Prizes versus Contracts as Incentives for Innovation*

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Abstract

The procurement of an innovation involves motivating a research effort to generate a new idea and then implementing that idea efficiently. If research efforts are unverifiable and implementation costs are private information, a trade-off arises between the two objectives. The optimal mechanism resolves the tradeoff via two instruments: a monetary prize and a contract to implement the project. The optimal mechanism favors the innovator in contract allocation when the value of innovation is above a certain threshold, and handicaps the innovator in contract allocation when the value of innovation is below that threshold. A monetary prize is employed as an additional incentive but only when the value of innovation is sufficiently high.

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1 Introduction

When buyers have specific needs that the products and services available on the market cannot readily satisfy, they may procure an innovation. This is, however, considerably more involved than procuring a readily available product. While the latter process can simply focus on identifying the most qualified provider, the former involves identifying and motivating potential innovators to engage in costly research to generate an idea and subsequently identifying the best implementor (which may or may not be the innovator). Questions then arise as to how to incentivize potential innovators. Should the buyer offer a prize? Or should the buyer favor innovators when awarding the contract to implement the project?

A lack of consensus on this issue is apparent in the treatment of unsolicited proposals. According to Hodges and Dellacha (2007), public procurement authorities in most countries would consider unsolicited proposals, but they follow different approaches in rewarding the proposers. In some countries, public authorities are explicitly prevented from directly rewarding unsolicited project proposals; hence, the incentive to submit a project proposal can only come from participation in the tender for its implementation should the public authority decide to pursue the project. By contrast, other countries, such as Chile, Korea, Italy, and Taiwan, have adopted specific procedures for unsolicited proposals that grant the proposer an advantage at the implementation stage.

A similar lack of consensus is found in the public procurement of innovation. For example, Europe is currently developing an explicit strategy of using public procurement to foster demand for innovative goods and services, but how contract rights should be assigned remains an open question. In this regard, the European Commission has outlined two different procurement models. Under the "Pre-commercial procurement" (PCP) model, the public authority procures R&D activities from the solution exploration phase to prototyping and testing, but it reserves the right to tender competitively the newly developed products or services.[1] By contrast, under the "Innovation Partnerships" model, research and production are procured through one single tender, with the innovator also obtaining the contract rights over the production of the innovation.[2]

To study the interplay between *ex ante* R&D incentives and *ex post* implementation efficiency, we develop a framework where an innovation can be of value to a single buyer. Innovations that increase the quality or reduce costs of public services can have high degree of specificity because of the public good nature of the good or service to which the innovation

[1] See EC (2007) and https://ec.europa.eu/digital-agenda/en/pre-commercial-procurement
[2] See EU (2014a) at paragraph 49 and Article 31.

is targeted (e.g., or traffic management systems). Innovations that improve the functionality of parts with product-specific design also fall within this category.

We first consider a baseline model in which a single innovator can generate an idea by investing in a costly research effort: higher research effort leads (stochastically) to better ideas. Not only does this model clearly reveal our insights, but the single-innovator assumption is also often relevant as many innovative projects procured by public agencies are unsolicited, and arrive at a given point in time. The innovation gives rise to a project that can be implemented by a number of firms, including the innovator. The value of the proposal is verifiable and the innovator can thus be rewarded with a monetary prize as well as through the contract for the implementation of the project.

As monetary prizes have no impact on allocative efficiency, they may appear to be superior to contract rights for providing incentives for innovation. Indeed, prizes would be preferable in the absence of any agency problem at the implementation stage. The optimal mechanism would then reward the innovator with a monetary prize when the buyer finds its project worthy of pursuing, but the right to implement the project would be based purely on the implementors' merits. We show that the situation is different, however, when agency problems generate some rents at the implementation stage, as these rents could then be used to motivate the innovator at no additional cost to the buyer; in this case, contract rights can be a central tool for providing incentives for innovation.

To capture this, we suppose that potential implementors are privately informed about their costs. The optimal mechanism then provides incentives for research effort through a combination of monetary prizes and implementation contract rights. The intuition is simple: a standard procurement auction would pick the implementor with the lowest cost but would also give it an information rent; it is instead optimal to use this rent to incentivize the innovator. Hence, when the proposal is of sufficient merit, the innovator is favored in the implementation tender and may thus win the contract even if its cost is not the lowest. Furthermore, the project may be implemented even when, because of high costs, it would not be implemented in a standard procurement auction. Conversely, the optimal mechanism biases the implementation tender against the innovator when the project has a low value. Monetary prizes, however, remain valuable: if the project is of particular merit, the innovator may receive a financial compensation in addition to being favored in the implementation tender. Finally, whenever such a prize is awarded, it is awarded regardless of whether or not the innovator eventually wins the contract.

Comparative statics reflect the same insights. When information rents are significant, it can be optimal for the buyer to rely solely on contract rights to incentivize innovation.

By contrast, the optimal mechanism involves a prize (that is, contract rights alone are not sufficient) when costs are not very heterogeneous or when the number of firms is high (as the innovator's information rent is low) or when the value of innovation is high (as the information rent may be insufficient to motivate effort).

Equipped with these findings, we extend the model to allow for multiple innovators. This situation is relevant when the buyer has a clear sense about the type of innovation she needs and its feasibility. We show that the above insights carry over. First, our analysis confirms the optimality of the use of contract rights for rewarding innovations: the project values affect the optimal allocation of contract rights, with the proposer of a high-value project being favored at the implementation stage. Second, a "winner-takes-all" principle holds, in the sense that at most one project is awarded a prize; this monetary prize is now warranted when the project is particularly valuable and/or when an innovator's research effort is particularly worth incentivizing.

Our analysis also clarifies whether the selection of the project and the choice of the implementor should be independent or joint decisions. When the choice of the project does not affect implementors costs, the project can be selected independently of the choice of the implementor; a project is then chosen based solely on its merit, without regard to which firm will implement it. Still, as in the single-innovator case, the choice of the implementor remains biased in favor of or against the innovator, depending on the value of its proposal. When instead the choice of the project affects implementors' costs, it can be optimal to distort the project selection as well; in this case, both the project selection and the choice of implementor should depend on the values of the proposals as well as on implementation costs.

The paper is organized as follows. In Section 2, we discuss the related literature. In Section 3, we study the case of a single innovator. Section 3.1 establishes the model, Section 3.2 presents a number of benchmarks, and section 3.3 develops the main analysis. In Section 4, we extend the analysis to the case of multiple innovators. In Section 5, we discuss the insights that our analysis offers for the approaches used in practice for unsolicited proposals and innovation procurement. In Section 6, we make some concluding remarks.

2 Related Literature

On prizes versus property rights to motivate innovation. Our issue of prize vs. contract is reminiscent of the well-known debate about patent systems as an effective method of motivating innovation – see Maurer and Scotchmer (2004) and Cabral *et al.* (2006) for reviews. Just as in our model, the patent system involves *ex post* distortion (in terms of both too little quantity and foreclosure on competing firms), making prizes apparently preferable – see, e.g., Kremer, 1998. Yet the literature has shown that, as in this paper, *ex post* distortion can be an optimal way to motivate *ex ante* innovation. The difference lies in the motivation for the *ex post* distortion. In the case of Weyl and Tirole (2012), for instance, the supplier has private information at the *ex ante* (innovation) stage; he obtains property rights to facilitate information revelation. In our case, the private information in the *ex post* implementation stage, coupled with limited liability, forces the buyer to give up rents to the winning supplier. These rents can be harnessed as incentives for innovation, but only when the allocation of the contract rights is shifted in favor the innovator. That is, the distortion in the allocation of contracts rights arises a way to incentivize innovation.

On bundling sequential tasks. Our analysis is related to the literature on whether two tasks should be allocated to the same agent ("bundling") or to two different agents ("unbundling"). The existing literature finds that this choice can be driven by problems of adverse selection (see, e.g., Ghatak, 1997; Armendariz and Gollier, 1998), monitoring (Besley and Coate, 1995; Armendariz, 1999; Rai and Sjöström, 2004), moral hazard (Stiglitz, 1990; Varian 1990; Holmstrom and Milgrom, 1991; Itoh, 1993), or agents' limited liability (Laffont and Rey, 2003). More recently, a second strand of literature has focused more specifically on sequential tasks, highlighting the role of externalities across tasks (Bennett and Iossa, 2006), budget constraints (Schmitz, 2013), information on the *ex post* value of the second task (Tamada and Tsai, 2007), or competition among agents (Li and Yu, 2011). Our paper contributes to this literature by showing that the implementation decision should depend on the value of the proposed project(s) as well as on the implementor's characteristics. Full unbundling is therefore typically not optimal, while pure bundling is optimal only under rather specific conditions – namely, when the innovator is in a much better position to implement its project.

On discrimination and bidding parity in auctions. Our analysis is also related to the literature on discrimination in auctions, which finds it optimal to distort the allocation to reduce the information rents accruing to the bidders: discrimination against efficient types helps level the playing field and elicit more aggressive bids from otherwise stronger bidders (Myerson, 1981, and McAfee and McMillan, 1985). In a similar vein, when bidders can invest in cost reduction, an *ex post* bias in the auction design can help to foster bidders' *ex ante* investment incentives (Bag 1997) or to prevent the reinforcement of asymmetry among market participants (Arozamena and Cantillon, 2004). Likewise, manipulating the auction rules can help motivate selfish investment in cost reduction by an incumbent firm (Laffont and Tirole, 1988) or favor the adoption of an efficient technology by an inefficient firm (Branco, 2002). We contribute to this literature by showing that when investment is "cooperative" (in the sense of Che and Hausch, 1999) and directly benefits the buyer, both favoritism and handicapping are optimal, depending on the value of the proposed project and on the bidders' costs.

On relational contracting. A large literature shows how long-term relations can be used to build trust and spur efforts and innovation. Interestingly, in a recent paper, Calzolari *et al.* (2015) emphasize that trust and rents from reduced supplier competition provide substitute ways of encouraging innovation. Using a large dataset on the German car manufacturing industry, they find that higher levels of trust are associated not only with higher investment levels, but also with more competitive procurement.

3 Unsolicited Proposals

We consider here the case in which a single innovator may propose a project, as is often the case with unsolicited proposals. The decision facing the buyer is whether to adopt the project and, if so, how to implement it via competition among multiple firms.

3.1 Model

A principal (buyer) oversees a project involving two stages: Innovation and Implementation.

In the first stage, an innovator, say firm 1, invests in developing the project proposal: exerting effort $e \geq 0$ costs the innovator $c(e)$ and leads to a project whose value for the principal is a random variable v. We assume that $c(\cdot)$ is increasing, strictly convex and twice differentiable, and satisfies $c'(0) = 0$. The variable v is drawn from $V := [\underline{v}, \bar{v}]$ according to a c.d.f. $F(\cdot|e)$, which admits a twice-differentiable density $f(\cdot|e)$ in the interior. We assume that raising e shifts the distribution $F(\cdot|e)$ in the sense of a Monotone Likelihood Ratio Property:

$$\frac{f(v'|e')}{f(v|e')} > \frac{f(v'|e)}{f(v|e)}, \text{ for any } v' > v \text{ and } e' > e. \tag{$MLRP$}$$

We assume that although the effort choice e by the innovator is unobservable to all parties other than firm 1, the value v of the resulting project is publicly observable and verifiable, once the project proposal is unveiled. The verifiability of v is a reasonable assumption in many procurement contexts, where projects can be described using precise functional and performance terms. For example, in the case of technology improvements for faster medical tests, transport units with lower energy consumption, or information and communication technology (ICT) systems with interoperability characteristics, v may respectively capture the speed increase for the medical test, the degree of energy efficiency of the transport unit, or the technical functionalities of the ICT system verified in submitted prototypes.

In the second stage, n potential firms, including the innovator, compete to implement the project. Each firm $i \in N := \{1, ..., n\}$ faces a cost θ_i, which is privately observed and drawn from $\Theta := [\underline{\theta}, \overline{\theta}]$ according to a c.d.f. $G_i(\cdot)$, which admits density $g_i(\cdot)$ in the interior. We assume that $\underline{\theta} < \overline{v}$ and $G_i(\theta_i)/g_i(\theta_i)$ is nondecreasing in θ_i, for each $i \in N$.

Suppose that the innovator generates an innovation worth v. If the project is not implemented, all parties obtain zero payoff. If instead the principal pays t for the project, then the principal internalizes the welfare of

$$v - t.$$

By the revelation principle, we can formulate the problem facing the principal as that of choosing a direct revelation mechanism. A direct mechanism is denoted by: $(x, t) : V \times \Theta^n \to \Delta^n \times \mathbb{R}^n$, which specifies the probability $x_i(v, \theta)$ that firm i implements the project and the transfer payment $t_i(v, \theta)$ that it receives, when the project developed by firm 1 has value v and firms report types $\theta := (\theta_1, ..., \theta_n)$, where $\Delta^n := \{(x_1, ..., x_n) \in [0, 1]^n | \sum_{i \in N} x_i \in [0, 1]\}$. The dependence of the mechanism on the project value v reflects its verifiability, whereas the absence of the argument e arises from its unobservability to the principal.

For each $v \in V$, let

$$u_i(v, \theta_i' | \theta_i) := \mathbb{E}_{\theta_{-i}} \left[t_i(v, (\theta_i', \theta_{-i})) - \theta_i x_i(v, (\theta_i', \theta_{-i})) \right]$$

denote the *interim* expected profit that firm i could obtain by reporting a cost θ_i' when it actually faces a cost θ_i, and let

$$U_i(v, \theta_i) := u_i(v, \theta_i | \theta_i)$$

denote firm i's expected payoff under truthful revelation when its type is θ_i.

Then, the revelation principle requires the direction mechanism (x, t) to satisfy *incentive compatibility:*

$$U_i(v, \theta_i) \geq u_i(v, \theta_i' | \theta_i), \quad \forall i \in N, \forall v \in V, \forall (\theta_i, \theta_i') \in \Theta^2. \qquad (IC)$$

The timing of the game is as follows:

1. The principal offers a direct revelation mechanism specifying the allocation decision (i.e., whether the project will be implemented and, if so, by which firm) and a payment to each firm, as functions of firms' reports on their costs.

2. The innovator chooses e; the value v is realized and observed by all parties.

3. Firms observe their costs and decide whether to participate.

4. Participating firms report their costs, the project is implemented (or not), and transfers are made according to the mechanism.

Note that firms participate only after learning their cost. In particular, this means that the principal cannot force the firms to participate before the project is developed by the innovator. This is a natural assumption in many settings and particularly so in the case of unsolicited proposals because until the nature of the project—its value and the costs of implementing it—is determined, the identities of the candidates capable of executing the project will not be known. This makes it difficult for the principal to solicit the relevant firms and to force them to buy in. This feature requires the direct mechanism (x, t) to satisfy *individual rationality*:

$$U_i(v, \theta_i) \geq 0, \quad \forall i \in N, \forall v \in V, \forall \theta_i \in \Theta, \tag{IR}$$

It will be seen that, together with (IC), this requirement will cause the principal to leave information rents to the selected implementor.[3]

We also assume that the principal must at least break even for each realized value v of the project. In other words, a feasible mechanism (x, t) must satisfy *limited liability*:

$$\mathbb{E}_\theta \left[w \left(v, \theta \right) \right] \geq 0, \quad \forall v \in V, \tag{LL}$$

where

$$w \left(v, \theta \right) := \sum_{i \in N} \left[x_i \left(v, \theta \right) v - t_i \left(v, \theta \right) \right]$$

denotes the principal's surplus upon realizing the value v of the project. Public projects are scrutinized by various stake-holders such as legislative body, project evaluation authority, consumer advocacy groups, and media, which reject a project that is likely to run a loss.

[3]In the absence of the individual rationality constraint, the principal could implement the project without any rents accruing to the firms because the firms could be required to "buy-in" to a contract via an upfront fee. As a result, the first best could be achieved at the implementation stage, and there would be no gain from using contract rights to reward the innovator; monetary prizes would indeed be preferable.

Limited liability may arise from such a political feasibility constraint. It is not crucial that the constraint is of the particular form assumed in (LL); the general thrust of our analysis carries through as long as there there is some cap on either the maximum loss the principal can sustain or the maximum payment she can make to the firm.[4]

Finally, as the innovator chooses effort e in its best interest, the mechanism must also satisfy the following *moral hazard* condition:

$$e \in \arg\max_{\tilde{e}} \left\{ \mathbb{E}_{v,\theta} \left[U_1(v, \theta_1) \mid \tilde{e} \right] - c(\tilde{e}) \right\}. \tag{MH}$$

The principal's problem is to choose an optimal mechanism satisfying these constraints. More formally, she solves the problem:

$$[P] \qquad \max_{x,t,e} \qquad \mathbb{E}_{v,\theta} \left[w\left(v, \theta\right) \mid e \right],$$

$$\text{subject to } (IR), (IC), (LL) \text{ and } (MH)$$

3.2 Benchmarks

Before solving $[P]$, it is useful to begin with two benchmarks.

No adverse selection *ex post* In this benchmark, we shut off the adverse selection problem by assuming that the principal observes the firms' implementation costs. Formally, the problem facing the principal will then be the same as $[P]$, except that the constraint (IC) is absent. We label such the relaxed problem $[P - FB]$, where "FB" refers to first-best implementation efficiency. In this problem, once the principal approves the project, she can have any firm i implement the project by simply paying its true cost θ_i. In other words, the implementing firm does not command any information rents. As will be formally stated shortly, this feature implies that the principal finds no reason to rely on contracting rights to provide incentive for the innovator. Hence, to deal with the moral hazard problem (MH), the principal will solely rely on the monetary prize. The logic, which conforms to the conventional wisdom, is clear: monetary prize is distortion-free, while contracting rights do involve distortion.

Thus, the solution to $[P - FB]$ is characterized as follows:

[4]Without any constraint on the maximum payment the principal can make to the firm, the optimal mechanism would not be well defined: the principal would find it desirable to pay an arbitrarily large bonus to the innovator only for a vanishingly set of projects with values close to $\overline{v}$. Such a scheme may be of theoretical interest but is unreasonable and unrealistic.

Proposition 1. *(First-Best) There exist λ^{FB} and e^{FB}, both strictly positive, such that the optimal mechanism solving $[P-FB]$ awards firm i a contract to implement the project with probability:*

$$x_i^{FB}(v,\theta) := \begin{cases} 1 & \text{if } \theta_i < \min\{v, \min_{j\neq i}\theta_j\}, \\ 0 & \text{otherwise,} \end{cases}$$

with a transfer that simply compensates the winning firm's cost, except that firm 1 is paid additionally a monetary prize equal to

$$\rho_1^{FB}(v) := \begin{cases} \mathbb{E}_\theta\left[\sum_{i\in N} x_i^{FB}(v,\theta)(v-\theta_i)\right] > 0 & \text{if } v > \hat{v}^{FB}, \\ 0 & \text{if } v < \hat{v}^{FB}, \end{cases}$$

where $\hat{v}^{FB}$ is such $\underline{v} < \hat{v}^{FB} < \bar{v}$ and solves $\beta^{FB}(v) = 1$, where

$$\beta^{FB}(v) := \lambda^{FB}\frac{f_e(v|e^{FB})}{f(v|e^{FB})},$$

and e^{FB} satisfies (MH).

Proof. See Appendix A. □

The manner in which the optimal mechanism awards the cash prize is intuitive, and follows the familiar logic from the moral hazard literature (e.g., Mirrlees (1975); Holmstrom (1979)). The realized project value v is an informative signal for the innovator's effort, and paying an additional dollar to the innovator for a project with value v relaxes (MH) by $\frac{f_e(v|e^{FB})}{f(v|e^{FB})}$. Multiplied with the shadow value λ^{FB} of relaxing (MH), $\beta^{FB}(v) = \lambda^{FB}\frac{f_e(v|e)}{f(v|e)}$ gives the *incentive benefit* to the principal by relaxing (MH) at the optimum. Naturally, the optimal mechanism calls for paying the maximal feasible prize to the innovator if $\beta^{FB}(v) > 1$ and zero prize otherwise. Given ($MLRP$), the incentive benefit $\frac{f_e(v|e^{FB})}{f(v|e^{FB})}$ is strictly increasing in v, so the threshold value $\hat{v}^{FB}$ is well-defined. Simply put, the optimal mechanism calls for paying as much as possible to the innovator whenever the project value v is high enough to indicate that the incentive benefit exceeds the cost, and nothing otherwise. In the former case, (LL) must be binding, so the maximal feasible prize is given by the net surplus the project generates after reimbursing the implementing firm.

In sum, given no adverse selection, the principle never uses contracting rights to motivate the innovating firm.

No moral hazard *ex ante* In this benchmark, we shut off the moral hazard problem by assuming that the project value follows some exogenous distribution $F(v)$ which does not depend on effort. Formally, the problem facing the principal in this benchmark coincides with $[P]$ except that the moral hazard constraint (MH) is absent and the distribution function $F(v|e)$ is replaced by some exogenous distribution $F(v)$. The resulting problem, labeled $[P-SB]$, conforms to the standard optimal auction design problem, except for the (LL) constraint. Ignoring the latter, the optimal auction solution, labeled **the optimal second-best mechanism**, is familiar from Myerson (1981). One can easily see that this solution satisfies (LL), and thus constitutes an optimal solution to $[P-SB]$ as well. As the associated analysis is standard, we provide the characterization of the optimal solution without a proof.

Proposition 2. *(Myerson) The optimal second-best mechanism awards firm i the contract to implement the project with probability:*

$$x_i^{SB}(v,\theta_i) := \begin{cases} 1 & \text{if } J_i(\theta_i) \leq \min\{v, \min_{j\neq i} J_j(\theta_j)\}, \\ 0 & \text{otherwise,} \end{cases}$$

where $J_i(\theta_i) := \theta_i + \frac{G_i(\theta_i)}{g_i(\theta_i)}$ is firm i's virtual cost.

3.3 Optimal Mechanism

We now consider problem $[P]$, in which the principal faces *ex post* adverse selection with respect to firms' implementation costs as well as an *ex ante* moral hazard problem with respect to the innovator's effort. Throughout the analysis, we assume that an optimal mechanism exists, which induces an interior effort level e^*. The following Proposition characterizes this optimal mechanism:

Proposition 3. *There exists $\lambda^* > 0$ such that the optimal mechanism solving $[P]$ is characterized as follows:*

(i) The mechanism assigns a contract to firm $i = 1, ..., n$ to implement the project with probability

$$x_i^*(v,\theta) = \begin{cases} 1 & \text{if } K_i^*(v,\theta_i) \leq \min\{v, \min_{j\neq i} K_j^*(v,\theta_j)\}, \\ 0 & \text{otherwise,} \end{cases}$$

where

$$K_i^*(v,\theta_i) := \begin{cases} J_i(\theta_i) - \min\{\beta^*(v), 1\} \frac{G_i(\theta_i)}{g_i(\theta_i)} & \text{if } i = 1 \\ J_i(\theta_i) & \text{if } i \neq 1 \end{cases}, \text{ with } \beta^*(v) := \lambda^* \frac{f_e(v|e^*)}{f(v|e^*)}.$$

(ii) The mechanism awards firm i a transfer

$$t_i^*(v,\theta_i) := \rho_i^*(v) + \int_{\theta_i}^{\overline{\theta}} X_i^*(v,s)ds,$$

where:

- *the second term reflects the information rent generated by the expected probability of contract assignment $X_i^*(v,\theta_i) = \mathbb{E}_{\theta_{-i}}[x_i^*(v,(\theta_i,\theta_{-i}))]$, and*
- *the first term corresponds to a "cash prize," which is zero for a non-innovator (i.e., $\rho_i^*(v) := 0$ for $i \neq 1$) and, for the innovator ($i = 1$), is equal to*

$$\rho_1^*(v) := \begin{cases} \mathbb{E}_\theta\left[\sum_{i\in N} x_i^*(v,\theta)\left[v - J_i(\theta_i)\right]\right] > 0 & \text{if } \beta^*(v) > 1, \\ 0 & \text{if } \beta^*(v) < 1. \end{cases}$$

(iii) The effort e^* satisfies $e^* > 0$ and

$$\int_v \int_\theta \left[\rho_1^*(v) + \frac{G_1(\theta)}{g_1(\theta)} x_1^*(v,\theta)\right] g(\theta)d\theta f_e(v|e^*)dv = c'(e^*).$$

Proof. See Appendix B. □

To gain more intuition about this characterization, it is useful to decompose the principal's payment to each firm into two components. The first component is the information rent that she must pay to elicit the firm's private information. By the standard envelope theorem argument, this component is uniquely tied to—and should therefore be interpreted as being necessitated by—the awarding of the contract to a firm. We thus call this *contract payment*. The second component is the constant amount paid to all cost types of a firm, including the highest cost type $\overline{\theta}$. As this component is not warranted by contract assignment, we call it the *cash prize* and denote it by $\rho_i^*(v)$. Obviously, the principal would never pay any cash prizes to non-innovating firms $i = 2, ..., n$. For the innovating firm $i = 1$, however, a cash prize may be necessary. The question is how the principal should combine these two types of payments to encourage innovation.

The key observation in answering this question hinges on the incentive benefit $\beta^*(v) = \lambda^* \frac{f_e(v|e)}{f(v|e)}$. As explained earlier, this term represents the value of paying a dollar to the innovator for developing a project worth v—more precisely, the effect $\frac{f_e(v|e)}{f(v|e)}$ of relaxing (MH) and its worth λ^* to the principal. If moral hazard were not a concern, then we would have $\lambda^* = 0$ and thus $\beta^*(v) = 0$, and the optimal mechanism would reduce to the optimal second-best auction mechanism described in Proposition 2. However, the above characterization shows that the moral hazard constraint is binding. The simple intuition is that, while the second-best auction gives away some information rents when the project is implemented and would thus induce the innovator to exert some effort (so as to increase the likelihood of implementation), these rents do not depend on the realized value of the project;[5] as a result, the procurer, who does not bear the cost of effort, wishes to induce a higher effort so as to increase the expected value (net of these rents) of the project.

Given $\lambda^* > 0$, the incentive benefit $\beta^*(\cdot)$ is nonzero, and the optimal mechanism departs from the optimal second-best mechanism. In particular, the contract assignment now depends on the realized value of the contract, through the shadow cost $K_i^*(v, \theta_i)$. For a non-innovating firm (i.e., $i \neq 1$), the shadow cost is the same as its virtual cost, $J_i(\theta_i)$, just as in the second-best benchmark. For the innovator, however, the shadow cost differs from its virtual cost by a term, $\beta^*(v)\frac{G_1(\theta_1)}{g_1(\theta_1)}$, which reflects the need to incentivize the innovator: Awarding the contract to the innovator with type θ by an additional unit of probability necessitates giving information rent to types below θ, paying $\frac{G_1(\theta_1)}{g_1(\theta_1)}$ in expectation to the innovator *ex ante*, and each dollar paid to the innovator yields the incentive benefit of $\beta^*(v)$.

Intuitively, rewarding the innovator for a low-value project (evidence of low effort) hurts innovation incentives, whereas rewarding the firm for a high-value project (evidence of high effort) enhances its incentive for innovation. Indeed, by (*MLRP*), $\beta^*(v) = \lambda^* \frac{f_e(v|e^*)}{f(v|e^*)}$ increases in v, and there exists a unique $\tilde{v} \in (\underline{v}, \overline{v})$ such that $\beta^*(\tilde{v}) = 0$. Thus, when $v < \tilde{v}$, rewarding the innovator reduces its innovation incentive: $\beta^*(v) < 0$. Hence, it is not optimal for the principal to award a cash prize to the innovator. For the same reason, each dollar paid as information rents harms the innovator's incentive, causing the shadow cost $K_1^*(v; \theta_1)$ of assigning the contract to the innovator to *exceed* its virtual cost $J_1(\theta_1)$, by $-\beta^*(v) G_i(\theta_i)/g_i(\theta_i)$ (> 0). Hence, the optimal mechanism calls for biasing the contract allocation against the innovator in comparison with the second-best.

When instead $v > \tilde{v}$, there are two possibilities. If $v < \hat{v} := \sup\{v \in V \mid \beta^*(v) \leq 1\}$, then the incentive benefit $\beta^*(v)$ of paying a dollar to the innovator is positive but less than one. Hence, it is still optimal to award no cash prize, as this would entail a net loss for the principal. However, a fraction $\beta^*(v)$ of the information rent accruing to the innovator goes toward its innovation incentive, which *reduces* the shadow cost $K_1^*(v, \theta_1)$ of assigning the contract to the innovator below its virtual cost $J_1(\theta_1)$ by $\beta^*(v) G_i(\theta_i)/g_i(\theta_i)$. Hence, compared with the second-best benchmark, the optimal mechanism distorts allocation of the contract in favor of the innovator.

[5] That is, increasing the value of the project increases the probability of its implementation, but does not affect the rents obtained when it is actually implemented.

If instead $v > \hat{v}$, then a dollar payment to the firm yields more than a dollar incentive benefit. A cash prize is then clearly beneficial, which is why $\rho_1^*(v) > 0$. Hence, it pays the principal to transfer any surplus she collects, either through the cash prize or through the information rent; that is, (LL) is binding. Furthermore, any increase in information rents for the innovator simply crowds out the cash prize by an equal amount; it follows that the incentive benefit of the information rent paid to the innovator is at most one 1 (and not $\beta^*(v) > 1$), and the shadow cost $K_1^*(v, \theta_1)$ reduces to the production cost θ_1. That is, compared with the second-best, the optimal mechanism distorts allocation of the contract in favor of the innovator to such an extent that the innovator is treated as an "in-house" supplier. Any further distortion in favor of the innovator reduces the total "pie" – and thus the cash prize to the innovator – more than it increases the information rent to that firm, and it is thus suboptimal.

We state these observations more formally as follows:

Corollary 1. *There exists $\tilde{v}$ and $\hat{v}$, with $\underline{v} < \tilde{v} < \hat{v} \leq \overline{v}$, such that the optimal mechanism has the following characteristics:*

- *If $v < \tilde{v}$, then no prize is awarded and $x_1^*(v, \theta) \leq x_1^{SB}(v, \theta)$, whereas $x_i^*(v, \theta) \geq x_i^{SB}(v, \theta)$ for all $i \neq 1$;*
- *If $\tilde{v} < v < \hat{v}$, then no prize is awarded but $x_1^*(v, \theta) \geq x_1^{SB}(v, \theta)$, whereas $x_i^*(v, \theta) \leq x_i^{SB}(v, \theta)$ for all $i \neq 1$;*
- *If $v > \hat{v}$, then a prize is awarded to the innovator and $x_1^*(v, \theta) \geq x_1^{SB}(v, \theta)$, whereas $x_i^*(v, \theta) \leq x_i^{SB}(v, \theta)$ for all $i \neq 1$.*

Whether it is optimal to award a monetary prize (i.e., $\hat{v} < \bar{v}$) depends on how much efforts need to be incentivized and on how much incentive would already have been provided by the information rents stemming from a standard second-best auction. For instance, we show in Appendix C that awarding a prize can be optimal when the range of project values is large (as innovation incentives then matter a lot) as well as when there is either little cost heterogeneity or a large number of firms (in which case the procurement auction does not generate much in information rents).

Corollary 1 shows that the optimal mechanism departs from a standard second-best auction in different ways for high-value and low-value projects. This mechanism can moreover be easily implemented in practice using simple variants to common procurement designs.

- $v > \tilde{v}$: *Bonus.* In this range, the contract allocation is biased in favor of the innovator, so the innovator may implement the project despite not being the most efficient firm. In practice, this could be achieved by giving the innovator a bidding credit in the tendering procedure. Bidding credits can take many forms, but most commonly, they consist of additional points in the score of the original proponent's bid or of financial support for bidding purposes. This system is, for example, adopted in Chile and Korea.

- $v < \tilde{v}$: *Handicap.* In this range, the contract allocation is biased against the innovator, who may not implement the project despite being the most efficient firm. We are not aware of the use of such a bias for procuring innovative projects; however, such handicap systems are used, for example, when governments want to favor domestic industries.[6] We discuss this further below (see the remark on handicaps).

We note further that in the region where a monetary prize is optimal, the mechanism can be implemented in a very familiar and simple manner:

- $v > \hat{v}$: *Full delegation.* In this region, the innovator is awarded a monetary prize $\rho_1^*(v)$ equal to the full value of the project (net of information rents) and is allocated the contract if $\theta_1 < \min\{v, \min_{i\neq 1} J_i(\theta_i)\}$. This can be achieved by delegating the procurement to the innovator for a fixed price equal to the value of the project. Indeed, suppose that the principal offers a payment v to the innovator to deliver the project either by itself or by subcontracting with a different firm. The innovator then acts as a prime contractor with the authority to assign production. Facing the price $v > \hat{v}$ and given θ_1, the innovator chooses $(x(v,\cdot), t(v,\cdot)) : \Theta^n \to \Delta \times \mathbb{R}^{n-1}$ so as to solve:

$$\max_{x,t} \ \mathbb{E}_{\theta_{-1}} \left[(v-\theta_1)x_1(v,\theta_1,\theta_{-1}) + \textstyle\sum_{i\neq 1} \{ v x_i(v,\theta_1,\theta_{-1}) - t_i(v,\theta_1,\theta_{-1}) \} \right],$$

subject to (IR) and (IC)

The standard procedure of using the envelope theorem and changing the order of integration results in the optimal allocation x solving

$$\max_{x,t} \mathbb{E}_{\theta_{-1}} \left[(v-\theta_1)x_1(v,\theta_1,\theta_{-1}) + \sum_{i\neq 1} [v - J_i(\theta_i)]\, x_i(v,\theta_1,\theta_{-1}) \right],$$

which is exactly the allocation x^* for the case of $v > \hat{v}$.

[6] Under "preferential price margins", purchasing entities accept bids from domestic suppliers over foreign suppliers as long as the difference in price does not exceed a specific margin of preference. The price preference margin can result from an explicit "buy local policy," e.g., the "Buy America Act."

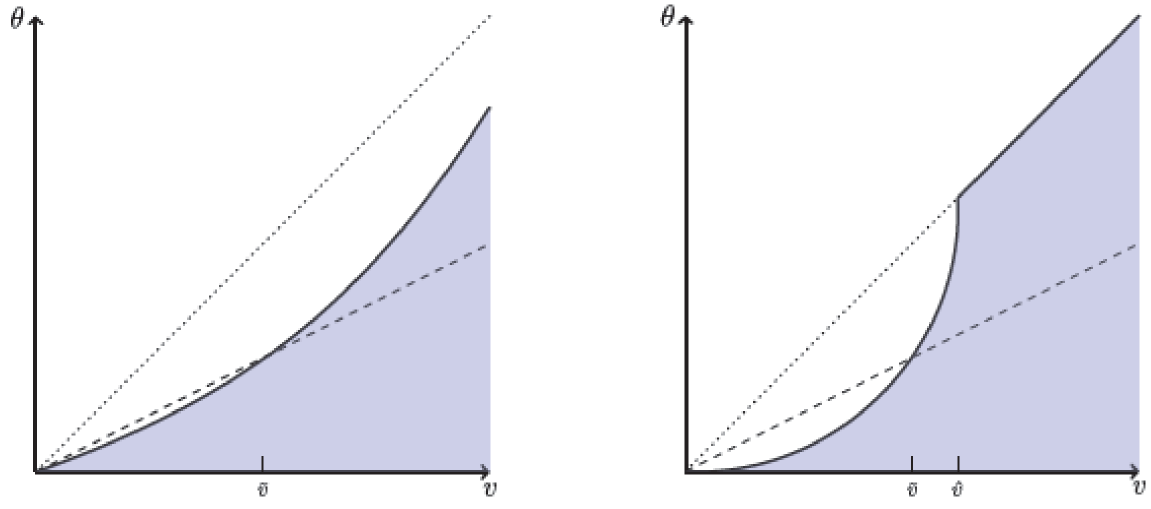
The above results also have implications for whether or not to implement a project. For instance, for $n = 1$, we have:

- For $v < \tilde{v}$, $K(v, \theta) > J(\theta)\,(> \theta)$: Compared with the first-best, there is a downward distortion – under-implementation of the project – which is even more severe than in the standard second-best.
- For $\tilde{v} < v < \hat{v}$, $J(\theta) < K(v, \theta) < \theta$: There is still a downward distortion compared with the first-best, but it is less severe than in the standard second-best.
- For $v \geq \hat{v}$, we have $J(\theta) < K(v, \theta) = \theta$: There is no distortion anymore; the project is implemented whenever it should be, from a first-best standpoint.

Illustration. To illustrate the above insights, consider the following example: (i) implementation costs are uniformly distributed over $\Theta = [0, 1]$; (ii) the innovator can exert an effort $e \in [0, 1]$ at cost $c(e) = \gamma e$; and (iii) the value v is distributed on $[0, 1]$ according to the density $f(v|e) = e + (1 - e)\,2\,(1 - v)$; that is, exerting effort increases value in the MLRP sense, from a triangular density peaked at $v = 0$ for $e = 0$ (where, in particular, $f(1|0) = 0$) to a better (in fact, uniform) distribution for $e = 1$. Note that $f_e(v|e) = 2v - 1 \gtrless 0 \iff v \gtrless \tilde{v} = 1/2$.

The linearity of the cost and benefits ensures that it is optimal to induce maximal effort ($e^* = 1$) as long as the unit cost γ is not too high. Conversely, as long as $e^* = 1$, the Lagrangian multiplier λ^* increases with the cost γ. For exposition purposes, we will use different values of λ^* (reflecting different values of γ) to illustrate the role of innovation incentives.

Consider first the case in which only the innovator can implement its project (i.e., $n = 1$). Figure 1 depicts the range of the firm's costs for which the project is implemented under optimal contract for different project values v. Figure 1-(a) depicts the case of $\lambda^* = 0.8$, whereas Figure 1-(b) shows the case of $\lambda^* = 4$. As the cost is uniformly distributed, the highest cost for which the project is implemented also equals the probability of the project being implemented, $p^*(v) := \mathbb{E}_\theta[x_1^*(v, \theta)]$. Compared with the second best, depicted by the dashed line, the optimal mechanism implements the project for a smaller range of project costs (thus with a lower probability) when the project has a low value (namely, when $v < \tilde{v} = 1/2$) and for a larger range of costs (thus with a higher probability) when the project has a high value ($v > \tilde{v}$). When $\lambda^* = 4$ (Figure 1-(b)), stronger innovation incentives are required and there is a range of values $v > \hat{v}$ (where $\hat{v} = 5/8 > \tilde{v}$) for which ($LL$) is binding: the procurer then exhausts the contracting as means of incentive and starts offering a cash prize. As noted, in such a case the optimal assignment coincides with the first-best, depicted by the dotted line.



(a) λ "small"

(b) λ "large"

Figure 1

Sticking to the case $\lambda^* = 4$ (where $\underline{v} = 0 < \tilde{v} < \hat{v} < 1$), Figure 2 illustrates the situation where a second firm can implement the project as well (i.e., $n = 2$), with the same cost distribution (i.e., firm 2's cost is also uniformly distributed over $\Theta = [0, 1]$):

- For $v < \tilde{v}$ (see Figure 2-(a), where $v = 1/4$): Compared with the first-best or the standard second-best, it is again optimal to bias the allocation of the contract against the innovator. This is now achieved in two ways, however. As before, the project is implemented less often than in the second-best (and thus, *a fortiori*, than in the first-best); the optimal mechanism shifts the vertical boundary of project implementation to the left of the second best (depicted by the dashed line). But in addition, when the project is implemented, the innovator obtains the contract less often than in the first-best or the standard second-best, where the more efficient implementor would be selected; graphically, this is reflected by the triangular shaded area.

- For $\tilde{v} < v < \hat{v}$ (see Figure 2-(b), where $v = 7/12$): Compared with the standard second-best (depicted by the dashed lines) it is now optimal to reward the innovator, both by implementing the project more often (rectangular shaded area) and by favoring the innovator in the competition with its rival (triangular shaded area).

- Finally, for $v \geq \hat{v}$ (see Figure 2-(c), where $v = 4/5$), the innovator's shadow cost then corresponds to its actual cost: The allocation of the contract thus favors the innovator even more over the other firm, and the project is also implemented substantially more often than in the standard second-best (in particular, it is now implemented whenever $\theta_1 < v$); still, it is implemented less often than in the first-best (e.g., when $\theta_2 < v < \theta_1$ and $J(\theta_2) = 2\theta_2$); graphically, the rectangular and triangular shaded areas further expand.

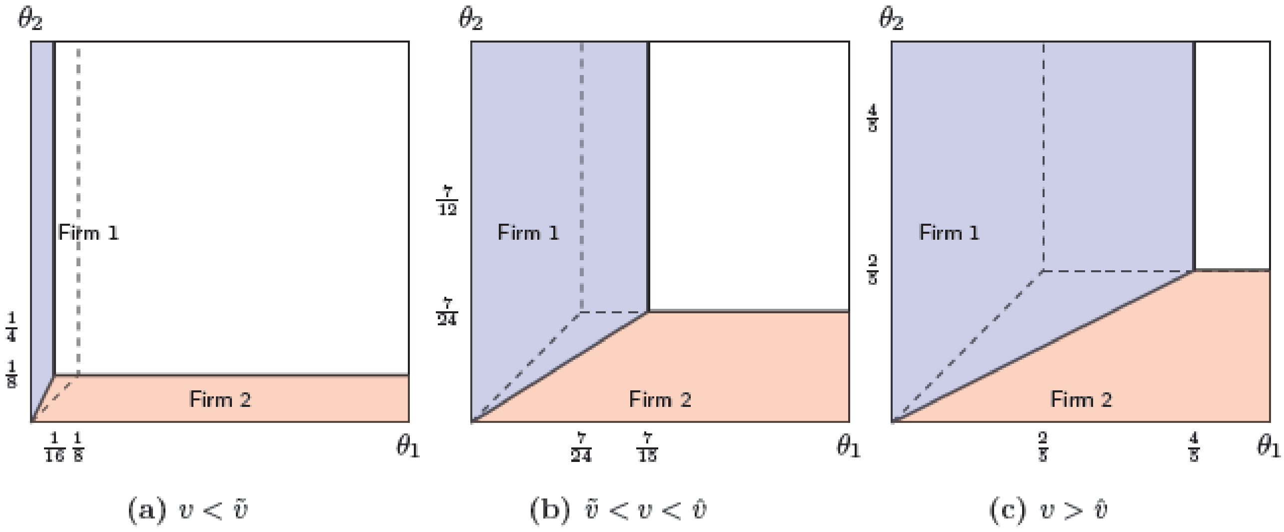


Figure 2

Remark: On the feasibility of handicaps. The optimal mechanism relies on a "stick and carrot" approach: it rewards good proposals by conferring an advantage in the procurement auction (possibly together with a monetary prize) and punishes weak proposals with a handicap in the procurement auction. Yet in practice, while many innovation procurement mechanisms involve innovation prizes or distort the contract allocation in favor of the innovators, handicaps for weak projects do not appear to be used. This may stem from the risk of manipulation: an innovator with a low-value project may, for instance, strategically choose to participate in the implementation tender through a different firm to avoid the handicap.

To see how the mechanism would need to be adjusted if handicaps were explicitly ruled out, suppose that the innovator cannot be left worse off than under the standard second best allocation.[7] That is, the mechanism must take into account the additional constraint:

$$x_1(v,\theta) > x_1^{SB}(v,\theta).$$

In Online Appendix A, we show that, keeping constant the multiplier λ for the innovator's incentive constraint, ruling out handicaps has no impact on the contract right for high-value projects (namely, those with $v \geq \tilde{v}$), as $x_1^*(v,\theta) > x_1^{SB}(v,\theta)$ in this case. By contrast, for low-value projects (i.e., those with $v < \tilde{v}$), the no-handicap constraint is binding and the contract right increases from $x_1^*(v,\theta)$ to $x_1^{SB}(v,\theta)$. Interestingly, the no-handicap constraint does not affect the size of the prize. Of course, removing the "stick" raises the cost of providing innovation incentives, and thus we would expect an increase in the multiplier of the incentive constraint λ (implying that the favorable bias for a high-value project is larger and that the monetary prize is more often awarded) and a reduction in the optimal innovation effort.

Remark: On simple rules. The above mechanism makes the contract allocation contingent on the realized value of the project. In practice, this value can be difficult to measure objectively or costly to verify, which in turn calls for simpler rule. Yet, we show in Online Appendix B that our main insight carries over when the buyer can assess the value of the project, but is required to use the same auction rules (that is, $x(v,\theta) = x(\theta)$ and $\rho(v) = \rho$) *whenever* deciding to implement the project: as long as the project is not always implemented, the optimal auction rule introduces a bias in favor of the innovator. Interestingly, handicaps are never optimal in this case; in addition, as long as the principal observes the value of the project, such mechanism can be used regardless of whether this value is also observed by the firms, or can be verified by third parties such as courts.

4 Procuring Innovation from Multiple Suppliers

We now assume that several firms may innovate and propose projects, as well as implement them. This case captures the problem of a buyer who has a specific need that readily available products or services cannot satisfy, and who decides to procure an innovation to satisfy this need. Illustrative examples include the Norwegian Department of Energy procuring a new technology for carbon capture and storage;[8] or the Scottish Government procuring low-cost, safe and effective methods of locating, securing and protecting electrical array cables

[7] It can be checked that this indeed ensures that the innovator is never worse off than a pure contractor – see Online Appendix A.

[8] http://ted.europa.eu/udl?uri=TED:NOTICE:214787-2011:TEXT:EN:HTML&src=0

in Scottish sea conditions.[9] In both instances, the public authority put out a RFP, and multiple firms responded with different projects.

For the sake of exposition, we will suppose from now on that each firm $k = 1, ..., n$ can come up with a project of value v^k, which is publicly observable and distributed over V according to a c.d.f. $F^k(v^k|e^k)$ with density $f^k(v^k|e^k)$, where e^k denotes firm k's innovation effort.[10] We assume that firms decide on these efforts simultaneously, and denote by $\mathbf{e} = (e^1, ..., e^n)$ the profile of the efforts. The alternative projects correspond to competing ways to fulfill the same need, so they are perfect substitutes, meaning that the buyer will choose one project at most. The previous setting corresponds to the special case where F^k is concentrated on $\underline{v}$ for all $k \neq 1$.

In practice, a firm's cost of implementing a project may depend on who proposed it. In some cases, the innovator has cost advantages in implementing the project. In other cases, the innovator has cost disadvantages in implementation; an example is when the innovator is an R&D specialty firm that lacks manufacturing capabilities necessary for implementing its R&D. To accommodate such an interdependency between innovation and implementation, we assume that firm i's cost of implementing project k is given by $\theta_i + \psi_i^k$, where:

- as before, θ_i is an idiosyncratic shock, privately observed by firm i and distributed according the c.d.f. G_i;
- ψ_i^k represents an additional cost, potentially both project- and firm-specific, which for simplicity is supposed to be common knowledge.

Without loss of generality, we consider a direct revelation mechanism that specifies an allocation and a payment to each firm as a function of realized project values, $\mathbf{v} = (v^1, ..., v^n)$, and of reported costs. Note that an allocation involves a decision as to which project is selected as well as who implements that project.

A mechanism is thus of the form $(x, t) : V^n \times \Theta^n \to \Delta^{n^2} \times \mathbb{R}^n$. The problem facing the principal can therefore be expressed as:

$$\max_{x,t,\mathbf{e}} \mathbb{E}_{\mathbf{v},\theta} \left[w(\mathbf{v}, \theta) \mid \mathbf{e} \right],$$

where the *ex post* net surplus is now equal to

$$w(\mathbf{v}, \theta) = \sum_{k,i \in N} \left[v^k x_i^k (\mathbf{v}, \theta) - t_i (\mathbf{v}, \theta) \right],$$

[9] http://ted.europa.eu/udl?uri=TED:NOTICE:436615-2013:TEXT:EN:HTML&src=0

[10] While formally all implementors are also innovators, the case of "pure contractors" can be accommodated by setting the density to zero for $v > \underline{v}$.

subject to (IR) and (IC), where now firms' *interim* expected profits when lying and reporting the truth are respectively given by

$$u_i(\mathbf{v},\theta_i'|\theta_i) = \mathbb{E}_{\theta_{-i}}[t_i(\mathbf{v},\theta_i',\theta_{-i}) - \sum_{k\in N}(\theta_i+\psi_i^k)x_i^k(\mathbf{v},\theta_i',\theta_{-i})] \text{ and } U_i(\mathbf{v},\theta_i) = u_i(\mathbf{v},\theta_i|\theta_i),$$

and are also subject to limited liability and moral hazard constraints, which respectively become:

$$\mathbb{E}_\theta\left[w\left(\mathbf{v},\theta\right) \mid \mathbf{e}\right] \geq 0, \quad \forall \mathbf{v} \in V^n,$$

$$e^i \in \arg\max_{\tilde{e}^i} \mathbb{E}_{\mathbf{v},\theta}\left[U(\mathbf{v},\theta_i) \mid \tilde{e}^i, e^{-i}\right] - c_i(\tilde{e}^i), \quad \forall i \in N.$$

As in the previous section, we assume that an optimal mechanism exists, which induces an interior profile of efforts $\mathbf{e}^*$. The following Proposition then partially characterizes this optimal mechanism:

Proposition 4. *There exists* $\boldsymbol{\lambda}^* = \left(\lambda^{1*}, ..., \lambda^{n*}\right) \geq 0$ *such that the optimal mechanism solving* $[P]$*:*

- *selects firm i to implement project k with probability*

$$x_i^{k*}(\mathbf{v},\theta) = \begin{cases} 1 & \text{if } v^k - K_i^*(\mathbf{v},\theta_i) - \psi_i^k \geq \max\left\{0, \max_{(l,j)\neq(k,i)} v^l - K_j^*(\mathbf{v},\theta_j) - \psi_j^l\right\}, \\ 0 & \text{otherwise,} \end{cases}$$

  where

$$K_i^*(\mathbf{v},\theta_i) := J_i(\theta_i) - \left(\frac{\beta^i(v^i)}{\max\{\max_k \beta^k(v^k),1\}}\right)\left(\frac{G_i(\theta_i)}{g_i(\theta_i)}\right), \text{ and } \beta^i(v^i) := \lambda^i \frac{f_e^i(v^i|e^{i*})}{f^i(v^i|e^{i*})}.$$

- *awards each firm i a transfer*

$$t_i^*(\mathbf{v},\theta) := \rho_i^*(\mathbf{v}) + \sum_{k\in N}\psi_i^k X_i^{k*}(\mathbf{v},\theta_i) + \int_{\theta_i}^{\overline{\theta}} \sum_{k\in N} X_i^{k*}(\mathbf{v},s)ds,$$

  where $X_i^{k*}(\mathbf{v},\theta_i) = \mathbb{E}_{\theta_{-i}}[x_i^{k*}(\mathbf{v},\theta_i,\theta_{-i})]$; and the transfer includes a cash prize

$$\rho_i^*(\mathbf{v}) := \mathbb{E}_\theta\left[\sum_{k,i\in N} x_i^{k*}(\mathbf{v},\theta)\left\{v^k - \psi_i^k - J_i(\theta_i)\right\}\right],$$

  which is positive only if $\beta^i(v^i) > \max\left\{\max_{j\in N}\beta^j(v^j), 1\right\}$.

Proof. See Appendix D. □

To interpret this characterization, consider first the case in which known implementation cost differences are additively separable across implementors and projects: $\psi_i^k = \psi_i + \psi^k$ for all i and k. Then, the project selection is simply based on the "net values" of the projects, $v^k - \psi^k$, without regard for who is awarded the contract to implement the chosen project.[11] Hence, there is no need to wait until the realization of the costs before selecting the project. However, the realized project values still do affect the choice of the implementor through their impact on virtual costs – the $K_i^*(\mathbf{v}, \theta_i)$s, which depend on all realized values, including those of unselected projects.[12] In particular, an increase in v^k raises both the probability that project k is selected and the probability that firm k is chosen to implement it, even when a project $j \neq k$ is selected. In this sense, there is again a bias in favor of innovators with high-value projects.

If the separability condition is not satisfied, the choices of the project and of the implementor are more intimately linked. Suppose for instance that $\psi_k^k = 0 < \psi_i^k = \bar{\psi}$ for all k and $i \neq k$: that is, each firm has a cost advantage of $\bar{\psi}$ for the project it proposes vis-à-vis other firms. If two firms i and j are such that $v^i > v^j$, but θ_j is significantly lower than θ_i, the desire to exploit this cost advantage may lead the principal to choose project j over project i.

A few other observations are worth making. First, as intuition suggests, the optimal allocation $x_i^{k*}(\mathbf{v}, \theta)$ is nondecreasing in (v^i, θ_{-i}) and nonincreasing in $(\mathbf{v}^{-i,k}, \theta_i)$. In addition, as all firms are now potential innovators, each virtual cost $K_i^*(\mathbf{v}, \theta_i)$ is characterized by two cutoffs, $\tilde{v}^i$ and $\hat{v}^i$, defined as in the previous section but with somewhat different implications. As before, each innovator benefits from a bias at the implementation stage when $v^i > \tilde{v}^i := \beta_i^{-1}(0)$ and is instead handicapped when $v^i < \tilde{v}^i$. To what extent a firm will actually benefit from this bias or be harmed by the handicap, however, now depends on the relative magnitude of the shadow values $\beta^i(v^i)$ across firms and thus depends also on the values brought by the other projects, $\mathbf{v}^{-i}$.

Second, a "winner-takes-all" principle holds in the sense that generically at most one firm is awarded a prize. As in the case of single innovator, a prize is worth giving only when the incentive benefit $\beta^i(v^i)$ exceeds one. But with multiple innovators, there may be

[11] To see this, note that the difference in surplus when a contractor i implements project k or project l is given by

$$\left(v^k - K_i - \psi^k - \psi_i\right) - \left(v^l - K_i - \psi^l - \psi_i\right) = \left(v^k - \psi^k\right) - \left(v^l - \psi^l\right),$$

and thus does not depend on which contractor i is selected.

[12] Note that for a "pure contractor," $K_i(\mathbf{v}, \theta_i) = J_i(\theta_i)$, as in a standard second-best auction.

several firms i for which $\beta^i(v^i) > 1$. Due to the limited liability of the buyer, an additional dollar paid to a firm is one less dollar available to reward another firm. As the incentive benefit of a dollar is proportional to $\beta^i(v^i)$, the marginal benefit of the prize is maximized by concentrating the prize to firm with the highest shadow value of incentive $\beta^i(v^i)$. Splitting the available cash across firms is never optimal for the same reason that it was never optimal to give less than the maximal prize to the innovator in the single innovator case. In the same vein, even if $\beta^i(v^i) > 1$ for several firms, only firm $\hat{\imath} := \max_{i\in N}\{\beta^i(v^i)\}$ will face undistorted virtual cost $K^*_{\hat{\imath}}(\mathbf{v},\theta_{\hat{\imath}}) = \theta_{\hat{\imath}}$; the others will instead face a distorted virtual cost equal to

$$K^*_i(\mathbf{v},\theta_i) = \theta_i + \left[1 - \frac{\beta^i(v^i)}{\beta^{\hat{\imath}}(v^{\hat{\imath}})}\right]\frac{G_i(\theta_i)}{g_i(\theta_i)} > \theta_i.$$

Note that if firms are *ex ante* symmetric (i.e., $f^k(.) = f(.)$ and $\psi^k_i = \psi$), then from MLRP, the highest $\beta^i(v^i)$ corresponds to the highest v^i; hence, the best project is selected, and only that project can ever be awarded a prize. By contrast, if firms are not *ex ante* symmetric, then the prize will instead be given to the firm whose effort was most worth incentivizing (i.e., the firm with the highest $\beta^i(v^i)$), even if it is not the one with the best project (i.e., the highest v^i).

In the same vein, the receiver of the prize is not necessarily the firm whose project is selected. For instance, if innovators are better placed to implement their own projects, then if firms are otherwise *ex ante symmetric* (so that $\beta^1(.) = \beta^2(.) = \beta(.)$), the firm with the best project may receive a prize (if the value of its projects exceeds $\hat{v} = \beta^{-1}(1)$), and yet cost considerations may lead the principal to select another project.[13]

Remark: On innovation strategies. We have so far assumed that firms' R&D efforts only affected the values of their projects. In practice, firms may also adopt innovation strategies targeting projects on which they are more likely to be better positioned for their implementation. This would further reinforce the need for taking into account the values of the projects when deciding over the allocation of contract rights. While a standard procurement auction would actually encourage such self-serving innovation strategies, introducing a bias in favor of innovators with high-value projects would mitigate these incentives and encourage instead the adoption of more valuable innovation strategies.

[13] Consider for example the case $n = 2$, and suppose that $\psi^1_1 = \psi^2_2 = 0 < \psi^1_2 = \psi^2_1 = +\infty$ (that is, a firm can only implement its own project). In this case, if $v^1 > \max\{v^2, \hat{v}\}$ but $\theta_1 < \theta_2$, then firm 1 receives a prize but firm 2's project is selected if the cost difference is large enough.

5 Discussion

5.1 Unsolicited Proposals

As mentioned in the Introduction, some countries do not allow public authorities to directly reward unsolicited proposals. Our analysis suggests instead that it can be optimal to reward valuable proposals through contract rights and possibly by monetary prizes. Hodges and Dellacha (2007) describe three alternative ways used in practice:

∘ Bonus system. The system gives the original project proponent a bonus in the tendering procedure. A bonus can take many forms but most commonly involves additional points in the score of the original proponent's technical or financial offer. This system is, for example, adopted in Chile and Korea. For example, the first two unsolicited proposals for airport concessions in Chile obtained a bonus equal to 20 percent points of the allowed score, whilst the third airport proposal received 10 percent points.[14]

∘ Swiss challenge system. The Swiss challenge system gives the original proposer the right to counter-match any better offers. It is most common in the Philippines and is also used in Guam, India, Italy, and Taiwan. Under this procedure, the original proposer will counter the lowest rival bid and win the contract if and only if its cost is less than that bid. Anticipating this, the rival bidders will optimally shade their bids above their costs in response. Hence, in equilibrium the system distorts the contract allocation in favor of the proposer (who wins the contract even when its cost is above the rivals' costs, and for sure when its cost is less than theirs). But the degree of the favoritism implemented by the system does not match that required by our optimal mechanism (see Proposition 3); the system often goes too far in favoring the proposer.[15]

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[14]These projects concerned respectively the expansion of the airports of Puerto Montt (June 1995), Iquique (August 1995) and Calama (Ocober 1997).

[15]To illustrate this point, suppose $n = 2$ and firm 1 is the innovator as in our baseline model. In that case, firm 1 counters any bid b_2 by firm 2 if and only if $\theta_2 < b_2$. Knowing this, firm 2 picks its bid b_2 to solve

$$\max_{b_2} (1 - G(b_2))(b_2 - \theta_2),$$

whose first-order condition is:

$$b_2 - \frac{1 - G(b_2)}{g(b_2)} = \theta_2.$$

Given firm 1's behavior, this means that the contract is allocated to firm 1 if and only if

$$\theta_1 - \frac{1 - G(\theta_1)}{g(\theta_1)} < \theta_2,$$

conditional on the project being implemented at all. In the case G is uniform on $[0, 1]$, this boils down

∘ Best and final offer system. Here, the key element is multiple rounds of tendering, in which the original proponent is given the advantage of automatically participating in the final round. It is used in Argentina and South Africa.

Our analysis suggests that these mechanisms have some merit, as biasing the implementation stage in favor of the innovator may indeed promote innovation. The bonus system has the additional merit of allowing the advantage to be linked to the value of the proposed project, with higher project values resulting in greater advantages. By contrast, the Swiss challenge system and the best and final offer system grant an unconditional advantage to the innovator, which in our analysis is sub-optimal. Note that none of these systems provides for explicit handicapping.

5.2 Bundling R&D and Implementation

In the practice of innovation procurement, we observe two polar cases.

First, there is *pure bundling,* where the firm whose project is selected also implements it. This approach was, for instance, followed in US Defense Procurement in the 1980s, where the winner of the technical competition for the best prototype was virtually assured of being awarded the follow-on defense contract (see Lichtenberg, 1990; and Rogerson, 1994). More recently, the European Procurement Directive 2014/24/EU has introduced so-called "Innovation Partnerships" for the joint procurement of R&D services and large-scale production.

Second, there is *unbundling,* where project selection and implementation are kept entirely separate; therefore, the firm whose project was selected is treated exactly the same way as any other firm at the implementation stage. Examples include research contests or the European Pre-commercial Procurement (PCP) model. In both cases, firms compete for innovative solutions at the innovation stage, and the best solution(s) may receive a prize. The procurer does not commit itself to acquire the resulting innovations.

Our analysis identifies the circumstances in which the two extreme cases are optimal.

Corollary 2. 1. *Pure bundling is optimal if for each* $k \in N$, $\psi_k^k = 0 < \psi_i^k = \bar{\psi}$ *for* $i \neq k$, *where* $\bar{\psi} > \sup_{\theta_i, \theta_j, i,j \in N}(J_i(\theta_i) - \theta_j)$.

2. *Unbundling is optimal if there exists* $N_1, N_2 \subset N$ *with* $N_1 \cup N_2 = N$ *and* $N_1 \cap N_2 = \emptyset$ *such that, for each* $i = N_1$, $\psi_i^j = \infty, \forall j$ *and that for each* $k \in N_2, i \in N_1$, $\psi_j^k = \infty, \forall j$ *and* $\psi_k^i = 0$. *In this case, the optimal mechanism selects the project* k *from* N_1 *with*

to $\theta_1 < \frac{1+\theta_2}{2}$, or $\theta_2 > 2\theta_1 - 1$. Comparing with Figure ??, the resulting allocation tends to involve more excessive favoritism toward firm 1 than the optimal allocation.

the highest value v^k *if* $v^k \geq \min_j J_j(\theta_j)$*, rewards the innovator* $i \in N_1$ *with the highest* $\beta^i(v^i) > 1$*, and awards the implementation contract to a firm* $j \in N_2$ *with the lowest virtual cost* $J_j(\theta_j) < v^k$*.*

Pure bundling can be optimal when there are large economies of scope between R&D and implementation as described by the condition in (i). For example, in the procurement of complex IT systems, the knowledge advantage of the software developer typically translates into a considerable cost advantage on the management and upgrading of the software. In this case, selecting the same firm for both R&D and implementation is likely to be better. However, even in that case, our results stress that project selection should be based on both value and cost considerations.

By contrast, unbundling is optimal when firms specialize in either innovation or in implementation (e.g., manufacturing or construction). Corollary 2-(ii) describes such a case: firms are partitioned into two groups so that one specializes in innovation and the other specializes in implementation. In that case, the optimal mechanism selects the project and rewards the innovator from the former group, according to the first-best scheme in Proposition 1, and awards the implementation contract to a firm in the second group according to the second-best scheme in Proposition 2.

Unbundling is sometimes prescribed as an affirmative action policy toward so-called *small and medium enterprises* (*SMEs*). In both Europe and the US, procurement programs aimed at stimulating R&D investment from *SMEs* provide for separation between the R&D stage and the implementation stage, with funding provided based on firms's project proposals. The Small Business Innovation Research (SBIR) program in the US or the UK's Small Business Research Initiative (SBRI) are characterized by this separation between project selection and implementation.[16] Such a policy can be justified based on Corollary 2-(ii) on the ground that small or medium R&D firms often lack manufacturing capabilities and thus would at a clear disadvantage when the R&D competition is bundled with the contract implementation. For instance, the SMEs may comprise group N_1 and non-SMEs may comprise N_2, in which case the government wishes to promote research effort specifically from SMEs and bans non-SMEs from proposing a project (as under SBIR and SBRI).

A similar reasoning suggests that when base university research may play a key role in R&D activities, separation between selection and implementation may also help to promote their participation. When instead innovators are also likely to play a role at the implementation stage, unbundling is never optimal.

[16] See, respectively, http://www.sbir.gov/ and https://sbri.innovateuk.org.

6 Conclusions

Procuring innovative projects requires incentivizing potential innovators' research efforts as well as an efficient implementation of the selected projects. Our analysis highlights a trade-off between these two objectives when implementors have private information about their costs. To solve this trade-off, the optimal mechanism relies on contract rights (possibly combined with monetary prizes).

A number of issues are worth exploring further. First, we have focused on situations in which the value of the proposals can be contracted upon. This is a plausible assumption when, for instance, the proposal involves a prototype or when performance measures – operational or productivity indicators, energy consumption, emissions, etc. – are available and can be used in the tender documents; yet another possibility is to rely on evaluation committees. In other situations (e.g., base research), however, the difficulty of describing the project and/or non-verifiability issues may make it impossible to contract *ex ante* on the *ex post* value of the projects. When *ex post* the value is verifiable, the literature suggests that the procurer can replicate the above mechanism (see, e.g., Maskin and Tirole (1999)); a similar remark applies when *ex post* the value of the project is observed by the parties but is non-verifiable by third parties such as courts (see, e.g., Maskin (1999) and Moore and Repullo (1988)). The situation is different when the value of the project is private information (e.g., only the buyer observes it). Yet, the spirit of our insights carries over when for instance the procurer must use the same auction rules whenever she decides to implement the project (see the remark on simple rules at the end of Section 3.3). Characterizing the optimal mechanism under private information is beyond the scope of this paper but clearly constitutes an interesting avenue for future research.

Second, we have ignored the costs of participating in procurement tenders. In practice, submitting a tender bid may require tender development costs (e.g., complex estimations and legal advice) that involve significant economic resources, in which case biasing the tender in favor of the innovator may discourage potential implementors from participating in the tender. It would therefore be worth endogenizing participation in the tender and exploring how the optimal mechanism should be adjusted to account for these development costs. More generally, accounting for endogenous entry is a promising research avenue.[17]

Likewise, we have assumed that the procurer was benevolent. In practice, corruption concerns and, more generally, institution design may matter, which may call for limiting the

[17] For recent work on the role of discrimination in auctions with endogenous entry, see, e.g., Jehiel and Lamy (2015).

discretion given to the procuring agency. Balancing this with the provision of innovation incentives constitutes another promising research avenue.

Finally, we have focused on a situation where the innovation is valuable to a single buyer – and thus has no "market" value. An interesting extension would be to consider multiple buyers, so as to allow for the possibility that extra contractual incentives for research effort arise from the commercialization of the innovation. Exploring the role of market forces would also help to shed light on possible anti-competitive effects of alternative mechanisms for public procurement of innovation.[18]

[18] See the 2014 European State Aid framework for research, development and innovation (EU 2014b).

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Appendix

A Proof of Proposition 1

To solve $[P - FB]$, we focus on the relaxed problem:

$$[P' - FB] \qquad \max_{x,t} \mathbb{E}_{v,\theta}[v \sum_{i \in N} x_i(v,\theta) - t_i(v,\theta)|e]$$

subject to (LL), (MH) and

$$\mathbb{E}_\theta[t_i(v,\theta) - \theta_i x_i(v,\theta)] \geq 0, \quad \forall v, i. \tag{IR'}$$

This problem is a relaxation of $[P - FB]$ since (IR') requires (IR) to hold only on average. At the same time, whenever a mechanism satisfies (IR'), one can construct at least one mechanism that satisfies (IR), without affecting other constraints. Hence, there is no loss in restricting attention to $[P' - FB]$. To solve $[P' - FB]$, we first observe that for each $i \neq 1$, the constraint (IR') must bind. If not, one can always lower the expected payment to increase the value of the objective without tightening any constraints. Next, define

$$\rho_1(v,\theta) := \mathbb{E}_\theta[t_1(v,\theta) - \theta_1 x_1(v,\theta)].$$

Then, we can weaken $[P' - FB]$ further to:

$$[P'' - FB] \qquad \max_{x,t} \mathbb{E}_{v,\theta}[\sum_{i \in N}(v - \theta_i)x_i(v,\theta) - \rho_1(v)|e]$$

subject to

$$\rho_1(v) \geq 0, \quad \forall v, \tag{IR''}$$

$$\mathbb{E}_\theta[\sum_{i \in N}(v - \theta_i)x_i(v,\theta)] \geq \rho_1(v), \quad \forall v, \tag{LL''}$$

$$\frac{\partial}{\partial e}\mathbb{E}_v\left[\rho_1(v)|e\right] \geq c'(e). \tag{MH''}$$

Note that the weakening occurs with the moral hazard constraint: (MH'') is a first-order necessary condition of (MH).

Let $\nu(v)$, $\mu(v)$, and λ denote the multipliers for constraints (IR''), (LL'') and (MH''), respectively. Then, the Lagrangian (more precisely its integrand) is given by:

$$L(v,\theta,e) := [1 + \mu(v)]\left\{\sum_{i \in N} x_i(v,\theta)\left(v - \theta_i\right)\right\} - \rho_1(v)\left[1 + \mu(v) - \nu(v) - \beta\left(v\right)\right] - \lambda c'\left(e\right),$$

where

$$\beta(v) := \lambda \frac{f_e(v|e)}{f(v|e)}.$$

The optimal solution $(e^{FB}, x^{FB}(v,\theta), \rho_1^{FB}(v), \lambda^{FB}, \mu^{FB}(v), \nu^{FB}(v))$ must satisfy the following necessary conditions.

First, since the Lagrangian is linear in x_i's, the optimal solution $x_i^{FB}(v,\theta)$ is as defined in Proposition 1. Next, the Lagrangian L is also linear in $\rho_1(v)$; hence, its coefficient must be equal to zero:

$$1 + \mu^*(v) - \beta^*(v) - \nu^*(v) = 0. \tag{1}$$

Next, the optimal effort e^{FB} must satisfy

$$\frac{\partial}{\partial e} \mathbb{E}_{v,\theta}[L(v,\theta,e)|e] \bigg|_{e=e^{FB}} = 0. \tag{2}$$

Finally, complementary slackness implies that, for each v,

$$\nu^{FB}(v)\rho_1^{FB}(v) = 0, \tag{3}$$

$$\mu^{FB}(v) \left\{ \mathbb{E}_\theta[\sum_{i\in N} x_i^{FB}(v,\theta)(v-\theta_i) - \rho_1^{FB}(v) \right\} = 0, \tag{4}$$

and

$$\lambda^{FB} \left[\int_v \rho_1^{FB}(v) f_e(v|e^{FB}) dv - c'(e^{FB}) \right] = 0. \tag{5}$$

We first prove that $\lambda^{FB} > 0$. Suppose not. Then, $\beta^{FB}(v) = 0$ for all $v \in V$. It then follows from (1) that $\nu^{FB}(v) > 0$ for all $v \in V$. By (3), this means that $\rho_1^{FB}(v) \equiv 0$. As $x_i = x_i^{FB}$, it then follows from (4) that for any $v > \underline{\theta}$, $\mu^{FB}(v) = 0$. Collecting these facts together, we conclude that

$$\mathbb{E}_\theta[L(v,\theta,e)] = \mathbb{E}_\theta[\max\{0, v - \min_i \{\theta_i\}],$$

which is increasing in v (and strictly so for a positive measure of v). By (*MLRP*), this means that

$$\frac{\partial}{\partial e} \mathbb{E}_{v,\theta}[L(v,\theta,e)|e] > 0,$$

a contradiction to (2). We thus conclude that $\lambda^{FB} > 0$.

If $v < \hat{v}^{FB}$, then $\beta^{FB}(v) < 1$, and thus $1 + \mu^{FB}(v) - \beta^{FB}(v) > 0$. Hence, by (1), $\nu^{FB}(v) > 0$ and, by (3), we have $\rho_1^{FB}(v) = 0$. It in turn follows from (4) that $\mu^{FB}(v) = 0$ provided that $v > \underline{\theta}$.

If instead $v > \hat{v}^{FB}$, then $\beta^{FB}(v) > 1$, and thus $1 - \beta^{FB}(v) - \nu^{FB}(v) < 0$. Hence, by (1), $\mu^{FB}(v) > 0$. But then, by (4), we must have

$$\rho_i^{FB}(v) = \mathbb{E}_\theta \left[\sum_{i \in N} x_i^{FB}(v, \theta) (v - \theta_i) \right],$$

as claimed in Proposition 1.

Next, we show that $\underline{v} < \hat{v}^{FB} < \bar{v}$. First, by (*MLRP*), $\beta^{FB}(v)$ is strictly increasing in v, and there exists $\tilde{v} \in (\underline{v}, \bar{v})$ such that $\beta^{FB}(\tilde{v}) = 0\,(< 1)$; it follows that $\hat{v}^{FB} > \tilde{v}\,(> \underline{v})$. Second, we must have $v > \hat{v}^{FB}$ with positive probability (i.e., λ^{FB} cannot be too small). Suppose to the contrary that $\beta^{FB}(v) < 1$ for all $v \in V$. Then, as argued above $\rho_1^{FB}(v) = \mu^{FB}(v) = 0$ for all $v \in V$. In this case, by the convexity of $c(\cdot)$, we must have $e^{FB} = 0$, or else we obtain a contradiction to (5). But then, we get

$$L(v, \theta, e) = \max\{0, \max_{\theta_i}(v - \theta_i)\} - c'(e).$$

As the first term is increasing in v (and strictly so for a positive measure of v), and $c'(0) = 0$, we thus get a contradiction to (2).

Finally, we prove that $e^{FB} > 0$. Given $\lambda^{FB} > 0$, it follows from (5) that

$$\int_v \rho_1^{FB}(v) f_e(v|e^{FB}) dv = c'(e^{FB}).$$

As $v > \hat{v}^{FB}$ for a positive measure of v, the left side is strictly positive. This implies that $e^{FB} > 0$, or else the right-hand side vanishes as $c'(0) = 0$.

B Proof of Proposition 3

To solve $[P]$, we first reformulate (*IC*) in terms of interim allocation and payment rules. For each $i \in N$ and for any $v \in V$ and any $\theta_i \in \Theta_i$, let $X_i(v, \theta_i) := \int_{\theta_{-i}} x_i(v, \theta)\, dG_{-i}(\theta_{-i})$ and $T_i(v, \theta_i) := \int_{\theta_{-i}} t_i(v, \theta)\, dG_{-i}(\theta_{-i})$ denote the interim allocation and payment for firm i and

$$U_i(v, \theta_i) := T_i(v, \theta_i) - \theta_i X_i(v, \theta_i) \tag{6}$$

denote firm i's expected profit. For each $i \in N$, (*IC*) then can be stated as

$$T_i(v, \theta_i) - \theta_i X_i(v, \theta_i) \geq T_i(v, \theta_i') - \theta_i X_i(v, \theta_i'), \forall v, \theta_i, \theta_i'.$$

The associated envelope condition then yields

$$U_i(v, \theta_i) = \rho_i(v) + \int_{\theta_i}^{\overline{\theta}} X_i(v, \theta) d\theta, \tag{7}$$

where

$$\rho_i(v) := U_i(v, \overline{\theta})$$

is the rent enjoyed by firm i when its cost is highest. Using (7), we can express firm i's expected rent as

$$\begin{aligned}
\int_{\theta_i} U_i(v,\theta_i) dG_i(\theta_i) &= \int_{\theta_i} \left[\rho_i(v) + \int_{\theta_i}^{\overline{\theta}} X_i(v,s)ds\right] dG_i(\theta_i) \\
&= \rho_i(v) + \int_{\theta_i} X_i(v,\theta_i) \frac{G_i(\theta_i)}{g_i(\theta_i)} dG_i(\theta_i). \qquad (8)
\end{aligned}$$

For each $i \neq 1$, the rent $\rho_i(v)$ does not help to relax any constraint and reduces the surplus for the principal, so it is optimal to set $\rho_i(v) = 0$ for all v.

Using (6) and (8), the total expected transfer to the firms can be expressed as:

$$\begin{aligned}
\int_{\theta} \sum_{i \in N} t_i(v,\theta) dG(\theta) &= \sum_{i \in N} \int_{\theta_i} T_i(v,\theta_i) dG_i(\theta_i) \\
&= \sum_{i \in N} \int_{\theta_i} \left[U_i(v,\theta_i) + \theta_i X_i(v,\theta_i)\right] dG_i(\theta_i) \\
&= \sum_{i \in N} \left\{\rho_i(v) + \int_{\theta_i} X_i(v,\theta_i) J_i(\theta_i) dG_i(\theta_i)\right\} \\
&= \rho_1(v) + \int_{\theta} \sum_{i \in N} x_i(v,\theta_i) J_i(\theta_i) dG(\theta), \qquad (9)
\end{aligned}$$

where $J_i(\theta_i) := \theta_i + \frac{G_i(\theta_i)}{g_i(\theta_i)}$ denotes firm i's virtual cost.

Substituting (9) into the principal's objective function, we can rewrite (LL) as follows:

$$\forall v \in V, \quad \int_{\theta} \left\{\sum_{i \in N} x_i(v,\theta) \left[v - J_i(\theta_i)\right]\right\} dG(\theta) \geq \rho_1(v). \qquad (\widehat{LL})$$

Let $\mu(v) \geq 0$ denote the multiplier associated with this constraint.

The innovating firm's individual rationality simplifies to

$$\forall v \in V, \quad \rho_1(v) \geq 0. \qquad (\widehat{IR})$$

Let $\nu(v) \geq 0$ denote the multiplier associated with this constraint.

We next focus on the first-order condition for the effort constraint.

$$\int_v \int_{\theta} \left[\rho_1(v) + \frac{G_1(\theta_1)}{g_1(\theta_1)} x_1(v,\theta)\right] dG(\theta) f_e(v|e) dv \geq c'(e). \qquad (\widehat{MH})$$

Note that we formulate the condition as a weak inequality to ensure the nonnegativity of the multiplier. Let $\lambda \geq 0$ be the associated multiplier.

Then, $[P]$ can more succinctly be reformulated as follows:

$$\max_{e,x(v,\theta),\rho_1(v)} \int_v \left\{ \int_\theta \left[\sum_{i\in N} x_i(v,\theta) \left[v - J_i(\theta_i) \right] \right] dG(\theta) - \rho_1(v) \right\} f(v|e) dv$$

$$\text{subject to } (\widehat{LL}), (\widehat{IR}) \text{ and } (\widehat{MH})$$

The integrand of the Lagrangian is given by:

$$L(v,\theta,e) := [1+\mu(v)] \left\{ \left[v - \theta_1 - \left(1 - \frac{\beta(v)}{1+\mu(v)} \right) \frac{G_1(\theta_1)}{g_1(\theta_1)} \right] x_1(v,\theta) + \sum_{\substack{j\in N \\ j\neq 1}} [v - J_j(\theta_j)] x_j(v,\theta) \right\}$$
$$- \rho_1(v) [1 + \mu(v) - \nu(v) - \beta(v)] - \lambda c'(e),$$

where

$$\beta(v) := \lambda \frac{f_e(v|e)}{f(v|e)}.$$

The optimal solution $(e^*, x^*(v,\theta), \rho_1^*(v), \lambda^*, \mu^*(v), \nu^*(v))$ must satisfy the following necessary conditions. First, observe that the Lagrangian L is linear in $\rho_1(v)$; hence, its coefficient must be equal to zero:

$$1 + \mu^*(v) - \beta^*(v) - \nu^*(v) = 0. \tag{10}$$

The Lagrangian is also linear in x_i's, so the optimal allocation must satisfy, for every i, v, θ:

$$x_i^*(v,\theta) = \begin{cases} 1 & \text{if } i \in \arg\min_j \left\{ \tilde{K}_j(v,\theta_j) \right\} \text{ and } \tilde{K}_i(v,\theta_i) \leq v, \\ 0 & \text{otherwise,} \end{cases}$$

where

$$\tilde{K}_i(v,\theta_i) := \begin{cases} J_i(\theta_i) - \frac{\beta^*(v)}{1+\mu^*(v)} \frac{G_i(\theta_i)}{g_i(\theta_i)} & \text{if } i = 1, \\ J_i(\theta_i) & \text{if } i \neq 1, \end{cases}$$

where $\beta^*(v) = \lambda^* \frac{f_e(v|e^*)}{f(v|e^*)}$.

Next, the optimal effort e^* must satisfy

$$\frac{\partial}{\partial e} \int_v \int_\theta L(v,\theta,e^*) f(v|e^*) dG(\theta) dv = 0. \tag{11}$$

Finally, complementary slackness implies that, for each v,

$$\nu^*(v) \rho_1^*(v) = 0, \tag{12}$$

$$\mu^*(v) \left\{ \int_\theta \sum_{i\in N} x_i^*(v,\theta) [v - J_i(\theta_i)] dG(\theta) - \rho_1^*(v) \right\} = 0, \tag{13}$$

and

$$\lambda^* \left[\int_v \int_\theta \left[\rho_1^*(v) + \frac{G_1(\theta_1)}{g_1(\theta_1)} x_1^*(v,\theta) \right] dG(\theta)\, f_e(v|e^*) dv - c'(e^*) \right] = 0. \tag{14}$$

We now provide the characterization. Consider first the case where $v < \underline{\theta}$. From (10), $\tilde{K}_i(v,\theta_i) \geq \theta_i$, and thus $\tilde{K}_i(v,\theta_i)$ and $K_i^*(v,\theta_i)$ both yield $x_i^*(v,\theta) = 0$ for every $i \in N$; furthermore, ($\widehat{LL}$) and ($\widehat{IR}$) together imply

$$\rho_1^*(v) = 0 = \int_\theta \sum_{i \in N} x_i^*(v,\theta) \left[v - J_i(\theta_i) \right] dG(\theta).$$

Hence, the characterization of $x_i^*(v,\theta)$ given in Proposition 3 is correct.

We now focus on the range $v > \underline{\theta}$. Again, there are two cases depending on the value of v. Consider first the case $v < \hat{v}^*$, where $\beta^*(v) < 1$. Hence, $1 + \mu^*(v) - \beta^*(v) > \mu^*(v) \geq 0$, and (10) thus implies $\nu^*(v) > 0$. The complementary slackness condition (12) then yields $\rho_1^*(v) = 0$. This, together with Lemma 4 (see Appendix D) and the complementary slackness condition (13), implies that $\mu^*(v) = 0$. Hence, $\tilde{K}_1(v,\theta_1) = J_1(\theta_1) - \beta^*(v)\, G_1(\theta_1)/g_1(\theta_1) = K_1^*(v,\theta_1)$.

Let us now turn to the case $v > \hat{v}^*$, where $\beta^*(v) > 1$. Hence, $1 - \beta^*(v) - \nu^*(v) < 0$, and (10) thus implies that $\mu^*(v) > 0$; from the complementary slackness condition (13), we thus have

$$\rho_1^*(v) = \int_\theta \sum_{i \in N} x_i^*(v,\theta) \left[v - J_i(\theta_i) \right] dG(\theta).$$

Suppose $\nu^*(v) > 0$. Lemma 4 then implies $\rho_1^*(v) > 0$, contradicting the complementary slackness condition (12). Therefore, $\nu^*(v) = 0$. It follows now from (10) that $1 + \mu^*(v) = \beta^*(v)$. We therefore conclude that $\tilde{K}_1(v,\theta_1) = \theta_1 = K_1^*(v,\theta_1)$.

The transfer payment t^* follows from (9), with $\rho_1^*(v)$ as described above and $\rho_j^*(v) = 0$ for all $j \neq 1$. The above characterization is valid only when the optimal allocation is monotonic (another necessary condition from incentive compatibility). This follows the assumption that $\frac{G_i(\theta_i)}{g_i(\theta_i)}$ is nondecreasing in θ_i, which implies that $K_i^*(v,\theta_i) = J_i(\theta_i)$, for $i \neq 1$, and

$$K_1^*(v,\theta_1) = J_1(\theta_1) - \min\{1, \beta(v)\} \frac{G_1(\theta_1)}{g_1(\theta_1)} = \theta_1 + \max\{0, 1 - \beta(v)\} \frac{G_1(\theta_1)}{g_1(\theta_1)},$$

are all nondecreasing in θ_i.

We next prove that $\lambda^* > 0$. Suppose $\lambda^* = 0$. Then, $\beta^*(\cdot) = 0$, so (10) again implies that $\nu^*(\cdot) > 0$ and $\mu^*(\cdot) = \rho_1^*(\cdot) = 0$. Hence,

$$L(v,\theta,e^*) = \max\{0, v - \min_i J_i(\theta_i)\},$$

which increases for a positive measure of v. It follows that

$$\frac{\partial}{\partial e}\int_{\underline{v}}^{\overline{v}}\int_{\theta}L(v,\theta,e)dG(\theta)f(v|e)dv\bigg|_{e=e^*}=\int_{\underline{v}}^{\overline{v}}\int_{\theta}\max\{0,v-\min_i J_i(\theta_i)\}dG(\theta)f_e(v|e^*)dv>0,$$

which contradicts (11).

Next, we show that $e^*>0$. It follows from (14) and $\lambda^*>0$ that

$$\int_v\int_\theta\left[\rho_1(v)+\frac{G_1(\theta)}{g_1(\theta)}x_1^*(v,\theta)\right]g(\theta)d\theta f_e(v|e)dv=c'(e).$$

The left-hand side is strictly positive, which implies that $e^*>0$, or else the right side vanishes since $c'(0)=0$.

C On the Optimality of Offering a Prize ($\hat{v}<\bar{v}$)

As mentioned, whether it is optimal to award a monetary prize (i.e., $\hat{v}<\bar{v}$) depends on how much innovation incentives are required and on how much would already be provided by the standard second-best auction. We show in this Appendix that a monetary prize is optimal when: (i) there is either little cost heterogeneity (see Section C.1) or a large number of firms (see Section C.2), as the procurement auction does not generate much information rents, and thus provides little innovation incentives; or (ii) the range of project values is large (see Section C.3), so that innovation incentives then matter a lot.

Throughout this Appendix, we start with an environment for which there exists an optimal mechanism with no monetary reward, and then consider variations of this environment for which the optimal mechanism must involve a prize.

The baseline environment, for which there exists an optimal mechanism with no monetary reward, consists of a distribution $F(\cdot|e)$ for the value v and a distribution $G_i(\cdot)$ for the cost of each firm $i\in N$, such that $\rho^*(\cdot)=0$, which amounts to $\hat{v}^*>\bar{v}$, or

$$\lambda<\bar{\lambda}:=\frac{f(\bar{v}|e)}{f_e(\bar{v}|e)},\tag{15}$$

and implies that $\mu^*(\cdot)=0$. The optimal allocation is therefore such that $x_i^*(v,\theta)=0$ for any $v\le\underline{\theta}$ and, for $v>\underline{\theta}$:

$$x_1^*(v,\theta)=\begin{cases}1 & \text{if } K_1^*(\theta_1)<\min\{v,J_2(\theta_2),...,J_n(\theta_n)\},\\ 0 & \text{otherwise.}\end{cases}$$

For further reference, it is useful to note that the objective of the principal, as a function of e, can be expressed as:

$$\begin{aligned}\int_{\underline{\theta}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} & \sum_{i \in N} x_i^* (v, \theta) [v - J_i (\theta_i)] \, dG (\theta) \, dF (v|e) \\ + & \lambda \left\{ \int_{\underline{\theta}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} X_1^* (v, \theta_1) \, G_1 (\theta_1) \, f_e(v|e) d\theta_1 dv - c' (e) \right\},\end{aligned}$$

where the innovator's expected probability of obtaining the contract is given by:

$$X_1^* (v, \theta_1) = \int_{\theta_{-1}} x_1^* (v, \theta) \, dG_{-1} (\theta_{-1}) \, .$$

The first-order condition with respect to e yields:

$$\begin{aligned}\int_{\underline{\theta}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} & \sum_{i \in N} x_i^* (v, \theta) [v - J_i (\theta_i)] \, dG (\theta) \, f_e (v|e^*) \, dv \\ = & \lambda \left\{ c'' (e^*) - \int_{\underline{\theta}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} X_1^* (v, \theta_1) \, G_1 (\theta_1) \, f_{ee}(v|e^*) d\theta_1 dv \right\}. \qquad (16)\end{aligned}$$

The optimal effort e^* moreover satisfies the innovator's incentive constraint $c' (e^*) = b (e^*)$, where the innovator's expected benefit is given by:

$$b (e) := \int_{\underline{\theta}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} X_1^* (v, \theta_1) \, G_1 (\theta_1) \, f_e(v|e) d\theta_1 dv.$$

C.1 Reducing Cost Heterogeneity

Suppose first that costs become increasingly less heterogeneous: the cost of each firm $i \in N$ becomes distributed according to $G_i^m (\theta_i)$ over the range $\Theta_i^m = \left[\underline{\theta}, \bar{\theta}^m = \underline{\theta} + \left(\bar{\theta} - \underline{\theta}\right) / m\right]$. For each $m \in \mathbb{N}^*$, we will denote by e^m, λ^m, $K_1^m (\theta_1)$ and $X_1^m (v, \theta_1)$ the values associated with the optimal mechanism. We now show that, for m large enough, this optimal mechanism must include a monetary prize.

We first note that as m goes to infinity, the innovator's effort tends to the lowest level, $\underline{e}$:

Lemma 1. *e^m tends to $\underline{e}$ as m goes to infinity.*

Proof. The innovator's expected benefit becomes

$$b^m (e) := \int_{\underline{\theta}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}^m} X_1^m (v, \theta_1) \, G_1^m (\theta_1) \, f_e(v|e) d\theta_1 dv,$$

and satisfies:

$$|b^m(e)| \leq \int_{\underline{\theta}}^{\bar{v}} \int_{\underline{\theta}}^{\underline{\theta}+\frac{\bar{\theta}-\underline{\theta}}{m}} d\theta_1 |f_e(v|e)| dv = \frac{(\bar{\theta}-\underline{\theta}) \int_{\underline{\theta}}^{\bar{v}} |f_e(v|e)| dv}{m}.$$

Therefore, as m goes to infinity, the expected benefit $b^m(e)$ converges to 0, and the innovator's effort thus converges to the minimal effort, $\underline{e}$. □

Furthermore:

Lemma 2. *As m goes to infinity:*

- *The left-hand side of (16) tends to*

$$B^\infty := \int_{\underline{\theta}}^{\bar{v}} (v-\underline{\theta}) f_e(v|\underline{e}) dv > 0.$$

- *In the right-hand side of (16), the terms within brackets tend to $c''(\underline{e})$.*

Proof. The left-hand side of (16) is of the form $\int_{\underline{\theta}}^{\bar{v}} h_1^m(v) dv$, where

$$h_1^m(v) := f_e(v|e^m) \int_{\underline{\theta}}^{\bar{\theta}^m} \sum_{i \in N} x_i^m(v,\theta) [v - J_i(\theta_i)] dG^m(\theta).$$

Furthermore, for any $v > \underline{\theta}$, $\hat{J}(\theta) := \min_{i \in N} \{J_i(\theta_i)\} < v$ for m is large enough (namely, for m such that $\bar{\theta}^m < v$ or $m > (\bar{\theta}-\underline{\theta})/(v-\underline{\theta})$), and so

$$h_1^m(v) = f_e(v|e^m) \int_{\underline{\theta}}^{\bar{\theta}^m} \left[v - \hat{J}(\theta)\right] dG^m(\theta),$$

which is bounded:

$$|h_1^m(v)| < \left|\max_e f_e(v|e^m)\right| \max\{v-\underline{\theta}, 0\},$$

and converges to

$$\lim_{m \longrightarrow \infty} h_1^m(v) = (v-\underline{\theta}) f_e(v|\underline{e}).$$

Using Lebesgue's dominated convergence theorem, we then have:

$$\lim_{m \longrightarrow \infty} \int_{\underline{\theta}}^{\bar{v}} h_1^m(v) dv = \int_{\underline{\theta}}^{\bar{v}} \lim_{m \longrightarrow \infty} h_1^m(v) dv = B^\infty.$$

We now turn to the right-hand side (16). The terms within brackets are

$$c''(e^m) - \int_{\underline{\theta}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}^m} X_1^m(v,\theta_1) G_1^m(\theta_1) f_{ee}(v|e^m) d\theta_1 dv,$$

where the first term tends to $c''(\underline{e})$ and the second term is of the form $\int_{\underline{\theta}}^{\bar{v}} h_2^m(v)\,dv$, where

$$h_2^m(v) = f_{ee}(v|e^m) \int_{\underline{\theta}}^{\bar{\theta}^m} X_1^m(v,\theta_1)\, G_1^m(\theta_1)\, d\theta_1$$

satisfies:

$$|h_2^m(v)| < \max_e |f_{ee}(v|e)| \int_{\underline{\theta}}^{\bar{\theta}^m} d\theta_1 = \frac{(\bar{\theta} - \underline{\theta}) \max_e |f_{ee}(v|e)|}{m}$$

and thus tends to 0 as m goes to infinity. □

To conclude the argument, suppose that the optimal mechanism never involves a prize. Condition (16) should thus hold for any m, and in addition, the Lagrangian multiplier λ^m should satisfy the boundary condition (15). We should thus have:

$$\int_{\underline{\theta}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}^m} \sum_{i \in N} x_i^m(v,\theta)\,[v - J_i(\theta_i)]\, dG(\theta)\, f_e(v|e^m)\, dv$$
$$< \frac{f(\bar{v}|e^m)}{f_e(\bar{v}|e^m)} \left\{ c''(e^m) - \int_{\underline{\theta}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}^m} X_1^m(v,\theta_1)\, G_1(\theta_1)\, f_{ee}(v|e^m) d\theta_1 dv \right\}.$$

Taking the limit as m goes to infinity, this implies:

$$B^\infty = \int_{\underline{\theta}}^{\bar{v}} (v - \underline{\theta})\, f_e(v|\underline{e})\, dv < \frac{f(\bar{v}|\underline{e})}{f_e(\bar{v}|\underline{e})} c''(\underline{e}),$$

which is obviously violated when the return on effort is sufficiently high (e.g., $c''(\underline{e})$ is low enough).

C.2 Increasing the Number of Firms

Let us now keep the cost distributions fixed, and suppose instead that m additional firms are introduced in the environment with the same cost distribution as the innovator: $G_k(\theta_k) = G_1(\theta_k)$ for $k = n+1, ..., n+m$. Letting again denote by e^m, λ^m, $K_1^m(\theta_1)$ and $X_1^m(v,\theta_1)$ the values associated with the optimal mechanism, we now show that the optimal mechanism must involve a prize for m large enough.

By construction, $K_1^m(\theta_1)\,(> \theta_1) > \underline{\theta}$ for any $\theta_1 > \underline{\theta}$, whereas the lowest $J_j(\theta_j)$ becomes arbitrarily close to $J_1(\underline{\theta}) = \underline{\theta}$ as m increases; it follows that the probability of selecting the innovator, $X_1^m(v,\theta_1)$, tends to 0 as m goes to infinity:

Lemma 3. *$X_1^m(v,\theta_1)$ tends to 0 as m goes to infinity.*

Proof. The probability of selecting the innovator satisfies:

$$\begin{aligned} X_1^m(v,\theta_1) &\le \Pr\left[K_1^m(\theta_1) \le \min_{j=n+1,\ldots,n+m}\{J_1(\theta_j)\}\right] \\ &\le \Pr\left[\theta_1 \le \min_{j=n+1,\ldots,n+m}\{J_1(\theta_j)\}\right] \\ &= \left[1 - G_1\left(J_1^{-1}(\theta_1)\right)\right]^m, \end{aligned} \tag{17}$$

where the second inequality stems from $K_1^m(\theta_1) \ge \theta_1$, and the last expression tends to 0 when m goes to infinity. □

It follows that Lemma 1 still holds, that is, the innovator's effort tends to the lowest level, $\underline{e}$, as m goes to infinity. To see this, it suffices to note that the innovator's expected benefit, now equal to

$$b^m(e) = \int_{\underline{\theta}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} X_1^m(v,\theta_1)\, G_1(\theta_1)\, f_e(v|e) d\theta_1 dv,$$

satisfies:

$$|b^m(e)| \le \int_{\underline{\theta}}^{\bar{v}} h(v)\, dv,$$

where

$$h(v) := |f_e(v|e)| \int_{\underline{\theta}}^{\bar{\theta}} X_1^m(v,\theta_1)\, d\theta_1$$

is bounded (by $\left(\bar{\theta} - \underline{\theta}\right) \max_{v,e}\{|f_e(v|e)|\}$) and, from the previous Lemma, tends to 0 as m goes to infinity. Hence, as m goes to infinity, the expected benefit $b^m(e)$ converges to 0, and the innovator's effort thus tends to $\underline{e}$.

Likewise, Lemma 2 also holds; that is,

- The left-hand side of (16) tends to B^∞. To see this, it suffices to follow the same steps as before, noting that $h_1^m(v)$, now given by

$$h_1^m(v) = \int_{\underline{\theta}}^{\bar{\theta}} \sum_{i \in N} x_i^m(v,\theta)\left[v - J_i(\theta_i)\right] dG(\theta)\, f_e(v|e^m),$$

  is still bounded:

$$|h_1^m(v)| < \max\{v - \underline{\theta}, 0\} \left|\max_e f_e(v|e)\right|,$$

  and tends to $v - \underline{\theta}$ for any $v > \underline{\theta}$:

– $\hat{J}(\theta) = \min_{i\in N}\{J_i(\theta_i)\}$ is almost always lower than v when m is large enough. Indeed, for any $\varepsilon > 0$, we have:

$$\begin{aligned}
\Pr\left[\hat{J}(\theta) \leq \underline{\theta} + \varepsilon\right] &\geq \Pr\left[\min_{i=n+1,\ldots,n+m}\{J_i(\theta_i)\} \leq \underline{\theta} + \varepsilon\right] \\
&= \Pr\left[\min_{i=n+1,\ldots,n+m}\{\theta_i\} \leq J_1^{-1}(\underline{\theta} + \varepsilon)\right] \\
&= 1 - \left[1 - F\left(J_1^{-1}(\underline{\theta} + \varepsilon)\right)\right]^m,
\end{aligned}$$

where the last expression converges to 1 as m goes to infinity. Therefore, for any $\varepsilon > 0$, there exists $\hat{m}_1(\varepsilon)$ such that for any $m \geq \hat{m}_1(\varepsilon)$,

$$\Pr\left[\hat{J}(\theta) \leq \underline{\theta} + \varepsilon\right] \geq 1 - \varepsilon.$$

– Hence, for $m \geq \hat{m}_1(\varepsilon)$:

$$v - \underline{\theta} \geq \int_{\underline{\theta}}^{\bar{\theta}} \sum_{i\in N} x_i^m(v,\theta)\left[v - J_i(\theta_i)\right] dG(\theta) \geq (1-\varepsilon)(v - \underline{\theta} - \varepsilon),$$

where the right-hand side converges to $v - \underline{\theta}$ as ε tends to 0.

The conclusion then follows again from Lebesgue's dominated convergence theorem.

- In the right-hand side of (16), the terms within brackets tend to $c''(\underline{e})$. To see this, it suffices to note that $h_2^m(v)$, now given by

$$h_2^m(v) = f_{ee}(v|e^m)\int_{\underline{\theta}}^{\bar{\theta}} X_1^m(v,\theta_1)\, G_1(\theta_1)\, d\theta_1$$

– is still bounded:

$$\begin{aligned}
\left|h_2^m(v)\right| &< \max_e\left|f_{ee}(v|e)\right|\int_{\underline{\theta}}^{\bar{\theta}} X_1^m(v,\theta_1)\, d\theta_1 \\
&\leq \max_e\left|f_{ee}(v|e)\right|\int_{\underline{\theta}}^{\bar{\theta}} \left[1 - G_1\left(J_1^{-1}(\theta_1)\right)\right]^m d\theta_1.
\end{aligned}$$

– and converges to 0: Indeed, for any $\varepsilon > 0$,

$$\begin{aligned}
\left|h_2^m(v)\right| &< \max_e\left|f_{ee}(v|e)\right|\left\{\int_{\underline{\theta}}^{\underline{\theta}+\frac{\varepsilon}{2}} d\theta_1 + \int_{\underline{\theta}+\frac{\varepsilon}{2}}^{\bar{\theta}} \left[1 - G_1\left(J_1^{-1}(\underline{\theta}+\varepsilon)\right)\right]^m d\theta_1\right\} \\
&< \max_e\left|f_{ee}(v|e)\right|\left\{\frac{\varepsilon}{2} + \left(\bar{\theta} - \underline{\theta}\right)\left[1 - G_1\left(J_1^{-1}(\underline{\theta}+\varepsilon)\right)\right]^m\right\}.
\end{aligned}$$

But there exists $\hat{m}_2(\varepsilon)$ such that, for any $m \geq \hat{m}_2(\varepsilon)$:

$$\left(\bar{\theta}-\underline{\theta}\right)\left[1-G_1\left(J_1^{-1}\left(\theta_1\right)\right)\right]^m \leq \frac{\varepsilon}{2},$$

and thus

$$\left|h_2^m(v)\right| < \max_e \left|f_{ee}(v|e)\right| \varepsilon.$$

– It follows that the second term converges again to 0:

$$\lim_{m\longrightarrow\infty} \int_{\underline{\theta}}^{\bar{v}} h_2^m(v)\, dv = \int_{\underline{\theta}}^{\bar{v}} \lim_{m\longrightarrow\infty} h_2^m(v)\, dv = 0.$$

The conclusion follows, using the same reasoning as in Section C.1.

C.3 Increasing the Value of the Innovation

Let us now keep the supply side (number of firms and their cost distributions) fixed and suppose instead that:

- v is initially distributed over $V = [\underline{v}, \bar{v}]$; for the sake of exposition, we assume $\underline{v} \gg \bar{\theta}$,[19] so that the innovation is always implemented.
- For every $m \in \mathbb{N}^*$, the value v^m becomes distributed over $V^m = [\underline{v}, \bar{v}^m = \underline{v} + m(\bar{v} - \underline{v})]$, according to the c.d.f. $F^m(v^m|e) = F(\underline{v} + (v^m - \underline{v})/m|e)$.

As before, letting e^m, λ^m, $K_1^m(\theta_1)$, and $X_1^m(v, \theta_1)$ denote the values associated with the optimal mechanism, we now show that this optimal mechanism must involve a prize for m large enough.

We first note that the virtual costs remain invariant here: $K_i^m(v^m, \theta_i) = K_i(v, \theta_i) = J_i(\theta_i)$ for $i > 1$ and, as

$$\beta^m(v^m) = \lambda \frac{f_e^m(v^m|e)}{f^m(v^m|e)} = \lambda \frac{f_e(v|e)}{f(v|e)},$$

we also have

$$\begin{aligned} K_1^m(v^m, \theta_1) &= J_1(\theta_1) - \min\{\beta^m(v^m), 1\} \frac{G_1(\theta_1)}{g_1(\theta_1)} \\ &= J_1(\theta_1) - \min\{\beta(v), 1\} \frac{G_1(\theta_1)}{g_1(\theta_1)} \\ &= K_1(v, \theta_1). \end{aligned}$$

[19] Namely, $\underline{v} > \min_{i\in N}\left\{K_i\left(\underline{v}, \bar{\theta}\right)\right\}$.

As by assumption, the innovation is always implemented in this variant, the probability of obtaining the contract only depends on these virtual costs and thus also remains invariant: $x_i^m(v^m, \theta) = x_i^*(v, \theta)$ for any $i \in N$. It follows that, in the right-hand side of (16), the terms within brackets also remained unchanged: using $X_1^m(v^m, \theta_1) = X_1^*(v, \theta_1)$ and $f_{ee}^m(v^m|e)\,dv^m = f_{ee}(v|e)\,dv$, we have:

$$c''(e) - \int_{\underline{v}}^{\bar{v}^m} \int_{\underline{\theta}}^{\bar{\theta}} X_1^m(v^m, \theta_1)\, G_1(\theta_1)\, f_{ee}^m(v^m|e)\, d\theta_1 dv^m = \Gamma^*(e),$$

where

$$\Gamma^*(e) := c''(e) - \int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} X_1^*(v, \theta_1)\, G_1(\theta_1)\, f_{ee}(v|e)\, d\theta_1 dv.$$

By contrast, the left-hand side of (16) is unbounded as m goes to infinity: using $\sum_{i \in N} x_i^*(v, \theta) = 1$ (as by assumption, the innovation is always implemented here), $f_e^m(v|e)\,dv^m = f_e(v|e)\,dv$ and $\int_{\underline{v}}^{\bar{v}} f_e(v|e)\,dv = 0$, we have:

$$\begin{aligned} \int_{\underline{v}}^{\bar{v}^m} \int_{\underline{\theta}}^{\bar{\theta}} & \sum_{i \in N} x_i^m(v^m, \theta)\,[v^m - J_i(\theta_i)]\, dG(\theta)\, f_e^m(v^m|e)\, dv^m \\ &= \int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} \sum_{i \in N} x_i^*(v, \theta)\,[\underline{v} + m(v - \underline{v}) - J_i(\theta_i)]\, dG(\theta)\, f_e(v|e)\, dv \\ &= mB^*(e) - C^*(e), \end{aligned}$$

where:

$$\begin{aligned} B^*(e) &= \int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} \sum_{i \in N} x_i^*(v, \theta)\, v dG(\theta)\, f_e(v|e)\, dv = \int_{\underline{v}}^{\bar{v}} v f_e(v|e)\, dv, \\ C^*(e) &= \int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} \sum_{i \in N} x_i^*(v, \theta)\, J_i(\theta_i)\, dG(\theta)\, f_e(v|e)\, dv. \end{aligned}$$

To conclude the argument, suppose that the optimal mechanism never involves a prize. Condition (16) should thus hold for any m, and in addition, the Lagrangian multiplier λ^m should satisfy the boundary condition (15). We should thus have:

$$mB^*(e) < C^*(e) + \frac{f(\bar{v}|e)}{f_e(\bar{v}|e)} \Gamma^*(e),$$

which is obviously violated for a large enough m.

D Proof of Proposition 4

As earlier, the incentive compatibility constraint can be replaced by the envelope condition:

$$U_i(\mathbf{v}, \theta_i) = \rho_i(\mathbf{v}) + \int_{\theta_i}^{\overline{\theta}} X_i(\mathbf{v}, \theta_i) d\theta_i, \quad \forall (\mathbf{v}, \theta_i) \in V^N \times \Theta, \forall i \in N, \tag{18}$$

where

$$X_i(\mathbf{v}, \theta_i) = \mathbb{E}_{\theta_{-i}} \left[\sum_{k \in N} x_i^k(\mathbf{v}, \theta_i, \theta_{-i}) \right].$$

Using condition (18), we can rewrite the limited liability constraint as:

$$\mathbb{E}_\theta \left[\sum_{k,i \in N} x_i^k(\mathbf{v}, \theta) \left\{ v^k - J_i(\theta_i) \right\} \right] \geq \sum_{i \in N} \rho_i(\mathbf{v}), \quad \forall \mathbf{v} \in V^n. \tag{LL}$$

Let $\mu(\mathbf{v}) \geq 0$ denote the multiplier associated with this constraint.

Also, from (18), individual rationality boils down to

$$\rho_i(\mathbf{v}) \geq 0, \quad \forall \mathbf{v} \in V^n, \forall i \in N. \tag{IR}$$

We let $\nu_i(\mathbf{v}) \geq 0$ denote the multiplier associated with this constraint.

Finally, using (7), firm i's expected rent and the total expected transfer to the firms can respectively be expressed as

$$\int_{\theta_i} U_i(\mathbf{v}, \theta_i) dG_i(\theta_i) = \rho_i(\mathbf{v}) + \int_{\theta_i} X_i(v, \theta_i) \frac{G_i(\theta_i)}{g_i(\theta_i)} dG_i(\theta_i), \tag{19}$$

and

$$\int_\theta \sum_{i \in N} t_i(\mathbf{v}, \theta) dG_i(\theta_i) = \sum_{i \in N} \rho_i(\mathbf{v}) + \int_\theta \sum_{i \in N} x_i^k(\mathbf{v}, \theta_i)(J_i(\theta_i) + \psi_i^k) dG_i(\theta_i), \tag{20}$$

and the moral hazard constraint can be replaced by the associated first-order condition, which, using (18), (19), and (20), can be expressed as:[20]

$$\mathbb{E}_{\mathbf{v},\theta} \left[\rho_i(\mathbf{v}) + \frac{G_i(\theta_i)}{g_i(\theta_i)} \sum_{k \in N} x_i^k(\mathbf{v}, \theta) \middle| e^i, e^{-i} \right] \geq c'(e^i), \quad \forall i \in N. \tag{MH}$$

We formulate again these conditions as weak inequalities to ensure the nonnegativity of the associated multipliers, which we will denote by $\boldsymbol{\lambda} = \left(\lambda^1, ..., \lambda^n\right)$.

[20] For simplicity, we normalize the firms' efforts in such a way that firms face the same cost $c(e)$; any asymmetry can, however, be accommodated through the distributions $F^k(v^k|e)$.

The principal's problem can then be more succinctly reformulated as follows:

$$[P] \qquad \max_{\mathbf{x},(\rho_i),\mathbf{e}} \; \mathbb{E}_{\mathbf{v},\theta}\left[\textstyle\sum_{k,i\in N} x_i^k(\mathbf{v},\theta)\left(v^k - J_i(\theta_i) - \psi_i^k\right) - \sum_{i\in N}\rho_i(\mathbf{v})\middle|\, \mathbf{e}\right]$$
$$\text{subject to } (LL),\ (IR), \text{ and } (MH).$$

The analysis of this problem follows the same steps as for the case of a single innovator, and we only sketch them here. The integrand of the Lagrangian is now given by:

$$L(\mathbf{v},\theta,e) := [1+\mu(\mathbf{v})]\left\{\sum_{k,i\in N}\left[v^k - \theta_i - \left(1 - \frac{\beta^i(v^i)}{1+\mu(\mathbf{v})}\right)\frac{G_i(\theta_i)}{g_i(\theta_i)} - \psi_i^k\right] x_i^k(\mathbf{v},\theta)\right\}$$
$$- \sum_{i\in N}\rho_i(\mathbf{v})\left[1+\mu(\mathbf{v}) - \nu_i(\mathbf{v}) - \beta^i(v^i)\right] - \sum_{i\in N}\lambda^i c'(e^i),$$

where

$$\beta^i(v^i) := \lambda^i \frac{f_e^i(v^i|e)}{f(v^i|e)}.$$

The first-order conditions for the monetary prize $\rho_i(\mathbf{v})$ and for the probability $x_i^k(\mathbf{v},\theta)$ yield, respectively:

$$1+\mu^*(\mathbf{v}) - \nu_i^*(\mathbf{v}) - \beta^{i*}(v^i) = 0, \quad \forall \mathbf{v}\in V^n, \forall i\in N, \tag{21}$$

and

$$x_i^{k*}(\mathbf{v},\theta) = \begin{cases} 1 & \text{if } v^k - \tilde{K}_i(\mathbf{v},\theta_i) - \psi_i^k \geq \max\left\{\max_{(l,j)\neq(k,i)} v^l - \tilde{K}_j(\mathbf{v},\theta_j) - \psi_j^l, 0\right\}, \\ 0 & \text{otherwise,}\end{cases} \tag{22}$$

where

$$\tilde{K}_i(\mathbf{v},\theta_i) := J_i(\theta_i) - \frac{\beta^{i*}(v^i)}{1+\mu^*(\mathbf{v})}\frac{G_i(\theta_i)}{g_i(\theta_i)}.$$

Note that $\tilde{K}_i(\mathbf{v},\theta_i)$ can be expressed as

$$\theta_i + \left[1 - \frac{\beta^{i*}(v^i)}{1+\mu^*(\mathbf{v})}\right]\frac{G_i(\theta_i)}{g_i(\theta_i)},$$

where (21) and $\nu_i^*(\mathbf{v}) \geq 0$ together imply that the term within brackets is non-negative. It follows that

$$\tilde{K}_i(\mathbf{v},\theta_i) \geq \theta_i \tag{23}$$

and that $\tilde{K}_i(\mathbf{v},\theta_i)$ increases with θ_i.

The complementary slackness associated with (LL) implies that for every $\mathbf{v}\in V^n$,

$$\mu^*(\mathbf{v})\left\{\mathbb{E}_\theta\left[\sum_{k,i\in N} x_i^{k*}(\mathbf{v},\theta)\left\{v^k - J_i(\theta_i)\right\}\right] - \sum_{i\in N}\rho_i^*(\mathbf{v})\right\} = 0, \tag{24}$$

whereas the complementary slackness associated with (IR) implies that for every $i \in N$ and every $\mathbf{v} \in V^n$,

$$\nu_i^*(\mathbf{v})\rho_i^*(\mathbf{v}) = 0. \tag{25}$$

We now prove the following result:

LEMMA 4. *Fix any* $\mathbf{v}$ *such that* $\max_{k,i}\left\{v^k - \psi_i^k\right\} > \underline{\theta}$. *We have*

$$\mathbb{E}_\theta\left[\sum_{k,i\in N} x_i^{k*}(\mathbf{v},\theta)\left[v^k - \psi_i^k - J_i(\theta_i)\right]\right] > 0, \tag{26}$$

if either (i) $n \geq 2$ *or (ii)* $n = 1$ *and either* $v^1 - \psi_1^1 > \overline{\theta}$ *or* $\nu_1\left(v^1\right) > 0$.

PROOF. We first focus on the case in which $n \geq 2$. Fix any $\mathbf{v}$ such that $v^l - \psi_j^l - \underline{\theta} > 0$ for some l, j. Further, fix any k such that $\sum_i x_i^k(\mathbf{v},\theta) > 0$ for a positive measure of θs (a project that does not satisfy this property is never adopted with positive probability and can be ignored).

Consider first the particular case in which project k is always implemented and allocated to the same firm i: $x_i^k(\mathbf{v},.) = 1$ (this can, for instance, happen when v^k is large and ψ_j^k is prohibitively high for $j \neq i$). In that case:

$$\begin{aligned}
&\mathbb{E}_\theta\left[\sum_{i\in N} x_i^k(\mathbf{v},\theta)\left[v^k - \psi_i^k - J_i(\theta_i)\right]\right] \\
=&\int_{\underline{\theta}}^{\overline{\theta}}\left[v^k - \psi_i^k - J_i(\theta_i)\right] dG_i(\theta_i) \\
=&v^k - \psi_i^k - \overline{\theta} \\
>&0,
\end{aligned}$$

where the inequality stems from (22), applied to $\theta_i = \overline{\theta}$,[21] and (23).

Let us now turn to the case in which no firm is selected with probability 1 to implement project k (because project k is not always implemented and/or different firms are selected to implement it). By (22), the optimal allocation rule is then such that

$$X_i^k(\mathbf{v},\theta_i) := \mathbb{E}_{\theta_{-i}}\left[x_i^k(\mathbf{v},\theta_i,\theta_{-i})\right]$$

[21] Generically, this condition implies $v^k - \psi_i^k > \tilde{K}_i\left(\mathbf{v},\overline{\theta}\right)$; we ignore here the non-generic case $v^k - \psi_i^k = \tilde{K}_i\left(\mathbf{v},\overline{\theta}\right)$.

is nonincreasing in θ_i for all $\theta_i \leq v^k - \psi_i^k$ and equals zero for any $\theta_i > v^k - \psi_i^k$. Further, it is strictly decreasing in θ_i for a positive measure of θ_i if $X_i^k(\mathbf{v}, \theta_i) > 0$, and by the choice of k, there is at least one such firm.

Now, for every i define

$$\bar{X}_i^k(\mathbf{v}, \theta_i) = \begin{cases} \bar{z}_i^k & \text{if } \theta_i \leq v^k - \psi_i^k \\ 0 & \text{if } \theta_i > v^k - \psi_i^k, \end{cases}$$

where $\bar{z}_i^k$ is a constant in $(0,1)$ chosen so that

$$\int_{\underline{\theta}}^{\overline{\theta}} \bar{X}_i^k(\mathbf{v}, \theta_i) dG_i(\theta_i) = \bar{z}_i^k G_i(v^k - \psi_i^k) = \int_{\underline{\theta}}^{\overline{\theta}} X_i^k(\mathbf{v}, \theta_i) dG_i(\theta_i).$$

Clearly, $\bar{z}_i^k$, and hence $\bar{X}_i^k(\mathbf{v}, \cdot)$, is well defined.

We have:

$$\begin{aligned}
&\mathbb{E}_\theta \left[\sum_{i \in N} x_i^k(\mathbf{v}, \theta) \left[v^k - \psi_i^k - J_i(\theta_i) \right] \right] \\
=&\sum_i \int_{\underline{\theta}}^{\overline{\theta}} X_i^k(\mathbf{v}, \theta_i) \left[v^k - \psi_i^k - J_i(\theta_i) \right] dG_i(\theta_i) \\
=&\sum_i \int_{\underline{\theta}}^{\min\{\overline{\theta}, v^k - \psi_i^k\}} X_i^k(\mathbf{v}, \theta_i) \left[v^k - \psi_i^k - J_i(\theta_i) \right] dG_i(\theta_i) \\
>&\sum_i \int_{\underline{\theta}}^{\min\{\overline{\theta}, v^k - \psi_i^k\}} \bar{X}_i^k(\mathbf{v}, \theta_i) \left[v^k - \psi_i^k - J_i(\theta_i) \right] dG_i(\theta_i) \\
=&\sum_i \bar{z}_i^k \int_{\underline{\theta}}^{\min\{\overline{\theta}, v^k - \psi_i^k\}} \left[v^k - \psi_i^k - J_i(\theta_i) \right] dG_i(\theta_i) \\
=&\sum_i \bar{z}_i^k \left(\max\{v^k - \psi_i^k - \overline{\theta}, 0\} \right) \\
\geq& 0.
\end{aligned}$$

The second equality stems from the fact that $X_i^k(\mathbf{v}, \theta_i) = 0$ for $\theta_i > v^k - \psi_i^k$, and the strict inequality follows from the fact that $v^k - \psi_i^k - J_i(\theta_i)$ is strictly decreasing in θ_i and, in the relevant range $\left[\underline{\theta}, \min\{\overline{\theta}, v^k - \psi_i^k\}\right]$, $X_i^k(\mathbf{v}, \theta_i)$ is nonincreasing in θ_i and strictly decreasing in θ_i for a positive measure of θ_i for some i, whereas by construction, $\bar{X}_i^k(\mathbf{v}, \cdot)$ is constant and

$$\int_{\underline{\theta}}^{\min\{\overline{\theta}, v^k - \psi_i^k\}} \bar{X}_i^k(\mathbf{v}, \theta_i) dG_i(\theta_i) = \int_{\underline{\theta}}^{\min\{\overline{\theta}, v^k - \psi_i^k\}} X_i^k(\mathbf{v}, \theta_i) dG_i(\theta_i).$$

Summing the above string of inequalities over all k, we obtain the desired result.

Next consider the case in which $n = 1$. In this case, $X_1^1(\mathbf{v}, \theta_1) = x_1^1(\mathbf{v}, \theta_1) = 1$ for $\tilde{K}_1(v^1, \theta_1) \le v^1$ and zero otherwise. Because $X_i^k(\mathbf{v}, \theta_i)$ is constant when it is strictly positive, the strict inequality above does not follow from the above argument. But the strict inequality does still hold if $v^1 - \psi_1^1 > \overline{\theta}$ or if $\nu_1(v^1) > 0$.

In the former case, the last inequality above becomes strict, thus yielding the desired result. To consider the latter case, assume without loss $v^1 - \psi_1^1 \le \overline{\theta}$. Because $\nu_1(v^1) > 0$, we have $\beta^1(v^1) < 1 + \mu(v^1)$, so $\tilde{K}_1(v^1, \theta_1) > \theta_1$, which implies that there exists $\tilde{\theta} < v^1 - \psi_1^1$ such that $x_1^1(\mathbf{v}, \theta_1) = 1$ for $\theta_1 < \tilde{\theta}$ and $x_1^1(\mathbf{v}, \theta_1) = 0$ for $\theta_1 > \tilde{\theta}$. Let $\check{\theta} := \sup\{\theta \le \overline{\theta} | v^1 - \psi_1^1 - J_1(\theta) \ge 0\}$. If $\tilde{\theta} \le \check{\theta}$, then

$$\mathbb{E}_\theta \left[x_1^1(\mathbf{v}, \theta) \left[v^1 - \psi_1^1 - J_1(\theta_1) \right] \right] = \int_{\underline{\theta}}^{\tilde{\theta}} \left[v^1 - \psi_1^1 - J_1(\theta_1) \right] dG(\theta_1) > 0.$$

If $\tilde{\theta} < \check{\theta}$, the same result holds because

$$\begin{aligned}
& \mathbb{E}_\theta \left[x_1^1(\mathbf{v}, \theta) \left[v^1 - \psi_1^1 - J_1(\theta_1) \right] \right] \\
=& \int_{\underline{\theta}}^{\tilde{\theta}} \left[v^1 - \psi_1^1 - J_1(\theta_1) \right] dG(\theta_1) \\
>& \int_{\underline{\theta}}^{\tilde{\theta}} \left[v^1 - \psi_1^1 - J_1(\theta_1) \right] dG_1(\theta_1) + \int_{\tilde{\theta}}^{v^1 - \psi_1^1} \left[v^1 - \psi_1^1 - J_1(\theta_1) \right] dG_1(\theta_1) \\
=& \int_{\underline{\theta}}^{v^1 - \psi_1^1} \left[v^1 - \psi_1^1 - J_1(\theta_1) \right] dG_1(\theta_1) \\
=& 0,
\end{aligned}$$

where the strict inequality holds because $v^1 - \psi_1^1 - J_1(\theta_1) < 0$ for $\theta_1 \in (\tilde{\theta}, v^1 - \psi_1^1)$ (which in turn holds because $\check{\theta} < \tilde{\theta} < v^1 - \psi_1^1$), and the last equality follows from integration by parts. □

Without loss of generality, assume $n \ge 2$ (otherwise, there would be a single innovator, a case studied earlier). There are two cases. Consider first the case in which $\beta^i(v^i) < 1$ for every $i \in N$. By (21), we must then have

$$\nu_i^*(\mathbf{v}) = 1 + \mu^*(\mathbf{v}) - \beta^{i*}(v^i) > 0,$$

and the complementary slackness condition (25) thus yields $\rho_i^*(\mathbf{v}) = 0$ for every firm $i \in N$. This, together with (26) and the complementary slackness condition (24), implies that

$\mu^*(\mathbf{v}) = 0$, and thus

$$\tilde{K}_i(\mathbf{v}, \theta_1) = J_i(\theta_i) - \beta^{i*}(v^i)\frac{G_i(\theta_i)}{g_i(\theta_i)} := K_i^*(v, \theta_1).$$

Consider next the case in which $\max_{i \in N}\left\{\beta^{i*}(v^i)\right\} > 1$. Let $\hat{I} = \arg\max_{i \in N}\left\{\beta^{i*}(v^i)\right\}$ for the firms that have the highest $\beta^{i*}(v^i)$. Applying (21) to $i \in \hat{I}$ then yields

$$\mu^*(\mathbf{v}) = \nu_i^*(\mathbf{v}) + \beta^{i*}(v^i) - 1 > \nu_i^*(\mathbf{v}) \geq 0, \tag{27}$$

whereas applying (21) to firm $j \notin \hat{I}$ yields

$$1 + \mu^*(\mathbf{v}) - \nu_i^*(\mathbf{v}) = \beta^{i*}(v^i) > \beta^{j*}(v^j) = 1 + \mu^*(\mathbf{v}) - \nu_j^*(\mathbf{v}).$$

It follows that $\nu_j^*(\mathbf{v}) > \nu_i^*(\mathbf{v}) \geq 0$ for $i \in \hat{I}, j \notin \hat{I}$. Therefore, by complementary slackness (25), $\rho_j^*(\mathbf{v}) = 0$, so that only firms $i \in \hat{I}$ can receive a positive monetary prize: $\rho_j^*(\mathbf{v}) = 0$ for $j \notin \hat{I}$. Finally, the complementary slackness condition (24) yields

$$\sum_{i \in \hat{I}} \rho_i^*(\mathbf{v}) = \sum_{j \in N} \rho_j^*(\mathbf{v}) = \mathbb{E}_\theta\left[\sum_{k,i \in N} x_i^{k*}(\mathbf{v}, \theta)\left\{v^k - J_i(\theta_i)\right\}\right].$$

By Lemma 4, the total prize must be strictly positive for all $\mathbf{v}$ such that $v^k > \psi_i^k + \underline{\theta}$ for some k, i. Given the atomlessness of $F_i(\cdot|e)$ for all e, $\hat{I}$ is a singleton with probability one. Hence, for any $\mathbf{v}$ such that $v^k > \psi_i^k + \underline{\theta}$ for some k, i, and $\max_i\{\beta^{i*}(v^i)\} > 1$, with probability one only one firm receives the monetary prize.

Last, we derive the characterization of the optimal allocation rule. By the above argument, there exists at least one firm $i \in \hat{I}$ such that $\rho_i^*(\mathbf{v}) > 0$, and for that firm, (25) yields $\nu_i^*(\mathbf{v}) = 0$. However, then (21) applied to all $j \in \hat{I}$ along with the fact that $\beta^{i*}(v^i) = \beta^{j*}(v^j)$ for $i, j \in \hat{I}$ means that $\nu_i^*(\mathbf{v}) = 0$ for all $i \in \hat{I}$. It then follows that

$$1 + \mu^*(\mathbf{v}) = \max_i\{\beta^{i*}(v^i)\}.$$

We thus conclude that

$$\tilde{K}_i(\mathbf{v}, \theta_1) = J_i(\theta_i) - \left(\frac{\beta^{i*}(v^i)}{\max_k \beta^{k*}(v^k)}\right)\left(\frac{G_i(\theta_i)}{g_i(\theta_i)}\right) := K_i^*(v, \theta_1).$$

Online Appendix
Not for publication

A Forbidding Handicaps

We explore here how the optimal mechanism is modified when handicaps are ruled out. Specifically, we suppose that the innovator cannot be handicapped compared to the standard second best allocation. That is, for every v and θ:

$$x_1(v,\theta) \geq x_1^{SB}(v,\theta), \tag{NH}$$

where:

$$x_1^{SB}(v,\theta) := \begin{cases} 1 & \text{if } J_1(\theta_1) \leq \min\left\{v, \min_{j\neq 1} J_j(\theta_j)\right\}, \\ 0 & \text{otherwise.} \end{cases}$$

Letting $\alpha(v,\theta) \geq 0$ be the multiplier of the no-handicap constraint (NH), the Lagrangian becomes

$$L(v,e) := [1+\mu(v)] \left\{ \left[v - J_1(\theta_1) + \frac{\beta(v)}{1+\mu(v)} \frac{G_1(\theta_1)}{g_1(\theta_1)} + \frac{\alpha(v,\theta)}{1+\mu(v)} \right] x_1(v,\theta) + \sum_{\substack{j\in N \\ j\neq 1}} [v - J_j(\theta_j)]\, x_j(v,\theta) \right\}$$
$$- \rho_1(v)\left[1+\mu(v)-\nu(v)-\beta(v)\right] - \lambda c'(e) + \alpha(v,\theta)[x_1(v,\theta) - x_1^{SB}(v,\theta)]$$

and the additional complementary slackness is

$$\alpha(v,\theta)\left[x_1(v,\theta) - x_1^{SB}(v,\theta)\right] = 0. \tag{28}$$

The Lagrangian is still linear in x_i's, so the optimal allocation must satisfy, for every i, v, θ:

$$\bar{x}_i(v,\theta) = \begin{cases} 1 & \text{if } i \in \arg\min_j \left\{\bar{K}_j(v,\theta_j)\right\} \text{ and } \bar{K}_i(v,\theta_i) \leq v, \\ 0 & \text{otherwise,} \end{cases}$$

where the shadow cost is now given by:

$$\bar{K}_i(v,\theta_i) := \begin{cases} J_i(\theta_i) - \frac{\beta(v)}{1+\mu(v)}\frac{G_i(\theta_i)}{g_i(\theta_i)} - \frac{\alpha(v,\theta)}{1+\mu(v)} & \text{if } i = 1, \\ J_i(\theta_i) & \text{if } i \neq 1, \end{cases} \quad \text{with } \beta(v) := \lambda \frac{f_e(v|e)}{f(v|e)}.$$

When $v > \tilde{v}$, $\bar{K}_1(v,\theta_1) < J_1(\theta_1)$, and we can thus ignore the constraint (NH); hence $\alpha(v,\theta) = 0$, implying $\bar{K}_1(v,\theta_1) = K_1(v,\theta_1)$ and $\bar{x}_1(v,\theta) = x_1^*(v,\theta)$. Let us now consider the case $v < \tilde{v}$. If $\alpha(v,\theta) = 0$, the above characterization yields again $\bar{x}_1(v,\theta) = x_1^*(v,\theta)$, and $v < \tilde{v}$ then implies $\bar{K}_1(v,\theta_1) > J_1(\theta_1)$ and thus $\bar{x}_1(v,\theta) < x_1^{SB}(v,\theta)$ for at least some θs, contradicting (NH); therefore, we must have $\alpha(v,\theta) > 0$, and the complementary slackness condition (28) thus implies $\bar{x}_1(v,\theta) = x_1^{SB}(v,\theta)$, and thus $\bar{K}_1(v,\theta_1) = J_1(\theta_1)$.

The other constraints are unaffected; thus the optimal effort e must satisfy

$$\frac{\partial}{\partial e}\int_v\int_\theta L(v,\theta,e)f(v|e)dvdG(\theta) = 0,$$

and complementary slackness implies that, for each v,

$$\nu(v)\rho_1(v) = 0,$$
$$\mu(v)\left\{\int_\theta\sum_{i\in N}\bar{x}_i(v,\theta)\left[v - J_i(\theta_i)\right]dG(\theta) - \rho_1(v)\right\} = 0,$$

and

$$e\left[\int_v\int_\theta\left[\rho_1(v) + \frac{G_1(\theta)}{g_1(\theta)}\bar{x}_1(v,\theta)\right]g(\theta)d\theta f_e(v|e)dv - c'(e)\right] = 0.$$

Going through the same steps as before and summing up, we have:

- Ruling out handicaps implies that contract rights are allocated according to the standard second-best for low-value projects: For $v < \tilde{v}$, $\alpha(v,\theta) = -\beta(v)G_1(\theta_1)/g_1(\theta_1)\ (> 0)$ and $\bar{K}_i(v,\theta_i) = J_i(\theta_i)$ for all i (and thus, $\bar{x}_i(v,\theta) = x^{SB}(\theta)$ for all i as well).
- Ruling out handicaps has instead no impact on optimal contract rights for high-value projects: For $v > \tilde{v}$, $\alpha(v,\theta) = 0$ and $\bar{x}_i(v,\theta) = x_i^*(v,\theta)$ for all i.

In addition, forbidding handicaps does not affect the size of the monetary prize when such a prize is given:

- For $v < \hat{v}$, $\nu(v) = 1 - \beta(v) > 0$ and thus $\rho_1(v) = 0$ and $\mu(v) = 0$.
- For $v > \hat{v}$, $\nu(v) = 0$ and $\beta(v) = 1 + \mu(v)$, and thus $\bar{K}_1(v,\theta_1) = \theta_1$ and thus $\bar{x}_1(v,\theta) = x_1^*(v,\theta)$, based on $K_1(v,\theta_1) = \theta_1$ and $K_i(v,\theta_i) = J_i(\theta_i)$ for $i \neq 1$; it follows that

$$\rho_1(v) = \int_\theta\sum_{i\in N}x_i^*(v,\theta)\left[v - J_i(\theta_i)\right]dG(\theta) = \rho_1^*(v).$$

Note however that ruling out handicaps can affect the conditions under which a prize is given: banning handicaps alters the multiplier λ, which in turn affects the threshold $\hat{v}$, which is determined by the condition $\lambda f_e(v|e)/f(v|e) = 1$.

B Fixed allocation

We show here that our main insight carries over when the procurer is required to use the same tender rules whenever she decides to implement the project. The optimal mechanism relies on contract rights (possibly combined with monetary prizes) to induce the innovator to exert effort. Indeed, as long as the project is not always implemented, it is optimal to bias the implementation auction in favor of the innovator (handicaps instead should never be used).

Specifically, we consider a setup where, should the procurer wish to implement the project, the mechanism $(x, t) \in \Delta^n \times \mathbb{R}^n$ cannot depend on v. We can then simply denote by $x_i(\theta)$ the probability that firm i implements the project and by $t_i(\theta)$ the transfer payment that it receives.

The timing of the game is now as follows:

1. The buyer offers a mechanism specifying the allocation x and a payment t_i to each firm i.
2. The innovator chooses e; the value v is then realized.
3. The buyer observes v and decides whether to implement the project, in which case firms observe their costs and decide whether to participate.
4. Participating firms report their costs, the project is allocated (or not), and transfers are made according to the mechanism (x, t).

If the procurer decides to implement the project, firm i's expected profit no longer depends on the project value v, and can thus be written as

$$U_i(\theta_i) := T_i(\theta_i) - \theta_i X_i(\theta_i)$$

where $X_i(\theta_i) := \int_{\theta_{-i}} x_i(\theta)\, dG_{-i}(\theta_{-i})$ and $T_i(\theta_i) := \int_{\theta_{-i}} t_i(\theta)\, dG_{-i}(\theta_{-i})$. Using incentive compatibility, this expected profit can be expressed as

$$U_i(\theta_i) = \rho_i + \int_{\theta_i}^{\overline{\theta}} X_i(\theta) d\theta,$$

where

$$\rho_i := U_i(\overline{\theta}).$$

is the rent enjoyed by firm i when its cost is highest. As before, it is optimal to set $\rho_i = 0$ for $i \neq 1$, and thus the total expected transfer to the firms is given by

$$\int_\theta \sum_{i \in N} t_i(\theta)\, dG(\theta) = \rho_1 + \int_\theta \sum_{i \in N} x_i(\theta_i)\, J_i(\theta_i) dG(\theta),$$

where $J_i(\theta_i) := \theta_i + \frac{G_i(\theta_i)}{g_i(\theta_i)}$ denotes firm i's virtual cost. It follows that the procurer chooses to implement the project when:

$$\int_\theta \left\{ \sum_{i \in N} x_i(\theta) \left[v - J_i(\theta_i)\right] \right\} dG(\theta) \geq \rho_1.$$

As the left-hand side strictly increases with v, there exists a unique $\check{v} \in [\underline{v}, \bar{v}]$ such that this constraint is strictly satisfied if $v > \check{v}$, and violated if $v < \check{v}$.

Obviously, if $\check{v} = \bar{v}$, then the project is never implemented, and thus the innovator has no incentive to provide any effort. The assumption that $\bar{v} > \underline{\theta}$ guarantees that this is not optimal. Conversely, if $\check{v} = \underline{v}$ then the project is always implemented. This could be optimal if even low-value projects were still sufficiently desirable, but implies again the innovator has no incentive to provide any effort, as it obtains for sure the same information rents, regardless of the realized value of the project. From now on, we will focus on the case where the optimal threshold is interior, i.e., $\check{v} \in (\underline{v}, \bar{v})$. Let $\mu \geq 0$ denote the multiplier associated with the above constraint for $v = \check{v}$:

$$\int_\theta \left\{ \sum_{i \in N} x_i(\theta) \left[\check{v} - J_i(\theta_i)\right] \right\} dG(\theta) \geq \rho_1. \tag{$\widehat{LL}$}$$

The innovating firm's individual rationality boils down here to

$$\rho_1 \geq 0. \tag{$\widehat{IR}$}$$

Let $\nu \geq 0$ denote the multiplier associated with this constraint.

Finally, the first-order condition for the effort constraint becomes:

$$\int_{v \geq \check{v}} \int_\theta \left[\rho_1 + \frac{G_1(\theta_1)}{g_1(\theta_1)} x_1(\theta)\right] dG(\theta) f_e(v|e) dv \geq c'(e). \tag{$\widehat{MH}$}$$

Let $\lambda \geq 0$ be the associated multiplier.

The buyer's problem can then be formulated as follows:

$$\max_{e, \check{v}, x(\theta), \rho_1} \quad \int_{v \geq \check{v}} \left\{ \int_\theta \left[\sum_{i \in N} x_i(\theta) \left[v - J_i(\theta_i)\right]\right] dG(\theta) - \rho_1 \right\} f(v|e) dv$$

subject to ($\widehat{LL}$), ($\widehat{IR}$) and ($\widehat{MH}$)

The Lagrangian is given by:

$$
\begin{aligned}
L &= \int_{v\geq\check{v}} \left\{ \int_\theta \left[\sum_{i\in N} x_i(\theta) \left[v - J_i(\theta_i) \right] \right] dG(\theta) - \rho_1 \right\} f(v|e)dv \\
&+\mu \left[\int_\theta \left\{ \sum_{i\in N} x_i(\theta) \left[\check{v} - J_i(\theta_i) \right] \right\} dG(\theta) - \rho_1 \right] \\
&+\nu\rho_1 + \lambda \left[\int_{v\geq\check{v}} \int_\theta \left[\rho_1 + \frac{G_1(\theta_1)}{g_1(\theta_1)} x_1(\theta) \right] dG(\theta) f_e(v|e)dv - c'(e) \right].
\end{aligned}
$$

Re-arranging terms, it can be expressed as $L = \int_\theta L(\theta, e, \check{v}) dG(\theta)$, where

$$
\begin{aligned}
L(\theta, e, \check{v}) &:= \left[1 - F(\check{v}|e) + \mu\right] \left\{ \left[\check{v}^e - J_1(\theta_1) + \frac{\beta^e}{1+\check{\mu}} \frac{G_1(\theta_1)}{g_1(\theta_1)} \right] x_1(\theta) + \sum_{\substack{i\in N \\ i\neq 1}} \left[\check{v}^e - J_i(\theta_i) \right] x_i(\theta) \right\} \\
&\quad - \left[1 - F(\check{v}|e)\right] \rho_1 \left(1 - \beta^e + \check{\mu} - \check{\nu}\right) - \lambda c'(e).
\end{aligned}
$$

where:

$$
\check{\mu} := \frac{\mu}{1 - F(\check{v}|e)} \text{ and } \check{\nu} := \frac{\nu}{1 - F(\check{v}|e)}
$$

denote the weighted value of the Lagrangian multipliers μ and ν (weighted by the probability of implementing the project), and:

$$
\beta^e := \lambda \int_{v\geq\check{v}} \frac{f_e(v|e)}{1 - F(\check{v}|e)} dv \text{ and } \check{v}^e := \frac{v^e + \check{\mu}\check{v}}{1+\check{\mu}},
$$

where

$$
v^e := \int_{v\geq\check{v}} v \frac{f(v|e)}{1 - F(\check{v}|e)} dv.
$$

The optimal solution $(e^*, \check{v}^*, x^*(\theta), \rho_1^*, \lambda^*, \mu^*, \nu^*)$ must satisfy the following necessary conditions. First, observe that the Lagrangian L is linear in $\rho_1(v)$; hence, its coefficient must be equal to zero:

$$
1 - \beta^{e*} + \check{\mu}^* - \check{\nu}^* = 0, \tag{29}
$$

where $\check{\mu}^*$ and $\check{\nu}^*$ denote the optimal values of the weighted multipliers, and

$$
\beta^{e*} := \frac{\lambda^*}{1 - F(\check{v}|e^*)} \int_{v\geq\check{v}} f_e(v|e^*) dv.
$$

The Lagrangian is also linear in x_i's, so the optimal allocation must satisfy, for every i, v, θ:

$$
x_i^*(\theta) = \begin{cases} 1 & \text{if } i \in \arg\min_j \left\{ \tilde{K}_j(\theta_j) \right\} \text{ and } \tilde{K}_i(\theta_i) \leq \frac{v^e + \check{\mu}^*\check{v}}{1+\check{\mu}^*}, \\ 0 & \text{otherwise,} \end{cases}
$$

where

$$\tilde{K}_i(\theta_i) := \begin{cases} J_i\left(\theta_i\right) - \frac{\beta^{e*}}{1+\breve{\mu}^*}\frac{G_i(\theta_i)}{g_i(\theta_i)} & \text{if } i = 1, \\ J_i\left(\theta_i\right) & \text{if } i \neq 1. \end{cases} \tag{30}$$

We next prove that $\lambda^* > 0$. Suppose $\lambda^* = 0$, which implies $\beta^{e*} = 0$. Together with (30) and (29), this yields

$$L(\theta, e, \breve{v}^*) = [1 - F(\breve{v}^*|e) + \mu^*] \int_\theta \max\left\{0, \breve{v}^{e*} - \min_i J_i\left(\theta_i\right)\right\} dG\left(\theta\right),$$

and thus

$$\frac{\partial}{\partial e}\int_\theta L(\theta, e, \breve{v}^*) dG(\theta)\bigg|_{e=e^*} = -F_e(\breve{v}^*|e^*) \int_\theta \max\left\{0, \breve{v}^{e*} - \min_i J_i\left(\theta_i\right)\right\} dG(\theta), \tag{31}$$

where, in the right-hand side:

- The first term, $-F_e(\breve{v}^*|e^*)$, as

$$-F_e(\breve{v}|e) > 0. \tag{32}$$

  for any $\breve{v}^* \in (\underline{v}, \bar{v})$. To see this, note that

$$-F_e(\breve{v}|e) = \frac{\partial}{\partial e}[1 - F(\breve{v}|e)] = \int_{v \geq \breve{v}} f_e(v|e) dv,$$

  where from (*MLRP*), $f_e(v|e) > 0$ for $v > \tilde{v}$ and $f_e(v|e) < 0$ for $v < \tilde{v}$. Therefore, if $\breve{v} \geq \tilde{v}$, then $\int_{v \geq \breve{v}} f_e(v|e) dv > 0$. If instead $\breve{v} < \tilde{v}$, then $\int_{v < \breve{v}} f_e(v|e) dv < 0$; but by construction,

$$\int_{v \geq \breve{v}} f_e(v|e) dv + \int_{v < \breve{v}} f_e(v|e) dv = \frac{\partial}{\partial e}\int f(v|e) dv = 0,$$

  implying again that $\int_{v \geq \breve{v}} f_e(v|e) dv > 0$.

$$\int_\theta \sum_{i \in N} x_i(\theta)\left[\breve{v} - J_i(\theta_i)\right] dG(\theta) \geq \rho_1$$

- The second term is also positive. Indeed, we have:

$$\begin{aligned} \int_\theta \max\left\{0, \breve{v}^{e*} - \min_i J_i\left(\theta_i\right)\right\} dG(\theta) &> \int_\theta \max\left\{0, \breve{v} - \min_i J_i\left(\theta_i\right)\right\} dG(\theta) \\ &\geq \int_\theta \sum_{i \in N} x_i^*(\theta)\left[\breve{v} - J_i(\theta_i)\right] dG(\theta) \\ &\geq 0, \end{aligned}$$

  where the first inequality stems $\breve{v}^e > \breve{v}$ for any $\breve{v}^* < \bar{v}$, and the last one follows from (*LL*) and (*IR*).

It follows that the right-hand side of (31) is positive, and thus

$$\frac{\partial}{\partial e}\int_\theta L(\theta, e, \check{v}^*)dG(\theta)\Big|_{e=e^*} > 0,$$

which violates the optimality of e^*. We thus conclude that $\lambda^* > 0$.

Given $\lambda^* > 0$, (32) implies $\beta^{e*} > 0$. It then follows from (30) that *the innovator benefits from a favorable bias in the allocation of the contract rights.*

Finally, complementary slackness implies that, for each v,

$$\check{\nu}^*\rho_1^* = 0, \tag{33}$$

$$\check{\mu}^*\left\{\int_\theta \sum_{i\in N} x_i^*(\theta)\left[v - J_i(\theta_i)\right]dG(\theta) - \rho_1^*\right\} = 0, \tag{34}$$

When $(0 <)\,\beta^{e*} < 1$, we have:

$$1 - \beta^{e*} + \check{\mu}^* > \check{\mu}^* \geq 0,$$

and (29) thus implies $\check{\nu}^* > 0$. The complementary slackness condition (33) then yields $\rho_1^* = 0$.

When instead $\beta^{e*} > 1$ we have:

$$1 - \beta^{e*} - \check{\nu}^* < 0,$$

and (29) thus implies that $\check{\mu}^*(v) > 0$; from the complementary slackness condition (34), we thus have

$$\rho_1^*(v) = \int_\theta \sum_{i\in N} x_i^*(\theta)\left[\check{v}^* - J_i(\theta_i)\right]dG(\theta).$$