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WALRAS ON CAPITAL: A SUGGESTED INTERPRETATION OF THE ROOTS OF HIS CONTRADICTION

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Abstract

An article by Bortkiewicz, stimulated by Walras himself, gives an important indication, until now neglected, of why Walras was for so long unable to understand that his vectorial endowment of capital goods was incompatible with the uniform rate of return on supply price that he was assuming. The insight can also help explain the introduction of the famous 'bons' in the fourth edition.

Keywords: Walras, capital, Bortkiewicz

JEL: B13, B31

I. Introduction.

In the *Preface* to the fourth edition (1900) of the *Eléments d'économie politique pure* Walras supplies a list of changes introduced in that edition, but one important change he leaves unmentioned: the new admission, in two unobtrusive passages (Walras 1954, pp. 294, 308; 1988, pp. 401, 430-31; when I do not indicate the edition, I mean the 4th one), that generally the condition of a uniform rate of return on the supply price of capital goods cannot be assumed to hold for all capital goods; the (not clearly indicated) reason being that, since the given endowments of capital goods are arbitrary, some capital goods may be present in such abundance that their demand price (capitalized value of their rentals) is too low to make it convenient to produce more of them; so that – Walras indicates in the first of these new passages – since these capital goods will not be present among the newly produced capital goods, the equations imposing for them the uniform-rate-of-return-on-supply-price condition (URRSP for brevity; i.e. the equality between supply price and demand price) must be eliminated. This change (which has received attention only after it was stressed by Garegnani 1958, 1960, (1962)2008, but is again not mentioned in the editorial Annexe I, "Histoire des différentes éditions", of the 1988 variorum edition of the *Eléments*) shows that Walras at last realized the *contradiction* between his fully traditional long-period condition of a uniform rate of return on the supply price of all capital goods[1], and his specification of the equilibrium's given endowment of capital as an arbitrary *vector* of endowments of the several capital goods: a contradiction avoided by the other founders of the marginal approach, who treated capital as a single factor of variable 'form' embodied in the heterogeneous capital goods, leaving the equilibrium 'form' (i.e. the equilibrium proportions between the endowments of the several capital goods) to be determined *endogenously* by the URRSP condition.

Why did Walras *not* wish to draw attention to this new admission? Probably we will never know. Perhaps he was conscious that the issue raised problems requiring more reflection than he felt capable of giving it[2] (as Donald Walker, 1996, pp. 205, 321-3, has stressed, Walras was by

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1 On the long-period nature of Walras's original conception of equilibrium see Petri (2004, pp. 140-150).

2 Walras deals with the issue *very* hurriedly. The capitalization equations are left unchanged from the earlier edition. In §258 Walras writes that the condition of URRSP "serait satisfait à celles des équations du système [8] qui subsisteraient après élimination des capitaux neufs qu'il n'y a pas lieu de produire" (1988 p. 401), that is, "would be satisfied for those equations of system [8] that would survive after elimination of the new capital goods that there is no reason to produce", a statement bound to look incomprehensible to readers, since it is not further explained at all, and no hint has been given up to then that not all capital goods will be generally reproduced; for something more on the issue the reader must

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then suffering from grave mental fatigue). Perhaps he was embarrassed at admitting a mistake. Or perhaps he did not perceive the implications of what he had finally realized and the limits of the solution he was now proposing. It does not seem, for example, that he was brought to doubt the legitimacy of the neglect of possible changes of relative prices over time in the equilibrium equations: and yet his admission that some capital goods might not be produced, implying a rapid decrease of their endowments, should have made evident to him the impermanence of the equilibrium prices he was determining, and should accordingly have made him doubt the right to interpret the equilibrium thus formalized as a centre of gravitation of day-by-day market prices and quantities; but Walras shows no such doubt, for example he maintains the passage, present from the first edition:

> "the market is like a lake agitated by the wind, where the water is incessantly seeking its level without ever reaching it. But whereas there are days when the surface of a lake is almost smooth, there never is a day when the effective demand for products and services equals their effective supply and when the selling price of products equals the cost of the productive services used in making them. The diversion of productive services from enterprises that are losing money to profitable enterprises takes place in various ways, the most important being through credit operations, *but at best these ways are slow*." (Walras §322, 1954 p. 380; 1988 p. 580; emphasis added).

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wait until §267, where it is admitted that "Dans une société, come celle que nous avons supposée, établissant son équilibre *ab ovo*, l'égalité des taux des revenus nets n'existerait probablement pas" (1988, p. 431), that is, "In a society, like the one we have assumed, that establishes its equilibrium *ab ovo* [= from scratch], the equality of net rates of return would probably not exist", again with no explanation of why; the reason is left to the reader to deduce, from the indication that savings will be utilized first for the purchase of the capital goods offering the highest prospective yields. I refer to Garegnani ((1962)2008) for a careful discussion of Walras's equations of 'capitalization' and of these two new passages.

Here the admitted slowness of the tendency towards equilibrium implies that the equilibrium is conceived by Walras as endowed with considerable persistence, otherwise it could not have the same role as the normal level of water in a lake.

Given the impossibility to advance other than unsupported conjectures on this first question, the present paper concentrates on a second question that concerns the same novelty, where a so far unnoticed evidence is of help. The question is: why did Walras take so long to discover the contradiction between the URRSP condition and the vectorial given capital endowment?

Certainly, why an author did not realize, or did not realize earlier, certain weaknesses of his/her analysis, is a difficult question, to which it may be even nonsensical to look for an answer; but in this case the question appears legitimate. After all, it should be obvious that a uniform rate of return on the supply price of capital goods cannot be established on arbitrarily given endowments of capital goods: if some significant change in demand composition or in optimal production methods has just occurred, then certainly some of the endowments of capital goods, especially of durable capital goods, will result excessive, and there will be no incentive to produce new capital goods of the same type until the wear and tear of the existing ones reduces their endowments sufficiently to raise their rentals to a level that allows earning the normal rate of return on the cost of producing them, causing some capital goods even to disappear completely if associated with no longer convenient production methods. And it is puzzling that Walras did not admit this fact earlier, especially when one remembers that Walras shared the universally agreed view of the process bringing about a uniform rate of return on supply price, a process based on variations of the relative endowments of the several capital goods:

> "Capital goods proper are artificial capital goods; they are products and their prices are subject to the law of cost of production. If their selling price is greater than their cost of production, the quantity produced will increase and their selling price will fall; if their

selling price is lower than their cost of production the quantity produced will diminish and their selling price will rise. In equilibrium their selling price and their cost of production are equal." (Walras 1954, §238, p. 271; 1988, p. 353; unchanged from the second to the last edition of the *Eléments*).[3]

The new evidence that may contribute to answer my question consists of Walras's wholehearted endorsement of an article by Ladislaus Bortkiewicz which he himself had stimulated, where the determination of the quantities produced of *new* capital goods in the *Eléments* is treated as if it amounted to determining the composition of the *endowment* of capital. The relevant portion of this article is reported in the next Section; the answer it appears to imply to my question is explained in Sections III and IV where it is argued that it is supported by one more novelty of the 4th edition that Walras omits to highlight in the *Preface*. Appendix 1 suggests a connection with the introduction of *bons* in that same edition. Appendix 2 argues that Walras's "theorem of maximum utility of new capital goods" might further support the proposed interpretation.

II. Edgeworth and Bortkiewicz.

In September 1889 Edgeworth published in the English journal *Nature* a review of the second edition of Walras' *Eléments*, where, after an initial praise of Walras as co-discoverer of the role of marginal utility, he criticized several aspects of Walras' work. Walras got angry at the criticisms, which he judged totally mistaken, but preferred not to reply directly, and persuaded

[3] "Selling price" is Walras's term for demand price, the capitalized value of the future rentals the factor will earn. This passage is present in all editions.

the young Bortkiewicz to rebut the criticisms in an article, formally another review of Walras's book but in fact essentially aimed at replying to Edgeworth. I have nothing to add to Roberto Marchionatti's (2007) synthesis of what then happened:

> In early December [1889] Walras received Bortkievicz's paper and was very satisfied of it: 'I found a man capable of reading me attentively and understanding me perfectly, and capable of defending my point of view as well as I can, if not better' ('J'ai trouvé un homme capable de me lire attentivement, de me comprendre parfaitement et de défendre mon point de vue aussi bien, sinon mieux, que je pourrais le faire moi-même') (letter of 8 December 1889). He sent it (with a few changes) to Gide, the editor of the *Revue d'économie politique*. He wrote: 'I am sending you separately an excellent paper that offers an exact idea of my work in the form of a rejoinder (incontestable, according to me) to Edgeworth's criticism' ('Je vous envoie sous ce pli séparé un article excellent qui, sous forme d'une réponse (tout à fait irréfutable, selon moi) aux critiques d'Edgeworth, donne une idée parfaitement exacte de mon ouvrage') (26 December). Bortkievicz's paper was published at the beginning of 1890. On 20 February 1890, Walras wrote proudly to Edgeworth: 'This is the answer to your critiques' ('Voici la réponse à vos critiques'). (Marchionatti, 2007, p. 293)

Now, Bortkiewicz's 1890 article (actually signed Ladislas Bortkévitch) contains the following passage on pp. 84-85 (translated immediately below), whose relevance for the interpretation of Walras' views on capital before the 4th edition appears to have escaped attention so far:

> M. Edgeworth ne distingue pas nettement le marché des produits du marché des services ou, ce qui revient au même, l'équilibre de l'échange de l'équilibre de la

production. Ce n'est pas le moindre mérite de M. Walras d'avoir insisté sur cette distinction importante.

Mais M. Edgeworth ne distingue pas mieux l'équilibre de la capitalisation de celui de la production qu'il ne distingue l'équilibre de la production de celui de l'échange. Il croit qu'il ne sert à rien de traiter spécialement le problème de l'utilité maxima des capitaux neufs, vu que, «le prix du capital étant déterminé par concurrence, il résulte de la théorie générale de l'offre et de la demande que l'utilité maxima de toutes les parties intéressées se réalise dans le même sens que dans les autres marchés» (p. 435, col. 1 et 2). On peut objecter au critique anglais : 1° que le concept de l'utilité des capitaux n'est pas le même que celui de l'utilité des produits consommables, l'utilité des capitaux étant en quelque sorte dérivée de celle des revenus auxquels les capitaux donnent naissance; 2° que la théorie de la capitalisation s'occupe du problème relatif aux quantités fabriquées des capitaux neufs, tandis que ces mêmes quantités sont considérées comme données dans la théorie de la production. Voilà donc un troisième problème tout nouveau qui ne saurait être traité comme un cas particulier d'aucun des problèmes résolus dans les chapitres précédents du livre de M. Walras. Il devient évident que M. Edgeworth n'a pas du tout saisi la corrélation existante entre les trois parties du système des *Éléments d'économie politique pure* – Dans la théorie de l'échange, il s'agit de déterminer les prix des produits, étant données les quantités fabriquées de ces produits. -- Dans la théorie de la production, ces quantités de produits figurent à titre d'inconnues qui se déduisent des quantités données de capitaux fonciers, personnels et mobiliers. Quant aux premiers (les terres), leurs quantités sont toujours des donnees du problème et non des inconnues. Les facultés personnelles des hommes ne dépendent pas non plus du mouvement de la production industrielle, mais de celui de la population (Éléments, p. 266). -- Restent les capitaux mobiliers (artificiels), ou capitaux proprements dits, dont les quantités peuvent être considérées comme des inconnues; il s'agit de démontrer comment elles se déterminent, et c'est là l'objet propre de la théorie de la capitalisation.

In the following translation I italicize the lines relevant to my argument.

Mr. Edgeworth does not clearly distinguish the market for products from the market for services, or, what comes to the same thing, the equilibrium of exchange from the equilibrium of production. It is not the least merit of Mr. Walras to have insisted on this important distinction.

But Mr. Edgeworth no better distinguishes the equilibrium of capitalization from the equilibrium of production than the equilibrium of production from the equilibrium of exchange. He believes that it is of no use to discuss specifically the problem of the maximum utility of new capital goods, since "the price of capital being determined by competition, it results from the general theory of supply and demand that the maximum utility of all interested parties is obtained in the same sense as in the other markets" (p. 435, col. 1 and 2). One can object to the English critic : 1° that the concept of utility of new capital goods is not the same as that of utility produced by consumable products, since the utility of capital goods is in some way derived from the one of the revenues to which the capital goods give birth; 2° *that the theory of capitalization deals with the problem of the quantities manufactured of new capital goods, while these same quantities are considered as given in the theory of production*. Here we have then a third totally new problem which it would be impossible to treat as a special case of any of the problems solved in the preceding chapters of Mr. Walras's book. It becomes evident that Mr. Edgeworth has not at all grasped the correlation existing among the three parts of the system of *Éléments d'économie politique pure*. -- In the theory of exchange, the issue is to determine the prices of products, the quantities manufactured of these products being given. -- In the theory of production, these quantities of products are treated as

> unknowns deduced from the given quantities of land capitals, personal capitals and mobile capitals. As to the first ones, their quantities are always among the givens of the problem and not among the unknowns. The people's personal faculties in the same way do not depend on the movement of industrial production, but from that of population (Éléments, p. 266). -- *There remain the mobile (artificial) capital goods, or capital goods proper, whose quantities can be considered as unknown; the issue is to demonstrate how they [p. 85] are determined, and there lies the proper object of the theory of capitalization*.

The sentences that I have italicized strongly suggest that, according to Bortkiewicz, Walras' theory of capitalization aims at determining *the quantities in existence, i.e. the endowments*, of capital goods. In the first italicized sentence Bortkiewicz writes that the theory of capitalization aims at determining the quantities *manufactured* of capital goods, while *these same* quantities are taken as given in the theory of production: now, the quantities of capital goods taken as given in the theory of production are their *endowments*. It would seem then that Bortkiewicz says here that the theory of capitalization aims at turning into *unknowns to be determined endogenously* the endowments of the several capital goods that in the theory of production are taken as given. The same identification of the determination of the quantities produced of new capital goods with the determination of their endowments is explicit in the second italicized sentence: after listing the quantities of land-capitals, personal capitals, and capital goods proper as *given* in the theory of production, Bortkiewicz goes on to say that the endowments of lands and of personal faculties are legitimately treated as given because not depending "du mouvement de la production industrielle", while those of capital goods proper "peuvent être considérées comme des inconnues; il s'agit de démontrer comment elles se déterminent". The symmetry in the treatment of the three types of 'capital' implies that Bortkiewicz is talking of the *endowments* of capital goods proper when he says that they "can be considered unknown" and that the problem is to show "how they are determined". Thus, again Bortkiewicz writes as if the determination of the quantities produced of *new* capital goods amounted to rendering their *endowments* unknown magnitudes that the model determines endogenously. The fact that in the model there are *given* endowments of capital goods, and that it is these given endowments that determine the rentals of capital goods, disappears from sight.

Now, we have seen that Walras wrote (and there is no reason to think that he was lying) that Bortkiewicz' article offered "a perfectly exact idea of my work".

III. Implications.

If the determination of the production of new capital goods is seen as turning the capital endowments into endogenously determined unknowns, then it seems logically to follow that the *raretés*, the scarcities, of capital goods, and therefore their rentals, will be seen as depending on these endogenously determined endowments. In other words, the above suggests that before the fourth edition Walras may have tended to confuse the *rareté* of the capital endowments he was taking as given, with the *rareté* of the endowments that would *result* from the production of new capital goods.

And indeed in the first three editions of the *Eléments* Walras justifies the establishment of a uniform rate of return on the supply price of capital goods with the argument that if a capital good offers a higher rate of return on supply price than the others, its production will increase, and this will tend to bring supply price and demand price into equality because of two effects: first, the increase in the production of a capital good will slightly ('légèrement') increase the rentals of the factors employed in its production and thus will increase its supply price ('prix de revient')[4]; second, *that increase will appreciably decrease the demand price* ('prix de vente')[5]. This second effect is the application of the traditional view accepted by Walras in the passage quoted here at the end of Section I: Walras does not seem to realize that this effect of the increased production of a new capital good can only come about if that increased production is allowed to alter the scarcity, i.e. the endowment, of that capital good, while his equations determine factor rentals on the basis of the *existing* scarcity of factors, i.e. on the basis of their *given* endowments. The production of new capital goods is treated here as altering the scarcity of the existing endowments of capital goods[6].

I suggest then that, before the fourth edition, Walras confusedly thought that the equilibrium rentals of capital goods resulted from the endowments that the production of new capital goods *would bring about*, and believed *because of this* that his initially given endowments of capital goods did not prevent the establishment of long-period prices associated with URRSP. (One might even suspect that, since the tendency of investment to correct the composition of capital could be assumed to have been going on already in previous periods, Walras somehow

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4 With a strange contradiction, Walras writes that this 'slight' increase in the factor rentals will *appreciably* ('sensiblement') increase the supply price of the capital good: he does not seem to realize that if the rentals increase 'légèrement', the total cost of production increases 'légèrement' too.

5 Cf.: "If we increase or decrease the quantity D_k [quantity produced of new capital good k] … we decrease or increase appreciably ... the selling price of the capital good" (Walras, 1988, p. 396, §253 of second edition, our transl.; unchanged in the third edition; also cf. Walras, 1954, p. 594, Jaffé's collation note [*n*]). The same idea is in the passage quoted at the end of Section I. Also cf. § 254 of eds. 2-3 (1988, p. 398), where an increase in the quantity produced of capital k causes "une basse du prix p_k" where p_k is the *rental* earned by the capital good.

6 After putting forth this interpretation in Petri (2006) on the basis of Walras's wholehearted endorsement of Bortkiewicz's article, I have discovered that Pierangelo Garegnani had analogously suggested: "It may be interesting to speculate about the origin of the ... error, which appears due to an imperfect realization that outputs of capital goods in the given period can affect stocks only in a subsequent period" ((1962)2008, p. 12). In the light of Walras's approval of Bortkiewicz's article, Garegnani's suggestion no longer appears to be just a speculation.

thought that at any time the given composition of capital would be already largely adjusted to the URRSP requirements.)

Then Walras's treatment of equilibrium prices as considerably persistent[7] appears less surprising: it follows from the interpretation I am suggesting that Walras must have thought that the reaching of equilibrium left little reason for further significant changes in the composition of capital (the sole plausible reason for rapid endogenous changes of relative prices over time)[8].

In conclusion, the answer I would give to the question I have posed is that Walras's inability (before the fourth edition) to perceive the contradiction between the given endowments of capital goods and his assumption of URRSP derived from a confused tendency to consider the capital endowments relevant for the determination of equilibrium rentals as *having already undergone* a tendency to adjust their composition toward the one required for URRSP.

IV. Another little noticed novelty of the fourth edition.

The admission in the fourth edition that the URRSP condition is not generally achievable appears due precisely to a realization by Walras that he could not treat the increase in the *production* of a capital good as reducing the scarcity of the *given stock* of that capital good. This

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[7] This emerges particularly clearly in how Walras determines the purchase price of land, which he obtains by dividing the rental of land by the rate of interest, the discounting appropriate to a rental and rate of interest constant over the indefinite future, cf. *Eléments* §236 (1954, p. 270; 1988, p. 352).

[8] As particularly evident in the description of the determination of the rate of interest in the *Preface* to the 4th edition (*Eléments*, 1954 pp. 45-6; 1988 pp. 18-19), in that edition Walras still treats the rentals expected on the newly produced capital goods as equal to the ones earned by the existing capital goods, in spite of the explicit indication that the newly produced capital goods "ne fonctionnent que dans une période subséquente à celle considérée" (1988, p. 377; 1954, p. 283) and of the admission that investment can become concentrated on only a few capital goods, something that clearly would alter subsequent rentals.

is suggested by another novelty of the fourth edition, again not mentioned by Walras in the *Preface*, again first stressed by Garegnani (1958; 1960; (1962)2008), and again not mentioned in the "Histoire des différentes éditions" of the 1988 variorum edition: a changed justification of the tendency toward a uniform rate of return on supply price.

Walras states now that the increase, during the tâtonnement, in the quantity produced of a particularly profitable new capital good (K) tends to bring the supply price into equality with the demand (or selling) price exclusively owing to a rise in the supply price caused by a rise of the rentals of the factors used in its production[9]; the indication, present in the corresponding paragraph of editions 2-3, that the rental of (K) will decrease[10] has completely disappeared; Walras indeed writes, in striking contrast with his previous views, that the increase in the quantity produced of (K) and the changes in quantities produced of other new capital goods "n'ont pu modifier qu'insensiblement le taux de revenu net et, par conséquent, le prix de vente", translated by Jaffé as "can hardly affect the selling price in any significant degree" (§ 258; 1954, p. 293; 1988, p. 399)[11]. Walras appears to have now realized that his earlier idea of a significant negative influence of the production of a capital good on its current rental would require a change of its endowment, which he cannot allow.

Having realized his mistake on this issue, it was easy for Walras to see that the URRSP condition could not be generally satisfied for all capital goods, because it was not difficult to realize that exclusive reliance on changes in the supply prices of capital goods was generally insufficient to eliminate differences in rates of return on supply price: for example, if a capital

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9 Even in this edition Walras neglects the possibility that production of a capital good may utilize that same capital good as an input and in a higher-than-average proportion – in which case an increase in the production of that capital good motivated by a higher-than-average rate of return on its supply price would raise its rental more than its cost of production, raising the rate of return on it as determined by Walras, and causing therefore instability.

10 Cf. footnote 5 above.

11 However, Walras does not eliminate the passage quoted at the end of Section I.

good's given endowment was so abundant as to render it impossible to utilize it fully, the rental would be zero, making it impossible to obtain on that capital good a positive rate of return.

Appendix 1. On a possible connection with the introduction of the 'bons'.

The argument of the main text induces one to wonder whether there was any connection between the two unobtrusive novelties of the 4th edition remembered in the main text, and the introduction of *bons* in that same edition.

It is possible that, before becoming better conscious of the implications of his treatment of the endowments of the several capital goods as given, Walras had not realized that for the economy with capitalization he could not describe the tâtonnement as entailing actual disequilibrium productions and sales at each *reprise*, since production of new capital goods and consumption of the existing ones would alter capital endowments. He may have been helped in realizing this difficulty by the extension of the theory to include circulating capital and inventories, another novelty of the 4th edition. The potential alterations of inventories due to sales and productions may have made it more evident to Walras that the adjustments toward equilibrium had to be conceived as excluding, not only actual exchanges, but also actual productions, since the inventories of circulating capital goods were among the equilibrium's data. In fact it is precisely in a Note at the end of the paper *Equations de la Circulation* (1899), the first discussion by Walras of circulating capital and inventories which would then become a new chapter of the 4th edition, that Walras proposes for the first time, and still tentatively, that the "tâtonnements préliminaires ... porraient être supposés fait sur *bons*", that is, "one might suppose that the preliminary tâtonnements are realized on the basis of pledges", motivating this assumption as a way more sharply to distinguish, "above all if one assumes them to be in succession, the following three phases: 1° The phase of preliminary tâtonnements; 2° The phase of the actual reaching *ab ovo* of the *static equilibrium* concerning the delivery of productive services and of products during the time period under consideration, at the agreed conditions, without change in the data of the problem; 3° A phase of *dynamic equilibrium*, with change in these data"[12]. To these sentences he immediately adds that "il doit être bien entendu que les

[12]. "...surtout si on les suppose successives, les trois phases suivantes: 1° La phase des *tâtonnements préliminaires*; 2° La phase de l'établissement effectif *ab ovo* de l'*équilibre statique* relatif à

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quantités ... de *capitaux neufs fixes ou circulants* livrés pendant la seconde phase ... ne fonctionnent que dans la troisème phase" ((1899)1993 pp. 581-2), that is, "it must be clearly understood that the quantities ... of fixed and circulating new capital goods delivered during the second phase ... only become operative in the third phase". This emphatic warning (repeated in the 4th edition with almost identical words at the end of Lesson 24 and then again in Lesson 28: 1954, pp. 283, 319) that the newly produced capital goods remain inactive until equilibrium decisions are accomplished is clearly intended to dispel suspicions that their production might alter the given capital endowments and therefore the equilibrium itself before equilibrium is reached. A similarly clear warning is absent in the second and third editions, where the description of the tâtonnement with capitalization is not totally unambiguous on the issue (see 1954, p. 590, collation note [f]) and in fact presents a problem: the description assumes the production of new capital goods to be initially random "and then increased or decreased according to circumstances", but if, as I will argue later (in accord with a majority of interpreters), in the 2nd edition the tâtonnement involves actual productions, then this description only makes sense if production of new capital goods is conceived as a *flow* to be compared with a flow of demand for new capital goods, the first flow being increased or decreased according as it is less or more than the second flow; but then one must be thinking of a process going on over a considerable time period; then an assumption, that the newly produced capital goods only come into use in a period subsequent to the establishment of the equilibrium under discussion, would appear unacceptable[13]; but the alternative is that the endowments of capital goods change during the tâtonnement. A greater awareness of this difficulty, stimulated also by the explicit consideration of circulating capital goods and inventories, seems a very plausible explanation for the 1899 emphatic warning and for the tentative idea of *bons*, an idea that then becomes a firm assumption one year later in the 4th edition, evidently because further reflection confirmed to Walras that there was a problem and he could not think of another way of overcoming it.

Therefore I suggest that up to the 3rd edition Walras had not fully realized the problem posed by production with, and of, capital goods for his description of the tâtonnement, and that

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la livraison des services producteurs et des produits pendant la période de temps considérée, aux conditions convenues, sans changement dans les données du problème; 3° Une phase d'*équilibre dynamique*, avec changement dans ces données"

[13] Furthermore, even if new capital goods were supposed unutilized until a subsequent period, the repeated productions would anyway consume capital goods and hence would still alter the initial endowments of capital goods.

the introduction of *bons* was motivated by the need to surmount this problem, and was therefore another effect of that better (although still insufficient) appreciation of the implications of the given vectorial capital endowment, evidenced by the other two novelties of the 4th edition that were discussed in the main text[14].

I will now briefly discuss two other reasons for the introduction of *bons*, suggested in recent papers. I will argue that these suggested reasons are not more convincing than mine.

Bridel and Huck (2002) argue that even in the production economy Walras had a serious problem with “distributional effects” (redistributions of the endowments among individuals), which he needed to avoid in order not to have the equilibrium changed by disequilibrium transactions (Bridel and Huck, 2002: 521, 2002b: 563). This need had already induced him in the second edition of the *Eléments* to assume no exchanges at disequilibrium prices in his exchange model. The *bons* satisfy the same need for the more complex production and capitalization economy. They advance two arguments in support of their claim: first, in order to determine the selling prices of the given quantities produced at each round of the tâtonnement in the production economy, Walras assumes given demand functions for the produced goods, while disequilibrium transactions would alter them; second, as long as the prices of produced goods differ from their costs of production, entrepreneurs make profits or losses and this alters their wealth, hence it cannot be assumed that the wealth of each individual remains unaltered during the tâtonnement. But the first argument is weakened by Walras’s opinion, expressed in the discussion of the exchange economy, that *as a matter of empirical fact* once the quantity of a product to be exchanged on a market is given the market is usually able to find the equilibrium price very quickly and therefore his assumption of no disequilibrium transactions in the exchange economy

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14 Pierangelo Garegnani too in (1960, p. 104, footnote 5) argues that Walras seems to have realized only with the 4th edition that he could not carry over the description of the tâtonnement for the production economy, that included actual disequilibrium productions, to the tâtonnement of the capitalization economy. On why disequilibrium productions could be admitted in the tâtonnement without capitalization, it may be useful to remember that in the production economy only consumption goods are produced, and there is no consideration of amortization and insurance of capital goods, which amounts to treating capital goods as indestructible, cf. *Eléments* §201 (1988, pp. 302-3); then production does not alter factor endowments, and the outputs produced at each round of the tâtonnement, since they consist only of consumption goods, can be conceived no longer to be there at the next round, having been consumed; therefore at each *reprise* the data of equilibrium are the same.

is *realistic*[15]. As to the second argument, nowhere does Walras discuss the possibility of redistributions of the existing property of nonlabour factors; anyway the profits and losses of entrepreneurs, even if they entailed some such redistribution, would be no more important than other possible redistributions of wealth during the tâtonnement, e.g. due to sales and purchases of lands at disequilibrium prices, but Walras, like – to the best of my knowledge – all other marginalist economists, never mentions these changes as a possible significant effect of disequilibrium, evidently considering them of secondary relevance.

Rebeyrol (1999, 2002) argues that Walras never thinks of the tâtonnement as admitting the actual implementation of disequilibrium exchanges and productions. No doubt for the exchange model Rebeyrol is right, Walras explicitly assumes that any discrepancy between supply and demand entails a suspension of exchanges, but I have already indicated his justification, that, Walras admits, does not extend to production. Coming then to the production economy, Rebeyrol relies on the fact that the production tâtonnement as described by Walras in the second and third editions[16] determines, on the basis of given factor rentals, the quantities

[15] "The rapidity and reliability of the practical solution leave no room for improvement. It is a matter of daily experience that even in big markets where there are neither brokers nor auctioneers, the current equilibrium price is determined within a few minutes, and considerable quantities of merchandise are exchanged at that price within half or three quarters of an hour" (Walras 1954, p. 106; 1988, p. 93; present from the first to the last ed.). So disequilibrium exchanges are excluded because such an assumption is a good approximation to reality, "the distributional neutrality of *tâtonnement*" being rather a consequence than a needed assumption. No analogous appeal to realism could be advanced in the 4th edition for the elimination of disequilibrium productions and the neglect of the time required for production.

[16] This tâtonnement can be succinctly described as follows. Random initial factor service prices (factor rentals) are fixed, and random quantities are produced; the factor rentals, and hence the costs of production, remain fixed during a series of sub-tâtonnements on quantities produced: the initial random quantities are sold at the market prices that render demands for them equal to supplies (the incomes from which these demands derive are not made clear); these market prices will generally differ from costs of production; then in the next round of the sub-tâtonnement different quantities are produced, increased if the market price had been above cost of production, decreased in the opposite case; the new quantities determine new market prices which determine the further evolution of quantities produced, until for all products the market price comes to coincide with cost of production; the resulting quantities determine excess demands on factor markets, *some of which*, Walras writes, *will be positive and some negative*; now factor rentals are modified accordingly, and then again kept fixed during a new round of sub-tâtonnements

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produced that guarantee prices equal to average costs, and derives from these quantities the excess factor demands that determine the changes in factor rentals for the next *reprise* of the tâtonnement; some of these excess factor demands, Walras says, will be negative and some positive; but, Rebeyrol notes, outputs implying positive excess demands for some factors cannot all be produced. Rebeyrol concludes that Walras always conceived the production tâtonnement as hypothetical, provisional, *as if* 'sur *bons'*, so that the introduction of *bons* in the 4th edition only makes things more explicit.

And yet, there is little doubt that in the 2nd and 3rd editions Walras describes the production tâtonnement as a succession of rounds, in each one of which certain quantities of goods *are actually produced*; why otherwise would he write, in the *Preface* to the 4th edition: "En ce qui concerne la production, j'ai supposé les tâtonnements préliminaires pour l'établissement de l'équilibre faits non plus effectivement, mais *sur bons*" (1988, pp. 6-7)[17]. The "*non plus* effectivement" leaves no doubt that in the earlier editions the various disequilibrium productions had been described as actually taking place. Also, §203 of the 2nd edition, that introduces the tâtonnement for the production economy (modified in §207 of the 4th edition with the introduction of the *bons*), supposes "les données du problème invariables pendant tout le temps que dureront nos tâtonnements" (1954 p. 582 [*g*]; same words also in the corresponding passage on the tâtonnement of capital formation, cf. 1954, p. 590 [*e*]): thus here the tâtonnement takes time, and not a very short time if Walras esteems it necessary to stress the assumption that no data change during it – and the reason is clearly the immediately following description of the successive *reprises* of the tâtonnement as requiring new actual productions and sales. (In the 4th edition on the contrary Walras explicitly assumes that one can ignore time lags: §207; 1954 p.

on quantities produced, which goes on until again equality is again reached between cost of production and market price for all outputs; then factor rentals are again modified; and so on. A little noticed weakness of this tâtonnement is the absence of clear indication of what determines the incomes from which demands derive; the italicized statement seems to imply Walras' Law, that is, a value of total demand (and hence of total production) equal to the value (at the given factor rentals) of the intended supplies of factor services, but nothing justifies this equality in a realistic adjustment process: consumer incomes will depend on whether their factor supplies find purchasers; for example, unemployed workers have no income and cannot demand consumption goods. There is no evidence that Walras was ever aware of this problem, so the introduction of *bons* cannot have been motivated by it.

[17] "As far as production is concerned, I have assumed that the preliminary tâtonnements for the reaching of equilibrium take place no longer effectively, but through pledges" (my literal translation, where I adopt Walker's translation of *bons* as 'pledges'; for Jaffé's translation see 1954, p. 37).

242; and that production is instantaneous: §251; 1954, p. 282.)

Let me also note that a tâtonnement with repetition of actual productions agrees with the experimentations needed according to a letter to Barone dated 1895, where Walras states that the production function is not known a priori to the entrepreneur, and must be found out by experimentation (Walker, 1987, p. 771); furthermore it agrees with Walras's description of the adjustments toward equilibrium as "slow"[18]. These considerations suggest that it was only natural for Walras to conceive the tâtonnement as involving actual productions and taking time. The problem is rather, why up to the third edition Walras felt he could adopt such a description, but some time in 1899 he concluded he could not. The tâtonnement with *bons* leaves no room for slow adjustments, nor for experimentations. Walras must have had a good reason for passing to a description of the tâtonnement incompatible with his realistic perception of actual economic processes. The reason I suggest is the new realization of the analytical difficulty caused, for his specification of the data of equilibrium, by changes in capital endowments during disequilibrium adjustments involving actual productions.

An argument partly similar to Rebeyrol's is advanced by Donzelli (2005). The latter author does not deny that before the 4th edition the tâtonnement is described as involving actual productions, and he admits that the change in capital endowments *is* one of the two main difficulties of the tâtonnement of the second and third edition, but he argues that *the other* difficulty is the one "which in the last analysis explains both Walras's twistings in the first three editions of the *Eléments* and his final change of course in the fourth one" (2005, p. 35, fn. 23): namely, the possibility already noticed by Rebeyrol that the disequilibrium productions assumed to happen in the tâtonnement without 'bons' be in fact impossible because entailing factor demands different from (I suppose Donzelli meant, in excess of some) factor supplies. Donzelli adds in particular that *this* is the difficulty explaining why, in the first edition, Walras introduces in the production tâtonnement the assumption that productive services are initially bought on a foreign market (1988, p. 312) and passes, to assuming that they are bought in the same country where the products are sold, only when stipulating equilibrium on factor markets (1988, pp. 312 and 322). I doubt the correctness of this interpretation because, if Walras had really felt that, for the reason suggested by Donzelli and Rebeyrol, he *could not* assume the adjustments on factor markets of his tâtonnement to go on in the same country where the products were sold, then he would also have realized that the tâtonnement could not prove the stability of equilibrium for a closed economy, certainly a problem for his theory. Anyway, whatever the reason for the

[18] Remember the 'lake' quotation in Section I.

introduction of the foreign market in the first edition, its disappearance in the second edition must have been due to some reflection that brought Walras to conclude that he could do without it; so if Donzelli were right, Walras must have changed again his mind on this issue between the third and the fourth edition; but there is no indication of such a change of mind. Indeed, like Rebeyrol, Donzelli is unable to produce any textual evidence that Walras was aware of this difficulty[19], nor hence that the introduction of *bons* was motivated by it. As noted above, something very close to an explicit motivation *is* given by Walras for the introduction of *bons* in 1899, and it is the need to prevent changes in the capital endowments.

But why has this need to prevent data changes become more relevant than before by the time of the 4th edition? A plausible answer is the greater awareness of the givenness of the capital endowments, that emerges from the other two changes of the 4th edition discussed in the main text, and was most probably reinforced by the extension of the analysis to include circulating capital and inventories.

Appendix 2. The theorem of maximum utility of new capital goods.

Some further support for the interpretation advanced in the main text can perhaps be derived from Lessons 26 and 27 of the *Eléments*, that present a "theorem of maximum utility of new capital goods", first introduced in the second edition. Jaffé (1954, p. 595) reports in collation note [a] to Lesson 26 a long quote from a 1889 letter by Walras to Wicksteed, where Walras explains that "when I published the first edition I had beome aware of only one of the two problems of utility maximization connected with the services of new capital goods"[20], namely the problem of efficient allocation of any given vector of endowments of capital goods, solved by the proportionality of *raretés* to the prices *of services* of capital goods; now he has become aware of a second problem, of efficient allocation of net savings among the *increases* of the quantities of capital goods, solved – Walras argues – by the proportionality of *raretés* to the prices of the capital goods themselves. Walras concludes that the simultaneous solution of both problems implies that the utility of new capital goods is maximized when (leaving aside for simplicity

[19] The difficulty of the tâtonnement noted by Donzelli and Rebeyrol is no doubt there, but it is not so implausible to think that Walras was not aware of it; it would not be the only difficulty, evident to us, of which Walras seems unaware: footnotes 8, 9, 16 of the main text supply three more examples.

[20] "Quand je publiai ma première edition, je n'avais encore aperçu qu'un seul des deux problèmes de maximum d'utilité relatifs aux services des capitaux neufs".

differences in depreciation and insurance) the ratio of service price to purchase price is the same for all new capital goods, that is, when there is a uniform rate of return on supply price. A detailed interpretation and assessment of these chapters is not easy, but here only one point is of interest and it does not suffer from ambiguities: the *first* of the two problems of utility maximization connected with *new* capital goods is described as identical to the one arising for *already existing* capital goods and is solved by the *same* proportionality, because the prices of the services of new capital goods and of existing capital goods of the same type are the same, and the same holds for their marginal (indirect) utilities. Now, this is not easy to reconcile with the caveat, stated more than once in the 4th edition as noted in Appendix 1, that new capital goods only come into use in a period subsequent to the one for which the equilibrium is determined, a caveat accompanied by the statement (§274: 1954, p. 319) that their coming into use "constitutes the first change in the data of our problem" to be considered when passing from statics to dynamics. Here Walras admits that when new capital goods come into use the equilibrium's data are changed by their addition; but then prices will change too, and Walras has no right to assume that the new capital goods will earn the same rentals as the existing capital goods.

A way to explain this inconsistency is to interpret the theorem of maximum utility of capital goods as formulated when Walras tended to conceive the endowments of existing capital goods as including the quantities produced of new capital goods, as suggested in the main text; then the reproduction of the theorem without alterations in the 4th edition can be seen as simply one more instance of Walras's inability to realize the incompatibility of many of his older conceptions with his new (partial) awareness of difficulties with the URRSP assumption.

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