Vertical flexibility, outsourcing and the financial choices of the firm*

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Abstract

We investigate the relationship between the financial choices and the extent of vertical flexibility of a firm. By vertical flexibility we mean simply the opportunity to outsource a necessary input and to reverse the choice as input market conditions dictate. We consider a firm that has to select the shares of equity and debt and its vertical setting, i.e., whether and how much to outsource the production of a necessary input and how to finance all that. Debt is provided by a lender that requires the payment of a fixed coupon over time and, as a collateral, an option to buy out the firm in certain circumstances. Debt leads to the same level of flexibility acquired by an unlevered firm but to invest earlier in the flexible technology. An alternative to debt is the involvement of venture capital for the production of the input. We explore this second avenue finding that the extent of outsourcing adopted is lower than for the unlevered firm, but the firm invests earlier in the flexible technology.

Keywords: vertical integration, flexible outsourcing, debt, equity and venture capital.

JEL Classification: C61; G31; G32; L24.

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1 Introduction

The purpose of the ensuing pages is to analyse the relationship between the financial structure and the flexibility of the vertical organization of a corporate enterprise. By vertical flexibility we mean the opportunity for a firm to buy inputs from the market (outsourcing) in a variable and reversible manner, going back to internal production if necessary. The organizational aspects of outsourcing and flexibility are crucial because most firms buy inputs in different and variable proportions and change quite often the extension of their activity along the vertical chain of the producion process, i.e., the related degree of market utilization. Flexibility improves the ability to cope with uncertain scenarios and has considerable effects on competitiveness, scale of production and social efficiency. Outsourcing and flexibility do not come for free since the acquisition of inputs from the market usually requires the setting up of a supply chain with relevant logistic investment. If a vertically flexible firm decides to substitute an internally produced input with an externally provided one keeping the option of bringing back in-house (backsourcing or reshoring) the same production it must keep alive an internal facility and the associated know-how. Hence, flexibility may be quite dear. The costs are affected by technical progress, by efficiency of external markets, i.e., the opportunity of buying easily inputs from producers which may be specialized or located in low cost countries and, last but not the least, by the design of a proper capital budgeting. This final aspect is crucial to finance vertical flexibility in the best way in terms of the mix between equity, debt and other possible financial sources[1]. Unfortunately this latter aspect is often sidestepped in the analysis of both vertical relationships and flexibility since funding and organization themes are studied separately in financial[2], managerial, industrial organization and operations research literature[3]. This partly unexplored field lets us analyse jointly finance and vertical organization issues. On the real side we shall be concerned with the extent and the type of vertical flexibility, that can be secured by arms' length outsourcing of inputs or by involving a venture capitalist in the joint production of an input while maintaining in all cases a partial in-house prudential production. On the financial side we shall see how the mix between equity and debt or the participation of a venture capitalist may affect the extent of flexibility acquired. A venture capitalist may be seen as a risk sharing device making a flexible firm financially more agile and differs from raising fresh capital through an IPO even though there are some affinities. As a matter of fact, financial sources may be represented by new equity, debt[4] (convertible or nonconvertible) or by a venture capitalist. We exclude from our investigation new equity raised through an IPO (initial public offering) since it tends to sharply reduce the price of existing stock and may open the way to a loss of control[5].

Since flexible technologies reduce risk (profit volatility) they may be considered as a kind of (real) option and their price should reflect their (option) value (Amran and Kulatilaka, 1999, Ch. 16). As a result the presumption is that the value of a vertically flexible firm be weakly larger than the value of a correspondent non flexible firm. Yet, we shall see that this is not always the case whenever the cost of flexibility and the related financial aspects are taken into account.

Our investigation is prompted by broad casual observation and related press reports[6] showing

[1] Other financial channels can be activated by a firm. For the sake of simplicity we confine to debt, equity and the involvment of a venture capitalist.

[2] See, for a good survey of main issues, Tirole (2006).

[3] See Van Mieghen (1999), Wang, Liu and Wang (2007), Moretto and Rossini (2012) where a good deal of literature is surveyed and referred.

[4] This does not make justice of the wealth of instruments to finance new productive challenges. We confine to a simple choice between debt, equity and a JV.

[5] See Eckbo, Masulis, and Norli (2007).

[6] For instance Apple has recently increased the OS of some inputs while reducing and bringing back home other inputs. See for further examples: *The Economist* (2011, 2013), *Forbes* (2012).See also empirical assesments in Klein

that several firms change over time their vertical production structure, expanding and/or subsequently reducing (or the other way round) the extent of outsourcing of inputs[7]. For instance in the automotive industry most brands adopt partial outsourcing, i.e., concomitant internal production and purchase of engines from external sources. Moreover, the extent of outsourcing is frequently changed as witnessed by the variable level of value added over revenue found in balance sheets and everydays news. Since different organizational settings exhibit distinct degrees of risk it is worth seeing how each financial setting affects the degree of flexibility acquired.

Literature has recently examined vertical flexibility (Shy and Stenbacka, 2005; Alvarez and Stenbacka, 2007; Moretto and Rossini, 2012) scantly going into the relationship with capital structure. Contributions on the link between industrial decisions and financial structure may be found in Lederer and Singhal (1994), in Leland (1998), in Mauer and Sarkar (2005), Benaroch et al. (2012), Banerjee et al. (2014). Published literature does not address the specific question of vertical flexibility and outsourcing. However, most find that if organizational and strategic decisions are not taken simultaneously with financial choices inefficiency arises. Mauer and Sarkar (2005) focus on the agency cost of financing investment with debt in a dynamic stochastic framework. In a similar environment Leland (1998) addresses the same topic that was raised initially in the seminal paper of Jensen and Meckling (1976). Unlike Leland (1998), Mauer and Sarkar (2005) emphasize the inefficiency of debt in most cases. In the traditional Modigliani and Miller (1958) scenario the value of a firm is given by the sum of its liabilities. Equity and debt turn out to be quite close (in certain circumstances, perfect substitutes). However, equityholders and debtholders do not coincide and each group maximizes a different objective function. Shareholders maximize the equity value while debtholders maximize the debt value generating subadditive results. Only a "social planner" would rather maximize the sum of debt and equity pursuing a first best. Mauer and Sarkar (2005) calculate the agency cost of debt as the difference between the total value of a firm where each group of stakeholders maximize separately and the case where the whole value of the firm is jointly maximized. They find that equityholders, in a limited liability legal framework, tend to overinvest if they do not face the proper agency cost of debt confirming the old Jensen and Meckling (1976) wisdom. The issue of going back and forth from (complete) outsourcing to vertical integration is studied in Benaroch and al. (2012) who analyse the particular case of service production. Outsourcing may allow a firm facing volatile demand to avoid the risk of bearing fixed costs that cannot be easily covered. By (complete) outsourcing of services which are capital intensive the firm turns a fixed into a variable cost cutting risk and can go back to internal production bearing each time a fixed cost. While in our model we go through the privately optimal (hence, variable) extent of outsourcing according to the capital structure adopted, in the Benaroch et al. (2012) paper the main question is about the optimal switching from (complete) outsourcing to backsource and the value of the switching option, which increases with demand volatility and the skill intensity of the production process of the input.

In Banerjee et al. (2014) the investment in a new technology, such as a flexible vertical process, financed by an external subject is seen as a joint option. In that case the size of the investment, the timing of the exercise and the sharing rule concerning the returns from the investment have to be established jointly by the firm and by the financial investor. According to Banerjee et al. (2014) it is inefficient to specify a sharing rule before the venture is carried out. Recently Bakhtiari and Breunig (2014) at firm level on longitudinal data have empirically assessed outsourcing seen as a device to smooth demand uncertainty. They find an asymmetric link with demand fluctuations, i.e., outsourcing increases substantially during slumps while does not respond much to demand

(2005) and Rossini and Ricciardi (2005).

[7]Examples may found in Benaroch et al. (2012).

increases. Some scanty data investigation on the financial support of outsourcing is attempted but it is fairly inconclusive. OS appears definitely as a shield against market contraction, yet many questions remain unanswered.

In the ensuing pages we consider two alternative cases. In the first the control right over the investment decision is allocated to the firm (i.e., the shareholders), while in the second case the control belongs with an outside investor (i.e., a venture capitalist). As in Banerjee et al. (2014) both actors agree in advance over the sharing rule of the project value. While the timing of the investment is determined by one party the terms of the investment are determined by both parties. In both cases the level of outsourcing is always set by the operating party.

As to the financial terms of the investment, in the first case we shall be concerned with debt financing. To overcome the agency problem of debt, the lender is granted an option to buy out the firm if outsourcing becomes the main source of profits for shareholders. The alternative case considers a pure equity offer: ownership is shared with an outside investor (venture capitalist) without side payments (i.e., no debt service by the firm).

In our endeavor we shall couple two streams of contributions: one on vertical flexibility (Moretto and Rossini, 2012; Shy and Stenbacka, 2005; Alvarez and Stenbacka, 2007) and the other on financial choices of a firm in an uncertain dynamic framework (Leland, 1994; Lederer and Singhal, 1994; Banerjee et al., 2014; Triantis and Hodder, 1990).

With debt the firm may rush to adopt flexibility, but it may be hard to finance it by debt unless the lender gets as a collateral an option to buy the entire firm in case flexibility turns out not to be profitable enough. (Warranted) debt makes a firm invest earlier in the vertically flexible technology and resort to more outsourcing than a pure equity financed firm. When a venture capitalist is involved, the investment occurs later and the extent of outsourcing is lower than with pure equity.

The paper roadmap is the following: in section 2 we see the basic model, in section 3 we go through the value of a vertically flexible firm using both equity and debt as financial sources, in section 4 we present the optimal outsourcing share, in section 5 we study the optimal timing for the acquisition of debt, in section 6 we go through the case of a joint venture supported by private equity. The epilogue is in section 7.

2 The basic set up

We consider the internal organization of a *vertically flexible* enterprise that for each unit of output to be produced needs one unit of a perfectly divisible input (*perfect vertical complementarity*). The firm has to decide whether to buy a vertically flexible technology that allows to manufacture the input in-house at the constant marginal cost d, to resort (totally or partially) to outsourcing (OS). in case the market price of the input, c_t, is low enough, and to reverse the choice over time (backsourcing), if c_t goes up enough.

In the specific, the enterprise, at any time, can switch from totally making the input in-house when $\hat{c}_t \equiv \alpha c_t + (1-\alpha)d$ rises above d, to partially purchasing it if $\hat{c}_t$ falls below d and viceversa, where $\alpha \in (0,1]$ is the outsourced share (its complement to one is the home produced portion).[8] The instantaneous profit is:

$$\pi_t \equiv \max\{0,\ [p-d+\max(d-\hat{c}_t,0)]\} \tag{1}$$

[8] For the sake of tractability, we assume that there are no fixed costs in the production and/or the purchasing of the input. Fixed costs would not change qualitatively our conclusions. They just introduce a hysteresis interval in the option to switch from producing the input in-house to outsource it. See Benaroch et al. (2012) for the consideration of fixed costs.

where p is the output market price. Then, when $\alpha < 1$ the firm uses a linear combination of produced and procured input. It can go back to vertical integration if $\hat{c}_t$ becomes too high. In addition, to avoid default we assume that $p - d > 0$.[9]

The cost of the technology is simply given by:[10]

$$I(\alpha) = k_1 + \frac{k_2}{2}\alpha^2 \quad \text{for } \alpha \in (0, 1] \tag{2}$$

where k_1 is the direct sunk cost to keep internal facilities ready or going (i.e., the cost of maintenance and updating the process for the internal production of the input) with total or partial OS. The term $\frac{k_2}{2}\alpha^2$ is the organizational cost to design and run a system devoted to obtain a cost advantage from a vertically flexible technology and to procure the input from the market (Simester and Knez, 2002). That requires setting up a supply chain of subcontractors, monitoring input quality and contract enforcement, etc. While the costs to keep in operation the internal facilites are fixed the organizational cost grows more than proportionally as the extent of OS tends towards its maximal level. Note, however, that when $\alpha = 1$, the firm buys the input entirely[11] from an independent provider, while keeping the option of returning to complete internal manufacturing.[12]

The increasing cost of recurring to OS may be seen as the mirror image of a (specificity based) hold-up which grows with the share of OS as Transaction Cost Economics (TCE)[13] suggests. Of course generic inputs like, for instance, janitorial services do not require specific know how and cannot be modeled in this way (Anderson and Parker, 2002; Holmes and Thornton, 2008) while for other services flexibility of OS may matter a lot (Benaroch et al., 2012).

The scenario is one of dynamic uncertainty where the input market price c_t follows a geometric Brownian motion:

$$dc_t = \gamma c_t dt + \sigma c dz_t \tag{3}$$

where dz_t is the increment of a Wiener process (or Brownian motion) uncorrelated over time, σ the istantaneous volatility of the market input price and γ the drift parameter. [14]

Finally, as anticipated in the introduction, we assume that the firm may partially finance the required investment for the flexible technology in two alterantive ways. 1) By debt, which may become convertible since it contains an option on the existing shares. 2) By private equity recurring to a joint venture for the production of the input. In both cases the constant discount factor is r. Shareholders, lenders and venture capitalists are all assumed to be risk neutral.[15]

[9] Vertical flexibility, as stressed in the introduction, is an insurance against risk based on the maintanance of the know how and the facilities to produce the input. This assumption allows to focus on different ways to finance and to see how they affect the decision as to whether and when to invest in the flexible technology and the extent of OS.

[10] The sunk cost to build up the mixed technology is assumed quadratic only for the sake of simplicity. None of the results is altered if the investment cost is of type $I(\alpha) = k_1 + k_2\alpha^\delta$ with $\delta > 1$ as in Alvarez and Stenbacka (2007).

[11] This is the case of Benaroch and al. (2012) where entry and exit occurs always with $\alpha = 1$ and there is a fixed cost. In our framework, once the flexible technology has been acquired, the firm can enter and exit without further costs and at any level of α.

[12] We do not consider investment in capacity expansion. We are assuming that capacity is already employed to meet demand in the best way producing the input in-house. Therefore, without loss of generality, if $\alpha \to 0$, we may assume $k_1 = 0$. When $k_1 > 0$ with $\alpha = 0$ we have the standard case where the firm invests in a plant just to produce the input in house. We neglect this case.

[13] TCE emphasizes the role of input specificities and consequent hold-up in OS relationships, which may make input markets less efficient than internal production (Williamson, 1971; Joskow, 2005; Whinston, 2003).

[14] Input price uncertainty may be due to the exchange rate if the input is bought abroad (see Kogut and Kulatilaka, 1994; Dasu and Li, 1997; Kouvelis et al. 2001).

[15] Alternatively, under the assumption of complete capital markets, we can assume that there are some traded assets that can be used to hedge the input cost uncertainty z_t of (3). These traded assets with a riskless asset allow to construct a continuously re-balanced self-financing portfolio that replicates the value of the firm (Constantinides, 1978; Harrison and Kreps, 1979; Cox and Ross, 1976).

3 The benchmark case: an unlevered vertically flexible or pure equity enterprise

As a benchmark, we consider the optimal OS share and the optimal investment policy of a firm entirely financed by equityholders (i.e., the unlevered firm value).

3.1 The operating value

We go through the firm's value function in two distinct cases. In the first we consider a *vertically integrated* firm manufacturing the input in house, if $\hat{c}_t > d$, keeping the option of buying it. In the second case we see an enterprise which adopts *OS*, if $\hat{c}_t < d$, acquiring a share α of the input while making in-house the remaining $1-\alpha$, keeping the option to manufacture the whole input requirement, if $\hat{c}_t$ goes further up. Since for $\alpha > 0$, the condition $\hat{c}_t > d$ implies $c_t > d$, standard arguments lead to a general solution for the unlevered operating firm's value taking the following functional form (See Appendix A):

$$V^U(c_t;\alpha) = \begin{cases} \frac{p-d}{r} + \tilde{A}c_t^{\beta_2} & \text{if } c_t > d \\ \left(\frac{p-(1-\alpha)d}{r} - \frac{\alpha c_t}{r-\gamma}\right) + \tilde{B}c_t^{\beta_1} & \text{if } c_t < d. \end{cases} \tag{4}$$

where $\beta_2 < 0$ and $\beta_1 > 1$ are, respectively, the negative and the positive roots of the characteristic equation: $\Phi(\beta) \equiv \frac{1}{2}\sigma^2\beta(\beta-1) + \gamma\beta - r$.

Notice that $V^U(c_t;\alpha)$ is a convex, decreasing function of c_t, with $V^U(0;\alpha) = \frac{p-(1-\alpha)d}{r}$ and $\lim_{c\to\infty} V^U(c_t;\alpha) = \frac{p-d}{r}$. Moreover, $\frac{p-d}{r}$ and $\left(\frac{p-(1-\alpha)d}{r} - \frac{\alpha c_t}{r-\gamma}\right)$ are the present values of the firm associated to the two distinct vertical arrangements and, as it appears from (4), viable in-house production rules out any closure option or default. Additional terms $\tilde{A}c_t^{\beta_2}$ and $\tilde{B}c_t^{\beta_1}$ indicate respectively the value of the option to go from *vertical integration* to *OS* and the other way round. The constants $\tilde{A}$ and $\tilde{B}$ are positive and equal to:

$$\begin{cases} \tilde{B}(\alpha) = \alpha B \equiv \frac{\alpha}{\beta_1-\beta_2}(r-\gamma\beta_2)d^{1-\beta_1}\frac{1}{r(r-\gamma)} \\ \tilde{A}(\alpha) = \alpha A \equiv \frac{\alpha}{\beta_1-\beta_2}(r-\gamma\beta_1)d^{1-\beta_2}\frac{1}{r(r-\gamma)}. \end{cases} \tag{5}$$

If $\alpha \to 0$, the firm is vertically integrated, $\tilde{A} \to 0$ and also $\tilde{B} \to 0$. If $\alpha \to 1$ the input is bought entirely from an independent provider. Even in this extreme case, the firm has the option to switch to internal production that (represented by $\tilde{B}c_t^{\beta_1}$) makes for a larger value of the firm than without the reversal opportunity. That is, *vertical flexibility increases the value of an unlevered firm.*

3.2 The optimal outsourcing share and investment timing

Let's now derive both the optimal investment timing and the OS share. Since the firm optimally sets the proportion of OS once the investment in the flexible technology is carried out, by working backward we determine the optimal α. We consider the case of a firm manufacturing in-house the input, while holding the option to switch to OS, at a future date, if c_t becomes lower than d.[16] Then, with $c_t > d$, the problem is to select α that maximizes (4) minus the cost of setting up a dedicated production organization consistent with OS, i.e.:

[16] Adopting a different starting point would not make sense since the option to do OS exists only if the firm is not doing it.

$$\alpha^{*U} = \arg\max\left[NPV^U(c_t;\alpha)\right] \tag{6}$$

where $NPV^U(c_t;\alpha) \equiv \frac{p-d}{r} + \tilde{A}c_t^{\beta_2} - I(\alpha)$, and $I(\alpha)$ is given by (1). Solving (6) we get*:*

$$\alpha^{*U}(c_t) = \begin{cases} 1 & \text{if} \quad c_t \leq \tilde{c}^U \\ \frac{A}{k_2}c_t^{\beta_2} & \text{if} \quad c_t > \tilde{c}^U \end{cases} \tag{7}$$

where $\tilde{c}^U \equiv \left(\frac{k_2}{A}\right)^{1/\beta_2}$. Since $\frac{\partial\alpha^{*U}}{\partial c_t} < 0$, if c_t is low it is better to choose complete OS, while, as c_t increases α goes down and tends to zero for high values of c_t. In other words, as c_t rises it becomes less likely that it will fall enough to justify investment in flexibility.

Let's now turn to the optimal investment policy. Denoting by c^{*U} the input price triggering investment in the new technology the ex-ante firm's value is given by:

$$F^U(c_t) = \max_{T^{*U}} E_0[e^{-rT^{*U}}NPV^U(c^{*U}, \alpha^{*U}(c^{*U}))] \tag{8}$$

where $T^{*U} = \inf\{t \geq 0 \mid c_t = c^{*U}\}$ is the optimal investment timing, and $\alpha^{*U}(c^{*U})$ is the optimal OS share at entry**.** The standard method used for V^U can be applied again to find the general solution of (8) and to derive the optimal trigger c^{*U}. In particular assuming that the current value of c_t is sufficiently high so that immediate investment is not optimal, we can prove that:

Proposition 1 *The firm's value can be written in compact notation as:*

$$F^U(c_t) = \begin{cases} F^U c_t^{\beta_2} & \text{for } c_t > c^{*U} \\ \frac{p-d}{r} + \tilde{A}(\alpha^{*U}(c^{*U}))c_t^{\beta_2} - I(\alpha^{*U}(c^{*U})) & \text{for } d < c_t \leq c^{*U} \end{cases}. \tag{9.1}$$

where $F^U = \tilde{A}$*, while the optimal trigger is:*

$$c^{*U} = \left[\frac{\sqrt{2k_2\left(\frac{p-d}{r} - k_1\right)}}{A}\right]^{1/\beta_2} \tag{9.2}$$

and the outsourcing share is

$$\alpha^{*U} = \min\left[\sqrt{\frac{\left(\frac{p-d}{r} - k_1\right)}{k_2/2}}\ ,\ 1\right] \tag{9.3}$$

Proof See Appendix A

As the firm is still able to produce in-house, the ex-ante value of the option to invest in a vertically flexible technology allowing to use a linear combination of in-house produced and procured input is simply the value of the option to do OS, once the flexible technology has been adopted, i.e.:

$$F^U(c_t) = \tilde{A}(\alpha^{*U}(c^{*U}))c_t^{\beta_2} \quad \text{for } c_t > c^{*U}$$

Notice that a solution for c^{*U} exists if $\frac{p-d}{r} - k_1 > 0$[17] Further, recalling that a condition for entry with vertical integration is $c^{*U} > d$, it is evident from (9.2.) and (9.3) that a necessary condition for having $c^{*U} > \tilde{c}^U > d$ and then $\alpha^{*U} < 1$, is $\frac{p-d}{r} - k_1 < \frac{k_2}{2}$. Otherwise it is always better to set $\alpha^{*U} = 1$. In words, if in-house input production leads to profits sufficient to cover organizational costs to buy the entire input requirement by OS, the firm sets $\alpha^{*U} = 1$, otherwise it adopts partial OS. The unlevered firm has to be able to finance OS and the flexible technology out of the cash flow generated when it manufactures the input in-house.

Further, a necessary condition for entry with vertical integration is that the optimal trigger c^{*U} be larger than d, otherwise it would be better to switch to OS immediately. In addition, as $\frac{\partial \alpha^{*U}}{\partial c_t} < 0$, it is evident from (9.2.) and (9.3) that a necessary condition for having $c^{*U} > \tilde{c}^U > d$ and then $\alpha^{*U} < 1$, is $\frac{p-d}{r} - k_1 < \frac{k_2}{2}$. Otherwise it is always better to set $\alpha^{*U} = 1$. In words, if in-house input production leads to profits sufficient to cover organizational costs to buy the entire input requirement by OS, the firm sets $\alpha^{*U} = 1$. Otherwise it adopts partial OS. The unlevered firm has to be able to finance OS and the flexible technology out of the cash flow generated when it manufactures the input in-house.

Notice that, a non negative solution c^{*U} exists if $\frac{p-d}{r} - k_1 > 0$. Finally, we must assume that $\frac{k_1}{\frac{p-d}{r}} \in (0, \frac{1}{2}]$ or $\frac{p-d}{r} > 2k_1$. In other words the share of fixed investment expenditure k_1 should not be too high vis à vis the cash flow under vertical integration.

4 Debt funding with a take over option (warrant)

Going through the case of debt, we assume that the firm negotiates a contract with a (financial) investor to get the funds to cover part of the cost of the flexible technology paying a fixed coupon D per year. Unlike traditional (riskless) debt financing, the firm, so as to incentivate the investor[18] to enter the project, grants the lender a call option to buy out the firm. That may occur if operative profits become very high since the market input price has gone extremely low making and the flexible technology is expected to become useless. This option may be seen as a warrant on the debt, a kind of "sweetener" for the investor.[19].

Since production in-house gives (however small) positive profits, it seems reasonable to assume that the lender, who takes over, to minimize risk continues production with the optimal share decided by the incumbent shareholders.[20] Since the funding contract contains a specific covenant (the warrant) allowing the lender to buy out the firm in certain circumstances, a rational shareholder signs the contract only if the coupon $D < p - d$.

The sequence of moves in this case is the following: first the firm and the lender decide the terms of the contract (i.e., the coupon and the buy out option in the covenant). Then, the firm optimally sets both the level of flexibility α and the investment timing while the lender chooses how much to lend and when to buy out the firm[21].

[17] We assume that this always holds.

[18] And so as to fix the problem of agency associated to lending to a limited liability corporate.

[19] The loan may be considered as a convertible (into equity) debt. To some extent all kinds of debt may be liable to be considered as convertible into something else even if there are infinite types of conversion of debt according to the financial rules and the legal framework in which that occurs. After all each debt implies a collateral, i.e., some kind of pawn.

[20] All results hold even if the lender adopts $\alpha = 1$.

[21] Notice the relevance of the point concerning who sets the timing of the investment. The evaluation of debt may take place in different scenarios. We confine to a simple, realistic, framework where the lender buys out the entire equity adopting previous outsourcing. Of course this is not the only possible scenario, since we may consider cases

4.1 The operating value

As for the benchmark case, we first compute the market value of the production facility which is given by the sum of the market value of equity and of debt. In this case, the instantaneous profit is:

$$\pi_t \equiv [p - d - D + \max(d - \hat{c}_t, 0)] \tag{2bis}$$

where the technology allows the firm to manufacture the input in-house with profits $p - d - D \geq 0$.

4.1.1 Equity

Defining $E(c_t; D)$ as the market value of the equity, the analogous general solution for the value of levered equity is:

Lemma 1 *The value of levered equity (for incumbent shareholders) is:*

$$E(c_t; \alpha) = \begin{cases} \frac{(p-d-D)}{r} + \hat{A}c_t^{\beta_2} & \text{if} \quad c_t > d, \\ \left(\frac{p-(1-\alpha)d-D}{r} - \frac{\alpha c_t}{(r-\gamma)}\right) + \hat{B}_1 c_t^{\beta_1} + \hat{B}_2 c_t^{\beta_2} & \text{if} \quad c^l < c_t < d \\ 0 & \text{if} \quad c_t \leq c^l \end{cases} \tag{10}$$

where $\beta_1 > 1$ and $\beta_2 < 0$ are the roots of the characteristic equation $\Phi(\beta)$ and c^l the level of the input price that triggers the buy out by the lender.

Proof See Appendix B

As before, the terms $\hat{A}c_t^{\beta_2}$and $\hat{B}_1 c_t^{\beta_1}$ indicate respectively the value of the option to go from *vertical integration* to *OS* and the other way round. Differently, the term $\hat{B}_2 c_t^{\beta_2}$ is the loss for the incumbent shareholders when the firm is bought out, therefore $\hat{B}_2 < 0$. This loss can be seen as a kind of *agency cost* (as in Mauer and Sarkar, 2005), that the equity has to pay to the lender. In the absence of any agency fee shareholders would excessively increase debt since they are protected by limited liability. That puts a boundary on losses which cannot exceed equity while leaving to shareholders the opportunity of getting the upside cream, i.e., profits, in bonanza times. In other words, the option of the lender to buy out the leveraged firm decreases the equity market value. $\hat{B}_2 c_t^{\beta_2}$, representing the loss due to the threat of buy out by the lender, is equal to the value of the call option in the hands of the lender who has the right to buy out the firm if c_t goes below c^l.[22]

Furthermore, by the value matching and the smooth pasting conditions at $c_t = d$, we are able to show that (see Appendix B):

$$\hat{A} = \tilde{A} + \tilde{B}_2, \quad \hat{B}_1 = \tilde{B} \quad \text{and} \quad \hat{B}_2 < 0 \tag{11}$$

The constant $\hat{B}_1$ is the same regardless of whether the firm has to decide only the extent of vertical flexibility or the capital structure as well. In other words, **once the investment is undertaken,** the option value of flexibility to go from *OS* to *vertical integration* remains the same regardless of the way it is financed. On the contrary the option value to go OS differs with respect to the unlevered firm, since it carries the risk of being taken over. Now, the constant $\hat{A}$, may even turn out negative. Here, the novelty concerns $\hat{B}_2$ which takes into account the possible buy out by the

in which the option is not to buy the entire equity but just a chunk or cases in which the lender decides to keep flexibility without constraining to entire OS for ever.

[22] c^l must be lower than the internal cost of production d for the buy out to make sense.

lender if the input price goes below the threshold c^l. If the take over threat is not high (i.e., $c^l \to 0$) the option value of OS is definitely positive while, if the threat is quite high, it is not profitable to do OS and the relative option suffers. Then, the firm must consider the effect on its equity value of financing OS and flexibility with debt.

4.1.2 Debt

Defining with $D(c_t;\alpha)$ the market value of debt, since it has no stated maturity, will be given by:

Lemma 2 *The value of debt is:*

$$D(c_t;\alpha)=\begin{cases}\frac{D}{r}+Cc_t^{\beta_2} & \text{if} \quad c_t>c^l,\\ \frac{p-(1-\alpha)d}{r}-\frac{\alpha c_t}{r-\gamma} & \text{if} \quad c_t\le c^l.\end{cases} \tag{12}$$

where $C=-\frac{1}{\beta_2-1}\left[\frac{p-(1-\alpha)d-D}{r}\right](c^l)^{-\beta_2}$, *while the buy out trigger is:*

$$c^l=\frac{\beta_2}{\beta_2-1}\frac{(r-\gamma)}{\alpha}\left[\frac{p-(1-\alpha)d-D}{r}\right]>0 \tag{13}$$

Proof See Appendix B

Since the term $Cc_t^{\beta_2}$ indicates the lender's value of the option to acquire the firm, the constant C must be positive. The take over occurs when the flexible technology is expected to become useless and the new owners produce with the optimal share decided earlier by the firm. The lender buys out the firm when the market input price has gone substantially low to suggest that it will be better to buy the input, rather than producing it, for ever. The fresh owner that has bought out the firm by exercising the option will behave like the former shareholders in terms of optimal strategies adopted by the firm. The assumption that the new enterpreneur sticks to the vertical setting of its predecessors is a simplication. Other possible scenarios may be featured.

Some comparative statics shows that:

$$\frac{\partial c^l}{\partial \alpha}<0 \qquad \text{and} \qquad \frac{\partial c^l}{\partial D}<0$$

The negative relationship between c^l and α shows the countervailing interests of the firm vis à vis the lender. If the firm sets a low level of α (i.e., it tends to be vertically integrated), the lender would find it profitable to buy the firm, i.e., $c^l \to d$. On the contrary, if the firm adopts a high α (i.e., the input is bought mainly from an independent provider), the lender prefers not to bear the risk and sticks to the coupon D. If α is high the benefit of keeping the facility to produce in-house has a low value. As for the second comparative statics inequality it appears that an increase in the coupon (the benefit for the lender) lets the trigger price decrease, i.e., the take over becomes less likely. With a larger coupon the lender gets a higher compensation that relaxes the take over threat and is less eager to buy out the firm by converting debt into equity. The assumption $p-d-D>0$ guarantees that both c^l and C are positive.

Finally, by Lemma 1 and 2, the market value of the levered firm is given by:

$$\begin{aligned} V^L(c_t;\alpha) &= E(c_t;\alpha)+D(c_t;\alpha) \qquad (14)\\ &= \begin{cases}\frac{p-d}{r}+\hat{A}c_t^{\beta_2}+Cc_t^{\beta_2} & \text{if} \quad c_t>d,\\ \frac{p-(1-\alpha)d}{r}-\frac{\alpha c_t}{r-\gamma}+\hat{B}_1c_t^{\beta_1}+\hat{B}_2c_t^{\beta_2}+Cc_t^{\beta_2} & \text{if} \quad c^l<c_t<d\\ \frac{p-(1-\alpha)d}{r}-\frac{\alpha c_t}{r-\gamma} & \text{if} \quad c_t\le c^l\end{cases}\end{aligned}$$

where, using (11) and (12), we are now able to isolate the constants $\hat{A}$ and $\hat{B}_2$. In particular we get (See Appendix B):

$$\hat{A} = \tilde{A} - \tilde{B}c^{l(\beta_1-\beta_2)} - C, \quad \text{and} \quad \hat{B}_2 = -\tilde{B}c^{l(\beta_1-\beta_2)} - C \tag{15}$$

Note that, if $c^l \to 0$ (the firm is never bought by the lender), then $\hat{A} \to \tilde{A} > 0$ is always positive and $\hat{B}_2 \to 0$. We are back to the unlevered firm as in Section 4. On the contrary, if $c^l \to d$ (the firm is bought the first time it does OS), then we have $\hat{A} \to -\alpha d\frac{\gamma}{(r-\gamma)}\frac{1}{r}d^{-\beta_2} - C < 0$ which is always negative as well as $\hat{B}_2$. This is a crucial result: even if an option value is, by definition, always non negative, it is possible that the cost of obtaining such an option exceeds its benefits making the strategic value of the option negative. In this case, the cost of the option handed over to the lender may rub out the value of the option to go from *vertical integration* to *outsourcing* reducing the equity value of the firm for shareholders.This is evident by substituting (15) in (14), i.e.:

$$V^L(c_t;\alpha) = V^U(c_t;\alpha) - E_t[e^{-r(T^l-t)}]\tilde{B}c^{l\,\beta_1} \qquad \text{for } c_t > d, \tag{16}$$

where $T^l = \inf\{t \geq 0 \mid c_t = c^l\}$ is the buy out timing.[23] The levered firm is equal to the unlevered firm minus the discounted value of the option to go from *outsourcing* to *vertical integration* calculated at the buy out time. As expected, the value of the firm does not depend on debt but on the covenant contained in the contract which corresponds to a shut down option. If this option is canceled the value of the firm in the two cases of debt and equity is the same, i.e., $V^L(c_t;\alpha) = V^U(c_t;\alpha)$ since the coupon disappears when we sum debt and equity values to obtain the entire value of the firm.

The value of the firm is lower with debt due to the (strategic) interaction between the lender and the shareholders. If such an interaction did not exist the value of the firm would be the sum of debt and equity and the use of debt would not erode the value of equity. The cost of the collateral option that the lender requires reduces the value of the equity since the lender does not accept to be paid back simply with the coupon. If the OS decision was to be taken by a social planner this cost would disappear since the equity and the debt holder would coincide and the equity would not have any incentive to invest too much as it occurs in the case of limited liability.

4.2 The optimal outsourcing share and the investment timing

Since equityholders control both the decision about the outsourcing share and the timing of the investment we proceed stating first α^{*L} and then the optimal investment trigger c^{*L}. To get the optimal α^{*L}, equityholders maximize (10) minus the cost of setting up the production organization with partial outsourcing:

$$\alpha^{*L} = \arg\max\left[NPV^L(c_t,\alpha)\right] \tag{17}$$

where $NPV^L(c_t,\alpha) = E(c_t;\alpha) - (I(\alpha) - k)$ and $k \leq k_1$ is the share of the investment expenditure paid by the lender who controls the amount to loan and the buy out timing. A rational investor will not agree to finance the firm unless k is a (financially) fair price for the debt, then we set

[23] The expected present value $E_t[e^{-r(T^l-t)}] = \left(\frac{c_t}{c^l}\right)^{\beta_2}$, can be determined by using dynamic programming (see e.g. Dixit and Pindyck, 1994, pp. 315-316).

$k(c_t) = D\,(c_t;\alpha)$ for $c_t > c^l$.[24] Substituting, we get:

$$\alpha^{*L} = \arg\max\left[V^L(c_t;\alpha) - k_1 - \frac{k_2}{2}\alpha^2\right] \tag{18}$$

where $V^L(c_t;\alpha)$ is given by (14).

As before, let's consider a firm manufacturing in-house the input, while holding the option to switch to partial outsourcing. Solving (18) the optimal outsourcing share is given by:

$$[A - S(\alpha^{*L})]c_t^{\beta_2} - k_2\alpha^{*L} = 0 \tag{19}$$

where A is as in (5) and $S(\alpha) = Bc^{l\beta_1-\beta_2}\left(1-(\beta_1-\beta_2)\frac{p-d-D}{p-(1-\alpha)d-D}\right) < 0$. Since $\frac{\partial\alpha^{*L}}{\partial c_t} < 0$, if c_t is low it is better to choose complete outsourcing, while, as c_t increases α goes down and tends to zero for high values of c_t. Further, if $c^l \to 0$, $S(\alpha) \to 0$, and then $\alpha^{*L} \to \alpha^{*U}$.

Defining with $F^L(c_t)$ the value of the option to invest in the vertically flexible technology this is equal to (8) with $T^{*L} = \inf\{t \geq 0 \mid c_t = c^{*L}\}$ as the optimal investment timing. Then, going through the same steps as before, we can prove that:

Proposition 2 *The firm's value can be written as:*

$$F^L(c_t) = \begin{cases} F^L c_t^{\beta_2} & \text{for } c_t > c^{*L} \\ \frac{p-d}{r} + \hat{A}(\alpha^{*L}(c^{*L}))c_t^{\beta_2} + C(\alpha^{*L}(c^{*L}))c_t^{\beta_2} - I(\alpha^{*L}(c^{*L})) & \text{for } d < c_t \leq c^{*L} \end{cases}. \tag{20.1}$$

where $F^L = [\tilde{A}(\alpha^{*L}(c^{*L})) - \tilde{B}(\alpha^{*L}(c^{*L})](c^l)^{(\beta_1-\beta_2)}$*, while the optimal trigger is:*

$$c^{*L} = \left[\frac{\sqrt{2k_2\left(\frac{p-d}{r} - k_1\right)}}{A - S(\alpha^{*L})}\right]^{1/\beta_2} \tag{20.2}$$

and α^{*L}*, by (19), becomes*

$$\alpha^{*L} = \min\left[\sqrt{\frac{\left(\frac{p-d}{r} - k_1\right)}{k_2/2}}\ ,\ 1\right] \tag{20.3}$$

Proof : See Appendix C

Substituting F^L in (20.2), we can write the ex-ante value of the firm in the form:

$$F^L(c_t) = \tilde{A}(\alpha^{*L}(c^{*L}))c_t^{\beta_2} - E_t[e^{-r(T^l-t)}]\tilde{B}(c^l)^{\beta_1} \quad \text{for } c_t > c^{*L} \tag{21}$$

Notice that, unlike the case of pure equity, if shareholders keep the possibility to decide both the optimal OS and the timing of the investiment, the value of investing in the new technology comes from the value of the option to do OS minus the value of the option to exit held by debt holders. Finally, by direct inspection of (9.2), (21.2) and (9.3), (21.3) the following proposition summarizes the comparison with respect to the unlevered firm.

[24] First the deal between the two parties is signed and the coupon is defined. Then, the lender chooses the amount of the loan at the moment the firm decides to invest. As in Mauer and Sarkar (2005) the contract may be seen as a revolving credit line where the firm decides when to use it.

Proposition 3 *The levered firm invests always earlier than the unlevered firm, i.e. :*

$$c^{*L} \geq c^{*U} \tag{22.1}$$

but adopts the same proportion of outsourced input, i.e.:

$$\alpha^{*L} = \alpha^{*U} \tag{22.2}$$

Proof : *Straightforward*

As it appears, while the levered firm adopts at least the same proportion of outsourced input it invests earlier. Motivations are quite simple. If the levered firm decides both α^{*L} and c^{*L} by maximizing the value of equity, it does not care of the risk carried by the lender. Since part of the investment is paid by the lender the risk born by the equityholders is just the buy out option in the hands of the lender. In this case the equityholders have an incentive to invest as soon as possible to get a higher loan and reap the profits of outsourcing. The lender is a kind of contingent equity holder since he owns an option on the entire firm. If the option were only on a share of the firm α would increase.

5 Venture capitalist involvment

Now let us assume that the firm offers to an outside investor (a venture capitalist or simply to a private equity), a share of profits $\psi \in (0, 1)$ without side payments if the venture capitalist finances the flexible technology for an amount up to k_1. This is just a take or leave offer. The venture capitalist may accept the offer together with the option to optimally decide when to initiate the deal. The firm, i.e., the incumbent equity holders decide both ψ and α. As it appears the decision setting and its distributive features change with respect to the case of debt seen before, where the incumbent equity holders retained the decision on the timing of the investment, while here this belongs with the venture capitalist.

Let's go through the sequence of moves: equityholders offer ψ. The venture capitalist decides when to enter and invest accepting that the incumbent equity holders set α. We assume that entry takes place as usual at $c_t > d$ with the firm initially producing the input in-house with the option to activate outsourcing and revert to backsourcing later on if required by market circumstances.

As usual, we proceed backward: equityholders decide the outsourcing share conditional on c_t. The venture capitalist knows the reaction function $\alpha^{*V}(c_t)$ and decides the optimal trigger c^{*V}. Equityholders may anticipate their offer of ψ that could be announced even before entry takes place, i.e. at $t = 0$.

The problem for the equityholders is to select the optimal α that maximizes (4) minus the cost of the technology after the financial cost:

$$\alpha^{*V} = \arg\max \left[(1-\psi)V^U(c_t;\alpha) - (I(\alpha) - k_1)\right] \tag{23}$$

Solving (23) yields:

$$\alpha^{*V}(c_t) = \begin{cases} 1 & \text{if} \quad c_t \leq \tilde{c}^V \\ \frac{(1-\psi)A}{k_2} c_t^{\beta_2} & \text{if} \quad c_t > \tilde{c}^V \end{cases} \tag{24}$$

where $\tilde{c}^V \equiv \left(\frac{k_2}{(1-\psi)A}\right)^{1/\beta_2} \leq \tilde{c}^U$.

Now, defining $F^V(c_t)$ as the value of the option to invest by the venture capitalist and $T^{*V} = \inf\{t \geq 0 \mid c_t = c^{*V}\}$ the venture capitalist's optimal investment timing, it is easy to prove that:

Proposition 4 *The firm's value is equal to:*

$$F^V(c_t) = \begin{cases} F^V c_t^{\beta_2} & \text{for } c_t > c^{*V} \\ \psi\left(\frac{p-d}{r} + \tilde{A}(\alpha^{*L}(c^{*L}))c_t^{\beta_2}\right) - k_1 & \text{for } c_t \le c^{*V} \end{cases}. \tag{26.1}$$

where $F^V = 2\psi\tilde{A}(\alpha^{*L}(c^{*L}))$*, while the optimal trigger is:*

$$c^{*V} = \left[\frac{\sqrt{k_2\left(\frac{p-d}{r} - \frac{k_1}{\psi}\right)}}{A}\right]^{1/\beta_2} \tag{26.2}$$

and the outsourcing share is

$$\alpha^{*V} = \min\left[\sqrt{\frac{\frac{1-\psi}{\psi}\left(\psi\frac{p-d}{r} - k_1\right)}{k_2}}\quad , \quad 1\right] \tag{26.3}$$

Proof : See Appendix D

As before substituting F^V in (26.1) we see that the value of the option to invest is equal to the option to go outsourcing multiplied by 2ψ. So only if $\psi = \frac{1}{2}$ the shareholders and the venture capitalist evenly split the value of the firm and the value of the option to invest is equal to the value of the option to outsource.

About the optimal trigger, notice that, unlike previous cases, the condition for its existence and finiteness is $\psi \ge \frac{k_1}{\frac{p-d}{r}}$, while the necessary condition for having $c^{*V} > \tilde{c}^V$ and then $\alpha^{*U} < 1$, is now $\psi\frac{p-d}{r} - k_1 < \frac{1-\psi}{\psi}\frac{k_2}{2}$.

Moreover, it is easy to show that $\frac{\partial c^{*V}}{\partial\psi} < 0$ and $\frac{\partial\alpha^{*V}}{\partial\psi} > 0$ for $\psi \in (\frac{k_1}{\frac{p-d}{r}}, \sqrt{\frac{k_1}{\frac{p-d}{r}}})$. On the contrary $\frac{\partial c^{*V}}{\partial\psi} \ge 0$ and $\frac{\partial\alpha^{*V}}{\partial\psi} \le 0$ for $\psi \in [\sqrt{\frac{k_1}{\frac{p-d}{r}}}, 1]$. The intuition suggests that if equity holders give to the venture capitalist a small profit share, this makes them to adopt a high level of flexibility since, for each level of investment, risk is transferred to the venture capitalist. The external investor will hedge by delaying investment up to the point where the market input price is sufficiently low. The opposite holds if the firm offers $\psi > \sqrt{\frac{k_1}{\frac{p-d}{r}}}$. This is summarized in the following proposition:

Proposition 5 *If the flexible technology is partially financed by a venture capitalist, then:*

$$\begin{aligned} c^{*V} &\ge c^{*U} \quad \text{for } \psi \in [\frac{k_1}{\frac{p-d}{r}}, \psi_1] \\ c^{*V} &< c^{*U} \quad \text{for } \psi \in (\psi_1, 1] \end{aligned} \tag{27.1}$$

where ψ_1 *is the positive root of* $\Psi(\psi) = 2\psi^2\left(\frac{p-d}{r} - k_1\right) - 2\psi\left(\frac{p-d}{r} - k_1\right) + \psi\frac{p-d}{r} - k_1$.

While, the optimal outsourcing level is:

$$\alpha^{*V} < \alpha^{*U} \quad \text{for all } \psi \in [\frac{k_1}{\frac{p-d}{r}}, 1] \tag{27.2}$$

Proof : See Appendix E

When ψ is low, i.e. $\psi \in [\frac{k_1}{\frac{p-d}{r}}, \psi_1]$, the joint venture enters earlier than the unlevered firm. In this case the venture capitalist is better off if he enters earlier to anticipate the time he will receive the profits (even if the share is small). On the contrary, if ψ is high, i.e., $\psi \in (\psi_1, 1]$, the venture capitalist enters later than the unlevered firm waiting for expected higher profits. That is, the larger is the share of the venture capitalist the more prudential the firm becomes as to investing in flexibility. Or, in other words, the value of waiting increases and the venture capitalist waits longer. In both cases, however, the shareholders choose a lower level of OS with respect to the unlevered firm because they will never reap the entire benefits of flexibility since as $\psi > 0$.

An open question is the determination of the share parameter ψ. The incumbent equityholders may set ψ, maximizing the portion of value they keep.[25] In this case the incumbent shareholders may announce ψ (i.e., static ultimatum game) at any time before the optimal investment timing chosen by the venture capitalist, i.e.,:

$$\max_{\psi} E_t \left[e^{-r(T^V - t)} \left((1-\psi)\frac{p-d}{r} + (1-\psi)\tilde{A}(\alpha^{*V}(c^{*V}))(c^{*V})^{\beta_2} - \frac{k_2}{2}(\alpha^{*V}(c^{*V}))^2 \right) \right].$$

.

Then we may write the following:

Proposition 6 *Under the assumption* $\frac{k_1}{\frac{p-d}{r}} \leq \frac{1}{2}$, *if the flexible tecnology is partially financed by a venture capitalist, while the incumbent shareholders decide the share of profits and the level of outsourcing, we obtain:*

$$\frac{k_1}{\frac{p-d}{r}} < \psi^* < \frac{1}{2}$$

$$\alpha^{*V} < \alpha^{*U}.$$

$$c^{*V} > c^{*U}$$

Proof : See Appendix F.

6 Epilogue

We have considered a firm that has to decide simultaneously the internal vertical setting and the financial structure in a dynamic stochastic framework. The firm we analyse is vertically flexible since it has an option to outsource entirely or partially a necessary input and it can reverse its choice by going back to in-house production, i.e., vertical integration. Unlike recent literature (Benaroch et al. 2012) we have not examined the choice of complete outsourcing vis à vis vertically integrating, yet we have gone through some financial aspects of a vertically flexible corporate organization where partial and reversible outsourcing are possible.

Flexibility comes with a cost required to set up a suitable supply chain and to keep alive the know how and the facilities to backsource the input in case market circumstances require to do so. We consider two possible financial avenues for the vertically flexible firm. First we study the case

[25] In a different environment Banerjee et al. (2014) introduce a bargaining as to the share parameter and find that it is inefficient to set it before the investment because of the emerging time inconsistency. Only a bargaining carried out after the investment may assure temporal efficiency.

of debt financing. A lender may be willing to finance the firm that invests in flexibility if she gets a suitable "sweetener" such as an option to buy out the firm in case flexibility becomes useless. The option is required to make the lender willing to finance the corporate firm where limited liability may induce the incumbent equityholders to overinvest. In such circumstances the firm rushes to invest earlier with respect to a corresponding pure equity firm which is not levered. The levered firm decides the level of outsourcing and the timing of the invrestment while the lender sets only the size of the investment and the buy out time. Vertical flexibility is a cushion against risk but it is costly. If financial providers require collaterals which are too expensive it may not be worth. In such a case the value of a levered flexible firm may be lower than the value of an unleverd vertically unflexible firm and the strategic value of the option to become flexible may turn negative. We went through a second possible financial arrangement for the vertically flexible firm considering a venture capitalist financing the production of the necessary input. In this case it appears that the level of ousoucing is lower than in the case of the unlevered firm and the investment takes place later. As the share of the firm offered to the venture capitalist decreases ($\psi- > 0$) the behaviour of the firm converges to the unlevered case.

In the end we may say that financing flexibility with a warranted debt - the only one that is consistent with an efficient allocation of debt in the presence of limited liability - induces the firm to invest earlier but not more than the unlevered firm. Then, debt makes a firm more eager to go flexible to anticipate reaping expected profits. This is consistent with common observation suggesting that debt may prompt innovation in organizational flexibility. As for the venture capital it tends to increase the prudential stance of the firm in terms of the amount committed to the investment in flexibility. If the venture capitalist has a high share of the project it will invest earlier than the unlevered firm, otherwise it starts later.

A Proof of Proposition 1

The standard arbitrage and hedging arguments require that the *vertically flexible* firm value, $V^U(c_t;\alpha)$, is the solution of the following dynamic programming problems:

$$\Gamma V^U(c_t;\alpha) = -(p-d), \qquad for \;\; c_t > d \tag{A.1}$$

and

$$\Gamma V^U(c_t;\alpha) = -(p-\alpha c_t-(1-\alpha)d), \qquad for \;\; c_t < d, \tag{A.2}$$

where Γ is the differential operator: $\Gamma = -r + \gamma c\frac{\partial}{\partial c} + \frac{1}{2}\sigma^2 c^2 \frac{\partial^2}{\partial c^2}$. The solution of the differential equations (A.1) and (A.2) requires the following boundary conditions:

$$\lim_{c\to\infty}\left\{V^U(c_t;\alpha) - \frac{p-d}{r}\right\} = 0 \qquad \text{if } c_t > d$$

and

$$\lim_{c\to 0}\left\{V^U(c_t;\alpha) - \left(\frac{p-(1-\alpha)d}{r} - \frac{\alpha c_t}{r-\gamma}\right)\right\} = 0, \quad \text{if } c_t < d$$

where $\frac{p-d}{r}$ is the present value of the firm “making” the input, while $\left(\frac{p-(1-\alpha)d}{r} - \frac{\alpha c_t}{r-\gamma}\right)$ is the present value when “buying” a share α of the input. Then, from the assumptions and the linearity of the differential equations (A.1) and (A.2), using the above boundary conditions, we get:

$$V^U(c_t;\alpha) = \begin{cases} \frac{p-d}{r} + \tilde{A}c_t^{\beta_2} & \text{if } c_t > d \\ \left(\frac{p-(1-\alpha)d}{r} - \frac{\alpha c_t}{r-\gamma}\right) + \tilde{B}c_t^{\beta_1} & \text{if } c_t < d. \end{cases} \tag{A.3}$$

where $\beta_2 < 0$ and $\beta_1 > 1$ are, respectively, the negative and the positive roots of the characteristic equation: $\Phi(\beta) \equiv \frac{1}{2}\sigma^2\beta(\beta-1) + \gamma\beta - r$. Finally, by the value matching and the smooth pasting conditions at $c_t = d$ we obtain the two constants (Dixit and Pyndyck,1994, p. 189):

$$\begin{cases} \tilde{B} = \alpha B \equiv \frac{\alpha}{\beta_1-\beta_2}(r-\gamma\beta_2)d^{1-\beta_1}\frac{1}{r(r-\gamma)} \\ \tilde{A} = \alpha A \equiv \frac{\alpha}{\beta_1-\beta_2}(r-\gamma\beta_1)d^{1-\beta_2}\frac{1}{r(r-\gamma)}. \end{cases} \tag{A.4}$$

which are always nonnegative and linear in α.

Since $\tilde{A} = \alpha A$, the optimal vertical arrangement is given by:

$$\begin{aligned} \alpha^{*U} &= \arg\max\left[NPV^{VI}(c_t,\alpha)\right] \\ &= \arg\max\left[\frac{p-d}{r} + \alpha A c_t^{\beta_2} - k_1 - \frac{k_2}{2}\alpha^2\right]. \end{aligned} \tag{A.5}$$

Then, the FOC is:

$$Ac_t^{\beta_2} - k_2\alpha = 0 \tag{A.6}$$

while the SOC is always satisfied. From (A.6) we obtain (7) in the text:

$$\alpha^{*U}(c_t) = \begin{cases} 1 & \text{if} \quad c_t \leq \tilde{c}^U \\ \frac{A}{k_2}c_t^{\beta_2} & \text{if} \quad c_t > \tilde{c}^U \end{cases}$$

where $\tilde{c}^U \equiv \left(\frac{k_2}{A}\right)^{1/\beta_2}$.

Let's now consider the firm's ex-ante value $F^U(c_t)$. In the range of c_t where the option to wait to invest is positive $F^U(c_t)$ is still given by the solution of the following dynamic programming problem:

$$\Gamma F^U(c_t) = 0, \qquad \text{for } \; c_t > c^{*U} \tag{A.7}$$

where c^{*U} is the threshold at which it is efficient to invest. Since when c_t approaches infinity $F^U(c_t)$ should go to zero, the solution of (A.7) requires the boundary condition, $\lim_{c\to\infty} F^U(c_t) = 0$. By the linearity of the differential equation (A.7) and using the boundary condition, we obtain:

$$F^U(c_t) = F^U c_t^{\beta_2}. \tag{A.8}$$

where β_2 is the negative root of $\Phi(\beta)$. To evaluate the constant F^U and the optimal entry trigger c^{*U}, the $F^U(c_t)$ must satisfy the matching value and smooth pasting conditions:

$$F^U(c^{*U}) = NPV^U(c^{*U}, \alpha^{*U}(c^{*U})), \tag{A.9.1}$$

$$F_c^U(c^{*U}) = NPV_c^U(c^{*U}, \alpha^{*U}(c^{*U})), \tag{A.9.2}$$

where the second equality follows from $NPV_\alpha^U(c^{*U}, \alpha^{*U}(c^{*U})) = 0$ by (A.5). Conditions (A.9.1) and (A.9.2) say that the optimal share of outsourcing α is set when the investment takes place. Substituting (A.8) into (A.9.1) and (A.9.2) we obtain:

$$F^U(c^{*U})^{\beta_2} = \frac{p-d}{r} + \tilde{A}(\alpha^{*U}(c^{*U}))(c^{*U})^{\beta_2} - k_1 - \frac{k_2}{2}(\alpha^{*U}(c^{*U}))^2$$

$$\beta_2 F^U(c^{*U})^{\beta_2 - 1} = \beta_2 \tilde{A}(\alpha^{*U}(c^{*U}))(c^{*U})^{\beta_2 - 1}$$

from which we get:

$$F^U = \tilde{A}(\alpha^{*U}(c^{*U})) \tag{A.10.1}$$

and

$$\frac{k_2}{2}(\alpha^{*U}(c^{*U}))^2 = \frac{p-d}{r} - k_1 \tag{A.10.2}$$

Simply substituting (A.6) in (A.10.2) we obtain:

$$c^{*U} = \left[\frac{\sqrt{2k_2\left(\frac{p-d}{r} - k_1\right)}}{A} \right]^{1/\beta_2} \qquad \text{and} \quad \alpha^{*U} = \min\left[\sqrt{\frac{\left(\frac{p-d}{r} - k_1\right)}{k_2/2}} \;,\; 1 \right] \tag{A.11}$$

from which it is easy to show that $c^{*U} > \tilde{c}^U$ if $\frac{p-d}{r} - k_1 < \frac{k_2}{2}$.

B Proof of Lemmas 1 and 2

The differential equation describing the market value of equity is the same as that in (A.1) and (A.2), except that the cash flow accruing to equityholders is now $p-d-D$ for $c_t > d$ and $p-D-\alpha c_t-(1-\alpha)d$ for the case $c^l \leq c_t < d$, where c^l is the input price triggering the lender to buy out the firm. The general solution can be expressed as:

$$p\frac{(p-d-D)}{r} + \hat{A}c_t^{\beta_2} \quad \text{if} \quad c_t > d, \tag{B.1}$$

and:

$$\left(\frac{p-(1-\alpha)d-D}{r} - \frac{\alpha c_t}{(r-\gamma)} \right) + \hat{B}_1 c_t^{\beta_1} + \hat{B}_2 c_t^{\beta_2} \quad \text{if} \quad c^l < c_t < d \tag{B.2}$$

By the value matching and the smooth pasting conditions at $c_t = d$ and the boundary condition $E\left(c^l;\alpha\right) = 0$, we get the system:

$$\frac{(p-d-D)}{r} + \hat{A}d^{\beta_2} = \left(\frac{p-(1-\alpha)d-D}{r} - \frac{\alpha d}{r-\gamma} \right) + \hat{B}_1 d^{\beta_1} + \hat{B}_2 d^{\beta_2} \tag{B.3.1}$$

and,

$$\beta_2 \hat{A} d^{\beta_2 - 1} = -\frac{\alpha}{r-\gamma} + \beta_1 \hat{B}_1 d^{\beta_1 - 1} + \beta_2 \hat{B}_2 d^{\beta_2 - 1} \tag{B.3.2}$$

and,

$$\left(\frac{p-(1-\alpha)d-D}{r} - \frac{\alpha c^l}{r-\gamma} \right) + \hat{B}_1 c^{l\beta_1} + \hat{B}_2 c^{l\beta_2} = 0 \tag{B.3.3}$$

Solving the system made by (B.3.1) and (B.3.2), we obtain:

$$\hat{B}_1 = \tilde{B} = \frac{1}{(\beta_1 - \beta_2)} \frac{r - \gamma\beta_2}{r} \frac{\alpha d^{1-\beta_2}}{(r-\gamma)} \tag{B.4}$$

$$\hat{A} = \tilde{A} + \hat{B}_2 = \frac{\alpha d^{1-\beta_2}}{(r-\gamma)} \left[\frac{r - \gamma\beta_1}{r(\beta_1 - \beta_2)} \right] + \hat{B}_2 \tag{B.5}$$

Let us consider now the debt. Similarly to equity, it must satisfy the following differential equations:

$$\Gamma D(c_t; \alpha) = -D, \qquad for \;\; c_t > c^l, \tag{B.6.1}$$

and

$$\Gamma D(c_t; \alpha) = -(p - \alpha c_t - (1-\alpha)d), \qquad for \;\; c_t \leq c^l, \tag{B.6.2}$$

with the following boundary conditions:

$$\lim_{c \to \infty} \left\{ D(c_t; \alpha) - \frac{D}{r} \right\} = 0$$

and

$$\lim_{c \to c^l} \left\{ D(c_t; \alpha) - \left(\frac{p - (1-\alpha)d}{r} - \frac{\alpha c_t}{r - \gamma} \right) \right\} = 0.$$

The solution is:

$$D(c_t; \alpha) = \begin{cases} \frac{D}{r} + C c_t^{\beta_2} & \text{if} \quad c_t > c^l, \\ \frac{p-(1-\alpha)d}{r} - \frac{\alpha c_t}{r-\gamma} & \text{if} \quad c_t \leq c^l. \end{cases} \tag{B.7}$$

By imposing the value matching and the smooth pasting conditions at $c_t = c^l$ we obtain:

$$c^l = \frac{\beta_2}{\beta_2 - 1} \frac{r-\gamma}{\alpha} \left[\frac{p - (1-\alpha)d - D}{r} \right] \tag{B.8}$$

and:

$$C = -\frac{1}{\beta_2 - 1} \left[\frac{p - (1-\alpha)d - D}{r} \right] (c^l)^{-\beta_2} > 0 \tag{B.9}$$

Finally, substituting (B.4) and (B.9) in (B.3.3) we get $C c^{l\beta_2} + \tilde{B} c^{l\beta_1} + \hat{B}_2 c^{l\beta_2} = 0$. Since the first and the second terms are positive, the equality is satisfied only if:

$$\hat{B}_2 < 0. \tag{B.10}$$

Finally, from (B.5) and (B.10) we are able to isolate $\hat{A}$ and $\hat{B}_2$ respectively, i.e.:

$$\hat{A} = \alpha A - \alpha B c^{l(\beta_1 - \beta_2)} - C. \tag{B.11}$$

and

$$\hat{B}_2 = -C - \alpha B c^{l(\beta_1 - \beta_2)} \tag{B.12}$$

If $c^l \to 0$ then $\hat{A} \to \alpha A > 0$ and $\hat{B}_2 \to 0 > 0$, we are back to the unlevered firm. If $c^l \to d$ we have:

$$\hat{A} \to -\alpha d \frac{\gamma}{(r-\gamma)} \frac{1}{r} d^{-\beta_2} + \frac{p - (1-\alpha)d - D}{r} \frac{d^{-\beta_2}}{\beta_2 - 1} < 0$$

which is always negative.

C Proof of Proposition 2

By (B.4), (B.5) and (B.9), we note that $\hat{A} + C = \tilde{A} + \hat{B}_2 + C = \alpha[A - Bc^{l(\beta_1-\beta_2)}]$. Then, by (14) the optimal vertical arrangement is given by:

$$\alpha^{*L} = \arg\max \left[\frac{p-d}{r} + \alpha[A - Bc^{l(\beta_1-\beta_2)}] - k_1 - \frac{k_2}{2}\alpha^2\right] \tag{C.1}$$

and the FOC is:

$$Ac_t^{\beta_2} - k_2\alpha - Bc^{l\beta_1-\beta_2}\left(1 - (\beta_1-\beta_2)\frac{p-d-D}{p-(1-\alpha)d-D}\right)c_t^{\beta_2} = 0 \tag{C.2}$$

Defining $S(\alpha) = Bc^{l\beta_1-\beta_2}\left(1 - (\beta_1-\beta_2)\frac{p-d-D}{p-(1-\alpha)d-D}\right) < 0$, we are able to reduce (C.2) to:

$$[A - S(\alpha^{*L})]c_t^{\beta_2} - k_2\alpha^{*L} = 0 \tag{C.3}$$

We go through the SOC:

$$\begin{aligned}\frac{\partial FOC}{\partial \alpha} &= -k_2 - \frac{r-\gamma\beta_2}{r}\frac{1}{(\beta_1-\beta_2)}\frac{1}{(r-\gamma)}d^{1-\beta_1}(\beta_1-\beta_2)c^{*\beta_1-\beta_2}c_t^{\beta_2}\\ &\quad \left[c^{*-1}\left(1-\left(\frac{p-d-D}{p-(1-\alpha)d-D}\right)\right) - \left(\frac{d(d+D-p)}{(p-(1-\alpha)d-D)^2}\right)\right]\end{aligned}$$

The sign depends on:

$$(1-\left(\frac{p-d-D}{p-(1-\alpha)d-D}\right)) - \left[\frac{d(d+D-p)}{(p-(1-\alpha)d-D)^2}\right]$$

which is always positive, making for a verified SOC.

We define $F^L(c_t)$ as the value of the option to invest by the levered firm. The constant F^L and the optimal trigger c^{*L} must satisfy the matching value and smooth pasting conditions:

$$F^L(c^{*L})^{\beta_2} = \frac{p-d}{r} + \tilde{A}(\alpha^{*L}(c^{*L}))(c^{*L})^{\beta_2} - \left(\frac{c^{*L}}{c^l}\right)^{\beta_2}\tilde{B}(\alpha^{*L}(c^{*L}))c^{l\beta_1} - k_1 - \frac{k_2}{2}(\alpha^{*L}(c^{*L}))^2 \tag{C.4}$$

$$F^L\beta_2(c^{*L})^{\beta_2-1} = \tilde{A}(\alpha^{*L}(c^{*L}))\beta_2(c^{*L})^{\beta_2-1} - \beta_2\left(\frac{c^{*L}}{c^l}\right)^{\beta_2-1}\frac{1}{c^l}\tilde{B}(\alpha^{*L}(c^{*L}))c^{l\beta_1} \tag{C.5}$$

from which we obtain:

$$F^L = [\tilde{A}(\alpha^{*L}(c^{*L})) - \tilde{B}(\alpha^{*L}(c^{*L})]c^{l(\beta_1-\beta_2)} \tag{C.6}$$

$$\frac{k_2}{2}(\alpha^{*L}(c^{*L}))^2 = \frac{p-d}{r} - k_1. \tag{C.7}$$

Simply, substituting (C.3) in (C.7) we obtain:

$$c^{*L} = \left[\frac{\sqrt{2k_2\left(\frac{p-d}{r}-k_1\right)}}{A - S(\alpha^{*L})}\right]^{1/\beta_2} \quad \text{and} \quad \alpha^{*L} = \min\left[\sqrt{\frac{\left(\frac{p-d}{r}-k_1\right)}{k_2/2}}\ ,\ 1\right] \tag{C.8}$$

D Proof of Proposition 4

Let's define $F^V(c_t)$ as the value of the option to invest by the venture capitalist. The optimal entry trigger c^{*V} must satisfy the matching value and smooth pasting conditions:

$$F^V(c^{*V}) = \psi V^U(c^{*V};\alpha^{*V}(c^{*V})) - k_1, \tag{D.1}$$

$$F_c^V(c^{*V}) = \psi\left[V_c^U(c^{*V};\alpha^{*V}(c^{*V})) + V_\alpha^U(c^{*V};\alpha^{*V}(c^{*V}))\frac{d\alpha^{*V}}{dc_t}_{|c_t=c^{*V}}\right] \tag{D.2}$$

where $\alpha^{*V}(c_t)$ is given by (24). Substituting for V^U we get:

$$F^V(c^{*V})^{\beta_2} = \psi\frac{p-d}{r} + \psi\alpha^{*V}(c^{*V})A(c^{*V})^{\beta_2} - k_1$$

$$\begin{aligned} F^V\beta_2(c^{*V})^{\beta_2-1} &= \psi\alpha^{*V}(c^{*V})\beta_2 A(c^{*V})^{\beta_2-1} + \psi A(c^{*V})^{\beta_2}\frac{d\alpha^{*V}}{dc_t}_{|c_t=c^{*V}} \\ &= 2\psi\alpha^{*V}(c^{*V})\beta_2 A(c^{*V})^{\beta_2-1} \end{aligned}$$

where the last equality follows from the fact that:

$$\frac{d\alpha^{*V}}{dc_t}_{|c_t=c^{*V}} = \frac{(1-\psi)\beta_2 A c_t^{\beta_2-1}}{k_2}_{|c_t=c^{*V}} = \beta_2 {c_t}^{-1}\alpha_{|c_t=c^{*V}} < 0.$$

By substituting it back in the matching value and smooth pasting conditions, we obtain:

$$F^V = 2\psi\alpha^{*V}(c^{*V})A \tag{D.3}$$

and

$$\frac{\psi}{1-\psi}k_2[\alpha^{*V}(c^{*V})]^2 = \psi\frac{p-d}{r} - k_1 \tag{D.4}$$

Simply, substituting (24) in (D.4) we obtain:

$$c^{*V} = \left[\frac{\sqrt{\frac{1-\psi}{\psi}k_2\left(\psi\frac{p-d}{r}-k_1\right)}}{(1-\psi)A}\right]^{1/\beta_2} \qquad \text{and } \alpha^{*V} = \min\left[\sqrt{\frac{\frac{1-\psi}{\psi}\left(\psi\frac{p-d}{r}-k_1\right)}{k_2}}\quad,\quad 1\right] \tag{D.5}$$

from which is easy to show that $c^{*V} > \tilde{c}^V$ if $\psi\frac{p-d}{r} - k_1 < \frac{1-\psi}{\psi}k_2$.

E Proof of Proposition 5

Consider condition $c^{*V} < c^{*U}$, i.e.:

$$\left[\frac{\sqrt{\frac{1-\psi}{\psi}k_2\left(\psi\frac{p-d}{r}-k_1\right)}}{(1-\psi)A}\right]^{1/\beta_2} < \left[\frac{\sqrt{2k_2\left(\frac{p-d}{r}-k_1\right)}}{A}\right]^{1/\beta_2} \tag{E.1}$$

Since $\beta_2 < 0$ this reduces to:

$$\Psi(\psi) = 2\psi^2 \left(\frac{p-d}{r} - k_1\right) - 2\psi \left(\frac{p-d}{r} - k_1\right) + \psi\frac{p-d}{r} - k_1 > 0 \tag{E.2}$$

where $\Psi(\psi)$ it is a parabola displaying a valley with $\Psi(1) = \frac{p-d}{r} - k_1 > 0,\ \Psi(\frac{1}{2}) = -\frac{1}{2}k_1 < 0$ and $\Psi(0) = -k_1 < 0$. Therefore, the two roots are : $\frac{1}{2} < \psi_1 < 1$, and $\psi_2 < 0$. Then:

$$\begin{aligned} c^{*V} &< c^{*U} \quad \text{for } \psi \in (\psi_1, 1] \\ c^{*V} &\geq c^{*U} \quad \text{for } \psi \in [0, \psi_1] \end{aligned}$$

In addtion, since ψ must be greater than $\frac{k_1}{\frac{p-d}{r}}$ and $\Psi(\frac{k_1}{\frac{p-d}{r}}) \leq 0$ over the whole interval $\frac{k_1}{\frac{p-d}{r}} \in (0,1]$ we may reduce the range to:

$$\begin{aligned} c^{*V} &< c^{*U} \quad \text{for } \psi \in (\psi_1, 1] \\ c^{*V} &\geq c^{*U} \quad \text{for } \psi \in [\frac{k_1}{\frac{p-d}{r}}, \psi_1]. \end{aligned} \tag{E.3}$$

Let us see now the the condition $\alpha^{*V} < \alpha^{*U}$, i.e.:

$$\frac{\sqrt{\frac{1-\psi}{\psi}\left(\psi\frac{p-d}{r} - k_1\right)}}{k_2} < \frac{\sqrt{2\left(\frac{p-d}{r} - k_1\right)}}{k_2}$$

from which it appears that

$$\Sigma(\psi) = -\psi^2\frac{p-d}{r} - \psi\left(\frac{p-d}{r} - 3k_1\right) - k_1 < 0 \tag{E.4}$$

where $\Sigma(\psi)$ is a parabola displaying a hill, with $\Sigma(1) < 0, \Sigma(0) < 0, \Sigma(\frac{k_1}{\frac{p-d}{r}}) < 0$, and $\Sigma'(\frac{k_1}{\frac{p-d}{r}}) = -2\frac{k_1}{\frac{p-d}{r}}\frac{p-d}{r} - \left(\frac{p-d}{r} - 3k_1\right) = -\frac{p-d}{r} + k_1 < 0$. Then, we may conclude that:

$$\alpha^{*V} < \alpha^{*U} \quad \text{for all } \psi \in [\frac{k_1}{\frac{p-d}{r}}, 1]. \tag{E.5}$$

F Proof of Proposition 6

The shareholders' problem is to set ψ to maximize the following function:

$$\begin{aligned} &\max_{\psi} E_0\left[e^{-rT^{*v}}\left((1-\psi)\frac{p-d}{r} + (1-\psi)\tilde{A}(\alpha^{*V}(c^{*V}))(c^{*V})^{\beta_2} - \frac{k_2}{2}(\alpha^{*V}(c^{*V}))^2\right)\right] \\ &\max_{\psi}\left[3(1-\psi)\frac{p-d}{r} - 2\frac{1-\psi}{\psi}k_1\right]\left(\frac{c_t}{c^{*V}}\right)^{\beta_2} \end{aligned} \tag{F.1}$$

where $E_0[e^{-rT^V}] = \left(\frac{c_t}{c^{*V}}\right)^{\beta_2}$.

The FOC becomes:

$$\left\{\left[-3\frac{p-d}{r}+\frac{2}{\psi^2}k_1\right]-\left[3(1-\psi)\frac{p-d}{r}-2\frac{1-\psi}{\psi}k_1\right]\beta_2\frac{1}{c^{*V}}\frac{dc^{*V}}{d\psi}\right\}=0 \tag{F.2}$$

By totally differentiating (D.4) we get:

$$-\psi((1-\psi))\beta_2\frac{1}{c^{*V}}\frac{dc^{*V}}{d\psi}=\frac{1}{2}\frac{1}{\psi\frac{p-d}{r}-k_1}\left[\psi^2\frac{p-d}{r}-k_1\right]$$

which, once substituted in (F.2), reduces the FOC to:

$$\begin{aligned} f(\psi) &\equiv \frac{1}{2}\left[3\psi\frac{p-d}{r}-2k_1\right]\left[\psi^2\frac{p-d}{r}-k_1\right]-\left[3\psi^2\frac{p-d}{r}-2k_1\right]\left[\psi\frac{p-d}{r}-k_1\right]=0 && \text{(F.3)}\\ &= -3\psi^3(\frac{p-d}{r})^2+4\psi^2\frac{p-d}{r}k_1+\psi\frac{p-d}{r}k_1-2k_1^2=0 && (1)\end{aligned}$$

and

$$f'(\psi)=\frac{1}{2}\frac{p-d}{r}\left[-9\psi^2\frac{p-d}{r}+8\psi k_1+k_1\right]. \tag{F.4}$$

Now it is easy to show that:

$$\begin{aligned} f(0) &= -2k_1^2<0\\ f(1) &= -\frac{1}{2}\left[3\frac{p-d}{r}-2k_1\right]\left[\frac{p-d}{r}-k_1\right]<0\\ f(\frac{k_1}{\frac{p-d}{r}}) &= \frac{1}{2}k_1^2\left[\frac{k_1}{(\frac{p-d}{r})}-1\right]<0\end{aligned}$$

and

$$\begin{aligned} f^{'}(0) &= \frac{1}{2}\frac{p-d}{r}k_1>0\\ f^{'}(1) &= \frac{1}{2}\frac{p-d}{r}\left[-9\frac{p-d}{r}+9k_1\right]<0.\\ f'(\frac{k_1}{\frac{p-d}{r}}) &= \frac{1}{2}\frac{p-d}{r}k_1\left[-\frac{k_1}{(\frac{p-d}{r})}+1\right]>0\end{aligned}$$

Therefore, in the interval $\psi\in[\frac{k_1}{\frac{p-d}{r}},1]$ there are two roots. The second is the maximum we are after. Hence $\frac{k_1}{\frac{p-d}{r}}<\psi^*<\frac{1}{2}$.

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