A cobweb model for electricity markets

Fausto Cavalli, DEMS University of Milan-Bicocca Email: fausto.cavalli@unimib.it

Ahmad Naimzada, DEMS University of Milan-Bicocca Email: ahmad.naimzada@unimib.it

Lucia Parisio, DEMS University Milan-Bicocca Email: lucia.parisio@unimib.it

Abstract

We study some dynamical features of electricity markets modelling demand and supply by means of a nonlinear cobweb model. We consider a periodically perturbed demand function to take into account the real world seasonalities (daily, weekly and yearly) while supply function can include a stochastic term to encompass possible shocks like outages and plants unavailability. Using adaptive expectations we investigate the effects on equilibrium prices in a perturbed and in an unperturbed model considering peak and off-peak market configurations. Starting from a well known partial equilibrium model, suitable to describe price dynamics of non-storable goods, we introduce a sigmoid supply function and a periodically perturbed demand function. Our simulative investigations confirm that the model is able to reproduce several effects observed in real price dynamics. We also focus on how the dynamics can be influenced by altering some market rules set by regulators, like price caps and floors. In particular, we show that taking into account periodical perturbations in the demand function can lead to the anticipation, with respect to the classical model, of chaotic dynamics. Moreover, we study how the dynamic is influenced if negative prices are allowed.

Keywords: Cobweb, electricity market, cyclic demand JEL classification: E32, C63, Q41

1 Introduction and Background

The electricity industry has radically changed during the last decades under the wave of privatizations and liberalizations. Now electricity is exchanged in whole-

sale markets where profit-maximising generators compete for the right of selling their product to eligible consumers who pay the cost minimizing equilibrium price. In the majority of electricity markets an auction takes place on a hourly basis and the equilibrium price is fixed at the intersection point of demand and supply. Decentralized production and price decisions introduced a new perspective with respect to the pre-liberalization monopolistic and vertically integrated scenario, where state-owned monopolists were responsible for all production and investment decisions. Now electricity prices are relevant since they constitute economic signals to producers and potential investors in the market.

Decentralized decision on production and investments based on expected profit maximization requires an accurate forecast of prices. Cycles and seasonalities are a peculiar characteristics of electricity market prices. Due to demand fluctuations, environmental conditions and supply shocks (plant outages, unavailability) we observe hourly, daily, weekly and seasonal fluctuations, whereas in the long run we observe cycles due to long term capacity decisions. System operators preserve the security of supply managing a capacity reserve to cover imbalances between generation and load due to random variations in demand, forecast errors, unplanned unavailability of generating capacity. The reserve margin, defined as the percentage difference of the installed capacity in excess of peak demand is used by [1] to measure the occurrence of cycles¹.

Economic conditions and regulatory provisions may influence the dynamics of electricity prices and their stability. In this paper we design a cobweb model that is able to replicate the main characteristics of an electricity market for demand and supply. Using this model we want to analyze the dynamics of electricity prices and in particular how their stability is influenced by deregulation, market structure and other regulatory rules like price caps and floors.

We follow the research perspective pioneered by [7] who studies the dynamic of prices in a cobweb model with nonlinearities and myopic expectations of players. He finds that chaotic behavior can occur even if demand and supply functions are monotonic. Adaptive expectations reduce the amplitude of fluctuations but at the same time price-quantity cycles may become unstable.

Economic literature on cycles has been applied to electricity markets mainly in models describing investment choices of producers under uncertainty about prices and profits. [2] and [3], used the electricity industry as an example of building booms with a simulated CCGT power plant construction which delivers more than enough new capacity to keep pace with growth in demand. From this point of view, the electricity industry shows analogies with other industries mainly due to long delays in plant constructions, sunk capital costs and non-storability. In this setting, cycles are induced by external shocks hitting the market with inelastic demand which generates price variability larger than the initial shock itself. Simulation models are based on some bounded rationality assumption for agents: they make long-term price expectations based on current information and cycles occur because of systematic forecast errors.

 $^{^1\}mathrm{The}$ reserve margin is considered adequate for system security when it ranges between 18% and 25%.

When rational expectations are assumed endogenous cycles are removed. Market structure is also relevant in determining investment and production choices as it can be seen in [8],who considers a model characterized by a long run increasing trend in demand and in [1], where deregulation of electricity markets can induce sustained fluctuations and a threat for the security of supply.

This research perspective is particularly interesting for analyzing deregulated electricity markets characterized by time varying demand and supply, which may also be subject to possible shocks.

2 Model

To describe the price behavior in an electricity market, we propose a theoretical model based on an adaption of the classical nonlinear cobweb model (for a detailed presentation see [6, 7]). The cobweb model, which describes a single market with one lag in supply, namely in which there exists a lag between the decision to produce a particular product and its actual production, is usually used to explain the cyclical nature of prices and quantities through time of a non-storable good. Let us suppose that p_t be the observed price of the good at time t and let p_t^e the expected price. If we consider the demand function $q_t^d = D(p_t)$ and the supply function $q_t^s = S(p_t^e)$, under the temporary equilibrium hypothesis $q_t^d = q_t^s$, we have that

$$p_t = D^{-1}(S(p_t^e)). (1)$$

Depending on how expectations are assumed, the previous dynamical equation describes the time adjustment of the prices. A possible choice is given by the so-called adaptive expectations

$$p_{t+1}^{e} = p_{t}^{e} + \omega(p_{t} - p_{t}^{e}), \qquad (2)$$

where $0 < \omega \leq 1$ is the expectation weight factor. We remark that, as explained in [6], rearranging the previous expression, the expected price with adaptive expectations can be expressed as a weighed average, with fading weights, of all past prices. Adaptive expectations can be considered as a suitable assumptions to describe agents' behavior in electricity markets where hourly auctions are conducted every day of the year (a total of 8760 auctions) between the same set of agents and under recurrent demand and supply conditions.

Combining (1) and (2) we obtain the cobweb model with adaptive expectations

$$p_{t+1}^e = p_t^e + \omega (D^{-1}(S(p_t^e)) - p_t^e).$$
(3)

In the classical nonlinear cobweb model both demand and supply functions are independent of t and the resulting model is an autonomous discrete dynamical equation. In the present work, to take into account the periodic variation of the demand function that occurs in electricity markets, we consider a periodically perturbed function $q_t^d = D(p_t, t)$. We stress that a perturbed supply function can be likewise considered, but we observed that the model with perturbation on just the demand function is able to reproduce all the qualitative dynamics of the doubly perturbed framework.

The resulting model consists of a non-autonomous discrete equation. For simplicity, we take into account a perturbation of period 2, which encompass the alternation of peak and off-peak settings of the actual demand/supply dynamic.

We assume that the periodic perturbation of the demand function is described by $D(p_t,t) = f(p_t) + \sigma(-1)^t$, where σ represents the size of the perturbation. When $\sigma = 0$, we indeed have no perturbations, while at even times $t = 2, 4, \ldots$ (resp. odd times $t = 1, 3, \ldots$) we have the peak (resp. off-peak) demand. In particular, we assume a linear perturbed demand function

$$D(p_t, t) = d_1 p_t + d_2 + \sigma (-1)^t.$$
(4)

In what follows, the model obtained considering $D(p_t, t)$ (resp. $D(p_t, 2), D(p_t, 1)$ and $\sigma = 0$) will be called perturbed (resp. peak, off-peak and unperturbed) cobweb model.

To reproduce the supply function, we make reference to the typical shape of the aggregate merit order which we observe, for example, in the Italian wholesale market. In Figure 1 we report an example of possible supply/demand configuration². Demand and supply are obtained from the merit order of buying and selling bids submitted to the Market Operator. We typically register some bids without price limit for demand (this means a completely inelastic portion of demand function) and bids at zero price for supply (this means a flat portion of the supply function). There is a regulatory cap on bids (and hence prices) equal to $\in 3,000$ so the supply function becomes horizontal at that price value. In the intermediate portion of supply (demand) we observe an increasing (decreasing) function. However, historical data show that the equilibrium price never reached the cap: the maximum equilibrium price registered in the wholesale market was \in 378,47 in year 2006 and \in 324,20 in 2012. The minimum value of equilibrium price is equal to zero in some hours of 2013. Therefore the empirical observation of the price range allows us to identify a relevant portion of the supply function which we will use for our simulative analysis (see 2).

For the above mentioned reasons, we choose for S the following analytical expression

$$q_t^s = S(p_t^e) = s_1 + \frac{1}{\lambda} \tanh^{-1}\left(\frac{p_t^e - s_2}{s_3}\right) + \varepsilon_t, \tag{5}$$

where s_i, λ are suitable parameters, \tanh^{-1} is the inverse of the hyperbolic tangent function and ε_t is a stochastic term that allows encompassing exogenous shocks in the supply function. We have that, using (4) and (5) in (3), the

 $^{^2\}mathrm{Hourly}$ graphical representations can be found in the Italian market operator website: www.mercatolelettrico.org



Figure 1: Example of supply (black) and demand (red) curves, from the Italian market.

resulting dynamical equation results

$$p_{t+1}^e = (1-\omega)p_t^e + \omega \left(\frac{s_1 + \frac{1}{\lambda} \tanh^{-1}\left(\frac{p_t^e - s_2}{s_3}\right) + \varepsilon_t - d_2 - \sigma(-1)^t}{d_1}\right)$$

For details about a classical cobweb model with stochastic perturbation we refer to [5]. Function (5) is an increasing smooth function taking values in $(s_2 - s_3, s_2 + s_3)$, with an inflection point at $p = s_2$ (see Figure 2). In particular, if we assume that prices can vary between the minimum p_m and the maximum p_M , we have that s_2, s_3 have to be chosen accordingly to

$$\begin{cases} s_3 = \frac{p_M - p_m}{2}, \\ s_2 = \frac{p_M + p_m}{2}, \end{cases}$$

so that function $S(p_t^e)$ defined in (5) can take values in (p_m, p_M) . We remark that $S(p_t^e)$ could be negative for some parameters choices and for some values of p, which is indeed unfeasible. This can be fixed by bounding S from below, so that the supplied quantity be non-negative. However, in all the simulative experiments, we focused on the price dynamics which gave only positive supply levels. Moreover, we notice that s_1 represent the supply level corresponding to the expected price $p_t^e = (p_m + p_M)/2$ and allows for shifting the supply curve. Finally, parameter $\lambda > 0$ regulates the slope of the supply curve. In Figure 2 we report some parameter configurations to which we will refer in Section 3. We consider two different couples of demand functions, which differ for the value of



Figure 2: Example of supply (black), peak (red)/off-peak (blue) and unperturbed (green) demand functions.

perturbation σ . In both cases we set $d_1 = -2, d_2 = 3$, while we considered a small perturbation $\sigma = 1$ for the former couple (solid colored lines) and and a larger one $\sigma = 2$ for the latter (dashed colored lines). Setting $p_M = 2, s_1 = 4$ and $\lambda = 0.6$, we consider two different supply functions, obtained respectively for $p_m = 0$ (solid black line) and $p_m = -0.15$ (dashed black line). The supply functions reported in Figure 2 are not stochastically perturbed.

We notice that the shape and the location of our assumed demand and supply function are consistent with real world observations. Electricity demand is completely inelastic in its upper portion and it can be approximated with a downward sloping function in the neighborhood of the equilibrium. Price and quantity data of the Italian electricity market suggest to model the possible equilibria, that we will consider in our simulative experiment, as in Figure 2. We observe that in the Italian wholesale market the equilibrium price never reached the cap level (Euro 3000) whereas it sometimes reached the floor of zero.

3 Simulative experiments

In this section we investigate the possible scenarios arising from the periodically perturbed cobweb model described in the previous section. We report and comment the results of several simulations, together with the parameters we used. In particular, we want to analyze the effects of demand perturbation on the stability of the dynamics, comparing the results with both the unperturbed and the peak/off-peak demands settings. We also comment the role of expectation weights. Unless specified, we do not add a stochastic perturbation to the supply. In all the simulations, we are going to use black, green, red and blue colors respectively for perturbed, unperturbed, peak, off-peak cobweb models.

In the first simulation we consider the parameters setting used in Figure 2 for the demand and supply functions represented by a solid line, which results in a moderate demand perturbation and a supply with only positive prices. In Figure 3, we compare the resulting price dynamics for different possible expectation weights. As we can see, if the expectation weight is too small, the resulting dynamic oscillates around the equilibrium of the unperturbed cobweb model, alternating two values which are different from the peak and off-peak equilibria. The unperturbed classical cobweb is able to reproduce only an "average" behavior of the market price dynamics and shows no oscillations. Increasing the expectation weight, the perturbed system exhibits a dynamic of two alternating values which are very close to the peak and off-peak equilibria. Increasing further the value of ω introduces instability in the system, and the resulting dynamics is chaotic for both the perturbed and the off-peak equation, while the peak dynamic still converges. As we can see, in all the three reported situations, the unperturbed model shows stable dynamics, while for $\omega = 0.076$ the perturbed model exhibits a chaotic dynamic which resembles price volatility. This is summarized Figure 4 (for details about bifurcation diagrams we refer to the book of Hommes [6]), in which we report the bifurcation diagrams of the perturbed, off-peak, peak and unperturbed models. As we can see, we have that the perturbed dynamics is less stable than the unperturbed one, as period doublings and chaotic dynamics occur for smaller values of ω . Moreover, the off-peak dynamic is the most unstable while the peak dynamic is the most stable one. If we compare the unperturbed cobweb model to the perturbed one, we can see that the unperturbed model has an "intermediate" behavior with respect to peak/off-peak scenarios, while introducing the periodical perturbation leads the dynamic to behave as the most unstable one, inheriting the stability behavior of the off-peak dynamic. We stress the fact that there is a range of expectation weights for which the dynamic is stable using the unperturbed classical cobweb model, but which already gives chaotic price trajectories in the perturbed mode. This suggests that the dynamics arising with a periodically perturbed demand can not be completely described by considering a simple classical model, as, for a fixed expectation weight, the two models may behave in different ways. Our findings seem to mimic, even if in a very different setup, a well known result

observed in empirical analysis of real world electricity prices. [4] found evidence of a direct leverage effect in the Italian market. This means that peak electricity prices are less volatile than the off-peak ones. In the simulative results, this is portrayed by the improved stability of the peak dynamic with respect to the off-peak one.

In Figure 5 we report the results of a simulation in which we considered the parameter setting used in Figure 2 for the demand represented by dashed lines (large perturbation) and the supply function represented by a solid line (positive prices). Moreover, we include a stochastic Gaussian perturbation term



Figure 3: Comparison of time series for different values of expectation weights.

for the supply, with null average and standard deviation 0.03. We can see that also in this case the perturbed scenario exhibits chaotic behavior in advance of the unperturbed one, while the peak scenario has further improved stability and the off-peak becomes unstable for smaller values of ω . In Figure 6 we report the time series obtained for $\omega = 0.053$, in which the dynamic, which is stochastically perturbed, qualitatively reproduces the period-two cycle of the peak/off-peak alternation As for the first simulation, we have that there are values of expectation weights for which the dynamic is stable for the unperturbed model while it is chaotic for the perturbed one.

In the last simulation we want to test the capability of the proposed approach to model the effects of possible variations in the price bounds p_m and p_M . In particular, we investigate the variation in the lower bound p_m , which is now set to -0.15 (see Figure 2, dashed supply curve). Negative bids (and prices) are allowed for example in the German market and also in Italy there is a regulatory debate on pro and cons of negative prices³. The remaining parameters are the

 $^{^{3}}$ In the presence of high shares of intermittent generation, conventional producers can find reasonable to be subject to a negative price to keep the production units operating.



Figure 4: Bifurcation diagrams of the perturbed, off-peak, peak and unperturbed model, showing that the route toward chaos occurs for different expectation weights. The stability behavior of the perturbed model is similar to that of the off-peak situation. For a range of values of ω , the unperturbed cobweb model presents a stable dynamic which, conversely, is already unstable for the perturbed one.



Figure 5: Bifurcation diagrams of the perturbed, off-peak, peak and unperturbed models with stochastic term ε , showing that the route toward chaos occurs for different expectation weights.

same used for the simulation reported in Figure 4. The resulting equilibrium prices are positive for the peak demand and negative for the off-peak one. As we can see comparing the results reported in Figure 7 with those of Figure 4, the



Figure 6: Prices time series of the periodically perturbed model with stochastic term ε , compared to the equilibria of the peak, off-peak and unperturbed model.



Figure 7: Bifurcation diagrams of the perturbed, off-peak, peak and unperturbed models, for negative off-peak equilibrium price.

dynamics obtained with a negative p_m are stable for a slightly larger interval of expectation weights (the bifurcation diagrams of Figure 7 are shifted to the right with respect to the corresponding ones of Figure 4)

4 Final Comments and Conclusions

We proposed a novel approach to model the price behavior in electricity markets, based on a classical cobweb model with a periodically perturbed demand and an increasing sigmoidal supply function, in which the two asymptotes represent the upper and lower bound on prices. We showed that such perturbed model describes dynamics which are different from both the unperturbed model, showing anticipation of chaotic dynamics, and the peak/off-peak dynamics. We studied the effect of adaptive expectations on price evolution and we showed that the classical cobweb model can exhibit stable dynamics for expectation weight values for which the dynamics of the perturbed one have already become chaotic. This highlights the importance to take into account the demand perturbation in the model, as the resulting dynamics may not be correctly described by the unperturbed model. In further investigations, we aim to refine the expectation formation, considering different kinds of expectations and endogenizing their formation. The simulative experiment conducted on price floors and caps shows that the introduction of negative prices in the wholesale electricity market may have a stabilizing effect on price dynamics. This result offers a new element in the debate among the regulator and producers in favor of the introduction of this new rule in the Italian market. We also found evidence of a possible direct leverage effect, already noticed in the Italian market, since the dynamic behavior of peak prices is less-unstable than the off-peak one. We aim to test our model using real world market data.

References

- Santiago Arango and Erik Larsen. Cycles in deregulated electricity markets: Empirical evidence from two decades. *Energy Policy*, 39(5):2457 – 2466, 2011.
- [2] A. Ford. Cycles in competitive electricity markets: A simulation study of the western united states. *Energy Policy*, 27(11):637–658, 1999. cited By 67.
- [3] A. Ford. Waiting for the boom: A simulation study of power plant construction in california. *Energy Policy*, 29(11):847–869, 2001. cited By 55.
- [4] A. Gianfreda, L. Grossi, and D. Olivieri. Volatility structures of the italian electricity market: An analysis of leverage and volume effects. In *Energy Market (EEM), 2010 7th International Conference on the European*, pages 1–6, June 2010.
- [5] C. Hommes and A. van Eekelen. Partial equilibrium analysis in a noisy chaotic market. *Economics Letters*, 53(3):275 – 282, 1996.
- [6] C. H. Hommes. Behavioral Rationality and Heterogeneous Expectations in Complex Economic Systems. Cambridge University Press, 2013.
- [7] Cars H. Hommes. Dynamics of the cobweb model with adaptive expectations and nonlinear supply and demand. Journal of Economic Behavior & Organization, 24(3):315 – 335, 1994.
- [8] Marek Kocan. Cyclic behavior in dynamic investment decisions for deregulated energy markets. Central European Journal of Operations Research, 16(1):67–78, 2008.