

# Equilibrium Analysis Under Ambiguity

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## EXTENDED ABSTRACT

In this paper we exhibit a link between two complementary literatures: the study of decision under uncertainty and the analysis of economies under uncertainty and asymmetric information. We study how the ambiguity aversion of decision makers (agents or traders) impacts on the set of ex ante efficient trades and on competitive equilibrium allocations. We do it by considering a very general framework which includes as particular case some well known models. Indeed, we use the general class of biseparable preferences, introduced by [8], without committing to a specific functional form, and we adopt the general framework of mixed (atomic) markets which encompasses finite as well as atomless economies. We also provide a new way to represent agents' information asymmetries by still ensuring that any efficient allocation is incentive compatible and hence enforceable.

In the past twenty years the theory of subjective expected utility (SEU) has been extended in order to allow ambiguity to matter for choice and the decision makers' (DM) attitude towards ambiguity to play a role in his choices. The two most popular approaches are due respectively to Schmeidler [20] and Gilboa-Schmeidler [11]. According to [20], the decision maker's beliefs are represented by non additive probabilities, called capacities, and his preferences by Choquet integral. His approach is called the theory of *Choquet expected utility* (CEU) maximization. The latter, due to [11], allows decision maker's beliefs to be represented by multiple probabilities and his preferences by the maxmin on the set of expected utilities. This generalization is called *Maximin expected utility* (MEU) theory. More generally, it can be also considered an  $\alpha$ -MEU preference which assigns some weight to both the worst case and the best case scenario. Here, we consider the biseparable preference model, introduced in [8], which is very general as it encompasses most of the known preference models, like the ones mentioned above. Therefore, we do not commit to

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a specific functional form according to the classic literature of general equilibrium (see [1] and [18] among others).

Recently a notion of absolute ambiguity aversion has been introduced (see [9]). It is based on two primitive assumptions: first, constant acts are the only acts that are unambiguous in any problem; second, a preference is ambiguity neutral if and only if it has a SEU representation. That is, the only preferences which are endogenously ambiguity neutral are SEU. Behind their notion of ambiguity aversion there is a clear intuition: if a decision maker prefers an unambiguous act to an ambiguous one, a more ambiguity averse one will do the same. We provide a new characterization of ambiguity aversion attitude, which allows us to address the following questions: how does agents' ambiguity aversion impact on the set of efficient trades? Can the degree of ambiguity aversion determine the size of the set of competitive equilibria? We will show that when aggregate uncertainty is absent and agents are ambiguity averse, the attitude towards ambiguity does not effect efficient trades, while the set of competitive equilibrium allocations expands as ambiguity aversion increases. A similar result has been recently obtained by Battigalli, Cerreia-Vioglio, Maccheroni and Marinacci [4] in terms of selfconfirming equilibrium (SCE).

For our analysis we deeply use the following result: once there is no aggregate uncertainty and agents' preferences are ambiguity averse, any efficient allocation is unambiguous. More is true: in one good economies, efficiency is equivalent to full insurance, that is an allocation is efficient if and only if it is constant across states of nature. The same result is obtained by Rigotti, Shannon and Strzalecki [19], who use the general class of convex preferences, but it cannot be deduced from their theorem since the only invariant biseparable preferences that are convex are MEU preferences, for which the equivalence has been proved in [5]. Actually Rigotti, Shannon and Strzalecki [19] obtain a stronger characterization of efficiency in one good convex economies. Precisely, they show that once aggregate uncertainty is absent, full insurance is efficient if and only if agents share some common subjective beliefs. A crucial instrument for their proof is the second welfare theorem for which the convex axiom cannot be easily avoided. In the special case of MEU preferences, our main assumption is equivalent to the existence of a common prior among agents. Some extensions have been recently obtained in [12] for non-convex preferences.

Our analysis covers finite, atomless economies as well as mixed markets. Indeed, the set of agents is decomposed by an atomless sector, representing the ocean of small traders; and a countable union of disjoint atoms representing few influential agents. This model seems to be the most realistic way to represent a real economic market since atoms formalize the presence of agents concentrating in their hands a large initial endowment compared with the totality of resources in the market (oligopoly), or the presence of some groups of traders deciding to act only together (cartels, syndicates).

The same comparative statics, made for economy with only uncertainty, are also proved in differential information model in which agents' private information are partitions of the set of states of nature. In this framework, we also provide a new way to formalize the

presence of asymmetrically informed agents, so that efficiency is not reduced and still any Pareto optimal allocation is incentive compatible.

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