Long-run Unemployment and Macroeconomic Volatility

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Abstract

This paper develops a DSGE model with downward nominal wage rigidity, in which aggregate price and productivity dynamics are exogenously determined by independent Brownian motions with drift. As a result, the long-run expected value of unemployment depends positively on the drift coefficients and negatively on the volatility coefficients of both price and productivity growth processes. Model prescriptions are empirically tested by using a dataset including a wide sample of OECD countries from a period spanning from 1961 to 2011. Panel regressions with fixed effects and time dummies confirm the expected relation of inflation and productivity with unemployment at low frequencies. Long-run unemployment is negatively correlated with the levels of inflation and productivity growth, and positively with their volatilities.

Keywords: Long-run unemployment, Downward Nominal Wage Rigidity, Volatility, Inflation targeting, DSGE model, Cross-country panel data.

JEL codes: E12, E24, E31, C23

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1. Introduction

One of the fundamental of the Neoclassical Synthesis is the dichotomy between monetary policy and real aggregate variables in the long-run. According to the Synthesis, variations of nominal variables might have real effects only in the short-run, when adjustments in the economy are prevented by different types of rigidity. When in the long-run these rigidities vanish, both prices and wages are free to fluctuate, thereby employment and output converge to their natural levels. The relation between real variables and inflation becomes so vertical and the monetary policy loses its potential effectiveness. The Classical Dichotomy between real and nominal sides of the economy has been challenged by several contributions, which conversely argue that the two dynamics are not necessary independent in the long-run. Starting with Tobin (1972), a huge strand of literature has stressed in particular on the long-run relation between inflation and unemployment, that is on the long-run Phillips curve.[1] This paper contributes to this literature exploring the relationship at low frequencies, between unemployment and the dynamics of the nominal and real growth of the economy. Such relation is firstly studied in the theory, through a dynamic stochastic general equilibrium model featured by i) downward rigidity of nominal wages and ii) exogenous growing processes for prices and productivity. The main result of the model is a long-run Phillips curve in closed-form, which relates expected unemployment with inflation and productivity growth. Importantly, this long-run relationship disentangles the effects of the levels of inflation and productivity growth from the effects of the volatilities of the same processes. According to the theoretical model, unemployment is negatively related to the levels of price and productivity growth, whereas it is positively related to their volatilities. The underlying intuition is that unemployment increases above its natural level, only when downward rigidities prevent nominal wages from falling in response to a negative shock. This means that the less the nominal wages are prone to fall in response to a negative shock, the lower is the probability that labor input overshoots the full employment level in the short-run. In a long-run prospective, the expected unemployment is lower as well. The negative association of unemployment at low frequencies with the trends of price and productivity growth is explained by the fact that, *ceteris paribus*, the latter push upward the level of nominal wages, making them less exposed to negative realizations of exogenous shocks. The positive association of unemployment with the volatilities of price and productivity growth is instead explained because these volatilities affect directly the variability of nominal wages. For any given trends, a higher volatility in either nominal or real processes makes then nominal wages more inclined to being constrained after a

[1] Some works dealing with the long-lasting effects of inflation on unemployment are Fisher and Seater (1993), King and Watson (1994), Akerlof et al. (1996, 2000) and Fair (2000). Ball (1997 and 1999) notes instead that the natural unemployment rate increased among the OECD countries during disinflationary periods. More recently, Svensson (2015) argues on the long-run unemployment costs due to the undershooting of inflation target.

negative shock. As an implication for the long-run, the macroeconomic volatility affects positively the expected unemployment rate. The theoretical prescriptions of the model are tested empirically in the second part of the paper. The empirical section uses panel data to present a cross-country analysis aimed to capture the international evidence on the associations at low frequencies between unemployment and the moments of price and productivity growth. A linear version of the long-run Phillips curve found in the theory, is estimated using data provided by a sample of 33 members of OECD countries for the period spanning from 1961 to 2011. Unemployment mean is the endogenous variable, whereas mean and standard deviation of both inflation and productivity growth are the regressors.[2] The results of panel regressions support the implications of the theory, suggesting that unemployment at low frequencies, is negatively correlated with the levels of price and productivity growth, whereas it is positively correlated with their volatilities. Remarkably, these results are robust not only by using different measures of inflation and productivity, but also when the estimates keep track of the degree of rigidity of the countries analyzed. In the countries with highest values of price inflation mean, in which nominal wages are generally less likely to be affected by downward rigidity, the long-run relations are weaker. Thus, only the coexistence of downward nominal wage rigidity with growth processes for both real and nominal sides of the economy, allows to make long-run expected unemployment endogenously determined by the drifts and the volatilities of the processes that lead the economy.

Both the assumptions of nominal wage rigidities and non-stationary processes for price and productivity are widely discussed in the literature. As regards the former, many contributions at micro and macro level, argue on the fact that nominal wages adjust upwardly more easily than downwardly. In the micro literature, the wage rigidity is well-documented using data not only at firm- and industrial-level,[3] but also on survey basis.[4] Using survey-based data on US firms, Bewley (1999) among others, explains the downward stickiness of nominal wages, contrasting the common view that rigidities come from the reluctance of workers in accepting wage cuts. His survey indicates that the firms are scarcely inclined to cut nominal wages because it would hurt the

[2] The empirical analysis focuses on the long-run, so to extrapolate the moments at low-frequencies of the series, the full interval of time is firstly divided into 10-year rolling windows, then for each window, it is calculated the average or the standard deviation of the variable of interest. All series used in regressions are thus compiled with single points, which correspond to the average or the standard deviation respectively, of the previous 10 years.

[3] Some examples dealing with US firms data are Akerlof et al. (1996), Kahn (1997), Card and Hyslop (1997), Altonji and Devereux (2000), Lebow et al. (2003), Elsby (2009), Fagan and Messina (2009), Kim and Ruge-Murcia (2009) and Daly et al. (2012). Still using firm-level data, Dickens et al. (2007) make a comparison among different countries. Differently, using industrial-level data Holden and Wulfsberg (2008) and Messina et al. (2010) provide a multi-country analysis of downward nominal wage rigidity.

[4] International evidence based on surveys are for example provided by Holden (2004), Knoppik and Beissinger (2009), Babecky et al. (2010) among others.

workers morale and eventually their productivity. In the macro literature most of the contributions on wage rigidities focus on their relevance in explaining the business cycle fluctuations with labor markets featured by search and matching frictions, as in Hall (2005) and Shimer (2005).[5] As regards instead the assumption of growing dynamics of price and productivity levels, it is in line with the experience for most of the developed countries, at least before the onset of the Great Recession. In particular, the introduction on a process with drift on prices allows to replicate the behavior of price level under a monetary policy of strict positive inflation targeting. Under this policy, the central bank is able to pursue a certain inflation level, unless unpredictable shocks that move the price growth from the target. The finding that positive level of inflation might reduce the expected unemployment in the long-run relates this paper to the literature that formalizes the long-lasting effects of price growth on labor market outcomes.[6] Adding downward nominal wage rigidity in a dynamic stochastic general equilibrium model, Kim and Ruge-Murcia (2008), Fagan and Messina (2009), Fahr and Smets (2010) investigate the so-called *greasing* effects of inflation,[7] while Akerlof et al. (1996), and more recently Benigno and Ricci (2011), derive a closed-form solution for the long-run Phillips curve.[8] Starting from an environment similar to Benigno and Ricci (2011), the model economy of this paper separates the processes that lead the real and the nominal growth.[9] This separation allows to relate expected unemployment contemporaneously to drifts and volatilities of both real and nominal processes. The positive relation between long-run unemployment and the macroeconomic volatility links this paper with the contributions that analyzes separately the real effects of the variability of inflation[10]

[5] More recent contributions in search literature with wage rigidities are Christoffel et al. (2009), Gertler and Trigari (2009), Barnichon (2010), Blanchard and Gali (2010), and Abbritti and Fahr (2011).

[6] Considering a non-zero inflation level in the long-run, this paper is also related to the macro literature that focuses on positive value of steady state inflation, that is the literature on *trend inflation*. See for instance Ascari (2004), Ascari and Sbordone (2014).

[7] Loboguerrero and Panizza (2006) find that the *greasing* effects of inflation are more relevant in those countries where the labor market is highly regulated. Restricting the analysis to Switzerland for the 1990s, Fehr and Gotte (2005) stress on the role of nominal rigidity for the long-run Phillips curve. They show that during the periods featured by low inflation, unemployment rate is higher in those Swiss cantons more affected by downward nominal wage rigidity.

[8] With a dynamic general equilibrium model featured by downward nominal wage rigidities similar to Benigno and Ricci (2011) and Fagan and Messina (2009), Daly and Hobijn (2014) study instead the long-run Phillps curve by solving for the non-linear dynamics of the model.

[9] Benigno and Ricci (2011) consider indeed a single exogenous process on nominal spending, that by definition, joins the dynamics of real and nominal sides of the economy.

[10] The role of inflation volatility in explaining real variables dynamics is studied in several papers that compare different monetary policy rules with US data as Clarida et al. (1999), Svensson (1999), Taylor (1999). In cross-countries analysis instead, Fischer (1993), Judson and Orphanides (1999) argue on the negative effects of inflation volatility on real growth.

and of productivity.[11] This work shares with those contributions the detrimental effects of either nominal or real volatility on the labor market outcomes. The finding instead of a negative relation between long-run unemployment and the level of productivity links this paper with the literature that claims that technology progress creates new jobs, rather than destroys them.[12]

The rest of the paper proceeds as follows. Section 2 illustrates the theoretical model, which provides a closed-form solution for the long-run Phillips curve. Section 3 discusses the data and the results of panel regressions. Section 4 describes the robustness checks that supports the empirical analysis. Specifically, panel regressions are repeated firstly, with alternative measures of inflation and productivity, secondly with different sub-periods, and thirdly with the inclusion of interaction terms that capture the role of nominal rigidities. Section 5 concludes.

2. Theoretical model

The theoretical framework used to study the long-run relationship between unemployment and nominal growth is a dynamic stochastic general equilibrium (DSGE) model featured by downward nominal wage rigidity (DNWR). For simplicity, the rigidity here considered is extreme, in the sense that nominal wages are assumed to be totally prevented from falling. This rigidity might be conveniently interpreted as a social norm that does not allow cuts in nominal wages. The non-negative constraint on nominal wage dynamics can be written as,

$$d \ln W_t \geqslant 0 \tag{1}$$

The model economy follows Benigno and Ricci (2010) and Benigno, Ricci and Surico (2015), but differently from them, here are introduced two independent processes on prices and productivity growth. The (logs of) price P_t and labor productivity A_t follow indeed two geometric Brownian motions,

$$d \ln P_t = \mu dt + \sigma_P dB_{P,t} \ , \tag{2}$$

$$d \ln A_t = g dt + \sigma_A dB_{A,t} \ , \tag{3}$$

where μ and g are the drift coefficients of price and productivity processes; σ_P and σ_A are the corresponding volatility coefficients; $B_{P,t}$ and $B_{A,t}$ are the independent Wiener processes.

[11]For instance in different settings, Hairault et al. (2010) and Benigno et al. (2015) emphasize on the positive impact on unemployment of the second moments of respectively, total factor productivity and labor productivity.

[12]The huge literature that supports the positive association between productivity and long-run employment, includes among others, Bruno and Sachs (1985), Phelps (1994), Blanchard et al.(1995), Blanchard and Wolfers (2000), Staiger, Stock, and Watson (2001), Pissarides and Vallanti (2007), Shimer (2010).

The rest of the model is very standard. The economy is composed by a continuum of infinitely lived households, which derive utility from consuming goods and disutility from supplying labor. Each household j, with $j \in [0,1]$, is heterogenous given that all its members, i.e. the workers, provide to firms a specific kind of job $l_t(j)$. Workers from different households compete in a labor market featured by monopolistic competition. Conversely, firms are homogenous and determine the labor demand by choosing both the aggregate level of labor and the optimal allocation between the different types of labor. Since prices adjust instantaneously, representative firm simply chooses the aggregate labor demand by maximizing the following static profit function with respect to the labor L_t,

$$P_t Y_t - W_t L_t. \tag{4}$$

In profit maximization, the only constraint faced by firms is the production function, $Y_t = A_t L_t^{\alpha}$, which with $\alpha < 1$ admits decreasing returns to scale for the labor input. The first order condition of firm problem equates the nominal wage to the value of marginal labor productivity,

$$W_t = \alpha P_t A_t L_t^{\alpha-1}. \tag{5}$$

The aggregate labor demand L_t is determined from the wage schedule (5). In turn, this aggregate labor demand is composed by the individual demands for any types of labor, through the following CES aggregate,

$$L_t \equiv \left[\int_0^1 l_t(j)^{\frac{\theta_\omega - 1}{\theta_\omega}} di \right]^{\frac{\theta_\omega}{\theta_\omega - 1}}, \tag{6}$$

where $\theta_\omega > 1$ represents the elasticity of substitution between labor types. In choosing the optimal demand for any kind of labor, representative firm considers the following Dixit-Stiglitz aggregate wage index, which takes as given all individual nominal wages $W_t(j)$ chosen by the households,

$$W_t \equiv \left[\int_0^1 W_t(j)^{1-\theta_\omega} di \right]^{\frac{1}{1-\theta_\omega}}.$$

The optimal labor allocation for the representative firm gives the following individual demand schedule for labor type j,

$$l_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\theta_\omega} L_t. \tag{7}$$

Such downward-sloping individual labor demand is considered among the constraints for household j, which sets autonomously the wage she gains, as in Erceg et al. (2000). Households j maximizes indeed the present discount value of instantaneous utilities with respect to both consumption goods C_t^j and nominal wage $W_t(j)$. The objective function can be written as,

$$E_{t_0}\left[\int_{t_0}^{\infty} e^{-\rho(t-t_0)}\left(\ln C_t^j - \frac{l_t(j)^{1+\eta}}{1+\eta}\right)dt\right], \tag{8}$$

where $\rho > 0$ is the preference discount rate and η is the inverse of the elasticity of labor supply with respect to nominal wage. Utility maximization is subject to the individual labor demand (7) and to the nominal intertemporal budget constraint,

$$E_{t_0}\left\{\int_{t_0}^{\infty} Q_t P_t C_t^j dt\right\} = E_{t_0}\left\{\int_{t_0}^{\infty} Q_t W_t(j)\, l_t(j)\, dt\right\}, \tag{9}$$

with Q_t the stochastic discount factor in capital markets, where claims to monetary units are traded. The household intertemporal allocative problem is solved by the first order conditions with respect to consumption at time t and $t+1$, which give the following standard Euler equation,

$$\frac{1}{R_t} = e^{-\rho} E_t\left[\frac{C_t}{C_{t+1}}\frac{P_t}{P_{t+1}}\right], \tag{10}$$

where R_t is the gross nominal interest rate. Since it is assumed that capital markets are complete, the consumption level C_t is uniform among the households. The first order condition with respect to nominal wage determines its optimal level W_t^* as,

$$W_t^* = \mu_\omega P_t C_t L_t^\eta. \tag{11}$$

The optimal nominal wage is equal to the marginal rate of substitution between consumption and leisure, unless a term $\mu_\omega \equiv \frac{\theta_\omega}{\theta_\omega - 1}$, which represents the mark-up due to worker monopoly power. The constraint on nominal wages (1) implies that the current level of nominal wage is not necessary equal to the optimal one. Whenever the optimal wage is lower than the wage prevailing in the previous period, i.e. $W_t^* < W_{t-1}$, the current level of nominal wage remains fixed to the level of previous period, resulting therefore higher than the optimal level for the current period,

$$W_t = W_{t-1} > W_t^*$$

Importantly so far, households are assumed to be myopic with respect to the social norm that prevents nominal wage from falling. In others words, it is assumed that households do not consider the non-negative constraint in choosing the optimal wage. Hence the wage schedule for the myopic households is given by the following,

$$\begin{aligned} \text{if } W_t^* &\geqslant W_{t-1} \Longrightarrow W_t = \mu_\omega P_t C_t L_t^\eta = W_t^*, \\ \text{if } W_t^* &< W_{t-1} \Longrightarrow W_t = W_{t-1} > W_t^*, \end{aligned} \tag{12}$$

which states again that the current level of nominal wage equates the current optimal level *if and only if* the latter is not lower than the wage prevailing in the previous period. As shown in Appendix A.1, the wage schedule for households would be slightly different, if they consider the non-negative bound on nominal wages (1) among the constraints of the utility maximization problem. Although the inclusion of the lower bound on nominal wages among the constraints of households problem, complicates the derivation of the wage supply schedule, it does not modify substantially the main result of the theory, that is a long-run Phillips curve in a closed-form. For this reason the main text continues to consider the case of myopic households. The long-run relation between expected unemployment and the moments of price and productivity growth processes prevailing in this environment is derived analytically in the following section.

2.1 Long-run Phillips curve

As in Gali (2011), unemployment is defined as the difference between the (logs of) notional labor supply and the current labor demand, $u_t \equiv \ln(L_t^n) - \ln(L_t^d)$. The former is the labor supply provided by workers under a perfect competitive labor market with flexible wages. Its value is recovered by equating the real wage to the marginal rate of substitution between consumption and leisure. In log-terms, it is

$$\ln W_t - \ln P_t = \ln C_t + \eta \ln(L_t^n). \tag{13}$$

Since all goods produced by the firms are consumed, it is possible to combine the goods market clearing condition, the labor demand determined by (5) and the notional labor supply determined by (13) in order to rewrite the unemployment rate as,

$$u_t = \chi \left(\ln W_t - \ln P_t - \ln A_t - \bar{\alpha} \ln \alpha\right), \tag{14}$$

where χ and $\bar{\alpha}$ are defined as $\chi \equiv \frac{1+\eta}{\eta(1-\alpha)}$ and $\bar{\alpha} \equiv \frac{\alpha+\eta}{1+\eta}$. Conveniently, the (log of) notional supply L_t^n can be rewritten as the sum of the (log of) labor supply L_t^*, provided by workers under a monopolistically competitive labor market with flexible wages, and the constant term $\frac{1}{\eta}\ln \mu_\omega$, which defines the natural unemployment as in Gali (2011),

$$\ln\left(L_t^n\right) = \ln\left(L_t^*\right) + \frac{1}{\eta}\ln \mu_\omega, \tag{15}$$

Plugging (14) into (15) gives the gap between supply and demand in a labor market featured by monopolist competition,

$$\ln\left(L_t^*\right) - \ln\left(L_t^d\right) = \chi \left(\ln W_t - \ln P_t - \ln A_t - \bar{\alpha} \ln \alpha\right) - \frac{1}{\eta}\ln \mu_\omega, \tag{16}$$

The wage norm assumed in the model ensures that, whenever nominal wage is equal to its optimal level, i.e. $W_t = W_t^*$, the economy behaves like with flexible wages, thereby labor market clears, i.e. $\ln(L_t^*) = \ln(L_t^d)$. Considering (16) and (14), this means that in that case unemployment is constant and equal to its natural level. Whenever instead, the current nominal wage is higher than its optimal level, unemployment is no more constant, but rises above its natural level. In this case, the labor gap is positive and unemployment follows a dynamics led by the underlying processes of prices and productivity. Taking the unemployment equation (14) in differential terms, it is straightforward to note that whenever lower constraint binds, i.e. $dW_t = 0$, unemployment variations are proportional to $d\ln P_t$ and $d\ln A_t$. Specifically, above the lower barrier $\frac{1}{\eta}\ln\mu_\omega$, unemployment moves like a geometric Brownian motion with a drift $-\chi(\mu+g)$ and volatility coefficient $-\chi(\sigma_P+\sigma_A)$. Unemployment follows thus a regulated Brownian motion with a negative drift, given that $\chi > 0$. Standard results guarantee that unemployment has a stationary distribution depending on trend and volatility coefficients of price and productivity processes,[13]

$$f(x) = \frac{2\vartheta}{\chi\tilde{\sigma}^2} e^{\frac{2\vartheta}{\chi\tilde{\sigma}^2}(x-\bar{u})}, \tag{17}$$

where $\bar{u}$, ϑ and $\tilde{\sigma}^2$ are respectively defined as $\bar{u} \equiv \frac{1}{\eta}\ln\mu_\omega$, $\vartheta \equiv \mu+g$ and $\tilde{\sigma}^2 \equiv \sigma_P^2+\sigma_A^2$. From the stationary distribution (17), it is possible to determine the long-run expected value for unemployment,

$$E[u_\infty] = \bar{u} + \frac{\chi\tilde{\sigma}^2}{2\vartheta}, \tag{18}$$

Equation (18) is the key equation of the model. It shows that, apart from the natural level of unemployment $\bar{u}$, the long-run expected value of unemployment depends positively on the quantity $\tilde{\sigma}^2$, that represents the sum of the variability coefficients of price and productivity processes, and negatively on the quantity ϑ, that represents the sum of the trend coefficients. The assumption of two separated processes on price and productivity allows to disentangle the contributions of nominal and real dynamics on expected unemployment. However, since the moments of inflation and productivity growth enters into equation (18) symmetrically, their contributions are equal from a qualitatively point of view. Interestingly, equation (18) states that long-run unemployment has a positive expected value. As a result, the differential of long-run unemployment du_∞ has to be null in expectation terms. Taking then, for both sides of unemployment equation (14), i) the long-run differentials and ii) the expected values, it derives the following,

$$E[d\ln W_\infty] = \mu + g. \tag{19}$$

that states that the expected long-run dynamics of nominal wages, that is the expected long-run nominal wage inflation, is given by the sum of the trends in price inflation and productivity. Combining equations (19) and (18) yields the expected long-run

[13] See for more details Harrison (1985).

unemployment depending directly on the expected long-run nominal wage inflation,

$$E[u_\infty] = \bar{u} + \frac{\chi \tilde{\sigma}^2}{2E[d \ln W_\infty]}. \tag{20}$$

Equation (20) is a long-run Phillips curve (LRPC) that highlights the negative relation of expected unemployment with the level of nominal wage inflation and the positive relation with the macroeconomic volatility. Equation (20) emphasizes that, in an environment featured by downward nominal wage rigidity, the expected unemployment at low frequencies depends on how much nominal wages grows on average. The higher is the trend in nominal wage inflation, the lower is the expected value of unemployment. Considering indeed a certain period of time, the higher is the level of wage inflation, the fewer will be on average the episodes during which negative realizations of normally distributed shocks need a fall in optimal nominal wages. Moreover, when optimal nominal wage needs effectively to fall, on average it will decrease less, because the higher wage inflation will alleviate the impact of any shock This has a consequence on the short-run unemployment, which will increase above its natural level at a lower frequency and for lower amounts than it would do with lower levels of wage inflation. This implies that expected value of unemployment rate in the long-run will be lower.

3. International evidence

This section tests on a cross-country basis, the main conclusions of the theoretical model. The empirical analysis aims to study the relationship between long-run unemployment and the moments of both inflation and productivity growth processes. The associations this analysis wishes to investigate, are provided by equation (18), that makes the expected long-run unemployment depending on the *normalized* variances of price inflation and productivity growth, with normalization rate given by the sum of the nominal and real trends. To test if the theoretical prescriptions are confirmed in the data even with a simple empirical strategy, the panel regression model considered is linear and static. Equation (18) is indeed approximated by a specification that makes long-run unemployment depending linearly on the moments of both inflation and productivity dynamics. The individual contributions of trends and volatilities are studied using an international dataset, that presents two important features. The first one is that, along with the variability of cross-sectional dimension, the dataset includes the variability of times-series dimension, since it collects observations for each country on annually basis from 1961 to 2011. Although the analysis is focused on the long-run, considering just the simple averages of the variables for the full sample interval do not allow to take into account the structural changes, that have affected the variables dynamics during the Great Moderation and the Great Recession. The mean and the standard deviation of different variables are instead calculated on 10-year rolling windows basis, in order to keep track of the time-variability of the data. The second feature of the dataset concerns the sample of countries analyzed. The international comparison is limited to advanced countries, that traditionally exhibit moderate variability in both real and nominal terms.[14] This restriction is motivated by the necessity of collecting sufficiently long series for all variables. For each country, there are calculated yearly series i) for the levels of unemployment rate, inflation rate and productivity growth rate and ii) for the standard deviations of inflation and productivity rate. For both i) and ii), any value of the series corresponds respectively, to the average and to the standard deviation of the variable levels observed in the previous 10 years. The dataset so compiled, that is however unbalanced, is used in panel regressions with fixed effect and time dummies. The next Section 3.1 describes in details the data used, while the following Section 3.2 discusses the results of panel regressions.

3.1 Data

The unemployment rate is the endogenous variable for all specifications considered below. The unemployment rate series is recovered by OECD Annual Labor Force Statistics, as the yearly percentage of unemployed workers on civilian labour force (YGTT06PC_ST).

To measure the mean and the volatility of inflation at low frequencies, two alternative variables are compared in the baseline specifications. The first one is the growth rate of CPI index for all items, which is the variable that probably best fits the growing process for prices assumed in the theory. As argued above, under an exogenous process with drift, the price level has a pattern similar to the one that it would follow if the monetary authority pursued a policy of strict inflation targeting. Since generally, the central banks that have adopted an inflation targeting, have taken the growth rate of consumer prices as the benchmark on which decided the target, the choice of CPI increments to measure level and volatility of price inflation is very reasonable. Such measure of price inflation is also used in other empirical contributions on the real effects of inflation, as Fischer (1993) and Judson and Orphanides (1996). Despite the analysis of these contributions focus on the short-run effects of inflation on growth, they shares with this paper the emphasis on the relation between real variables and inflation volatility.[15] The series for consumer inflation are recovered by the OECD Consumer price indices database, as the

[14] The countries considered in the sample are Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Israel, Italy, Japan, South Korea, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Russian Federation, Slovenia, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States.

[15] Differently in particular, from Judson and Orphanides (1996), in this paper the inflation volatility is not computed using quarterly observations, because annually data appears more informative for an analysis at low frequencies.

percentage changes on the previous year of CPI index for all items (CPALTT). The alternative variable used to measure long-run inflation moments is the GDP deflator, whose availability of data is sufficiently long to make feasible the comparison with the CPI index for all goods. Such comparison is useful because, although CPI index and GDP deflator are both commonly used in literature to measure the price inflation, they differ significantly with respect to the variability of the series. Increments of GDP deflator, which are provided by the OECD Annual National Accounts as the percentage changes on the previous year, result in particular much less volatile than the increments of CPI index.[16]

Since the theoretical model presented above, considers the labor input as the single factor used in the production process, the mean and the volatility of productivity growth are measured through the variations of the marginal contribution of labor. Given that observations on total hours are very short for several countries in the sample, the marginal contribution of labor is measured in terms of output per employed workers. The labor productivity growth is obtained by the yearly increments of the ratio between the real gross value added at basic prices and the total employment. For the former, data are provided by OECD Annual National Accounts, through the series of gross domestic product calculated according to output approach at constant prices and constant PPPs. For the employment, data are given by Annual Labor Force Statistics, through the series of total employment (YGTT04L1_ST) and of civilian employment (YGTT08L1_ST). When data available are sufficiently long, the series on total employment are preferred.[17] Since for most of the countries, the total employment series start in the Seventies,[18] the interval of time actually analyzed in the panel regressions for those countries, is restricted of about ten years.

3.2 Panel regressions

In the baseline specification, unemployment rate $u_{i,t}$ for country i at time t, is regressed over level and volatility terms of inflation and labor productivity growth, as the following equation shows,

$$u_{i,t} = \beta_1 \mu_{i,t} + \beta_2 g_{i,t} + \beta_3 (\sigma_P)_{i,t} + \beta_4 (\sigma_A)_{i,t} + \alpha_i + \lambda_t + \varepsilon_{i,t} \ , \tag{21}$$

where $\mu_{i,t}$ and $(\sigma_P)_{i,t}$ indicate respectively, the mean and the standard deviation of inflation. Analogously, $g_{i,t}$ and $(\sigma_A)_{i,t}$ indicate the mean and standard deviation of

[16] Considering for instance the full database, the standard deviation for the CPI inflation series is $0,54$ versus $0,20$ for GDP deflator variations.

[17] Civilian employment is taken as the measure of labor input for Austria, Chile, Greece, Israel, Japan, South Korea, Mexico, Poland, Slovenia, Sweden, Switzerland.

[18] The only countries in the sample with total employment series starting before than 1970 are Denmark, France, South Korea, Netherlands, United Kingdom.

labor productivity growth. The further regressors α_i and λ_t indicate the country and time fixed effects. These dummy variables capture the effects on unemployment due to factors related to the structural characteristics of any country and to the specific events happened during the years analyzed.

The estimation results are shown in Table 1. In specifications 1 and 2, unemployment at low frequencies is regressed only over inflation terms. With either CPI inflation or GDP deflator increments, unemployment results to depend negatively on inflation mean and positively on inflation volatility. Similarly, in specification 4 unemployment is regressed only on labor productivity terms. Still the signs of the estimated coefficients for productivity are in line with the theory, that is negative for the mean and positive for the volatility, but they are not statistically significant. When in specifications 5-6, both real and nominal moments are included into regressions, all the four associations of unemployment with drift and volatility of price and productivity growth are statistically significant and with the signs predicted by the theory. Independently from how inflation is measured, in specifications 5 and 6 unemployment is affected negatively by the levels and positively by the volatilities of both price and productivity growth.

Importantly, all specifications that include mean and standard deviation of inflation present an appropriated R^2 for fixed effect models, i.e. the LSDV-R^2 -where LSDV indicates the regression method of Least Squares Dummy Variables-, higher than $0,75$. The fitness of the estimates confirm that, albeit the specifications used in panel regressions are only linear approximations of the non-linear long-run Phillips curve (18), the data well support the opposite effects of drifts and volatilities on long-run unemployment. These effects are especially remarkable for the inflation terms, because even at low frequencies, the level and the volatility of price growth are highly positively correlated.[19] This positive correlation does not however prevent the level and the variability of inflation from having an opposite impact on the long-run unemployment mean.[20]

4. Robustness

In the following paragraphs the panel regressions are repeated in order to check the robustness of the estimation results under different conditions. The relation between

[19]Considering the full dataset, the correlation among mean and standard deviation computed over 10-year rolling windows is $0,95$ and $0,88$ respectively, for the increments of CPI index and of deflator index.

[20]To mitigate the positive correlation among level and volatility of price growth in the short-run, Judson and Orphanides (1996) consider an intra-year measure of inflation volatility that uses quarterly data. To check the robustness with quarterly data of the results here obtained, panel regressions described in Section 3.2 have been repeated with the same intra-year measure of inflation volatility used by Judson and Orphanides (1996). Estimation results are analogous to the ones obtained with annually data and are available upon request.

unemployment and the moments of price and productivity growth is tested firstly, considering alternative measures of inflation and productivity, secondly, dividing the full sample interval into two sub-periods, and thirdly, taking into account the different degree of nominal rigidity between the countries.

4.1 Alternative measures of inflation and productivity

As discussed in Section 2.1, the unemployment equation (18) can be conveniently rewritten as equation (20) that makes the expected long-run unemployment depending directly on the expected long-run nominal wage inflation. To test this alternative relation, estimations are repeated using mean and standard deviation of nominal wage inflation instead of price inflation in the linear regression model (21). The dynamics of nominal wage is calculated by collecting data on labor compensation. The series considered is provided by the OECD Annual National Accounts, as the compensation of labor at current prices and current PPPs. In Table 1 are shows with specification 3, the results obtained by regressing unemployment mean over average and standard deviation of wage inflation, whereas with specification 7, the results obtained by regressing unemployment mean over averages and standard deviations of both wage inflation and labor productivity growth. In both specifications the estimated coefficient related to the level of wage inflation is negative as the theory suggests. Still the estimated coefficient of the wage inflation variability is negative, but it is not statistically significant when labor productivity terms are included.[21]

To check also the robustness of the baseline estimations with an alternative measures of productivity, the panel regressions are run considering the total factor productivity (TFP) instead of the marginal contribution of labor. The TFP is a more general measure of productivity and thus, might be read more easily as the last force that leads the real growth. Using the empirical model (21), the regressions are repeated with the series of mean and standard deviation of TFP growth, calculated as in Pissarides and Vallanti (2007) as follows,

$$d\ln A_t = \frac{1}{\alpha}\left[d\ln Y_t - (1-\alpha)\, d\ln K_t - \alpha d\ln L_t\right] \ ,$$

where A_t is the level of TFP, Y_t is the GDP at constant price and national currencies, K_t is the capital stock, L_t is the total employment.[22] As shown in Table 2, independently

[21] The fact that both level and variability of nominal wage inflation affect negatively long-run unemployment is remarkable to the extent that, differently from price inflation, the correlation between mean and standard deviation of nominal wage inflation is lower and equals to $0,31$.

[22] In details, for the real output Y and the capital stock K there are used the gross domestic product (expenditure approach) and the gross capital formation. Both are taken at constant prices and constant PPPs. For labor input L, there are used series of the total employment (YGTT04L1_ST) when

from how it is measured the price inflation, when nominal and real moments are considered contemporaneously, as in specifications 9 and 10, all estimated coefficients are in line with the theory. Furthermore there are all statistically significant, but the TFP growth mean when inflation is measured with GDP deflator. In specification 11, the nominal wage inflation terms substitute the price inflation ones and the regression with TFP confirms the results obtained with labor productivity. Unemployment is negatively related to the wage inflation mean and positively related to the real volatility. Remarkably, although the substitution of labor productivity with TFP reduces significantly the number of observations used in the regressions, the appropriate R^2 remains high, above than $0,8$ for all specifications.

4.2 Controlling for sub-periods

To assess how the empirical evidence evolves during the time interval analyzed, panel regressions are run for sub-periods of the full interval considered in previous sections. The full sample interval spans a long period that includes the Great Moderation, during which the dynamics of aggregate variables has changed deeply, especially in developed countries. To control for the structural break that reduced heavily the volatility of aggregate variables in the 1980s, the full interval is divided into two sub-periods. Following Kim and Nelson (1999) and Stock and Watson (2002), the cut-off that divides the full sample period and marks the beginning of Great Moderation is fixed at the end of 1983. For comparability with the benchmark estimations in Section 3.2, the panel regressions are repeated using price inflation and labor productivity. In Table 3 unemployment mean is regressed over average and standard deviation of CPI index variations and labor productivity growth. With specification 12 are used data up to 1983, while with specification 13 are used data from 1984 on. Specifications 14 and 15 are identical to the previous two, apart from the variable used to measure the price inflation, which is the GDP deflator and not the CPI index. Estimates for specifications that use data for the sub-period preceding the Great Moderation are never significant. Conversely, the estimates for specifications that use data for the following sub-period are significant and in line with the theory, at least for volatility of labor productivity and for mean and volatility of price inflation. These results indicate that, restricting the analysis to a period in which many countries experienced a strong reduction in the level and in the variability of price inflation, the long-run real effects of pure nominal dynamics are enhanced. The evidence of a relation between real and nominal dynamics during

available, otherwise the series of civilian employment (YGTT08L1_ST). The labor share α is simply defined as the ratio between aggregate labor compensation and GDP using the same variables in Pissarides and Vallanti (2007): i) the labor cost, measured as the compensation of employees at current prices and current PPPs; ii) the gdp deflator; iii) the labor input; iv) the total number of self-employed -which are given by Annual Labor Force Statistics (YGTT22L1_ST)- and v) the real output.

the last decades featured by moderate rate of inflation, supports the general validity of the underlying theoretical model. The model provides a long-run relation between unemployment and nominal dynamics, under the assumption of downward nominal wage rigidity. By consequence, the long-run relation linking nominal and real sides of the economy should be dampen, when the level and the variability of inflation are high and the downward rigidities on nominal wages are less likely to bind. According to the data, this is exactly what occurred before the onset of the Great Moderation. To investigate more on this point, the following section studies how the long-run relation is influenced by the different rate of inflation between the countries.

4.3 Nominal rigidity interactions

Along with the growing processes that lead real and nominal sides of the economy, the existence of long-run Phillips curve detected in Section 2.1 is guaranteed by the presence of downward nominal wage rigidity. Without a constraint on nominal wages variations, wages would be to fluctuate freely and unemployment would remain fixed at its natural level. Hence, price and productivity dynamics should play a predictable role for long-run unemployment only for those economies actually affected by downward rigidities on nominal wages. Unfortunately, the degree of wage rigidity across countries is not easily measurable through a single variable, but it is however possible to discriminate the countries over the rigidities in the labor market, by considering some proxies related to the frictions that prevent wages from fluctuating freely. One of these proxies is the average value of inflation rate. The level of inflation pushes indeed upward the nominal wages, making less relevant the presence of downward rigidities. It derives that the more inflated countries should exhibit a less constrained wage dynamics, and in turn, a weaker relationship between long-run employment and the moments of price and productivity processes. To assess if effectively this relationship varies according to how heavy are the nominal rigidities, the country sample is divided into two categories, that separate the high from the low inflated countries. Countries are differentiated according to the inflation mean over the full interval of time. Table 5 illustrates for each country, the full sample mean of price inflation, measured by both CPI index and GDP deflator, and the full sample mean value of wage inflation, measured by the nominal compensation for employees. A dummy variable $\delta_{\pi,i}$ assigns 1 to those countries with the inflation mean above the median of the sample, and 0 to those with the inflation mean below the median. This dummy enters into the empirical model through the interactions terms with the baseline regressors discussed above. The linear regression model used for estimations becomes the following,

$$\begin{aligned} u_{i,t} &= \beta_1 \mu_{i,t} \left(1 + \beta_5 \delta_{\pi,i}\right) + \beta_2 g_{i,t} \left(1 + \beta_6 \delta_{\pi,i}\right) + \\ &\quad + \beta_3 \left(\sigma_P\right)_{i,t} \left(1 + \beta_7 \delta_{\pi,i}\right) + \beta_4 \left(\sigma_A\right)_{i,t} \left(1 + \beta_8 \delta_{\pi,i}\right) + \alpha_i + \lambda_t + \varepsilon_{i,t} \quad , \end{aligned} \tag{22}$$

Table 6 shows the estimates obtained from three specifications that share the regression equation (22), but use different variables to measure the inflation. Results are very clear for specifications 16 and 17, that use data of price inflation. As in specification 5 and 6, for mean and standard deviation of the price growth and for mean of the labor productivity growth, the estimated direct impacts on long-run unemployment are statistically significant and with signs coherent with the theory. Still for the interactions of mean and standard deviation of the price inflation with the inflation dummy, the estimated coefficients are statistically significant, but they present opposite signs. For high inflated countries, which face a regression equation (22) with $\delta_{\pi,i} = 1$, it is positive the additional contribution of inflation and productivity mean, whereas it is negative the additional contribution of inflation volatility. It follows that for high inflated countries, the overall impact of price inflation and productivity growth on long-term unemployment are substantially dampen with respect to the overall impact for less inflated countries, whose additional contributions of interaction terms are null, given that $\delta_{\pi,i} = 0$. Therefore, the estimations highlight that the explicative role of price and productivity growth for long-run unemployment is weaker for those countries in which prices pressure is heavier and the nominal rigidities are less likely to bind. The empirical results supports the theory even when inflation is measured by the nominal wage growth, as in specification 18. Focusing on the direct effect of wage inflation level on unemployment at low frequencies, the estimated correlation is statistically significant and negative like in specification 7. That correlation changes the sign becoming positive, when it is considered the additional effect of wage inflation level reserved to the countries with relative high nominal wage inflation. Like in the case of price growth, it means that the overall effect of wage inflation level on long-run unemployment is mitigated for those countries, where wages increase more on average and the downward rigidities are less likely to bind.

5. Conclusions

This paper studies the long-run relation between unemployment and the last forces that drive the overall growth, that is productivity and inflation that in turn affect respectively, the real and the nominal dimension of the economy. This paper emphasizes that drifts and volatilities of productivity and inflation have opposite effects on expected unemployment in the long-run, especially in those economies more affected by downward nominal wage rigidities. As regards the drifts indeed, they are negative related to expected unemployment, because by fostering the nominal growth, they reduce the probability that for a given negative shock, i) nominal wages are dragged to the lower bound, ii) the labor margin compensates, and iii) the expected unemployment in the long-run lies above the natural level. On the other side, the volatilities of productivity and inflation are positively related to the expected unemployment, because for any given trend in nominal growth, they amplify variables fluctuations making less likely wages to be free to fluctuate and, in a longer prospective, more likely unemployment rate to be higher than the natural level. These associations are firstly derived from the theory through a dynamic stochastic general equilibrium model featured by i) Brownian motions with drift for productivity and price growth and ii) downward rigidity for nominal wages. The theoretical model allows to obtain a long-run Phillips curve in closed form, which relates the long-run expected unemployment with the growth dynamics of productivity and prices. The same associations are confirmed empirically through panel estimations with fixed effects, which consider a linear approximation of the Phillips curve derived in the theory. Long-run mean of unemployment is regressed over averages and standard deviations at low frequencies of productivity and price growth. Regressions are run using a database that encompass annual observations for the most of OECD members from 1961 to 2011. Panel estimations suggest that whether the presence of nominal wage rigidities is heavier, especially when the inflation pressures are sluggish, the relationship derived from the theory between unemployment and growth dynamics of productivity and prices is well established in the data.

Despite the paper does not include a policy analysis, the main findings support the effectiveness of monetary policy in the long-run. Focusing indeed, on the nominal dynamics of the economy, the analysis here developed suggests that any measure aimed to foster the nominal growth might contribute to clear the labor market. Meanwhile, any measure of price stabilization allows to limit the harmful effects of macroeconomic volatility. Since the price level contributes to sustain the nominal growth, as the productivity level does for the real growth, policymakers should not be then particularly adverse to measures aimed to raise the inflation level, still in a long-run prospective. Although the role of inflation on labor market outcomes has been widely discussed in the literature at business cycle frequencies, its role has been less analyzed on a prospective of long horizon. Nevertheless, if the structure of labor market is featured by nominal wage rigidities, as the micro literature suggests for many advanced economies, even the long-run relation between unemployment and inflation level deserves more attention. To shed the light on this issue, a promising way could be to study the relation in a different framework that admits unemployment at the steady state, as the models with search and matching frictions in the labor market. On the empirical side instead, the study of the jointly dynamics at low frequencies of macroeconomic volatility and labor market outcomes is on my research agenda. A possible way to tackle with this kind of analysis is to consider a time varying parameters VAR model with stochastic volatility. Time varying parameters and heteroskedastic variance of VAR innovations allows to recover not only estimated measures of long-run mean of endogenous variables, but also of their volatilities. Moreover through the spectral analysis, that investigate the empirical model on the frequency domain, it is possible to discern the contribution of long-run components to the estimated variance of variables. Thereby the introduction among the endogenous variables of some proxy of macroeconomic volatility, as for instance the economic policy uncertainty provided by Baker et al. (2015), allows to explore directly the comovement at low frequencies of macroeconomic volatility and unemployment rate.

References

Abbritti, Mirko & Stephan Fahr, (2011). "Macroeconomic implications of downward wage rigidities," Working Paper Series 1321, European Central Bank.

Akerlof, George A. & William R. Dickens & George L. Perry, (1996). "The Macroeconomics of Low Inflation," Brookings Papers on Economic Activity, Economic Studies Program, The Brookings Institution, vol. 27(1), pages 1-76.

Akerlof, George A. & William R. Dickens & George L. Perry, (2000). Near-Rational Wage and Price Setting and the Long-Run Phillips Curve. Brookings Papers on Economic Activity 1:1—60.

Altonji, Joseph G., & Paul J. Devereux (2000), "The extent and consequences of downward nominal wage rigidity," in (ed.) 19 (Research in Labor Economics, Volume 19), Emerald Group Publishing Limited, 383-431.

Ascari, Guido, (2004). "Staggered Prices and Trend Inflation: Some Nuisances," Review of Economic Dynamics, Elsevier for the Society for Economic Dynamics, vol. 7(3), pages 642-667, July.

Ascari, Guido & Argia M. Sbordone, (2014). "The Macroeconomics of Trend Inflation," Journal of Economic Literature, American Economic Association, vol. 52(3), pages 679-739, September.

Babecky, Jan & Philip Du Caju & Theodora Kosma & Martina Lawless & Julián Messina & Tairi Room, (2010). "Downward Nominal and Real Wage Rigidity: Survey Evidence from European Firms," Scandinavian Journal of Economics, Wiley Blackwell, vol. 112(4), pages 884-910, December.

Baker, Scott R. & Nicholas Bloom & Steven J. Davis, (2015). "Measuring Economic Policy Uncertainty," NBER Working Papers 21633, National Bureau of Economic Research, Inc.

Ball, Laurence. (1997). Disinflation and the NAIRU. In C.D. Romer and D.H. Romer (eds.) Reducing Inflation: Motivation and Strategy (pp. 167-194). Chicago: University of Chicago Press.

Ball, Laurence. (1999). Aggregate demand and long-run unemployment. Brooking Papers on Economic Activity 2: 189-251.

Ball, Laurence & Gregory Mankiw & David Romer, (1988). The New Keynesian Economics and the Output-Inflation Trade-off, Brookings Papers on Economic Activity, pp. 1-82.

Ball, Laurence & Robert Moffitt, (2002). Productivity Growth and the Phillips Curve. In The Roaring Nineties: Can Full Employment be Sustained?, edited by A. B. Krueger and R. Solow. New York: Russell Sage Foundation.

Barnichon, Regis (2010). "Productivity and Unemployment over the Business Cycle," Journal of Monetary Economics 57 (2010), 1013-1015.

Benigno, Pierpaolo & Luca Antonio Ricci, (2011). "The Inflation-Output Trade-

Off with Downward Wage Rigidities," American Economic Review, American Economic Association, vol. 101(4), pages 1436-66, June.

Benigno Pierpaolo & Luca Antonio Ricci & Paolo Surico, (2015). "Unemployment and Productivity in the Long Run: The Role of Macroeconomic Volatility," The Review of Economics and Statistics, MIT Press, vol. 97(3), pages 698-709, July.

Bewley, Truman F. (1995) "A Depressed Labor Market as Explained by Participants," American Economic Review, 85, 250-254.

Bewley, Truman F. (1999). Why Wages Don't Fall During a Recession, Cambridge, MA: Harvard University Press.

Blanchard, Olivier & Jordi Galí, (2010). "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment," American Economic Journal: Macroeconomics, American Economic Association, vol. 2(2), pages 1-30, April.

Blanchard, Olivier & Robert Solow & Beth Anne Wilson, (1995). "Productivity and Unemployment," mimeo Massachusetts Institute of Technology.

Blanchard, Olivier & Justin Wolfers, (2010). "The Role of Shocks and Institutions in the Rise of European Unemployment: The Aggregate Evidence," Economic Journal 110, 1-33.

Bruno, Michael & Jeffrey. D. Sachs, (1985). "Economics of Worldwide Stagflation,".Cambridge, Massachusetts, Harvard University Press.

Christoffel, Kai & Keith Kuester & Tobias Linzert, 2009). The role of labor markets for euro are a monetary policy, European Economic Review, 53(8), 908 — 936

Daly, Mary C.& Bart Hobijn, (2014), "Downward Nominal Wage Rigidities Bend the Phillips Curve". Journal of Money, Credit and Banking, 46: 51–93.

Daly, Mary C.& Bart Hobijn & Brian T. Lucking (2012) "Why Has Wage Growth Stayed Strong?", FRB SF Economic Letter 2012-10, April 2, 2012.

Dickens, William T. & Lorenz F. Goette & Erica L. Groshen & Steinar Holden & Julian Messina & Mark E. Schweitzer & Jarkko Turunen & Melanie E. Ward-Warmedinger, (2007) "How Wages Change: Micro Evidence from the International Wage Flexibility Project," Journal of Economic Perspectives, 21, 195-214.

Dixit, Avinash (1991), "A Simplified Treatment of the Theory of Optimal Regulation of Brownian Motion," Journal of Economic Dynamics and Control 15, 657-673.

Dumas, Bernard (1991), "Super Contact and Related Optimality Conditions: A Supplement to Avinash Dixit's: A simplified Exposition of Some Results Concerning Regulated Brownian Motion," Journal of Economic Dynamics and Control, vol. 15(4), pp. 675-685.

Erceg, Christopher J.& Dale W. Henderson & Andrew T. Levin (2000), "Optimal Monetary Policy with Staggered Wage and Price Contracts," Journal of Monetary Economics, 46 (2), pp. 281-313.

Elsby, Michael W.L. (2009) "Evaluating the Economic Significance of Downward Nominal Wage Rigidity," Journal of Monetary Economics, 56, 154-169.

Fagan, Gabriel & Julian Messina, (2009). "Downward wage rigidity and optimal steady-state inflation," Working Paper Series 1048, European Central Bank.

Fahr, Stephan & Frank Smets, (2010). "Downward Wage Rigidities and Optimal Monetary Policy in a Monetary Union," Scandinavian Journal of Economics, Wiley Blackwell, vol. 112(4), pages 812-840, December.

Fair, R.C. (2000). Testing the NAIRU model for the United States. The Review of Economics and Statistics 82 (1): 64-71.

Fehr, Ernst, & Lorenz Goette, (2005). Robustness and Real Concequences of Nominal Wage Rigidity. Journal of Monetary Economics, 52(4), 779–804.

Fischer, Stanley (1993), The Role of Macroeconomic Factors in Growth, Journal of Monetary Economics, December , 485-512.

Fisher, Mark E. & Seater, John.J. (1993). Long-run neutrality and supeneutrality in an ARIMA framework. The American Economic Review, 83: 402-415.

Gertler, Mark & Antonella Trigari, (2009). "Unemployment Fluctuations with Staggered Nash Wage Bargaining," Journal of Political Economy, University of Chicago Press, vol. 117(1), pages 38-86, 02.

Hairault, Jean-Olivier & Francois Langot & Sophie Osotimehin, (2010). "Matching frictions, unemployment dynamics and the cost of business cycles," Review of Economic Dynamics, Elsevier for the Society for Economic Dynamics, vol. 13(4), pages 759-779, October.

Hall, Robert. E. (2005). "Employment Fluctuations with Equilibrium Wage Stickiness,"American Economic Review 95 (2005), 50-65.

Harrison, Michael J., (1985). Brownian Motion and Stochastic Flow Systems, John Wiley and Sons, New York.

Holden, Steinar (2004). The Costs of Price Stability—Downward Nominal Wage Rigidity in Europe. Economica, 71, 183–208.

Holden, Steinar & Wulfsberg Fredrik, (2008). "Downward Nominal Wage Rigidity in the OECD," The B.E. Journal of Macroeconomics, De Gruyter, vol. 8(1), pages 1-50, April.

Judson, Ruth & Athanasios Orphanides, (1999). "Inflation, Volatility and Growth," International Finance, Wiley Blackwell, vol. 2(1), pages 117-38, April.

Kahn, Shulamit, (1997). "Evidence of Nominal Wage Stickiness from Microdata," American Economic Review, American Economic Association, vol. 87(5), pages 993-1008, December.

Kim, Chang-Jin & Charles R. Nelson, (1999). "Has The U.S. Economy Become More Stable? A Bayesian Approach Based On A Markov-Switching Model Of The Business Cycle," The Review of Economics and Statistics, MIT Press, vol. 81(4), pages 608-616, November.

Kim, Jinill & Ruge-Murcia, Francisco J., (2009). "How much inflation is necessary to grease the wheels?," Journal of Monetary Economics, Elsevier, vol. 56(3), pages

365-377, April.

King, R.G. & Watson, M.W. (1994) The post-war U.S. Phillips curve: a revisionist econometric history. Carnegie-Rochester Conference Series on Public Policy 41: 157-219.

Knoppik, Christoph & Thomas Beissinger, (2009). "Downward nominal wage rigidity in Europe: an analysis of European micro data from the ECHP 1994–2001," Empirical Economics, Springer, vol. 36(2), pages 321-338, May.

Lebow, David A & Raven E. Saks & Beth Anne Wilson (2003), "Downward Nominal Wage Rigidity: Evidence from the Employment Cost Index," B.E. Journal of Macroeconomics: Advances, 3, issue 1.

Loboguerrero, Ana Maria & Ugo Panizza (2006). "Does Inflation Grease the Wheels of the Labor Market?," Contributions to Macroeconomics, Berkeley Electronic Press, vol. 6(1), pages 1450-1450.

Messina, Julián & Cláudia Filipa Duarte & Mario Izquierdo & Philip Du Caju & Niels Lynggård Hansen, (2010). "The Incidence of Nominal and Real Wage Rigidity: An Individual-Based Sectoral Approach," Journal of the European Economic Association, MIT Press, vol. 8(2-3), pages 487-496, 04-05.

Phelps, Edmund. S. (1994). Structural Slumps, The Modern Equilibrium Theory of Unemployment, Interest and Assets, Cambridge MA: Harvard University Press.

Pissarides, Christopher A. & Giovanna Vallanti, (2007). "The Impact Of Tfp Growth On Steady-State Unemployment," International Economic Review, Department of Economics, University of Pennsylvania and Osaka University Institute of Social and Economic Research Association, vol. 48(2), pages 607-640, 05.

Shimer, Robert (2005). "The Cyclical Behavior of Equilibrium Unemployment and Vacancies,"American Economic Review 95(1), 25-49.

Shimer, Robert, (2012). "Wage rigidities and jobless recoveries," Journal of Monetary Economics, Elsevier, vol. 59(S), pages S65-S77.

Staiger, Douglas & James H. Stock & Mark W. Watson, (2001). "Prices, Wages and the U.S. NAIRU in the 1990s," NBER Working Papers 8320, National Bureau of Economic Research, Inc.

Stokey, Nancy L. (2006), Brownian Models in Economics, mimeo, University of Chicago.

Stock, James H. & Mark W. Watson, (2003). "Has the Business Cycle Changed and Why?," NBER Chapters, in: NBER Macroeconomics Annual 2002, Volume 17, pages 159-230 National Bureau of Economic Research, Inc.

Svensson, Lars E O. (2015). "The Possible Unemployment Cost of Average Inflation below a Credible Target." American Economic Journal: Macroeconomics, 7(1): 258-96.

Tobin, James (1972), "Inflation and Unemployment," American Economic Review, 62, 1-18.

Appendix

A.1 Wage decision for forward-looking workers

In this appendix, the wage decision problem is solved for not-myopic households, that is for households that are aware of the lower bound on nominal wage changes, i.e. $dW_t \geqslant 0$. In this case, each j household chooses a sequence of optimal nominal wages, among the ones belonging to the space Ω of non-decreasing stochastic processes $\{W_t(j)\}$. In details each j household maximizes the following objective function,

$$E_t \left\{ \int_t^{\infty} e^{-\rho(t-t_0)} \pi \left(W_t(j), W_t, P_t, A_t \right) dt \right\}, \tag{23}$$

where the surplus $\pi(W_t(j), W_t, P_t, A_t)$ for period t can be defined as $\pi(W_t(j), W_t, P_t, A_t) \equiv \left(\frac{1}{P_t C_t}\right) W_t(j)\, l_t(j) - \left(\frac{l_t(j)^{1+\eta}}{1+\eta}\right)$.[23] Objective function (23) is concave over a convex set. The set Ω is convex because for any $x \in \Omega$ and $y \in \Omega$ then $\tau x + (1-\tau) y \in \Omega$ for each $\tau \in [0,1]$. Objective function is concave in $W_t(j)$ because it is an integral of functions $\pi(\cdot)$ that are concave in the first-argument. The value function $V(\cdot)$ associated to household problem is given by

$$V\left(W_t(j), W_t, P_t, A_t\right) = \max_{\{W_\tau(j)\}_{\tau=0}^{\infty} \in \Omega} E_t \left\{ \int_t^{\infty} e^{-\rho(t-t_0)} \pi \left(W_t(j), W_t, P_t, A_t \right) dt \right\}. \tag{24}$$

The related Bellman equation for the wage-setter problem can be written as

$$\rho V\left(W_t(j), W_t, P_t, A_t\right) dt = \max_{dW_t(j)} \pi\left(W_t(j), W_t, P_t, A_t\right) dt + E_t\left[dV\left(W_t(j), W_t, P_t, A_t\right)\right], \tag{25}$$

subject to $dW_t(j) \geqslant 0$. Or,

$$\max_{dW_t(j)} \pi\left(W_t(j), W_t, P_t, A_t\right) dt + E_t\left[dV\left(W_t(j), W_t, P_t, A_t\right)\right] - \lambda_t\left(-dW_t(j)\right), \tag{26}$$

where λ_t is the multiplier associated to the inequality constraint $dW_t(j) \geqslant 0$. The first order condition gives

$$V_{W(j)}\left(W_t(j), W_t, P_t, A_t\right) + \lambda_t = 0, \tag{27}$$

[23] Using both aggregate labor demand (5) and individul labor demand (7), the household surplus becomes

$$\pi\left(W_t(j), W_t, P_t, A_t\right) = \alpha \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_\omega} - \frac{1}{1+\eta} \left(\left(\frac{W_t(j)}{W_t}\right)^{-\theta_\omega} \left(\frac{\alpha P_t A_t}{W_t}\right)^{\frac{1}{1-\alpha}} \right)^{1+\eta}.$$

with $\lambda_t(-dW_t(j)) = 0$ as complementary slackness condition. The first term on the LHS of (27) comes from the expected value of $dV(W_t(j), W_t, P_t, A_t)$. Indeed, from the Ito's lemma it holds,

$$
\begin{aligned}
E_t[dV(W_t(j), W_t, P_t, A_t)] &= E_t\left[V_{W_t(j)}(W_t(j), W_t, P_t, A_t)\, dW_t(j)\right] + \\
&+ E_t[V_W(W_t(j), W_t, P_t, A_t)\, dW_t] + \\
&+ \frac{1}{2} E_t\left[V_{WW}(W_t(j), W_t, P_t, A_t)\, dW_t^2\right] + \\
&+ E_t[V_P(W_t(j), W_t, P_t, A_t)\, dP_t] + \\
&+ \frac{1}{2} E_t\left[V_{PP}(W_t(j), W_t, P_t, A_t)\, dP_t^2\right] + \\
&+ E_t[V_A(W_t(j), W_t, P_t, A_t)\, dA_t] + \\
&+ \frac{1}{2} E_t\left[V_{AA}(W_t(j), W_t, P_t, A_t)\, dA_t^2\right] + \\
&+ E_t[V_{WP}(W_t(j), W_t, P_t, A_t)\, dW_t dP_t] + \\
&+ E_t[V_{WA}(W_t(j), W_t, P_t, A_t)\, dW_t dA_t], \qquad (28)
\end{aligned}
$$

where to obtain (28) it is considered that $dW_t(j)$ has finite variance, that is it holds $dW_t(j)^2 = dW_t(j)\, dW_t = dW_t(j)\, dP_t = dW_t(j)\, dA_t = 0$. From (2) and (3), the processes that lead dynamics of price and productivity levels are,

$$
\begin{aligned}
dP_t &= \left(\mu + \frac{\sigma_P^2}{2}\right) P_t dt + \sigma_P P_t dB_{P,t}\ , \qquad (29) \\
(dP_t)^2 &= \sigma_P^2 P_t^2 dt, \qquad (30)
\end{aligned}
$$

and

$$
\begin{aligned}
dA_t &= \left(g + \frac{\sigma_A^2}{2}\right) A_t dt + \sigma_A A_t dB_{A,t}\ , \qquad (31) \\
(dA_t)^2 &= \sigma_A^2 A_t^2 dt. \qquad (32)
\end{aligned}
$$

The two processes are independent, thereby $E_t[dP_t dA_t] = 0$. Plugging (29)-(32) into (28), it becomes

$$
\begin{aligned}
E_t[dV(W_t(j), W_t, P_t, A_t)] &= E_t\left[V_{W_t(j)}(\cdot)\, dW_t(j)\right] + \\
&+ E_t[V_W(\cdot)\, dW_t] + \frac{1}{2} E_t\left[V_{WW}(\cdot)\, dW_t^2\right] + \\
&+ V_P(\cdot)\left(\mu + \frac{\sigma_P^2}{2}\right) P_t dt + \frac{1}{2} V_{PP}(\cdot)\, \sigma_P^2 P_t^2 dt + \\
&+ V_A(\cdot)\left(g + \frac{\sigma_A^2}{2}\right) A_t dt + \frac{1}{2} V_{AA}(\cdot)\, \sigma_A^2 A_t^2 dt + \\
&+ E_t[V_{WP}(\cdot)\, dW_t dP_t] + E_t[V_{WA}(\cdot)\, dW_t dA_t]. \qquad (33)
\end{aligned}
$$

Finally, taking the first derivative of (33) with respect to $dW_t(j)$, it is gives the first term on the LHS of (27).

Importantly equation (27) and complementary slackness condition ensure the following two possible scenarios,

$$\begin{aligned} \text{if} \quad dW_t(j) &> 0 \Longrightarrow V_{W(j)}(W_t(j), W_t, P_t, A_t) = 0, && (34) \\ \text{if} \quad dW_t(j) &= 0 \Longrightarrow V_{W(j)}(W_t(j), W_t, P_t, A_t) \leq 0. && (35) \end{aligned}$$

Focusing only on the case with growing nominal wages, i.e. $dW_t(j) > 0$, the first term on the RHS of (33) cancels out and the Bellman equation (25) becomes,

$$\begin{aligned} \rho V(W_t(j), W_t, P_t, A_t)\,dt &= \pi(\cdot)\,dt + \\ &+E_t[V_W(\cdot)\,dW_t] + \frac{1}{2}E_t\left[V_{WW}(\cdot)\,dW_t^2\right] + \\ &+\left[V_P(\cdot)\left(\mu + \frac{\sigma_P^2}{2}\right)P_t + \frac{1}{2}V_{PP}(\cdot)\,\sigma_P^2 P_t^2\right]dt + \\ &+\left[V_A(\cdot)\left(g + \frac{\sigma_A^2}{2}\right)A_t + \frac{1}{2}V_{AA}(\cdot)\,\sigma_A^2 A_t^2\right]dt \\ &+E_t[V_{WP}(\cdot)\,dW_t dP_t] + E_t[V_{WA}(\cdot)\,dW_t dA_t]. \qquad (36) \end{aligned}$$

Differentiating then both sides of (36) with respect to $W_t(j)$, it holds

$$\begin{aligned} \rho V(W_t(j), W_t, P_t, A_t)\,dt &= \pi_{W(j)}(\cdot)\,dt + E_t\left[V_{W(j)W} dW_t\right] + \frac{1}{2}E_t\left[V_{W(j)WW} dW_t^2\right] + \\ &+\left[V_{W(j)P}(\cdot)\left(\mu + \frac{\sigma_P^2}{2}\right)P_t + \frac{1}{2}V_{W(j)PP}(\cdot)\,\sigma_P^2 P_t^2\right]dt + \\ &+\left[V_{W(j)A}(\cdot)\left(g + \frac{\sigma_A^2}{2}\right)A_t + \frac{1}{2}V_{W(j)AA}(\cdot)\,\sigma_A^2 A_t^2\right]dt \\ &+E\left[V_{W(j)WP} dW_t dP_t\right] + E\left[V_{W(j)WA} dW_t dA_t\right]. \qquad (37) \end{aligned}$$

Since the objective is concave and the set of constraints is convex, the optimal choice for $W_t(j)$ is unique. It follows that $W_t(j) = W_t$ for each j. Also W_t has a finite variance, so that $dW_t^2 = dW_t dP_t = dW_t dA_t = 0$. Moreover, super-contact conditions[24] require that when $dW_t(j) > 0$, the following holds

$$\begin{aligned} V_{W_t(j)W_t(j)}(W_t, P_t, A_t) &= 0, && (38) \\ V_{W_t(j)W}(W_t, P_t, A_t) &= 0, && (39) \\ V_{W_t(j)P}(W_t, P_t, A_t) &= 0, && (40) \\ V_{W_t(j)A}(W_t, P_t, A_t) &= 0. && (41) \end{aligned}$$

[24] See Dixit (1991) and Dumas (1991)

Equation (37) reduces then to

$$\begin{aligned}\rho V_W(\cdot) &= \pi_W(\cdot) + \\ &+ \left[V_{WP}(\cdot)\left(\mu + \frac{\sigma_P^2}{2}\right)P_t + \frac{1}{2}V_{WPP}(\cdot)\,\sigma_P^2 P_t^2\right] + \\ &+ \left[V_{WA}(\cdot)\left(g + \frac{\sigma_A^2}{2}\right)A_t + \frac{1}{2}V_{WAA}(\cdot)\,\sigma_A^2 A_t^2\right], \qquad (42)\end{aligned}$$

where $\pi_W(W_t, P_t, A_t) = \left(\frac{1-\theta_\omega}{W_t}\right)\left[\alpha - \mu_\omega\left(\frac{\alpha P_t A_t}{W_t}\right)^{\frac{1+\eta}{1-\alpha}}\right]$. Defining $V_W(\cdot) \equiv \upsilon(\cdot)$, (42) can be rewritten as a differential equation of second order,

$$\begin{aligned}&-\frac{1}{2}\upsilon_{PP}(W_t, P_t, A_t)\,\sigma_P^2 P_t^2 W_t - \frac{1}{2}\upsilon_{AA}(W_t, P_t, A_t)\,\sigma_A^2 A_t^2 W_t \\ &-\upsilon_P(W_t, P_t, A_t)\left(\mu + \frac{\sigma_P^2}{2}\right)P_t W_t - \upsilon_A(W_t, P_t, A_t)\left(g + \frac{\sigma_A^2}{2}\right)A_t W_t \\ &+\rho\upsilon(W_t, P_t, A_t)\,W_t \\ = \;& (1-\theta_\omega)\left[\alpha - \mu_\omega\left(\frac{\alpha P_t A_t}{W_t}\right)^{\frac{1+\eta}{1-\alpha}}\right]. \qquad (43)\end{aligned}$$

Now let's assume that $\upsilon(W_t, P_t, A_t)\,W_t = v\left(\tilde{\Gamma}_t\right)$ with $\tilde{\Gamma}_t \equiv \frac{P_t A_t}{W_t}$.[25] Equation (43)

[25] Indeed, derivative terms of (43) become

$$\begin{aligned}\upsilon_P &= \frac{\partial}{\partial P_t}\upsilon(W_t, P_t, A_t) = \frac{\partial}{\partial P_t}\left[v\left(\frac{P_t A_t}{W_t}\right)\frac{1}{W_t}\right] \\ &= \frac{\partial}{\partial \tilde{\Gamma}_t}v\left(\tilde{\Gamma}_t\right)\frac{A_t}{W_t^2} = v_\Gamma\frac{A_t}{W_t^2};\end{aligned}$$

$$\begin{aligned}\upsilon_A &= \frac{\partial}{\partial A_t}\upsilon(W_t, P_t, A_t) = \frac{\partial}{\partial A_t}\left[v\left(\frac{P_t A_t}{W_t}\right)\frac{1}{W_t}\right] \\ &= \frac{\partial}{\partial \tilde{\Gamma}_t}v\left(\tilde{\Gamma}_t\right)\frac{P_t}{W_t^2} = v_\Gamma\frac{P_t}{W_t^2};\end{aligned}$$

$$\begin{aligned}\upsilon_{PP} &= \frac{\partial^2}{\partial P_t^2}\upsilon(W_t, P_t, A_t) = \frac{\partial}{\partial P_t^2}\left[v\left(\frac{P_t A_t}{W_t}\right)\frac{1}{W_t}\right] \\ &= \frac{\partial}{\partial \tilde{\Gamma}_t}\left(\frac{\partial}{\partial \tilde{\Gamma}_t}v\left(\tilde{\Gamma}_t\right)\frac{A_t}{W_t}\right)\frac{A_t}{W_t^2} = v_{\Gamma\Gamma}\frac{A_t^2}{W_t^3}; \\ \upsilon_{AA} &= \frac{\partial^2}{\partial A_t^2}\upsilon(W_t, P_t, A_t) = \frac{\partial}{\partial A_t^2}\left[v\left(\frac{P_t A_t}{W_t}\right)\frac{1}{W_t}\right] \\ &= \frac{\partial}{\partial \tilde{\Gamma}_t}\left(\frac{\partial}{\partial \tilde{\Gamma}_t}v\left(\tilde{\Gamma}_t\right)\frac{P_t}{W_t}\right)\frac{P_t}{W_t^2} = v_{\Gamma\Gamma}\frac{P_t^2}{W_t^3}.\end{aligned}$$

can be read as,

$$
\begin{aligned}
& -\left(\frac{\sigma_P^2}{2}+\frac{\sigma_A^2}{2}\right) v_{\Gamma\Gamma}\tilde{\Gamma}_t^2-\left(\mu+g+\frac{\sigma_P^2}{2}+\frac{\sigma_A^2}{2}\right) v_{\Gamma}\tilde{\Gamma}_t+\rho v \\
= & \ (1-\theta_\omega)\left[\alpha-\mu_\omega\left(\alpha\tilde{\Gamma}_t\right)^{\frac{1+\eta}{1-\alpha}}\right]. \qquad (44)
\end{aligned}
$$

Equation (44) is a Cauchy-Euler equation, which through a change of variables can be transformed into a constant-coefficient equation. Defining indeed $\tilde{\Gamma}_t \equiv e^{x_t}$, it is true that

$$
\frac{\partial}{\partial x_t} v\left(\tilde{\Gamma}_t\right)=\frac{\partial v\left(\tilde{\Gamma}_t\right)}{\partial \tilde{\Gamma}_t} e^{x_t}=v_\Gamma e^{x_t}=v_\Gamma \tilde{\Gamma}_t, \qquad (45)
$$

and

$$
\begin{aligned}
\frac{\partial^2}{\partial x_t^2} v\left(\tilde{\Gamma}_t\right) & = \frac{\partial^2 v\left(\tilde{\Gamma}_t\right)}{\partial \tilde{\Gamma}_t^2} e^{2x_t}+v_\Gamma \frac{\partial}{\partial x_t} e^{x_t} \\
& = v_{\Gamma\Gamma} e^{2x_t}+v_\Gamma e^{x_t}=v_{\Gamma\Gamma}\tilde{\Gamma}_t^2+v_\Gamma\tilde{\Gamma}_t. \qquad (46)
\end{aligned}
$$

Plugging (45) and (46) into Equation (44), it boils down to,

$$
\begin{aligned}
& -\left(\frac{\sigma_P^2+\sigma_A^2}{2}\right)\frac{\partial^2}{\partial x_t^2} v\left(\tilde{\Gamma}_t\right)-(\mu+g)\frac{\partial}{\partial x_t} v\left(\tilde{\Gamma}_t\right)+\rho v\left(\tilde{\Gamma}_t\right) \\
= & \ (1-\theta_\omega)\frac{\alpha}{\mu_p}-\theta_\omega\left(\frac{\alpha}{\mu_p}\tilde{\Gamma}_t\right)^{\frac{1+\eta}{1-\alpha}}. \qquad (47)
\end{aligned}
$$

The LHS of (47) is the characteristic equation, which can be solved by finding the two roots $\iota_{1,2}$,

$$
\iota_{1,2}=\frac{1}{\sigma_P^2+\sigma_A^2}\left(-(\mu+g) \pm \sqrt{(\mu+g)^2+2\rho\left(\sigma_P^2+\sigma_A^2\right)}\right). \qquad (48)
$$

Given that $v\left(W_t, P_t, A_t\right)=\frac{v\left(\tilde{\Gamma}_t\right)}{W_t}$, the complementary solution in terms of $v(\cdot)$ has the following form,

$$
v^c\left(W_t, P_t, A_t\right)=\left(\kappa_1\left|\frac{P_t A_t}{W_t}\right|^{\iota_1}+\kappa_2\left|\frac{P_t A_t}{W_t}\right|^{\iota_2}\right) W_t^{-1}. \qquad (49)
$$

As regards the particular solution, it is used the method of undetermined coefficients. Given the RHS of (47), a possible solution might assume the form $\upsilon^p = A + B\tilde{\Gamma}_t^{\frac{1+\eta}{1-\alpha}}$. Constant A and B are then determined as,

$$A = (1-\theta_\omega)\frac{\alpha}{\rho\mu_p}, \tag{50}$$

$$B = \theta_\omega\left(\frac{\alpha}{\mu_p}\right)^{\frac{1+\eta}{1-\alpha}}\left(\left(\frac{\sigma_P^2}{2}+\frac{\sigma_A^2}{2}\right)\left(\frac{1+\eta}{1-\alpha}\right)^2+(\mu+g)\left(\frac{1+\eta}{1-\alpha}\right)-\rho\right)^{-1}. \tag{51}$$

The particular solution in terms of $\upsilon(\cdot)$ can then be written as,

$$\upsilon^p(W_t,P_t,A_t) = (1-\theta_\omega)\frac{\alpha}{\rho\mu_p}\frac{1}{W_t} + -\frac{\theta_\omega}{W_t}\left(\rho-\left(\frac{\sigma_P^2}{2}+\frac{\sigma_A^2}{2}\right)\left(\frac{1+\eta}{1-\alpha}\right)^2-(\mu+g)\left(\frac{1+\eta}{1-\alpha}\right)\right)^{-1}\left(\frac{\alpha}{\mu_p}\frac{P_tA_t}{W_t}\right)^{\frac{1+\eta}{1-\alpha}} \tag{52}$$

Since when $W \to \infty$ and/or $P_tA_t \to 0$, the length of time until the next wage adjustment can be made arbitrarily long with probability arbitrarily close to one,[26] then it should be the case that

$$\lim_{W\to\infty}\left[\upsilon(W_t,P_t,A_t)-\upsilon^p(W_t,P_t,A_t)\right] = 0, \tag{53}$$

$$\lim_{P_tA_t\to 0}\left[\upsilon(W_t,P_t,A_t)-\upsilon^p(W_t,P_t,A_t)\right] = 0, \tag{54}$$

which both require that ι should be positive. The general solution is finally obtained by the sum of particular solution (52) and complementary one (49),

$$\upsilon(W_t,P_t,A_t) = (1-\theta_\omega)\frac{\alpha}{\rho W_t}-\frac{\theta_\omega}{W_t}\Delta\left(\alpha\frac{P_tA_t}{W_t}\right)^{\frac{1+\eta}{1-\alpha}}+\kappa\left(\frac{P_tA_t}{W_t}\right)^{\iota}\frac{1}{W_t},$$

where Δ is defined as $\Delta = \left(\rho-\left(\frac{\sigma_P^2}{2}+\frac{\sigma_A^2}{2}\right)\left(\frac{1+\eta}{1-\alpha}\right)^2-(\mu+g)\left(\frac{1+\eta}{1-\alpha}\right)\right)^{-1}$, whereas κ is a constant to be determined.

To find explicitly the optimal wage W_t, it necessary to define a function $W(P_t,A_t)$ such that if nominal wages are constant, i.e. $dW_t(j)=0$, it holds $\upsilon(W(P_t,A_t),P_t,A_t)\leq 0$, while if they are increasing, i.e. $dW_t(j)>0$, the followings are verified,

$$\upsilon(W(P_t,A_t),P_t,A_t) = 0, \tag{55}$$

$$\upsilon_W(W(P_t,A_t),P_t,A_t) = 0, \tag{56}$$

$$\upsilon_P(W(P_t,A_t),P_t,A_t) = 0, \tag{57}$$

$$\upsilon_A(W(P_t,A_t),P_t,A_t) = 0. \tag{58}$$

[26] See Stokey (2006).

Evaluating then the general solution at $W(P_t, A_t)$, it holds

$$(1-\theta_\omega)\frac{\alpha}{\rho} - (1-\theta_\omega)\Delta\mu_\omega\left(\alpha\frac{P_tA_t}{W_t(P_t,A_t)}\right)^{\frac{1+\eta}{1-\alpha}} + \kappa\left(\frac{P_tA_t}{W_t(P_t,A_t)}\right)^{\iota} = 0. \tag{59}$$

From (56), it holds

$$-(1-\theta_\omega)\frac{\alpha}{\rho}+\left(\frac{2+\eta-\alpha}{1-\alpha}\right)(1-\theta_\omega)\Delta\mu_\omega\left(\alpha\frac{P_tA_t}{W_t(P_t,A_t)}\right)^{\frac{1+\eta}{1-\alpha}} +- (1+\iota)\kappa\left(\frac{P_tA_t}{W_t(P_t,A_t)}\right)^{\iota} = 0. \tag{60}$$

From (57), it holds

$$\left(\frac{1+\eta}{1-\alpha}\right)\left(\frac{1-\theta_\omega}{\iota}\right)\Delta\mu_\omega\left(\frac{\alpha}{\mu_p}\right)^{\frac{1+\eta}{1-\alpha}}\left(\frac{P_tA_t}{W_t(P_t,A_t)}\right)^{\frac{1+\eta}{1-\alpha}} = \kappa\left(\frac{P_tA_t}{W_t(P_t,A_t)}\right)^{\iota}. \tag{61}$$

From (58), it holds

$$\left(\frac{1+\eta}{1-\alpha}\right)\left(\frac{1-\theta_\omega}{\iota}\right)\Delta\mu_\omega\left(\frac{\alpha}{\mu_p}\right)^{\frac{1+\eta}{1-\alpha}}\left(\frac{P_tA_t}{W_t(P_t,A_t)}\right)^{\frac{1+\eta}{1-\alpha}} = \kappa\left(\frac{P_tA_t}{W_t(P_t,A_t)}\right)^{\iota}. \tag{62}$$

Equations (61) and (62) are equal, while from equations (59) and (60) it is determined the constant κ

$$\begin{aligned}\kappa\left(\frac{P_tA_t}{W_t(P_t,A_t)}\right)^{\iota} &= -\left(\frac{2}{2+\iota}\right)\left((1-\theta_\omega)\frac{\alpha}{\rho}\right) + \\ &\quad + \left(\frac{3+\eta-2\alpha}{1-\alpha}\right)\left(\frac{1-\theta_\omega}{2+\iota}\right)\Delta\mu_\omega\alpha^{\frac{1+\eta}{1-\alpha}}\left(\frac{P_tA_t}{W_t(P_t,A_t)}\right)^{\frac{1+\eta}{1-\alpha}}. \end{aligned}\tag{63}$$

Plugging (63) into (61) or (62) and then substituting $\Delta \equiv \left(\rho - \left(\frac{\sigma_P^2+\sigma_A^2}{2}\right)\left(\frac{1+\eta}{1-\alpha}\right)^2 - (\mu+g)\left(\frac{1+\eta}{1-\alpha}\right)\right)^{-1}$ and $\rho = \left(\frac{\sigma_P^2+\sigma_A^2}{2}\right)\iota^2 + (\mu+g)\iota$, the optimal wage for forward-looking workers is finally determined as

$$W_t(P_t,A_t) = \left(-\frac{\rho}{\iota(1-\alpha)}\frac{1+\eta-\iota(1-\alpha)}{\rho - \left(\frac{\sigma_P^2+\sigma_A^2}{2}\right)\left(\frac{1+\eta}{1-\alpha}\right)^2 - (\mu+g)\left(\frac{1+\eta}{1-\alpha}\right)}\right)^{\frac{1-\alpha}{1+\eta}} \alpha^{\frac{\alpha+\eta}{1+\eta}}\mu_\omega^{\frac{1-\alpha}{1+\eta}}P_tA_t, \tag{64}$$

or alternatively,

$$W_t(P_t,A_t) \equiv \tilde{W}_t^* = c\left(\vartheta,\tilde{\sigma}^2,\alpha,\eta,\iota\right)W_t^*, \tag{65}$$

where

$$c\left(\vartheta,\tilde{\sigma}^{22},\alpha,\eta,\iota\right) \equiv \left(\frac{\frac{\tilde{\sigma}^2}{2}\iota+\vartheta}{\frac{\tilde{\sigma}^{22}}{2}\iota+\vartheta+\frac{\tilde{\sigma}^{22}}{2}\left(\frac{1+\eta}{1-\alpha}\right)}\right)^{\frac{1-\alpha}{1+\eta}},$$

$$\vartheta \equiv \mu + g, \qquad \tilde{\sigma}^2 \equiv \sigma_P^2 + \sigma_A^2,$$

and

$$W_t^* = \alpha^{\frac{\alpha+\eta}{1+\eta}} \mu_\omega^{\frac{1-\alpha}{1+\eta}} P_t A_t.$$

The optimal nominal wage for non-myopic workers is proportional to the nominal wage W_t^* prevailing without rigidity. The latter can be conveniently recovered by equating aggregate labor demand (5) with the labor supply (11) provided by myopic workers in case of positive variations of nominal wages, i.e. $dW_t > 0$. Since $c\left(\vartheta, \tilde{\sigma}^2, \alpha, \eta, \iota\right) \in [0,1]$, optimal wage $\tilde{W}_t^*$, or desired wage following Benigno and Ricci (2010), for workers aware of the non-negative bound on wage changes is lower than the flexible wage W_t^*. Intuitively, forward-looking workers choose to minimize the increments of nominal wages in order to reduce the probability that the lower bound binds in the future. The lower is the growth in nominal wages, the lower is the probability that the constraint is hit for a given contraction in price or productivity level, and the lower is the probability that unemployment overshoot its natural level. Like in the case of myopic workers, the dynamics of optimal wage continues to be driven by the underlying processes of price and productivity level. Unemployment as well, whenever nominal wages are stuck to previous period level, continues to follow a geometric Brownian motion with drift and volatility coefficients given by the combination of inflation and productivity growth processes. The unemployment value prevailing when nominal wage adjust upward, i.e. the natural level of unemployment, is however lower than in the case of myopic workers. To prove that, the labor demand schedule (5) is firstly substituted in (65), in order to detect the employment level prevailing when $dW_t > 0$,

$$\alpha P_t A_t L_t^{\alpha-1} = c\alpha^{\frac{\alpha+\eta}{1+\eta}} \mu_\omega^{\frac{1-\alpha}{1+\eta}} P_t A_t,$$

or,

$$\tilde{L}_t^* = c^{-\frac{1}{1-\alpha}} \alpha^{\frac{1}{1+\eta}} \mu_\omega^{-\frac{1}{1+\eta}}. \tag{66}$$

Analogously, the employment level prevailing with myopic workers when $dW_t > 0$, is recovered by equating the labor demand schedule (5) to the labor supply schedule (11) evaluated when goods market clears,

$$\alpha P_t A_t L_t^{\alpha-1} = \mu_\omega P_t A_t L_t^{\alpha+\eta},$$

or,

$$L_t^* = \alpha^{\frac{1}{1+\eta}} \mu_\omega^{-\frac{1}{1+\eta}}. \tag{67}$$

Combining then (66) with (67), it holds

$$\tilde{L}_t^* = c^{-\frac{1}{1-\alpha}} L_t^*. \tag{68}$$

Plugging the labor supply schedule (11) and the equation (68) into (65), the desired wage can be obtained as,

$$
\begin{aligned}
\tilde{W}_t^* &= cW_t^* \\
&= c\mu_\omega P_t A_t \left(L_t^*\right)^{\alpha+\eta} \\
&= c^{1+\frac{\eta}{1-\alpha}} \mu_\omega P_t C_t \left(c^{-\frac{1}{1-\alpha}} L_t^*\right)^\eta ,
\end{aligned}
$$

or

$$
\frac{\tilde{W}_t^*}{P_t C_t} = c^{1+\frac{\eta}{1-\alpha}} \mu_\omega \left(\tilde{L}_t^*\right)^\eta .
$$

Therefore the notional labor supply (15) can be then rewritten as

$$
\ln\left(L_t^n\right) = \frac{1}{\eta}\left(1+\frac{\eta}{1-\alpha}\right)\ln c + \frac{1}{\eta}\ln\mu_\omega + \ln\left(\tilde{L}_t^*\right). \tag{69}
$$

To recover the unemployment rate defined as in Gali (2011), i.e. $u_t \equiv \ln\left(L_t^n\right) - \ln\left(L_t^d\right)$, the labor demand $\ln L_t^d$ is subtracted from the both sides of (69)

$$
u_t = \frac{1}{\eta}\left(\frac{1+\eta-\alpha}{1-\alpha}\right)\ln c + \frac{1}{\eta}\ln\mu_\omega + \ln\left(\tilde{L}_t^*\right) - \ln\left(L_t^d\right) \tag{70}
$$

Whenever nominal wage is equal to the optimal level, $W_t = \tilde{W}_t^*$, labor market clears, i.e. $\ln\left(\tilde{L}_t^*\right) = \ln\left(L_t^d\right)$, and unemployment is equal to its natural level. However, the natural level of employment level prevailing with forward-looking workers, is lower than the corresponding one with myopic workers, because $\eta > 1$, $\alpha < 1$ and $c \in [0,1]$.

Tables

Regressors	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ΔCPI_m	**-0,0909*****				**-0,0876*****		
t-statistics	*-7,530*				*-6,354*		
ΔCPI_std	**0,0443*****				**0,1094*****		
t-statistics	*4,613*				*3,219*		
ΔDEF_m		**-0,0857*****				**-0,0937*****	
t-statistics		*-6,123*				*-6,661*	
ΔDEF_std		**0,0756****				**0,0953****	
t-statistics		*2,010*				*2,379*	
ΔNW_m			**-0,3252*****				**-0,2230*****
t-statistics			*-6,529*				*-4,398*
ΔNW_std			**-0,1348****				-0,1131
t-statistics			*-2,078*				*-1,566*
ΔLabProd_m				-0,1187	**-0,2215*****	**-0,1868****	-0,0582
t-statistics				*-1,454*	*-2,744*	*-2,315*	*-0,628*
ΔLabProd_std				0,0798	**0,1427***	**0,1416***	**0,2101****
t-statistics				*1,115*	*1,938*	*1,963*	*2,463*
Observations	1044	899	821	819	816	815	765
LSDV R-squared	0,772	0,807	0,822	0,800	0,813	0,811	0,823

Table 1. Dependent variable is the unemployment rate u_t. Regressors are the mean and the standard deviation of CPI for all goods increments (ΔCPI_m and ΔCPI_std), of GDP deflator increments (ΔDEF_m and ΔDEF_std), of nominal labor compensation increments (ΔNW_m and ΔNW_std), of gross value added - employment ratio increments ($\Delta LabProd\_m$ and $\Delta LabProd\_std$). All specifications include intercepts and coefficients on time dummies that are not reported. p-value<0,01 is defined as ***, p-value <0.05 as **, p<0.1.is defined as *.

Regressors	(8)	(9)	(10)	(11)
ΔCPI_m		**-0,4732*** **		
t-statistics		*-9,999*		
ΔCPI_std		**0,4432*** **		
t-statistics		*6,006*		
ΔDEF_m			**-0,396*** **	
t-statistics			*-8,927*	
ΔDEF_std			**0,3602*** **	
t-statistics			*4,713*	
ΔNW_m				**-0,2976*** **
t-statistics				*-5,316*
ΔNW_std				**-0,2001*** **
t-statistics				*-2,593*
ΔTFP_m	-0,0504	**-0,1826***	-0,1164	0,0968
t-statistics	*-0,468*	*-1,800*	*-1,145*	*-0,873*
ΔTFP_std	**0,6655*** **	**0,5206*** **	**0,5664*** **	**0,7582*** **
t-statistics	*5,022*	*4,005*	*4,350*	*0,873*
Observations	664	660	660	664
LSDV R-squared	0,838	0,862	0,856	0,847

Table 2. Dependent variable is the unemployment rate u_t. Regressors are the mean and the standard deviation of CPI for all goods increments (ΔCPI_m and ΔCPI_std), of GDP deflator increments (ΔDEF_m and ΔDEF_std), of nominal labor compensation increments (ΔNW_m and ΔNW_std), of total factor productivity increments (ΔTFP_m and ΔTFP_std). All specifications include intercepts and coefficients on time dummies that are not reported. not shown. p-value<0,01 is defined as ***, p-value <0.05 as **, p<0.1.is defined as *.

Regressors	(12) up to 1983	(13) after 1983	(14) up to 1983	(15) after 1983
ΔCPI_m	0,1041	**-0,0841*** **		
t-statistics	*1,111*	*-6,211*		
ΔCPI_std	-0,1993	**0,0793** **		
t-statistics	*-1,653*	*2,395*		
ΔDEF_m			0,0028	**-0,0958*** **
t-statistics			*0,026*	*-6,902*
ΔDEF_std			0,0848	**0,0859** **
t-statistics			*0,565*	*2,183*
ΔLabProd_m	-0,0157	0,1280	-0,1737	**0,1475***
t-statistics	*-0,076*	*1,468*	*-0,743*	*1,737*
ΔLabProd_std	0,1058	**0,2283*** **	-0,0276	**0,2295*** **
t-statistics	*0,423*	*3,116*	*0,096*	*3,216*
Observations	112	704	108	707
LSDV R-squared	0,928	0,847	0,922	0,848

Table 3. Dependent variable is the unemployment rate u_t. Regressors are the mean and the standard deviation of CPI for all goods increments (ΔCPI_m and ΔCPI_std), of GDP deflator increments (ΔDEF_m and ΔDEF_std), of gross value added - employment ratio increments ($\Delta LabProd\_m$ and $\Delta LabProd\_std$). All specifications include intercepts and coefficients on time dummies that are not reported. not shown. p-value<0,01 is defined as ***, p-value <0.05 as **, p<0.1.is defined as *.

Country	ΔCPI	ΔDEF	ΔNW
Australia	**5,82**	5,91	7,74
Austria	3,66	3,40	5,99
Belgium	4,11	3,87	5,79
Canada	4,59	4,68	**6,39**
Chile	**35,06**	**8,64**	**6,62**
Czech Rep.	4,59	**7,04**	5,52
Denmark	5,18	5,36	**6,42**
Estonia	4,36	**9,91**	**8,98**
Finland	5,57	5,55	6,36
France	5,06	5,02	**7,29**
Germany	2,99	2,70	5,61
Greece	**11,03**	**11,33**	**7,03**
Hungary	**14,46**	**11,96**	6,35
Irland	4,82	**7,12**	**8,34**
Israel	**50,70**	**48,91**	3,77
Italy	**7,38**	**8,42**	5,39
Japan	3,54	2,08	**6,64**
South Korea	**9,00**	**9,93**	**12,42**
Luxembourg	3,92	4,00	**8,42**
Mexico	**30,77**	**30,06**	**6,53**
Netherlands	3,86	3,50	6,30
New Zealand	**6,96**	**7,13**	5,72
Norway	5,34	5,52	**7,45**
Poland	**15,79**	**13,10**	5,72
Portugal	**10,91**	**11,47**	**6,70**
Russia Fed.	**41,47**	**23,69**	**12,01**
Slovenia	**116,45**	**20,15**	5,62
Spain	**8,08**	**8,44**	**6,99**
Sweden	5,39	5,53	5,62
Switzerland	3,05	2,71	5,87
Turkey	**42,11**	**47,48**	5,98
United Kingdom	**6,22**	6,60	**6,70**
United States	4,53	3,93	1,93
Median	5,57	7,04	6,39
Mean	14,48	10,65	6,66

Table 4. Columns show for each country the mean values calculated over 10-year rolling windows for CPI index inflation, gdp deflator index growth and nominal labor compensation growth respectively. Reference period is 1961-2011. Mean values above the cross-country median are in bold.

Regressors	(16)	Regressors	(17)	Regressors	(18)
ΔCPI_m	-0,2034***	ΔDEF_m	-0,3345***	ΔNW_m	-0,3394***
t-statistics	*-3,427*	*t-statistics*	*-6,250*	*t-statistics*	*-5,318*
ΔCPI_std	0,4930***	ΔDEF_std	0,4012***	ΔNW_std	-0,3021**
t-statistics	*3,667*	*t-statistics*	*3,848*	*t-statistics*	*2,151*
ΔLabProd_m	-0,4773***	ΔLabProd_m	-0,3457***	ΔLabProd_m	0,3455**
t-statistics	*-4,093*	*t-statistics*	*-2,849*	*t-statistics*	*2,333*
ΔLabProd_std	-0,0474	ΔLabProd_std	0,1570	ΔLabProd_std	0,2503**
t-statistics	*-0,426*	*t-statistics*	*-1,474*	*t-statistics*	*2,344*
dΔCPI_m	0,1154**	dΔDEF_m	0,2464***	dΔNW_m	0,1850***
t-statistics	*-1,866*	*t-statistics*	*4,714*	*t-statistics*	*3,250*
dΔCPI_std	-0,4249***	dΔDEF_std	-0,3725***	dΔNW_std	0,1541
t-statistics	*-3,066*	*t-statistics*	*-3,285*	*t-statistics*	*0,491*
dΔLabProd_m	0,4070***	dΔLabProd_m	0,2082	dΔLabProd_m	-0,7438***
t-statistics	*2,805*	*t-statistics*	*1,402*	*t-statistics*	*-4,189*
dΔLabProd_std	0,2383	dΔLabProd_std	-0,1065	dΔLabProd_std	0,0892
t-statistics	*1,634*	*t-statistics*	*-0,744*	*t-statistics*	*0,491*
Observations	816		815		765
LSDV R-squared	0,817		0,817		0,830

Table 5. Dependent variable is the unemployment rate u_t. Regressors are the mean and the standard deviation of CPI for all goods increments (ΔCPI_m and ΔCPI_std), of GDP deflator increments (ΔDEF_m and ΔDEF_std), of nominal labor compensation increments (ΔNW_m and ΔNW_std), of gross value added - employment ratio increments ($\Delta LabProd\_m$ and $\Delta LabProd\_std$). The product terms between previous regressors and inflation dummies are $d\Delta CPI\_m$ and $d\Delta CPI\_std$ for CPI increments dummy, $d\Delta DEF\_m$ and $d\Delta DEF\_std$ for GDP deflator increments dummy, $d\Delta NW\_m$ and $d\Delta NW\_std$ for labor compensation increments dummy. The terms $d\Delta LabProd\_m$ and $d\Delta LabProd\_std$, which relate the labor productivity to the inflation dummy, change according to the measure of inflation considered. All specifications include intercepts and coefficients on time dummies that are not reported. not shown. p-value<0,01 is defined as ***, p-value <0.05 as **, p<0.1.is defined as *.