Optimal Cartel Prices in Two-Sided Markets*

Federico Boffa[†] Lapo Filistrucchi[‡]

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Abstract

We study optimal cartel prices in a two-sided market. We present a simple model showing that prices above the two-sided monopoly price may prevail on one side of a two-sided market as a means to enhance the sustainability of the cartel. We prove that in such a case a higher benefit from the network effect may compensate customers on that side of the market for the higher prices they are charged. We then provide both sufficient and necessary conditions for these results to hold in more complex models of two-sided markets. Our analysis extends to cartels in two-sided markets a result previously known for cartels selling complementary products, despite the fact that products in a two-sided market are not complements for customers, since customers typically buy only one of the two products (e.g. in the case of newspapers, advertisers buy advertising slots while readers buy content) and products on each side are substitutes (e.g. newspapers publishers compete for readers and for advertisers). However, while in the case of complement products, collusion is beneficial, in two-sided markets in which complementarity prevails, collusion may still damage consumers

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[†]Free University of Bolzano

[‡]Department of Economics and Management, University of Florence and CentER, TILEC, Tilburg University

1 Introduction

In a two-sided market firms sell two different products or services to two different categories of buyers. They recognize that demand from one group depends on demand from the other group and choose prices accordingly. Differently from the case of complement products, this interdependency among the two demands is not recognised by customers as they buy only one of the two products sold by the two-sided firm and hence take into account only the price of that product.[1] As a result, the indirect network effects mentioned above are often called externalities.

Typical examples of two-sided markets are media markets, where firms sell content to viewers (or readers or listeners) and advertising to advertisers, taking into account that advertisers willingness to pay for advertising space increases with the number of viewers (or readers or listeners) while also the latter may be affected by the amount of advertising that is provided together with content.

While the literature extensively analyzes the effects of multi-market contacts on collusive behavior (see, for instance, Bernheim & Whinston, 1990), the analysis of the incentives for collusion with the demand externalities typical of two-sided markets has been so far very limited, and the literature on the topic is still scarce.

Ruhmer (2011) shows how in two-sided markets the presence of indirect network externalities affects the incentives to collude and the welfare implications of collusion. She uses the single-homing model of Armstrong(2006) as a stage game of an infinitely repeated game to model a two-sided market where firms are differentiated on both sides and simultaneously choose both prices. Assuming firms collude adopting grim trigger strategies she finds that higher network externalities have two opposite effects: on the one hand they tend to raise incentives to collude as they increase the gain from collusion (collusive profits increase and competitive profits decline); on the other hand they tend to lower incentives to collude as they increase the gain from deviation (deviation profits increase and competitive profits decline). In her model the latter effect is always found to dominate. As a result, collusion becomes harder to sustain as indirect network effects between the two sides of the market increase. Furthermore, she finds that a higher asymmetry in the indirect network effects reduces the incentives to collude.

Dewenter, Haucap and Wenzel (2011) analyse the welfare consequence of collusion on the adver-

[1] Although many examples of two-sided markets where the network effects are both positive (e.g. the market for yellow pages or the payment cards' market) give the impression that a two-sided market is not different from a market in which firms sell two complementary products, there is in fact a substantial difference: when complement products are sold, the buyers of the two products usually buy both; when a two-sided firm sells two products, the buyers of the products only buy one of them. Consider the case of inkjet printers and ink jet cartridges: unless consumers are particularly naives, they can be expected to consider the price of both as they choose to buy one; they therefore internalize the network effect. As in a two-sided market the buyers buy only one of the two products, the network effects are in fact not internalized by them and are therefore network externalities.

tising tariffs only, in a duopoly newspaper market where firms first choose the advertising quantity and then the cover prices, while readers like advertising. Under these assumptions and the additional assumption of a linear demand for differentiated products on the readers side, they find that collusion on the advertising tariffs may not only lead to an increase in readers' welfare (since it may reduce readers' prices more than it reduces the value of the newspaper to readers by decreasing the quantity of ads) but it can also lead to a higher advertisers' welfare (as it increases advertising tariffs less than it increases the newspaper's value to advertisers due to higher circulation).

Antonielli and Filistrucchi (2011) analyse instead collusion in a newspaper market where two editors first choose the political position of their newspaper, then set cover prices and advertising tariffs. They build on the work of Gabszewicz, Laussel and Sonnac (2001, 2002), whose model of competition among newspaper publishers they take as the stage game of an infinitely repeated game, and investigate the incentives to collude and the properties of the collusive agreements in terms of welfare and pluralism. As Gabszewicz, Laussel and Sonnac (2001, 2002), they assume that readers are indifferent to advertising. They compare two forms of collusion: in the first, publishers cooperatively select both prices and political position; in the second, compete on the political position but cooperatively select prices. They show that collusion on prices only reinforces the tendency towards minimum differentiation discussed in Gabszewicz, Laussel and Sonnac (2001, 2002), while collusion on both prices and the political line tends to mitigate it. Their findings question the rationale for the special antitrust regime for Joint Operating Agreements (JOAs) among US newspapers, which allow publishers to cooperate in setting cover prices and advertising tariffs but not the editorial line. They also show that, whatever the form of collusion, incentives to collude first increase, then decrease as advertising revenues per viewer increase.

Among empirical works, Argentesi and Filistrucchi (2007) provide econometric evidence that daily newspapers in Italy have been colluding on the cover price but not on the advertising tariffs. Flath (2013) focuses instead on whether resale price maintenance is used to sustain collusion in the Japanese newspaper market. Fan (2013) proposes a structural econometric model which endogenises also the choice of newspaper quality. She then evaluates the effects of mergers among competing US newspapers, modelling newspapers engaged in JOAs as cooperatively setting cover prices and advertising tariffs but not newspaper quality. Finally, Gentzkow, Shapiro and Sinkinson (forthcoming) formulate a model of newspaper demand, entry and affiliation choice in which newspapers compete for both readers and advertisers. They use a combination of calibration and estimation to identify the model's parameters from data on US newspapers circulation, costs and revenues. They argue that allowing newspapers to form JOA, i.e. to collude on both circulation and advertising prices, increases diversity at no cost to economic welfare.

In this paper we study optimal cartel prices in a two-sided market. First, building on Peitz and Valletti (2008), we present a simple theoretical model, able to capture competition among two advertising financed TV channels, showing that prices above the two-sided monopoly price may prevail on one side of a two-sided market as a means to enhance the sustainability of the cartel. We prove that in such a case a higher benefit from the network effect may compensate customers on that side of the market for the higher prices they are charged. We then provide necessary and sufficient conditions for these results to hold in more complex models of two-sided markets, in which firms compete by setting quantities on both sides of the market. In particular, we find that, when firms compete à la Cournot, cartel stability may require producing a lower-than-monopoly output in the collusive arrangement on one of the two market-sides.[2] The conditions we provide are related to tthe relative size of ratio between the own and cross price elasticities and the ratio between the own an cross network elasticities on each side of the market.

This paper also relates to the strand of literature on sustainability of collusion (Deneckere, 1983, Ross, 1992). From this literature it is known that, when firms compete in quantities or in price and sell differentiated products, there may be cases in which, while joint profit maximization cannot be sustained as it is is not incentive-compatible, firms may still settle for a collusive outcome yielding them a lower profit than joint-profit maximization but a higher profit than competition. In other words, in a one-sided market, when products are substitutes, the Pareto-optimal (from the firms' standpoint) collusive outcome may prescribe an aggregate output above the level resulting from joint profit maximization, both under Cournot and under Bertrand competition. On the other hand, when products are complements, the Pareto-optimal (from the firms' standpoint) collusive outcome may prescribe an aggregate output below the level resulting from joint profit-maximization. Intuitively this is because, when products are complements, prices are strategic substitutes and quantities are strategic complements. However, the literature on collusion with differentiated goods in one-sided markets does not show clear results in terms of how different degrees of product substitutability/complementarity contribute to the stabilization of a cartel.

[2]Analogously, when firms compete à la Bertrand, it may be optimal to price above monopoly on one of the two market sides. The case of Bertrand competition on both sides of the market is however more complex, in that additional conditions are needed to ensure that there exists a unique set of quantities for each set of prices. Such conditions are discussed in Filistrucchi and Klein (2013) and they relate to the existence of a unique equilibrium in the consumers' coordination game. Such conditions are not necessary in models of competition à la Cournot because, the assumption of an elastic market demand is sufficient to guarantee the existence of a unique set of prices for each set of quantities. This is shown in sub-section 3.1.

Our analysis thus extends to cartels in two-sided markets a result previously known for cartels selling complementary products, despite the fact that products in a two-sided market are not complements for customers, since customers typically buy only one of the two products (e.g. in the case of newspapers, advertisers buy advertising slots while readers buy content) and products on the two sides are substitutes (e.g. newspapers publishers compete for both readers and advertisers).[3]

When the analysis turns to two sided markets, the interplay of network effects generates interesting departures from the one sided case. In two sided markets, even when products are substitutes on each side, the indirect network effects between the two sides may generate both an own-side and a cross-side complementarity. For instance, an increase in a firm's advertising quantity may end up increasing the advertising price of the rival or an increase in the viewers quantity may end up increasing the rival's advertising price. Given these complementarities, once network effects are taken into account, on one side prices may become strategic substitutes and quantities may become strategic complements. That advertising quantities can exhibit the properties of strategic complements is is not new to the literature. Reisinger, Ressner and Schmidtke (2009) discuss conditions for this to be the case in free-to-air TV market. Their focus however is on predicting the effects of entry on the level of advertising in the market. In fact, when advertising quantities are strategic complements, entry of an additional TV channel increases advertising levels in existing TV channels. We focus instead on optimal cartel prices. In addition, we provide more general conditions which are valid also for two-sided markets in which customers on both sides pay for joining the platform. This is the case of the Pay-TV or Pay-per-View examples we discuss in the paper, but also of other marts, such as the traditional markets for newspapers and for magazines.

The analysis of how indirect network effects shape the collusive optimal price structure is the focus of our paper. Our results to this regard are new. In particular, they are not present in Rhumer (2011) as her model, based on the Hotelling model of Armstrong (2006), assumes inelastic aggregate demand on both sides of the market with respect to both the price and the network effect. In fact, our main results are obtained when aggregate demand is elastic on at least one of the two market sides.

The paper proceeds as follows: section 2 presents a simple model of competition and collusion applicable to advertising financed free-to-air TV; section 3 analyses which features of more complex two-sided models lead to results similar to the ones in section 3; section 4 concludes.

[3] See Filistrucchi, Geradin and van Damme (2013) p.38-39.

2 A simple model

We now present a simple model of competition and collusion in the market for free-to-air television. In such a market advertizers' demand depends on the own advertizing price and viewers' quantity, viewers' demand on the amount of advertizing on each channel. We assume free-to-air TV is exogenously free. [4]

2.1 The demand-side

Our simple model encompasses demand for differentiated products on both the viewers' and the advertisers' side of the market. On the viewers' side TV channels are differentiated in terms of content (e.g. political content or amount of entertainement). We assume each broacaster owns only one TV channel.

2.1.1 Viewers' demand

Viewers single home. They are characterized by a taste β for a specific content.[5] β is uniformly located on a $(0,1)$ line. The market size is 1. Channel 1 and 2 are located at the two extremes of the line. Without loss of generality, we assume channel 1 is located at 0, and channel 2 is located at 1. Viewers bear a cost $t_V x$ when viewing a TV channel which is located at distance x from their preferred channel type β. [6] Content is associated to advertizing. We assume that viewers dislike advertising, but they cannot avoid it. Viewers incur a disutility γ, assumed to be invariant across viewers and across channels, for each unit of advertizing they have to bear. Hence, a user incurs an overall disutility $q^A\gamma$ as a result of advertizing. Finally, we assume that firms are confronted with the constraint that content is available for free. This constraint may be of technological or of legal nature. Therefore, the price a viewer pays to watch a channel $p^V = 0$ for both channels. The utility of a viewer V who decides to view channel $i = 1,2$ and is located at a distance x from the channel is given by:

$$u_1^V = k - \gamma q_1^A - t_V x$$

where k is the intrinsic value of viewing the channel, which is assumed equal across viewers and channels. Yet channels are not only horizontally differentiated but also vertically differentiated, in the eyes of viewers, since they broadcast a different level of advertising. Notice that, whereas horizontal

[4]See Calvano and Polo(2014) for why free-to-air Tv and pay-Tv should in fact be considered endogenous business models. This argument is particularly valid for digital TV as it is now possible for every TV station to encrypt the signal, a necessary condition for a successful pay-TV. This was not true however for traditional aerial TV. Since the simple model in this section is used only for expositional reasons, we maintain here this assumption. It will be released in Section 3.

[5]They may, for instance, have political preferences rannging from extreme left to extreme right.

[6]This can be seen, for instance, as the cost of reading a newspaper which does not have the ideal political position for the viewer.

differentiation is exogenous, vertical differentiation is endogenous, as the quantity of advertising is determined by the prices set by broadcasters. Except for their different locations and their different amounts of advertising, the two channels are assumed to be perfectly symmetric. Hence, they have the same cost function. We normalize both fixed and marginal costs to zero.

Viewers choose the channel that yields them the highest utility. We assume t_V is small enough that the viewers' market is fully covered when the two channels are owned by the same publisher.

The indifferent user is located at $\widehat{\beta}$ that solves the following equation:

$k - \gamma q_1^V - t_V \widehat{\beta} = k - \gamma q_2^V - t_V \left(1 - \widehat{\beta}\right)$

All the users to the left of $\widehat{\beta}$ buy from platform 1, while all users to the right of $\widehat{\beta}$ buy from platform 2.

Demands on the viewers' side are given by

$$q_i^V = \frac{\gamma\left(q_j^A - q_i^A\right)}{2t_V} + \frac{1}{2} \vee i,j = 1,2 : i \neq j$$

The allocation of viewers across the two channels is thus exclusively determined by the amount of advertizing.

2.1.2 Advertisers' demand

The advertizers side is modeled along the lines of Peitz and Valletti (2009). Advertizing is a necessary condition for producers to be able to sell. Each advertizer sells its product to a portion λ of consumers who have seen an ad. Each product has a quality α, which also reflects final consumer's willingness to pay for it. Consumers are perfectly informed on the product's quality, while producers have monopoly power on their product variety, and are therefore able to extract the full surplus α from a product of quality α. Products are produced at constant marginal costs, which without loss of generality are set equal to zero. The advertizers' side is of mass 1, and advertizers differ with respect to α, which is uniformly distributed on the $[0,1]$ interval. Advertizer α advertizes in platform i if the expected profit from advertizing $\alpha\lambda q_i^V$ exceeds its cost p_i^A, $i = 1,2$. Observe that advertizers may decide to advertize in both TV channels, since advertizers aim at reaching consumers and we assumed that viewers single home. The advertizer that is indifferent between advertising and not advertising in platform $i = 1,2$ is at $\widehat{\alpha}_i$ such that

$$\widehat{\alpha}_i \lambda q_i^V - p_i^A = 0$$

Hence,

$$\widehat{\alpha}_i = \frac{p_i^A}{\lambda q_i^V}$$

Since $\widehat{\alpha} = 1 - q_i^A$ then demands on the advertisers' side are given by

$$q_i^A = 1 - \frac{p_i^A}{\lambda q_i^V}$$

and inverse demand is given by

$$p_i^A = \left(1 - q_i^A\right) \lambda q_i^V$$

2.2 The supply-side

We assume TV broadcasters collude adopting grim trigger strategies. We assume no transfers are possible among cartel members.

We assume TV broadcasters engage in an infinitely repeated game. They set advertising quantities/ In each period they decide whether to compete à la Cournot or to collude. In case of collusion, we assume they adopt grim-trigger strategies involving reversion to the static Cournot outcome for ever following a defection. We assume no transfers are possible among cartel members. We assume that broadcasters discount the future at a rate δ.

The incentive compatibility constraint (IC) for sustainability of collusion is therefore:

$$\frac{\pi_{cls}}{1-\delta} \geq \pi_D + \frac{\delta \pi_{comp}}{1-\delta}$$

where π_{cls} identifies the collusive profit, π_D the unilateral deviation profit when broadcaster i chooses its optimal uni-periodal deviation from the agreed upon collusive outcome, and π_{comp} the competitive oligopolistic profits to which broadcasters revert after a deviation.

In the next sub-sections, we first derive broadcasters profits in case of joint-profit maximization, of oligopolistic competition and of deviation. We then move to the analysis of the sustainability of the cartel. Finally, we discuss the welfare implications of collusion.

2.2.1 Joint-profit maximization

As already mentioned, we assume t_V is low enough that the market is covered on the viewers' side. More precisely, we assume

$$q^A_{mon} \leq \frac{k - t_V q^V}{\gamma}$$

where the subscript *mon* stands for monopoly (or joint-profit maximization). As long as the above expression holds, a monopolist broadcaster is indifferent between inducing users to split into the two channels or concentrating them into a single one (although, the more users are distributed asymmetrically across the two channels, the more restrictive the condition for the market to be fully covered gets). However, a collusive agreement involving the closure of one of the two channels does not appear to be likely: it would leave one of the two cartel members (the one whose channel has been shut down) in the hands of the other one, particularly in the case of fixed costs typical of the broadcasting market. We thus assume the joint-profit maximization agreement does not involve closing down any of the two channels.

We then focus on the symmetric agreement, which gives at least as high a profit as any other agreement involving an asymmetric allocation of viewers across the two platforms. Indeed, such an agreement would seem the easier one to agree on when broadcasters have the same cost functions and, as we assumed, no transfers are possible among carte members.

The joint profit maximization problem is therefore:

$$\max_{q_1^A, q_2^A} \left(1 - q_1^A\right) \lambda q_i^V q_i^A + \left(1 - q_2^A\right) \lambda q_j^V q_j^A$$
$$s.t.\, q_1^V = q_2^V = \tfrac{1}{2}$$

This yields $q_1^A = q_2^A = \frac{1}{2}$, $p_1^V = p_2^V = \frac{\lambda}{4}$, and $\pi_{1+2} = \frac{\lambda}{4}$

2.2.2 Oligopolistic competition

Under quantity competition, each broadcaster $i = 1, 2$ maximizes:

$$\begin{aligned} & \max_{q_i} \left(1 - q_i^A\right) \lambda q_i^V q_i^A \\ = \quad & \left(1 - q_i^A\right) \lambda \left(\left(\frac{\gamma q_j^A - \gamma q_i^A}{2 t_V} + \frac{1}{2} \right) \right) q_i^A \end{aligned}$$

This yields $q_1^A = q_2^A = q^A_{comp} = \frac{1}{2\gamma} \left(\gamma + 2t_V - \sqrt{\gamma^2 + 4t_V^2} \right)$. The amount of advertizing is unrelated to λ, the probability that a viewer buys the advertized product. The intuition behind this result is that an increase in λ increases proportionally the willingness to pay for each unit of advertizing; as a result, when the advertizing cost is normalized to zero, the tradeoff faced by each competitor is unaltered.

It follows that:

$$\begin{aligned} p_1^A = p_2^A = p_{comp}^A &= \frac{\lambda}{4\gamma}\left(\gamma - 2t_V + \sqrt{\gamma^2 + 4t_V^2}\right) \\ \pi_1 = \pi_2 = \pi_{comp} &= \frac{1}{2\gamma^2}\left(\lambda t_V \sqrt{\gamma^2 + 4t_V^2} - 2\lambda t_V^2\right) \end{aligned}$$

with $p_{comp}^A > p_{mon}^A = \frac{\lambda}{4}$, and $q_{comp}^A < q_{mon}^A = \frac{1}{2}$, where the subscript *comp* stands for competition.

2.2.3 Deviation

When broadcasters collude at the joint-profit maximizing outcome $q_{cls}^A = q_{mon}^A$, broadcaster i's unilaterally deviates by maximizing:

$$\max_{q_i^A} \left(1 - q_i^A\right) \lambda \left(\left(\frac{\frac{\gamma}{2} - \gamma q_i^A}{2t_V} + \frac{1}{2}\right)\right) q_i^A : i = 1,2$$

The optimal deviation for broadcaster $i = 1,2$ therefore prescribes:

$$\begin{aligned} q_i^A &= \frac{1}{6\gamma}\left(3\gamma + 2t_V - \sqrt{3\gamma^2 + 4t_V^2}\right) : i = 1,2 \\ \pi_i &= \frac{1}{432}\frac{\lambda}{\gamma^2 t_V}\left(4t_V + \sqrt{3\gamma^2 + 4t_V^2}\right)\left(6\gamma^2 - 8t_V^2 + 4t_V\sqrt{3\gamma^2 + 4t_V^2}\right) : i = 1,2 \end{aligned} \tag{1}$$

When broadcasters, instead, agree upon a generic $q_{cls}^A \neq q_{mon}^A$, the unilateral deviation profit is given by:

$$\pi_D = -\frac{1}{54}\frac{\lambda}{\gamma^2 t_V}\left(t_V + \gamma q_{cls}^A - 2\gamma - \chi\right)\left(2t_V - \gamma + \chi + 2\gamma q_A\right)\left(\gamma + t_V - \chi + \gamma q_{cls}^A\right)$$

where for simplicity we denote: $\chi = \sqrt{\gamma^2\left(q_{cls}^A\right)^2 + q_{cls}^A\left(2\gamma t_V - \gamma^2\right) + \left(\gamma - t_V\right)^2 + \gamma t_V}$

It can be shown that $\frac{\partial \pi_D}{\partial q_{cls}^A} > 0$. This reflects the already established fact that the two quantities are strategic complements in each platform's profit function.

2.2.4 Collusion and sustainability

When broadcasters agree upon collusion at the monopoly outcome, each broadcaster deviates according to 1.

The cartel is sustainable if the incentive compatibility constraint is not violated for either broadcaster, hence if $\delta > \delta_{mon}$

$$\begin{aligned} \frac{\lambda}{8(1-\delta)} \quad \geq \quad & \frac{1}{432}\frac{\lambda}{\gamma^2 t_V}\left(4t_V+\sqrt{3\gamma^2+4t_V^2}\right)\left(6\gamma^2-8t_V^2+4t_V\sqrt{3\gamma^2+4t_V^2}\right)+ \\ & +\frac{\delta}{1-\delta}\left(\frac{1}{2\gamma^2}\left(\lambda t_V\sqrt{\gamma^2+4t_V^2}-2\lambda t_V^2\right)\right) \end{aligned} \qquad (2)$$

$$\delta > -\frac{8t_V^3-3\gamma^2\sqrt{3\gamma^2+4t_V^2}+9\gamma^2 t_V-4t_V^2\sqrt{3\gamma^2+4t_V^2}}{208t_V^3-108t_V^2\sqrt{\gamma^2+4t_V^2}+3\gamma^2\sqrt{3\gamma^2+4t_V^2}+18\gamma^2 t_V+4t_V^2\sqrt{3\gamma^2+4t_V^2}}=\delta_{mon} \qquad (3)$$

When $\delta < \delta_{mon}$, collusion at monopoly is not feasible, since platforms have incentives to deviate, as shown in **??**. Still, platforms may do better than competition, by agreeing upon a different (i.e., not monopolistic) cartel outcome. The relevant question is, though, in which direction (above or below monopoly) should q_{cls}^A move in order to relax platform's IC constraint, and expand the set of discount factors under which collusion can be sustained.

Tedious computations allow us to derive $\delta\left(q_{cls}^A\right)$, that is, the minimum discount factor at which collusion is sustainable as a function of the agreed-upon (symmetric) quantity $q_{i,cls}^A, i = 1,2$. While we already know that $\delta\left(q_{mon}^A\right) = \delta_{mon}$, we are interested in understanding the sign of the relation $\frac{\partial\delta\left(q_{cls}^A\right)}{\partial q_{cls}^A}$. $\delta\left(q_{cls}^A\right)$ is the value of δ that solves the following equation:

$$\frac{\lambda}{8(1-\delta)}=-\frac{1}{54}\frac{\lambda}{\gamma^2 t_V}\left(t_V+\gamma q_A-2\gamma-\chi\right)\left(2t_V-\gamma+\chi+2\gamma q_A\right)\left(\gamma+t_V-\chi+\gamma q_A\right)+\frac{\delta}{1-\delta}\left(\frac{1}{2\gamma^2}\left(\lambda t_V\sqrt{\gamma^2+4t_V^2}-2\lambda t_V^2\right.\right.$$

It can be shown that

$$\frac{\partial\delta\left(q_{cls}^A\right)}{\partial q_{cls}^A} > 0$$

that is, a lower advertising quantity expands the range of discount factors under which collusion is feasible, thereby helping to sustain the cartel. The intuition behind this result can again be easily grasped by considering that the two advertizing quantities are strategic complements in the platforms' profit function. Collusive profit is higher than competitive profit as long as q_{cls}^A is sufficieny high. Denote δ_{comp} the discount factor at which the best collusive outcome is exactly equivalent to the competitive outcome, so $\delta\left(q_{comp}\right)$. Therefore, we may conclude that there exists a range, $\delta_{comp} < \delta < \delta_{mon}$, in which broadcasters find it optimal to agree on a lower than monopoly quantity to sustain the collusive arrangement. We have therefore established the following:

Proposition 1. *When the platforms repeatedly interact in an infinite-horizon game, they are able to sustain joint-profit maximization if $d > d_{mon}$. When $d_{comp} < d < d_{mon}$, they sustain a cartel yielding them higher profit than competition, and prescribing a lower advertizing quantity with respect to joint-profit maximizing. Finally, when $d < d_{comp}$, no collusion is sustainable.*

2.2.5 Welfare

In order to characterize welfare in the collusive arrangement, we analyze welfare in the monopoly and in the competitive situations.

Consumers' welfare is the aggregate of the demand side welfare in each of the two markets served by the platform. In general terms, consumer welfare is:

$$\begin{aligned} W^V &= \int_0^{\frac{1}{2}} \left(k - \gamma q_1^A - t_V x\right) dx + \int_{\frac{1}{2}}^1 \left(k - \gamma q_2^A - t_V (1-x)\right) dx = \\ W^A &= \int_{\frac{p_1^A}{\lambda q_1^V}}^1 \left(\alpha\lambda q_1^V - p_1^A\right) d\alpha + \int_{\frac{p_2^A}{\lambda q_2^V}}^1 \left(\alpha\lambda q_2^V - p_2^A\right) d\alpha = \end{aligned}$$

for the viewers and for the advertizers side respectively.

Welfare under monopoly is:

$$\begin{aligned} W_{mon}^V &= k - \frac{1}{2}\gamma - \frac{1}{4}t_V \\ W_{mon}^A &= \frac{\lambda}{8} \end{aligned}$$

Welfare under competition is:

$$\begin{aligned} W_{comp}^V &= k - \frac{1}{2}\gamma - \frac{5}{4}t_V + \frac{1}{2}\sqrt{\gamma^2 + 4t_V^2} \\ W_{comp}^A &= \frac{1}{8}\frac{\lambda}{\gamma^2}\left(\gamma^2 - 2t_V\sqrt{\gamma^2 + 4t_V^2} + 4t_V^2 + 2\gamma t_V - \gamma\sqrt{\gamma^2 + 4t_V^2}\right) \end{aligned}$$

Observe that users' welfare is higher under competition, while advertizers' welfare is higher under monopoly. This result may be quite counterintuitive at least at a first glance. It is entirely due to the reduction in advertizing brought about by the increase in competition. Users are valuable for the advertizing side, and they are attracted by reducing quantity.

Competition for viewers drives the amount of advertizing down under competition. The amounts of ads of the two companies thefore become *strategic complements* in the profit function. When a broadcaster reduces its ads, the rival has an incentive to do the same. This reduces welfare in the advertizers' market.

Overall consumers' welfare cannot be ranked a-priori. Indeed, its effects depend on the sign of the profit function.

$$
\begin{aligned}
W^{A+V}_{comp} &= k - \frac{1}{2}\gamma - \frac{5}{4}t_V + \frac{1}{2}\sqrt{\gamma^2+4t_V^2} + \frac{1}{8}\frac{\lambda}{\gamma^2}\left(\gamma^2 - 2t_V\sqrt{\gamma^2+4t_V^2} + 4t_V^2 + 2\gamma t_V - \gamma\sqrt{\gamma^2+4t_V^2}\right) = \\
&= \frac{1}{8\gamma^2}\left(4\gamma^2\sqrt{\gamma^2+4t_V^2} - 4\gamma^3 + 8k\gamma^2 + \lambda\gamma^2 + 4\lambda t_V^2 - 10\gamma^2 t_V - \lambda\gamma\sqrt{\gamma^2+4t_V^2} - 2\lambda t_V\sqrt{\gamma^2+4t_V^2} + 2\lambda\gamma t_V\right) \\
W^{A+V}_{mon} &= k - \frac{1}{2}\gamma - \frac{1}{4}t_V + \frac{\lambda}{8}
\end{aligned}
$$

Which of the two is larger depends on the sign of $\left(-4\gamma^2+\lambda\gamma+2\lambda t_V\right)$.

If $\left(-4\gamma^2+\lambda\gamma+2\lambda t_V\right)>0$, then $W^{A+V}_{comp} < W^{A+V}_{mon}$. Hence, monopoly welfare is larger than competitive welfare. This implies that advertizers' welfare is relatively more important, and that t_V (differentiation on the viewers' side) and λ (external effect) are relatively large with respect to γ (negative effect of advertizing on viewers).

If, instead, $\left(-4\gamma^2+\lambda\gamma+2\lambda t_V\right)<0$, then $W^{A+V,M} < W^{A+V,C}$, because the negative effect of advertizing on viewers is large.

Profits are clearly superior in the monopoly solution. In terms of total welfare, therefore, this relatively increases welfare in the monopoly solution.

Our results can therefore be summarized in the following:

Proposition 2. *When broadcasters are constrained not to charge a price on the viewers' side, advertizing quantities are strategic complements in their profit functions. Competition reduces advertizing quantities. Competition reduces advertisers welfare while increasing viewers welfare. The effect on the aggregate consumer surplus is ambiguous: when differentiation on the viewers side and advertizers willingness to pay to reach viewers are relatively large with respect to the nuisance cost of advertising, consumer surplus is larger under monopoly; viceversa, when the nuisance cost of advertising is relatively large, consumer surplus is larger under competition.*

Remember that, when collusion at the monopoly outcome cannot be sustained, and $\delta_{comp} < \delta < \delta_{mon}$, our Proposition 2 shows that platforms reduce the advertizing quantity with respect to monopoly. In this case, with respect to a monopolistic cartel, advertizers' welfare decreases, viewers' welfare increase, and the net effect on consumer welfare depends on the interaction between λ, t_V and γ, as illustrated above.

3 A more complex model

3.1 The demand-side

Consider two TV channelss facing demand functions for advertising and viewership that are linear in all prices on the same side and all quantities on the other side,

$$\begin{aligned} q_1^a &= \alpha_1^a - \beta_{11}^a p_1^a + \beta_{12}^a p_2^a + \gamma_{11}^a q_1^V + \gamma_{21}^a q_2^V \\ q_2^a &= \alpha_2^a + \beta_{21}^a p_1^a - \beta_{22}^a p_2^a + \gamma_{21}^a q_1^V + \gamma_{22}^a q_2^V \\ q_1^V &= \alpha_1^V - \beta_{11}^V P_1^V + \beta_{21}^V p_2^V + \gamma_{11}^V q_1^a + \gamma_{12}^V q_2^a \\ q_2^V &= \alpha_2^V + \beta_{21}^V p_1^V - \beta_{22}^V p_2^V + \gamma_{21}^V q_1^a + \gamma_{22}^V q_2^a, \end{aligned} \tag{4}$$

with positive price coefficients $\beta_{11}^a, \beta_{12}^a, \beta_{21}^a, \beta_{22}^a$ and $\beta_{11}^V, \beta_{12}^V, \beta_{21}^V, \beta_{22}^V$

This system can be written in matrix notation as

$$q = \alpha + Bp + \Gamma q,$$

with vectors

$$q = \begin{pmatrix} q^a \\ q^V \end{pmatrix} = \begin{pmatrix} q_1^a \\ q_2^a \\ q_1^V \\ q_2^V \end{pmatrix}, \alpha = \begin{pmatrix} \alpha^a \\ \alpha^V \end{pmatrix} = \begin{pmatrix} \alpha_1^a \\ \alpha_2^a \\ \alpha_1^V \\ \alpha_2^V \end{pmatrix}, p = \begin{pmatrix} p^a \\ p^V \end{pmatrix} = \begin{pmatrix} p_1^a \\ p_2^a \\ p_1^V \\ p_2^V \end{pmatrix}$$

and block-diagonal matrixes

$$\Gamma = \begin{pmatrix} 0 & \Gamma^a \\ \Gamma^V & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \gamma_{11}^a & \gamma_{12}^a \\ 0 & 0 & \gamma_{21}^a & \gamma_{22}^a \\ \gamma_{11}^V & \gamma_{12}^V & 0 & 0 \\ \gamma_{21}^V & \gamma_{22}^V & 0 & 0 \end{pmatrix}.$$

$$B=\begin{pmatrix} B^a & 0 \\ 0 & B^V \end{pmatrix}=\begin{pmatrix} -\beta_{11}^a & \beta_{12}^a & 0 & 0 \\ \beta_{21}^a & -\beta_{22}^a & 0 & 0 \\ 0 & 0 & -\beta_{11}^V & \beta_{12}^V \\ 0 & 0 & \beta_{21}^V & -\beta_{22}^V \end{pmatrix}$$

Assuming firms set quantities on both sides of the market, from

$q=\alpha+Bp+\Gamma q$

it follows that

$Bp=-\alpha+(I-\Gamma)q$

Hence, if total demand is elastic (in the sense that a rise in price decreases own demand more than it increases competitors demand), then B has a dominant diagonal and is invertible and one can derive the inverse demand functions

$p=-B^{-1}\alpha+B^{-1}(I-\Gamma)q$

where $q=\begin{pmatrix} q^a \\ q^V \end{pmatrix}=\begin{pmatrix} q_1^a \\ q_2^a \\ q_1^V \\ q_2^V \end{pmatrix}$, $\alpha=\begin{pmatrix} \alpha^a \\ \alpha^V \end{pmatrix}=\begin{pmatrix} \alpha_1^a \\ \alpha_2^a \\ \alpha_1^V \\ \alpha_2^V \end{pmatrix}$, $p=\begin{pmatrix} p^a \\ p^V \end{pmatrix}=\begin{pmatrix} p_1^a \\ p_2^a \\ p_1^V \\ p_2^V \end{pmatrix}$ $\Gamma=\begin{pmatrix} 0 & \Gamma^a \\ \Gamma^V & 0 \end{pmatrix}=$
$\begin{pmatrix} 0 & 0 & \gamma_{11}^a & \gamma_{12}^a \\ 0 & 0 & \gamma_{21}^a & \gamma_{22}^a \\ \gamma_{11}^V & \gamma_{12}^V & 0 & 0 \\ \gamma_{21}^V & \gamma_{22}^V & 0 & 0 \end{pmatrix}$ $.B=\begin{pmatrix} B^a & 0 \\ 0 & B^V \end{pmatrix}=\begin{pmatrix} -\beta_{11}^a & \beta_{12}^a & 0 & 0 \\ \beta_{21}^a & -\beta_{22}^a & 0 & 0 \\ 0 & 0 & -\beta_{11}^V & \beta_{12}^V \\ 0 & 0 & \beta_{21}^V & -\beta_{22}^V \end{pmatrix}$

In that case,

$B^{-1}=\begin{pmatrix} B^{a^{-1}} & 0 \\ 0 & B^{V^{-1}} \end{pmatrix}=\begin{pmatrix} \frac{-\beta_{22}^a}{b^a} & \frac{-\beta_{12}^a}{b^a} & 0 & 0 \\ \frac{-\beta_{21}^a}{b^a} & \frac{-\beta_{11}^a}{b^a} & 0 & 0 \\ 0 & 0 & \frac{-\beta_{22}^V}{b^V} & \frac{-\beta_{12}^V}{b^V} \\ 0 & 0 & \frac{-\beta_{21}^V}{b^V} & \frac{-\beta_{11}^V}{b^V} \end{pmatrix}$ where $b^a=\beta_{11}^a\beta_{22}^a-\beta_{12}^a\beta_{21}^a>0$ and $b^V=\beta_{11}^V\beta_{22}^V-\beta_{12}^V\beta_{21}^V>0$

while

$$I-\Gamma=\begin{pmatrix} I & -\Gamma^a \\ -\Gamma^V & I \end{pmatrix}\begin{pmatrix} 1 & 0 & -\gamma_{11}^a & -\gamma_{12}^a \\ 0 & 1 & -\gamma_{21}^a & -\gamma_{22}^a \\ -\gamma_{11}^V & -\gamma_{12}^V & 1 & 0 \\ -\gamma_{21}^V & -\gamma_{22}^V & 0 & 1 \end{pmatrix}$$

and

$$\frac{\partial p}{\partial q} = B^{-1}(I-\Gamma) = \begin{pmatrix} B^{a^{-1}} & 0 \\ 0 & B^{V^{-1}} \end{pmatrix} \begin{pmatrix} I & -\Gamma^a \\ -\Gamma^V & I \end{pmatrix} = \begin{pmatrix} B^{a^{-1}} & -B^{a^{-1}}\Gamma^a \\ -B^{V^{-1}}\Gamma^V & B^{V^{-1}} \end{pmatrix}.$$

Hence, since all the elements of $B^{a^{-1}}$ and $B^{V^{-1}}$ are negative, products on each side will always be substitutes (in the sense that a higher quantity of one always decreases price). However, across sides products may be complements or substitutes.

In fact, the matrix of the cross-side effects of viewership on advertising is

$$-B^{a^{-1}}\Gamma^a = \frac{1}{b^a} \begin{pmatrix} -\beta_{22}^a & -\beta_{12}^a \\ -\beta_{21}^a & -\beta_{11}^a \end{pmatrix} \begin{pmatrix} -\gamma_{11}^a & -\gamma_{12}^a \\ -\gamma_{21}^a & -\gamma_{22}^a \end{pmatrix} = \begin{pmatrix} \beta_{22}^a\gamma_{11}^a + \beta_{12}^a\gamma_{21}^a & \beta_{22}^a\gamma_{12}^a + \beta_{12}^a\gamma_{22}^a \\ \beta_{21}^a\gamma_{11}^a + \beta_{11}^a\gamma_{21}^a & \beta_{21}^a\gamma_{12}^a + \beta_{11}^a\gamma_{22}^a \end{pmatrix}.$$

The first element in the first row is positive as $\gamma_{11}^a > 0$ (and $\gamma_{21}^a < 0$) and because $|\beta_{22}^a\gamma_{11}^a| > |\beta_{12}^a\gamma_{21}^a|$. Similarly for the second element in the second row. The sign of the off-diagonal elements is instead uncertain. Since $\gamma_{22}^a > 0$ (and $\gamma_{12}^a < 0$) second element in the first row is negative if $|\beta_{22}^a\gamma_{12}^a| > |\beta_{12}^a\gamma_{22}^a|$or equivalently $\left|\frac{\gamma_{12}^a}{\gamma_{22}^a}\right| > \left|\frac{\beta_{12}^a}{\beta_{22}^a}\right|$. Similarly, since $\gamma_{11}^a > 0$ (and $\gamma_{21}^a < 0$), the first element in the second row is negative if $|\beta_{21}^a\gamma_{11}^a| < |\beta_{11}^a\gamma_{21}^a|$ or$\left|\frac{\gamma_{21}^a}{\gamma_{11}^a}\right| > \left|\frac{\beta_{21}^a}{\beta_{11}^a}\right|$. The conditions $\left|\frac{\gamma_{12}^a}{\gamma_{22}^a}\right| > \left|\frac{\beta_{12}^a}{\beta_{22}^a}\right|$ and $\left|\frac{\gamma_{21}^a}{\gamma_{11}^a}\right| > \left|\frac{\beta_{21}^a}{\beta_{11}^a}\right|$can be interpreted as requiring (endogenous) vertical product differentiation to be more important than horizontal product differentiation.

Instead,

$$-B^{V^{-1}}\Gamma^V = \frac{1}{b^V} \begin{pmatrix} -\beta_{22}^V & -\beta_{12}^V \\ -\beta_{21}^V & -\beta_{11}^V \end{pmatrix} \begin{pmatrix} -\gamma_{11}^V & -\gamma_{12}^V \\ -\gamma_{21}^V & -\gamma_{22}^V \end{pmatrix} = \begin{pmatrix} \beta_{22}^V\gamma_{11}^V + \beta_{12}^V\gamma_{21}^V & \beta_{22}^V\gamma_{12}^V + \beta_{12}^V\gamma_{22}^V \\ \beta_{21}^V\gamma_{11}^V + \beta_{11}^V\gamma_{21}^V & \beta_{21}^V\gamma_{12}^V + \beta_{11}^V\gamma_{22}^V \end{pmatrix}.$$

If viewers like advertising, i.e. $\gamma_{11}^V > 0, \gamma_{22}^V > 0$ (and $\gamma_{21}^V < 0, \gamma_{12}^V < 0$) the pattern of the signs is the same as the one for $-B^{a^{-1}}\Gamma^a$ discussed above.

Let us then focus on the case in which viewers dislike advertising, i.e. $\gamma_{11}^V < 0, \gamma_{22}^V < 0$ (and $\gamma_{21}^V > 0, \gamma_{12}^V > 0$) . Then the first element in the first row is negative because $|\beta_{22}^V\gamma_{11}^V| > |\beta_{12}^V\gamma_{21}^V|$. Similarly for the second element in the second row. The sign of the off-diagonal elements is instead again uncertain. The second element in the first row is positive if $|\beta_{22}^V\gamma_{12}^V| > |\beta_{12}^V\gamma_{22}^V|$or equivalently $\left|\frac{\gamma_{12}^V}{\gamma_{22}^V}\right| > \left|\frac{\beta_{12}^V}{\beta_{22}^V}\right|$. Similarly, the first element in the second row is positive if $||\beta_{11}^V\gamma_{21}^V| > \beta_{21}^V\gamma_{11}^V|$ or equivalently$\left|\frac{\gamma_{21}^V}{\gamma_{11}^V}\right| > \left|\frac{\beta_{21}^V}{\beta_{11}^V}\right|$.The conditions $\left|\frac{\gamma_{12}^V}{\gamma_{22}^V}\right| > \left|\frac{\beta_{12}^V}{\beta_{22}^V}\right|$ and $\left|\frac{\gamma_{21}^V}{\gamma_{11}^V}\right| > \left|\frac{\beta_{21}^V}{\beta_{11}^V}\right|$can be interpreted as requiring, once again, (endogenous) vertical product differentiation to be more important than horizontal product differentiation.

Hence, when viewers dislike advertising, the signs in the matrix of the derivatives with respect to prices are

$$sign\left(\frac{\partial p}{\partial q}\right) = \begin{pmatrix} - & - & + & -/+ \\ - & - & -/+ & + \\ - & +/- & - & - \\ +/- & - & - & - \end{pmatrix}$$

where

$sign(\frac{\partial p_1^a}{\partial q_2^V}) = sign(\frac{\partial p_2^a}{\partial q_1^V}) = "-"$ if $\left|\frac{\gamma_{12}^a}{\gamma_{22}^a}\right| > \left|\frac{\beta_{12}^a}{\beta_{22}^a}\right|$ and $\left|\frac{\gamma_{21}^a}{\gamma_{11}^a}\right| > \left|\frac{\beta_{21}^a}{\beta_{11}^a}\right|$

and

$sign(\frac{\partial p_1^V}{\partial q_2^a}) = sign(\frac{\partial p_2^V}{\partial q_1^a}) = "+"$ if $\left|\frac{\gamma_{12}^V}{\gamma_{22}^V}\right| > \left|\frac{\beta_{12}^V}{\beta_{22}^V}\right|$ and $\left|\frac{\gamma_{21}^V}{\gamma_{11}^V}\right| > \left|\frac{\beta_{21}^V}{\beta_{11}^V}\right|$.

If such conditions hold, the own-product cross-side effects are such that "my viewers are a complement to my advertisers" but my "advertisers are a substitute to my viewers", whereas the cross-product cross-side effects are such that "viewers of my rival are a substitute to my advertisers" but "advertisers of my rival are a complement to my viewers".

Let us now consider the special case we presented in the previous section.

3.1.1 Back to the simple model

Our simple model of Section 2, features quantity competitition on the advertising side of the market, while no additional strategic instrument is available to broadcasters on the viewers' market. In addition, advertising demand is such that, as in most of the theoretical two-sided market literature[7], there are no cross price and cross network effects on the advertising side. This is a very special case of the one presented in Section 2.

We can rewrite the original demand system as

$$\begin{pmatrix} q^a \\ q^V \end{pmatrix} = \begin{pmatrix} \alpha^a \\ \alpha^V \end{pmatrix} + \begin{pmatrix} B^a & 0 \\ 0 & B^V \end{pmatrix} \begin{pmatrix} p^a \\ p^V \end{pmatrix} + \begin{pmatrix} 0 & \Gamma^a \\ \Gamma^V & 0 \end{pmatrix} \begin{pmatrix} q^a \\ q^V \end{pmatrix}.$$

where $\begin{pmatrix} q^a \\ q^V \end{pmatrix} = \begin{pmatrix} q_1^a \\ q_2^a \\ q_1^V \\ q_2^V \end{pmatrix}$, $\begin{pmatrix} \alpha^a \\ \alpha^V \end{pmatrix} = \begin{pmatrix} \alpha_1^a \\ \alpha_2^a \\ \alpha_1^V \\ \alpha_2^V \end{pmatrix}$, $\begin{pmatrix} p^a \\ p^V \end{pmatrix} = \begin{pmatrix} p_1^a \\ p_2^a \\ p_1^V \\ p_2^V \end{pmatrix}$ $\begin{pmatrix} 0 & \Gamma^a \\ \Gamma^V & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \gamma_{11}^a & \gamma_{12}^a \\ 0 & 0 & \gamma_{21}^a & \gamma_{22}^a \\ \gamma_{11}^V & \gamma_{12}^V & 0 & 0 \\ \gamma_{21}^V & \gamma_{22}^V & 0 & 0 \end{pmatrix}$

[7] See, among theoretical papers, Peitz and Valletti (2009), but also Gabszewitcz, Laussel and Sonnac (2001, 2002, 2004) and, among structural econometric papers, Rysman (2004) and Fan(2013). For the opposite assumption see instead, among theoretical contributions, Armstrong (2006) and Rhumer (2011) and, among structural econometric contributions, Kaiser and Wright92006) and Argentesi and Filistrucchi (2007).

$$\begin{pmatrix} -\beta^a_{11} & \beta^a_{12} & 0 & 0 \\ \beta^a_{21} & -\beta^a_{22} & 0 & 0 \\ 0 & 0 & -\beta^V_{11} & \beta^V_{12} \\ 0 & 0 & \beta^V_{21} & -\beta^V_{22} \end{pmatrix}$$

Under the assumption of the simple model, viewers demand is

$$q^V = \alpha^V + \Gamma^V q^a \tag{5}$$

while inverse advertisers' demand is [8]

$$p^a = -B^{a^{-1}} \left(\alpha^a + \Gamma^a \alpha^V\right) + B^{a^{-1}} \left(I - \Gamma^a \Gamma^V\right) q^a \tag{6}$$

Under the assumption of the simple model

$$B^{a^{-1}} = \begin{pmatrix} -\frac{1}{\beta^a_{11}} & 0 \\ 0 & -\frac{1}{\beta^a_{22}} \end{pmatrix}$$

[9]

$$\Gamma^a \Gamma^V = \begin{bmatrix} \gamma^a_{11} & 0 \\ 0 & \gamma^a_{22} \end{bmatrix} \begin{bmatrix} \gamma^V_{11} & \gamma^V_{12} \\ \gamma^V_{21} & \gamma^V_{22} \end{bmatrix} = \begin{pmatrix} \gamma^a_{11}\gamma^V_{11}, & \gamma^a_{11}\gamma^V_{12} \\ \gamma^a_{22}\gamma^V_{21}, & \gamma^a_{22}\gamma^V_{22} \end{pmatrix}$$

$$I - \Gamma^a \Gamma^V = \begin{pmatrix} 1 - \left(\gamma^a_{11}\gamma^V_{11}\right) & \gamma^a_{11}\gamma^V_{12} \\ \gamma^a_{22}\gamma^V_{21} & 1 - \gamma^a_{22}\gamma^V_{22} \end{pmatrix}$$

Hence,

[8] From $q^a = \alpha^a + B^a p^a + \Gamma^a q^V$, one obtains
$-B^a p^a = \alpha^a - q^a + \Gamma^a q^V$
or equivalently
$p^a = -B^{a^{-1}} \alpha^a + B^{a^{-1}} q^a - B^{a^{-1}} \Gamma^a q^V$
Substituting for q^V, one obtains
$p^a = -B^{a^{-1}} \alpha^a + B^{a^{-1}} q^a - B^{a^{-1}} \Gamma^a \left(\alpha^V + \Gamma^V q^a\right)$
or equivalently
$p^a = -B^{a^{-1}} \left(\alpha^a + \Gamma^a \alpha^V\right) + B^{a^{-1}} \left(q^a - \Gamma^a \Gamma^V q^a\right)$

[9] Using the notation in subsection 3.1, this can also be written as

$$B^{a^{-1}} = \frac{1}{b^a} \begin{pmatrix} -\beta^a_{22} & 0 \\ 0 & -\beta^a_{11} \end{pmatrix}$$

with $b^a = \beta^a_{11}\beta^a_{22} > 0$ or equivalently

$$
\begin{aligned}
\frac{\partial p^a}{\partial q^a} = B^{a^{-1}}\left(I - \Gamma^a \Gamma^V\right) &= \begin{pmatrix} -\frac{1}{\beta^a_{11}} & 0 \\ 0 & -\frac{1}{\beta^a_{22}} \end{pmatrix} \begin{pmatrix} 1 - \left(\gamma^a_{11}\gamma^V_{11}\right) & \gamma^a_{11}\gamma^V_{12} \\ \gamma^a_{22}\gamma^V_{21} & 1 - \gamma^a_{22}\gamma^V_{22} \end{pmatrix} \\
&= \begin{pmatrix} -\frac{1}{\beta^a_{11}}\left[1 - \left(\gamma^a_{11}\gamma^V_{11}\right)\right] & -\frac{1}{\beta^a_{11}}\left(\gamma^a_{11}\gamma^V_{12}\right) \\ -\frac{1}{\beta^a_{22}}\left(\gamma^a_{22}\gamma^V_{21}\right) & -\frac{1}{\beta^a_{22}}\left[1 - \left(\gamma^a_{22}\gamma^V_{22}\right)\right] \end{pmatrix}
\end{aligned} \tag{7}
$$

Since $\gamma^a_{11}, \gamma^a_{22} > 0, \gamma^V_{11,}\gamma^V_{22} < 0$ and $\gamma^V_{12,}\gamma^V_{21} > 0$, the diagonal elements in 7 will have a negative sign and off-diagonal elements will have a positive sign.

Hence, products on the advertisers' side of the market will be complements once network effects have been taken into account. As the price to the viewers is exogenously set to zero, broacasters will be able to gain profits on the advertisers side of the market only. They will thus be competing in advertising quantities with complement products in a one-sided market. It follows that, as in a one-sided market, quantities will be strategic complements. Monopoly, by incentivizing the internalization of these strategic effects, will lead to higher quantities, and lower prices, than competition. Similarly to a one-sided markets, when joint profit maximization is not sustainable as a cartel outcome, sustainability of the cartel can be obtained by pushing the collusive price towards the competitive level, thus raising cost of deviation. With complement products, this implies raising the collusive price (and lowering the collusive quantity) above the monopoly price (and below the collusive quantity) .

Let us now consider, for purely explanatory reasons, how strategic effects would vary if one introduced exogenously fixed viewers' prices. The main consequence would be that broacasters profits would depend not only on advertising revenues but also on viewers revenues. By setting advertising quantity broacasters would affect also viewers quantity and thus viewers revenues. Since viewers demand is given by 5, cross-side effects from adverting to viewership would be given by Γ^V. Given the assumption that viewers dislike advertising, then own advertising quantity will be a substitute for own viewership, while rivals advertising demand will be a complement for own viewership. In other words advertising quantity will play a similar role to prices on the viewers' side of the market. Thus, when broacasters set advertising quantities, they would not only compete in complement products on the advertising side of the market, as in the simple model above, but would also be competing with substitute products on the viewers' side of the market. Whether competition among complements or substitutes would prevail would depend on whether raising advertising quantities has a relatively greater complementary effect on rivals advertising profits or substitutary effect on rivals viewers profits. If firms set prices on the viewers side too, the picture would get more complicated but intuitively the crucial question will be whether features of competition among complements or among competitors would prevail.

3.2 The supply-side

When modelling boradcasters behaviour, to simplify calculations, we assume symmetry of the two platforms within each side of the market. We thus rewrite the linear demand system 4 as

$$\begin{aligned}
q_1^A &= A - \beta^A p_1^A + \beta_C^A p_2^A + \gamma^V q_1^V - \gamma_C^V q_2^V \\
q_2^A &= A - \beta^A p_2^A + \beta_C^A p_1^A + \gamma^V q_2^V - \gamma_C^V q_1^V \\
q_1^V &= V - \beta^V p_1^V + \beta_C^V p_2^V - \gamma^A q_1^A + \gamma_C^A q_2^A \\
q_2^V &= V - \beta^V p_2^V + \beta_C^V p_1^V - \gamma^A q_2^A + \gamma_C^A q_1^A
\end{aligned}$$

As in the previous section, we focus on the situation in which viewers have a positive effect on advertisers (captured by the parameter γ^V), while advertizers have a negative effect on viewers (captured by the parameter β^A). Parameters without a subscript measure the own-effects, while the subscript C denotes the cross-effects. Intuitively, the cross-effects have the opposite sign (and they are lower in absolute value) with respect to the corresponding direct effect. Hence, for instance, an increase in viewers by platform j has a negative effect on firm i's advertizing profit, but of smaller magnitude than the positive effect of an increase in i's viewers on i's advertizing profits, for $i, j = 1, 2, i \neq j$.

We study the relationship between monopoly and competition in terms of prices and quantities. In particular, we characterize the conditions under which the findings of our stylized model, that competition reduces consumers' welfare on one of the two sides (and possibly even aggregate consumers welfare), and that platforms may have an incentive to produce below-monopoly quantities on one side of the market in order to sustain a cartel (when a monopolistic cartel is not feasible), may require to price above monopoly on one side of the market (or producing below-monopoly quantities), may be replicated in a more general setting.

3.2.1 Cournot competition on both sides

We therefore consider the sustainability of a collusive cartel when platforms compete in quantities in both sides.

The demand system is thus rewritten as:

$$
\begin{aligned}
p_1^A &= \frac{A\left(\beta^A+\beta_C^A\right)-\beta_C^A q_2^A-\beta^A q_1^A+q_1^V\left(\gamma^V\beta^A-\gamma_C^V\beta_C^A\right)-q_2^V\left(\gamma_C^V\beta^A-\gamma^V\beta_C^A\right)}{\left(\beta^A\right)^2-\left(\beta_C^A\right)^2}\\
p_2^A &= \frac{A\left(\beta^A+\beta_C^A\right)-\beta^A q_2^A-\beta_C^A q_1^A-q_1^V\left(\gamma_C^V\beta^A-\gamma^V\beta_C^A\right)+q_2^V\left(\gamma^V\beta^A-\gamma_C^V\beta_C^A\right)}{\left(\beta^A\right)^2-\left(\beta_C^A\right)^2}\\
p_1^V &= \frac{V\left(\beta^V+\beta_C^V\right)-\beta^V q_1^V-\beta_C^V q_2^V-q_1^A\left(\beta^V\gamma^A-\beta_C^V\gamma_C^A\right)+q_2^A\left(\beta^V\gamma_C^A-\beta_C^V\gamma^A\right)}{\left(\beta^V\right)^2-\left(\beta_C^V\right)^2}\\
p_2^V &= \frac{V\left(\beta^V+\beta_C^V\right)-\beta_C^V q_1^V-\beta^V q_2^V+q_1^A\left(\beta^V\gamma_C^A-\beta_C^V\gamma^A\right)-q_2^A\left(\beta^V\gamma^A-\beta_C^V\gamma_C^A\right)}{\left(\beta^V\right)^2-\left(\beta_C^V\right)^2}
\end{aligned}
$$

In order to directly characterize the direction of change of optimal cartel prices when collusion at the monopoly level may not be sustained, we start by assuming that parameter values are such that a cartel is unable to coordinate on the monopoly outcome, because this would violate the firms' IC constraint.

3.2.2 Deviation profit

The crucial ingredient to understand the direction of change in optimal cartel prices from monopoly is the unilateral deviation profit, π_D, which, in a two-sided market, is composed of both the viewer's profit, π_D^V and the advertizer's profit, so that $\pi_D = \pi_D^V + \pi_D^A$.

We first study the sign of $\frac{\partial \pi_{D,i}}{\partial q_{cls}^V}$, that is, we analyze how the unilateral deviation profit for firm $i = 1,2$ changes when firm j, with $j = 1,2, j \neq i$ sticks to $q_j = q_{cls}^V$, that is, to the viewers' quantity agreed upon in the cartel increases. The derivative can be decomposed in the following way:

$$
\frac{\partial \pi_{D,i}}{\partial q_{cls}^V} = \frac{\partial \pi_{D,i}^V}{\partial q_{cls}^V} + \frac{\partial \pi_{D,i}^A}{\partial q_{cls}}
$$

By the envelope theorem, $\frac{\partial \pi_{D,i}^V}{\partial q_{cls}^V} = \frac{\partial p_i^V\left(q_i^V, q_j^V, q_i^A, q_j^A\right)}{\partial q_{cls}^V} q_i^V$, and $\frac{\partial \pi_{D,i}^A}{\partial q_{cls}^V} = \frac{\partial p_i^A\left(q_i^V, q_j^V, q_i^A, q_j^A\right)}{\partial q_{cls}^V} q_i^A$, hence

$$
\frac{\partial \pi_{D,i}}{\partial q_{cls}^V} = \frac{\partial p_i^V}{\partial q_{cls}^V} q_i^V + \frac{\partial p_i^A}{\partial q_{cls}^V} q_i^A \tag{8}
$$

8 can be rewritten as:

$$
\frac{\partial \pi_{D,i}}{\partial q_{cls}^V} = \frac{-\beta_C^V}{\left(\beta^V\right)^2-\left(\beta_C^V\right)^2} q_i^V + \frac{-\left(\gamma_C^V\beta^A-\gamma^V\beta_C^A\right)}{\left(\beta^A\right)^2-\left(\beta_C^A\right)^2} q_i^A
$$

Observe that $\frac{-\beta_C^V}{\left(\beta^V\right)^2-\left(\beta_C^V\right)^2} < 0$. Notice that an increase in the cartel output has only a direct effect on the viewer's profit (that is, there is no impact through the network effect), and it decreases the willingness to pay for the product of the deviating company.

The increase in viewership under the cartel directly hurts the rival firm through the network effect (by $-\frac{\gamma_C^V}{\beta^A}$); however, it also increases the rival firm's advertizing prices (by $\frac{\lambda^V}{\gamma^A}$), and this increases the firm's willingness to pay for the product (by $\frac{\gamma^V}{\beta^A}\frac{\beta_C^A}{\beta^A}$). The combination of the two effects can be positive or negative. For the effect to be positive, a necessary condition is that the rival's price increase is sufficiently large to offset the price reduction due to the (negative) network effect. In other words, it has to be that $-\frac{\gamma_C^V}{\beta^A}+\frac{\gamma^V}{\beta^A}\frac{\beta_C^A}{\beta^A}>0$, or $\gamma^V\beta_C^A-\gamma_C^V>0$. An intuitive way to state the condition is to say that the price increase generated by the network effect is sufficiently large to offset the price decrease entailed directly by the network effect itself. Another way to put it, is that the ratio of own to rival's network effect is larger than the ratio of own to rival's price effect, that is, the two network effects are less similar than the two price effects ($\frac{\gamma^V}{\gamma_C^V}>\frac{\beta^A}{\beta_C^A}$).

By reiterating the previous argument for the case of a change in the advertizing quantity, we find that an increase in the cartel output from advertizing has an unanbigously negative effect on the firm's deviation profit on the advertizing side; as a result, it has a positive effect on the sustainability of the cartel. At the same time, however, it has a positive network effect (and therefore a negative effect on the sustainability of the cartel) on the firm's profit from viewers. Combining the above effects, we have that:

$$\frac{\partial \pi_{D,i}}{\partial q_{cls}^A}=\frac{\partial \pi_{D,i}^A}{\partial q_{cls}^A}+\frac{\partial \pi_{D,i}^V}{\partial q_{cls}^A}=\frac{\beta_C^A}{\left(\beta_C^A\right)^2-\left(\beta^A\right)^2}q_i^A+\frac{-\gamma^V\left(\lambda_C^A-\lambda^A\right)}{\left(\gamma^V\right)^2-\left(\gamma_C^V\right)^2}q_i^V$$

While $\frac{\beta_C^A}{\left(\beta_C^A\right)^2-\left(\beta^A\right)^2}q_i^A<0$, $\frac{-\beta^V\left(\gamma_C^A-\gamma^A\right)}{\left(\beta^V\right)^2-\left(\beta_C^V\right)^2}>0$.

In this case, the increase in the cartel advertizing quantity entails a surge in the deviation profit on the viewer's side and a decline in the deviation profit on the advertizing side.

3.2.3 Sustainability of the cartel

After the analysis of the ingredients of the incentive compatibility constraint faced by the two individual firms, we are now ready to characterize the optimal deviation.

Proposition 3. *When the platforms repeatedly interact in a quantity supergame on both sides of the market, it may be profitable to set a lower than monopoly quantity on one of the two sides. In particular, it may be profitable to set a lower than monopoly quantity on the viewers' market if the deviation profit on the advertizing side increases with the collusive quanity of viewers, and the advertizers market is sufficiently larger than the viewers market. On the other hand, it may be profitable to produce below monopoly in the advertizers' markets if the deviation profit on the viewers side increases with the* *collusive quantity of advertizers, and the viewers market is sufficiently larger than the advertizers.*

The IC constraint for monopoly may be written as:

$$\frac{\pi_{cls,i}}{1-\delta} \geq \pi_{D,i} + \frac{\delta}{1-\delta}\frac{\pi_{couV,i}}{1-\delta}$$

where $\pi_{couV,i}$ is the continuation profit, equal to the Cournot profit, given the combination of the setting with quantity competition as well as our assumption of Nash reversion.

The constraint be decomposed into:

$$\frac{\pi_{cls,i}}{1-\delta} \geq \pi^V_{D,i} + \pi^A_{D,i} + \frac{\delta}{1-\delta}\frac{\pi_{couV,i}}{1-\delta}$$

Assume that the monopoly output cannot be sustained as an equilibrium.

We now check if the IC constraint faced by firm $i = 1,2$ is relaxed as a result of decreasing the individual TV channels sales q^V_{cls} below the monopoly level.

$$\frac{\partial\left(\frac{\pi_{cls,i}}{1-\delta} - \pi_{D,i} - \frac{\delta}{1-\delta}\frac{\pi_{couV,i}}{1-\delta}\right)}{\partial q^V_{cls}} = \frac{\frac{-\beta^V_C}{(\beta^V)^2-\left(\beta^V_C\right)^2}q^V_{cls,i} - \frac{-\left(\gamma^V_C\beta^A-\gamma^V\beta^A_C\right)}{(\beta^A)^2-\left(\beta^A_C\right)^2}q^A_{cls,i}}{1-\delta} - \left(\frac{-\beta^V_C}{(\beta^V)^2-\left(\beta^V_C\right)^2}q^V_{D,i} + \frac{-\left(\gamma^V_C\beta^A-\gamma^V\beta^A_C\right)}{(\beta^A)^2-\left(\beta^A_C\right)^2}q^A_{D,i}\right)$$

If $\left(\frac{-\beta^V_C}{(\beta^V)^2-\left(\beta^V_C\right)^2}q^V_{D,i} + \frac{-\left(\gamma^V_C\beta^A-\gamma^V\beta^A_C\right)}{(\beta^A)^2-\left(\beta^A_C\right)^2}q^A_{D,i}\right) > 0$, and larger, in absolute value, than $\frac{\frac{-\beta^V_C}{(\beta^V)^2-\left(\beta^V_C\right)^2}q^V_{cls,i} - \frac{-\left(\gamma^V_C\beta^A-\gamma^V\beta^A_C\right)}{(\beta^A)^2-\left(\beta^A_C\right)^2}q^A_{cls,i}}{1-\delta}$, then $\frac{\partial\left(\frac{\pi_{cls,i}}{1-\delta} - \pi_{D,i} - \frac{\delta}{1-\delta}\frac{\pi_{couV,i}}{1-\delta}\right)}{\partial q^V_j} < 0$, and a decrease in viewership softens the collusive constraint, and therefore helps stabilizing the cartel. Otherwise, the cartel is stabilized by an increase in the collusive number of viewers.

Analogously, after a deviation on the advertizing side, we study the effect of changing q^A_{cls} on the restrictiveness of the IC constraint, and, as a result, on the sustainability of the collusive constraint:

$$\frac{\partial\left(\frac{\pi_{cls,i}}{1-\delta} - \pi^D_i - \frac{\delta}{1-\delta}\frac{\pi_{couV,i}}{1-\delta}\right)}{\partial q^A_{cls}} = \frac{\frac{-\beta^A_C}{(\beta^A)^2-\left(\beta^A_C\right)^2}q^V_{cls,i} + \frac{\left(\beta^V\gamma^A_C-\beta^V_C\gamma^A\right)}{(\beta^V)^2-\left(\beta^V_C\right)^2}q^A_{cls,i}}{1-\delta} - \left(\frac{-\beta^A_C}{(\beta^A)^2-\left(\beta^A_C\right)^2}q^V_{D,i} + \frac{\left(\beta^V\gamma^A_C-\beta^V_C\gamma^A\right)}{(\beta^V)^2-\left(\beta^V_C\right)^2}q^A_{D,i}\right)$$

If $\left(\frac{-\beta^A_C}{(\beta^A)^2-\left(\beta^A_C\right)^2}q^V_{D,i} + \frac{\left(\beta^V\gamma^A_C-\beta^V_C\gamma^A\right)}{(\beta^V)^2-\left(\beta^V_C\right)^2}q^A_{D,i}\right) > 0$ and larger, in absolute value, than $\frac{\frac{\left(\beta^V\gamma^A_C-\beta^V_C\gamma^A\right)}{(\beta^V)^2-\left(\beta^V_C\right)^2}q^V_{cls,i} + \frac{\left(\beta^V\gamma^A_C-\beta^V_C\gamma^A\right)}{(\beta^V)^2-\left(\beta^V_C\right)^2}q^A_{cls,}}{1-\delta}$, then $\frac{\partial\left(\frac{\pi_{cls,i}}{1-\delta} - \pi_{D,i} - \frac{\delta}{1-\delta}\frac{\pi_{couV,i}}{1-\delta}\right)}{\partial q^A_{cls}} < 0$, and a decrease in advertizing quantity softens the collusive constraint, and therefore helps stabilizing the cartel. Otherwise, the cartel is stabilized by an increase in the collusive number of advertizements.

Observe that, when a decrease in the number of ads helps stabilizing the cartel, the amount of ads under competition is below the monopoly number of ads. In that case, q_1^A and q_2^A are strategic complements.

4 Conclusion

We studied optimal cartel prices in a two-sided market. We presented a simple model, applicable to a market for free channels or to a market for advertising financed TV, showing that prices above the two-sided monopoly price may prevail on one side of a two-sided market as a means to enhance the sustainability of the cartel. We proved that in such a case a higher benefit from the network effect may not compensate customers on that side of the market for the higher prices they are charged.

Our finding that firms may charge higher than monopoly prices to sustain collusion extends to cartels in two-sided markets a result previously known for cartels selling complementary products, despite the fact that products in a two-sided market are not complements for customers, since customers typically buy only one of the two products (e.g. in the case of TV channels, advertisers buy advertising slots while viewers buy content) and products on the two sides are substitutes (e.g. broacasters compete for viewers and for advertisers).

This is because products on the advertisers' side of the market may become complements once network effects have been taken into account. If such complementarity dominates, quantities will be strategic complements and prices will be strategic substitutes. Monopoly, by incentivizing the internalization of these strategic effects, will lead to higher quantities, and lower prices, than competition. Similarly to a one-sided markets, when joint profit maximization is not sustainable as a cartel outcome, sustainability of the cartel can be obtained by pushing the collusive price towards the competitive level, thus raising the cost of deviation. With complement products, this implies raising the collusive price (and lowering the collusive quantity) above the monopoly price (and below the collusive quantity). in our model the fact that advertising)

We provided necessary and sufficient conditions for these results to hold in more complex models of two-sided markets, encompassing the choice of quantities by platforms on at least one side of the market. In particular, it must be the case that, although products on each side of the market are substitutes or independent conditional on the network effects, when firms take into account the network effects, product on at least one side become complements. A necessary condition for this result to hold is that one of the two network effects between demands is negative (in our model the fact that viewers value a channel less the higher the amount of ads it broadcasts). Another necessary condition is that aggregate demand is elastic (unless as in our simple model demands for competiting products are, on one side, independent). [10]

Finally, with complement products, while in a one-sided market welfare under monopoly is higher than welfare under (imperfect) competition, in two-sided markets welfare under monopoly may be lower than welfare under competition. This is both because there are two market sides and because in two-sided market there are indirect network effects at work, whereby a lower price coupled with a lower network effect may lead to a utility loss for consumers on one side. In turn that loss may be higher than the consumers' gain on the other side of the market. As a consequence, when, in order to sustain collusion, prices on one side of the market are raised above (or quantities are lowered below) the monopoly level, welfare may either increase or decrease compared to monopoly, whereas in a one-sided market with complement products it would always decline.

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[10] In other words, these results are more likely the higher the differentiation .

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