Public debt expansions and the dynamics of the household borrowing constraint*

António Antunes
Banco de Portugal and NOVA SBE

Valerio Ercolani†
Banco de Portugal

May, 2016

Preliminary and Incomplete Version

Abstract

The literature based on models where households have a precautionary saving motive has stressed the ability of public debt to act as if it relaxed the household borrowing constraint. We specifically study the effects of temporary government debt expansions on this constraint. We show, within an incomplete-markets model featuring consumer credit, that these policies make the borrowing constraint persistently tighter. Because of the tightening, debt-constrained agents must deleverage, thus working (consuming) more (less). Unconstrained agents react similarly due to precautionary saving reasons. In the aggregate, both consumer credit and physical capital are crowded out. The dynamics of the borrowing limit explains a significant share of the aggregate reactions. For example, five years after the beginning of the debt expansions to finance realistic government spending policies, the tightening reinforces the fall in consumer credit by an average of 20%. Furthermore, it explains as much as half of both the reaction of physical capital and the dynamics of the fiscal multipliers. The quantitative effects generated by the tightening are largely independent of the calibrated share of debt-constrained agents. Finally, we show that the debt expansions implemented during a (simulated) financial crisis further tighten the borrowing limit and depress credit.

JEL classification: E21, E44, E62, H60.

Keywords: Government debt, endogenous borrowing constraints, precautionary saving motive, unsecured credit, heterogeneous households, government transfers and purchases.

*Acknowledgements: We particularly thank Pedro Teles for several discussions at different stages of the project. We are also grateful to Mark Aguiar, Pedro Amaral, George-Marios Angeletos, Marco Bassetto, Pedro Brinca, Craig Burnside, Francesco Carli, Dean Corbae, Isabel Correia, Jonathan Heathcote, João Victor Issler, Pat Kehoe, Tim Kehoe, Jesper Lindé, Fabrizio Mattesini, Ellen McGrattan, Ettore Panetti, Morten Ravn, Sérgio Rebelo, João Valle e Azevedo and Gianluca Violante for useful comments and suggestions. We thank participants to the seminars at Banco de Portugal, Bank of Italy, and Portuguese Catholic University and to the following conferences: the 3rd Workshop on Macro Banking and Finance in Pavia and the 4th *LuBra Macro* in Rio de Janeiro. All errors are ours. The views expressed are those of the authors and do not necessarily represent those of Banco de Portugal or the Eurosystem. A previous version of the paper circulated under the name of *'Debt-financed fiscal policies and the dynamics of the household borrowing constraint'*.

†Corresponding author: E-mail: valerio.ercolani@gmail.com

1 Introduction

Issuing debt is the key instrument to finance government spending policies. For example, the fiscal stimuli during the 2007-2009 period generated significant increases in public debt, both in the U.S. and in Europe (see Cogan et al. 2010, Van Riet 2010). A well established strand of the literature, including Woodford (1990) and Aiyagari and McGrattan (1998), highlights important linkages between public debt and the households' financial conditions. For example, Aiyagari and McGrattan (1998) put forward the view that, within and economy where households face borrowing constraints and the precautionary saving motive is active, public debt can act as if it relaxed the household borrowing constraint. That is, higher levels of public debt result in higher interest rates making assets more attractive to hold and, hence, enhancing households' self-insurance possibilities.

If, on the one hand, an increase in the interest rate contributes to a 'relaxation' of the borrowing limit, on the other hand the same increase makes borrowing more costly, thus generating *ceteris paribus* an actual tightening in the borrowing constraint, that is, a shrinking in the maximum amount of resources that households can borrow. Virtually, any endogenous borrowing constraint has the property of being proportional to the inverse of the interest rate or the borrowing cost; consider, for example, the *natural borrowing limit* in Aiyagari (1994) or the constraint with collateral in Kiyotaki and Moore (1997). Notice that the mentioned property is supported by the empirical evidence, that is, a significant correlation between proxies for borrowing constraints (lending standards) and interest rates is documented in the data. For example, Maddaloni and Peydró (2011) show that lending standards applied to households and firms become tighter when short-term interest rates increase, both in the U.S. and in the Euro area.[1]

The aim of this paper is then to study the effects of temporary government debt expansions, used to finance plausible increases in transfers or in purchases of goods and services, on the household borrowing constraint. To the best of our knowledge, we are the first in tracing out the dynamic reaction of the household borrowing limit to the mentioned fiscal policies. A further contribution to the literature is studying how and by how much the implied dynamics of the borrowing limit influences the households' reactions to these policies.

We perform the analysis within a general equilibrium, incomplete-markets model with physical capital which relies on the early contribution of Bewley (1977). Households are heterogeneous in terms of wealth and borrow or lend in order to self-insure against the occurrence of productivity shocks. The return to assets equals the cost of borrowing. Households also decide how much to consume and how much labor to supply in the intensive margin. We endogeneize the borrowing constraint by considering limited commitment for the repayment obligations of

[1]Maddaloni and Peydró (2011) use various indices for lending standards which are computed from surveys to commercial banks (see, e.g., the Bank Lending Service for the Eurosystem). Specifically, in their regressions, they show that 'the net percentage of responding banks indicating a tightening in lending standards in a given quarter' is positively affected by short-term interest rates, conditioning on a number of controls. The empirical relation holds for lending standards in mortgages, corporate and consumer loans. The authors primarily focus on the effect of nominal interest rates on these lending standards. However, the evidence can also be extended to real interest rates since, in their computations, the effect of inflation is controlled for.

the households. In particular, in case of default, they are excluded from intertemporal trade forever. We assume that honoring their debt is better than defaulting. Importantly, within our framework and consistently with the empirical evidence, the relative value of declaring bankruptcy decreases as the household's labor income increases due to the fact that financial markets are incomplete.[2] The economy is characterized by uncollateralized consumer credit. Concerning the fiscal authority, it can use debt and taxes to finance its spending policies. Public debt has the same return as claims on physical capital.

We calibrate the stationary distribution of our model at quarterly frequency for the U.S. economy. Then, we study the transition of the economy due to (unexpected) temporary public debt expansions. Specifically, we carry on two exercises which mimic two plausible or realistic policies. In one, the increase in debt finances a 1% GDP increase in transfers which then decay persistently according to the process estimated by Leeper et al. (2010). The transfers are evenly distributed across agents. In the second one, the debt finances an expansion in purchases of a magnitude similar to that in the *American Recovery and Reinvestment Act* (henceforth, *ARRA*), as simulated by Uhlig (2010).

We point out the following sets of results. First, issuing public debt, either to finance transfers or purchases, generates a persistent tightening of the borrowing constraint. After ten years, the borrowing limit is still far from its steady-state value. We provide a careful analysis for the process of the tightening, through the study of the dynamics of both equilibrium and autarky value functions. We show that the crucial factor responsible for the tightening is the increase in the interest rate, or, equivalently, in the borrowing cost. An additional observation is worth mentioning. Our baseline model considers fully flexible prices; we show that the tightening arises even within frameworks that are able to reproduce the effects of nominal stickiness.

Second, we describe the consequences of the tightening on the households' reactions to the fiscal policies. There are common features in the reactions to both policies. Because of the tightening, debt-constrained agents must deleverage, and they do so by working more and consuming less. Unconstrained agents behave in a qualitatively similar fashion but for a different reason, i.e., the precautionary saving motive. Indeed, they start accumulating precautionary wealth (or decrease their debt if they are borrowers) because, *ceteris paribus*, their asset position is (and will be) closer to the borrowing constraint.[3]

Third, we concentrate on the aggregate reactions to the policies and how the dynamics of the borrowing limit shapes them. Public debt expansions crowd out both consumer credit and physical capital. Importantly, the dynamic of the borrowing limit explains a significant part of the aggregate reactions. Considering both policies, five years after the beginning of the debt

[2]Kehoe and Levine (1993) and Alvarez and Jermann (2000) study the equilibrium allocation's properties of models characterized by limited commitment borrowing constraints, in the presence of a complete set of state-contingent securities. Furthermore, Zhang (1997), Ábrahám and Cárceles-Poveda (2010) and Antunes and Cavalcanti (2013) used these types of constraints within incomplete-markets models.

[3]Guerrieri and Lorenzoni (2015) highlight similar qualitative behaviors for both unconstrained and debt-constrained households as a reaction to an exogenous shift of the borrowing limit, within an incomplete-markets framework.

expansions, the tightening reinforces the fall in consumer credit by, on average, 20%. Further, the tightening explains as much as half of both the reaction of physical capital and the dynamics of the fiscal multipliers.

Interestingly, the quantitative effects generated by the tightening are largely independent of the calibrated share of debt-constrained agents which, in fact, represents only roughly 10% of the population. Indeed, focusing only on the reactions generated by the tightening, a significant share of these responses is explained by the sum of the reactions of the unconstrained households alone. For example, both in the transfers and in the purchases policy, the mentioned share goes from roughly 65% to 85% regarding labor, consumption and asset holdings' reactions. This result allows us to differentiate our paper from others using models where households face borrowing constraints but have access to a full set of state contingent securities (see, e.g., the borrower-saver model of Iacoviello 2005). In these frameworks, the absence of a precautionary saving motive does not let unconstrained agents to directly react to the tightening, hence, the latter would arguably create negligible quantitative effects.

Finally, we produce a 'crisis experiment' through which we study how an economic crisis characterized by a tightening in the borrowing limit, a fall in credit and a decrease in output can be affected by the implementation of the described debt-financed fiscal policies. We show that both policies contribute to a further tightening and a more marked fall in credit.

As mentioned above, our work is related to those papers that analyze the roles of public debt within incomplete-markets models, like for example Woodford (1990), Aiyagari and McGrattan (1998) and Challe and Ragot (2011). Our main contribution to this strand of the literature is to study how the actual dynamics of the household borrowing limit shapes the households' reactions and to provide related measurements. Regarding the modeling framework, our model belongs to the same class of models of Aiyagari and McGrattan (1998); it is a heterogeneous agents model (where the agents' wealth distribution evolves endogenously) with incomplete insurance markets. However, we differ from Aiyagari and McGrattan (1998) in several aspects. First, in the objective: they derive normative conclusions on the level of public debt, whereas the nature of our analysis is strictly positive. Second, in the modeling: they do not allow for private credit in the economy, and indeed, their main simulations are based on an exogenous credit limit that prevents households from borrowing. Third, their analysis is based on comparisons between steady states while ours studies the transitional dynamics; we believe that analyzing the transition of the economy is appropriate when studying the effects of policies within a given country.[4]

There is a recent stream of the literature that studies the effects of (i) taxes and monetary transfers and of (ii) government consumption within incomplete-markets frameworks. For example, Heathcote (2005), Ábrahám and Cárceles-Poveda (2010), Oh and Reis (2012), McKay

[4]Regarding the other two cited papers, Woodford (1990) derives the optimal level of public debt within a deterministic model featuring liquidity constrained agents. Challe and Ragot (2011) study the effects of a government spending stimulus within a stochastic model where households face collateralized borrowing constraints but their decisions are independent from the wealth distribution. The authors mainly focus on the ability of a debt-financed government spending shock to crowd in private consumption depending on the extent to which the fiscal policy enhances self-insurance possibilities.

and Reis (2013), Kaplan and Violante (2014), and Huntley and Michelangeli (2014) belong to the first class of papers, while Brinca et al. (2014) and Ercolani and Pavoni (2014) to the second one. Typically, these papers do not focus on the roles of public debt during the transitional dynamics.

Our work is also related to those papers studying the effects of a credit crunch, within frameworks of heterogeneous agents and incomplete markets, as, among others, Guerrieri and Lorenzoni (2015) and Buera and Moll (2012). We contribute to this literature by studying the interactions between the (endogenous) dynamics of the borrowing constraint and debt-financed fiscal policies.

Finally, there is a well-established stream of the literature that studies government spending stimuli within general equilibrium models, with complete markets and representative agent(s). The seminal contribution is represented by Baxter and King (1993). Furthermore, Galí et al. (2007), Fernández-Villaverde (2010), Christiano et al. (2011), Eggertsson and Krugman (2012), Corsetti et al. (2013), Bilbiie et al. (2013) and Rendhal (forthcoming) study the effects of fiscal policies in the presence of various financial frictions. Unlike them, our framework of analysis allows us to study how the combination of borrowing constraints, wealth heterogeneity and market incompleteness influences the households' reactions to public debt expansions.

The paper is structured as follows. Section 2 presents the model. Section 3 presents the results for the stationary distribution. Section 4 reports the transitional dynamics of the economy generated by the government debt expansions. Section 5 studies the effects of the debt expansions during a financial crisis. Section 6 concludes and highlights our research agenda.

2 Model

We consider a general equilibrium model with capital in which households differ by their wealth and productivity. Households choose how much to consume and to save or borrow. They can also vary their labor intensity. Our model belongs to the long-standing tradition of incomplete markets models like, for example, Bewley (1977) and Aiyagari (1994).

In the spirit of Kehoe and Levine (1993), Alvarez and Jermann (2000) and Zhang (1997), we endogeneize the borrowing constraint by allowing households to default on their debt (in this case, they go to autarky forever) and assuming that the value of honoring their debt is not less than defaulting. Two important characteristics feature this type of constraints. First, unlike in a complete market setting (see Kehoe and Levine 1993, Alvarez and Jermann 2000), using these constraints in an incomplete-market framework makes the relative value of declaring bankruptcy (or the temptation of declaring default) decreasing with the level of household's labor income. Second, unlike the standard *natural borrowing limit*, these constraints allow the model to generate realistic shares of both credit-to-output ratio and debt-constrained households. The details of both characteristics are described in Section 3.

In the model there is a fiscal authority that can collect lump sum, capital and labor taxes and issue debt, with the same return as physical capital, to finance either transfers or purchases programs.

Finally, we recall that, within our framework, the Ricardian equivalence does not hold even if the financing operates only through lump sum taxation. This is because the class of borrowing constraints used in our model are typically tighter than the *natural borrowing limit* (see, e.g., Ljungqvist and Sargent 2004, for detailed explanations).

2.1 Households and firms

There is a continuum of infinitely lived and *ex ante* identical households with measure one. As in Hall (2009), we denote the households' instantaneous utility function by:

$$u(c,n) = \frac{c^{1-\sigma}-1}{1-\sigma} - \chi\frac{n^{1+\psi}}{1+\psi},$$

where c and n are consumption and labor, respectively. The individual state vector is defined as $x = (a,z)$, where a and z are asset holdings and productivity, respectively. z follows a finite state Markov process with support $\mathcal{Z}$ and transition probability matrix $\mathcal{P} = \Pi(z,z^{'}) = \Pr(z^i_{t+1} = z^{'}|z^i_t = z)$.

The household problem in recursive form can be written as follows:

$$v(x,\theta) = \max_{c,n,a'} u(c,n) + \beta\mathbf{E}\left[v(x',\theta')|z\right] \tag{1}$$

subject to

$$\begin{aligned} c + a' &= (1 + r(1-\tau_k\mathcal{I}_{a\geq 0}))a + wnz(1-\tau_w) + \mathrm{Tr} - \Gamma \\ v(x',\theta') &\geq \underline{v}(z',\theta'), \quad \text{all } z' \in B(z) \qquad (2) \\ \underline{v}(z,\theta) &= \max_n u(\gamma\, wnz(1-\tau_w) + \mathrm{Tr} - \Gamma, n) + \beta\mathbf{E}\left[\underline{v}(z',\theta')|z\right] \qquad (3) \\ \theta' &= H(\theta)\,. \end{aligned}$$

In these expressions, $\mathcal{I}_{a\geq 0}$ is an indicator function that takes 1 if $a \geq 0$ and 0 otherwise, and $B(z) = \{\xi \in \mathcal{Z} : \Pi(z,\xi) > 0\}$ is the set of possible next period idiosyncratic states given that the current state is z. θ is the measure of households, defined in a set of possible asset holdings and idiosyncratic shocks. It subsumes all relevant aggregate variables taken as given by the household. $H(\theta)$ is the forecasting function used by households in predicting next period's measure. Tr represents transfers from the government to households, while Γ are lump sum taxes. We need to distinguish among these two lump sum transfers to allow for the coexistence of an exogenous policy of lump sum transfers to households and a rule-based lump sum tax. The net return on capital, or the borrowing cost, is r and the wage rate for labor efficiency units is w. Capital income is taxed at rate τ_k and labor income is taxed at rate τ_w.

Equation (2) represents the individual rational constraint in that households have the option to go bankrupt. If so, they renege on all existing debts, their capital is seized and they are excluded from the future participation to capital and credit markets, while their human capital is inalienable. Equation (3) defines the value of being in autarky, i.e., $\underline{v}(z,\theta)$, where γ is a pecuniary cost of having tainted credit status, as in Chatterjee et al. (2007). Notice that constraint (2) guarantees that it is never in the household's best interest to default. Since $v(x',\theta')$ is non decreasing in a while $\underline{v}(z',\theta')$ is independent of a, equation (2) defines a set of endogenous lower bounds on borrowing, denoted by $\underline{\hat{a}}(z',\theta')$, such that, conditional on each level of z, $a' \geq \underline{\hat{a}}(z',\theta')$. Formally, we define $\underline{\hat{a}}(z',\theta')$ as the lowest, or most negative, possible asset level conditional on each level of z, i.e.:

$$\underline{\hat{a}}(z,\theta) = \inf\{a \in \mathbb{R} : v(a,z,\theta) \geq \underline{v}(z,\theta)\} . \tag{4}$$

In practice, as we will see in Section 3, given the characteristics of our $\mathcal{P}$, the relevant endogenous borrowing limit is unique and generated by the inequality (2) parameterized in the lowest z, i.e., the tightest among the borrowing limits in (4).

A representative firm with $Y = AK^{\alpha}N^{1-\alpha}$ chooses efficient labor, N, and capital, K, taking factor prices as given, according to:

$$r^K = \alpha A \left(\frac{N}{K}\right)^{1-\alpha}, \text{ where } r = r^K - \delta \tag{5}$$

$$w = (1-\alpha) A \left(\frac{K}{N}\right)^{\alpha}, \tag{6}$$

where A is total factor productivity (TFP).

2.2 Government

We will assume a fiscal sector similar to Uhlig (2010). We first consider the gap to finance in each period as the following variable,

$$D = G + \mathrm{Tr} + (1+r)B - \tau_k r \int_{a\geq 0} a \, d\theta - \tau_w w N \tag{7}$$

where B and G is the current government debt and the level of government consumption, respectively. We assume that D is to be financed through lump sum taxes Γ and newly issued debt. It follows that

$$D = \Gamma + B' . \tag{8}$$

There is a fiscal rule whereby lump sum taxes are imposed based on the difference between the steady-state level of the gap to finance, $\bar{D}$, and its current level, D, so that when this difference is zero lump sum taxes remain at their steady-state level, $\bar{\Gamma}$. Formally,

$$\Gamma - \bar{\Gamma} = \phi(D - \bar{D}) . \tag{9}$$

If ϕ is one, then all the gap is financed through lump sum taxes. If ϕ is close to zero but large enough so as to ensure stability of the debt level, then the gap is largely financed through issuing debt, with taxation being postponed into the future. The second case is of great interest for us, hence our simulations will be conditioned on very low levels of ϕ.

2.3 Equilibrium

The steady-state equilibrium in this economy is standard. Given a transition matrix $\mathcal{P}$ for idiosyncratic productivity, a set of government policies $(\tau_k, \tau_w, \mathrm{Tr}, B)$, and assuming that any deviation to default is not coordinated among households, we define a *recursive competitive equilibrium* as a belief system H, a pair of prices (r, w), a measure defined over the set of possible states θ, a government consumption G, a pair of value functions $v(x, \theta)$ and $\underline{v}(z, \theta)$, and individual policy functions $(a', c, n) = (a(x, \theta), c(x, \theta), n(x, \theta))$ such that:

1. Each agent solves the optimization problem (1).

2. Firms maximize profits according to (5) and (6).

3. The government balances its budget according to (7) and (8).

4. All markets clear:

$$K' + B' = \int a(x, \theta)\, d\theta \tag{10}$$

$$N = \int n(x, \theta)\, z\, d\theta \tag{11}$$

$$\int c(x, \theta)\, d\theta + K' + G = (1 - \delta)K + K^{\alpha} N^{1-\alpha}\,. \tag{12}$$

5. The belief system H is consistent with the aggregate law of motion implied by the individual policy functions.

6. The measure θ is constant over time.

The definition of an equilibrium with a transition follows naturally from the previous one although at the cost of a heavier notation, so we economize on space and omit it. Briefly, as it will be stressed in Section 4, our transition is triggered by the unexpected introduction of a perfectly credible and deterministic change in the trajectory of both government spending and transfers, along with a fiscal rule. We assume that in the transition agents can perfectly foresee the evolution of aggregate variables, including the borrowing limits, thus making sure that off-equilibrium paths are not possible.

3 Steady-state calibration

The computational procedures for the calculation of the stationary distribution are described in Appendix A. Here below, the most important details.

The first point to discuss regards the setting of the borrowing limit. In order to do that, we need to say the following. We calibrate the model at quarterly frequency and present the relevant calibration targets in Table 1. Given the endogenous labor supply, we borrow the time-varying component of labor productivity from Floden and Lindé (2001):

$$\log(z_t) = \rho \log(z_{t-1}) + \eta_t \, ,$$

where ρ defines the persistence of the process and η_t is a serially uncorrelated and normally distributed perturbation with variance σ^2_η. The parameters ρ and σ^2_η are set as to match the yearly autocorrelation and variance of labor productivity process, which are 0.9136 and 0.0426, respectively (as estimated by Floden and Lindé 2001). In order to discretize the productivity process, we use the Rouwenhorst method (see Kopecky and Suen 2010) with 7 levels of productivity. The implied transition probability matrix, $\mathcal{P}$, is characterized by all non-zero entries, meaning that the most productive household in period t can have the lowest productivity in $t+1$ (though with a very small probability). This implies that the only borrowing limit such that all households will be able to repay back their debt irrespective of the productivity shock that will hit them in the next period is the tightest one among the limits defined in (4), i.e., the borrowing limit parameterized in the lowest level of z. We define this borrowing limit to be:

$$\underline{a}(\theta') = \sup\{\hat{\underline{a}}(z', \theta') : z' \in B(z)\} \, . \tag{13}$$

Since in our model there is only unsecured credit, we calibrate the model's credit using data for total revolving credit; as in Antunes and Cavalcanti (2013), we target a credit-to-output ratio of 8%, which is the average pre-crisis period, fixing γ at 0.956.

Given the targeted credit-to-output ratio, our economy is characterized by roughly 10% of agents at the borrowing constraint and a total of roughly 24% of borrowers, which are values close to the actual ones in the U.S. economy (see Ábrahám and Cárceles-Poveda 2010, Jappelli 1990).[5] As a consequence, the unconstrained agents represent roughly 90% of the population; obviously, the great part of them holds positive levels of assets (they are lenders), the other part holds debt (they are borrowers). Further, the implied borrowing limit, $\underline{a}(\theta')$, is such that the average income household can borrow up to roughly 45% of his yearly income. In particular, Figure 1 shows a graphical representation of this limit which is indeed identified by the vertical line. As stated above, the latter is generated by the intersection between the equilibrium and the autarky value functions parameterized in the lowest level of z. Given that the figure reports a measure of assets in the x-axis, the autarky value function is flat, whereas the equilibrium one has a positive slope.[6]

Using the standard *natural borrowing limit* as in Aiyagari (1994) instead of the previously described endogenous borrowing limit would yield values for the credit-to-output ratio and the percentage of households at the constraint considerably far from their actual values. The first

[5] As explained in Appendix A, we use a grid for the asset holdings throughout our computations. We define agents to be debt-constrained if they seat in the grid points which are in a eye-ball of 5% around the borrowing limit. Or, equivalently, debt-constrained agents are defined to be the ones seating in the two grid points nearest to the borrowing limit; see Appendix A for details.

[6] If we wanted to express the borrowing limit in percentage terms of the (quarterly) average total income, the number would become roughly 190%, which is actually the value reported in the x-axis of Figure 1.

Table 1: Steady-State Calibration.

Parameter	Value	Observation/Target
A	1	Normalization
α	0.36	Share of capital in production
δ	0.025	Capital-to-output ratio of 2.6 (yearly)
σ	2	Standard in the literature
ψ	0.67	Hall (2009) base number
β	0.9894	Real interest rate of 1%
ρ	0.977	Floden and Lindé (2001)
σ_η	0.11	Floden and Lindé (2001)
χ	0.4	Average labor supply normalized to 1
τ_w	0.27	Domeij and Heathcote (2004)
τ_k	0.4	Domeij and Heathcote (2004)
γ	0.956	Credit-to-output ratio is 8% (yearly)

measure would be around 30% and the percentage of debt-constrained households would be virtually nil. This is due to the way the natural limit is implemented; it is engineered so that households would consume zero at the constraint, conditional on a long string of realizations of the worst productivity shock.

Regarding the households' temptation to default, we need to refer again to Figure 1. The figure shows pairs of value functions associated with the three lowest levels of z, as formalized in (2).[7] It can be seen that, as households' productivity increases, both types of value functions move up; however, the equilibrium value function moves up more than the autarky one. Hence, the temptation of declaring default, i.e., choosing autarky relative to remaining in the market, decreases with the households' productivity, for any level of asset holdings. This is due to the incompleteness of the market; *ceteris paribus*, high income households would loose more by defaulting than low income ones because the opportunity cost of a permanent preclusion from self-insuring is higher for the former.[8]

Figure 2 shows the agents' policy functions associated with consumption and labor, for different levels of idiosyncratic productivity, i.e., the lowest, the median and the highest level. Focusing on the consumption policy function parameterized in the lowest z level, we see that it exhibits more curvature the closer the households are to the borrowing limit. This is typical in models with precautionary saving motives and borrowing limits (Zeldes 1989, Carroll and Kimball 1996). As expected, the curvature diminishes as the level of the idiosyncratic productivity increases. The labor policy functions mirror the ones of consumption; more specifically, borrowers and wealth-poor households at the lowest level of productivity are the most responsive

[7]For the sake of presentation, we do not report the value functions associated with the higher productivity levels, which, in fact, follow the same pattern as the ones reported.

[8]Another way to interpret Figure 1 is by noting that the higher the agent's productivity is, the looser the agent's borrowing limit could be. Ábrahám and Cárceles-Poveda (2010) proof that the positive relationship between labor income and credit limits is a property of a model similar to ours. They have a heterogenous agent model with capital and inelastic labor supply, where households are subject to endogenous borrowing limits and idiosyncratic productivity shocks.

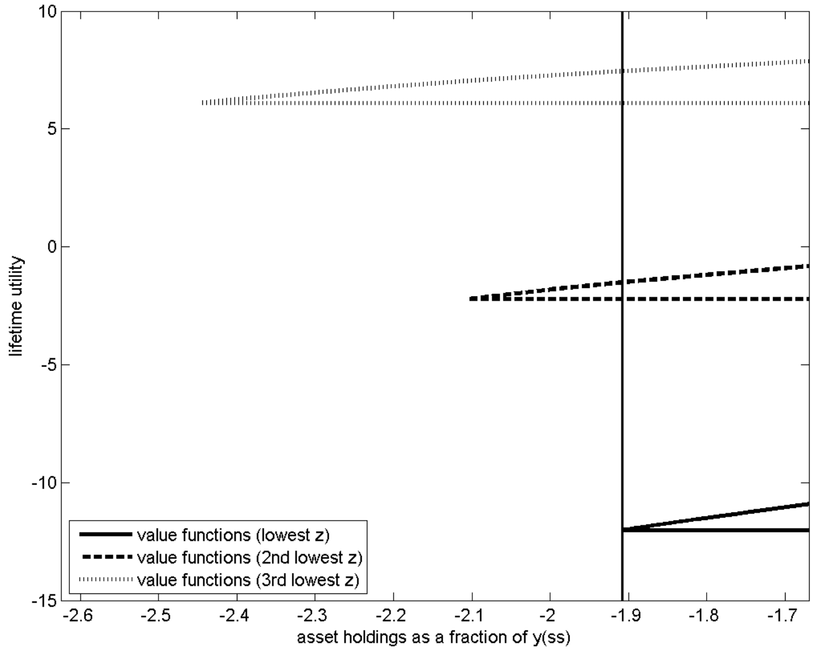

Figure 1: Selected value functions and the borrowing limit in the steady state.
The flat lines correspond to the value functions in autarky, $\underline{v}(z', \theta')$, for different levels of productivity. Similarly, the lines with a positive slope refer to the equilibrium value functions, $v(x', \theta')$, for different levels of productivity. The relevant borrowing limit, $\underline{a}(\theta')$, is identified by the vertical line. y(ss) stands for steady-state output.

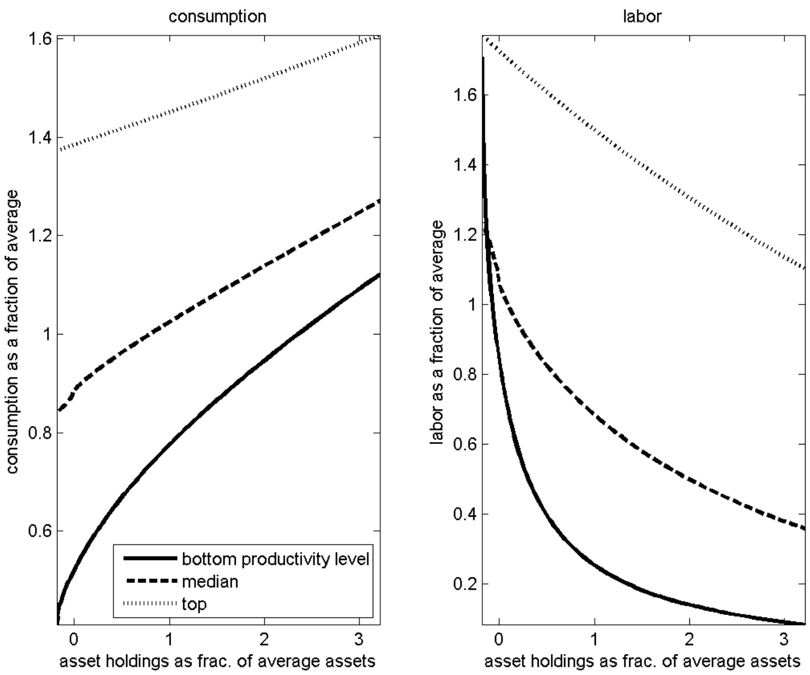

Figure 2: Consumption and labor policy functions in the steady state, conditional on different levels of idiosyncratic productivity.

Table 2: Distribution of wealth: U.S. economy (Castañeda et al. 2003) vis-à-vis the model.

	Gini index	Quintiles				
		First	Second	Third	Fourth	Fifth
Data	0.78	-0.39	1.7	5.7	13.4	79.5
Model	0.67	-3.0	0.85	10.6	26.7	65.0

in terms of labor. A marginal decrease in their wealth produces a positive and large reaction in their labor supply.

Table 2 shows the comparison of our wealth distribution with one for the U.S. as reported by Castañeda et al. (2003). The profile of the wealth distribution in our model does a reasonable job at mimicking the U.S. wealth distribution. In particular, as in the data, households in the first quintile hold negative wealth and those in the fifth quintile hold most of the available wealth. However, as usual in this type of model and with the specific assumptions about the stochastic behavior of the idiosyncratic shocks, the model does not generate enough inequality, especially in the upper tail of the asset distribution.[9]

Notice that in the stationary equilibrium the government budget is balanced and public debt is assumed to be zero; hence, the gap to finance, D, is zero as well. Moreover, $\mathrm{Tr} = \Gamma = 0$. Importantly, we also perform simulations starting from positive and large levels of public debt; see Sections 4.1.1 and 4.2.3 for details. Given the chosen values for the tax rates, we identify a government consumption of around 21% of steady-state output, which is close to the actual measure in the U.S. data.

4 The transition

In this section, we present the results for the transitional dynamics. We perform two sets of exercises. In one, issuing debt finances a uniform transfers policy to households; in the second one, it finances an increase in purchases. The policies are unexpected by the households.

Technically, we set the simulation horizon to 600 periods, where, as already stressed, every period represents a quarter. We then iterate on the path of prices and of the set of time-dependent policy functions, under the assumption of perfect foresight, until we have a fixed point in these objects. Appendix A describes the details of the computation of the transition.

We first report and explain the effects of the two mentioned policies on the borrowing constraint. Then, we describe the consequences –in terms of households' reactions– of the constraint's movement.

[9]We should also refer that the data counterpart of our model's wealth is different from the one reported by Castañeda et al. (2003). Indeed, they calculate a wealth distribution based on net worth whereas our model features the type of wealth represented by net financial assets.

4.1 The tightening of the borrowing limit

More specifically, the two policies under study are the following. First, we simulate a debt expansion that finances an increase in transfers which is uniform across agents. On impact, which we define to occur at $t = 1$, transfers increase by 1% of steady-state output and then decay following an AR(1) with persistence 0.95, as estimated by Leeper et al. (2010). Second, we simulate a debt expansion that finances an increase in purchases similar to that set in the *ARRA*. We borrow the process for G from Uhlig (2010). On impact, the stimulus amounts to around 0.3% of GDP, reaching its maximum (around 0.8% of GDP) after 6–7 quarters.[10] Under both policies, in order to postpone lump sum taxation in the future, we set the parameter ϕ in the fiscal rule (9) at a very low level, 0.02, that is still large enough so as to ensure stability of the debt level.[11] As a result, the quarterly debt-to-GDP ratio increases up to a maximum of 10–12% after around 30 quarters and then slowly comes back to the steady state. In yearly terms, the peak of the ratio reaches 2.5–3%. The evolution of the fiscal variables can be seen in the top panels of both Figures 3 and 5.

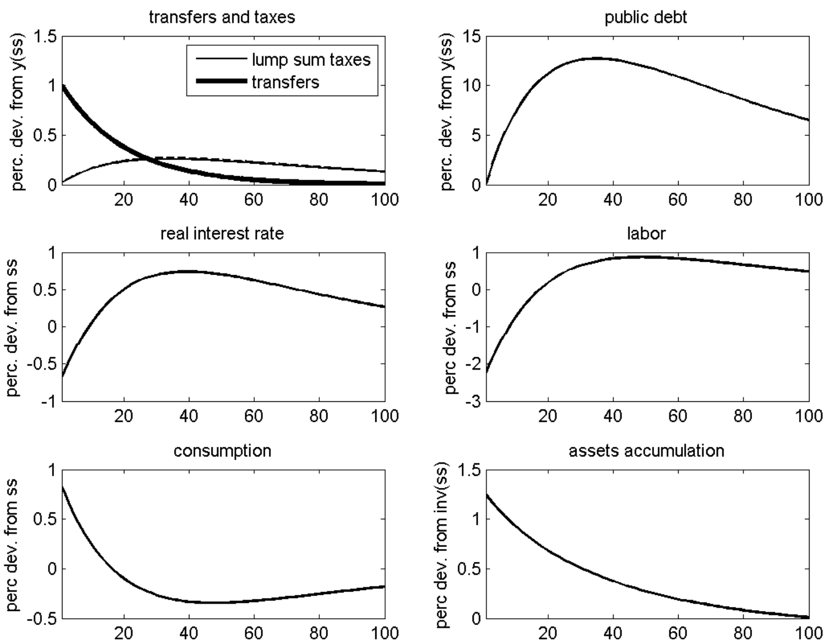

Figure 3: Selected reactions to the transfers policy of those households holding negative assets and characterized by the lowest productivity level. The evolution of the fiscal variables together with the one of the interest rate are reported as well.
The 'assets accumulation' refers to the change in assets between period $t + 1$ and t. y(ss) and inv(ss) stand for steady-state output and investment, respectively. The x-axis is in quarters.

In order to study the effect of the policies on the household borrowing constraint, we need

[10]Notice that the G process in Uhlig (2010) is characterized by a zero increase of G on impact, and a 0.3% of GDP increase in the second period. We start in the second period of that process in order to avoid unbounded multipliers on impact. The G path follows an AR(2) process, with the coefficients on the first and the second autoregressive term being equal to 1.653 and -0.672, respectively.

[11]We use lump sum taxation to finance the spending programs because we would avoid the distortive effects of income taxes to mix with the channel under scrutiny. However, using labor taxes does not change significantly our quantitative results (see Section 4.1.1 for details).

to analyze their impact on the equilibrium and autarky value functions. Specifically, we need to consider the behavior of the value functions—parameterized in the lowest z—in the region where assets are close to the borrowing limit, or perhaps negative. To gain some intuition on these value functions' movements, we describe the average reactions of the lowest productive households holding negative assets together with the evolution of the interest rate. Figure 3 reports these reactions in the case of the transfers policy.[12] After the policy change, on average, these households consume more and work less. This is expected given the shape of the consumption and labor policy functions presented in Figure 2. The interest rate is positively affected by the increase in public debt but negatively affected by the decrease of the labor supply. On impact, the interest rate decreases, but after roughly two years is already higher than its steady-state level and reaches its peak in 10 years. After that, it slowly comes back to the steady-state level. These households, who are borrowers, forecast that borrowing will be more costly and hence they start to deleverage immediately.[13]

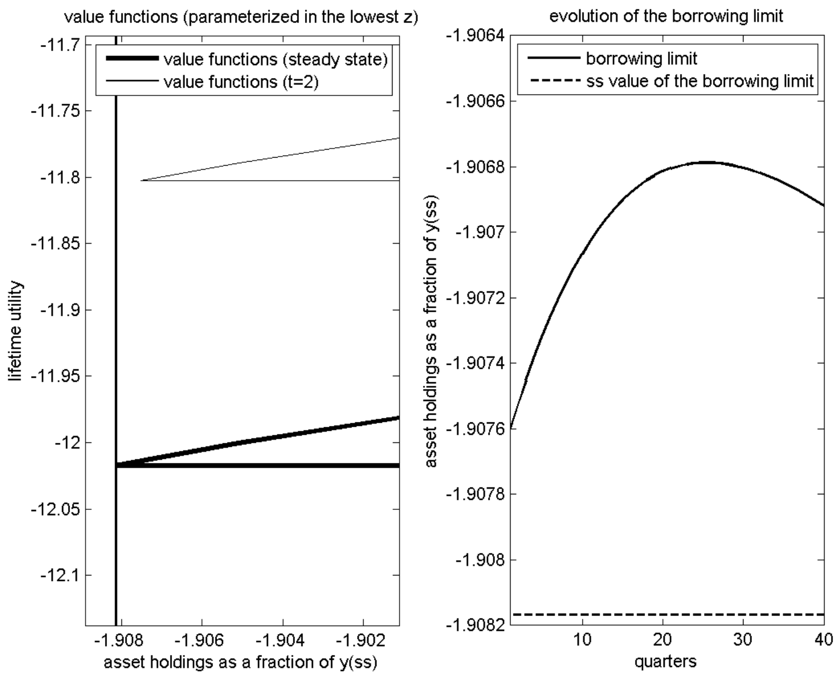

Figure 4: Effects of the transfers policy on the borrowing constraint, i.e., on $\underline{a}(\theta')$.
In the left panel, the two flat lines corresponds to the value functions in autarky, i.e., $\underline{v}(z', \theta')$, both in the steady state (thick lines) and in the second period of the transition (thin lines). Similarly, the other two lines refer to the equilibrium value functions, i.e., $v(x', \theta')$. The value functions are parameterized in the lowest z. The steady state borrowing limit is identified by the vertical line. y(ss) stands for steady-state output. In the right panel, the solid line corresponds to the evolution of the borrowing limit over time, while the dashed line identifies the constraint at its steady-state level.

[12] The reaction of labor, say, for the least productive borrowers (that is, those with the lowest productivity level z_1) in the first period of the transition is calculated as $\int_{a \leq 0 \wedge z=z_1} \left(\frac{n_1(x,\theta)\, z - n(x,\theta)\, z}{n(x,\theta)\, z} \right) d\theta$, where $n_1(x, \theta)$ is the labor policy function in the first period of the transition and $n(x, \theta)$ is the policy function in the steady state. The reactions in the following periods are calculated in the same fashion. The same logic is used to reproduce the reactions for consumption and asset holdings.

[13] Obviously, a part of these reactions is generated by the movement in the borrowing constraint. For the sake of clarity, we will postpone the detailed description of the effects of the constraint dynamics in Section 4.2.

The effects of the transfers policy on the borrowing constraint, $\underline{a}(\theta')$, are visible in Figure 4. The left panel presents the movement of the borrowing limit from the steady-state value to its value in $t = 2$, that is, the period after the occurrence of the fiscal shock.[14] Consistently with the households' reactions reported in Figure 3, both value functions $\upsilon(x', \theta')$ and $\underline{\upsilon}(z', \theta')$ move up on impact. Crucially, the increase in the borrowing cost affects (negatively) only the dynamics of $\upsilon(x', \theta')$ which, indeed, moves up by less than what $\underline{\upsilon}(z', \theta')$ does. This fact generates a tightening of the borrowing limit at $t = 2$, meaning that the maximum attainable level of borrowing decreases. The right panel of Figure 4 shows the dynamics of the borrowing limit over time. Its evolution is characterized by a high degree of persistence; after ten years from the occurrence of the shock the limit is still far from its steady-state level. Figure 16 in Appendix B shows the evolution of the borrowing limit for a larger number of periods.

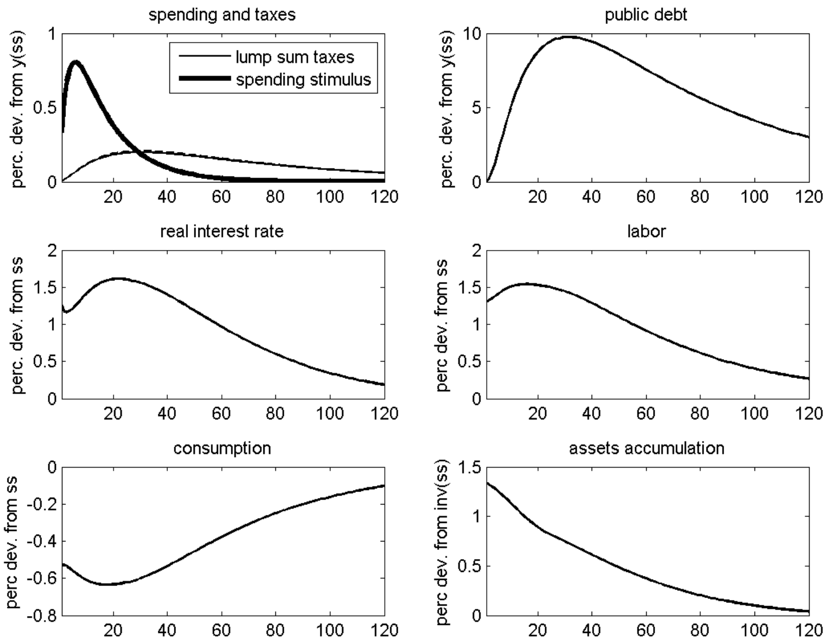

Figure 5: Selected reactions to the purchases policy of those households holding negative assets and characterized by the lowest productivity level. The evolution of the fiscal variables together with the one of the interest rate are reported as well.
The 'assets accumulation' refers to the change in assets between period $t + 1$ and t. y(ss) and inv(ss) stand for steady-state output and investment, respectively. The x-axis is in quarters.

The variable that is ultimately responsible for the tightening seems to be the interest rate. To get further evidence on this issue, we simulate a fixed-prices version of the model in which the interest rate, and hence the borrowing cost, is kept fixed throughout the transition.[15] Because of this, $\upsilon(x', \theta')$ moves up more than in the case with flexible prices (for any period of the transition). Initially, this dampens the tightening of the borrowing limit, and, after few quarters, the limit starts loosening. The loosening is highly persistent. The results are shown

[14]We report the value functions at $t = 2$ because these define the maximum amount of borrowing which is relevant for the asset holdings' decision at $t = 1$, i.e., when the shock occurs.

[15]Notice that keeping constant the interest rate implies also a constant wage rate. Practically, we iterate on policy functions conditional on factor prices kept fixed at their steady-state levels.

in Figure 18 in Appendix C.

Regarding the other policy, issuing public debt to finance an increase in purchases produces a tightening of the constraint as well. Interestingly, unlike in the transfer policy, the value functions move downward.[16] However, similarly to the transfers policy, because the interest rate affects only the equilibrium value function, the latter moves downward more than the autarky value function. Hence, a tightening is eventually generated. Figures 5, 6 and 19 report the borrowers' reactions (characterized by the lowest z), the dynamics of the borrowing constraint in the general equilibrium setting, and the dynamics of the borrowing constraint under fixed prices, respectively. Specifically, Figure 19 in Appendix C confirms that simulating a fixed-prices version of the model produces a loosening in the borrowing limit even in the case of the purchases policy. Further, Figure 17 in Appendix B shows the evolution of the borrowing limit for a larger number of periods.

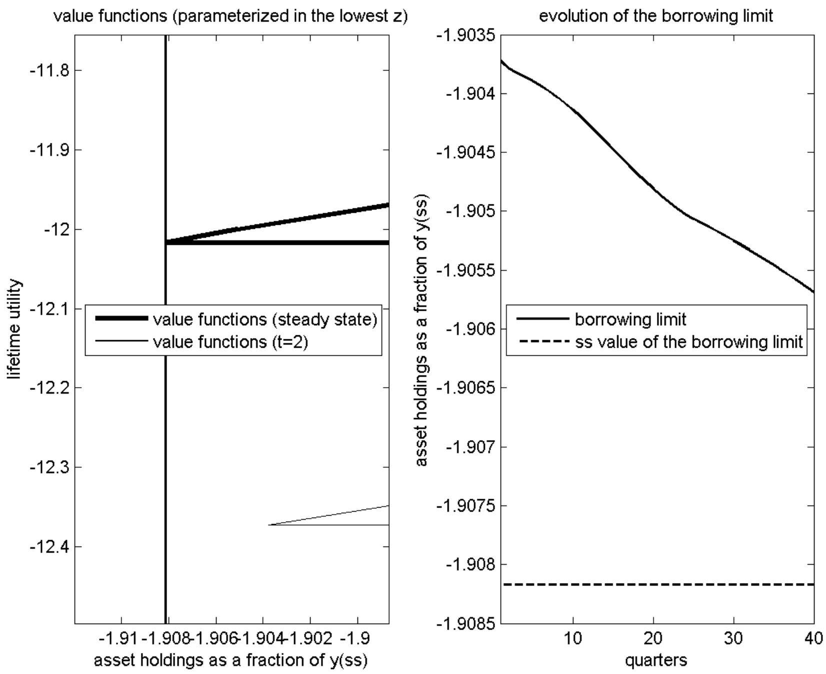

Figure 6: Effects of the purchases policy on the borrowing constraint, i.e., on $\underline{a}(\theta')$.
In the left panel, the two flat lines corresponds to the value functions in autarky, i.e., $\underline{v}(z', \theta')$, both in the steady state (thick lines) and in the second period of the transition (thin lines). Similarly, the other two lines refer to the equilibrium value functions, i.e., $v(x', \theta')$. The value functions are parameterized in the lowest z. The steady state borrowing limit is identified by the vertical line. y(ss) stands for steady-state output. In the right panel, the solid line corresponds to the evolution of the borrowing limit over time, while the dashed line identifies the constraint at its steady-state level.

4.1.1 Robustness exercises

We check if the tightening is produced even if we simulate the policies in other environments.

[16]The downward movement of both value functions is consistent with the reactions presented in Figure 5. Indeed, conditional on this policy, consumption goes down while labor up because of the well-known negative wealth effect generated by the increase in G.

An alternative borrowing limit. We simulate the debt-financed transfers policy using a borrowing limit which represents a modified version of the *natural borrowing limit* in Aiyagari (1994). Specifically, we let the borrowing limit be $\underline{a} = -\eta * \left(\frac{w}{r}\right)$ where η has the same role that γ had in the original calibration. That is, we use η in order to match the credit-to-output ratio in the data, i.e., 8%. The steady-state calibration obtained with this alternative borrowing limit is the same we had using the original model.[17] Figure 20 in Appendix D shows that a tightening is produced. More specifically, initially, the borrowing limit loosens because it is directly affected by the impact fall in the interest rate. Then, the limit persistently tightens before to slowly come back to its steady-state level. The simulation of the debt-financed purchases policy delivers similar results, which are available upon request.

Different steady-state levels of public debt. In the baseline simulations we start from a steady state of zero public debt. We recalibrate our economy and produce two additional steady states, one with a debt-to-output ratio of 60% and another one with 120% in yearly terms. We then simulate our policies starting from the two new steady states and obtain very similar results for the dynamics of the borrowing constraint. In particular, Figure 22 in Appendix E shows the evolution of the borrowing constraint under the transfers policy starting from a steady state of 60% for the debt-to-output ratio. Notice that this level of public debt was roughly the one charaterizing the U.S. prior to the so called great recession. The simulations in the cases of a steady state of 120% for the debt-to-output ratio and of the purchases policy deliver similar results, which are available upon request.

Accounting for price stickiness. Our model is characterized by flexible prices. This could have an impact in the adjustment of the borrowing limit. Hence, we build a version of the model with countercyclical markups in the line of Hall (2009), which is able to replicate the typical reactions to the government spending shock obtained in a model where price stickiness is properly modeled.[18] For example, in our version of the model with countercyclical markups wages react positively, unlike in the flexible prices version, to the G shock. Despite that, the process of the tightening is hardly affected in the new specification. Again, the increase in the interest rate is crucial for the process of the tightening. Figure 24 in Appendix F shows the evolution of the borrowing constraint under the transfer policy, using the version of our model with countercyclical markups. The simulation of the debt-financed purchases policy delivers similar results, which are available upon request.

Additional exercises. We simulate our model using a different value for the parameter in the fiscal rule, i.e., ϕ; we use the value proposed by Uhlig (2010), i.e, 0.05. Both under the transfers

[17]We recall that in the standard version of the *natural borrowing limit* η would represent the lowest level of the idiosyncratic productivity.

[18]Briefly, this version of the model consider an additional equation for the evolution of the markup (κ), i.e., $\kappa = \tilde{\kappa} Y^{-\omega}$ where $\tilde{\kappa}$ is the markup in steady state and a positive ω implies a countercylycal markup; following Hall (2009), we set $\tilde{\kappa}$ and ω to 1.1 and 0.7, respectively. Accordingly, (5) and (6) become $r^K = A\alpha \left(\frac{1}{\kappa}\right) \left(\frac{N}{K}\right)^{1-\alpha}$ and $w = A(1-\alpha) \left(\frac{1}{\kappa}\right) \left(\frac{K}{N}\right)^{\alpha}$, respectively. The resulting profits are evenly distributed among households.

and the purchases policies, a tightening of the borrowing constraint is produced. Further, we simulate different profiles for the the policies, e.g., only a one-period increase in transfers or in purchases. The tightening is always produced. Finally, we check that using labor taxes instead of lump sum taxes (to finance the debt expansions) produces a tightening across all simulations. All the mentioned simulations are available upon request.

4.2 The consequences of the tightening

In this section, we present the consequences of the tightening of the borrowing constraint. In order to isolate the effects of the tightening, both qualitatively and quantitatively, we compare the results generated by simulating two different models. One is our baseline model, where the borrowing limit is allowed to evolve endogenously. The other model is conditional on keeping the limit fixed throughout the transition, implying that the relevant borrowing constraint becomes $a' \geq \underline{a}$, where $\underline{a}$ is the endogenously determined borrowing limit using the baseline model in steady state. The latter specification is labeled as fixed-constraint model.

We first gain some insights by comparing the decision rules in the two models. Then, we study the households' reactions both at a disaggregate and an aggregate level.

4.2.1 Policy functions

Figures 7 and 8 compare policy functions with endogenous or fixed borrowing limit and under the two fiscal policies. Specifically, in the two figures, each line reports the difference for the labor, consumption and asset policy functions obtained within the baseline model from the respective ones generated within the fixed-constraint model, conditional on different levels of productivity. These lines are calculated when the shock occurs, that is, at $t = 1$.

The main message of these figures is that the decision rules, on impact, are different between the two considered models. Interestingly, if we consider low productivity levels, the differences are larger, in absolute value, in the assets region close to the borrowing limit. On the contrary, when we consider the highest productivity level, the differences are homogenous along the whole grid of assets.

Further, it can be seen that the qualitative effects of the tightening on the decision rules are similar in the two fiscal policies.

4.2.2 Households' reactions

In this section, we study the reactions of different categories of households, specifically, the debt-constrained households and the unconstrained ones. We investigate on how the tightening shapes the behaviour of these two categories. Finally, we aggregate the individual reactions and measure the percentage of the aggregate reactions explained by the tightening. In the following figures, the solid lines correspond to the reactions obtained within the baseline model, while the dashed lines represent those generated by the fixed-constraint model. The gap between these two sets of reactions quantifies the part of the reactions attributable to the tightening.

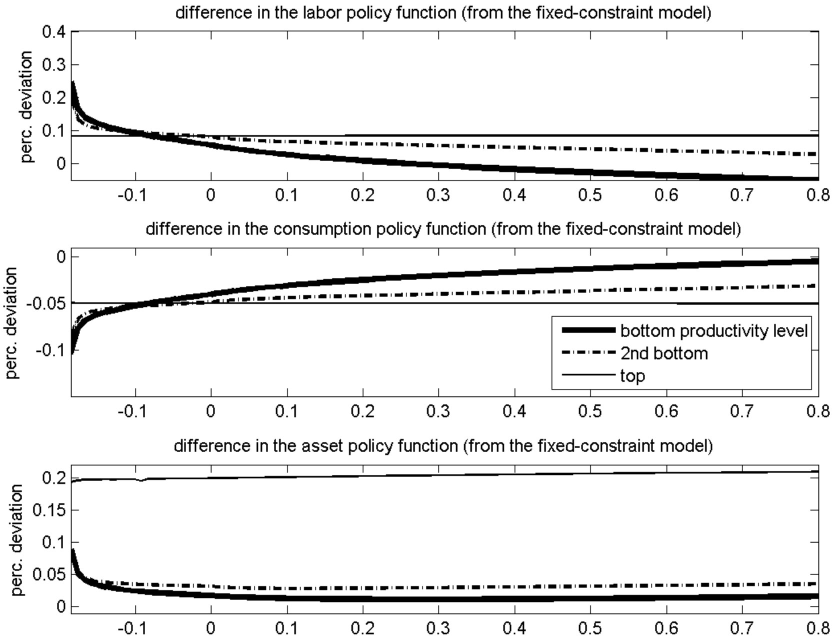

Figure 7: Effects of the transfers policy on labor, consumption, and asset decision rules, conditional on different levels of productivity.
In particular, each line shows the difference between the policy function obtained in the baseline model and that obtained with the fixed-constraint model, calculated at $t = 1$. The x-axis is asset holdings as a fraction of average assets.

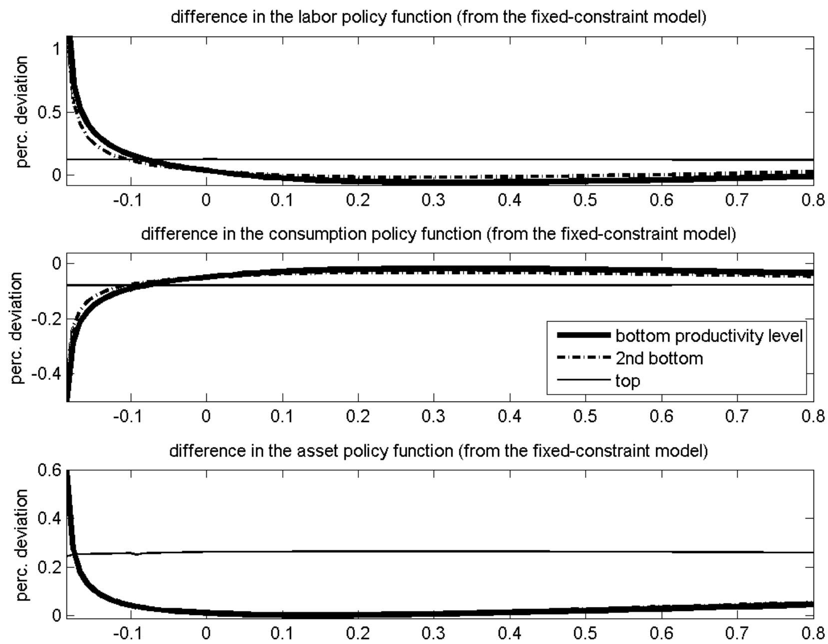

Figure 8: Effects of the purchases policy on labor, consumption, and asset decision rules, conditional on different levels of productivity.
In particular, each line shows the difference between the policy function obtained in the baseline model and that obtained with the fixed-constraint model, calculated at $t = 1$. The x-axis is asset holdings as a fraction of average assets.

Figure 9 shows the average reactions in assets, labor and consumption for both debt-constrained and unconstrained households, under the transfers policy.[19] Let us focus on the reactions of the debt-constrained under the fixed-constraint model, which are represented by the thick-dashed lines in the figure. Debt constrained households use the transfer received from the government to increase their consumption given that their marginal propensity to consume is the highest in the economy. They also decrease their labor supply. Regarding the asset accumulation, they deleverage because they can now rely more on the received transfers (as opposed to borrowing) for targeting the desired level of consumption. Obviously, they deleverage also because the borrowing cost is higher.

Let us now concentrate on the reactions of the unconstrained households under the fixed-constraint model, which are represented by the thin-dashed lines in Figure 9. The unconstrained households react less in terms of consumption and labor relative to debt-constraint agents, indeed the former ones prefer to smooth their consumption knowing that they will have to pay higher taxes in the future. As a consequence, they use a consistent part of the received transfers to buy assets (if they are lenders), or to decrease their indebtedness (if they are borrowers).[20] The rise in saving occurs even because of the increasing profile of its return. As highlighted by Aiyagari and McGrattan (1998), this helps households to comply with self-insurance purposes given that their asset position happens to be, on average, farther away form the borrowing constraint.

What does the tightening add to these reactions? The comparison between solid (baseline model) and dashed lines (fixed-constraint model) in Figure 9 gives the answer. Because of the tightening, debt-constrained households must deleverage. They do so by working more and consuming less relative to the case in which the borrowing limit is fixed. Unconstrained agents realize that, all else equal, their asset position will be closer to the borrowing limit, hence their precautionary saving motive is reinforced. As a consequence, the unconstrained agents who are lenders will save more, while the unconstrained ones who are borrowers will cut on their debt. Similarly to the debt-constrained agents, they achieve a higher saving by working more and consuming less.

Figure 10 shows the reactions in assets, labor and consumption for both debt-constrained and unconstrained households, under the purchases policy. For the reasons explained in Section 4.1, the sign of the reactions for consumption and labor are different with respect to those generated by the transfers policy. However, the role of the tightening is similar to that described under the transfers policy. Debt-constrained households are forced to deleverage and unconstrained households accumulate (decumulate) assets (debt). They both do so by working more and consuming less.

[19] The reaction of labor in the first period of the transition for the debt-constrained, say, is calculated as $\int_{a\in\mathcal{V}(\underline{a})}\left(\frac{n_1(x,\theta)\,z-n(x,\theta)\,z}{n(x,\theta)\,z}\right)d\theta$, where $n_1(x,\theta)$ is the labor policy function in the first period of the transition, $n(x,\theta)$ is the policy function in the steady state and $\mathcal{V}(\underline{a})$ is a tight neighborhood of $\underline{a}$ (as defined in Section 3). The reactions in the following periods are calculated in the same fashion. Finally, the same logic is used to reproduce the heterogeneous responses for consumption and asset holdings.

[20] Of course, the unconstrained borrowers will decrease their indebtedness also because of higher borrowing costs.

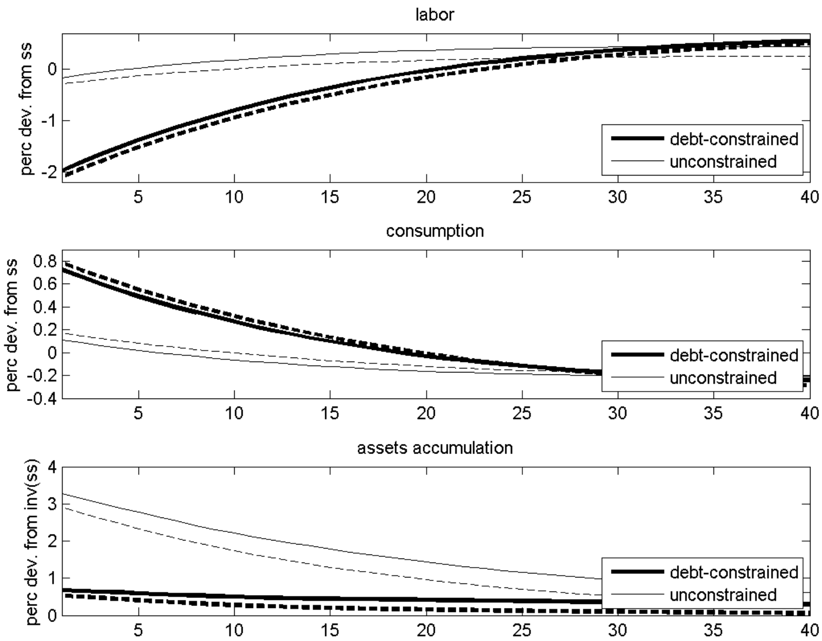

Figure 9: Heterogeneous effects of the transfers policy.
Both the reactions of debt-constrained (thick lines) and unconstrained (thin lines) agents are presented. Solid lines are generated with the baseline model and dashed lines with the fixed-constraint model. The term 'assets accumulation' refers to the change in assets between period $t + 1$ and the steady-state level and inv(ss) stands for steady-state investment. The x-axis is in quarters.

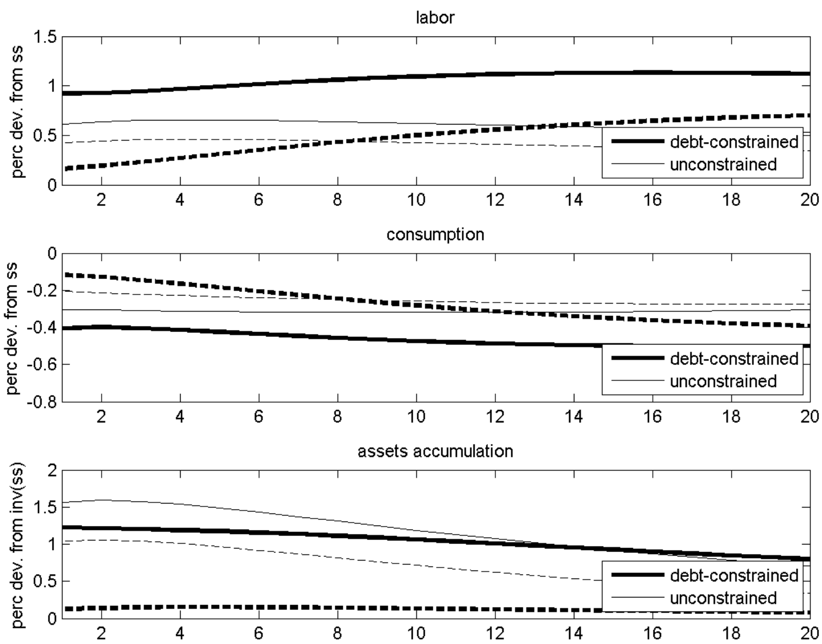

Figure 10: Heterogeneous effects of the purchases policy.
Both the reactions of debt-constrained (thick lines) and unconstrained (thin lines) agents are presented. Solid lines are generated with the baseline model and dashed lines with the fixed-constraint model. The term 'assets accumulation' refers to the change in assets between period $t + 1$ and the steady-state level and inv(ss) stands for steady-state investment. The x-axis is in quarters.

Figures 11 and 12 show selected aggregate reactions, including those of physical capital and consumer credit, generated by the two policies. Because of the increase in the interest rate, capital is crowed-out. Furthermore, consistently with the heterogeneous reactions outlined above consumer credit persistently falls. The tightening explains a significant share of these aggregate reactions. Indeed, considering both policies, five years after the beginning of the debt expansions, the tightening reinforces the fall in consumer credit by, on average, 20%. Conversely, the tightening attenuates the fall in physical capital explaining around half of the physical capital's reaction.

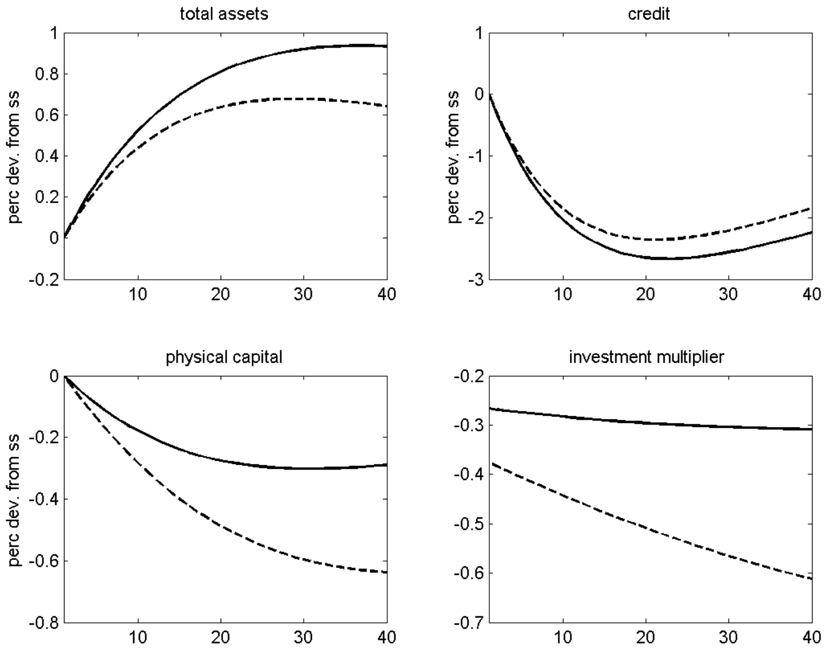

Figure 11: Aggregate effects of the transfers policy.
Solid lines are generated with the baseline model and dashed lines with the fixed-constraint model. 'Total assets' is the sum of (claims of) physical capital and public debt. The cumulative dynamic multipliers are calculated following Uhlig (2010). The x-axis is in quarters.

Figures 13 and 14 show the rest of the aggregate reactions generated by the two policies. Consistently with the heterogeneous reactions, aggregate labor reacts more in the baseline model relative to the model where the borrowing limit is fixed. A direct consequence is that output, and hence, the output multiplier, is bigger in the baseline model, both in the transfers and in the purchases policies. Furthermore, the fact that physical capital is crowed out less in the baseline model sustains the bigger output multipliers in this specification. Around half of the five-year (cumulative) output multipliers is explained by the dynamics of the borrowing limit, in both policies.[21] In addition, the reaction of consumption is more negative in the baseline model.

A final comment is about the quantitative importance of the reactions of the unconstrained households. The sum of the reactions of the unconstrained households alone explains the

[21] Notice that the output multiplier associated to the purchases policy is mildly larger than one on impact. This is due to the hump-shaped process of G.

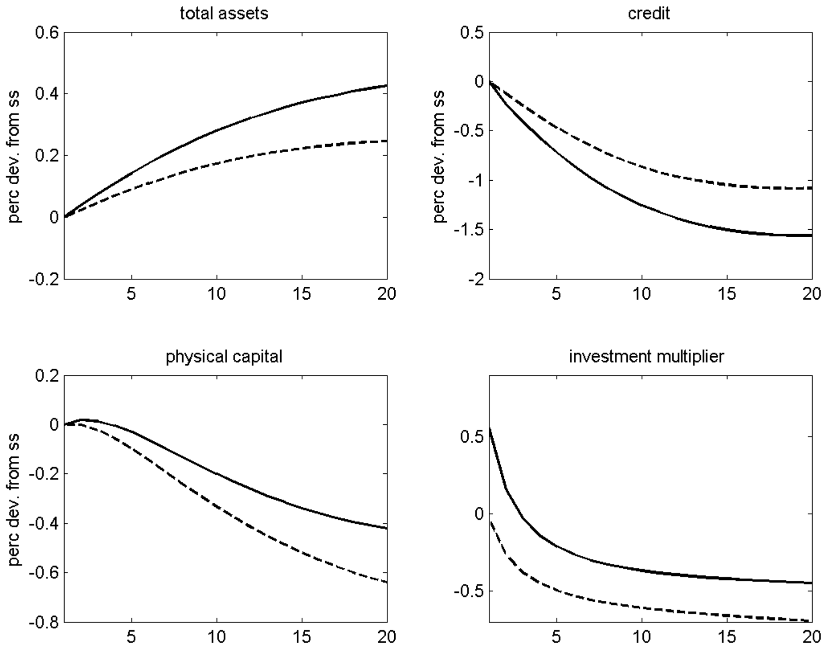

Figure 12: Aggregate effects of the purchases policy.
Solid lines are generated with the baseline model and dashed lines with the fixed-constraint model. 'Total assets' is the sum of (claims of) physical capital and public debt. The cumulative dynamic multipliers are calculated following Uhlig (2010). The x-axis is in quarters.

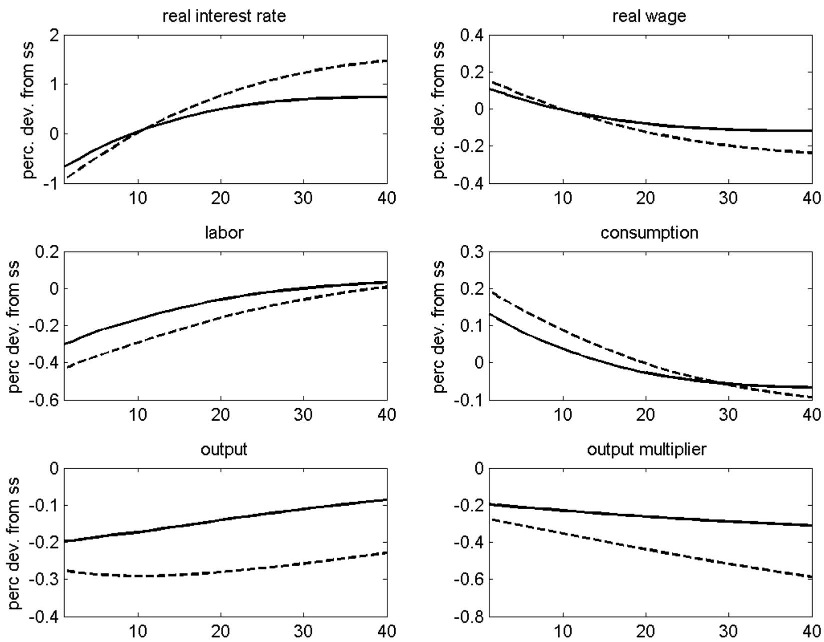

Figure 13: Aggregate effects of the transfers policy.
Solid lines are generated with the baseline model and dashed lines with the fixed-constraint model. 'Total assets' is the sum of (claims of) physical capital and public debt. The cumulative dynamic multipliers are calculated following Uhlig (2010). The x-axis is in quarters.

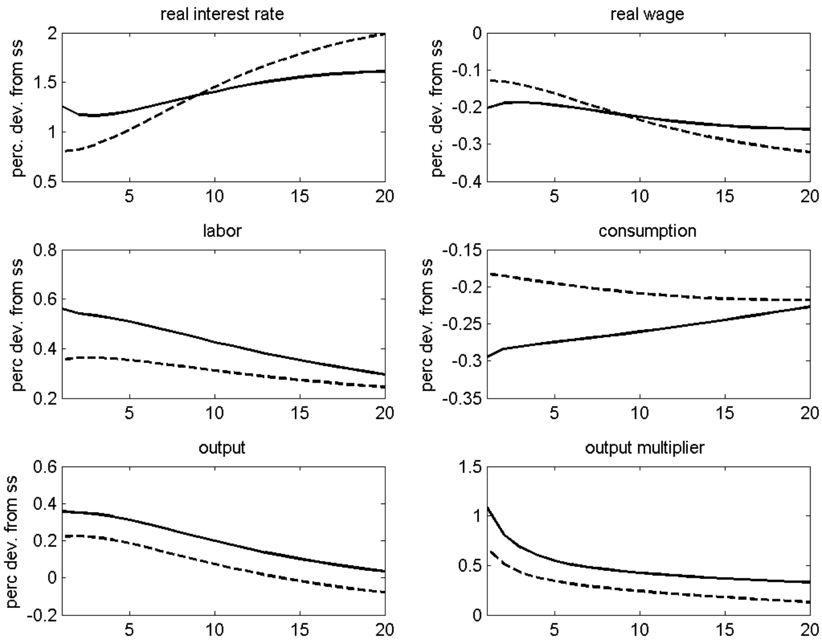

Figure 14: Aggregate effects of the purchases policy.
Solid lines are generated with the baseline model and dashed lines with the fixed-constraint model. 'Total assets' is the sum of (claims of) physical capital and public debt. The cumulative dynamic multipliers are calculated following Uhlig (2010). The x-axis is in quarters.

greatest share of the quantitative effects associated with the tightening. For example, under both policies, the mentioned share goes from roughly 65% to 85% regarding labor, consumption and asset holdings' reactions. Hence, the aggregate effects generated by the tightening are largely independent of the calibrated share of debt-constrained agents.

4.2.3 Robustness exercises

Following the exercises in Section 4.1.1, we check what are the effects produced by the tightening if (i) we use the alternative borrowing limit (the modified *natural borrowing limit*), (ii) we start from the 60% level of debt-to-GDP ratio, and (iii) we use model with countercylcycal markups. For ease of space, we here present the simulations under the transfers policy, however the simulations under the purchases policy deliver similar results and are available upon request.

Figure 21 in Appendix D shows the comparison between the reactions produced by the model with the alternative borrowing limit and those generated by the fixed-constraint model, for selected variables, under the transfers policy. The effects generated by the tightening in Figure 21 are undoubtedly larger than those obtained in our baseline simulations using the rational borrowing constraints (see Figures 11 and 13). This can give rise to the conclusion that the effects generated by the dynamics of the household borrowing constraint are dampened if the borrowing limit depends not only on prices (like the typical *natural borrowing limit*) but also on the value functions which discount the future path of the model's variables.

Figure 23 in Appendix E shows the effects generated by the tightening within an economy with 60% level of debt-to-GDP ratio in steady state, for selected variables, under the transfers policy. These effects have a similar magnitude of those ones produced in the context where the steady-state level of public debt was nil.

Figure 25 shows the effects generated by the tightening, using the model with countercylical markups which replicates the effect of the nominal stickiness, under the transfers policy. Though the effects generated by the dynamics of the borrowing limit are similar to the ones of the baseline model, one thing is worth noting. Unlike the baseline model with fully flexible prices, the model with countercylical markups allows changes in aggregate demand to translate into output. Hence, the overall positive effects on output generated by the tightening is now dampened by the fact that consumption is lower in the baseline model relative to the fixed-constrained model.

5 A crisis experiment

The purpose of this section is to see how public debt expansions affect the behavior of agents during an economic crisis characterized by both a fall in output and in private credit.

In order to generate such a crisis, we simulate a negative technology shock (or a decrease in TFP) within our baseline model, namely, a fall in A as defined in Section 2.1. In particular, after an initial shock A follows the stochastic process $A' = 1-\rho^a+\rho^a A$, where we set the initial level of A to 0.965 and $\rho^a = 0.85$. Within our baseline model specification, we unexpectedly implement the mentioned deterministic process for A. Solid lines in Figure 15 show selected reactions to such a shock. All else equal, the marginal product of capital decreases. Therefore, the real interest rate and capital fall. Households consume and work less. The shock produces an initial fall in output of roughly 4%.[22] On impact, a tightening of the borrowing constraint is generated because the borrowing cost, though initially decreasing, is higher than its steady-state level after roughly two years and remains positive for more than ten years. Hence, because of the TFP shock, both value functions (parameterized in the lowest z) fall; however, the equilibrium value function falls more than the autarky one because of the mentioned dynamics of the borrowing cost and this produces a shrinking in the maximum amount of borrowing.[23] This contributes to generate a fall in credit.

The effects generated by the TFP shock alone can be seen as a 'no intervention' scenario where the fiscal authority does not implement any discretionary policy. In order to see how the crisis is affected by the fiscal policies we proceed as follows. At the same time of the occurrence of the TFP shock, we simulate, in turn, the two debt expansions described in Section 4.1. The dashed lines represent the variables' reactions to the TFP shock plus the debt-financed transfers policy. The dotted lines represent the variables' reactions to the TFP shock plus the debt-financed purchase policy. Though the quantitative effects of the two fiscal policies are somehow different, the qualitative ones show several similarities. For example,

[22]Notice that we calibrate the initial fall in A in order to get the 4% fall in output. Such a fall is the same observed for the U.S. GDP during the Great Recession, specifically, between 2008:Q2 and 2009:Q2. Regarding ρ^a, we used different values around the chosen one and the results hardly change.

[23]For lack of space, we do not report the graph showing the movement of the value functions generated by the TFP shock. It is available upon request.

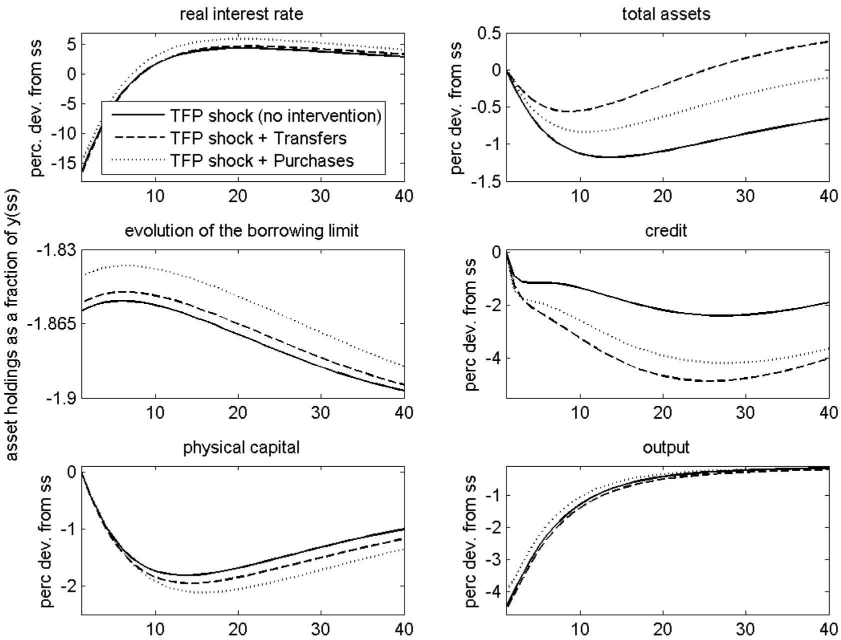

Figure 15: Selected reactions within a crisis experiment.
All reactions are generated using the baseline model. Solid lines represent the reactions to a (negative) TFP shock with no intervention from the fiscal authority. Dashed lines are generated by the occurrence of both the TFP shock and the transfers policy. Dotted lines are generated by the occurrence of both the TFP shock and the purchases policy. 'Total assets' is the sum of (claims of) physical capital and public debt. The x-axis is in quarters.

both policies produce a higher level of the borrowing cost with respect to the 'no intervention' scenario. This is consistent with the fact that the implementation of the policies contributes to a further tightening of the borrowing limit. Both policies generate a fall in credit: after 5 years from the occurrence of the shocks, the fall in credit is, on average, twice as much the one obtained in the 'no intervention' case. Furthermore, because of the behaviour of the interest rate, physical capital is crowded out more if the fiscal policies are implemented. Finally, the effects of the policies on output are modest; in particular, the implementation of the purchases policy alleviates the recession by roughly 0.5% of output on impact.

6 Conclusions and research agenda

An important view in the literature affirms that, in incomplete-markets frameworks, issuing public debt acts as if it relaxed the borrowing constraint because additional assets are provided to the household sector that can use them for self-insurance purposes. In this paper, we focus on an additional channel generated by government debt expansions: the increase in the interest rate generated by the policy represents also an increase in the borrowing cost which, in turn, produces an actual tightening of the household borrowing constraint, or equivalently, a reduction in the maximum quantity that households can borrow.

We show, within an incomplete-markets model featuring both consumer credit and endogenous borrowing constraints, that debt expansions have the potential to persistently tighten the household borrowing constraint. Then, we show that this tightening shapes the households' reactions to the fiscal policies. For example, the dynamics of the borrowing limit strongly interacts with capital accumulation, consumer credit dynamics, and fiscal multipliers. Importantly, the contribution of the tightening to the aggregate reactions is mainly driven by the sum of the reactions of unconstrained agents; hence, the quantitative effects generated by the tightening are largely independent of the calibrated share of debt-constrained agents. Finally, we produce a 'crisis experiment' through which we see that the fiscal policies under scrutiny, implemented while a financial crisis occurs, reinforce both the tightening of the borrowing limit and the credit crunch.

Our results are obtained using a model which considers only unsecured consumer credit. As we conjectured in the Introduction, our channel survives even if we consider a borrowing constrained characterized by collateralized credit. However, it would be interesting to study how ad by how much the inclusion of a collateral would change the present results. This is left for future research.

References

Ábrahám, Á. and Cárceles-Poveda, E. (2010), 'Endogenous trading constraints with incomplete asset markets', *Journal of Economic Theory* **145**(3), 974–1004.

Aiyagari, S. R. (1994), 'Uninsured idiosyncratic risk and aggregate saving', *Quarterly Journal of Economics* pp. 659–684.

Aiyagari, S. R. and McGrattan, E. R. (1998), 'The optimum quantity of debt', *Journal of Monetary Economics* **42**(3), 447–469.

Alvarez, F. and Jermann, U. J. (2000), 'Efficiency, equilibrium, and asset pricing with risk of default', *Econometrica* **68**(4), 775–797.

Antunes, A. A. and Cavalcanti, T. V. (2013), 'The welfare gains of financial liberalization: capital accumulation and heterogeneity', *Journal of the European Economic Association* **11**(6), 1348–1381.

Baxter, M. and King, R. G. (1993), 'Fiscal policy in general equilibrium', *American Economic Review* pp. 315–334.

Bewley, T. (1977), 'The permanent income hypothesis: A theoretical formulation', *Journal of Economic Theory* **16**(2), 252–292.

Bilbiie, F. O., Monacelli, T. and Perotti, R. (2013), 'Public debt and redistribution with borrowing constraints', *Economic Journal* **123**(566), F64–F98.

Brinca, P. S., Holter, H. A., Krusell, P. and Malafry, L. (2014), Fiscal multipliers in the 21st century, Research Paper 2014/119, Robert Schuman Centre for Advanced Studies.

Buera, F. J. and Moll, B. (2012), Aggregate implications of a credit crunch, Technical report, National Bureau of Economic Research.

Carroll, C. D. and Kimball, M. S. (1996), 'On the concavity of the consumption function', *Econometrica* pp. 981–992.

Castañeda, A., Díaz-Giménez, J. and Ríos-Rull, J.-V. (2003), 'Accounting for the u.s. earnings and wealth inequality', *Journal of Political Economy* **111**(4), 818–857.

Challe, E. and Ragot, X. (2011), 'Fiscal policy in a tractable liquidity-constrained economy', *Economic Journal* **121**(551), 273–317.

Chatterjee, S., Corbae, D., Nakajima, M. and Ríos-Rull, J.-V. (2007), 'A quantitative theory of unsecured consumer credit with risk of default', *Econometrica* **75**(6), 1525–1589.

Christiano, L., Eichenbaum, M. and Rebelo, S. (2011), 'When is the government spending multiplier large?', *Journal of Political Economy* **119**(1), 78–121.

Cogan, J. F., Cwik, T., Taylor, J. B. and Wieland, V. (2010), 'New keynesian versus old keynesian government spending multipliers', *Journal of Economic dynamics and control* **34**(3), 281–295.

Corsetti, G., Kuester, K., Meier, A. and Müller, G. J. (2013), 'Sovereign risk, fiscal policy, and macroeconomic stability', *Economic Journal* **123**(566), F99–F132.

Domeij, D. and Heathcote, J. (2004), 'On the distributional effects of reducing capital taxes', *International Economic Review* **45**(2), 523–554.

Eggertsson, G. B. and Krugman, P. (2012), 'Debt, deleveraging, and the liquidity trap: A fisher-minsky-koo approach*', *Quarterly Journal of Economics* **127**(3), 1469–1513.

Ercolani, V. and Pavoni, N. (2014), The precautionary saving effect of government consumption, Discussion Paper 10067, CEPR.

Fernández-Villaverde, J. (2010), 'Fiscal policy in a model with financial frictions', *American Economic Review* pp. 35–40.

Floden, M. and Lindé, J. (2001), 'Idiosyncratic risk in the united states and sweden: Is there a role for government insurance?', *Review of Economic Dynamics* **4**(2), 406–437.

Galí, J., López-Salido, J. D. and Vallés, J. (2007), 'Understanding the effects of government spending on consumption', *Journal of the European Economic Association* **5**(1), 227–270.

Guerrieri, V. and Lorenzoni, G. (2015), Credit crises, precautionary savings, and the liquidity trap, Technical report, National Bureau of Economic Research.

Hall, R. E. (2009), By how much does gdp rise if the government buys more output?, Technical report, National Bureau of Economic Research.

Heathcote, J. (2005), 'Fiscal policy with heterogeneous agents and incomplete markets', *Review of Economic Studies* **72**(1), 161–188.

Huntley, J. and Michelangeli, V. (2014), 'Can tax rebates stimulate consumption spending in a life-cycle model?', *American Economic Journal: Macroeconomics* **6**(1), 162–189.

Iacoviello, M. (2005), 'House prices, borrowing constraints, and monetary policy in the business cycle', *American Economic Review* pp. 739–764.

Jappelli, T. (1990), 'Who is credit constrained in the us economy?', *Quarterly Journal of Economics* **105**(1), 219–234.

Kaplan, G. and Violante, G. L. (2014), 'A model of the consumption response to fiscal stimulus payments', *Econometrica* **82**(4), 1199–1239.

Kehoe, T. J. and Levine, D. K. (1993), 'Debt-constrained asset markets', *Review of Economic Studies* **60**(4), 865–888.

Kiyotaki, N. and Moore, J. (1997), 'Credit cycles', *Journal of Political Economy* **105**(2), 211–248.

Kopecky, K. A. and Suen, R. (2010), 'Finite state markov-chain approximations to highly persistent processes', *Review of Economic Dynamics* **13**(3), 701–714.

Leeper, E. M., Plante, M. and Traum, N. (2010), 'Dynamics of fiscal financing in the united states', *Journal of Econometrics* **156**(2), 304–321.

Ljungqvist, L. and Sargent, T. J. (2004), *Recursive macroeconomic theory*, MIT press.

Maddaloni, A. and Peydró, J.-L. (2011), 'Bank risk-taking, securitization, supervision, and low interest rates: Evidence from the euro-area and the us lending standards', *Review of Financial Studies* **24**(6), 2121–2165.

McKay, A. and Reis, R. (2013), The role of automatic stabilizers in the u.s. business cycle, Technical report, National Bureau of Economic Research.

Oh, H. and Reis, R. (2012), 'Targeted transfers and the fiscal response to the great recession', *Journal of Monetary Economics* **59**, S50–S64.

Rendhal, P. (forthcoming), 'Fiscal policy in an unemployment crisis', *Review of Economic Studies* **forthcoming**.

Uhlig, H. (2010), 'Some fiscal calculus', *American Economic Review: Papers and Proceedings* pp. 30–34.

Van Riet, A. (2010), Euro area fiscal policies and the crisis, Technical report, European Central Bank.

Woodford, M. (1990), 'Public debt as private liquidity', *American Economic Review* pp. 382–388.

Zeldes, S. P. (1989), 'Optimal consumption with stochastic income: Deviations from certainty equivalence', *Quarterly Journal of Economics* pp. 275–298.

Zhang, H. H. (1997), 'Endogenous borrowing constraints with incomplete markets', *Journal of Finance* **52**(5), 2187–2209.

A Computational procedures

The objective of this section is to explain the details for calculating both the stationary distribution and the transition of our economies.

A.1 The solution method

As explained in Sections 2.1 and 3 the relevant borrowing limit in our economy will be unique and independent of z (since it is parameterized in the lowest productivity level). This fact allows us to solve a simplified version of the household's problem instead of the one stated in (1):

$$v(a,z,\theta) = \max_{c,n,a'} u(c,n) + \beta \mathbf{E}\left[v(a',z',\theta')|z\right] \tag{14}$$

subject to

$$c + a' = (1 + r(1-\tau_k \mathcal{I}_{a\geq 0}))a + wnz(1-\tau_w) + \mathrm{Tr} - \Gamma \tag{15}$$

$$a' \geq \underline{a}(\theta)\,. \tag{16}$$

We use a direct solution method for solving problem (14). The problem is a mixed-constrained optimal control problem because of the coexistence of equality and inequality constraints. As already stressed, the borrowing constraint is unique, but the solution method is robust to the consideration of borrowing limits contingent on different z's. We will economize on notation and drop explicit reference to measure θ. Consider the Lagrangian function:

$$\begin{aligned}\mathcal{L}(c,n,a') = {} & u(c,n) + \beta\,\mathrm{E}\left[v(a',z')|z\right] + \\ & \left((1+(1-\tau_k\mathcal{I}_{a\geq 0})r)a + w(1-\tau_w)nz + \mathrm{Tr} - \Gamma - c - a'\right)\lambda + \left(\underline{a} - a'\right)\mu + \left(\overline{a} - a'\right)\gamma\end{aligned} \tag{17}$$

where we assume that there is an absolute upper level for asset holdings, $\overline{a}$. This will have to be confirmed once we solve the problem numerically. In practice it suffices to set it to a sufficiently high positive value. The necessary conditions for an optimum of the above problem are:

$$u_c(c,n) - \lambda = 0 \tag{18}$$

$$u_n(c,n) + \lambda w(1-\tau_w)z = 0 \tag{19}$$

$$\beta\,\mathrm{E}\left[v_a(a',z')|z\right] - \lambda - \mu - \gamma = 0 \tag{20}$$

$$\mu \leq 0 \tag{21}$$

$$(\underline{a} - a')\mu = 0 \tag{22}$$

$$\gamma \geq 0 \tag{23}$$

$$(\overline{a} - a')\gamma = 0 \tag{24}$$

$$\overline{a} \geq a' \tag{25}$$

plus the two equations (15) and (16). Using an envelope result yields

$$v_a(a,z) = \lambda(1 + (1 - \tau_k \mathcal{I}_{a\geq 0})r)\,. \tag{26}$$

Hence, equation (20) becomes

$$\beta\,\mathrm{E}\,[\lambda'(1 + (1 - \tau_k \mathcal{I}_{a'\geq 0}))r')|z] - \lambda - \mu - \gamma = 0 \tag{27}$$

where we have assumed without loss of generality that tax rates stay constant over time. Given the simplification of the notation, the previous expressions are relevant for the steady state and the transition.

A.2 Numerical solution

In general, our numerical procedure aim at precisely calculating the policy functions by iterating on the above first-order conditions and constraints. This method can be used in computing both the steady state and the transition with slight adaptations that will be described below.

We first set up a grid $\mathcal{A}$ on assets with overall negative and positive asset holdings $\underline{a}_{\min}$ and $\overline{a}$, and ensure that they are not binding in any of the calibrations. The grid oversamples negative holdings. A sampling scheme where about one fifth of the grid points are negative is used. With the scale parameter of the production function normalized to 1, we set these bounds to -20 and 600. We use 250 points for $\mathcal{A}$ and 7 points for $\mathcal{Z}$. The stochastic process of idiosyncratic productivity is modeled after using Rouwenhorst's method (Kopecky and Suen 2010).[24]

The procedure for calculating the steady state is conditional on some combination of the fiscal variables satisfying the intertemporal government budget described by equations (7) and (8). This means that B, G, Tr, Γ, τ_k and τ_w must satisfy the steady-state relationship

$$\Gamma + \tau_k r \int_{a\geq 0} a\, d\theta + \tau_w w N = G + \mathrm{Tr} + rB\,. \tag{28}$$

There are two levels of iterations. The inner iteration is on individual policy functions. The outer iteration, which will be indexed by i, is on the aggregate prices and quantities.

The measure θ can be defined in a grid considerably finer than $\mathcal{X}$ in the first dimension. Without loss of generality, assume a grid $\tilde{\mathcal{X}} = \tilde{\mathcal{A}} \times \mathcal{Z}$, where $\tilde{\mathcal{A}}$ is denser than $\mathcal{A}$ but contains all its elements.[25]

1. Start with a first guess for next period's aggregate capital, value function, autarky value, borrowing limit and consumption policy function, and a first guess for current period's

[24] Ábrahám and Cárceles-Poveda (2010) iterate on the first order conditions of the household's problem and use different approaches concerning the state space.

[25] In the solutions computed in the paper, the two grids are the same because $\mathcal{A}$ already contains a sufficiently large number of points. In even more numerically intensive applications one should decrease the number of points in $\mathcal{A}$ to a much lower number (say, 30) and keep a large number of elements for $\tilde{\mathcal{A}}$.

aggregate labor supply, joint distribution of assets and shocks and the asset policy functions,

$$(K^0_{\text{next}}, N^0, v^0_{\text{next}}(a,z), \underline{v}^0_{\text{next}}(z), \underline{a}^0_{\text{next}}, \theta^0, a^0(a,z), c^0_{\text{next}}(a,z)) .$$

Set the outer iteration index i to 0.

2. If solving for the steady state, set $N^i_{\text{next}} = N^i$.

3. Compute prices $(r^i_{\text{next}}, w^i_{\text{next}})$ using (5) and (6). If solving for the steady state, set $r^i = r^i_{\text{next}}$ and $w^i = w^i_{\text{next}}$.

4. Given $\underline{v}^i_{\text{next}}(z)$, compute the autarky value, $\underline{v}^i(z)$ for all values in $\mathcal{Z}$ using (3). The maximization problem in the expression is well-behaved and yields an interior solution in terms of n.

5. Given policy function $a^i(a,z)$, solve equations (15), (18) and (19) to obtain policy functions

$$(n^i(a,z), c^i(a,z), \lambda^i(a,z)) .$$

6. Using equation (18), compute $\lambda^i_{\text{next}}(a,z) = u_c(c^i_{\text{next}}(a,z), \cdot)$, where the second argument is irrelevant given separability of the arguments of the utility function. Compute

$$\Phi(a,z) = \beta\, \mathrm{E}\left[\lambda^i_{\text{next}}(a^i(a,z), z')(1 + (1 - \tau_k \mathcal{I}_{a^i(a,z)\geq 0}))r^i_{\text{next}})|z\right] - \lambda^i(a,z)$$

using linear interpolation where necessary.

7. Use equation (27) to set $\mu^i(a,z) = \Phi(a,z)$ in all points of set $S_1 = \{(a,z) \in \mathcal{X} : a^i(a,z) < \underline{a}^i_{\text{next}} + \epsilon_1\}$, for small $\epsilon_1 > 0$, and zero elsewhere.

8. Use equation (27) to set $\gamma^i(a,z) = \Phi(a,z)$ in all points of set $S_2 = \{(a,z) \in \mathcal{X} : a^i(a,z) > \overline{a}^i_{\text{next}} - \epsilon_2\}$, for small $\epsilon_2 > 0$, and zero elsewhere.

9. Partition the grid into five mutually exclusive sets:

$$\begin{aligned}
R_1 &= \left\{(a,z) \in S_1 : \mu^i(a,z) > 0\right\} \\
R_2 &= \left\{(a,z) \in S_2 : \gamma^i(a,z) < 0\right\} \\
R_3 &= \left\{(a,z) \in S_1 : \mu^i(a,z) \leq 0\right\} \\
R_4 &= \left\{(a,z) \in S_2 : \gamma^i(a,z) \geq 0\right\} \\
R_5 &= \mathcal{X} \setminus \{R_1 \cup R_2 \cup R_3 \cup R_4\} .
\end{aligned}$$

Increase $a^i(a,z)$ in all points of R_1; decrease $a^i(a,z)$ in all points of R_2; set $a^i(a,z)$ equal to $\underline{a}^i_{\text{next}}$ in all points of R_3; set $a^i(a,z)$ equal to $\overline{a}$ in all points of R_4; increase $a^i(a,z)$ in the points of R_5 such that $\Phi(a,z) > 0$; and decrease $a^i(a,z)$ in the points of R_5 such that $\Phi(a,z) < 0$. Set the values of $a^i(a,z)$ larger than $\overline{a}$ (if any) to $\overline{a}$ and the values smaller than $\underline{a}^i_{\text{next}}$ (if any) to $\underline{a}^i_{\text{next}}$.

10. If computing for the steady state, set $c^i_{\text{next}}(a,z) = c^i(a,z)$; compute v^i iterating on expression (14) with policy functions $n^i(a,z)$ and $c^i(a,z)$.

11. Go back to stage 5 until changes in $a^i(a,z)$ in stage 9 are small enough for all points in $\mathcal{X}$; all the above necessary conditions should be satisfied within a small error.

12. Compute the borrowing limit for the current period by solving $v^i(\zeta,z) = \underline{v}^i(z)$ with linear interpolation and pick the tightest of these limits, $\underline{a}^i$.

13. Given the policy function $a^i(a,z)$, compute next period's measure $(\theta^i)'$ defined in set $\tilde{\mathcal{X}}$ using linear interpolation to extend the policy function to the denser space. In particular, next period's density in state $(\tilde{a},z) \in \tilde{\mathcal{X}}$ is computed taking into account the distance of next period's asset holdings to the two points of the denser grid. To that effect, define the value in the denser grid of assets $\tilde{\mathcal{A}}$ immediately above the policy function $a^i(\tilde{a},z)$ at a generic point as $a^u(\tilde{a},z) = \max\{\tilde{a} \in \tilde{\mathcal{A}} : a^i(\tilde{a},z) \leq \tilde{a}\}$ and similarly for the value immediately below, $a^l(\tilde{a},z) = \min\{\tilde{a} \in \tilde{\mathcal{A}} : a^i(\tilde{a},z) \geq \tilde{a}\}$. Further define the weights associated to these two points as

$$w^l(\tilde{a},z) = \begin{cases} \frac{a^u(\tilde{a},z)-a(\tilde{a},z)}{a^u(\tilde{a},z)-a^l(\tilde{a},z)} & \text{if } a^u(\tilde{a},z) \neq a^l(\tilde{a},z) \\ \frac{1}{2} & \text{otherwise} \end{cases}$$

and $w^u(\tilde{a},z) = 1 - w^l(\tilde{a},z)$.

The next period measure can be computed from the current period measure with generic density $\theta^i_{(\tilde{a},z)}$ as:

$$\left(\theta^i\right)'_{(\tilde{a}',z')} = \sum_{(\tilde{a},z)\in H^u(\tilde{a}')\times\mathcal{Z}} \Pi(z,z')\theta^i_{(\tilde{a},z)} w^u(\tilde{a},z) + \sum_{(\tilde{a},z)\in H^l(\tilde{a}')\times\mathcal{Z}} \Pi(z,z')\theta^i_{(\tilde{a},z)} w^l(\tilde{a},z)$$

where $H^u(\tilde{a}') = \{(\tilde{a},z) \in \tilde{\mathcal{X}} : a^u(\tilde{a},z) = \tilde{a}'\}$ and $H^l(\tilde{a}') = \{(\tilde{a},z) \in \tilde{\mathcal{X}} : a^l(\tilde{a},z) = \tilde{a}'\}$.

14. Compute the desired level of assets in the next period and the supply of labor in the current period given by expressions (10) and (11), whose discrete counterparts are:

$$K^i_s = \sum_{\tilde{\mathcal{X}}} a^i(\tilde{a},z)\theta^i_{(\tilde{a},z)}$$
$$N^i_s = \sum_{\tilde{\mathcal{X}}} n^i(\tilde{a},z)\theta^i_{(\tilde{a},z)}\,.$$

15. If $K^i_{\text{next}} + B - K^i_s$ and[26] $N^i - N^i_s$ are both small enough in absolute terms, one should have a solution for the problem and stop here.

16. Otherwise, set K^{i+1}_{next} to a number between K^i_s and K^i_{next}, and similarly for N^{i+1}. Set θ^{i+1} to $(\theta^i)'$, $a^{i+1}(a,z)$ to $a^i(a,z)$, $c^{i+1}_{\text{next}}(a,z)$ to $c^i(a,z)$, $\underline{a}^{i+1}_{\text{next}}$ to $\underline{a}^i$, $\underline{v}^{i+1}_{\text{next}}(z)$ to $\underline{v}^i(z)$ and $v^{i+1}_{\text{next}}(a,z)$ to $v^i(a,z)$. Increment i by one and go back to stage 2.

[26]Notice that we are solving for a constant, exogenous level of government debt, B.

Regarding the transition, we point out the following. The transition exercise consists of calculating the evolution of the economy starting with a certain distribution of assets and shocks θ^{Init} that is potentially different from the steady-state distribution of the economy and a certain path of a few exogenous variables, like for example government spending.

Set the simulation horizon, T, to a large number, say 600 periods. Instead of guesses for aggregate capital, labor supply, the policy and value functions, the autarky value, the joint distribution of assets and shocks, and the borrowing limit, we need to have a first guess for the entire path of those quantities. In practical terms, a good first guess for the paths of these quantities is, for all T periods, their values at the final steady state.

We then have to proceed in the following way. Identify the iteration label i with the time period $t+1$ and the results for the next iteration, denoted by $i+1$, with time t. We start in moment t equal to $T-1$, so that i is 0. Run the procedure above using the same computational routines as for the steady state. Then, update t to $T-2$ and repeat this cycle until t is 1. The next part of the problem is to update the distributions of assets and shocks given the policy functions just calculated. Use the distribution λ^{Init} and the policy functions to update the entire path of the joint distribution.

At this stage it is also necessary to compute updates for the fiscal variables. To do that, one has to compute the path of the fiscal variables implicit in a known fiscal policy. One such policy is described by the fiscal rule (9) together with equations (7) and (8) and particular assumptions about the paths of fiscal variables. One possibility is to fix the path of G exogenously while keeping the tax rates on capital income and labor income constant. This implies that, for a sufficiently high parameter ϕ, government debt B_t and lump sum taxes Γ_t follow a deterministic path that converges to a certain steady-state level.

For each of the T periods compare the computed aggregate capital with the guess, and likewise for aggregate labor, following the general idea in stage 15. Update the paths of these two quantities. Repeat the entire procedure until the differences between the computed aggregate capital and aggregate labor and their guesses are sufficiently small in all periods.

The level of debt in each period must be consistent with the exogenous path for the policy under analysis (for example, an exogenous path for government spending or an exogenous path for transfers), given the fiscal rule.

B The evolution of the household borrowing constraint

Figures 16 and 17 show the evolution of the household borrowing limit under the transfers and the purchases policies, respectively.

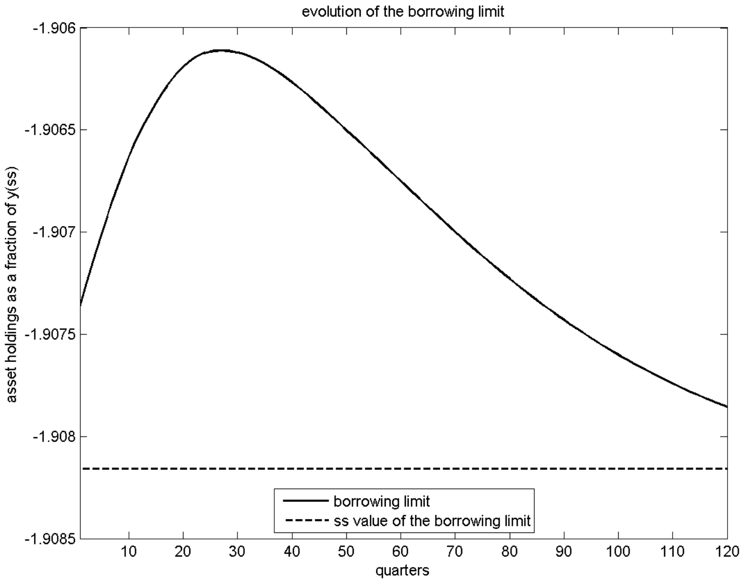

Figure 16: Evolution of the household borrowing limit under the transfers policy.

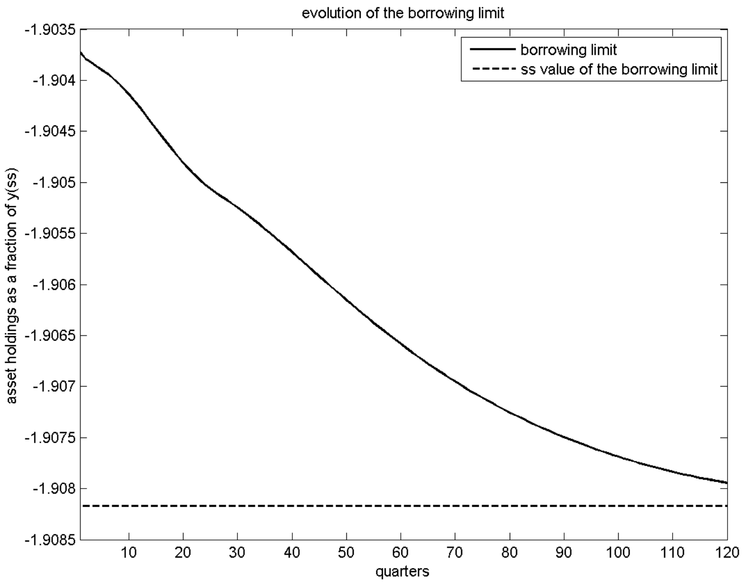

Figure 17: Evolution of the household borrowing limit under the purchases policy.

C Results under fixed (factor) prices

Figures 18 and 19 present the effects of the two fiscal policies on the borrowing limit, within a version of the model in which factor prices are fixed. Crucially, as explained in the main text, $v(x', \theta')$ moves less relative to what happens in the exercises based on the flexible-prices version of the model. Hence, under both policies, the borrowing limits become looser over time prior to coming back to the its steady-state level.

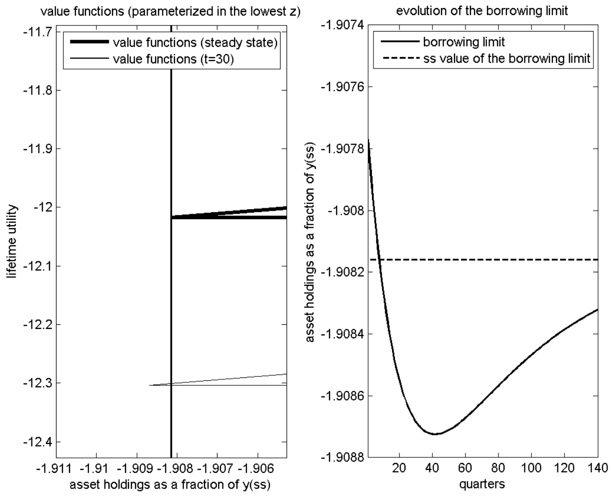

Figure 18: Effects of the transfers policy on the borrowing constraint, i.e., on $\underline{a}(\theta')$, under fixed prices. In the left panel, the two flat lines correspond to the value functions in autarky, $\underline{v}(z', \theta')$, both in the steady state (thick lines) and in the second period of the transition (thin lines). Similarly, the other two lines refer to the equilibrium value functions, $v(x', \theta')$. The value functions are parameterized in the lowest z. The steady-state borrowing limit is identified by the vertical line. y(ss) stands for steady-state output. In the right panel, the solid line corresponds to the evolution of the borrowing limit over time, while the dashed line identifies the constraint at its steady-state level.

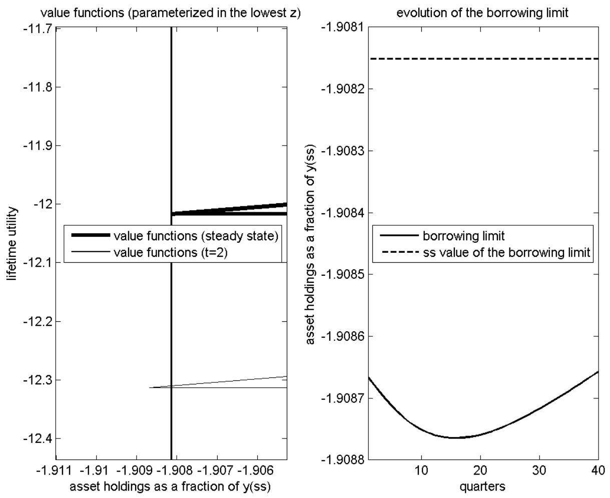

Figure 19: Effects of the purchases policy on the borrowing constraint, i.e., on $\underline{a}(\theta')$, under fixed prices. In the left panel, the two flat lines correspond to the value functions in autarky, $\underline{v}(z', \theta')$, both in the steady state (thick lines) and in the second period of the transition (thin lines). Similarly, the other two lines refer to the equilibrium value functions, $v(x', \theta')$. The value functions are parameterized in the lowest z. The steady-state borrowing limit is identified by the vertical line. y(ss) stands for steady-state output. In the right panel, the solid line corresponds to the evolution of the borrowing limit over time, while the dashed line identifies the constraint at its steady-state level.

D Results with an alternative borrowing constraint

Figure 20 presents the reaction of the household borrowing constraint to the debt-financed transfers policy. The used borrowing limit is a modified version of the *natural borrowing limit*. As explained in the main text, this borrowing limit initially loosens and then persistently tightens over time before to come back to its steady state.

Figure 21 show selected aggregate reactions to the transfers policy using a modified version of the *natural borrowing limit*.

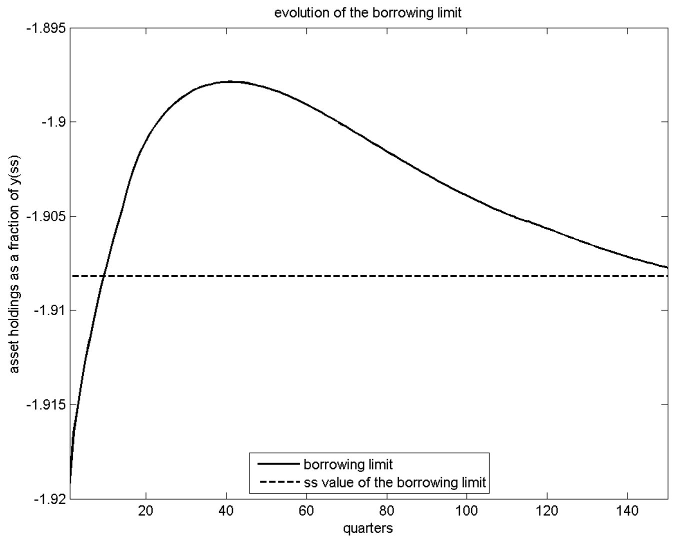

Figure 20: Evolution of an alternative borrowing limit, i.e., $-\eta * \left(\frac{w}{r}\right)$, under the transfers policy.

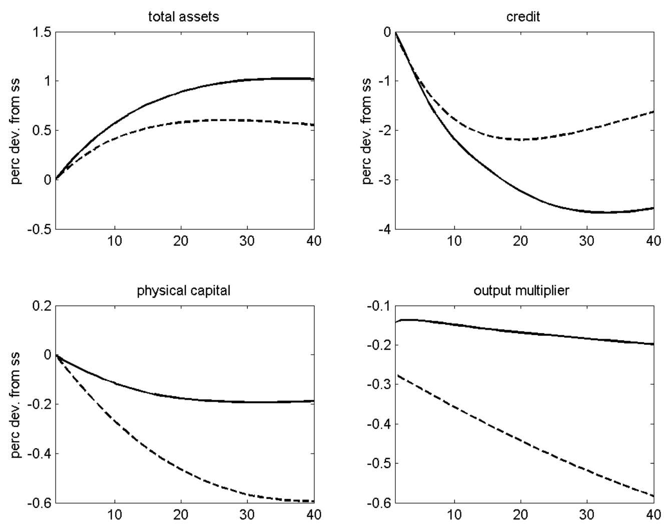

Figure 21: Aggregate effects of the transfers policy under the alternative borrowing limit.
Solid lines are generated with the model that uses the alternative borrowing limit, i.e., $-\eta * \left(\frac{w}{r}\right)$; dashed lines are generated with the fixed-constraint model. 'Total assets' is the sum of (claims of) physical capital and public debt. The cumulative dynamic multipliers are calculated following Uhlig (2010). The x-axis is in quarters.

E Results with a positive level of steady-state public debt

Figure 22 presents the reaction of the household borrowing constraint to the transfers policy, conditional on a steady-state level of public debt-to-output ratio of 60%. The tightening manifests itself even in this framework.

Figure 23 show selected aggregate reactions to the debt-financed transfers policy, conditional on a steady-state level of public debt-to-output ratio of 60%.

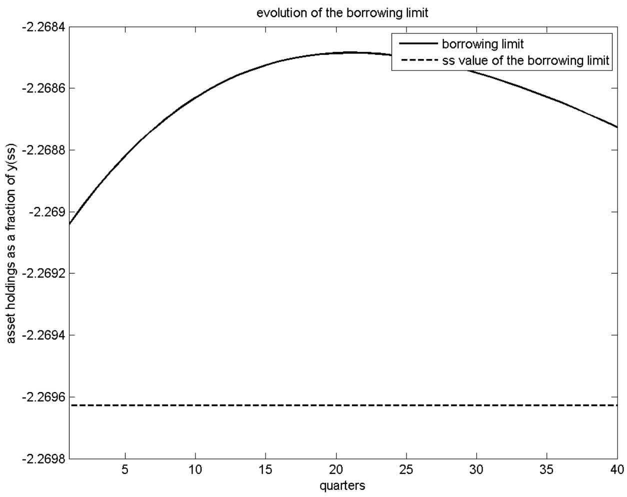

Figure 22: Evolution of an alternative borrowing limit, conditional on a steady-state level of public debt-to-output ratio of 60%.

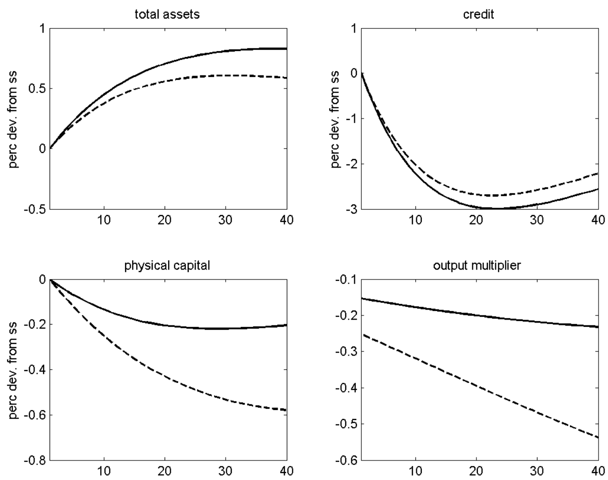

Figure 23: Aggregate effects of the transfers policy, conditional on a steady-state level of public debt-to-output ratio of 60%.
Solid lines are generated with the baseline model and dashed lines with the fixed-constraint model. 'Total assets' is the sum of (claims of) physical capital and public debt. The cumulative dynamic multipliers are calculated following Uhlig (2010). The x-axis is in quarters.

F Results with a model mimicking price stickiness

Figure 24 presents the reaction of the household borrowing constraint to the transfers policy, using the model with contercylcical markups which replicates the effects of price stickiness as described in the main text. The tightening manifests itself even in this framework.

Figure 25 show selected aggregate reactions to the debt-financed transfers policy, using the model with contercylcical markups.

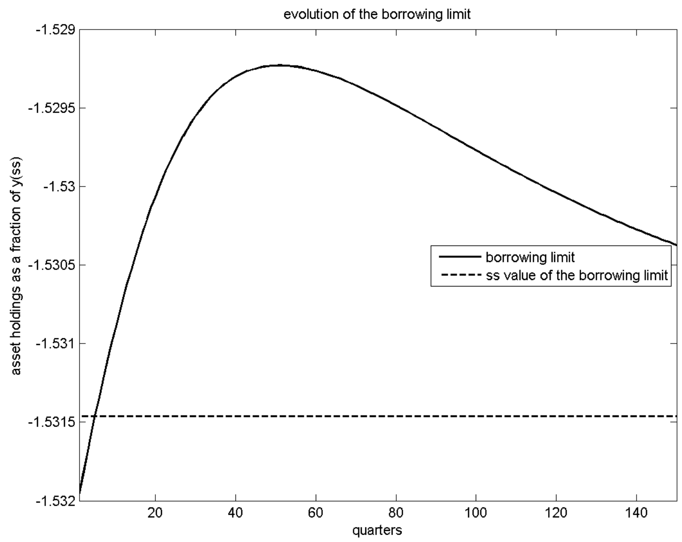

Figure 24: Evolution of the husehold borrowing limit, using the model with countercylcical markups.

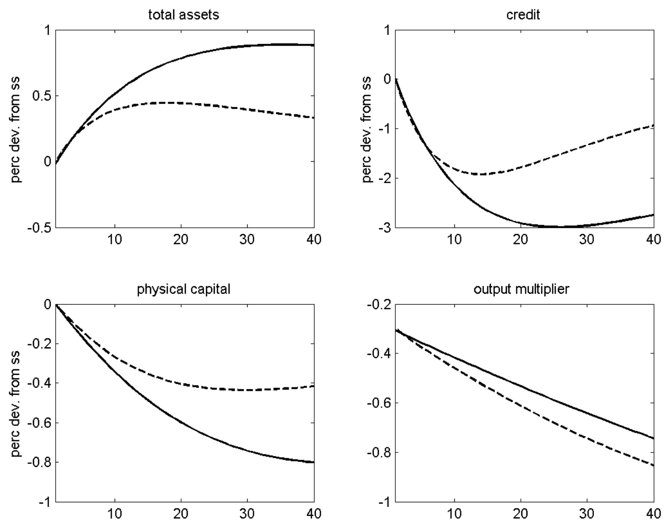

Figure 25: Aggregate effects of the transfers policy, using the model with countercylcical markups. Solid lines are generated with the baseline model and dashed lines with the fixed-constraint model. 'Total assets' is the sum of (claims of) physical capital and public debt. The cumulative dynamic multipliers are calculated following Uhlig (2010). The x-axis is in quarters.