Trade tariff, wage gap and public spending

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Abstract

This paper studies the interplay between wage gap and government spending in a small open economy facing a liberalization of commodities trade with the external world. We consider a developing economy with two sectors: an export sector, which uses capital and unskilled labour, and an import-competing sector, which uses capital and skilled labour. In this specific factor model, the return to capital is the link between the two sectors. We show that there exists a direct relation between trade liberalization, which decreases the skilled-unskilled wage gap, and the level of government expenditure. However, either an unbalanced distribution of political bargaining power, or tariff revenue co-financing of public spending may break this direct relation.

Key words: wage gap; trade liberalization; positive political economy.

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1 Introduction

This paper studies the interplay between wage gap and government spending in a small open economy facing a liberalization of commodities trade with the external world.

The aim of this analysis is to explore the link between tariff policies and the size of public spending for public goods. We develop a model where public spending is a result of a collective decision-making mechanism where the equilibrium is a compromise among conflicting collective interests.

We consider an economy with two sectors: an export sector, which uses capital and unskilled labour, and an import-competing sector, which uses capital and skilled labour. In this specific factor model, the return to capital is the link between the two sectors. Trade liberalization decreases the skilled-unskilled wage gap as it decreases the skilled and increases the unskilled wages.

Workers are also voters. We capture the conflicting redistributive interests of skilled and unskilled workers by allowing them to elect a representative, one for each group, who is responsible of negotiating the size of public spending in the government.

The paper proceeds as follows. Section two develops the economic environment and describes the relationship between trade policy and wages in a developing economy. Section three finds the government equilibrium outcome for a given trade policy. Section four investigates the consequences for public spending of changes in tariff policy when public spending for public goods provision is or is not cofinanced by tariff revenue. We also study the consequences of skilled and unskilled workers having different political influence in the government decision. Section five finds the conditions under which both trade policy and the distribution of political influence leads to efficiency in the provision of public goods. Section six concludes. The Appendix contains derivations and proofs.

2 The economy

We consider a small open developing economy producing two goods, an export good x using unskilled labour, L, and capital, K, and an import-competing good y using skilled labour, S, and capital. Both goods are produced with a constant return to scale technology. The government protects the import-competing sector by imposing an "ad valorem" tariff, denoted by $t \in [0, 1]$. Flexible wages, denoted by w and w_s for unskilled and skilled workers respectively, and flexible rate of return to capital, r, coupled with perfectly competitive markets ensure full employment of labour and capital.

The small open economy cannot influence the world prices of these goods, which are respectively denoted by P_X^W and P_Y^W.

We now state the set of equations that describes the economy.

Prices are such that

$$P_X^W = a_{LX}w + a_{KX}r \tag{1}$$

$$(1+t)\,P_Y^W = a_{SY}w_S + a_{KY}r \tag{2}$$

where, a_{LX} (a_{SY}) denotes the requirement of unskilled (skilled) labour to produce one unit of the export (import-competing) good x (y); a_{KX} (a_{KY}) denotes the requirement of capital to produce one unit of good x (y). Equations (1) and (2) indicate that the domestic producers earn zero profit due to perfect competition and free entry, which equalise price to unit cost. Notice that wile the price of good x is equal to that prevailing at the world level, the price of good y is $P_Y = (1+t)\,P_Y^W$.

The following set of equations gives us the least-cost input choice:

$$a_{LX} = a_{LX}\left(\frac{w}{r}\right), a_{SY}\left(\frac{w_s}{r}\right), a_{KX} = a_{KX}\left(\frac{w}{r}\right), a_{KY} = a_{KY}\left(\frac{w_s}{r}\right). \tag{3}$$

The full employment conditions ensure that:

$$S = a_{SY}Y \tag{4}$$

and

$$L = a_{LX}X, \tag{5}$$

where, X and Y, denote the output levels of good x and y respectively; and the terms L, S denote fixed endowments of unskilled and skilled labour respectively. For what follows, we also assume $L \geq S$.

Finally, full capital employment is given by

$$K = a_{KX}X + a_{KY}Y, \tag{6}$$

where, K denotes fixed endowment of capital.

Given the world commodity prices and tariff rate, the above set of nine equations together determine three factor prices, four input choices, and two output levels. It is evident that the wages consistent with zero-profit and full-employment conditions vary with the trade policy choice as captured here by the tariff rate.[1] Clearly, exogenous changes in the

[1] This is a typical specific factor model *a' la* Jones (1971), where the intersectoral mobile factor between the import competing sector and the export sector is capital as in Acharyya and Kar (2013).

tariff rate would change the equilibrium value of wages. The above set of conditions can be used to derive the precise nature and magnitude of changes in wages due to changes in tariff rates, as described below (the proof is in Appendix A),

$$\widehat{w} = -\frac{\theta_{KX}}{\theta_{LX}}\frac{\alpha_Y}{\alpha}\widehat{T} \tag{7}$$

for the unskilled wage, and

$$\widehat{w}_S = \frac{\theta_{KY}}{\theta_{SY}}\left(\frac{1-\theta_{KY}\frac{\alpha_Y}{\alpha}}{\theta_{KY}}\right)\widehat{T} \tag{8}$$

for the skilled wage. In the above equations, the "hat" over a variable denotes its proportional change (e.g., $\widehat{w}_S = \frac{dw_S}{w_S}$); $T = (1+t)$ and $\widehat{T} = \frac{t}{1+t}\widehat{t}$,[2] $\theta_{LX} \equiv \frac{a_{LX}w}{P_X^W}$ is the share of unskilled labour in unit cost of producing good x; cost shares θ_{KY}, θ_{KX} and θ_{SY} are similarly defined; $\alpha \equiv \frac{\lambda_{KY}}{\theta_{SY}}\sigma_Y + \frac{\lambda_{KX}}{\theta_{LX}}\sigma_X = \alpha_Y + \alpha_X$ where, $\alpha_Y \equiv \frac{\lambda_{KY}}{\theta_{SY}}\sigma_Y$ and $\alpha_X \equiv \frac{\lambda_{KX}}{\theta_{LX}}\sigma_X$, with $\lambda_{KY} \equiv \frac{a_{KY}Y}{K}$ and $\lambda_{KX} \equiv \frac{a_{KX}X}{K}$ respectively denoting the share of sector y and x in total employment of capital and σ_j is the elasticity of factor substitution in sector j $(j = x, y)$, as described in Appendix A.

Change in the rate of return to capital is given by:

$$\widehat{r} = \frac{\alpha_Y}{\alpha}\widehat{T}. \tag{9}$$

It follows that the equilibrium wages change asymmetrically with a reduction of the tariff rate. We write the functional relationship between wages and tariff as follows:

$$\frac{\partial w(t)}{\partial t} < 0 \text{ and } \frac{\partial w_S(t)}{\partial t} > 0. \tag{10}$$

The intuition is simple. Consider an initial situation in which factor and good prices and output levels are at their equilibrium values. A tariff reduction contracts the domestic production in the import-competing sector through an increased competition from importers as it reduces the price P_Y. Capital employed in the sector producing y decreases as the value of its marginal product decreases and, consequently, the wage of skilled workers employed in the production of y declines. Capital released from the import competing sector creates scope for expansion in the export sector, where production increases. The increase in the use of capital in the export sector increases the demand for unskilled workers raising their wage under full employment. Therefore, a reduction in tariff has asymmetric effects on skilled and unskilled wages.

[2] Note that $\widehat{T} = \frac{dT}{1+t} = \frac{dt}{1+t}\frac{t}{t} = \frac{t}{1+t}\widehat{t}$.

3 Government spending

The government is compounded of the representatives of the two groups of workers. The representatives negotiate over the level of public spending for a public good g, to be financed through both a proportional tax τ on wages, with $\tau \in [0,1[$, and eventually by tariff revenue denoted by $V = tI_y(t)$. In the latter equation, $I_y(t)$ represents the value of imports, which follows a Laffer curve shape. The government budget constraint is:

$$g = \tau(wL + w_s S) + \beta t I_y(t) \Rightarrow \tau = \frac{g - \beta t I_y(t)}{wL + w_s S} \tag{11}$$

where, $g - \beta t I_y(t) \geq 0$. The parameter β can take either a value of one or zero and captures the contribution of tariff revenue to the financing of g: when $\beta = 1$, tariff revenue cofinances the provision of g; instead, when $\beta = 0$, public good provision is only financed through tax revenue.

Tax paid by an unskilled individual is $\tau w = w\frac{g-\beta t I_y(t)}{wL+w_s S}$ and tax paid by a skilled individual is $\tau w_s = w_s \frac{g-\beta t I_y(t)}{wL+w_s S}$.

The utility of an unskilled individual is

$$u = (1-\tau)w - f(P) + ln(g+1) = \tag{12}$$

$$= w - f(P) - \frac{w}{Lw + Sw_s}(g - \beta t I_y(t)) + ln(g+1). \tag{13}$$

Similarly, the utility of a skilled individual is

$$u_s = w_s - f(P) - \frac{w_s}{Lw + Sw_s}(g - \beta t I_y(t)) + ln(g+1).$$

In the legislature, the representatives of both groups, skilled and unskilled, form a government where they negotiate over the size of public spending g. In order to approve government budget and implement public policy an agreement is needed. In case of disagreement, wages cannot be taxed and we obtain $\tau = g = 0$. Therefore, even if tariff revenue is positive, it cannot be used for the provision of the public good under government impasse, as the government cannot decide. Hence, the disagreement utility is given by private consumption only:

$$u^d = w - f(P) \text{ and } u_s^d = w_s - f(P). \tag{14}$$

According to the Nash bargaining axiomatic approach, an agreement will occur if and only if $\phi = u - u^d = -\frac{w}{Lw+Sw_s}(g - \beta t I_y(t)) + ln(g+1) \geq 0$ and $\phi_s = u_s - u_s^d = -\frac{w_s}{Lw+Sw_s}(g - \beta t I_y(t)) + ln(g+1) \geq 0$.

In order to find the equilibrium policy outcome, we maximise the following Nash bargaining product:

$$g = argmax \left[-\frac{w\left(g-\beta t I_y\left(t\right)\right)}{Lw+Sw_s} + ln\left(g+1\right) \right]^{\gamma} \left[-\frac{w_s\left(g-\beta t I_y\left(t\right)\right)}{Lw+Sw_s} + ln\left(g+1\right) \right]^{1-\gamma} \quad (15)$$

where, $\gamma \in [0,1]$ is the political bargaining leverage of the unskilled workers, and when $\gamma = 1/2$ skilled and unskilled workers have the same bargaining power.

The first order condition with respect to g is

$$\gamma \frac{-\frac{w}{Lw+Sw_s} + \frac{1}{g+1}}{-\frac{w}{Lw+Sw_s}\left(g-\beta t I_y\left(t\right)\right) + ln\left(g+1\right)} + \left(1-\gamma\right) \frac{-\frac{w_s}{Lw+Sw_s} + \frac{1}{g+1}}{-\frac{w_s}{Lw+Sw_s}\left(g-\beta t I_y\left(t\right)\right) + ln\left(g+1\right)} = 0, \quad (16)$$

which implies $-\frac{w}{Lw+Sw_s} + \frac{1}{g+1} \geq 0$ and $-\frac{w_s}{Lw+Sw_s} + \frac{1}{g+1} \leq 0$, given that we assume that technology is such that $w \leq w_s$;[3] this implies that, in equilibrium, the unskilled voters would like to have higher taxation and public spending than the skilled voters, the reason being that the unskilled workers bear a lower marginal cost; i.e., $MC = \frac{w}{Lw+Sw_s} \leq \frac{w_s}{Lw+Sw_s} = MC_s$.

Furthermore, we denote by $\epsilon = \gamma \frac{-\frac{w}{Lw+Sw_s}+\frac{1}{g+1}}{-\frac{w}{Lw+Sw_s}(g-\beta t I_y(t))+ln(g+1)}$ and $\epsilon_s = \left(1-\gamma\right) \frac{-\frac{w_s}{Lw+Sw_s}+\frac{1}{g+1}}{-\frac{w_s}{Lw+Sw_s}(g-\beta t I_y(t))+ln(g+1)}$ the elasticities of the net gains from implementing policy g of unskilled and skilled workers respectively. It follows that in the Nash bargaining equilibrium (16), skilled and unskilled workers are equally elastic in absolute value; i.e., $\epsilon = -\epsilon_s$, which implies $|\epsilon| = |\epsilon_s|$.

4 The relation between tariff and public spending

In this section, we study how exogenous changes in the tariff policy t influences government spending g. Our analysis is based on the following Lemma.

Lemma 1 *Tariff policy influences government spending as specified in the following relation:*

$$\frac{dg^*}{dt} \geq 0 \ if \ \epsilon_s \left[\left(g-V\right)\left(\frac{S}{\phi}+\frac{L}{\phi_s}\right) + V'\left(\frac{w}{\phi}-\frac{w_s}{\phi_s}\right)\frac{Lw+Sw_s}{ww_s'-w_s w'} \right] \geq \left(1-\gamma\right)\frac{L}{\phi_s} - \gamma\frac{S}{\phi} \quad (17)$$

[3]Note that in the limit case where $t=0$, $L=S$ and the thecnology used in the two sectors is the same, we obtain $w=w_s$.

with: $V = \beta t I_y(t)$ and $V' = \partial V/\partial t$. The proof is in Appendix B.

We use the above Lemma to study how changes in the tariff t affects the equilibrium level g^* of public spending in the following cases: when public good provision is (or is not) cofinanced by the tariff revenue and representatives have the same bargaining power and when public good provision is (or is not) cofinanced by the tariff revenue and representatives have different bargaining power.

First, we consider the case where public good provision is not cofinanced by the tariff revenue and representatives have the same bargaining power; that is, $\beta = 0$ and $\gamma = 1/2$. The results are described in the following Proposition.

Proposition 1 *Assume the representatives of skilled and unskilled workers have the same bargaining power, $\gamma = 1/2$, and the public good provision is not cofinanced by the tariff revenue, $\beta = 0$, then there exists a direct relation between trade liberalization and the size of government spending for public good g; that is, a lower tariff protection leads to both a lower wage gap and a larger public spending.*

The proof follows from Lemma 1 after considering that (17) leads to

$$\frac{dg^*}{dt} \leq 0 \text{ if } \epsilon_s g \left(\frac{S}{\phi} + \frac{L}{\phi_s}\right) \leq \frac{1}{2}\left(\frac{L}{\phi_s} - \frac{S}{\phi}\right)$$

which is the case when $\beta = 0$ and $\gamma = 1/2$.

A higher tariff increases the wage gap between skilled and unskilled workers and this worsens the redistributive conflicts between the two working classes. As a result, government provision of public good declines. The opposite relation is also true. This, in turn, implies that government spending for public good provision increases under free-trade ($t = 0$) as this leads to a lower domestic wage disparity.

We now describe the role of a different political influence in the legislature where $\beta = 0$ and $\gamma \neq 1/2$.

Proposition 2 *Assume $\beta = 0$. The direct relation between trade liberalization and the size of government spending may not hold when $\gamma > 1/2$. Instead, it holds when $\gamma < 1/2$.*

Proposition 2 states that, when tariff revenue does not contribute to the financing of g and the bargaining power is asymmetrically distributed in favour of the unskilled workers, government spending for public good provision may either increase or decrease when there is a process of trade liberalization. Therefore, when the bargaining power is sufficiently higher for the unskilled, they may use their leverage to obtain either an increase or a decrease in g. Recall that the unskilled have a higher demand than the skilled workers for public good consumption, which is due to their lower marginal contribution to the financing of g. In the case under consideration, when tariffs are reduced, the unskilled face a trade-off between a higher demand and a higher marginal cost of g. Indeed, since trade liberalization increases their income, their marginal cost of g also increases. As a result, the unskilled have to balance this trade-off, which leads to ambiguous policy outcomes.

Instead, the direct relation between trade liberalization and the size of government spending holds when the skilled workers have the highest political influence in the government. The reason is that trade liberalization leads to a lower marginal cost of public good provision for the skilled.

Now, we study what happens when tariff revenue co-finances the public good provision, while the two representatives have the same bargaining power; i.e.: $\beta = 1$ and $\gamma = 1/2$.

Proposition 3 *Assume the representatives of skilled and unskilled workers have the same bargaining power, $\gamma = 1/2$, and public good provision is cofinanced by the tariff revenue, $\beta = 1$, then trade liberalization leads to a lower wage gap, while public spending increases if the tariff revenue increases and is ambiguous otherwise.*

In order to prove the Proposition consider that the relation in Lemma 1 becomes

$$\frac{dg^*}{dt} \geq 0 \text{ if } \epsilon_s \left[(g-V)\left(\frac{S}{\phi}+\frac{L}{\phi_s}\right) + V'\left(\frac{w}{\phi}-\frac{w_s}{\phi_s}\right)\frac{Lw+Sw_s}{ww_s'-w_sw'} \right] \geq \frac{1}{2}\left(\frac{L}{\phi_s}-\frac{S}{\phi}\right), \tag{18}$$

which leads to Proposition 3.

According to Proposition (3), when tariff revenue co-financies public spending, government spending increases as tariff decreases if this leads to a larger tariff revenue ($V' = \frac{\partial V}{\partial t} \leq 0$) and is ambiguous otherwise. Clearly, an increase in tariff revenue mitigates the redistributive conflict between skilled and unskilled workers and facilitates an increase in government spending.

Now, what happens when skilled and unskilled workers have different political influence in the legislature and public spending is cofinanced by tariff revenue? We answer this question in the following Proposition.

Proposition 4 *Assume $\beta = 1$ and $\gamma < 1/2$, trade liberalization leads to an increase in public spending when tariff revenue increases and is ambiguous otherwise. On the contrary, when $\gamma > 1/2$ the impact on public spending of changes in t is always ambiguous.*

The proof is a straightforward application of Lemma 1.

Accordingly, consider a trade liberalization policy where tariff t decreases exogenously leading to an increase in tariff revenue, then government spending increases if the skilled workers have a higher bargaining leverage than the unskilled ones and is ambiguous otherwise.

Proposition 4 suggests that a higher tariff revenue may not be able to mitigate the redistributive conflict when political influence is asymmetrically distributed between the two groups of workers. Under trade liberalization and tariff revenue co-financing, an increase in tariff revenue produces an increases in the provision of g if political influence is distributed in favour of the skilled workers. The reason being that lower skilled wages reduces the marginal cost for the skilled workers, who will be more willing to increase the size of g. However the change in g is ambiguous when either tariff revenue declines or political bargaining power is distributed in favour of the unskilled workers.

5 Efficiency

In this section we study the efficient policy outcome and compare it with the Nash bargaining solution. After solving the following maximization problem

$$\begin{aligned} g = & argmax \ \ (Su + Lu_s) = \\ & = argmax \ \gamma S \left[w - f(P) - \frac{w\left(g - \beta t I_y\left(t\right)\right)}{wL + w_s S} + ln\left(g+1\right) \right] + \\ & + \left(1-\gamma\right) L \left[w_s - f(P) - \frac{w_s\left(g - \beta t I_y\left(t\right)\right)}{wL + w_s S} + ln\left(g+1\right) \right], \end{aligned} \tag{19}$$

we find that the efficient policy outcome, g^e, is given by

$$g^e = \frac{\gamma S + \left(1-\gamma\right) L}{\gamma S w + \left(1-\gamma\right) L w_s}\left(wL + w_s S\right) - 1, \tag{20}$$

which does not depend on the co-financing of tariff revenue.

Furthermore, we notice that the efficiency condition (20) and the Nash bargaining condition tend to equalise under free trade ($t = 0$)

when the two groups have the same bargaining power ($\gamma = 1/2$) and the technology is such that wages equalize under free trade. For the proof, see Appendix C.

6 Concluding remarks

This paper links models of international trade with models of public finance.

In our framework, trade liberalization reduces the redistributive conflicts between the two working groups. As a result, government spending increases unless there are some kinds of perturbation due to either an unbalanced distribution of bargaining power or tariff revenue co-financing of public spending.

In a world with balanced political influence, when the government co-finances the provision of public goods through both a direct wage tax and tariff revenues, an increase in the tariff will decrease public spending when it decreases the value of imports. The reason being that both an increase in the wage gap coupled with a decrease in the tariff revenue worsen the internal redistributive conflict. However, a higher tariff may increase public spending when the worsening in the wage-gap is more then compensated by an increase in the tariff revenue.

However, there may be reasons why the two groups of voters have different bargaining leverage in the government. The unskilled workers, for instance, are the largest group in the economy, and this might give them more political weight. On the other hand, the smaller but richer group might have more political influence.

We find that free-trade coupled with a balanced political power, when it produces convergence in wages, increases the efficiency of public spending.

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7 Appendix A

From the full employment condition for unskilled labour (5), we obtain $d\overline{L} = da_{LX}X + a_{LX}dX; 0 = \frac{da_{LX}}{a_{LX}}Xa_{LX} + Xa_{LX}\frac{dX}{X}; 0 = Xa_{LX}\left(\widehat{a}_{LX} + \widehat{X}\right)$, which leads to the following proportional change:

$$\widehat{a}_{LX} + \widehat{X} = 0. \tag{21}$$

Similarly, from the full employment condition for skilled labour (4), we obtain $\overline{dS} = da_{SY}Y + a_{SY}dY; 0 = \frac{da_{SY}}{a_{SY}}Ya_{SY} + Ya_{SY}\frac{dY}{Y}$, which leads to

$$\widehat{a}_{SY} + \widehat{Y} = 0. \tag{22}$$

From the full employment condition for capital (6), we obtain

$d1 = da_{KX}\frac{X}{K} + a_{KX}d\frac{X}{K} + da_{KY}\frac{Y}{K} + \frac{a_{KY}}{K}dY; \widehat{a}_{KX}\frac{a_{KX}X}{K} + \frac{a_{KX}X}{K}\widehat{X} + \widehat{a}_{KY}\frac{a_{KY}Y}{K} + \frac{a_{KY}Y}{K}\widehat{Y} = 0;$

$\widehat{a}_{KX}\lambda_{KX} + \lambda_{KX}\widehat{X} + \widehat{a}_{KY}\lambda_{KY} + \lambda_{KY}\widehat{Y} = 0$;this leads to

$$\left(\widehat{a}_{KX} + \widehat{X}\right)\lambda_{KX} + \left(\widehat{a}_{KY} + \widehat{Y}\right)\lambda_{KY} = 0. \tag{23}$$

Substitution of (21) and (22) in (23) yields

$$\left(\widehat{a}_{KX} - \widehat{a}_{LX}\right)\lambda_{KX} + \left(\widehat{a}_{KY} - \widehat{a}_{SY}\right)\lambda_{KY} = 0.$$

After using the definition of factor substitution elasticity in sector X and Y, $\sigma_X = \frac{\widehat{a}_{KX} - \widehat{a}_{LX}}{\widehat{w} - \widehat{r}}$ and $\sigma_Y = \frac{\widehat{a}_{KY} - \widehat{a}_{SY}}{\widehat{w}_s - \widehat{r}}$, the above equation boils down to:

$$\sigma_X\left(\widehat{w} - \widehat{r}\right)\lambda_{KX} + \sigma_Y\left(\widehat{w}_s - \widehat{r}\right)\lambda_{KY} = 0. \tag{24}$$

Now, we use the zero profit conditions (2) and (1) in order to obtain the relationship between changes in wages and in the rate of return to capital, as follows:

$(1+t)P_Y^W = a_{SY}w_S + a_{KY}r; d(1+t)P_Y^W = da_{SY}w_S + da_{KY}r + a_{SY}dw_S + a_{KY}dr$; since it must be $da_{SY}w_S + da_{KY}r = 0$ (see Krugman, Obstfeld and Meliz, 2015, pp. 449-450), we can write

$d(1+t) = \frac{a_{SY}}{P_Y^W}dw_S + \frac{a_{KY}}{P_Y^W}dr; \frac{d(1+t)}{1+t}(1+t) = \frac{a_{SY}w_S}{P_Y^W}\frac{dw_S}{w_S} + \frac{a_{KY}r}{P_Y^W}\frac{dr}{r};$

$\widehat{(1+t)}(1+t) = \frac{a_{SY}w_S}{P_Y^W}\widehat{w}_s + \frac{a_{KY}r}{P_Y^W}\widehat{r}; \widehat{(1+t)}(1+t) = \theta_{SY}\widehat{w}_s + \theta_{KY}\widehat{r};$

now, since $d(1+t) = dt \Rightarrow \frac{d(1+t)}{1+t}(1+t) = \frac{dt}{t}t \Rightarrow \widehat{(1+t)}(1+t) = \widehat{t}t \Rightarrow \widehat{(1+t)} = \widehat{t}\frac{t}{(1+t)};$

therefore, we can now write $\widehat{t}\frac{t}{(1+t)} = \theta_{SY}\widehat{w}_s + \theta_{KY}\widehat{r}$; that is,

$$\widehat{w}_s = \frac{\widehat{T}}{\theta_{SY}} - \frac{\theta_{KY}}{\theta_{SY}}\widehat{r}, \tag{25}$$

with $T = 1 + t$ and $\widehat{T} = \widehat{t}\frac{t}{1+t}$.

Furthermore, from the zero profit conditions (1), we obtain

$1 = \frac{1}{P_X^W}\left(a_{LX}w + a_{KX}r\right); d1 = \left(a_{LX}w + a_{KX}r\right)d\frac{1}{P_X^W} + \frac{1}{P_X^W}d\left(a_{LX}w + a_{KX}r\right);$

$\frac{1}{P_X^W}d\left(a_{LX}w + a_{KX}r\right) = 0; \frac{1}{P_X^W}\left(da_{LX}w + a_{LX}dw + da_{KX}r + a_{KX}dr\right) = 0;$

since it must be $da_{LX}w + da_{KX}r = 0$ (see Krugman, Obstfeld and Meliz, 2015, pp. 449-450), we can write

$\frac{a_{LX}}{P_X^W}dw + \frac{a_{KX}}{P_X^W}dr = 0; \theta_{LX}\widehat{w} + \theta_{KX}\widehat{r} = 0; \Rightarrow$

$$\widehat{w} = -\frac{\theta_{KX}}{\theta_{LX}}\widehat{r}. \tag{26}$$

Substitution of these values in (24) yields the change in the rate of return to capital (9), as shown below

$\sigma_X\left(\widehat{w} - \widehat{r}\right)\lambda_{KX} + \sigma_Y\left(\widehat{w}_s - \widehat{r}\right)\lambda_{KY} = 0;$

$\sigma_X\left(-\frac{\theta_{KX}}{\theta_{LX}}\widehat{r} - \widehat{r}\right)\lambda_{KX} + \sigma_Y\left(\frac{\widehat{T}}{\theta_{SY}} - \frac{\theta_{KY}}{\theta_{SY}}\widehat{r} - \widehat{r}\right)\lambda_{KY} = 0;$

$-\widehat{r}\sigma_X\lambda_{KX}\left(\frac{\theta_{KX}}{\theta_{LX}} + 1\right) + \frac{\sigma_Y\lambda_{KY}\widehat{T}}{\theta_{SY}} - \widehat{r}\sigma_Y\lambda_{KY}\left(\frac{\theta_{KY}}{\theta_{SY}} + 1\right) = 0;$

$\frac{\sigma_Y\lambda_{KY}\widehat{T}}{\theta_{SY}} = \widehat{r}\left[\frac{\sigma_Y\lambda_{KY}}{\theta_{SY}}\left(\theta_{KY} + \theta_{SY}\right) + \frac{\sigma_X\lambda_{KX}}{\theta_{LX}}\left(\theta_{KX} + \theta_{LX}\right)\right];$

since $\left(\theta_{KY} + \theta_{SY}\right) = 1$ and $\left(\theta_{KX} + \theta_{LX}\right) = 1$ (see Krugman, Obstfeld and Meliz, 2015, p. 451), we can write $\frac{\sigma_Y\lambda_{KY}\widehat{T}}{\theta_{SY}} = \widehat{r}\left(\frac{\sigma_Y\lambda_{KY}}{\theta_{SY}} + \frac{\sigma_X\lambda_{KX}}{\theta_{LX}}\right)$; recalling that $\alpha \equiv \frac{\lambda_{KY}}{\theta_{SY}}\sigma_Y + \frac{\lambda_{KX}}{\theta_{LX}}\sigma_X = \alpha_Y + \alpha_X$, with $\alpha_Y \equiv \frac{\lambda_{KY}}{\theta_{SY}}\sigma_Y$ and $\alpha_X \equiv \frac{\lambda_{KX}}{\theta_{LX}}\sigma_X$, we obtain $\alpha_Y\widehat{T} = \widehat{r}\alpha$, which leads to equation (9).

Finally, substitution of (9) in (25) and (26) yields the changes in the domestic wages (8) and (7).

8 Appendix B

Denote by Z the left hand side of the first order condition:

$$Z = \gamma\frac{-\frac{w}{Lw+Sw_s} + \frac{1}{g+1}}{-\frac{w}{Lw+Sw_s}\left(g - \beta tI_y\left(t\right)\right) + ln\left(g+1\right)} + \left(1-\gamma\right)\frac{-\frac{w_s}{Lw+Sw_s} + \frac{1}{g+1}}{-\frac{w_s}{Lw+Sw_s}\left(g - \beta tI_y\left(t\right)\right) + ln\left(g+1\right)} = 0$$

Clearly, the second order condition is negative; that is,

$$Z_g < 0. \tag{27}$$

Therefore, the sign of $\frac{dg}{dt} \equiv -\frac{Z_t}{Z_g}$ depends on the sign of Z_t.

$$Z_t = \gamma \frac{S\frac{(ww_s' - w_s w')}{(Lw+Sw_s)^2}\phi - \left[S\frac{(ww_s' - w_s w')}{(Lw+Sw_s)^2}\left(g - \beta t I_y(t)\right) + \frac{w}{Lw+Sw_s}\beta\left(I_y(t) + tI_y'(t)\right)\right]\phi'}{\phi^2} +$$

$$+ (1-\gamma)\frac{-L\frac{ww_s' - w_s w'}{(Lw+Sw_s)^2}\phi_s + \left[L\frac{ww_s' - w_s w'}{(Lw+Sw_s)^2}\left(g - \beta t I_y(t)\right) - \frac{w_s}{Lw+Sw_s}\beta\left(I_y(t) + tI_y'(t)\right)\right]\phi_s'}{\phi_s^2}$$

$Z_t > 0 \Rightarrow$

$$\frac{(ww_s' - w_s w')}{(Lw+Sw_s)^2}\gamma \frac{S\phi - \left[S\left(g - \beta t I_y(t)\right) + \frac{w(Lw+Sw_s)}{(ww_s' - w_s w')}\beta\left(I_y(t) + tI_y'(t)\right)\right]\phi'}{\phi^2} +$$

$$+\frac{ww_s' - w_s w'}{(Lw+Sw_s)^2}(1-\gamma)\frac{-L\phi_s + \left[L\left(g - \beta t I_y(t)\right) - \frac{w_s(Lw+Sw_s)}{(ww_s' - w_s w')}\beta\left(I_y(t) + tI_y'(t)\right)\right]\phi_s'}{\phi_s^2} > 0$$

After substituting $\beta t I_y(t) = V$ and $\beta\left(I_y(t) + tI_y'(t)\right) = V'$ and rearranging, we obtain

$$\epsilon_s \left\{ \frac{S(g-V) + \frac{w(Lw+Sw_s)}{(ww_s' - w_s w')}V'}{\phi} + \frac{L(g-V) - \frac{w_s(Lw+Sw_s)}{(ww_s' - w_s w')}V'}{\phi_s} \right\} > (1-\gamma)\frac{L}{\phi_s} - \gamma\frac{S}{\phi},$$

which leads to (17).

Now, since $\epsilon_s \leq 0$, the left hand side is always weakly negative when $V' \leq 0$. Instead, the right hand side is always negative when $\gamma \leq 1/2$, as $L \geq S$ and $\phi_s \leq \phi$ given $w \leq w_s$.

9 Appendix C

In order to calculate the efficiency condition, we maximise the additive social welfare function (19). The first order condition is

$$\gamma S\left(-\frac{w}{wL + w_s S} + \frac{1}{g+1}\right) + (1-\gamma) L\left(-\frac{w_s}{wL + w_s S} + \frac{1}{g+1}\right) = 0, \tag{28}$$

which leads to the efficient policy outcome (20).

Now, we compare the efficiency condition (20) with the Nash bargaining equilibrium condition (16). The two conditions equalise when

$t = 0$, $w = w_s$[4] and $\gamma = 1/2$. Under these assumptions, equations (20) and (16) become

$$g = L + S - 1;$$

that is, the normative and the positive solutions become equal.

[4]Note that, in order to simplify the discussion we are assuming that the technology is such that wages equalise under free-trade.