BEHAVIOR-BASED PRICE DISCRIMINATION WITH DOUBLY HETEROGENOUS CONSUMERS

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Abstract

We develop a model of behavior-based discriminatory pricing where consumers are heterogeneous both in tastes and in price sensitivity. Each firm is able to distinguish between the consumers that have bought from it and those that have bought from the rival. Furthermore, each firm learns the price sensitivity of the own consumers, but not that of the rival's. We show that using this additional information yields higher profits with respect to the case where this information is not available and firms can only discriminate between own and rival's consumers, and it may yield higher profits than uniform pricing provided that consumers are heterogeneous enough with respect to price sensitivity. We also discuss the pricing policy equilibrium in a game where firms choose in advance which type of pricing policy to adopt; two types of equilibria are possible: either each firm chooses uniform pricing, or both choose behavior-based discriminatory pricing with additional information about price sensitivity.

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1. Introduction

Behavior-based price discrimination (BBPD hereafter) is a widely observed practice in many markets. BBPD is a peculiar form of price discrimination: in a nutshell, the consumers are charged with different prices depending on their purchase histories. Therefore, in the case of repeated purchases, the current price paid by a consumer may depend on which firm he has patronized before. Among the many examples of industries where BBPD is now commonly adopted, supermarkets, web-retailers, air travel, financial services, telecommunication, electricity, gas and insurance companies stand out for a wide use of BBPD techniques.[1]

The literature on BBPD has increased in the last years, following the real world trend of increasingly use of BBPD techniques by the firms. A widely discussed question is whether adopting price discrimination techniques is *profitable* or not for firms. In a context of "standard" price discrimination,[2] higher flexibility in setting prices is usually associated with higher profits in monopoly or when the rival sets a uniform price, but it is associated with lower profits when all firms are endowed with the same ability in setting flexible prices (Thisse and Vives, 1988). Results are more ambiguous when BBPD is considered. Villas-Boas (1999 and 2004) considers an infinite-horizon duopoly market with overlapping generations of consumers and demonstrates that BBPD hurts the firms when the consumers are patient. Fudenberg and Tirole (2000), in a two-period model with no switching costs, show that BBPD hurts the firms in the second period, but benefits them in the first period: the overall

[1] Common examples of firms operating in various industries that price discriminate across consumers on the basis of their purchase histories are for example Sky, Amazon, Barners and Nobles, Digg, eBay, iTunes, Netflix, YouTube, Air Canada, Societè Nanceienne Varin-Bernier Bank, Scandinavian Airlines and Le Monde, to name a few (see also Pazgal and Soberman, 2008, Mahmood, 2014, Esteves, 2014 and 2015 for other examples).

[2] By "standard" price discrimination we refer to the ability of the firms to directly identify the characteristics of the consumers, without basing their knowledge on the past behavior of the consumers.

impact of BBPD on the profits is negative. Shaffer and Zhang (2000) consider two firms with asymmetric market shares and show that BBPD may benefit one or both firms depending on the degree of asymmetry in the industry. Pazgal and Soberman (2008) consider the profitability of BBPD when the firms may add value in the second period and find that the profits under BBPD are lower than under uniform pricing.

Up to now, the contributions about BBPD are characterized by the fact that when engaging in BBPD each firm has no additional information about the own consumers with respect to the rival.[3] At the beginning of the story the firms have no information about the consumers, but after the first purchase of consumers, each firm is able to recognize whether a consumer has bought from it or from the rival.[4] Therefore, no firm has more information than the rival, because each firm knows the initial choice of the consumer, and this is the *only* information that firms are able to retain after the first-period purchase by the consumers (see for example Fudenberg and Tirole, 2000, Pazgal and Soberman, 2008, Jeong and Maruyama, 2009, Esteves and Reggiani, 2014, among the others). However, firms are often able to retain additional information about a consumer that has bought from it. For example, a firm can collect consumer's personal and demographic information through user's account; it can conduct post-purchase satisfaction surveys; or it can infer the consumer's preferences by analysing his behavior during the search period (see Hauser et al., 2009, and Reinecke and Bernstein, 2013, for further examples of firms' strategies to retain information about own consumers). If a firm is able to retain additional

[3] The only exceptions we are aware of are Zhang (2011) and Shin and Sudhir (2010) in the marketing literature. The former considers a situation where there is no competition in the second period of consumers' purchase (each firm is a monopolist in its own turf) and the consumers are heterogeneous only with respect to one characteristic. The latter consider customer heterogeneity in purchase quantities and allow for changing preferences across periods, which are excluded from the present analysis.

[4] For updated surveys on BBPD models, refer to Fundenberg and Villas-Boas (2007) and Esteves (2009).

information about its own consumers, an information gap is generated between the firms with regard to each group of consumers.

A crucial aspect for marketing practitioners when defining the marketing mix is to know the consumer's price sensitivity (see for example Miller et al. 2011, Loffler, 2015).[5] In this respect, BBPD represents a great opportunity to learn about the price sensitivity of the own consumers, and hence to properly design targeted prices. Consider for example the following situation. Mr X and Mr Y have the same taste toward the good produced by Firm *A*, but they have different price sensitivity. In particular, suppose that Mr X is richer than Mr Y. Consequently, Mr *X* is likely to be less sensitive to price variations than Mr *Y*.[6] After the initial purchase of the consumers, Firm *A* may get two pieces of information: *i*) it knows that Mr *X* and Mr *Y* have bought from it and not from a rival, and *ii*), it can infer the price sensitivity of their consumers through data analytics (hard for competitor to observe). Whereas the current literature on BBPD has commonly focused on the first piece of information and has disregarded the second one, it is a matter of fact that firms devote a lot of effort not only to keep track of own consumers (first piece of information), but also to learn about the characteristics of the own consumers (second piece of information).[7] Indeed, Liu et al. (2009) and Loffler, (2015) document that firms make frequently use of information about own consumers to learn their price sensitivity. Furthermore, firms invest in customer relationship management (CRM) with the aim to obtain an information advantage with respect to rivals about existing customers (see for example Lewis, 2005).

[5] With "price sensitivity" we mean the extent to which a variation of the price of the good affects the utility of a consumer buying that good (see for example Erdem et al., 2002).

[6] As shown by Tirole (1988), a high level of income is typically associated with lower price sensitivity.

[7] A rather classical example is Amazon. When a consumer buys on Amazon.com, he gives to Amazon two relevant pieces of information: *i*) Amazon learns that the consumer bought from it, and *ii*) it learns (some of) the consumers' characteristics. In contrast, a rival of Amazon only learns the first piece of information, i.e., that the consumer has bought from Amazon and not from it.

The aim of this article is to extend the analysis of BBPD to include the possibility for firms to retain additional information about the price sensitivity of the own consumers. This allows us to answer the following question. Is this practice profitable for firms when strategic aspects of competition are taken into consideration? Quite surprisingly, we show that the consumers' heterogeneity with respect to price sensitivity is crucial for the profitability of BBPD. Indeed, we show that having additional information increases the profits with respect to the case of BBPD without additional information, but it lowers the profits with respect to uniform pricing if consumers are scarcely heterogeneous in terms of price sensitivity. Therefore, a necessary condition for BBPD with additional information about consumers' price sensitivity (from now onward simply "additional" BBPD) to be profitable is that consumers' heterogeneity is sufficiently high. Indeed, using additional information yields higher profits than classic BBPD as it does not increase first-period competition with respect to classic BBPD as price sensitivity is neutral for first-period choice of consumers, whereas it allows better targeting of prices in the second period. On the other hand, additional BBPD increases the fierceness of competition in the first period with respect to uniform pricing. When consumers' heterogeneity in terms of price sensitivity is low, the latter effect outweighs the better targeting of prices in the second period: in this case, additional BBPD yields lower profits than uniform pricing. In contrast, when consumers' heterogeneity is sufficiently high, the possibility to properly design the second-period prices on the basis of consumers' price sensitivity outweighs the increased competition in the first period: in this case, additional BBPD yields higher profits than uniform pricing.

The second issue we address is which type of pricing policy would emerge in an endogenous choice model where each firm chooses which type of pricing policy to adopt (see for example Pazgal and Soberman, 2008). We show that, in the case of a simultaneous choice of the pricing policy, multiple equilibria arise: one equilibrium consists in both firms choosing uniform pricing, whereas in the other equilibrium both firms choose additional BBPD. In the case of sequential choice of the pricing policy, there is a unique pricing policy equilibrium, either additional BBPD or uniform pricing: additional BBPD (resp. uniform pricing) arises under the same conditions under which it yields higher profits than under uniform pricing (resp. additional BBPD).

Finally, we discuss the implications of additional information about consumers' price sensitivity on consumer surplus. In particular, we find that the impact of BBPD depends on the consumers' type. Indeed, additional BBPD always increases the surplus of high-sensitivity consumers with respect to both classic BBPD and uniform pricing. In contrast, it always decreases the surplus of low-sensitivity consumers with respect to classic BBPD, whereas it may increase or decrease the surplus with respect to uniform pricing.

Our results have both managerial and policy implications. First, we find that the profits are higher with discrimination in both history and price sensitivity than in case with discrimination just in history (Proposition 1). In addition, we find that discriminating in both history and price sensitivity may make BBPD profitable when it would otherwise not be (Proposition 2). Moreover, we show that a situation where all firms adopt BBPD with additional information may emerge endogenously in a more realistic game where the firms choose whether or not to adopt BBPD with or without additional information before setting the prices (Proposition 4). Furthermore, from a policy perspective, we show that allowing firms to discriminate within the set of the own consumers helps those consumers with high price sensitivity, but it damages low-sensitivity consumers: for the latter, the best situation consists in price discrimination between own and rival's consumers (Proposition 6). Therefore, the optimal policy should take into account the different impact of additional BBPD depending on the consumers' characteristics.

This article also complements other papers where the role of the information available to firms is discussed. Chen et al. (2001) and Esteves (2014) consider a situation where a firm receives a noisy signal about the true preferences of the consumers. In these models, better information translates into a less noisy signal. Liu and Serfes (2004) and Colombo (2011) classify the consumers into different groups on the basis of their preferences: firms are able to recognize to which group a consumer belongs, but they are not able to distinguish between the consumers belonging to the same group. In this case, better information means a finer partition of consumers. Colombo (2016) considers a BBPD model where the information that the firms are able to collect about consumers' past purchase may be incomplete, i.e., some consumers may be not classified into own consumers or rival's consumers: hence, greater information means a wider scope of the information gathered by firms. In contrast, in the present model, better information means that the firms, after the consumers' initial purchase, are able to identify their own consumers *and* to recognize their price sensitivity. Hence, with respect to Chen et al. (2001) and Esteves (2014), in our model better information does not entail a less noisy signal about consumers' characteristics, but, rather, it means the possibility for the firms to collect one additional piece of information (consumer's price sensitivity). On the other hand, differently from Liu and Serfes (2004) and Colombo (2011), we do not consider higher or lower precision of consumer's segmentation with respect to a certain characteristic. In other words, while the above mentioned papers consider to what extent firms know about one specific consumers' characteristic, we consider how many pieces of information a firm has about consumers (where they bought, and, within the set of the own consumers, which is their price sensitivity). Furthermore, neither Chen et al. (2001) and Esteves (2014), nor Liu and Serfes (2004) and Colombo (2011), consider a dynamic model of competition, as they assume a one-period game. By contrast, we consider here a two-period model where the composition of the first-period turf is determined by the first-period choice of firms and in turn it influences the competition in the second period. Finally, Colombo (2016) considers a two-period BBPD model as the present one, but in that paper the consumers are heterogeneous only with respect to one attribute.[8]

This article proceeds as follows. In Section 2 we introduce a model of BBPD where the consumers are differentiated both in terms of tastes and in terms of price sensitivity. In Section 3 we find the equilibrium profits in the various cases and we compare them. In Section 4 we extend the model to the case of asymmetric pricing policies, and then we derive endogenously the pricing policy equilibria. Section 5 discusses consumer surplus and welfare implications. Section 6 concludes. The Appendix contains the proofs of the propositions.

2. The model

We consider a product characteristic space represented by a segment of length one. There are two firms, Firm *A* and Firm *B*, competing in the market. Firm *A* is located at the left endpoint of the segment, whereas Firm *B* is located at the right endpoint. The firms produce at constant marginal production costs, which are normalized to zero. Fixed costs are disregarded. Consumers are

[8] Another couple of papers should be mentioned. Shin et al. (2012) consider a monopolist firm which, after the initial purchase of customers, is able to recognize which of them are more costly to serve, and it might deliberately decide not to serve them in the subsequent periods (buy setting an excessive price on them). Subramanian et al. (2014) extend Shin et al. (2012) to a duopoly context, and find that high-cost customers may be strategically valuable, as long as they discourage poaching by the rivals. Both papers consider customer heterogeneity and BBPD, by focusing on the cost of serving customers rather then their price sensitivity.

uniformly distributed along the segment. We assume density one, and we denote by $x \in [0,1]$ the location of each consumer in the product space. Therefore, *x* indicates the preferred variety of a given good for a consumer located at *x*: the more distant from *x* is the variety produced, the lower is the utility that the consumer derives from consuming the good. Furthermore, we depart from the literature by assuming that at any location *x* there are two types of consumers, consumers with a high price sensitivity (*H*), and consumers with a low price sensitivity (*L*). In particular, at each point of the product characteristic space there is a fraction $\lambda \in [0,1]$ of type-*H* consumers, and a fraction $1-\lambda$ of type-*L* consumers.

There are two periods: period 1 and period 2. In each period, each consumer can buy either one unit of the variety sold by Firm *A* or one unit of the variety sold by Firm *B*, but not both. The parameters of the utility function of the consumers (see later) are such that, in equilibrium, all consumers buy one unit of the good in each period. We denote the first-period price by q^I, where $I = \{A, B\}$. Let us denote by $\mathbf{q}$ the vector of the first-period prices, that is: $\mathbf{q} \equiv \{q^A, q^B\}$. Furthermore, let us denote by M^I the set of consumers that in period 1 buy from Firm $I = \{A, B\}$. We shall refer to M^I also as the "turf" of Firm *I*.

Let us introduce now the consumers' preferences. When buying from Firm $I = \{A, B\}$ in period $j = \{1, 2\}$, the utility of a type-$Z = \{L, H\}$ consumer whose preferred variety is *x* is given by: $u^I_{j,Z}(x) = v - (1-\xi\gamma)\varphi^I - tT$, with $T \equiv \begin{cases} x & if \quad I = A \\ 1-x & if \quad I = B \end{cases}$ and $\xi \equiv \begin{cases} 1 & if \quad Z = L \\ -1 & if \quad Z = H \end{cases}$. Variable $\varphi^I \in \mathbf{\Phi^I}$ is the relevant price set by the firm. Therefore, the parameter $\gamma \in [0,1]$ measures price sensitivity. For type-*L* (resp. *H*) consumers, the higher is γ, the lower (resp. higher) is the sensitivity.

When $\gamma = 0$ or $\lambda \in \{0,1\}$ there is no heterogeneity of consumers in terms of price sensitivity, and the model collapses to the standard BBPD framework commonly considered in literature (see for example the Fudenberg and Tirole, 2000, seminal paper), as the two groups of consumers are identical or there is just one group of consumers. On the other hand, provided that $\lambda \in (0,1)$, the higher is γ the greater is the consumers' heterogeneity with respect to price sensitivity. The parameter $t > 0$ is a standard horizontal differentiation parameter and indicates the disutility cost from consuming a variety that is different from the preferred one. We suppose that v is large enough to guarantee that each consumer always buys the good in equilibrium. Finally, we require that the heterogeneity of consumers with respect to price sensitivity is not too high. Denote $\gamma_1 \equiv \frac{2(3-3\lambda-\sqrt{1+3\lambda})}{8-21\lambda+9\lambda^2}$. We assume that:

<u>Assumption 1</u>: $\gamma \leq \gamma_1$.

After the consumers' purchase in the first period, two turfs are generated. Since in period 1 the prices are uniform and the disutility costs are linear, for each consumer type there is a unique indifferent consumer in period 1 that determines $M^A(\mathbf{q})$ and $M^B(\mathbf{q})$. Denoting the indifferent consumer with $k_Z(\mathbf{q})$, $Z = \{L, H\}$, and provided that $k_Z(\mathbf{q}) \in [0,1]$,[9] we have that: $M^A(\mathbf{q}) = \{x : x \in [0, k_H(\mathbf{q})] \cup x \in [0, k_L(\mathbf{q})]\}$ and $M^B(\mathbf{q}) = \{x : x \in [k_H(\mathbf{q}), 1] \cup x \in [k_L(\mathbf{q}), 1]\}$.

[9] It is easy to show (see later) that, at the equilibrium prices, the condition $k_Z(\mathbf{q}) \in [0,1]$, $Z = \{L, H\}$, is always satisfied.

Suppose that both firms have access to a technology, available to the firms at zero costs, that provides the firms with information about consumers.[10] In particular, this technology may be of two different types:

T1) the technology allows the firm to keep track of the consumers that have bought from it. The firm has no other additional information about the characteristics of the own consumers. We refer to this kind of price discrimination as "classic" BBPD.

T2) the technology allows the firm to keep track of the consumers that have bought from it. Moreover, among the consumers that belong to the own turf, the firm is able to distinguish between type-*L* consumers and type-*H* consumers. We refer to this kind of price discrimination as "additional" BBPD.[11]

Therefore, in the second period, each firm distinguishes between two groups or three groups of consumers, depending on the type of information adopted. Consider first the case of T1, or classic BBPD. In the second period, each firm can set different prices for the own consumers and for the rival's consumers. Let us denote the former by $\hat{p}^I$ and the latter by $\tilde{p}^I$, with $I = \{A, B\}$.[12] Next, consider the case of T2, or additional BBPD. Under this type of information, each firm is able to distinguish, within its own turf, between type-*L* and type-*H* consumers. Let us indicate with $\hat{p}_L^I$ and $\hat{p}_H^I$ the price set by Firm $I = \{A, B\}$ on own type-*L* and type-*H* consumers, respectively, while the price set on the consumers belonging to the rival's turf is still indicated by $\tilde{p}^I$.[13]

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[10] A firm may retain information about the own consumers by using internal resources as internal data collection or by purchasing data from specialised sellers as DoubleClick, 24/7 Real Media, Modern Media and I-Behavior.

[11] We assume that the two types of information technologies have the same cost, which is normalized to zero, in order to avoid trivial effects connected to the cost.

[12] The possibility of arbitrage between the consumers is excluded.

[13] We assume that each firm cannot learn about the price sensitivity of the rival's consumers. In theory, a firm may have some chances to learn about the price sensitivity of the rival's consumers when information comes from external providers. In this article we follow Chen and Iyer (2002) and Deighton et al. (1994), that argue that it is much more easier for a firm to keep information about own consumers than rival's consumers.

To sum up, the set of possible prices of Firm $I=\{A,B\}$, depending on the period of competition and on the type of BBPD considered, is the following: if T1, then $\mathbf{\Phi^I}=\{q^I,\hat{p}^I,\tilde{p}^I\}$, whereas if T2, then $\mathbf{\Phi^I}=\{q^I,\tilde{p}^I,\hat{p}_L^I,\hat{p}_H^I\}$.[14] Finally, let us assume that the consumers and the firms evaluate the future with the same discount factor, which is equal to $\delta\in[0,1]$.

3. Prices and profits

In this section we derive the equilibrium prices and profits under the different types of BBPD, and we compare them.

3.1. Classic behavior-based price discrimination

We consider first the last stage of the game. In period 2, each firm is able to distinguish whether a consumer has bought from it or from the rival, but it has no other additional information about the consumer's type (*H* or *L*). Hence, it can set two different prices: one for the own consumers, and one for the rival's consumers. Consider a generic indifferent consumer in period 2. It is obtained by $u_x^A(\theta^A)=u_x^B(\theta^B)$, where $\theta^I\in\{\hat{p}^I,\tilde{p}^I\}$ depends on which turf is considered. Therefore, a generic indifferent consumer is given by:

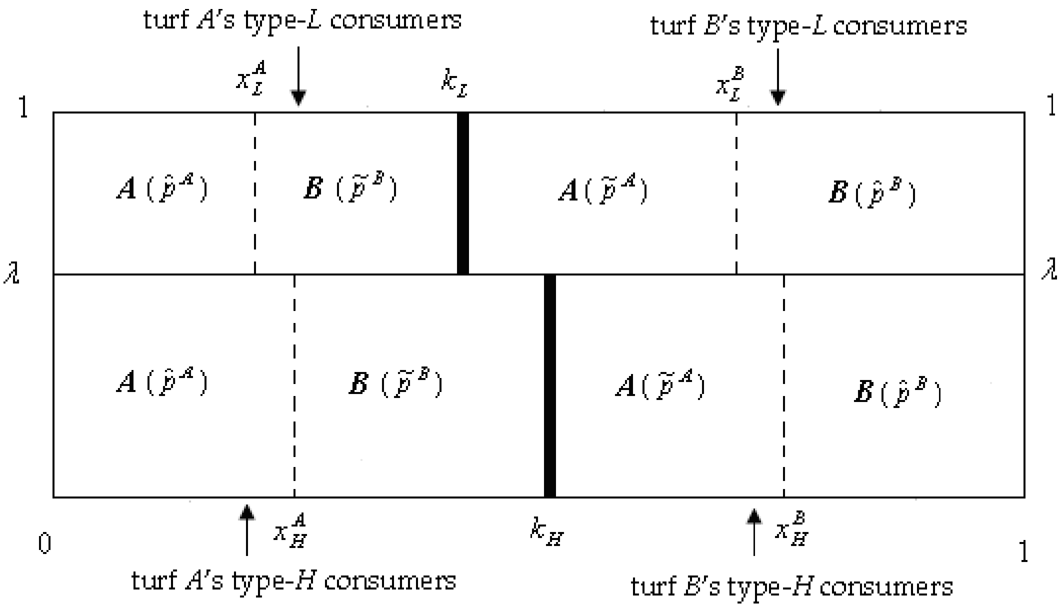
$$y=\frac{(\theta^B-\theta^A)(1-\xi\gamma)}{2t}+\frac{1}{2}. \tag{1}$$

Using (1), the four relevant indifferent consumers in period 2 are immediately derived. We indicate the period 2 indifferent consumers in Turf *A* (resp. *B*) by x_Z^A (resp. x_Z^B), with $Z=\{L,H\}$. Assume that, in equilibrium,

[14] Note that in T1, $\hat{p}^I=\tilde{p}^I$ is also possible. Similarly, in T2, $\tilde{p}^I=\hat{p}_L^I=\hat{p}_H^I$ is also possible. That is, a firm cannot commit not to discriminate once it owns the technology.

poaching occurs, that is $x_Z^A \in [0, k_Z]$ and $x_Z^B \in [k_Z, 1]$.[15] Figure 1 illustrates the market sharing between the firms in the second period of competition. In Figure 1 the large bold line separates the turfs of the two firms. The label **A** and **B** indicate whether the consumer is buying from Firm *A* or from Firm *B*, while in the brackets it is indicated the price paid by the consumers. Above the λ-line there are type-*L* consumers. Below the λ-line there are type-*H* consumers.[16]

Figure 1: classic BBPD with doubly heterogeneous consumers



Therefore, the second-period profits of each firm can be written as:

$$\pi_2^{A,cc} = \hat{p}^A[\lambda x_H^A + (1-\lambda)x_L^A] + \tilde{p}^A[\lambda(x_H^B - k_H(\mathbf{q})) + (1-\lambda)(x_L^B - k_L(\mathbf{q}))] \quad (2)$$

$$\pi_2^{B,cc} = \hat{p}^B[\lambda(1-x_H^B) + (1-\lambda)(1-x_L^B)] + \tilde{p}^B[\lambda(k_H(\mathbf{q}) - x_H^A) + (1-\lambda)(k_L(\mathbf{q}) - x_L^A)] \quad (3)$$

[15] In the Appendix is indeed verified that poaching emerges in equilibrium.

[16] Note that Figure 1 as well as the other following figures has been depicted for values of the critical cut-off consumers that may be out of the equilibrium path. Indeed, in equilibrium, given the symmetric nature of the problem, the location of the cut-off consumers is expected to be symmetric as well.

where the superscript "*cc*" is used here and later to indicate the case where both firms are adopting classic BBPD. Denote $\Gamma \equiv 1-\gamma(1-2\lambda)$. The next Lemma indicates the second-period prices, which are function of the vector of the first-period prices through $k_H(\mathbf{q})$ and $k_L(\mathbf{q})$:

Lemma 1. The optimal second-period prices in the case of classic BBPD are:

$$\hat{p}^{A,cc}(\mathbf{q}) = \frac{t[1+2k_L(\mathbf{q})(1-\lambda)+2\lambda k_H(\mathbf{q})]}{3\Gamma} \tag{4}$$

$$\tilde{p}^{A,cc}(\mathbf{q}) = \frac{t[3-4k_L(\mathbf{q})(1-\lambda)-4\lambda k_H(\mathbf{q})]}{3\Gamma} \tag{5}$$

$$\hat{p}^{B,cc}(\mathbf{q}) = \frac{t[3-2k_L(\mathbf{q})(1-\lambda)-2\lambda k_H(\mathbf{q})]}{3\Gamma} \tag{6}$$

$$\tilde{p}^{B,cc}(\mathbf{q}) = \frac{t[4k_L(\mathbf{q})(1-\lambda)+4\lambda k_H(\mathbf{q})-1]}{3\Gamma} \tag{7}$$

Proof. See the Appendix. ■

For future purposes, note that Γ increases (decreases) with the price sensitivity parameter, γ, if the fraction of high-sensitive consumers, λ, is higher (lower) than 1/2. Therefore, it can be easily checked that, when considering the neighbourhood of $k_L(\mathbf{q}) = k_H(\mathbf{q}) = 1/2$,[17] all prices increase with γ if type-*L* consumers are more than type-*H* consumers (i.e. $\lambda < 1/2$), whereas all prices decrease with γ if type-*L* consumers are less than type-*H* consumers (i.e. $\lambda > 1/2$). Indeed, since the firms cannot distinguish between consumers' types, the higher price sensitivity of type-*H* consumers dominates the lower price sensitivity of type-*L* if there are more type-*H* consumers than type-*L* consumers,

[17] Indeed, the impact may change when considering other values of $k_L(\mathbf{q})$ and $k_H(\mathbf{q})$. However, as the model considered in this section is symmetric, the first-period market segmentation for both types of consumers is expected to be symmetric as well, as indeed occurs in equilibrium.

and vice-versa. On the other hand, a larger turf makes competition milder, thus increasing prices, whereas a smaller turf makes competition fiercer, thus decreasing prices (see also Fudenberg and Tirole, 2000).

Consider the first period of competition. As the second-period profits depend on the first-period prices, each firm sets its price in period 1 in order to maximize the discounted sum of the profits in period 1 and period 2. We denote $D^A(\mathbf{q}) \equiv \lambda k_H(\mathbf{q}) + (1-\lambda)k_L(\mathbf{q})$ and $D^B(\mathbf{q}) \equiv \lambda(1-k_H(\mathbf{q})) + (1-\lambda)(1-k_L(\mathbf{q}))$. Therefore, the overall profits of Firm $I = \{A, B\}$ can be expressed as follows:

$$\Pi^{I,cc} = q^I D^I(\mathbf{q}) + \delta \pi_2^{I,cc} * (\mathbf{q}) \tag{8}$$

where $\pi_2^{I,cc} * (\mathbf{q})$ is obtained by plugging (4)-(7) into (2) and (3). Consider now the first-period consumers' decision. In the case of poaching in period 2, the cut-off consumers in period 1 must be indifferent between buying from Firm A in period 1 and then buying from Firm B in period 2, or buying from Firm B in period 1 and then buying from Firm A in period 2. Therefore, the indifferent type-$Z = \{L, H\}$ consumer in period 1, $k_Z(\mathbf{q})$, is the solution of:

$$(1-\xi\gamma)q^A + tk_Z(\mathbf{q}) + \delta[(1-\xi\gamma)\tilde{p}^{B,cc}(\mathbf{q}) + t(1-k_Z(\mathbf{q}))] = (1-\xi\gamma)q^B + t(1-k_Z(\mathbf{q})) + \delta[(1-\xi\gamma)\tilde{p}^{A,cc}(\mathbf{q}) + tk_Z(\mathbf{q})]] \tag{9}$$

From (9), we get the first-period type-$Z = \{L, H\}$ indifferent consumer:[18]

$$k_Z^{cc}(\mathbf{q}) = \frac{3(1-\xi\gamma)(q^B - q^A)}{2t(3+\delta)} + \frac{1}{2} \tag{10}$$

[18] Note that, as usual in this type of models, the more the consumers are forward-looking, the lower is the first-period demand elasticity. See Fudenberg and Tirole (2000) for an explanation.

Having derived an explicit expression for the demand in period 1, we use it into (8). A straightforward maximization of the profits with respect to q^A and q^B, yields the following equilibrium first-period prices:

$$q^{I,cc}* = \frac{t(3+\delta)}{3\Gamma} \tag{11}$$

As for the second-period prices, the first-period prices increases with γ if the fraction of high-sensitive consumers, λ, is lower than 1/2, and vice-versa. Having derived the first-period equilibrium prices, the equilibrium profits follow:[19]

$$\Pi^{I,cc}* = \frac{t(9+8\delta)}{18\Gamma} \tag{12}$$

It is immediate to observe that when $\gamma = 0$ or when $\lambda = \{0,1\}$ we recover the original model (see Fudenberg and Tirole, 2000), as we have not additional consumer heterogeneity. Not surprisingly, when the average price sensitivity goes down (i.e. λ decreases, or γ increases with $\lambda < 1/2$), the profits under classic BBPD increase. As we already showed, in this case both the second-period and the first-period prices increase. Indeed, the firms are able to extract higher profits from the fact that there are more type-*L* consumers (that is, consumers with low price sensitivity). At the opposite, when the average price sensitivity goes up (i.e. λ increases, or γ increases with $\lambda > 1/2$), the profits decrease. Finally, when $\lambda = 1/2$, the profits do not depend on γ: indeed, an

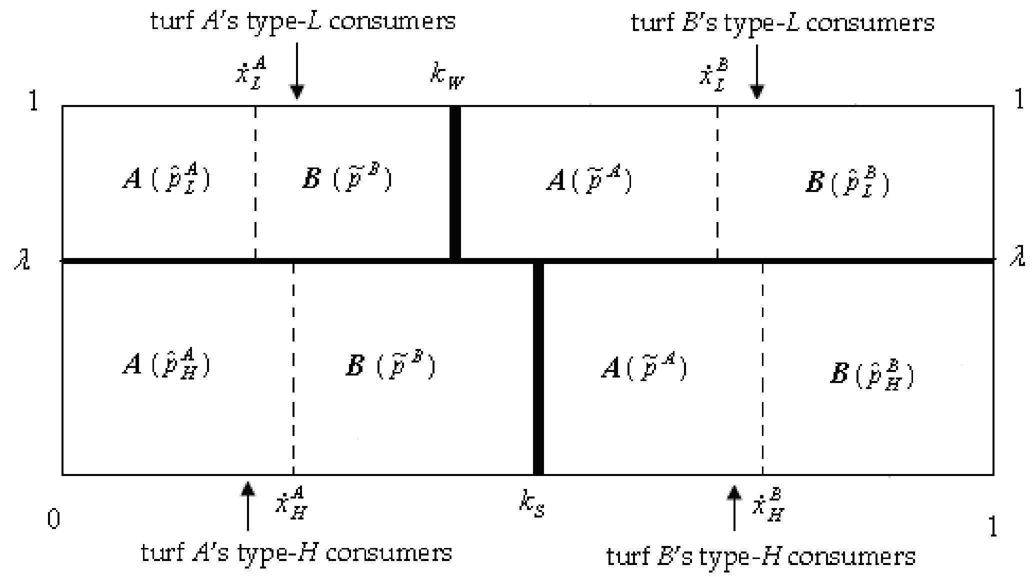
[19] The equilibrium first-period and the second-period equilibrium market shares, as well as the second-period equilibrium prices are reported in the Appendix.

increase of the price sensitivity of type-H consumers is exactly compensated by a decrease of the price sensitivity of type-L consumers.

3.2. Additional behavior-based price discrimination

We consider now the case of additional BBPD, i.e. a situation where each firm is able to get information about the price sensitivity of those consumers that have bought previously from it. Therefore, each firm can group its own consumers into two additional subgroups, based on their price sensitivity. We start from the last stage of the game. In period 2, each firm is able to distinguish whether a consumer belongs to its own turf or to the rival's turf. Moreover, each firm is able to distinguish within its own turf between type-L and type-H consumers. Hence, it can set three different prices: one for the own type-L consumers, one for the own type-H consumers and one for the rival's consumers. The relevant indifferent consumers, indicated by $\dot{x}_Z^I$, $I = \{A, B\}$ and $Z = \{L, H\}$, are still given by (1), where the set of the relevant prices is now given by $\theta^I \in \{\hat{p}_L^I, \hat{p}_H^I, \tilde{p}^I\}$.

Figure 2: additional BBPD with doubly heterogeneous consumers



By assuming poaching, Figure 2 illustrates the market sharing between the firms in the second period of competition. In Figure 2 the large bold line separates the turfs of the two firms. Moreover, the medium bold lines within turf $I=\{A,B\}$ indicate the information at the disposal of Firm *I*. Firm *A* (*B*) is able to distinguish between the type-*L* consumers and the type-*H* consumers within the set of the consumers located at the left (right) of k_Z.Therefore, the second-period profits of each firm can be written as:

$$\pi_2^{A,aa}(\mathbf{q})=\hat{p}_H^A\lambda\dot{x}_H^A+\hat{p}_L^A(1-\lambda)\dot{x}_L^A+\tilde{p}^A[\lambda(\dot{x}_H^B-k_H(\mathbf{q}))+(1-\lambda)(\dot{x}_L^B-k_L(\mathbf{q}))] \tag{13}$$

$$\pi_2^{B,aa}(\mathbf{q})=\hat{p}_H^B\lambda(1-\dot{x}_H^B)+\hat{p}_L^B(1-\lambda)(1-\dot{x}_L^B)+\tilde{p}^B[\lambda(k_H(\mathbf{q})-\dot{x}_H^A)+(1-\lambda)(k_L(\mathbf{q})-\dot{x}_L^A)] \tag{14}$$

where the superscript "*aa*" is used here and later to indicate the case where both firms use additional BBPD. The following Lemma indicates the optimal second-period prices in the case of additional BBPD.

Lemma 2. The optimal second-period prices in the case of additional BBPD are:

$$\hat{p}_L^{A,aa}(\mathbf{q})=\frac{t[2-2\gamma+4\lambda k_H(\mathbf{q})(1-\gamma)+6\lambda\gamma+4k_L(\mathbf{q})(1-\gamma-\lambda+\gamma\lambda)]}{6(1-\gamma)\Gamma} \tag{15}$$

$$\hat{p}_H^{A,aa}(\mathbf{q})=\frac{t[1+2k_L(\mathbf{q})(1+\gamma)(1-\lambda)+2\lambda k_H(\mathbf{q})-\gamma(2-3\lambda-2\lambda k_H(\mathbf{q}))]}{3(1+\gamma)\Gamma} \tag{16}$$

$$\tilde{p}^{A,aa}(\mathbf{q})=\tilde{p}^{A,cc}(\mathbf{q}) \tag{17}$$

$$\hat{p}_L^{B,AA}(\mathbf{q})=\frac{t[3-2k_L(\mathbf{q})(1-\gamma)(1-\lambda)-2\lambda k_H(\mathbf{q})-\gamma(3-3\lambda-2\lambda k_H(\mathbf{q}))]}{3(1-\gamma)\Gamma} \tag{18}$$

$$\hat{p}_H^{B,aa}(\mathbf{q})=\frac{t[3(1-\gamma+2\lambda\gamma)+(1+\gamma)(3-4k_L(\mathbf{q})(1-\lambda)-4\lambda k_H(\mathbf{q}))]}{6(1+\gamma)\Gamma} \tag{19}$$

$$\tilde{p}^{B,aa}(\mathbf{q})=\tilde{p}^{B,cc}(\mathbf{q}) \tag{20}$$

Proof. See the Appendix. ■

By considering the case of symmetric first-period market shares ($k_H(\mathbf{q}) = k_L(\mathbf{q}) = 1/2$), we observe that $\hat{p}_L^{I,aa}$ always increases with the price sensitivity parameter, γ, whereas $\hat{p}_H^{I,aa}$ is U-shape with γ if the fraction of high types, λ, is sufficiently low and strictly decreases if λ is sufficiently high. The intuition is the following. Consider type-*L* consumers. When price sensitivity diminishes, the discriminating firm can set a higher price to them. In contrast, we observe that the second-period price on type-*H* consumers is U-shape in the price sensitivity, provided that the fraction of high types is sufficiently low. The reason is the following. First, note that when the fraction of type-*H* consumers is low enough, the uniform price set by Firm $J \neq I$ in Turf *I* increases with price sensitivity, as there are many type-*L* consumers. Therefore, when price sensitivity increases two effects are at work. On one hand, Firm *I* would like to set a lower price to type-*H* consumers, as they are more sensitive to price. On the other hand, the higher uniform price set by Firm $J \neq I$ induces Firm *I* to set a higher price due to strategic complementarity of prices. For low levels of γ the first effect dominates, whereas for high levels of γ the second effect dominates, thus determining the U-shape impact of γ on $\hat{p}_H^{I,aa}$. On the other hand, the impact of the turf size is similar to the case of classic BBPD: the larger (smaller) is the turf, the higher (lower) is the price charged to the consumers. Not surprisingly, we observe that $\hat{p}_L^{I,aa}(\mathbf{q}) \geq \hat{p}_H^{I,aa}(\mathbf{q})$, $I = \{A, B\}$: the price set to type-*L* consumers is higher than the price set to type-*H* consumers. Importantly, note that $\tilde{p}^{I,aa}(\mathbf{q}) = \tilde{p}^{I,cc}(\mathbf{q})$. Indeed the weighted average of the discriminatory prices is equal to the uniform price in the own turf under classic BBPD, that is:

$\hat{p}^{I,cc} = \dfrac{(1-\gamma)(1-\lambda)\hat{p}_L^{I,aa} + (1+\gamma)\lambda\hat{p}_H^{I,aa}}{\Gamma}$. Consequently, the uniform poaching price is the same, both when the rival adopts classic BBPD and when it adopts additional BBPD.

Consider now the first period of competition. In period 1, the overall profits of Firm $I = \{A, B\}$ can be expressed as follows:

$$\Pi^{I,aa}(\mathbf{q}) = q^I D^I(\mathbf{q}) + \delta\pi_2^{I,aa} * (\mathbf{q}) \tag{21}$$

where $\pi_2^{I,aa} * (\mathbf{q})$ is obtained by plugging (15)-(20) into (13) and (14). Consider now the first-period consumers' decision. Clearly, as $\tilde{p}^{I,aa}(\mathbf{q}) = \tilde{p}^{I,cc}(\mathbf{q})$, it must be that $k_Z^{aa}(\mathbf{q}) = k_Z^{cc}(\mathbf{q})$, $Z = \{L, H\}$. This also implies that $q^{I,aa} * = q^{I,cc} *$. Indeed, the price sensitivity of the consumers does not impact on the first-period market sharing, as in equilibrium the two firms set the same price. In other words, price sensitivity is neutral to the first-period decision of consumers. Since the poaching second-period price is the same under classic BBPD and additional BBPD, the first-period indifferent consumer is also the same under the two pricing regimes. Therefore, by plugging (11) into the profits functions, we have

$$\Pi^{I,aa} * = \frac{t[9 + 8\delta - \gamma^2(9 + \delta(8 - 9\lambda + 9\lambda^2))]}{18(1-\gamma^2)\Gamma} \tag{22}$$

Note that the profits are U-shape with the price sensitivity parameter, γ, if the fraction of high-sensitivity consumers, λ, is higher than 1/2, whereas they strictly increase with γ if $\lambda \le 1/2$. In order to explain this relationship, recall that the first-period price and the second-period price in the rival's turf strictly increase (decrease) with γ if λ is lower (larger) than 1/2. In contrast, the second-period price for type-L consumers increases with γ, whereas the second-period price for type-H consumers is U-shape in γ if λ is low and strictly decreases if λ is high. Therefore, if there are few type-H consumers ($\lambda < 1/2$), the profits strictly increase with γ, following the impact of γ on the first-period price, the second-period poaching price and the second-period price on type-L consumers. In contrast, if there are enough type-H consumers in the population ($\lambda > 1/2$), the impact of γ on the profits initially is negative, following the impact of γ on the second-period price on type-H consumers, but eventually it becomes positive, because the effect on $\hat{p}_L^I$ dominates.

3.3. Comparison

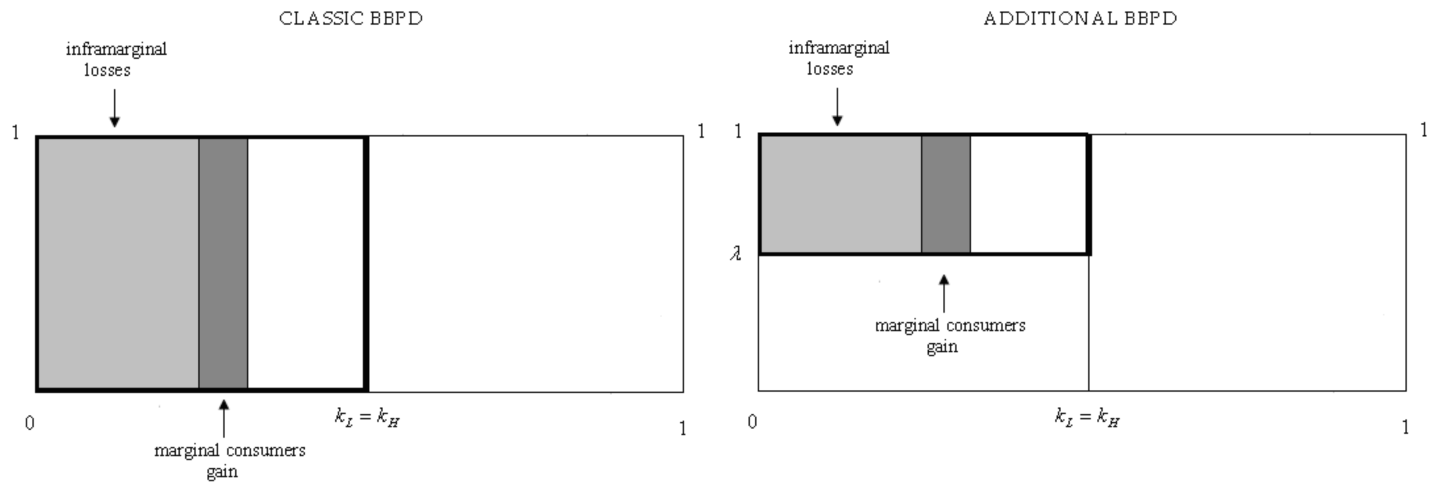
In this section we compare the profits under classic BBPD, additional BBPD, and uniform pricing. First, we can state the following proposition:

Proposition 1. *Additional BBPD yields higher profits than classic BBPD.*

Proof. The proof is given by the comparison of $\Pi^{I,aa}*$ and $\Pi^{I,cc}*$, $I = \{A, B\}$. ■

Proposition 1 shows that having additional information about consumers' price sensitivity allows the firms to get higher profits when they are engaging in BBPD. We already noted that the first-period consumers' decision is not affected by the existence of additional information available to firms in the second period. Indeed, price sensitivity of consumers is neutral to the first-period choice. Therefore, the difference between the profits must stem from the second-period prices. It can be seen that $\hat{p}_L^{I,aa} \geq \hat{p}^{I,cc}$, whereas $\hat{p}_H^{I,aa}$ may be higher or lower than $\hat{p}^{I,cc}$. Therefore, with respect to the case of classic BBPD, the existence of additional information allows the price discriminating firm to set a higher price on low-sensitivity consumers. Furthermore, discriminating on the basis of the price sensitivity of consumers does not increase the fierceness of competition. The reason is the following. We observe that the balance between inframarginal losses and gains of marginal consumers as a consequence of a price reduction in the second period is not altered by the presence of additional information about consumers' price sensitivity (see Figure 3).[20] Therefore, having additional information does not increase competition in the second period. In contrast, it allows the price discriminating firm to better targeting the prices to its consumers, thus obtaining higher second-period profits.

Figure 3: gains and losses from a price decrease under classic BBPD and additional BBPD



In the case of no heterogeneity of consumers with respect to price sensitivity, the literature has emphasized that BBPD decreases the profits with respect to uniform pricing (see for example Pazgal and Soberman, 2008, and Fudenberg and Tirole, 2000, among the others). With double heterogeneity of

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[20] Consider Firm *A*. In Figure 3, for each case, the relevant turf is highlighted by bold lines. Consider additional BBPD, where the box refers to the sub-turf of type-*L* consumers. By comparing with the case of classic BBPD, it can be noted that, given a certain price decrease, *both* the marginal consumers gain and the inframarginal losses are lower. Therefore, additional BBPD does not increase the competition in the second period with respect to the case of classic BBPD.

consumers, it is immediate to see that under uniform pricing the equilibrium price (which is identical in both periods) is equal to $q^{I,uu}* = t/\Gamma$. It follows that the equilibrium profits under uniform pricing are the following:

$$\Pi^{I,uu}* = \frac{t(1+\delta)}{2\Gamma} \tag{23}$$

where the superscript *"uu"* indicates the case where both firms price uniformly. Note that $\Pi^{I,uu}*$ strictly increase (decrease) with the price sensitivity parameter, γ, if the fraction of high-sensitivity consumers, λ, is lower (larger) than 1/2.

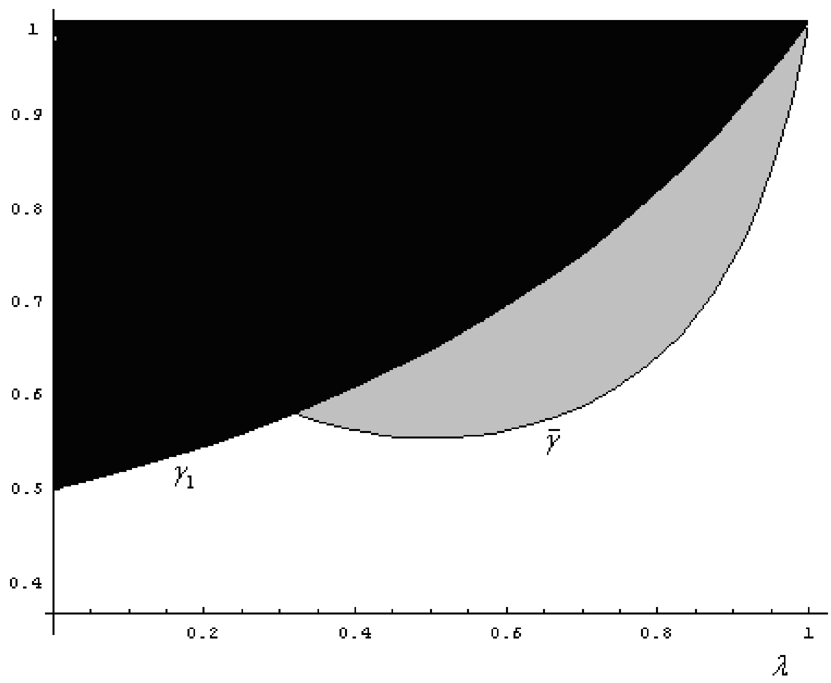
By comparing $\Pi^{I,uu}*$ and $\Pi^{I,cc}*$, it can be observed, not surprisingly, that classic BBPD reduces the profits with respect to uniform pricing even in the case of double heterogeneity of consumers. However, as additional BBPD yields higher profits than classic BBPD (Proposition 1), it is not obvious whether uniform pricing induces higher profits than additional BBPD. Let us denote $\bar{\gamma} \equiv \frac{1}{\sqrt{1+9\lambda(1-\lambda)}}$. We can state the following proposition:

Proposition 2. *Additional BBPD induces higher (lower) profits than uniform pricing if* $\gamma \geq (\leq)\, \bar{\gamma}$.

Proof. The proof is given by the comparison of $\Pi^{I,aa}*$ and $\Pi^{I,uu}*$, $I = \{A, B\}$. ■

Therefore, we have that when consumers are sufficiently heterogeneous in terms of price sensitivity, having additional information about the own consumers' price sensitivity allows the firms to get higher profits with respect to the case of uniform pricing, while the reverse holds when there is scarce heterogeneity. Figure 4 illustrates Proposition 2. The grey area indicates the parameter space where profits are larger under additional BBPD, the white area indicates the parameter space where the profits are larger under uniform pricing, whereas the black area is excluded from Assumption 1.

Figure 4: equilibrium profits under additional BBPD and uniform pricing



The intuition is the following. As indicated in the Introduction, the present model can be seen as a framework of increasing information accuracy, where the lowest level of information accuracy coincides with the case of uniform pricing, the intermediate level coincides with the case of classic BBPD, and the highest level coincides with the case of additional BBPD. As noted in the literature of information accuracy, when information accuracy increases two effects are at works. On one hand, greater information accuracy allows firms to extract more surplus from loyal consumers (i.e. those consumers that have innate preference toward the firm), thus increasing profits. On the other hand, there is also a strategic effect, as more information induces the firms to compete more fiercely, thus reducing profits.[21] In our model, intermediate information accuracy reduces the profits with respect to no-information, whereas maximum information may yield higher profits than no-information. Indeed, when switching from uniform pricing to classic BBPD, the surplus extraction effect in the second period is dominated by the strategic effect. Even if the first-period price is higher under classic BBPD than under uniform pricing, this is not sufficient to compensate for the second-period negative effect.[22] Consequently, profits are lower under classic BBPD than under uniform pricing.[23] However, when switching from classic BBPD to additional BBPD, the strategic effect remains the same,[24] whereas targeting of prices is more accurate, thus implying that the surplus extraction effect is greater. As already noted, this implies that when switching from classic BBPD to additional BBPD, profits increase. Moreover, the targeting of prices is highly efficient when the consumers are strongly differentiated with respect to price sensitivity. Therefore, when heterogeneity of consumers is large enough, the surplus extraction effect is high, and, together with the positive first-period effect, it is sufficient to outweigh to strategic effect and to provide higher profits under additional BBPD.

Figure 5 illustrates the impact of γ on the equilibrium prices under uniform pricing, classic BBPD and additional BBPD for the case where $\lambda > 1/2$.[25,26] First,

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[21] These effects are at work – with some differences – in Chen et al. (2001), Liu and Serfes (2004), Colombo (2011, 2016) and Esteves (2014).

[22] The reason is that the first-period demand elasticity is lower when BBPD is possible, as the consumers take into account that, after a decrease of the price in the first period, they will pay a higher price in the second period (see Fudenberg and Tirole, 2000, p. 641, for details).

[23] See also Fudenberg and Tirole (2000).

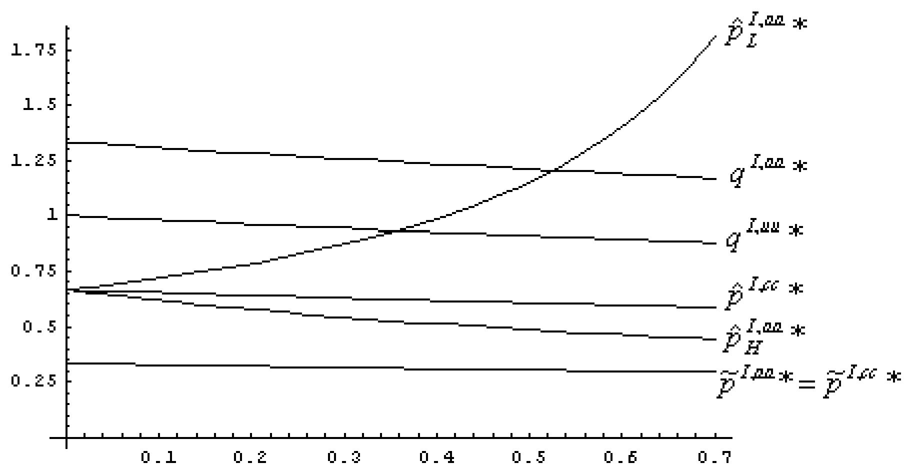
[24] Indeed, the second-period uniform price in the rival's turf is the same.

[25] In Figure 5, we set $\lambda = 3/5$, $\delta = 1$ and $t = 1$. Note that, under this parameter specification, $\gamma \leq 0.7$ by Assumption 1.

[26] The case where $\lambda < 1/2$ can be treated similarly, the only difference being that in this case all prices increases apart the second-period price for high-sensitivity consumers.

by comparing the prices under classic BBPD and additional BBPD, it can be observed that, when γ goes up, the increase of the price set on the own low-sensitivity consumers outweighs the reduction of the price set on the own high-sensitivity consumers. As the poaching price is the same under both pricing regimes, it follows that additional BBPD yields higher profits than classic BBPD (Proposition 1). Compare now additional BBPD and uniform pricing. For low levels of γ, $q^{I,uu}*$ is sufficiently higher than $\hat{p}_L^{I,aa}*$, $\hat{p}_H^{I,aa}*$ and $\tilde{p}^{I,aa}*$. Therefore, uniform pricing yields higher profits than additional BBPD, even if $q^{I,uu}*$ is lower than $q^{I,aa}*$. However, when γ increases, $q^{I,uu}*$ decreases, whereas $\hat{p}_L^{I,aa}*$ starts to increase: it follows that when γ is sufficiently high, the profits under additional BBPD outweigh those under uniform pricing (Proposition 2).[27,28]

Figure 5: the impact of γ on the equilibrium prices under uniform pricing, classic BBPD and additional BBPD



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[27] It is interesting to note that even when $\lambda = 1/2$ additional BBPD may yield higher or lower profits than uniform pricing depending on the value of γ (see Figure 4). This means that heterogeneity with respect to price sensitivity plays a crucial role, as when $\lambda = 1/2$, the average price sensitivity of consumers (i.e. $\lambda(1+\gamma)+(1-\lambda)(1-\gamma)$) is invariant with γ.

[28] Clearly, consumers' heterogeneity can be considered also with respect to the fraction of type-*H* consumers, λ. Not surprisingly, the closer is λ to 1/2 (i.e. the more the two consumers' types are equally distributed), the more likely is that the profits are larger under additional BBPD (see Figure 4), whereas the opposite holds when λ is closer to 0 or 1. Indeed, in the latter case there is scarce heterogeneity of consumers with respect to price sensitivity: almost all consumers are of type-*H* or almost all consumers are of type-*L*.

It is worth comparing the results stated in Proposition 1 and Proposition 2 with the other papers that investigate the impact of the information at the disposal of firms on the firms' profits. In particular, Esteves (2014) finds that the profits strictly decrease with the accuracy of information about consumers, whereas in Chen et al. (2001) there is U-shape relationship between information accuracy and equilibrium profits. However, even in Chen et al. (2001), when comparing the case of no-information with the case of maximal information, it is found that no information yields higher profits. In Liu and Serfes (2004) and Colombo (2011, 2016) the relationship between the level of information and the equilibrium is U-shape, and still, as in Esteves (2014) and Chen et al. (2001), the profits under maximal information are lower than under no information. In contrast with these papers, we have found that the profits may be higher when the firms have maximal information. The reason of this difference is the following. In the above mentioned papers, the consumers are differentiated with regard to only one attribute (i.e. the innate preference of each consumer toward each firm), and greater information regards this attribute. When information is very high, the strategic effect tends to be strong whereas the surplus extraction effect is weak. This implies that, when comparing the case of no information with the case of maximal information, the profits in the latter case are lower than in the former. In contrast, in our model more information involves a second attribute of consumers (i.e. the price sensitivity of each consumer), which is different from the taste of the consumers toward each firm (which is captured by the location of the consumers in the production characteristic space). At equal prices, we have that some consumers prefer one firm to the other on the basis of their innate preference for that firm, whereas all the consumers are indifferent between the firms on the basis of their second attribute (i.e. price sensitivity). This implies that adding information about price sensitivity does not increase competition between firms, but it allows better targeting of prices. In other words, the strategic effect is unchanged when passing from classic BBPD to additional BBPD, whereas the surplus extraction effect increases. For sufficiently high consumers' heterogeneity, this induces higher profits under additional BBPD than under uniform pricing.[29]

4. Pricing policies

In this section, we consider the case where each firm chooses its pricing policy before setting the prices. This allows investigating which pricing policy is expected to emerge endogenously.[30] Clearly, this requires that the decision to adopt BBPD (and which type of BBPD) must be observable. As keeping track about own consumers, with or without collecting additional information about their price sensitivity, is a costly choice for firms, we can introduce the pricing policy stage at the start of the game (see Pazgal and Soberman, 2008).

Therefore, we suppose that the game has an initial stage, where each firm independently chooses which pricing policy to adopt. Price competition then unfolds accordingly to the choice in the first stage.[31] There are three possible

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[29] The first-period profits effect is also crucial: indeed, in the absence of the first period of competition and setting $k_Z^{gg}* = 1/2$ with $gg = \{aa, cc, uu\}$ and $Z = \{L, H\}$, the (second-period) profits under additional BBPD would be higher than under classic BBPD, but they would be lower than under uniform pricing.

[30] While the focus of this section is to find the pricing policy equilibria that emerge endogenously, asymmetric situations are also relevant *per se*. Indeed, asymmetric situations may be motivated by the fact that one firm may be in the market from a longer period than the rival. Therefore, while the former has been able to develop marketing techniques that allows to record past consumers' purchases, the latter has not been able yet. Literature has also emphasized that asymmetric pricing policies can emerge as a consequence of regulatory or competition policy interventions (Bouckaert et al., 2013). See also Jeong and Maruyama (2009) and Gehrig et al. (2011 and 2012) for other asymmetric BBPD models.

[31] Note that if a firm has chosen uniform pricing, it is unable to price discriminate across consumers in the second period. On the other hand, if a firm has chosen classic BBPD or additional BBPD, in the second period it will price discriminate accordingly to the pricing policy it has chosen. As shown in Section 3, this happens even if the firm is not constrained to price discriminate after having chosen a price discriminating policy, as it is free to price uniformly. In other words, once a firm decides to implement a price discriminating practice, the implementation of this practice in the second period of competition is self-enforcing.

pricing policies among which each firm may choose: uniform pricing, classic BBPD and additional BBPD. Furthermore, in addition to Assumption 1, we impose the following parameters' restriction. By denoting $\gamma_2 \equiv 1/(3-4\lambda)$:

<u>Assumption 2</u>: when $\lambda \le 8/25$, we assume that $\gamma \le \gamma_2$.

Assumption 2 guarantees that poaching emerges in equilibrium in the asymmetric cases considered in this section. We first discuss the case where one firm, say Firm *A*, chooses additional BBPD whereas the other firm, Firm *B*, chooses classic BBPD. We indicate this case with the superscript "*ac*". Clearly, the second-period prices in turf *A* are given by (15), (16) and (20), whereas those in turf *B* are given by (5) and (6). The first-period indifferent consumers are given by (9), and, consequently, the first-period equilibrium prices are given by (11). It follows that the equilibrium profits are given by $\Pi^{A,ac}* = \Pi^{A,aa}*$ and $\Pi^{B,ac}* = \Pi^{B,cc}*$. We can state the following proposition:

Proposition 3. *Classic BBPD never emerges in equilibrium.*

Proof. It follows directly from Proposition 1. ■

Therefore, Proposition 3 shows that additional BBPD dominates classic BBPD, which therefore would never arise in equilibrium. In words, once the rival discriminates on the basis of the past purchase history of consumers (with or without additional information), it is always better to have additional information about the consumers' price sensitivity. Indeed, the first-period equilibrium prices are not affected by additional information about consumers' price sensitivity in the second period. Therefore, having additional information in the second period allows better targeting of prices without inducing fiercer competition in the first period, and then it induces greater profits.

Since classic BBPD does not emerge in equilibrium, only additional BBPD and uniform pricing may emerge in equilibrium. Therefore, we may consider the following 2x2 matrix:

Table 1: the pay-off matrix in the pre-competition stage

B A	uniform pricing	additional BBPD
uniform pricing	$\Pi^{A,uu}*;\ \Pi^{B,uu}*$	$\Pi^{A,ua}*;\ \Pi^{B,ua}*$
additional BBPD	$\Pi^{A,au}*;\ \Pi^{B,au}*$	$\Pi^{A,aa}*;\ \Pi^{B,aa}*$

where the superscript "*au*" ("*ua*") indicates the case where Firm *A* (*B*) adopts additional BBPD whereas Firm *B* (*A*) commits to uniform pricing. Consider case *au*.[32] The second-period profits of Firm *A* are given by (13), whereas the profits of Firm *B* are given by:

$$\pi_2^{B,au}(\mathbf{q}) = \tilde{p}^B[\lambda(1-\dot{x}_H^B)+(1-\lambda)(1-\dot{x}_L^B)+\lambda(k_H(\mathbf{q})-\dot{x}_H^A)+(1-\lambda)(k_L(\mathbf{q})-\dot{x}_L^A)] \tag{24}$$

The following Lemma indicates the second-period optimal prices:

Lemma 3. The optimal second-period prices when Firm *A* adopts additional BBPD whereas Firm *B* adopts uniform pricing are:

$$\hat{p}_L^{A,au}(\mathbf{q}) = \frac{t[4+\lambda k_H(\mathbf{q})+k_L(\mathbf{q})(1-\lambda)(1-\gamma)-\gamma(4-\lambda(6-k_H(\mathbf{q})))]}{6(1-\gamma)\Gamma} \tag{25}$$

$$\hat{p}_H^{A,au}(\mathbf{q}) = \frac{t[4+\lambda k_H(\mathbf{q})+k_L(\mathbf{q})(1-\lambda)(1+\gamma)-\gamma(2-\lambda(6+k_H(\mathbf{q})))]}{6(1+\gamma)\Gamma} \tag{26}$$

[32] Clearly, the case *ua* is symmetric.

$$\tilde{p}^{A,au}(\mathbf{q}) = \frac{t[4-5(1-\lambda)k_L(\mathbf{q}) - 5\lambda k_H(\mathbf{q})]}{6\Gamma} \tag{27}$$

$$\tilde{p}^{B,au}(\mathbf{q}) = \frac{t[1+k_L(\mathbf{q})(1-\lambda) + \lambda k_H(\mathbf{q})]}{3\Gamma} \tag{28}$$

Proof. See the Appendix. ■

The first-period indifferent consumers are given by (9) once $\tilde{p}^{A,cc}$ is substituted by $\tilde{p}^{A,au}$ and $\tilde{p}^{B,cc}$ is substituted by $\tilde{p}^{B,au}$. The overall profits of Firm $I = \{A, B\}$ are $\Pi^{I,au} = q^I D^I(\mathbf{q}) + \delta\pi_2^{I,au}*(\mathbf{q})$, where $\pi_2^{I,au}*(\mathbf{q})$ is obtained by using (25)-(28). By maximizing the profits, we get the first-period equilibrium prices: $q^{A,au}* = \dfrac{t(3-\delta)}{3\Gamma}$ and $q^{B,au}* = \dfrac{t(12-\delta)}{12\Gamma}$.[33] The equilibrium profits are:

$$\Pi^{A,au}* = \frac{t[24+7\delta-\gamma^2(24+\delta(7-24\lambda+24\lambda^2))]}{48(1-\gamma^2)\Gamma} \tag{29}$$

$$\Pi^{B,au}* = \frac{t(12+5\delta)}{24\Gamma} \tag{30}$$

We can state the following proposition:

Proposition 4. *Two equilibria exist, where either both firms choose additional BBPD or both firms choose uniform pricing.*

Proof. Observe that $\Pi^{A,au}* \leq \Pi^{A,uu}*$ and $\Pi^{B,aa}* \geq \Pi^{B,au}*$. ■

Therefore, we show that a multiplicity of equilibria arises: either both firms choose to engage in BBPD with additional information about consumers' price

[33] The other equilibrium variables are reported in the Appendix.

sensitivity, or both firms commit to uniform pricing. Indeed, suppose first that the rival has not invested in gathering information about own consumers (i.e. it committed to uniform price). By choosing not to discriminate too, the focal firm reduces the fierceness of competition, thus inducing greater profits. On the other hand, suppose that the rival has invested to keep track of the own consumers and to learn their price sensitivity. In this case it is better for the focal firm to mimic the strategy of the rival, in such a way to be as flexible as the rival in targeting the prices on the consumers' characteristics.

It can be shown that the multiplicity of equilibria does not arise in the case of sequential moves. Suppose that the firms decide the pricing policy in different stages. We can state the following proposition.

Proposition 5. *When the firms decide sequentially which pricing policy to adopt, if* $\gamma \geq (\leq)\, \bar{\gamma}$*, the unique pricing policy equilibrium consists in both firms choosing additional BBPD (uniform pricing).*

Proof. See the Appendix. ■

Therefore, in the case of a sequential choice of the pricing policies, the firms endogenously coordinate on the pricing policies that yield higher profits.

5. Welfare

In this section, we consider the welfare implications of additional BBPD. First, we consider the impact on consumer welfare, and then we consider the impact on overall welfare. We consider only the case of symmetric pricing policies.

Consider first the consumer surplus. Since the firms are symmetric, we can focus on the turf of Firm A only. Consider cases $gg=\{aa,cc\}$. The surplus of type-$Z=\{L,H\}$ consumers is given by:

$$CS_Z^{gg}*=\int_0^{k_Z^{gg}*}[v-(1-\xi\gamma)q^{A,gg}*-tx]dx+\delta\int_0^{x_Z^{A,gg}*}[v-(1-\xi\gamma)\hat{p}_Z^{A,gg}*-tx]dx+ \\ +\delta\int_{x_Z^{A,gg}*}^{k_Z^{gg}*}[v-(1-\xi\gamma)\tilde{p}^{B,gg}*-t(1-x)]dx \tag{31}$$

where the first term indicates the utility in the first period for any consumer, the second term indicates the utility in the second period for those consumers that do not shift, and the third term indicates the utility in the second period for those consumers that shift. On the other hand, in the case of uniform pricing, *uu*, the consumer surplus of type-$Z=\{L,H\}$ consumers can be expressed as follows:

$$CS_Z^{uu}*=(1+\delta)\int_0^{k_Z^{uu}*}[v-(1-\xi\gamma)q^{A,uu}*-tx]dx \tag{32}$$

The following proposition illustrates the impact of BBPD with additional information about consumers' price sensitivity on the consumer surplus with respect to the case of uniform pricing and classic BBPD.

Proposition 6. *Consider type-H consumers. The consumer surplus under additional BBPD is higher than under classic BBPD, which in turn is higher than under uniform pricing. Consider type-L consumers. The consumer surplus is maximum under classic BBPD, whereas it is higher (lower) under additional BBPD than under uniform pricing when γ is low (high).*

Proof. See the Appendix. ■

Consider type-H consumers. Proposition 6 claims that consumer surplus is highest under additional BBPD and lowest under uniform pricing. This result can be explained as follows. By comparing the second-period equilibrium prices under additional BBPD and classic BBPD, we have that prices are lower under additional BBPD for non-switching consumers and identical for switching consumers.[34] Indeed, when a firm can target the price on the basis of the consumer's price sensitivity (additional BBPD), high-sensitivity consumers are better off. Moreover, there are fewer switching consumers under additional BBPD than under classic BBPD, as $\dot{x}_H^{A,aa}* \geq x_H^{A,cc}*$. It follows that the overall surplus of type-H consumers must be higher under additional BBPD.[35] On the other hand, when comparing case *cc* and case *uu*, we observe that the first-period equilibrium price is lower under uniform pricing, but the second-period equilibrium prices are lower under classic BBPD. Even if some consumers switch under classic BBPD, the impact of the second-period price dominates, and the surplus of type-H consumers is higher under classic BBPD than under uniform pricing.

Consider now type-L consumers. Clearly, the surplus of consumers is higher under classic BBPD than under additional BBPD. Indeed, at the opposite of high-sensitivity consumers, the low-sensitivity consumers are charged with a higher price when the prices can be targeted on the basis of the consumer's price sensitivity. In addition to this, there are more switching consumers under additional BBPD than under classic BBPD, as $\dot{x}_L^{A,aa}* \leq x_L^{A,cc}*$. Compare now the case of additional BBPD and the case of uniform pricing. The price charged to non-switching consumers under additional BBPD is lower than the uniform

[34] We showed in Section 3 that the first-period equilibrium price is the same in both situations.
[35] Moreover, the surplus is higher under additional BBPD than under classic BBPD for *any* type-H consumer.

price only if γ is low enough.[36] Indeed, when γ is high enough, the price sensitivity of type-*L* consumers is quite low, and a price discriminating firm sets a high price. Therefore, the surplus of type-*L* consumers under additional BBPD is higher than under uniform pricing if γ is sufficiently low, and vice-versa.

Proposition 6 has an immediate policy implication. If the authority takes care of the consumer surplus implications of BBPD, it has to distinguish between different types of BBPD (with or without additional information about the price sensitivity of consumers) and different types of consumers (with high or low price sensitivity). On one hand, additional BBPD favours high-sensitivity consumers, because a firm targeting its price on them is induced to set a lower price. On the other hand, additional BBPD damages low-sensitivity consumers, and this is particularly true when the heterogeneity of consumers with respect to this characteristic is quite high, as the surplus of high-sensitivity consumers under additional BBPD is lower than the surplus under both classic BBPD and under uniform pricing. With regard to classic BBPD, this pricing regime maximizes the surplus of low-sensitivity consumers, but it yields a lower surplus for high-sensitivity consumers with respect to additional BBPD. Therefore, we can conclude that an overall ban to BBPD certainly damages high-sensitivity consumers. On the other hand, low-sensitivity consumers would be certainly better off if classic BBPD would be allowed, but they would be worse off if additional BBPD would be allowed.

Finally, we briefly discuss the impact of additional BBPD on total welfare. Total welfare in cases $ff = \{aa, cc, uu\}$ is given by the following function: $W^{ff} = 2[\Pi^{A,ff} * + \lambda CS_H^{ff} * + (1-\lambda) CS_L^{ff} *]$. We can state the following proposition:

[36] Indeed, by comparing $\hat{p}_L^{A,aa} *$ and $q^{A,uu} *$, we have that $\hat{p}_L^{A,aa} * \leq (\geq)\, q^{A,uu} *$ if $\lambda \leq (\geq)\, 1/(1+3\lambda)$.

Proposition 7. *Welfare is minimum under classic BBPD, whereas it is higher (lower) under additional BBPD than under uniform pricing when* γ *is high (low) and/or* λ *is intermediate (high or low).*

Proof. See the Appendix. ■

On one hand, when comparing additional BBPD and classic BBPD, we have that the profits are higher under additional BBPD than under classic BBPD (Proposition 1), whereas the consumer surplus may be higher or lower depending on the consumers' type (Proposition 6). The effect on the profits dominates the effect on the consumer surplus. It follows that total welfare is higher under additional BBPD. On the other hand, when comparing additional BBPD and uniform pricing, we have that when the price sensitivity parameter, γ, is high, the profits are higher under additional BBPD (Proposition 2), but the surplus of low-sensitivity consumers is lower (Proposition 6). When γ is low, the impact on the consumer surplus dominates and the welfare is higher under uniform pricing, whereas when γ is high, the effect on the profits dominates, and the welfare is higher under additional BBPD.

6. Conclusions

In this paper we analyse BBPD in the case of doubly heterogeneous consumers. On one hand, the consumers are heterogeneous with regard to their tastes; on the other hand, the consumers are also heterogeneous with regard to their price sensitivity. After the initial purchase of each consumer, each firm is able to recognize whether a consumer has bought from it or from the rival. In addition, we admit the possibility that a firm is able to recognize whether an own consumer is a high-sensitivity-to-price one, or a low-sensitivity-to-price one, and then to price discriminate accordingly. We indicate this pricing behaviour as "additional" BBPD to distinguish it from the "classic" BBPD case usually considered in the literature.

We show that additional BBPD yields always higher profits than classic BBPD, whereas it may yield higher or lower profits than uniform pricing. That is, discriminating in both history and price sensitivity (additional BBPD) is always better than discriminating in just history (classic BBPD). Moreover, discriminating in both history and price sensitivity may make BBPD more profitable than uniform pricing when it would not be otherwise. In particular, this happens when consumers are sufficiently heterogeneous in terms of their price sensitivity, whereas the opposite holds when consumers are scarcely heterogeneous. We also extend the model to consider the endogenous choice of the pricing policy, and we show that, in the case of a simultaneous choice by firms, multiple equilibria arise, where both firms choose either additional BBPD or uniform pricing, whereas in the case of a sequential choice of the pricing policy, the firms endogenously coordinate on the pricing policies that yield higher profits.

The present framework could be extended in many ways. For example, a widely debated question in BBPD literature regards the impact of the switching costs sustained by consumers when they buy from different firms in different periods (see for example Chen, 1997, Taylor, 2003). Extending the present model to include switching costs of consumers seems a reasonable path for future research. Second, the present set-up considers a situation where the two firms are ex-ante identical. However, one may consider a situation where one firm offers a higher quality product than the rival.[37] How quality asymmetry of

[37] See for example Gehrig et al. (2008) and Thomadsen and Rhee (2016) for a model of BBPD with vertically differentiated products. In both papers, the consumers are heterogeneous only with respect to their preference for quality. See also Carroni (2015), where firms offer products of different qualities but consumers are only horizontally heterogeneous.

firms interacts with double heterogeneity of consumers in a context of BBPD is an intriguing question for future research. Third, one may consider the possibility that consumers may act strategically by hiding their initial purchase or their type. In this case, the level of information accuracy (and then the price discrimination possibilities of the firms) would emerge endogenously as a consequence of the consumers' choice.

Appendix

Proof of Lemma 1. Consider Turf *A*. The first-order condition is given by the solution of the following system: $\begin{cases} \partial\pi_2^{A,cc} / \partial\hat{p}^A = 0 \\ \partial\pi_2^{B,cc} / \partial\tilde{p}^B = 0 \end{cases}$, which yields $\hat{p}^{A,cc}$ and $\tilde{p}^{B,cc}$.

Note that the second-order conditions are always satisfied, as $\partial^2\pi_2^{A,cc} / \partial\hat{p}^{A\,2} = \partial^2\pi_2^{B,cc} / \partial\tilde{p}^{B\,2} = -\Gamma/t < 0$. However, we have additionally to check that Firm *A* and Firm *B* has no incentive to deviate from $\hat{p}^{A,cc}$ and $\tilde{p}^{B,cc}$, respectively. Indeed, a firm may have the incentive to serve only the lowly-sensitive consumers and setting a higher price. Suppose that Firm *A* deviates. In Turf *A*, it maximizes $\overline{\pi}_2^{A,cc}(\tilde{p}^{B,cc}) = p^A(1-\lambda)x_L^A$ with respect to p^A given $\tilde{p}^{B,cc}$, which yields $\overline{p}^{A,cc} = \dfrac{t[1+2(1-\gamma)(1-\lambda)k_L + 2\lambda k_H - \gamma(1-3\lambda+2\lambda k_H)]}{3(1-\gamma)\Gamma}$. The profits of Firm *A* in Turf *A* in the case of a deviation are then $\overline{\pi}_2^{A,cc}(\overline{p}^{A,cc}, \tilde{p}^{B,cc})$. By comparing with $\pi_{2/A}^{A,cc}(\hat{p}^{A,cc}, \tilde{p}^{B,cc}) = \hat{p}^{A,cc}[\lambda x_H^A + (1-\lambda)x_L^A]$, it can be noted that $\pi_{2/A}^{A,cc}(\hat{p}^{A,cc}, \tilde{p}^{B,cc}) > \overline{\pi}_2^{A,cc}(\overline{p}^{A,cc}, \tilde{p}^{B,cc})$ in the around of $k_L = k_H = 1/2$ under Assumption 1.[38] Consider now the case of a deviation by Firm *B*. In Turf *A*, it maximizes $\overline{\pi}_2^{B,cc}(\hat{p}^{A,cc}) = p^B(1-\lambda)(k_L - x_L^A)$ with respect to p^B given $\hat{p}^{A,cc}$, which yields $\overline{p}^{B,cc} = \dfrac{t[k_H\lambda - 1 + k_L(4-\gamma(4-7\lambda)-\lambda) + \gamma(1-\lambda(3+k_H))]}{3(1-\gamma)\Gamma}$. The profits of Firm *B*

[38] This first-period market sharing emerges in equilibrium in all the cases considered.

in Turf A in the case of a deviation are then $\overline{\pi}_2^{B,cc}(\hat{p}^{A,cc},\overline{p}^{B,cc})$. By comparing with $\pi_{2/A}^{B,cc}(\hat{p}^{A,cc},\tilde{p}^{B,cc})=\tilde{p}^{B,cc}[\lambda(k_H-x_H^A)+(1-\lambda)(k_L-x_L^A)]$, it can be noted that $\pi_{2/A}^{B,cc}(\hat{p}^{A,cc},\tilde{p}^{B,cc})>\overline{\pi}_2^{B,cc}(\hat{p}^{A,cc},\overline{p}^{B,cc})$ always holds in the around of $k_L=k_H=1/2$. The case of Turf B can be treated symmetrically. ■

Equilibrium variables under classic BBPD. By using (11), we have: $k_Z^{cc}*=1/2$, $x_L^{A,cc}*=1-x_L^{B,cc}*=\dfrac{1-\gamma(1-3\lambda)}{3\Gamma}$, $x_H^{A,cc}*=1-x_H^{B,cc}*=\dfrac{1-\gamma(2-3\gamma)}{3\Gamma}$, $\tilde{p}^{I,cc}*=\dfrac{t}{3\Gamma}$ and $\hat{p}^{I,cc}*=2\tilde{p}^{I,cc}*$, $I=\{A,B\}$ and $Z=\{L,H\}$. It can be noted that poaching emerges in equilibrium under Assumption 1, as $1\geq x_Z^{B,cc}*\geq k_Z^{cc}*\geq x_Z^{A,cc}*\geq 0$.

Proof of Lemma 2. Consider Turf A. The first-order condition is given by the solution of the following system: $\begin{cases}\partial\pi_2^{A,aa}/\partial\hat{p}_H^A=0\\ \partial\pi_2^{A,aa}/\partial\hat{p}_L^A=0\\ \partial\pi_2^{B,aa}/\partial\tilde{p}^B=0\end{cases}$, which yields $\hat{p}_H^{A,aa}$, $\hat{p}_L^{A,aa}$ and $\tilde{p}^{B,aa}$. Note that the second-order conditions are always satisfied, as $\partial^2\pi_2^{A,aa}/\partial\hat{p}_L^{A\,2}=-(1-\gamma)(1-\lambda)/t$, $\partial^2\pi_2^{A,aa}/\partial\hat{p}_H^{A\,2}=-(1+\gamma)\lambda/t$ and $\partial^2\pi_2^{B,aa}/\partial\tilde{p}^{B\,2}=-\Gamma/t$. However, we have additionally to check that Firm B has no the incentive to deviate from $\tilde{p}^{B,aa}$. Suppose that Firm B deviates. It maximizes $\overline{\pi}_2^{B,aa}(\hat{p}_L^{A,aa})=p^B(1-\lambda)(k_L-x_L^A)$ with respect to p^B given $\hat{p}_L^{A,aa}$, which yields $\overline{p}^{B,aa}=\dfrac{t[2k_H\lambda-2+2k_L(4-\gamma(4-7\lambda)-\lambda)+\gamma(2-\lambda(3+2k_H))]}{6(1-\gamma)\Gamma}$. The profits of Firm B in Turf A in the case of a deviation are then $\overline{\pi}_2^{B,aa}(\hat{p}_L^{A,aa},\overline{p}^{B,aa})$. By comparing with $\pi_{2/A}^{B,aa}(\hat{p}_H^{A,aa},\hat{p}_L^{A,aa},\tilde{p}^{B,aa})=\tilde{p}^{B,aa}[\lambda(k_H-x_H^A)+(1-\lambda)(k_L-x_L^A)]$, it can be noted that $\pi_{2/A}^{B,aa}(\hat{p}_H^{A,aa},\hat{p}_L^{A,aa},\tilde{p}^{B,aa})>\overline{\pi}_2^{B,aa}(\hat{p}_L^{A,aa},\overline{p}^{B,aa})$ holds in the around of $k_L=k_H=1/2$ under Assumption 1. The case of Turf B can be treated symmetrically. ■

Equilibrium variables under additional BBPD. By using (11), we get: $k_Z^{aa}*=1/2$, $x_L^{A,aa}*=1-x_L^{B,aa}*=[2-\gamma(2-3\lambda)]/6\Gamma$, $x_H^{A,aa}*=1-x_H^{B,aa}*=[2-\gamma(1-3\lambda)]/6\Gamma$, $\hat{p}_L^{I,aa}*=\dfrac{t(2+3\lambda\gamma-2\gamma)}{3(1-\gamma)\Gamma}$, $\hat{p}_H^{I,aa}*=\dfrac{t(2-\gamma+3\lambda\gamma)}{3(1+\gamma)\Gamma}$ and $\widetilde{p}^{I,aa}*=\dfrac{t}{3\Gamma}$, $I=\{A,B\}$ and $Z=\{L,H\}$. It can be noted that emerges in equilibrium under Assumption 1, as $1\geq x_Z^{B,aa}*\geq k_Z^{aa}*\geq x_Z^{A,aa}*\geq 0$.

Proof of Lemma 3. The first-order condition is given by the solution of the following system: $\begin{cases}\partial\pi_2^{A,au}/\partial\hat{p}_H^A=0\\ \partial\pi_2^{A,au}/\partial\hat{p}_L^A=0\\ \partial\pi_2^{A,au}/\partial\widetilde{p}^A=0\\ \partial\pi_2^{B,au}/\partial\widetilde{p}^B=0\end{cases}$, which yields $\hat{p}_H^{A,au}$, $\hat{p}_L^{A,au}$, $\widetilde{p}^{A,au}$ and $\widetilde{p}^{B,au}$.

Note that the second-order conditions are always satisfied, as shown in Lemma 2 and given that $\partial^2\pi_2^{B,au}/\partial\widetilde{p}^{B\,2}=-2\Gamma/t$. We check that Firm *B* has no the incentive to deviate from $\widetilde{p}^{B,au}$. Suppose that Firm *B* deviates. It maximizes $\overline{\pi}_2^{B,au}(\hat{p}_H^{A,au},\hat{p}_L^{A,au},\widetilde{p}^{A,au})=p^B(1-\lambda)(k_L-x_L^A+1-x_L^B)$ with respect to p^B given $\hat{p}_H^{A,au}$, $\hat{p}_L^{A,au}$ and $\widetilde{p}^{A,au}$. We get $\overline{p}^{B,au}=\dfrac{t[4-2k_H\lambda+2k_L(2-\gamma(2-5\lambda)+\lambda)-\gamma(4-\lambda(3+2k_H))]}{12(1-\gamma)\Gamma}$.

The profits of Firm *B* in the case of a deviation are then $\overline{\pi}_2^{B,au}(\hat{p}_H^{A,au},\hat{p}_L^{A,au},\widetilde{p}^{A,au},\overline{p}^{B,au})$. By comparing with $\pi_2^{B,au}(\hat{p}_H^{A,au},\hat{p}_L^{A,au},\widetilde{p}^{A,au},\widetilde{p}^{B,au})$, it can be noted that $\pi_2^{B,au}(\hat{p}_H^{A,au},\hat{p}_L^{A,au},\widetilde{p}^{A,au},\widetilde{p}^{B,au})>\overline{\pi}_2^{B,au}(\hat{p}_H^{A,au},\hat{p}_L^{A,au},\widetilde{p}^{A,au},\overline{p}^{B,au})$ holds in the around of $k_L=k_H=1/2$ under Assumption 1. Suppose that Firm *A* deviates in Turf *B*. It maximizes $\overline{\pi}_2^{A,au}(\widetilde{p}^{B,au})=p^A(1-\lambda)(x_L^B-k_L)$ with respect to p^A given $\widetilde{p}^{B,au}$. We get $\overline{p}^{A,au}=\dfrac{t[4+k_H\lambda-k_L(5-\gamma(5-11\lambda)+\lambda)-\gamma(4-\lambda(6-k_H))]}{6(1-\gamma)\Gamma}$.

Therefore, the profits of Firm *A* in Turf *B* in the case of a deviation are $\overline{\pi}_2^{A,au}(\overline{p}^{A,au},\widetilde{p}^{B,au})$. Without deviation, the profits of Firm *A* in Turf *B* are $\pi_{2/B}^{A,au}(\widetilde{p}^{A,au},\widetilde{p}^{B,au})=\widetilde{p}^{A,au}[\lambda(x_H^B-k_H)+(1-\lambda)(x_L^B-k_L)]$. By comparing in the around of

$k_L = k_H = 1/2$, we have that $\pi_{2/B}^{A,au}(\tilde{p}^{A,au}, \tilde{p}^{B,au}) > \bar{\pi}_2^{A,au}(\bar{p}^{A,au}, \tilde{p}^{B,au})$ holds under Assumption 1. ■

Equilibrium variables in the case au. Using $q^{A,au}*$ and $q^{B,au}*$, we get: $k_L^{au}* = k_H^{au}* = \frac{1}{2}$, $x_L^{A,au}* = \frac{3-3\gamma+4\lambda\gamma}{8\Gamma}$, $x_L^{B,au}* = \frac{5-5\gamma+8\lambda\gamma}{8\Gamma}$, $x_H^{A,au}* = \frac{3-\gamma+4\lambda\gamma}{8\Gamma}$, $x_H^{B,au}* = \frac{5-3\gamma+8\lambda\gamma}{8\Gamma}$, $\hat{p}_L^{A,au}* = \frac{t(3+4\lambda\gamma-3\gamma)}{4(1-\gamma)\Gamma}$, $\hat{p}_H^{A,au}* = \frac{t(3-\gamma+4\lambda\gamma)}{4(1+\gamma)\Gamma}$, $\tilde{p}^{A,au}* = \frac{t}{4\Gamma}$ and $\tilde{p}^{B,au}* = \frac{t}{2\Gamma}$. It can be noted that poaching emerges in equilibrium under Assumption 1 and 2, as $1 \geq x_Z^{B,au}* \geq k_Z^{au}* \geq x_Z^{A,au}* \geq 0$, where $Z = \{L, H\}$.

Proof of Proposition 5. From Proposition 4, we have that the follower chooses the pricing policy *a* (resp. *u*) when the leader has chosen the pricing policy *a* (resp. *u*). From Proposition 2, we know that $\Pi^{I,aa}* \geq (\leq) \Pi^{I,uu}*$ if $\gamma \geq (\leq) \bar{\gamma}$. Therefore, if $\gamma \geq (\leq) \bar{\gamma}$ the leader chooses *a* (resp. *u*), thus inducing *aa* (resp. *uu*) as the unique equilibrium. ■

Proof of Proposition 6. The proof is obtained by comparing $CS_Z^{aa}*$, $CS_Z^{cc}*$ and $CS_Z^{uu}*$, with $Z = \{L, H\}$. First, in the case of type-*H* consumers, we have $CS_H^{aa}* \geq CS_H^{cc}* \geq CS_H^{uu}*$. Second, in the case of type-*L* consumers, we have a critical threshold of γ, say $\gamma_3 \equiv \frac{1+6\lambda-3\sqrt{7}\lambda}{1+12\lambda-27\lambda^2}$ such that $CS_L^{cc}* \geq CS_L^{uu}* \geq CS_L^{aa}*$ if $\gamma \geq \gamma_3$ and $CS_L^{cc}* \geq CS_L^{aa}* \geq CS_L^{uu}*$ if $\gamma \leq \gamma_3$. ■

Proof of Proposition 7. The proof is obtained by comparing $W^{aa}*$, $W^{cc}*$ and $W^{uu}*$. An explicit solution of the critical threshold of γ when comparing $W^{aa}*$ and $W^{uu}*$ cannot be obtained. However, for example, when $\lambda = 0.5$, the critical threshold is $\gamma = 0.418$, such $W^{aa}* \geq W^{uu}* \geq W^{cc}*$ if $\gamma \geq 0.418$ and $W^{uu}* \geq W^{aa}* \geq W^{cc}*$ if $\gamma \leq 0.418$. Other numerical examples confirm this result. Furthermore, it can be shown that the critical threshold is U-shape in λ. ■

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