}{a - bq} \\
\Longrightarrow &= bq
\end{aligned}$$

$$\frac{\partial p(q)}{\partial q} = \left(\frac{1}{1 + \frac{1}{\varepsilon_{q/p}}}\right)' c'(q) + \frac{1}{\left(1 + \frac{1}{\varepsilon_{q/p}}\right)} c''(q)$$

Using the explicit functions

$$
\begin{aligned}
\frac{\partial p\left(q\right)}{\partial q} &= \left(\frac{1}{1-\frac{bq}{a-bq}}\right)' c'(q) + \left(\frac{1}{1-\frac{bq}{a-bq}}\right) c''(q) \\
&= -\frac{ab}{\left(a-2bq\right)^2}\cdot c'(q) + \frac{a-bq}{a-2bq}\cdot c\text{''}(q) \qquad (2) \\
&= -\frac{ab}{\left(a-2bq\right)^2}\cdot \gamma k q^{(\gamma-1)} + \frac{a-bq}{a-2bq}\cdot \gamma\left(\gamma-1\right) k q^{(\gamma-2)} \qquad (3)
\end{aligned}
$$

Now:

I. if $C(q) = kq + F$, i.e. $\gamma = 1$:

$$\frac{\partial p\left(q\right)}{\partial q} = -\frac{ab}{\left(a-2bq\right)^2}\cdot k < 0$$

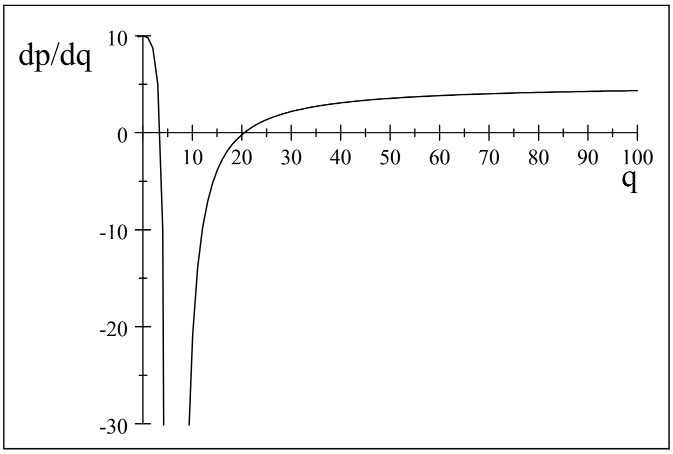
because there is only the downward sloping demand effect. The marginal cost is constant.

II. if $C(q) = kq^2 + F$, i.e. $\gamma > 1$:

$$\frac{\partial p\left(q\right)}{\partial q} = -\frac{ab}{\left(a-2bq\right)^2}\cdot k + \frac{a-bq}{a-2bq}\cdot 2kq = (-) + (+) > 0$$

Ex. $\gamma = 2$

$$
\begin{aligned}
\frac{\partial p\left(q\right)}{\partial q} &= -\frac{ab}{\left(a-2bq\right)^2}\cdot 2kq^{(2-1)} + \frac{a-bq}{a-2bq}\cdot 2\left(2-1\right) k q^{(2-2)} \\
&= -\frac{2abk}{\left(a-2bq\right)^2} q + 2k\frac{a-bq}{a-2bq}
\end{aligned}
$$



The example shows that an initial positive sign due to the sum of positive marginal costs, then a dominating role of downward sloping demand and marginal costs not too increasing, but after a quantity threshold, the increasing is soon dominated by decreasing demand curve.

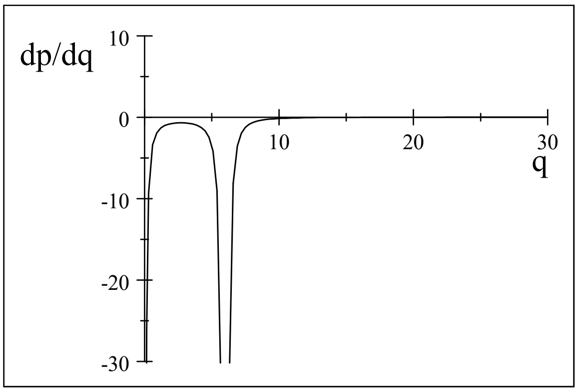
III. if $C(q) = k\sqrt{q} + F$, i.e. $\gamma < 1$:

$$
\begin{aligned}
\frac{\partial p\left(q\right)}{\partial q} &= -\frac{ab}{\left(a-2bq\right)^2}\cdot \gamma k q^{(\gamma-1)} + \frac{a-bq}{a-2bq}\cdot \gamma\left(\gamma-1\right) k q^{(\gamma-2)} \\
&= -\frac{ab}{\left(a-2bq\right)^2}\cdot k + \frac{a-bq}{a-2bq}\cdot \frac{\left(\gamma-1\right)k}{2\sqrt{q}} = (-) + (-) < 0
\end{aligned}
$$

decreasing, where the first term is due to decreasing demand curve and the second term due to decreasing costs, oscillating in the dominance of the different parameters.

Ex. $\gamma = 0.5$:

$$\begin{aligned}\frac{\partial p(q)}{\partial q} &= -\frac{ab}{(a-2bq)^2}\cdot 0.5\cdot kq^{(0.5-1)} + \frac{a-bq}{a-2bq}\cdot 0.5\,(0.5-1)\,kq^{(0.5-2)} \\ &= -\frac{12\cdot 1}{(12-2q)^2}\cdot 0.5\cdot 5q^{(0.5-1)} + \frac{12-q}{12-2}\cdot 0.5\,(0.5-1)\,5q^{(0.5-2)}\end{aligned}$$



3.3.3 Pass-through due to cost function variations

Following the standard maximizing procedure, the monopoly price is a mark-up over the marginal cost

$$p(q) = \frac{1}{\left(1+\frac{1}{\varepsilon_{q(p)/p}}\right)}c'(q)$$

We define

Definition 2 *The "**Pass-through**" represents how much the final price changes following a marginal cost change (Note. Not the quantity change, discussed above).*

$$\frac{\partial p}{\partial c'(q)}$$

We look for

$$\frac{\partial p}{\partial c'(q)} = \frac{1}{\left(1+\frac{1}{\varepsilon_{q/p}}\right)}c''(q) \gtreqless 0$$

while the term $1/\left(1+\frac{1}{\varepsilon_{q/p}}\right)$ is constant, because the quantity is constant.

Using the explicit parametric functions, we have that

$$\frac{\partial p}{\partial c'(q)} = \frac{1}{\left(1+\frac{1}{\varepsilon_{q/p}}\right)}\cdot\alpha\,(\gamma-1)\,kq^{(\gamma-2)} \tag{4}$$

Defining $\frac{\partial p}{\partial c'(q)}$ as $\phi(\gamma)$, the pass-on $\phi'(\gamma)$ is non decreasing, constant or not decreasing in q as $\gamma \gtreqless 2$.

The paper explores the results for the only "constant marginal cost" assumption.

4 Integrated Production Chain Cournot Equilibrium with n firms

Let's recall the Cournot Equilibrium results for n symmetric firms and one integrated production chain.

We define the profit function of the i^{th} integrated firm, $\pi_{I,i}$ as

$$\begin{aligned}\max_{q_i} \pi_{I,i}(Q) &= (a - b(n-1)q_{-i} - bq_i)q_i - kq_i - F_{Ii} \\ &= aq_i - b(n-1)q_{-i}q_i - bq_i^2 - kq_i - F_{Ii}\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi_{I,i}(Q)}{\partial q_i} &: \quad a - b(n-1)q_{-i} - k - 2bq_i = 0 \\ &\Longrightarrow \quad q_i = \frac{a-k}{2b} - \frac{(n-1)}{2}q_{-i}\end{aligned} \tag{5}$$

To avoid to solve a system of first order conditions, using the symmetric output assumption, we use the fact that $q_{-i} = q_i$, and write

$$a - b(n+1)q_i - k = 0$$

so that

$$q_{I,i}^*(n) = \frac{a-k}{(n+1)b} \tag{6}$$

In computing the $p^*(Q)$ we want to explicit the marginal cost alone

$$\begin{aligned}p_{I,i}^*(Q(n)) &= a - bnq_i^* = a - b\frac{n(a-k)}{(n+1)b} = \frac{a+nk}{n+1} \\ &= \frac{a+nk}{n+1} + \frac{k}{n+1} - \frac{k}{n+1} = \frac{a}{n+1} + \frac{k(n+1)}{n+1} - \frac{k}{n+1}\end{aligned}$$

$$p_{I,i}^*(Q(n)) = k + \frac{(a-k)}{n+1} \tag{7}$$

Inserting solutions in the profit function

$$\begin{aligned}\pi_{I,i}^*(n) &= [p(Q) - k]q_i^* = \left(k + \frac{(a-k)}{n+1} - k\right)\frac{a-k}{(n+1)b} - F_{I,i} \\ &= \frac{(a-k)^2}{b(n+1)^2} - F_{I,i}\end{aligned}$$

$$\pi_{I,i}^*(n) = \frac{(a-k)^2}{b(n+1)^2} - F_{I,i} \tag{8}$$

By considering the limit for $n \longrightarrow \infty$, we obtain the perfect competition solution, where the optimal quantity produced by a single firm is infinitesimal in comparison with the total industry

$$\lim_{n \longrightarrow \infty} q_{I,i}^*(n) = 0,$$

the optimal price tend to the marginal cost

$$\lim_{n \longrightarrow \infty} p_{I,i}^*(Q(n)) = k$$

and profits tend to zero and unable to support fixed costs.

$$\lim_{n \longrightarrow \infty} \pi_{I,i}^*(n) = 0 - F_{I,i}$$

5 Vertical Control: 1 upstream firm, 1 downstream firm

Let's recall now the other classical result: the double marginalization.

The first step is to consider 1 firm at each production stage, so $m = 1$ and $n = 1$, *i.e.* double monopoly.

The integrated firm results that produces within itself both productions stages are the ones above, $(6), (7)$ and (8) just using $n = 1$.

5.1 Decentralized solution: $m = 1, n = 1$.

5.1.1 Downstream solution

The downstream results are again the $(6), (7)$ and (8), where the cost is not k but the still unknown intermediate price w. Equilibrium quantities have subscript D.

$$q_D^*(w) = \frac{a-w}{2b} \tag{9}$$

$$p_D^*(Q(w)) = \frac{a+w}{2} \tag{10}$$

$$\pi_D^*(w) = \frac{(a-w)^2}{2^2 \cdot b} - F_D \tag{11}$$

5.1.2 Upstream solution

News are that in the production chain, the upstream firm cannot independently decide the output quantity, because both quantity and final price are determined at downstream level, as a function of the gross (intermediate) price w. The industry profit, determined by downstream quantity and price as a function of the upstream cost, here marginal cost k, is shared by the upstream and downstream firms as a function of the market regime in the two segments. The more competitive a market segment is, the more the price tends to the cost. We start here by assuming double monopoly. In the next paragraph we will enlarge the result to any number of firms upstream (m) and downstream (n).

The upstream firm maximizes his monopoly profits to increase his profit share in the full production chain, by searching the optimal intermediate price w, while bearing linear costs in the quantity

$$\begin{aligned}
\max_w \pi_U(w) \quad &: \quad (w-k)\, q_D^*(w) - F_U \\
&= \quad (w-k)\left(\frac{a-w}{2b}\right) - F_U = \frac{1}{2}aw - \frac{1}{2}ak + \frac{1}{2}kw - \frac{1}{2}w^2 - F_U
\end{aligned}$$

$$\frac{d\pi_U(q)}{dw} : \frac{a}{2} - w + \frac{k}{2} = 0$$

$$\Longrightarrow w^* = \frac{a+k}{2} \tag{12}$$

$$\begin{aligned}
\Longrightarrow \quad \pi_U^* &= (w-k)\left(\frac{a-w}{2b}\right) - F_U = \\
= \quad & \left(\frac{a+k}{2} - k\right)\left(\frac{a - \frac{a+k}{2}}{2b}\right) - F_U =
\end{aligned}$$

$$\pi_U^* = \frac{(a-k)^2}{8b} - F_U \tag{13}$$

Reinserting the w^* solution, we can determine q^D, p^D and therefore π^D

$$\Longrightarrow \quad q_D^* = \frac{a - w^*}{2b} = \frac{a - \frac{a+k}{2}}{2b} = \frac{a-k}{4b} \tag{14}$$

$$\Longrightarrow \quad p_D^* = \frac{a + w}{2} = \frac{a + \frac{a+k}{2}}{2} = \frac{3a+k}{4} \tag{15}$$

$$\Longrightarrow \quad \pi_D^* = \frac{(a-w)^2}{2^2 b} - F_V = \frac{\left(a - \frac{a+k}{2}\right)^2}{2^2 b} - F_V = \frac{(a-k)^2}{2 \cdot 2^2} - F_V \tag{16}$$

Remark 3 *Final quantity in a double monopoly production chain and linear function is reduced by half, implying a higher price and halved profits.*

5.2 Welfare results: $m = 1, n = 1$.

Integrated Solution:

$$\pi_I^* = \frac{(a-k)^2}{4b} - F_I$$

Decentralized Solution:

$$\pi_U^* + \pi_D^* = \frac{(a-k)^2}{8b} + \frac{(a-k)^2}{16b} - (F_M + F_V) = \frac{3}{16b}(a-k)^2 - (F_M + F_V) < \pi_I^*$$

where profits sum is smaller than in the integrated solution, and so is for the Producer Surplus PS :

$$PS_{Dec} \equiv \frac{3}{16}(a-k)^2 < \frac{4}{16}(a-k)^2 \equiv PS_{Int}$$

The ratio is

$$\frac{PS^{Dec}}{PS^{Int}} = \frac{\frac{3(a-k)^2}{16}}{\frac{(a-k)^2}{4}} = \frac{3}{4}$$

assuming $F_I = (F_M + F_V)$.

Remark 4 *In the double marginalization, when there is monopoly in both production segments, Total Producer Surplus is reduced by 1/4.*

Consumer Surplus:

$$CS_I^* = \frac{1}{2}\left(a - \frac{a+k}{2}\right)\frac{a-k}{4b} = \frac{1}{b}\left(\frac{a-k}{4}\right)^2$$

Decentralized Solution:

$$CS_D^* = \frac{1}{2}\left(a - \frac{3a+k}{4}\right)\frac{a-k}{4b} = \frac{1}{2b}\left(\frac{a-k}{4}\right)^2 = \frac{1}{2}CS_I^*$$

Remark 5 *In the double marginalization, when there is monopoly in both production segments, Consumer Surplus is halved.*

6 Vertical Control: m upstream firm, n downstream firm

6.1 Integrated Solution $(m = 0, n)$

The integrated firm producing within itself both productions stages obtains the following equilibrium results, parametric in the number of firms, are the ones above, (6), (7) and (8).

6.2 Decentralized solution (m, n)

6.2.1 Downstream with n firms

The downstream results are again the $(6), (7)$ and (8), where the cost is not k but the still unknown intermediate price w. Equilibrium results have subscript D.

$$q_{i,D}(w; n, m) = \frac{a - w}{b(n+1)} \tag{17}$$

$$p_D(w; n, m) = \frac{a + nw}{(n+1)} \tag{18}$$

$$\pi_D(w; n, m) = \frac{(a-w)^2}{(n+1)^2 \cdot b} - F_D \tag{19}$$

6.2.2 Upstream with m firms

There are m firms on the upstream market and n firms on the downstream market. Equilibrium results have subscript U.

$$\begin{aligned} \max_w \pi_U(w) \quad &: \quad (w-k)\, q_D^*(w) - F_U \\ &= \quad (w-k)\left(\frac{a-w}{b(n+1)}\right) - F_U = \frac{1}{2}aw - \frac{1}{2}ak + \frac{1}{2}kw - \frac{1}{2}w^2 - F_U \end{aligned}$$

The Nash equilibrium on the upstream market shared by m firms, directly following from (7), determines a gross/intermediate price of

$$w^*(m, n) = \frac{a + mk}{m+1} = k + \frac{(a-k)}{m+1} \tag{20}$$

and

$$\lim_{m \longrightarrow \infty} w^*(m, n) = k$$

Definition 6 *The "**mark-up**" on the upstream output price is*

$$\frac{(a-k)}{m+1}$$

and it's inversely proportional to the number of firms and it disappears, when the segment tends to perfect competition $(m \longrightarrow \infty)$.

By inserting it in the downstream Nash equilibrium quantity with n firms

$$\begin{aligned} q_D^*(n, m) &= \frac{a - w^*}{(n+1)\, b} \\ &= \frac{a - \frac{a+mk}{m+1}}{(n+1)\, b} \\ &= \frac{1}{b}\frac{m}{m+1}\left(\frac{a-k}{n+1}\right) \end{aligned} \tag{21}$$

Remark 7

$$q_D^*(n, m) = \frac{m}{m+1} \cdot q_I^*(m, n)$$

Downstream quantity decreases in the number of the downstream firms n and it is almost insensitive to the number of the upstream firms m, when "many" (when $m \simeq m + 1$).

$$
\begin{aligned}
p_D(n,m) &= a - nb\left(\frac{1}{b}\frac{m(a-k)}{(m+1)(n+1)}\right) \\
&= \frac{(a+am+an+kmn)}{(m+1)(n+1)} \\
&= \frac{(m+n+1)a+mn\cdot k}{m+n+1+mn} + \frac{(m+n+1)k}{m+n+1+mn} - \frac{(m+n+1)k}{m+n+1+mn}
\end{aligned}
$$

$$
p_D(n,m)^* = k + \frac{(m+n+1)}{(m+1)(n+1)}(a-k) \tag{22}
$$

where the second term is the "pass-through".

Result 8 *The "mark-up" on the downstream output price*

$$
\frac{(m+n+1)}{(m+1)(n+1)}(a-k)
$$

depends on the relative number of the upstream and downstream firms and it survives, unless both segments have the number of firms increasing to infinity,i.e. $m \longrightarrow \infty$ *and* $n \longrightarrow \infty$.

Result 9 *The downstream **"pass-on"** is larger, the larger* n *and/or* m.

$$
\lim_{\substack{m\longrightarrow\infty \\ n\longrightarrow\infty}} p_D^*(n,m) \simeq k + 1(a-k) = a >> k
$$

The mark-up in each segment is higher the smaller the number of firms in the segment, and therefore the effect on the retail price is higher the smaller the number of firms, i.e. the market power, at each segment. On the contrary, the possibility to “pass-through” the cost increase is higher the larger the number of firms. We are generalizing the well known result that cost increases, under competitive regimes, are completely transferred downstream.

Using the results (20), (21) ,and (22), we compute equilibrium downstream profits

$$
\begin{aligned}
\pi_D^*(n,m) &= \left(p_D(n,m)^* - w^*(m)\right)\cdot q_D^*(n,m) \\
&= \left(k + \frac{(m+n+1)}{m+n+1+mn}(a-k) - k - \frac{(a-k)}{m+1}\right)\left(\frac{1}{b}\frac{m(a-k)}{(m+1)(n+1)}\right) \\
&= \left(\frac{m}{m+1}\right)^2 \frac{1}{b}\left(\frac{a-k}{n+1}\right)^2
\end{aligned}
$$

Remark 10

$$
\pi_D^*(n,m) = \left(\frac{m}{m+1}\right)^2 \cdot \pi_I^*(m,n)
$$

Downstream profits grow in the number of upstream firms m, because this means a lower input price.

Upstream profits

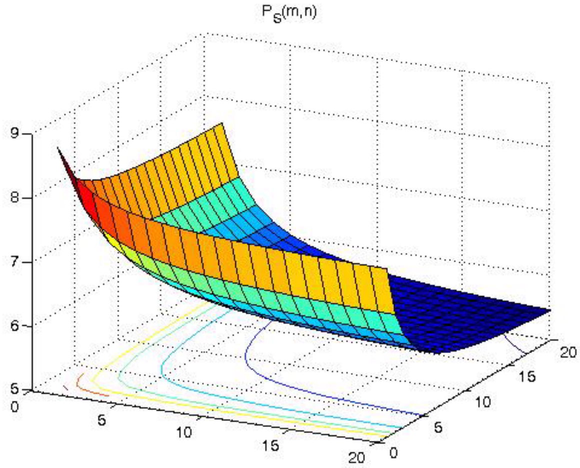
$$
\begin{aligned}
\pi_U^*(n,m) &= (w^* - k)\, q_D^*(m,n) \\
&= \left(k + \frac{(a-k)}{m+1} - k\right)\cdot\left(\frac{1}{b}\frac{m(a-k)}{(m+1)(n+1)}\right) \\
&= \frac{m}{n+1}\cdot\frac{1}{b}\left(\frac{a-k}{m+1}\right)^2
\end{aligned}
$$

Remark 11

$$\pi_U^*(n,m) = \frac{m}{n+1} \cdot \pi_{Cournot}^*(m)$$

When $m = n$, upstream profits are smaller than normal oligopoly profits. In the benchmark case of $m = n = 1$, $\pi_U^*(n,m) = (1/2) \cdot \pi_{Olig}^*(m)$.

Fig. 1. P(m,n)



In fig.1. prices as functions of m and n are shown. As we saw in the formula the iso-contours are hyperbolas, meaning that the price is due to the products $m * n$, no matter the position of the number in the sequence.

P(m,n)

m \ n	1	2	3	4	5	6	7	8	9	10
1	0,75	0,67	0,63	0,60	0,58	0,57	0,56	0,56	0,55	0,55
2	0,67	0,56	0,50	0,47	0,44	0,43	0,42	0,41	0,40	0,39
3	0,63	0,50	0,44	0,40	0,38	0,36	0,34	0,33	0,33	0,32
4	0,60	0,47	0,40	0,36	0,33	0,31	0,30	0,29	0,28	0,27
5	0,58	0,44	0,38	0,33	0,31	0,29	0,27	0,26	0,25	0,24
6	0,57	0,43	0,36	0,31	0,29	0,27	0,25	0,24	0,23	0,22
7	0,56	0,42	0,34	0,30	0,27	0,25	0,23	0,22	0,21	0,20
8	0,56	0,41	0,33	0,29	0,26	0,24	0,22	0,21	0,20	0,19
9	0,55	0,40	0,33	0,28	0,25	0,23	0,21	0,20	0,19	0,18
10	0,55	0,39	0,32	0,27	0,24	0,22	0,20	0,19	0,18	0,17

7 Welfare Results (m,n)

7.1 Producer Surplus (m,n)

7.1.1 Integrated Solution

$$\pi_i^{I*}(n) = \frac{(a-k)^2}{b(n+1)^2} - F_i$$

so

$$PS_I^*(m,n) = n\left(\frac{(a-k)^2}{b(n+1)^2} - F_I\right)$$

7.1.2 Decentralized Solution

$$\begin{aligned}\pi_U^* + \pi_D^* &= m\left(\frac{m}{n+1}\cdot\frac{1}{b}\left(\frac{a-k}{m+1}\right)^2\right) + n\left(\left(\frac{m}{m+1}\right)^2\frac{1}{b}\left(\frac{a-k}{n+1}\right)^2\right) \\ &= \frac{1}{b}m^2(2n+1)\frac{(a-k)^2}{(m+1)^2(n+1)^2}\end{aligned}$$

Neglecting fixed costs

$$PS_{Dec} \equiv m^2(2n+1)\frac{(a-k)^2}{b(m+1)^2(n+1)^2} < n\frac{(a-k)^2}{b(n+1)^2} \equiv PS_{Int}$$

The ratio is

$$\frac{PS_{Dec}}{PS_{Int}} = \frac{\frac{m^2(2n+1)}{(m+1)^2}}{\frac{(m+1)^2 n}{(m+1)^2}} = \frac{m^2}{m^2+(2m+1)}\frac{2n+1}{n} = \frac{\frac{m^2}{m^2+(2m+1)}}{\frac{n}{2n+1}}$$

Assuming $m = n = 1$,

$$\frac{PS_{Dec}(m=1,n=1)}{PS_{Int}(m=1,n=1)} = \frac{\frac{n^2}{n^2+(2n+1)}}{\frac{n}{2n+1}} = \frac{3}{4}$$

as before.

Remark 12 *In the double marginalization, when there is monopoly in both production segments, total producer surplus is reduced by 1/4.*

7.1.3 Consumer Surplus (m,n)

Integrated Solution

$$CS_I^* = \frac{1}{2}\left(a-k-\frac{(a-k)}{n+1}\right) = n\frac{1}{2}\frac{(a-k)}{(n+1)}$$

Decentralized Solution

$$\begin{aligned}CS_D^* &= \frac{1}{2}\left(a - p_D^*(n,m)\right) \\ & \frac{1}{2}\left(a-\left(k+\frac{(m+n+1)}{(m+1)(n+1)}(a-k)\right)\right) \\ &= \frac{m}{(m+1)}\frac{1}{2}\frac{n(a-k)}{(n+1)} = \frac{m}{(m+1)}CS_I^*\end{aligned}$$

Remark 13 *In the decentralized solution, consumer surplus is decreased by the ratio* $m/(m+1)$ *with respect to the integrated solution and this ratio is maximized for* $m=1$, *i.e. having one single monopoly in the upstream segment.*

8 Generalized solution to S segments

Generalizing to S stages of production

$$Mark - up \propto \frac{\left(\sum_{s=1}^{S} n_s\right) + 1}{\prod_{s=1}^{S} (n_s + 1)} (a - k)$$

where

- Number of segments: $s = 1, \ldots, S$,
- Number of firms for each segment: n_s
- $a =$ highest willingness to pay,
- $k =$ constant unit cost =marginal cost,

$$Pass - Through :\approx \frac{\Delta p}{\Delta k} \approx \frac{\prod_{s=1}^{S} n_s}{\prod_{s=1}^{S} (n_s + 1)}$$

Given the different numbers n_s, for $n_s \longrightarrow \infty$, the "markup" size decreases much faster than the “pass-on” coefficient, because it has a sum and not a product in the numerator.

As expected, in perfect competition, the pass-through rate is 100%.

9 Vertical Control: The General Case - n$_s$ per segment

- Number of production stages $s = 1, ..., S$,
- Number of firms at each stage: $n_s = n_1, ..., n_S$, where the downstream stage is S.
- Each firm in the network is indexed by $\{i, s\}$, where $i = 1, ..., n_s$ is the position of the firm in that stage

9.1 Backward Solution

9.1.1 Stage S, with N_S firms

$$\begin{aligned}
q_{i,S}(w_{S-1}) &= \frac{(a - w_{S-1})}{(n_S + 1) b} \\
p_S &= a - n_S \cdot b \frac{(a - w_{S-1})}{(n_S + 1) b} \\
&= \frac{a + n_S w_{S-1}}{n_S + 1} \\
&= \frac{1}{n_S + 1} (a + n_S w_{S-1}) + \frac{w_{S-1}}{n_S + 1} - \frac{w_{S-1}}{n_S + 1} \\
&= w_{S-1} + \frac{a - w_{S-1}}{n_S + 1}
\end{aligned}$$

$$\begin{aligned}
\pi_{i,S} &= (p_S - w_{S-1}) q_{i,S} \\
&= \left(w_{S-1} + \frac{a - w_{S-1}}{n_S + 1} - w_{S-1}\right) \frac{(a - w_{S-1})}{(n_S + 1) b} \\
&= \frac{1}{b} \frac{(a - w_{S-1})^2}{(n_S + 1)^2}
\end{aligned}$$

9.1.2 Stage $S-1$

$$w_{S-1}\left(w_{S-2}\right)=\frac{a+n_{S-1}w_{S-2}}{n_{S-1}+1}$$

From which

$$\begin{aligned}
q_{i,S-1}\left(w_{S-2}\right) &= \frac{\left(a-w_{S-1}\right)}{\left(n_S+1\right)b}=\frac{\left(a-\frac{a+n_{S-1}w_{S-2}}{n_{S-1}+1}\right)}{\left(n_S+1\right)b} \\
&= \frac{n_{S-1}}{\left(n_{S-1}+1\right)}\frac{\left(a-w_{S-2}\right)}{\left(n_S+1\right)\cdot b}
\end{aligned}$$

$$\begin{aligned}
p_S\left(w_{S-2}\right) &= a-n_S\cdot b\frac{n_{S-1}}{\left(n_{S-1}+1\right)}\frac{\left(a-w_{S-2}\right)}{\left(n_S+1\right)b} \\
&= \frac{1}{\left(n_S+1\right)\left(n_{S-1}+1\right)}\left(a+an_S+an_{S-1}+n_Sn_{S-1}w_{S-2}\right) \\
&= \frac{1}{n_S+n_{S-1}+1+n_Sn_{S-1}}\left(\left(n_S+n_{S-1}+1\right)a+n_Sn_{S-1}w_{S-2}\right) \\
&= \frac{\left(n_S+1\right)\left(n_{S-1}+1\right)}{\left(n_S+1\right)\left(n_{S-1}+1\right)}w_{S-2}+\frac{\left(n_S+n_{S-1}+1\right)}{\left(n_S+1\right)\left(n_{S-1}+1\right)}\left(a-w_{S-2}\right) \\
&= w_{S-2}+\frac{\left(n_S+n_{S-1}+1\right)}{n_S+n_{S-1}+1+n_Sn_{S-1}}\left(a-w_{S-2}\right)
\end{aligned}$$

Remark 14 *The price is a weighted average of "a" the highest willigness to pay and the initial unit cost. The third line shows that the a component is more important the smaller n_S, n_{S-1}. I.e. the mark-up is less powerful the larger the number of firms in the production chain, confirming the "double-monopoly" problem.*

$$\begin{aligned}
\pi_{i,S-1} &= \left(p_S-w_{S-1}\right)q_{i,S} \\
&= \left(w_{S-2}+\frac{\left(n_S+n_{S-1}+1\right)}{n_S+n_{S-1}+1+n_Sn_{S-1}}\left(a-w_{S-2}\right)-\frac{a+n_{S-1}w_{S-2}}{n_{S-1}+1}\right)\cdot\frac{n_{S-1}}{\left(n_{S-1}+1\right)}\frac{\left(a-w_{S-2}\right)}{\left(n_S+1\right)b} \\
&= \frac{1}{b}\left(\frac{n_{S-1}}{n_S+1}\right)^2\frac{\left(a-w_{S-2}\right)^2}{\left(n_{S-1}+1\right)^2}=WRONG
\end{aligned}$$

$$\begin{aligned}
\pi_{i,S-1} &= \left(w_{S-1}-w_{S-2}\right)q_{i,S}\left(w_{S-2}\right) \\
&= \left(\frac{a+n_{S-1}w_{S-2}}{n_{S-1}+1}-w_{S-2}\right)\frac{n_{S-1}}{\left(n_{S-1}+1\right)}\frac{\left(a-w_{S-2}\right)}{\left(n_S+1\right)b} \\
&= \frac{1}{b}\frac{n_{S-1}}{n_S+1}\frac{\left(a-w_{S-2}\right)^2}{\left(n_{S-1}+1\right)^2}
\end{aligned}$$

9.1.3 Stage S-2

$$\begin{aligned}
\max\pi_{i,S-2} &= \left(w_{S-2}-w_{S-3}\right)q_{i,S} \\
&= \left(w_{S-2}-w_{S-3}\right)\frac{n_{S-1}}{\left(n_{S-1}+1\right)}\frac{\left(a-w_{S-2}\right)}{\left(n_S+1\right)b}
\end{aligned}$$

Cournot Price

$$w_{S-2}\left(w_{S-3}\right)=\frac{a+n_{S-2}w_{S-3}}{\left(n_{S-2}+1\right)}$$

$$
\begin{aligned}
q_{i,S-2}\left(w_{S-3}\right) &= \frac{\left(a-w_{S-2}\right)}{\left(n_{S-1}+1\right)\left(n_{S}+1\right)b} = \frac{\left(a-\frac{a+n_{S-2}w_{S-3}}{\left(n_{S-2}+1\right)}\right)}{\left(n_{S-1}+1\right)\left(n_{S}+1\right)b} \\
&= \frac{n_{S-2}\left(a-w_{S-3}\right)}{\left(n_{S}+1\right)\left(n_{S-1}+1\right)\left(n_{S-2}+1\right)b}
\end{aligned}
$$

$$
\begin{aligned}
p_{S}\left(w_{S-3}\right) &= a-n_{S}\cdot b\frac{\left(a-w_{S-3}\right)}{\left(n_{S}+1\right)\left(n_{S-1}+1\right)\left(n_{S-2}+1\right)b} \\
&= \frac{1}{\left(n_{S}+1\right)\left(n_{S-1}+1\right)\left(n_{S-2}+1\right)} \\
&\quad *\left(n_{S}w_{S-3}+a+an_{S-1}+an_{S-2}+an_{S}n_{S-1}+an_{S}n_{S-2}+an_{S-1}n_{S-2}+an_{S}n_{S-1}n_{S-2}\right) \\
&= \frac{\left(n_{S}n_{S-1}+n_{S}n_{S-2}+n_{S-1}+n_{S-2}+n_{S-1}n_{S-2}+n_{S}n_{S-1}n_{S-2}+1\right)a+n_{S}w_{S-3}}{\left(n_{S}+1\right)\left(n_{S-1}+1\right)\left(n_{S-2}+1\right)} \\
&= w_{S-3}+\frac{\left(n_{S}n_{S-1}+n_{S}n_{S-2}+n_{S-1}n_{S-2}+n_{S}n_{S-1}n_{S-2}+n_{S-1}+n_{S-2}+1\right)}{\left(n_{S}+1\right)\left(n_{S-1}+1\right)\left(n_{S-2}+1\right)}\left(a-w_{S-3}\right)
\end{aligned}
$$

$$
\begin{aligned}
\pi_{i,S-2}\left(w_{S-3}\right) &= \left(w_{s-2}-w_{S-3}\right)q_{i,S}\left(w_{S-3}\right) \\
&= \left(\frac{a+n_{S-2}w_{S-3}}{\left(n_{S-2}+1\right)}-w_{S-3}\right)\cdot\frac{n_{S-2}\left(a-w_{S-3}\right)}{\left(n_{S}+1\right)\left(n_{S-1}+1\right)\left(n_{S-2}+1\right)b} \\
&= \frac{1}{b}\frac{\left(n_{S}n_{S-1}+n_{S}n_{S-2}+n_{S-1}+n_{S-2}+n_{S-1}n_{S-2}+n_{S}n_{S-1}n_{S-2}+1\right)}{\left(n_{S}+1\right)^{2}\left(n_{S-1}+1\right)^{2}\left(n_{S-2}+1\right)^{2}}\left(a-w_{S-3}\right)^{2} \\
&= \frac{n_{S-2}}{\left(n_{S}+1\right)\left(n_{S-1}+1\right)}\cdot\frac{1}{b}\frac{\left(a-w_{S-3}\right)^{2}}{\left(n_{S-2}+1\right)^{2}}
\end{aligned}
$$

9.2 Sequence

9.2.1 Quantities

$$
\begin{aligned}
q_{i,S}\left(w_{S-1}\right) &= \frac{\left(a-w_{S-1}\right)}{\left(n_{S}+1\right)b} \\
q_{i,S-1}\left(w_{S-2}\right) &= \frac{n_{S-1}}{\left(n_{S}+1\right)}\frac{\left(a-w_{S-2}\right)}{\left(n_{S-1}+1\right)\cdot b} = \\
&= \frac{n_{S-1}}{\left(n_{S-1}+1\right)}\frac{\left(a-w_{S-2}\right)}{\left(n_{S}+1\right)\cdot b} = \frac{n_{S-1}}{\left(n_{S-1}+1\right)}\cdot q_{i,S-1}^{I}\left(w_{S-2}\right) \\
q_{i,S-2}\left(w_{S-3}\right) &= \frac{n_{S-2}}{\left(n_{S}+1\right)\left(n_{S-1}+1\right)}\frac{\left(a-w_{S-3}\right)}{\left(n_{S-2}+1\right)b} = \\
&= \frac{n_{S-2}}{\left(n_{S-1}+1\right)\left(n_{S-2}+1\right)}\cdot\frac{\left(a-w_{S-3}\right)}{\left(n_{S}+1\right)b} = \frac{n_{S-2}}{\left(n_{S-1}+1\right)\left(n_{S-2}+1\right)}\cdot q_{i,S-1}^{I}\left(w_{S-3}\right)
\end{aligned}
$$

$$
q_{i,S-t} = \frac{n_{S-t}}{\prod_{\tau=1}^{t}\left(n_{S-\tau}+1\right)}\cdot\frac{\left(a-w_{S-t}\right)}{\left(n_{S}+1\right)b} = \frac{n_{S-t}}{\prod_{\tau=1}^{t}\left(n_{S-\tau}+1\right)}\cdot q_{i,S-t+1}^{I}\left(w_{S-t}\right)
$$

9.2.2 Prices

$$
\begin{aligned}
p_{S}\left(w_{S-1}\right) &= \frac{a+n_{S}w_{S-1}}{n_{S}+1} = w_{S-1}+\frac{a-w_{S-1}}{n_{S}+1} = \\
&= \frac{a+n_{S}\frac{a+n_{S-1}w_{S-2}}{n_{S-1}+1}}{n_{S}+1} \\
&= \frac{1}{\left(n_{S}+1\right)\left(n_{S-1}+1\right)}\left(a+an_{S}+an_{S-1}+n_{S}n_{S-1}w_{S-2}\right)
\end{aligned}
$$

$$
\begin{aligned}
w_{S-1}\left(w_{S-2}\right) &= \frac{a+n_{S-1}w_{S-2}}{n_{S-1}+1} = \\
w_{S-2}\left(w_{S-3}\right) &= \frac{a+n_{S-2}w_{S-3}}{\left(n_{S-2}+1\right)} =
\end{aligned}
$$

$$
\begin{aligned}
p_S\left(w_{S-3}\right) &= \frac{a+n_S\frac{a+n_{S-1}\frac{a+n_{S-2}w_{S-3}}{\left(n_{S-2}+1\right)}}{n_{S-1}+1}}{n_S+1} \\
&= \frac{\left(a+an_S+an_{S-1}+an_{S-2}+an_Sn_{S-1}+an_Sn_{S-2}+an_{S-1}n_{S-2}+n_Sn_{S-1}n_{S-2}w_{S-3}\right)}{\left(n_S+1\right)\left(n_{S-1}+1\right)\left(n_{S-2}+1\right)} \\
&= \frac{\left(1+n_S+n_{S-1}+n_{S-2}+n_Sn_{S-1}+n_Sn_{S-2}+n_{S-1}n_{S-2}\right)\cdot a+n_Sn_{S-1}n_{S-2}\cdot w_{S-3}}{\left(n_S+1\right)\left(n_{S-1}+1\right)\left(n_{S-2}+1\right)} \\
&= \frac{7a+1w}{8} \\
&= \frac{\left(1+300+3\cdot 100\cdot 100\right)10+\left(100^3\right)\cdot 1}{101^3}
\end{aligned}
$$

$$
\begin{aligned}
w_{S-1}\left(w_{S-2}\right) &= \frac{a+n_{S-1}w_{S-2}}{n_{S-1}+1} = \frac{a+n_{S-1}\frac{a+n_{S-2}w_{S-3}}{\left(n_{S-2}+1\right)}}{n_{S-1}+1} \\
&= \frac{\left(1+n_{S-1}+n_{S-2}\right)a+\left(n_{S-1}n_{S-2}\right)w_{S-3}}{\left(n_{S-1}+1\right)\left(n_{S-2}+1\right)} \\
&= w_{S-3}+\frac{\left(1+n_{S-1}+n_{S-2}\right)}{\left(n_{S-1}+1\right)\left(n_{S-2}+1\right)}\left(a-w_{S-3}\right) \\
w_{S-2}\left(w_{S-3}\right) &= \frac{a+n_{S-2}w_{S-3}}{\left(n_{S-2}+1\right)}
\end{aligned}
$$

Proposition 15 *If all firms are symmetric and the initial cost is the same for them all, the final price is not affected by the "position of the market power" along the production chain. The effect is symmetric.*

$$
p_S\left(w_{S-t}\right) = \frac{a\left(\prod_{\tau=0}^{t-1}\left(n_{S-\tau}+1\right)-\prod_{\tau=0}^{t-1}n_{S-\tau}\right)+w_{S-t}\cdot\prod_{\tau=0}^{t-1}n_{S-\tau}}{\prod_{\tau=0}^{t-1}\left(n_{S-\tau}+1\right)} \tag{23}
$$

9.2.3 Profits

$$
\begin{aligned}
q_{i,S}\left(w_{S-3}\right) &= \frac{\left(a-w_{S-1}\right)}{\left(n_S+1\right)b} \\
&= \frac{1}{b}\frac{a-w_{S-3}}{n_S+1}\frac{n_{S-1}\cdot n_{S-2}}{\left(n_{S-1}+1\right)\left(n_{S-2}+1\right)}
\end{aligned}
$$

$$
q_S^*\left(w_{S-t}\right) = \frac{1}{b}\frac{\prod_{\tau=1}^{t-1}n_{S-\tau}}{\prod_{\tau=0}^{t-1}\left(n_{S-\tau}+1\right)}\left(a-w_{S-\tau}\right)
$$

$$
\begin{aligned}
\pi_S\left(w_{S-3}\right) &= \left[p_S\left(w_{S-3}\right)-w_{S-3}\right]q_S\left(w_{S-3}\right) \\
&= \frac{1}{b}\frac{\left(a-w_{S-3}\right)^2}{\left(n_S+1\right)^2}\frac{n_{S-1}n_{S-2}}{\left(n_{S-1}+1\right)^2\left(n_{S-2}+1\right)^2}\left(n_S+n_{S-1}+n_{S-2}+n_Sn_{S-1}+n_Sn_{S-2}+n_{S-1}n_{S-2}+1\right)
\end{aligned}
$$

$$
\pi_S^*\left(w_{S-\tau}\right) = \left(\frac{1}{b}\prod_{\tau=0}^{t-1}\left(n_{S-\tau}+1\right)^2-\prod_{\tau=1}^{t-1}n_{S-\tau}\right)\frac{\prod_{\tau=1}^{t-1}n_{S-\tau}}{\prod_{\tau=0}^{t-1}\left(n_{S-\tau}+1\right)^2}\left(a-w_{S-\tau}\right)^2
$$

10 Conclusions

In this paper, in a production chain producing a homogeneous good with homogeneous inputs at each stage of production, we have computed the mark-up and the pass-through component that upstream and downstream firms apply on their costs to determine their output price and generalized it to any number of firms existing at any number of production stages.

Results are mainly the following, where the first two are just a confirmation of the two stages results, the third is original:

1. Mark-up vanishes in the increasing number of firms, at any production stage.
2. Pass-through increases in the number of firms, existing at each production stage, because competition tranfers cost increases.
3. Results are indifferent to the stage at which the larger number of firms happen. In other words, results do not discount the position in the sequence.

Equation (23) is the algorithm able to generate any price sequence in a production chain starting from the initial cost an pricing the evolution of it. It is therefore able to generate the price differential generated by each stage of production, due to the number of existing firms at that stage.

The Classical comparisons between integrated and decentralized chain have been computed with computed consequences on welfare results.

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