Tullio Gregori
Università degli Studi di Trieste,
P.le Europa 1, 34127
Trieste Italy
(email: tgregori@units.it
tullio.gregori@deams.units.it)

International Value Added Trade: a subsystem approach

Abstract

The structure of international trade has changed dramatically in recent decades with the emergence of supply chains that span multiple countries. The growth of these chains is posing new challenges to studies of countries' competitiveness and the analysis of international trade based on the concept of vertical integration has trigged attention from both the empirical and theoretical perspective. A growing applied literature, that embraces the Leontief model, has produced a large body of interesting results, complementing the standard gross flow analyses. Value Added in Trade measures value added embodied in gross trade flows, while Trade in Value Added accounts for value added of one country directly and indirectly embodied in final consumption of another country. Such a task has been performed without using the device of representing world interindustry relationships via vertically integrated sectors or subsystems. Subsystems allow us to split the economic system into ideal self-sufficient sectors, which produce a single final commodity with all their necessary intermediate inputs. In this paper, I fill this gap showing how this approach provides additional information that can shed light on the process of international production fragmentation.

Keywords: Value added in trade, Trade in value added, Vertical integrated sectors, Subsystems.

DRAFT

1. Introduction

Over the past decades, world trade has grown faster than both world output and manufacturing value added. Actually, trade as a share of global output has tripled since the 1950s. "From the steam revolution till the mid or late 1980s, globalization was mostly about falling trade costs; this was globalisation's first unbundling" (Baldwin, 2013, p. 166). Reduction in transaction costs and better international environment and institutions fostered an unprecedented globalization. Current and capital account liberalizations facilitated trade beyond arm's-length operations opening the way to large foreign direct investment in developing and new accession countries too. However, the simple communication technology available until the mid-1980s forced most of manufacturing processes to be located within the same area. Proximity was essential to coordinating and all the stages of production had to be inside a single factory or industrial district. Most of the necessary competencies had to exist domestically. The digital revolution of the 1980s has entirely changed the picture. It allows supply chains to be increasingly sliced up into smaller and smaller components. This process has been called in the literature with a wide range of names, including fragmentation, outsourcing, offshoring, production disintegration or relocation, international product sharing or segmentation, intra-product specialization, multi-stage production, to quote a few. All of them indicate that many inputs are outsourced to specialized subcontractors that are located abroad. Nowadays, several products and services are no longer produced within a single country but made in global supply chains. Intra and inter industry trade accounts for a large part of trade growth, signaling the importance of such a phenomenon defined as "the second wave of global unbundling" (Baldwin, 2013). The increasing fragmentation is changing the nature of international labor division and it has radically transformed the landscape worldwide. International competition is no longer a country to country but a sector to sector or even a product to product contest.

Within this picture, it is generally believed standard trade figures that measure the value of imports and exports do not reflect any more what is really happening (Daudin et al., 2011, Timmer et. al., 2013, Amador and Di Mauro, 2015). Furthermore, it undermines the conventional indicators of competiveness and openness. Gross export is no longer a useful measure as flows are appraised on their market value when they cross borders. When these goods contain a large share of imported inputs, their market value can be very large with respect to local commodities. This is the likely reason why some countries exhibit excessive export-to-GDP ratio exceeding even 100% as in Singapore, Luxemburg, Malta and Estonia. This ratio and the share in world export market are the very first statistical indicator used to assess the importance of international trade in a country, but they tend to overestimate the competitiveness of economies that rely heavily on imported raw materials and intermediate inputs.

To overcome these issues the concepts of Trade in Value Added (VAiT) and Value Added in Trade (TiVA) have been put forward (Escaith and Inomata, 2011, Johnson and Noguera, 2012, Foster-McGregor and Stehrer, 2013, Koopman *et al.* 2014). The former accounts for the value added of one country directly and indirectly contained in final consumption of another nation. The latter calculates value added contained in gross trade flows. However, the national income accounting identity states the country's trade deficit or surplus must be the same and equal to the gross one (Benedetto 2012, Stehrer, 2012, Kuboniwa 2014). Bilateral TiVA is defined as the country's valued added induced by the destination country's final demand, excluding intermediate imports, while VAiT is the country's origin value added induced by its gross export to the destination country, including exports of intermediate commodities. Actually, they are equivalent when VAiT is measured by the value added content of trade as proposed by Trefler and Zhu (2010).

This literature shows there are large and growing differences between gross and value added exports, around 25%, quite heterogeneous across countries and bilateral partners (Johnson, 2014). Several indices and decomposition techniques have been proposed to address these differences (Koopman *et al.* 2014), international fragmentation (Los *et al.* 2015) or the "great trade collapse" (Nagengast and Stehrer, 2016). Value added international trade has never been addressed with the device of representing world interindustry relationships via vertically integrated sectors or subsystems as suggested by Sraffa (1960) and Pasinetti (1973). The method of the subsystems is a way to subdivide a natural system "*into as many parts as there commodities in its net product, in such a way that each part forms a smaller self-replacing system the net product of which consist of only one kind of commodity*" (Sraffa, 1960, p. 89). The analysis is no longer carried out in terms of industries or branches only, but in terms of vertically integrated sectors. Industry-subsystems tables show both gross and net (value added) inputs required directly and indirectly to support actual final demand in all the countries. It is also possible to introduce an operator that "*allows us to reclassify any variable linked to production from branches to subsystems and to attribute the value of this variable to each final commodity, observing the industries of origin thereof*" (Momigliano and Siniscalco, 1984, p. 260). In this paper I adopt the value added perspective, but we can use factor content too (Montresor and Vittucci Marzetti, 2011, Garbellini and Wickiermann, 2014). The present paper is organized as follows. Section 2 gives on overview of the standard measure of VAiT and TiVA. In section 3 I introduce the Sraffian approach and modify it in order to fit into the international trade framework. Section 4 presents a simple numerical example and section 5 concludes and illustrates scope for future research.

2. Standard measures of value added trade

We assume there are R countries each of which produces n commodities. We adopt the standard notation in multiregional analysis (Miller and Blair, 2006) so that z_{ij}^{cs} denote the value flow of goods from sector i in country c to sector j in region s. Just as the order of subscripts is "from–to" with respect to sectors, the order of superscripts indicates "from–to" with respect to geographic locations. The world industry-by-industry flow matrix is $\mathbf{Z} = \left[\mathbf{Z}^{cs}\right] = \left[z_{ij}^{cs}\right]$. Define the overall ($NR$×1) output and final demand vectors as $\boldsymbol{x}$ and $\boldsymbol{y}$, whose (N×1) country components are $\boldsymbol{x}^c$ and $\boldsymbol{y}^c$. Input coefficients are obtained from normalizing the industry columns $\mathbf{A} = \mathbf{Z}\,\hat{\boldsymbol{x}}^{-1}$. This yields the standard World Input Output model:

$$\boldsymbol{x} = \left(\mathbf{I} - \mathbf{A}\right)^{-1} \boldsymbol{y} = \mathbf{B}\sum_{i=1}^{R} \boldsymbol{f}^{\,i} \tag{1}$$

where:

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}^1 \\ \vdots \\ \boldsymbol{x}^c \\ \vdots \\ \boldsymbol{x}^R \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{A}^{11} & \cdots & \mathbf{A}^{1c} & \cdots & \mathbf{A}^{1R} \\ \vdots & & & & \\ \mathbf{A}^{c1} & \cdots & \mathbf{A}^{cc} & \cdots & \mathbf{A}^{cR} \\ \vdots & & & & \\ \mathbf{A}^{R1} & \cdots & \mathbf{A}^{Rc} & \cdots & \mathbf{A}^{RR} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{B}^{11} & \cdots & \mathbf{B}^{1c} & \cdots & \mathbf{B}^{1R} \\ \vdots & & & & \\ \mathbf{B}^{c1} & \cdots & \mathbf{B}^{cc} & \cdots & \mathbf{B}^{cR} \\ \vdots & & & & \\ \mathbf{B}^{R1} & \cdots & \mathbf{B}^{Rc} & \cdots & \mathbf{B}^{RR} \end{bmatrix},$$

and the overall final demand vector is obtained by the following country aggregation:

$$\mathbf{y}=\begin{bmatrix}\mathbf{y}^1\\ \vdots\\ \mathbf{y}^c\\ \vdots\\ \mathbf{y}^R\end{bmatrix}=\begin{bmatrix}\mathbf{y}^{11}\\ \vdots\\ \mathbf{y}^{c1}\\ \vdots\\ \mathbf{y}^{R1}\end{bmatrix}+\cdots+\begin{bmatrix}\mathbf{y}^{1c}\\ \vdots\\ \mathbf{y}^{cc}\\ \vdots\\ \mathbf{y}^{Rc}\end{bmatrix}+\cdots+\begin{bmatrix}\mathbf{y}^{1R}\\ \vdots\\ \mathbf{y}^{cR}\\ \vdots\\ \mathbf{y}^{RR}\end{bmatrix}=\boldsymbol{f}^1+\cdots+\boldsymbol{f}^c+\cdots+\boldsymbol{f}^R$$

Johnson and Noguera (2012) focus on the vector of output used both directly and indirectly to produce final goods absorbed in country c:

$$\boldsymbol{q}^c=\left(\mathbf{I}-\mathbf{A}\right)^{-1}\boldsymbol{f}^c \tag{2}$$

with $\boldsymbol{x}=\sum_{c=1}^{R}\boldsymbol{q}^c$. Actually, as suggested by Koopman et al. (2014), we can arrange the Leontief model in a different way:

$$\begin{bmatrix}\boldsymbol{q}^{11}\ \boldsymbol{q}^{12}\cdots\boldsymbol{q}^{1R}\\ \boldsymbol{q}^{21}\ \boldsymbol{q}^{22}\cdots\boldsymbol{q}^{2R}\\ \vdots\quad\vdots\ \cdots\ \vdots\\ \boldsymbol{q}^{R1}\ \boldsymbol{q}^{R2}\cdots\boldsymbol{q}^{RR}\end{bmatrix}=\begin{bmatrix}\mathbf{B}^{11}\ \mathbf{B}^{12}\cdots\mathbf{B}^{1R}\\ \mathbf{B}^{21}\ \mathbf{B}^{22}\cdots\mathbf{B}^{2R}\\ \vdots\quad\vdots\ \cdots\ \vdots\\ \mathbf{B}^{R1}\ \mathbf{B}^{R2}\cdots\mathbf{B}^{RR}\end{bmatrix}\begin{bmatrix}\boldsymbol{y}^{11}\ \boldsymbol{y}^{12}\cdots\boldsymbol{y}^{1R}\\ \boldsymbol{y}^{21}\ \boldsymbol{y}^{22}\cdots\boldsymbol{y}^{2R}\\ \vdots\quad\vdots\ \cdots\ \vdots\\ \boldsymbol{y}^{R1}\ \boldsymbol{y}^{R2}\cdots\boldsymbol{y}^{RR}\end{bmatrix} \tag{3}$$

or

$$\left[\boldsymbol{q}^1\cdots\boldsymbol{q}^c\cdots\boldsymbol{q}^R\right]=\left(\mathbf{I}-\mathbf{A}\right)^{-1}\left[\boldsymbol{f}^1\cdots\boldsymbol{f}^c\cdots\boldsymbol{f}^R\right] \tag{4}$$

$$\mathbf{Q}=\left(\mathbf{I}-\mathbf{A}\right)^{-1}\mathbf{F} \tag{4bis}$$

where both the gross output decomposition **Q** and final demand matrices **F** are ($NR\times R$). In (3) $\boldsymbol{q}^{cs}$ and $\boldsymbol{y}^{cs}$ are ($N\times1$) gross output and final demand vectors produced in country c and absorbed in s. Of course, $\boldsymbol{q}^c=\sum_{s=1}^{R}\boldsymbol{q}^{cs}$ $\boldsymbol{y}^c=\sum_{s=1}^{R}\boldsymbol{y}^{cs}$. Let $\boldsymbol{v}=\boldsymbol{u}_{RN}\left(\mathbf{I}\text{ - }\mathbf{A}\right)$, where $\boldsymbol{u}_{RN}$ is the ($1\times NR$) unit vector[1], and $\hat{\boldsymbol{v}}$ the ($NR\times NR$) diagonal matrix with direct value-added coefficients along the diagonal. Then, we obtain the value-added production matrix:

$$\hat{\boldsymbol{v}}\mathbf{BF}=\begin{bmatrix}\hat{\boldsymbol{v}}^1\ 0\cdots 0\\ 0\ \hat{\boldsymbol{v}}^2\cdots 0\\ \vdots\ \vdots\cdots\vdots\\ 0\ 0\cdots\hat{\boldsymbol{v}}^R\end{bmatrix}\begin{bmatrix}\boldsymbol{q}^{11}\ \boldsymbol{q}^{12}\cdots\boldsymbol{q}^{1R}\\ \boldsymbol{q}^{21}\ \boldsymbol{q}^{22}\cdots\boldsymbol{q}^{2R}\\ \vdots\quad\vdots\ \cdots\ \vdots\\ \boldsymbol{q}^{R1}\ \boldsymbol{q}^{R2}\cdots\boldsymbol{q}^{RR}\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{v}}^1\sum_{g=1}^{R}\mathbf{B}^{1g}\boldsymbol{y}^{g1} & \cdots & \hat{\boldsymbol{v}}^1\sum_{g=1}^{R}\mathbf{B}^{1g}\boldsymbol{y}^{gR}\\ \hat{\boldsymbol{v}}^2\sum_{g=1}^{R}\mathbf{B}^{2g}\boldsymbol{y}^{g1} & \cdots & \hat{\boldsymbol{v}}^2\sum_{g=1}^{R}\mathbf{B}^{2g}\boldsymbol{y}^{gR}\\ \vdots & & \vdots\\ \hat{\boldsymbol{v}}^R\sum_{g=1}^{R}\mathbf{B}^{Rg}\boldsymbol{y}^{g1} & \cdots & \hat{\boldsymbol{v}}^R\sum_{g=1}^{R}\mathbf{B}^{Rg}\boldsymbol{y}^{gR}\end{bmatrix} \tag{5}$$

whose block elements in the diagonal give each country's production of value added absorbed at home, while off diagonal block elements show value added produced in country and absorbed abroad. The rows

[1] In the following, I will drop subscript of the unit vector that conforms case by case.

of $\hat{\boldsymbol{v}}\mathbf{BF}$ sum to the total value added, while the columns add up to the total value added absorbed in a given country. Value added exports from country *c* to *s* and worldwide are:

$$\boldsymbol{e}_{VA}^{cs} = \hat{\boldsymbol{v}}^{c}\sum_{g=1}^{R}\mathbf{B}^{cg}\boldsymbol{y}^{gs} \tag{6}$$

$$\boldsymbol{e}_{VA}^{c} = \hat{\boldsymbol{v}}^{c}\sum_{s\neq c}^{R}\sum_{g=1}^{R}\mathbf{B}^{cg}\boldsymbol{y}^{gs} \tag{7}$$

Similarly, country *c* imports value added produced in country *s* and worldwide:

$$e_{VA}^{sc} = \hat{\boldsymbol{v}}^{s}\sum_{g=1}^{R}\mathbf{B}^{sg}\boldsymbol{y}^{gc} \tag{8}$$

$$\boldsymbol{m}_{VA}^{c} = \hat{\boldsymbol{v}}^{s}\sum_{s\neq c}^{R}\sum_{g=1}^{R}\mathbf{B}^{sg}\boldsymbol{y}^{gs} \tag{9}$$

Country *c* bilateral value added trade balance with country *s* is:

$$\boldsymbol{t}_{VA}^{cs} = \boldsymbol{u}\hat{\boldsymbol{v}}^{c}\sum\nolimits_{g=1}^{R}\mathbf{B}^{cg}\boldsymbol{y}^{gs} - \boldsymbol{u}\hat{\boldsymbol{v}}^{s}\sum\nolimits_{g=1}^{R}\mathbf{B}^{sg}\boldsymbol{y}^{gc}\ . \tag{10}$$

Finally, we can figure out country *c* gross exports to country *s*:

$$\boldsymbol{e}_{G}^{cs} = \mathbf{A}^{cs}\boldsymbol{x}^{cs} + \boldsymbol{y}^{cs} \tag{11}$$

and bilateral gross trade balance:

$$\boldsymbol{t}_{G}^{cs} = \boldsymbol{u}\left(\mathbf{A}^{cs}\boldsymbol{x}^{s} + \boldsymbol{y}^{cs}\right) - \boldsymbol{u}\left(\mathbf{A}^{sc}\boldsymbol{x}^{c} + \boldsymbol{y}^{sc}\right). \tag{12}$$

It is easy to check that:

$$\sum_{s=1}^{R}\boldsymbol{t}_{VA}^{cs} = \sum_{s=1}^{R}\boldsymbol{t}_{G}^{cs} \tag{13}$$

Kuboniwa (2014) calls it the fundamental theorem, but (13) simply reaffirms the national income accounting identity, as GDP minus domestic demand is equal to gross net trade. However, sectoral trade balances in value added can differ from those in gross terms even if (13) implies the overall sum is equal to zero:

$$\sum_{s=1}^{R}\left(\boldsymbol{t}_{VA}^{cs} - \boldsymbol{t}_{G}^{cs}\right) = 0 \tag{14}$$

Following Stehrer (2012) we can embrace a different approach to figure out value added exports and imports to and from all other countries:

$$e_{VA}^{c} = \begin{bmatrix} 0 & \cdots & \hat{v}^{c} & \cdots & 0 \end{bmatrix} \begin{bmatrix} \mathbf{B}^{11}\cdots\mathbf{B}^{1R} \\ \mathbf{B}^{21}\cdots\mathbf{B}^{2R} \\ \vdots \ \cdots \ \vdots \\ \mathbf{B}^{R1}\cdots\mathbf{B}^{RR} \end{bmatrix} \begin{bmatrix} \boldsymbol{y}^{11} + \cdots + \boldsymbol{y}^{1c-1} + 0 + \boldsymbol{y}^{1c+1} + \cdots \boldsymbol{y}^{1R} \\ \boldsymbol{y}^{21} + \cdots + \boldsymbol{y}^{2c-1} + 0 + \boldsymbol{y}^{2c+1} + \cdots \boldsymbol{y}^{2R} \\ \vdots \\ \boldsymbol{y}^{R1} + \cdots + \boldsymbol{y}^{Rc-1} + 0 + \boldsymbol{y}^{Rc+1} + \cdots \boldsymbol{y}^{RR} \end{bmatrix} \tag{15}$$

$$m_{VA}^{c} = \begin{bmatrix} \hat{v}^{1} & \cdots & \hat{v}^{c-1} & 0 & \hat{v}^{c+1} & \cdots & \hat{v}^{R} \end{bmatrix} \begin{bmatrix} \mathbf{B}^{11} \cdots \mathbf{B}^{1R} \\ \mathbf{B}^{21} \cdots \mathbf{B}^{2R} \\ \vdots \cdots \vdots \\ \mathbf{B}^{R1} \cdots \mathbf{B}^{RR} \end{bmatrix} \begin{bmatrix} \boldsymbol{y}^{1c} \\ \boldsymbol{y}^{2c} \\ \vdots \\ \boldsymbol{y}^{Rc} \end{bmatrix} \tag{16}$$

It is easy to check (16) and (7) are equivalent since:

$$e_{VA}^{c} = \begin{bmatrix} \hat{v}^{c}\mathbf{B}^{c1} & \cdots & \hat{v}^{c}\mathbf{B}^{cc} & \cdots & \hat{v}^{c}\mathbf{B}^{cR} \end{bmatrix} \begin{bmatrix} \sum_{s\neq c}^{R} \boldsymbol{y}^{1s} \\ \sum_{s\neq c}^{R} \boldsymbol{y}^{2s} \\ \vdots \\ \sum_{s\neq c}^{R} \boldsymbol{y}^{Rs} \end{bmatrix} = \hat{v}^{c} \sum_{g=1}^{R} \mathbf{B}^{cg} \sum_{s\neq c}^{R} \boldsymbol{y}^{gs} = \hat{v}^{c} \sum_{s\neq c}^{R} \sum_{g=1}^{R} \mathbf{B}^{cg} \boldsymbol{y}^{gs} \tag{17}$$

Actually, Stehrer (2012) deems special value added and final demand vectors:

$$\boldsymbol{v}_{D}^{c} = \begin{bmatrix} 0 & \cdots & \boldsymbol{v}^{c} & \cdots & 0 \end{bmatrix}, \ \ \boldsymbol{v}_{E}^{c} = \begin{bmatrix} \boldsymbol{v}^{1} & \cdots & \boldsymbol{v}^{c-1} & 0 & \boldsymbol{v}^{c+1} & \cdots & \boldsymbol{v}^{R} \end{bmatrix} \tag{18}$$

$$\boldsymbol{f}_{E}^{c} = \boldsymbol{f}^{1} + \cdots + \boldsymbol{f}^{c-1} + \boldsymbol{f}^{c+1} + \cdots \boldsymbol{f}^{R} \tag{19}$$

such that $\boldsymbol{v}^{c} = \boldsymbol{v}_{D}^{c} + \boldsymbol{v}_{E}^{c}$, $\boldsymbol{y} = \boldsymbol{f}^{c} + \boldsymbol{f}_{E}^{c}$. Nagengast and Stehrer (2016) call value added exports as:

$$\mathrm{VAX}^{c} = \boldsymbol{v}_{D}^{c}\mathbf{B}\boldsymbol{f}_{E}^{c} = \boldsymbol{u}\, e_{VA}^{c} \tag{20}$$

that is value added of country *c* which is absorbed in final demand abroad. Similarly, we can define imports and trade balance in compact form:

$$\mathrm{VAM}^{c} = \boldsymbol{v}_{E}^{c}\mathbf{B}\boldsymbol{f}^{c} = \boldsymbol{u}\, m_{VA}^{c} \tag{21}$$

$$t_{VA}^{c} = e_{VA}^{c} - m_{VA}^{c} = \boldsymbol{v}_{D}^{c}\mathbf{B}\boldsymbol{f}_{E}^{c} - \boldsymbol{v}_{E}^{c}\mathbf{B}\boldsymbol{f}^{c} = \boldsymbol{v}_{D}^{c}\boldsymbol{x} - \boldsymbol{v}\mathbf{B}\boldsymbol{f}^{c} = GDP^{c} - \boldsymbol{u}\ \boldsymbol{f}^{c} = NX^{c} \tag{22}$$

Eq. (22) states the national income accounting identity once more. With this notation, bilateral value added and net value exports can be written as:

$$\boldsymbol{u}_{NR} e_{VA}^{cs} = \boldsymbol{v}_{D}^{c}\mathbf{B}\boldsymbol{f}^{s}, \quad t_{VA}^{cs} = \boldsymbol{v}_{D}^{c}\mathbf{B}\boldsymbol{f}^{s} - \boldsymbol{v}_{D}^{s}\mathbf{B}\boldsymbol{f}^{c} \tag{23}$$

The latter equals a country's net exports in gross terms as in (10). The former can be obtained by (6) too. Los et al. (2015) adopt a similar approach to identify vectors of value added involved by a value chain as they deem $\hat{\boldsymbol{v}}\boldsymbol{B}\tilde{\boldsymbol{F}}\boldsymbol{u}$. They consider a particular final demand matrix $\tilde{\boldsymbol{F}}$ "*in which only the cells in the row representing final demand for the country-industry (i,j) have their actual values and all other final demand is set to* 0". Los et al. (2015, p. 88). This implies $\tilde{\boldsymbol{F}}\boldsymbol{u}$ is a vector with a single positive element, which is obtained by adding domestic and foreign final demand for (*i*,*j*)'s products.

Another strand of literature focuses on trade in factors. Following Trefler and Zuh (2010), the content of value added as contained in a country's gross exports and imports ("Value Added in Trade") is given by:

$$\begin{bmatrix} \hat{v}^1\mathbf{B}^{11} & \cdots & \hat{v}^1\mathbf{B}^{1c} & \cdots & \hat{v}^1\mathbf{B}^{1R} \\ \vdots & & & & \vdots \\ \hat{v}^c\mathbf{B}^{c1} & & \hat{v}^c\mathbf{B}^{cc} & & \hat{v}^c\mathbf{B}^{cR} \\ \vdots & & & & \vdots \\ \hat{v}^R\mathbf{B}^{R1} & \cdots & \hat{v}^R\mathbf{B}^{Rc} & \cdots & \hat{v}^R\mathbf{B}^{RR} \end{bmatrix} \begin{bmatrix} -\boldsymbol{e}_G^{1c} \\ \vdots \\ \sum_{g\neq c}^{R}\boldsymbol{e}_G^{cg} \\ \vdots \\ -\boldsymbol{e}_G^{Rc} \end{bmatrix} = \begin{bmatrix} -\hat{v}^1\mathbf{B}^{11}\boldsymbol{e}_G^{1c} + \cdots + \hat{v}^1\mathbf{B}^{1c}\boldsymbol{e}_G^{c} + \cdots - \hat{v}^1\mathbf{B}^{1R}\boldsymbol{e}_G^{Rc} \\ \vdots \\ -\hat{v}^c\mathbf{B}^{c1}\boldsymbol{e}_G^{1c} + \cdots + \hat{v}^c\mathbf{B}^{cc}\boldsymbol{e}_G^{c} + \cdots - \hat{v}^c\mathbf{B}^{cR}\boldsymbol{e}_G^{Rc} \\ \vdots \\ -\hat{v}^R\mathbf{B}^{R1}\boldsymbol{e}_G^{1c} + \cdots + \hat{v}^R\mathbf{B}^{Rc}\boldsymbol{e}_G^{c} + \cdots - \hat{v}^R\mathbf{B}^{RR}\boldsymbol{e}_G^{Rc} \end{bmatrix} \quad (24)$$

with $\boldsymbol{e}_G^c = \sum_{g\neq c}^{R}\boldsymbol{e}_G^{cg}$. However, Kuboniwa (2014) proves, in a G = 3 system, the output transfer equation implied in (24) admit a unique output solution, which is the same as in (1). This author also suggests the same holds for the general model, as his empirical analysis indeed shows. Hence, value added in trade and trade in value added are equivalent measures. In the following section, I will show how to expand the TiVA framework in order to assess which final commodities are actually exported in value added terms.

3. A Sraffian approach

Sraffa (1960) defines a subsystem as gross production needed to supply final demand to a single industry. Hence, we can rewrite (1) by considering a diagonal matrix of final demand, say $\hat{\boldsymbol{y}}$, instead of the column vector $\boldsymbol{y}$. The corresponding ($NR\times NR$) production matrix of vertically integrated sectors is:

$$\mathbf{X} = \left(\mathbf{I}-\mathbf{A}\right)^{-1}\hat{\boldsymbol{y}} = \mathbf{B}\,\hat{\boldsymbol{y}} \;. \quad (25)$$

Since each entry in the Leontief inverse measures impact in domestic output due to a unit change in final demand, any column in vertically integrated sector production matrix depicts output activated by a single component of final demand. There is a vertically integrated sector for each commodity produced in every country. If $\hat{\boldsymbol{y}}$ is formed by a zero column vector but one deviating element equal to one, as in Los et al. (2015), than any column in $\mathbf{X}$ is simply a multi-sector output multiplier, as each b_{ij}^{cs} specifies how much sector i in country c must produce in order to get one unit of the j-th final demand absorbed in country s. In other words, if we set $\hat{\boldsymbol{y}} = \mathbf{I}$, we obtain a decomposition by vertically integrated sector of the standard Type 1 multiplier (Schnabl, 1995). On the other hand, for any final demand vector, eq. (25) allows to get a useful decomposition of total production activated by a country's final demand bundle, since each column of the matrix $\mathbf{X}$ represents production needed, directly and indirectly, to satisfy final demand for the corresponding industry. It is interesting to notice that columns of $\mathbf{X}$ comprise subsystems of the economy, while rows show how output by a single sector is distributed to satisfy all final demand items.

Momigliano and Siniscalco (1982) introduced the **S**-operator. This operator is obtained dividing the matrix $\mathbf{X}$ row by row by the corresponding total output:

$$\mathbf{S} = \hat{\boldsymbol{x}}^{-1}\left(\mathbf{I}-\mathbf{A}\right)^{-1}\hat{\boldsymbol{y}} \;. \quad (26)$$

Therefore the **S**-operator is simply norming the production matrix $\mathbf{X}$ and each row of $\mathbf{S}$ adds up to one, showing "the proportions of the activity of each branch which comes under the various subsystems" (Momigliano and Siniscalco, 1982, p. 281). Columns represent a subsystem in terms of sectoral output shares directly and indirectly needed to sustain final demand for the corresponding commodity. The **S**-operator reclassifies any magnitude from sectors to subsystems. However, we are not allowed to figure out column sums because values are not homogenous. To overcome this problem we can premultiply (26) by an appropriate diagonal matrix such as $\hat{\boldsymbol{v}}$:

$$\mathbf{V}=\left[\mathrm{v}_{ij}^{cs}\right]=\hat{\boldsymbol{v}}\ \left(\mathbf{I}-\mathbf{A}\right)^{-1}\ \hat{\mathbf{y}}=\begin{bmatrix} \hat{v}^{1}\mathbf{B}^{11}\hat{\boldsymbol{y}}^{1} & \dots & \hat{v}^{1}\mathbf{B}^{1c}\hat{\boldsymbol{y}}^{c} & \dots & \hat{v}^{1}\mathbf{B}^{1R}\hat{\boldsymbol{y}}^{R} \\ \vdots & & & & \vdots \\ \hat{v}^{c}\mathbf{B}^{c1}\hat{\boldsymbol{y}}^{1} & & \hat{v}^{c}\mathbf{B}^{cc}\hat{\boldsymbol{y}}^{c} & & \hat{v}^{c}\mathbf{B}^{cR}\hat{\boldsymbol{y}}^{R} \\ \vdots & & & & \vdots \\ \hat{v}^{R}\mathbf{B}^{R1}\hat{\boldsymbol{y}}^{1} & \dots & \hat{v}^{R}\mathbf{B}^{Rc}\hat{\boldsymbol{y}}^{c} & \dots & \hat{v}^{R}\mathbf{B}^{RR}\hat{\boldsymbol{y}}^{R} \end{bmatrix}. \tag{27}$$

The resulting matrix **V** offers a new arrangement of net world flows. Each entry**,** say v_{ij}^{cs}, shows the amount of value added required, both directly and indirectly, from sector *i* in country *c* in order to satisfy final demand absorbed in sector *j* of country *s*. Since all entries are in homogeneous units (i.e. value added), we can sum entries in every row or column. Summation along rows gives sectoral value added, i.e. payments to production factors, while the sum of entries in a column is equal to final demand.

We can also figure out a net backward linkage indicator that allows us to identify key sectors when "*economy-wide output generated by final demand in j is larger than the amount of j's output that is generated by all the other industries' final demands. So industry j can be said to be more important for the others than the others are for industry j, and j would thus be identified as a key sector by this measure*" (Miller and Blair, 2009, p. 559). Actually, we are not using gross output, as suggested by these authors, but value added:

$$NM_{j}^{c}=\frac{\boldsymbol{u}_{j}^{c}\mathbf{V}\boldsymbol{u}^{T}}{\boldsymbol{u}\mathbf{V}\left(\boldsymbol{u}_{j}^{c}\right)^{T}}=\frac{\sum_{s=1}^{R}\sum_{k=1}^{N}\mathrm{v}_{kj}^{sc}}{\sum_{s=1}^{R}\sum_{k=1}^{N}\mathrm{v}_{jk}^{cs}} \tag{28}$$

where $\boldsymbol{u}_{j}^{c}$ is a ($NR\times1$) zero vector but one entry equal to one in correspondence of sector *j* in country *c*. It has been observed "*a net multiplier for industry j that is larger than one can be shown to indicate that the final demand in j generates more value added in other industries than all the other industries' final demands do generate value added in industry j.1 Hence, in terms of generating value added, industry j can be said to be more important for the others than the others are for industry j*" (Dietzenbacher, 2005, p. 423-424). If this ratio is larger than one sector *j* in country *c* will absorb more value added from other industries than the value added it provides to the other subsystems. We can use this index to identify sectors at the end of the global supply chains since a subsystem produces final goods by definition. The larger its value the relatively closer sector *j* in country *c* is to final demand, and viceversa.

The overall sum of all the entries in the value added matrix is, of course, world GDP:

$$\boldsymbol{u}\mathbf{V}\boldsymbol{u}' = GDP = \boldsymbol{u}\left(\hat{v}\ \mathbf{B}\ \hat{\mathbf{y}}\right)\boldsymbol{u}' =$$

$$= \boldsymbol{u}\begin{bmatrix} \hat{v}^1\mathbf{B}^{11} & \cdots & \hat{v}^1\mathbf{B}^{1c} & \cdots & \hat{v}^1\mathbf{B}^{1R} \\ \vdots & & & & \vdots \\ \hat{v}^c\mathbf{B}^{c1} & & \hat{v}^c\mathbf{B}^{cc} & & \hat{v}^c\mathbf{B}^{cR} \\ \vdots & & & & \vdots \\ \hat{v}^R\mathbf{B}^{R1} & \cdots & \hat{v}^R\mathbf{B}^{Rc} & \cdots & \hat{v}^R\mathbf{B}^{RR} \end{bmatrix}\begin{bmatrix} \hat{\boldsymbol{y}}^1 & \cdots & 0 & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & & \hat{\boldsymbol{y}}^c & & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & 0 & \cdots & \hat{\boldsymbol{y}}^R \end{bmatrix}\boldsymbol{u}' = \boldsymbol{u}\hat{\mathbf{y}}\boldsymbol{u}' \qquad (29)$$

as column sums in the left hand matrix are still equal to one.

Since row sums of the **S**-operator are meaningless, can resort to **V** as an alternative. Actually, Momigliano and Siniscalco (1982) propose to divide each cell by the total of the correspondent column:

$$\mathbf{C} = \mathbf{V}\left(\widehat{\boldsymbol{u}\mathbf{V}}\right)^{-1} \qquad (30)$$

Entry c_{ij}^{rs} is the share accounted by sector *i* in country *r* in the total value added required by subsystem *j* of country *s* in order to produce the output needed to satisfy that final demand. The **C**-operator allows to uncover commodities whose production required a substantial contribution by several sectors suggesting that outsourcing is at work. It is also interesting to notice these matrices are invariant to relative price changes. In case of the value added matrix, we can see that:

$$\mathbf{V} = \hat{\boldsymbol{v}}^{-1}\ \left(\mathbf{I}-\mathbf{A}\right)^{-1}\ \hat{\mathbf{y}} = \left(\hat{\bar{\boldsymbol{v}}}^{-1}\hat{\boldsymbol{p}}^{-1}\right)\ \left(\mathbf{I}-\hat{\boldsymbol{p}}\bar{\mathbf{A}}\hat{\boldsymbol{p}}^{-1}\right)^{-1}\ \left(\hat{\boldsymbol{p}}\hat{\bar{\mathbf{y}}}\right) = \left(\hat{\bar{\boldsymbol{v}}}^{-1}\right)\ \left(\mathbf{I}-\bar{\mathbf{A}}\right)^{-1}\ \left(\hat{\bar{\mathbf{y}}}\right) \qquad (31)$$

where values with a bar indicate physical quantities (Rampa, 1982, Momigliano and Siniscalco, 1982). Finally, the **S**-operator does not depend on sectoral labor productivities, while the **C**-operator on final demand structures (Montresor and Mazzetti, 2011).

Even if $\hat{v}^c\mathbf{B}^{cs}\hat{\boldsymbol{y}}^s$ shows value added flows from sectors in country *c* embodied in final demand of country *s*, it does not correspond to bilateral net trade as we can check in the numerical example provided in the next section. Actually, value added deliveries to another country do not fulfill the basic properties for VAiT: "*First, adding up a country's bilateral trade surpluses or deficits over all partner countries should result in this country's overall trade surplus or deficit. Second, a country's bilateral trade surplus (deficit) with another country must this other country's bilateral deficit (surplus), i.e. it's mirrored*" (Stehrer, 2012, p. 2). Similarly, value added produced in *c* and included in foreign final demand, i.e. $\sum_{s\neq c}^{R}\hat{v}^c\mathbf{B}^{cs}\hat{\boldsymbol{y}}^s$, is not equal to net exports worldwide, nor $\sum_{s\neq c}^{R}\hat{v}^s\mathbf{B}^{sc}\hat{\boldsymbol{y}}^c$ is total imports. In order to assess net trade in country *c* (with respect to the world) let's consider (22) within this approach:

$$\begin{aligned} NX^c &= GDP^c - \boldsymbol{u}\boldsymbol{f}^c = \boldsymbol{u}^c\left(\hat{v}\ \mathbf{B}\ \hat{\mathbf{y}}\right)\boldsymbol{u}' - \boldsymbol{u}F\left(\boldsymbol{u}_F^c\right)' = \\ &= \boldsymbol{u}\left(\hat{\boldsymbol{v}}_D^c\mathbf{B}\hat{\boldsymbol{f}}_E^c - \hat{\boldsymbol{v}}_E^c\mathbf{B}\hat{\boldsymbol{f}}^c\right)\boldsymbol{u}' = \boldsymbol{u}\left(\hat{\boldsymbol{v}}\mathbf{B}\hat{\boldsymbol{f}}_E^c - \hat{\boldsymbol{v}}_E^c\mathbf{B}\hat{\boldsymbol{y}}\right)\boldsymbol{u}' = \boldsymbol{u}\left(\hat{\boldsymbol{v}}_D^c\mathbf{B}\hat{\boldsymbol{y}} - \hat{\boldsymbol{v}}\mathbf{B}\hat{\boldsymbol{f}}^c\right)\boldsymbol{u}' \end{aligned} \qquad (32)$$

where $\boldsymbol{u}_F^c$ is a $(R\times1)$ vector with all zero entries but the ones corresponding to country *c*. The last right hand side in (32) explains why the value added matrix cannot be used to detect VAiT since $\hat{\boldsymbol{v}}_D^c\mathbf{B}\hat{\boldsymbol{y}}$ has zero entries but flows from sectors in country c as in $\mathbf{V}$. These figures are not exports as they include value added created by domestic demand too. Hence, we have to correct it by $\hat{\boldsymbol{v}}\mathbf{B}\hat{\boldsymbol{f}}^{c}$. The latter generates also imports and the difference is the net trade matrix whose entries depict value added flows due to foreign final demand pushing exports and domestic final demand activating imports. Actually, the net value added trade matrix is given directly by $\hat{\boldsymbol{v}}_{D}^{c}\mathbf{B}\hat{\boldsymbol{f}}_{E}^{c} - \hat{\boldsymbol{v}}_{E}^{c}\mathbf{B}\hat{\boldsymbol{f}}^{c}$ that show exports in the first item and imports in the last one.

The net value added trade matrix is also equal to $\hat{\boldsymbol{v}}\mathbf{B}\hat{\boldsymbol{f}}_{E}^{c} - \hat{\boldsymbol{v}}_{E}^{c}\mathbf{B}\hat{\boldsymbol{y}}$. The first one can be conceived as worldwide value added due to foreign final demand. Rows corresponding to country c are still exports embodied in all final demand items and the sum of these entries is equal to VAiT or TiVA. As above, row sums give foreign final demand. It is obvious that within global chains foreign sectors provide part of the value added needed to satisfy them and we have to correct these flows by $\hat{\boldsymbol{v}}_{E}^{c}\mathbf{B}\hat{\boldsymbol{y}}$, that is value added created outside country c due to total final demand.

A numerical example

In order to assess the approach introduced in the previous section, let's consider a system comprised by three countries in each of which three sectors (say Agriculture, Manufacturing, and Services) provide differentiated commodities. Macro data for these countries are presented in Table 1. Domestic demand, i.e. household and public consumption plus gross capital formation, is always equal to 300, while GDP may differ by about 10%. International trade is balancing in the last country, while the production deficit in the first nation is matched by a trade surplus in the second one. However, international trade is sizable in all of them as export propensities, import penetration, and openness ratios show.

Table 1– Macro data

	Value added	Domestic demand	Export	Import	Net export	Export prop.	Import penetr.	Openness Ratio
1	270	300	185	215	-30	69%	72%	148%
2	330	300	277	247	30	84%	82%	159%
3	300	300	228	228	0	76%	76%	152%
tot.	900	900	690	690	0	77%	77%	153%

These aggregate data are derived from the world Input Output table presented below. In the first country all the sectors have an identical value added and quite similar gross output. In country 2, manufacturing is the prevailing industry, while tertiary is the leader in the last nation. Value added coefficients ranges from 37% to 53%. Final demand is larger where there is a likely comparative advantage, while intermediate demand is relevant for manufacturing and the service sectors.

Table 2 – World Input Output table

	a_1	m_1	s_1	a_2	m_2	s_2	a_3	m_3	s_3	***tot.***	f_1	f_2	f_3	y
a_1	11	2	0	5	4	1	5	6	2	36	68	38	38	144
m_1	20	35	26	2	7	4	5	4	3	106	40	24	15	79

***s*₁**	25	25	28	0	2	1	0	2	1	84	70	5	11	86
***a*₂**	3	2	1	8	16	7	3	5	1	46	16	33	5	54
***m*₂**	10	10	8	16	62	38	2	22	8	176	65	79	60	204
***s*₂**	1	8	8	4	55	20	0	10	12	118	9	65	8	82
***a*₃**	2	2	0	5	4	2	8	8	12	43	10	17	30	57
***m*₃**	18	10	7	10	49	25	17	40	31	207	9	14	40	63
***s*₃**	0	1	2	0	1	2	10	73	52	141	13	25	93	131
tot.	90	95	80	50	200	100	50	170	122	36	300	300	300	900
va	90	90	90	50	180	100	50	100	150	900				
x	180	185	170	100	380	200	100	270	272	1857				

Table 3 and 4 show gross and net trade flows. The latter are from the TiVA approach. It is interesting to notice how net trade flows between country 1 and 3 are whimsical, while gross trade is significant. The first country is selling its agricultural output worldwide, and importing both manufactured goods and services. The opposite holds in country 2, while the last one is balancing agricultural imports with exports in secondary. Trade flows are quite large in all the nations, with a total VAiT that is half of world GDP.

Table 3 – Gross trade flows

EXP	A	M	S	tot.	IMP	A	M	S	tot.
1	99	64	22	185	1	36	137	42	215
2	36	185	56	277	2	76	135	36	247
3	42	142	44	228	3	65	119	44	228
tot.	177	391	122	690	tot.	177	391	122	690

EXP	1	2	3	tot.	NX	1	2	3	tot.
1		93	92	185	1		-48	18	-30
2	141		136	277	2	48		-18	30
3	74	154		228	3	-18	18		0
tot.	215	247	228	690	tot.	30	-30	0	0

Table 4 – Value added flows

EXP	A	M	S	tot.	IMP	A	M	S	tot.
1	50.1	46.5	30.0	126.6	1	25.5	82.9	48.2	156.6
2	24.2	109.5	45.3	179.0	2	40.9	58.0	50.1	149.0
3	26.1	58.0	61.5	145.6	3	34.1	73.1	38.4	145.6
tot.	100.4	214.0	136.8	451.2	tot.	100.4	214.0	136.8	451.2

EXP	1	2	3	tot.	NX	1	2	3	tot.
1		64.6	62.0	126.6	1		-30.8	0.8	-30.0
2	95.3		83.6	179.0	2	30.8		-0.8	30.0
3	61.2	84.4		145.6	3	-0.8	0.8		0.0
tot.	156.6	149.0	145.6	451.2	tot.	30.0	-30.0	0.0	0.0

Table 5 displays the value added matrix. Sums along columns are sectoral value added, while summations along rows provide final demand. The overall total is world GDP. Net multipliers are figure out in the last column. They are larger the one in all the primary sectors, and the maximum is achieved in the first case. This primary sector of country 1 is selling only 20% of its gross output as intermediate demand and the net multiplier is revealing it is quite close to final demand. Intermediate demand shares are large but below 50% in the agriculture and manufacturing of the second country, where net multiplier are still above one. We cannot state the net multipliers is conveying the same information as direct sales yet, as it includes indirect effects too. Take the service sector in country 1 where intermediate demand is slightly less than the final one, but the net multiplier is below one.

Table 5 – The **V** matrix

	a_1	m_1	s_1	a_2	m_2	s_2	a_3	m_3	s_3	va	NM
a_1	77.6	1.0	0.4	2.0	3.0	0.9	2.1	1.4	1.5	90	1.60
m_1	14.1	49.6	10.3	1.6	5.4	2.2	2.7	1.7	2.5	90	0.88
s_1	16.4	9.0	56.5	0.8	2.8	1.1	1.0	1.1	1.3	90	0.96
a_2	2.7	1.3	1.0	30.1	7.8	2.8	1.5	1.4	1.4	50	1.08
m_2	9.6	5.4	5.3	7.1	125.6	12.4	2.7	5.6	6.3	180	1.13
s_2	4.6	4.3	4.6	3.0	23.7	48.6	1.4	3.5	6.2	100	0.82
a_3	2.2	1.1	0.6	2.1	3.7	1.4	31.8	2.2	4.9	50	1.14
m_3	10.7	4.4	4.0	4.6	20.3	7.9	6.3	30.5	11.2	100	0.63
s_3	6.1	2.9	3.1	2.7	11.5	4.8	7.5	15.6	95.7	150	0.87
y	144	79	86	54	204	82	57	63	131	900	

Rows in matrix **V** indicate how payments to production factors are included in all the final demand items. For instance, summing up all the entries concerning countries' but 1 final demands. i.e. $\boldsymbol{u}\left(\mathbf{V}^{12}+\mathbf{V}^{13}\right)\boldsymbol{u}^T$, we get value added delivered by the first nation and included in other final demand items. This figure can shed light on how much other countries can indirectly activate production in the first nation, but does not concern exports, as its value is 35 only and much less than VAX. By the same token, $\boldsymbol{u}\left(\mathbf{V}^{21}+\mathbf{V}^{31}\right)\boldsymbol{u}^T$ is not VAM (it is equal to 74), nor the difference is net trade in value added. Nevertheless, these figures are important as they can signal how a country participate in global value chains.

It is immediate to check column shares are the **S**-operator. In the first country most of agricultural value added is absorbed by its own final demand (86.3%), while this share is smaller in tertiary (62.8%) and much less in manufacturing (55.1%). Actually, all sectors, but manufacturing in country 3, are largely activated by their own final demand. Not all the industries are focused in local markets yet. For instance, manufacturing value added of the last country is spread worldwide, with a sizeable quota in secondary final demand of country 2.

Table 6 – the **S**-operator

	a_1	m_1	s_1	a_2	m_2	s_2	a_3	m_3	s_3
a_1	86.3	1.1	0.5	2.2	3.4	1.0	2.3	1.5	1.7
m_1	15.6	55.1	11.4	1.8	6.0	2.4	3.0	1.9	2.7
s_1	18.2	10.0	62.8	0.9	3.1	1.2	1.1	1.2	1.5
a_2	5.4	2.5	2.1	60.1	15.6	5.5	3.0	2.8	2.8
m_2	5.4	3.0	3.0	3.9	69.8	6.9	1.5	3.1	3.5
s_2	4.6	4.3	4.6	3.0	23.7	48.6	1.4	3.5	6.2
a_3	4.3	2.2	1.2	4.2	7.4	2.8	63.5	4.5	9.8
m_3	10.7	4.4	4.0	4.6	20.3	7.9	6.3	30.5	11.2
s_3	4.1	2.0	2.1	1.8	7.7	3.2	5.0	10.4	63.8

As stated above, we cannot perform row sum in table 6, but we can use the **C**-operator as an alternative. These shares are informative. Values along the diagonal indicate the weight of each branch within its subsystem. They are all, but secondary in country 3, larger than 50% showing the importance of self-activation. Spillovers may be important too. For instance, $c_{sa}^{11} = c_{sm}^{11} = 11.4\%$ reveals that agricultural and manufacturing final demand in country 1 is activating local service providers a lot. We can notice that, even if all these industries are trade oriented, only about 25% of activities used in production come from outside the country. This share drops to 15% in the service sector of country 3.

Table 7 – the **C**-operator

	a_1	m_1	s_1	a_2	m_2	s_2	a_3	m_3	s_3
a_1	53.9	1.2	0.5	3.7	1.5	1.1	3.7	2.2	1.2
m_1	9.8	62.7	12.0	3.0	2.7	2.6	4.8	2.7	1.9
s_1	11.4	11.4	65.7	1.4	1.4	1.3	1.7	1.7	1.0
a_2	1.9	1.6	1.2	55.7	3.8	3.4	2.6	2.2	1.1
m_2	6.7	6.9	6.2	13.1	61.6	15.1	4.7	8.9	4.8
s_2	3.2	5.4	5.4	5.6	11.6	59.3	2.5	5.6	4.7
a_3	1.5	1.4	0.7	3.9	1.8	1.7	55.7	3.5	3.8
m_3	7.4	5.6	4.7	8.5	10.0	9.6	11.1	48.5	8.5
s_3	4.2	3.7	3.6	5.0	5.6	5.9	13.2	24.8	73.0

The **V** matrix, **S** and **C** operators are extremely useful to address subsystem compositions, disentangle structural changes and see which commodities are boosting production. They are not linked with exports and imports in value added yet. To address foreign trade we must resort to the **NVAT** matrix and its decompositions. Table 8 presents value added trade flows for the first country, where imports have a negative sign. Sums along each row are VAiT or TiVA (compare with table 4) and the overall value is also the gross trade balance that is a deficit of 30 units. Let's take the first sector. Agricultural gross exports add up to 99 units, 76 of which are delivered as final demand. VAiT is only 50 and tables 8 and 9 show which components of are activating these net exports. Actually, most of it is still included in its own products, as we can see from table 9. If agriculture is producing tomatoes only, about 82% of primary value added exports is due to tomatoes bought by foreign families. However, value added exports are also in other foreign final demand items, such as goods (tomato sauce) and services (restaurants serving tomato salads or pasta with tomato sauce) produced domestically or abroad. Figures along this row are not large, but findings are case dependent. As we can see from the third rows in tables 8 and 9, value added service exports are mostly included in other items, as only about one third is due to its own foreign final demand. Noticeably a larger share is due to the other domestic items, that is foreign final demand for local agricultural product (8.7 units, i.e. 29%) and manufactured goods (4.5 units or 15%).

Table 8 – **NVAT** matrix for country 1

	a_1	m_1	s_1	a_2	m_2	s_2	a_3	m_3	s_3	VAiT
a_1	41.0	0.5	0.1	1.4	2.1	0.8	1.7	1.2	1.4	50.1
m_1	7.4	24.5	1.9	1.1	3.7	1.9	2.3	1.4	2.2	46.5
s_1	8.7	4.5	10.5	0.5	1.9	0.9	0.8	0.9	1.2	30.0
a_2	-1.3	-0.6	-0.9	-8.9	-2.5	-0.3	-0.3	-0.2	-0.1	-15.1
m_2	-4.6	-2.8	-4.3	-2.1	-40.0	-1.4	-0.5	-0.8	-0.6	-57.0
s_2	-2.2	-2.2	-3.8	-0.9	-7.6	-5.3	-0.2	-0.5	-0.6	-23.3
a_3	-1.0	-0.6	-0.5	-0.6	-1.2	-0.2	-5.6	-0.3	-0.5	-10.4
m_3	-5.1	-2.2	-3.3	-1.4	-6.5	-0.9	-1.1	-4.4	-1.1	-25.9
s_3	-2.9	-1.5	-2.5	-0.8	-3.7	-0.5	-1.3	-2.2	-9.5	-24.9
tot.	40.1	19.6	-2.7	-11.6	-53.7	-4.8	-4.2	-4.9	-7.7	-30.0

Table 9 – VAiT shares for country 1

	a_1	m_1	s_1	a_2	m_2	s_2	a_3	m_3	s_3	tot.
a_1	81.8	0.9	0.2	2.8	4.1	1.7	3.5	2.4	2.7	1
m_1	16.0	52.6	4.1	2.5	8.0	4.1	4.8	3.1	4.8	1
s_1	28.9	14.9	35.1	1.8	6.4	3.2	2.7	3.0	4.0	1
a_2	8.5	4.2	5.7	59.1	16.5	2.0	1.7	1.3	0.9	1
m_2	8.0	4.8	7.6	3.7	70.2	2.4	0.8	1.4	1.1	1
s_2	9.4	9.3	16.1	3.9	32.5	22.9	1.1	2.2	2.6	1
a_3	9.8	5.4	4.8	6.0	11.3	1.5	53.5	3.1	4.7	1
m_3	19.6	8.7	12.7	5.3	25.0	3.3	4.3	16.9	4.3	1
s_3	11.5	6.0	10.2	3.2	14.7	2.1	5.3	8.9	38.1	1

Final demand activates production in all the sectors. Table 8 shows how to sustain a foreign final demand of 76 units, domestic manufacturing contributes with exports equal to 7.4 units and services with 8.7. Actually, all the entries in the first column of the $\hat{\mathbf{v}}\mathbf{B}\hat{\boldsymbol{f}}_E^c$ describe the subsystem connected with the foreign final demand for primary products. As the **C**-operator is invariant to changes in final demand, the structure is the same. However, this is not the focus of our analysis, as we are not interested in spillovers. In order to disclose net trades we have to turn to imported value added. This is due to domestic final demand. Let’s consider the primary sector again. Domestic final consumers buy 68 units. To produce this amount, the primary sector needs foreign intermediate inputs and negative entries in the first column show value added imports from foreign branches taking into account all the direct and induced effects. Goods produced by foreign industries are somehow important as, to sustain 68 units of domestic final demand, manufacturing provides about 10 units of value added from country 2 and 3. Total import is equal to 17 units and this sum is balancing the domestic components.

The first column in table 8 discloses net trade connected with agricultural products in the first nation. On one side, positive entries show value added exported by domestic industries, i.e. due to foreign demand for local agricultural products. A foreign final demand of 76 units has generated exports by 57, with a 75% ratio. On the other side, a domestic final demand of 68 units creates value added imports by 17 units, with a 25% ratio. The overall value is 40.1 units that is the trade surplus due to the different sources of agricultural final demand.

The service sector example is intriguing. Gross output is 170 with a value added equal to 90. Very little is sold to foreign final consumers (16 units) and its final demand is weakly activating domestic output and value added exports. The latter are whimsical in agriculture. Nonetheless, the third row of table 8 shows this sector is exporting much more, as value added exports total 30 units, while gross exports are equal to 22. This is due to foreigners, who like other domestic commodities, such as tomatoes, that absorb services a lot. Hence, tertiary appear much more competitive on VAiT basis than on standard measures. This is not a complete picture yet, since a large domestic final demand (70) requires inputs and value added imports (15.2). The overall balance, between exports due to foreign final demand and imports generated by the domestic one, is slightly negative (-2.7).

Finally, each row in table 9 shows the structure of value added imports too. For instance, value added imports from manufacturing in country 2 are mostly due to its own final demand (70%), while this share is much smaller for similar imports from country 3 (about 17%). Maybe the latter is producing components that are spread in almost all final demand items, while the former mostly cars that are directly sold to final consumers in country 1. Empirical analyses will disclose such cases.

4. Conclusions

Global Value Chains are probably the most prominent feature of the recent globalization wave. These vertical supply linkages are posing new challenges to analyses of international trade and countries' competitiveness as intermediates move across borders several times before being assembled into final goods. Double counting of trade may severely bias standard trade statistics. An example can shed some light on this point. "*The firm Burberry sends perfume bottles from France to Shanghai to be decorated with a Scottish pattern before bringing them back to be sold on the French market. Standard trade statistics suggest that France is exporting perfume bottles to China and China is exporting perfume bottles to France. Yet France does not export anything for Chinese consumption, as the perfume bottles are consumed in France. China simply exports decoration for French consumption. Suppose the pigments used for the decoration of the perfume bottles are imported by China from Japan. This Japan-China trade flow does not mean that China consumes Japanese products, as the final consumer is in France. Unravelling these long supply chains is impossible using simply trade statistics*" (Daudin, 2011, p. 1404). Several case studies have motivated the growing literature on global chains. These case study face obvious limitations, as the particular components of a product under consideration are themselves made of other components and so on. At the end, it is far from being easy to trace back a product to the ultimate producer: "*there is no simple answer to who makes the iPod or where it is made. The iPod, like many other products, is made in several countries by dozens of companies, with each stage of production contributing a different amount to the final value*" (Varian 2007).

Two measures are commonly used to address these issues: Trade in Value Added and Value Added in Trade. TiVa answers the question: 'how much value added of other countries is contained in the consumption of the country under examination?', while VAiT is tackling the question: 'How much of foreign value added does the gross exports of a country embody?' (Stehrer, 2012). Kuboniwa (2014) proves the equivalence theorem between TiVA and VAiT in terms of factor (value-added) content of trade as proposed by Trefler and Zhu (2014). As value added generated in a sector can be included in any kind of commodity, it is natural to wonder what the source of this output is. This problem can be easily solved adopting the Sraffian concept of vertically integrated sectors or subsystems. They allow us to split the economic system into ideal self-sufficient sectors, which produce a single final commodity with all their necessary intermediate inputs. This way I can provide an answer to Varian's question about who is actually making the iPod and how it is contributing to the trade balance too. The Value added matrix has been already used in several analyses concerning outsourcing and deindustrialization at the national level. Furthermore, World Input Output tables have been used to derive the distribution of value added by all countries involved in the production chain of a particular final good (Los et al. 2015). Unfortunately, the latter does consider backwards effects only, while the former is not coherent with standard international trade framework as neither value added included in foreign commodities is exports in value added nor foreign value added in domestic final demand corresponds to value added imports. However, adapting the methodology devised by Stehrer among others I formulate a new matrix, called NVAT, which shows value added bilateral trades distinguishing between exports and imports. The former is due to external final demand about all final commodities. The latter is linked with domestic demand that requires not only foreign intermediate inputs but foreign value added too. In the case of a Ferrari, whose last stage of production is in Maranello, Italy, an export is carried out when the car is sold outside Italy. Positive entries in NVAT show value added generates by these sales pinpointing the contribution of all the Italian industries directly and indirectly involved in Ferrari's components. However, some Italian customers buy Ferraris, whose making require foreign inputs and valued added too. Negative entries in NVAT show imported valued added. The overall sum is the valued added trade balance concerning these bilateral flows. NVAT fulfils the identity between the sum total of a country's value-added trade balances and gross trade balances (net "gross exports" or net exports) and may be an important tool to assess international competitiveness.

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