Fiscal policy and growth with debt financing of productive public spending

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Abstract

This paper develops an endogenous growth model with public debt and publicly financed infrastructure and human capital accumulation. In order to ensure debt sustainability, both the tax and the expenditure rates automatically adjust in response to the debt-GDP ratio deviating from a target. Conditions for the existence of a unique equilibrium and saddle-path stability are discussed: in a simplified version of the model these are ensured by a strong enough reaction to a debt increase. Then, we resort to numerical methods to simulate the impact of fiscal changes on an economy calibrated to approximate an average Euro Area country. Both long-run equilibrium effects and the transitional dynamics are considered. Fiscal consolidation turns out growth-enhancing in the long run, provided it is not pursued via spending cuts. In general, a trade-off emerges between short and long-run growth, but it is weaker when consolidation is implemented by reducing the debt-GDP target. Also reallocating funds from education to infrastructure and increasing the government size may boost economic growth. Indeed, when spending and tax rates are both raised, the discouraging effect on investment of the tax raise is more than compensated by the productivity increase arising from higher levels of public and human capital.

JEL Classification: E62, H54, H63, O41.

1 Introduction

Since the burst of the financial crisis, many developed countries have been suffering from increasing levels of public debt because of the combined effect of fiscal stimuli and (to a greater extent) falling revenues. Together with causing some of these countries into a sovereign debt crisis, this process led the fiscal policy discussion to shift toward the need for fiscal consolidation. This purpose may be pursued either by public spending cuts or by raising the tax rate; governments usually resort to a mix of the two policies. However, the combined effect of these measures may be controversial. On the one hand, by depressing economic activity, high tax rates may also depress fiscal revenues and, conversely, the higher growth rates resulting from a lower tax rate may increase the present value of future revenues. This possibility has been largely investigated after Laffer (1979) firstly proposed it. On the other hand, the effects of public spending cuts may as well be ambiguous if the government provides productivity enhancing services. Since Aschauer's (1989) seminal paper, a huge effort has been devoted to analyzing the growth effects of productive public spending at both empirical and theoretical level. Although the empirical evidence for such an effect is sparse, Barro (1990) and the large strand of literature spawned by it show that public services entering the production function[1] may promote growth through their effect on the productivity of factors and the return on capital. Assuming that public productive services are mainly carried out throughout infrastructure, Futagami et al. (1993), Baxter and King (1993), Turnovsky (2004), among others, treat them as a stock. However, since investment on human capital accumulation has been recognized as a key determinant of long-run growth, models of productive public spending should be expanded to include not only investment in public capital, but also current expenditure on education. Along this line Agenor (2011) develops a Lucas-type model of growth where publicly driven human capital accumulation is the engine of growth and public capital generates a positive externality in production.

Despite the literature on these topics being substantial, less attention has been devoted to the debt financing of public investment. Bruce and Turnovsky (1999) investigate if a reduction in the role of government, either in the form of a tax cut or combined tax and spending cuts, may improve the long-run fiscal balance, showing that the second policy is likely to be effective to this purpose. Greiner and Semmler (2000) investigate the growth effects of alternative budgetary regimes which impose constraints on the ability of the government to run a deficit. They find that less strict budget policies do not necessarily imply higher growth rates. Ghosh and Mourmouras (2004) extend this framework to include welfare analysis. Greiner (2008) analyzes the dynamic behavior of an endogenous growth model with human capital formation financed by public spending and allowing for the issue of government bonds. In order to ensure the sustainability of public debt, the primary surplus-GDP ratio is assumed to be a positive linear function of the debt-GDP ratio. This study finds that both too loose a fiscal policy and a too tight one are not compatible with sustained growth unless the government is a creditor. Futagami et al. (2012), using an endogenous growth model with public capital, show that reducing government debt by cutting public investment improves welfare and can also lead to higher long-run growth. The rationale for this result is that, when public debt is reduced, interest payments are lower and more resources can be devoted to public capital accumulation in the long run.

We extend this analysis to account for human capital accumulation as an engine of growth. Like Agenor (2011), we use an endogenous growth model with human capital where public infrastructure enter the production function, and human capital accumulation depends on the government providing both education services and infrastructure. We assume that the government can issue bonds to finance its expenditures but, since it is worried about debt sustainability, it sticks to a rule such that the tax rate increases and the spending rate decreases as the debt-GDP ratio exceeds a target level[2]. Bohn (1995, 1998) as

[1]With regard to this branch of literature, see for instance the works of Ireland (1994) and Bruce and Turnovsky (1999), where public spending is treated as a flow.

[2]The overall mechanism is similar to the one in Greiner (2007, 2008), with the primary

well as the Maastricht Treaty requiring EU member states to keep their government debt below 60% of GDP provide a rationale for employing such a rule.

This paper contributes to the literature on the fiscal policy of productive public spending with debt financing in two ways. First, we compare the consequences of alternative debt-reducing policies. Despite ensuring debt sustainability, the commitment to a 60% debt to GDP ratio may indeed not be strong enough to be actually fulfilled, so that fiscal consolidation may be necessary to approach the target. Then we evaluate the growth effects of fiscal policies that do not affect the long-run debt-GDP ratio. Both long-run equilibrium effects and the transitional dynamics are considered. Particular attention is devoted to calibrating the model to fit a benchmark developed economy.

Our analysis focuses on two aspects. First, we derive a sufficient condition for existence and uniqueness of an equilibrium. We also derive a necessary and sufficient condition for saddle-path stability in a simplified version of the model, where the debt-feedback rule only applies to the income tax rate[3]. Then, we resort to numerical methods to simulate the effects of several fiscal policy shocks with respect to our benchmark. With regard to the debt reduction, in contrast with Futagami et al. (2012), we find that reducing the investment spending would harm growth both in the short and in the long run, while either raising the tax rate or lowering the targeted debt-GDP ratio would eventually be growth-enhancing in the long run. As the debt to GDP ratio decreases, the income tax rate decreases too and the public spending share rises, as an effect of the budgetary rule applied by the government. Thus, when fiscal consolidation is pursued through higher taxes, in the long run the initial tax rate raise is partially offset while public investment increases. The increasing stock of public capital stimulates productivity and, in turn, private investment, so that the reduction in the disposable income affects the consumption level more than private capital accumulation and the economy ends up growing at a faster pace. Conversely, when public spending is cut, the relatively lower stock of infrastructure reduces productivity and discourages investment: thus the supplemental disposable income is mostly devoted to consumption and the debt reduction has no beneficial effect on long-run growth.

As to the debt-neutral policies, we find that the result of Agenor (2011) that a resource shift from education spending to public capital accumulation raises the long-run growth rate may still hold true when a developed economy is considered. Finally, also expanding the government size, although being contractionary in the short run, improves the equilibrium growth rate since the discouraging effect on investment of raising the tax rate is more than compensated by the productivity increase arising from higher levels of public and human capital. These results are robust with respect to changes in both the output elasticity to public capital and the parameters characterizing the human capital technology.

The remainder of the paper is structured as follows. Section 2 presents the basic model. In Section 3, after deriving the equilibrium, we discuss the dynamic properties of the simplified version of the model. Section 4 calibrates the model and examines the transitional dynamics associated with several fiscal policy shocks. Section 5 presents parameters sensitivity analysis. Section 6 concludes

surplus actually increasing in response to a rise in the debt-GDP ratio.

[3]Since labor supply is inelastic, the debt-related increase in the income tax is equivalent to a lump sum tax.

the paper.

2 The Model

The economy is populated by an infinitely-lived representative agent with perfect foresight. He produces a homogeneous good which can be used for consumption or investment. Population size remains fixed over time and is normalized to unity. Abstracting from labor-leisure choice, the representative household maximizes the discounted stream of utilities arising from consumption.

$$\int_0^\infty \log C e^{-\rho t} dt, \tag{1}$$

where C and $\rho(> 0)$ denote, accordingly, aggregate consumption and the discount rate. Utility maximization is subject to the following budget constraint:

$$\dot{W} = \dot{K}_p + \dot{B} = (1-\tau)(r_k - \delta)K_p + (1-tau)rB + (1-\tau_w)wH - C + Tr, \tag{2}$$

where W denotes assets, that is the sum of private capital (K_p) and government bonds (B), H stands for the efficiency units of labor (and since hereby we abstract from any labor-schooling choice, they coincide with the level of human capital), Tr are government transfers, r_k is the rental rate of private capital,r denotes the interest rate paid on government bonds, w is the wage rate, τ and τ_w[4] are the tax rates on capital and labor income, δ is the depreciation rate of physical capital. Taking the interest rate, rental rate of capital, wage rate and tax rates as given, the household chooses C and $\dot{W}$ so as to maximize (1) subject to (2). The agent's optimality conditions are:

$$1/C = \lambda \tag{3a}$$

$$\frac{\dot{\lambda}}{\lambda} - \rho = (1-\tau)(r_k - \delta) = (1-\tau)r \tag{3b}$$

where λ, the marginal utility of wealth, is equated in the first equation to the marginal utility of consumption. The second equation states the equality between the rate of return on consumption and the after tax return on savings, either in the form of rental rate of capital or bonds interest rate. Thus, it also sets the equilibrium interest rate equal to rental rate of capital net of depreciation.

The previous optimality conditions lead to the usual Euler equation:

$$\frac{\dot{C}}{C} = (1-\tau)r - \rho. \tag{4}$$

Moreover, the following transversality condition must hold:

$$\lim_{t\to\infty} C^{-1} W e^{-\rho t} = 0. \tag{5}$$

[4]The rationale for distinguishing between the two tax rates in the absence of labor-leisure choice will be made clear when introducing the government.

Production The production process employs private factors (physical and human capital) as well as publicly provided infrastructure (K_g). These include roads, bridges, water, electricity, telecommunications, and so on. Thus aggregate output (Y) is obtained, like in Agenor (2011), according to the following Cobb-Douglas technology:

$$Y = K_g^{\alpha}(K_p)^{\beta}H^{1-\alpha-\beta}, \tag{6}$$

where α, $\beta \in (0,1)$. The services derived from K_g are not subject to congestion, so that public capital is assumed to be a pure public good. The production function shows decreasing returns to scale in each factor, and constant returns to scale as a whole.

Profit maximization leads to the following first order conditions:

$$r_k = \beta K_g^{\alpha}K_p^{\beta-1}H^{1-\alpha-\beta} = \beta\frac{Y}{K_p}; \tag{7a}$$

$$w = (1-\alpha-\beta)\left(\frac{K_g}{H}\right)^{\alpha}\left(\frac{K_p}{H}\right)^{\beta} = (1-\alpha-\beta)\,\frac{Y}{H}. \tag{7b}$$

Public infrastructures are also necessary to the accumulation of human capital, whose production takes place in the public sector and does not require the representative agent to devote time to schooling. Indeed new human capital is only obtained by combining public infrastructure, government provided education services (G_e), and the existing stock of human capital according to a Cobb-Douglas function:

$$\dot{H} = G_e^{\kappa_1}K_g^{\kappa_2}H^{1-\kappa_1-\kappa_2}, \tag{8}$$

where κ_1, $\kappa_2 \in (0,1)$, $\kappa_1+\kappa_2<1$, and no skill depreciation and constant returns to scale are assumed. To a first approximation, this is not a bad description of school-specific factors if we think that private funding accounts for 17% of total educational expenditure on average in OECD countries[5].

Government As we've already seen, public expenditure is meant to the financing of infrastructures that are used for the production of goods and human capital, and to the provision of education services that are necessary to the accumulation of human capital. The government pays for its expenditure by issuing bonds as well as by raising taxes on both capital and labor income. Thus the government budget constraint is:

$$\dot{B} = rB - \tau rW - \tau_w wH + I, \tag{9}$$

where I is the overall amount of public expenditure. Implicit in here is the assumption that transfers to households are equated to the rent associated with public capital (which is equal to αY), thus they do not appear in the government budget constraint. Assuming the same depreciation rate for both private and public capital, the last one evolves according to:

$$\dot{K}_g = g_i I - \delta K_g, \tag{10}$$

[5] Source:OECD, available at http://www.oecd.org/education/skills-beyond-school/48630940.pdf.

where g_i is the share of public spending allocated to capital investment, while education spending is defined as $G_e = (g_e I)$; we do not require g_i and g_e to sum up to 1, for us to allow some unproductive expenditure (actually in the form of money wasting) to collect a variety of spending items that we are not explicitly accounting for in the model. We also assume that the government has a target level for the debt to output ratio ($\bar{b}$), so that public spending and the labor income tax rate are set according to the following rules:

$$I = \left[i - \phi \left(\frac{B}{Y} - \bar{b} \right) \right] Y; \tag{11a}$$

$$\tau_w = \tau + \phi \left(\frac{B}{Y} - \bar{b} \right). \tag{11b}$$

That is, public expenditure is set as a share of GDP, minus an adjustment (ϕ) proportional to the gap between the current debt-output ratio and the targeted one. Likewise, the labor income tax rate is given by the base tax rate arbitrarily set by the government (that is the same rate charged on capital income) plus the same correction coefficient applied to government spending. Thus, other than collecting income taxes, the government also raises a debt related non-distortionary tax.

Using the rules in (11) together with the optimality conditions in (7), we can rewrite the government budget constraint in (9) as:

$$\dot{B} = (1-\tau) \left(\beta \left(\frac{Y}{K_p} \right) - \delta \right) B - \left[(1-\alpha)\tau - i + (2-\alpha-\beta)\phi \left(\frac{B}{Y} - \bar{b} \right) \right] Y + \tau \delta K_p. \tag{12}$$

Thus the ratio of the primary surplus to GDP in this model is a positive linear function of the debt to GDP ratio, which has also been observed in real economies like the US (Bohn 1995, 1998) and some Euro Area countries (Bettina and Alfred (2011)). Therefore, it is easy to show that $(2 - \alpha - \beta)\phi$ is a sufficient condition for the government budget constraint to be fulfilled[6].

3 Equilibrium

We define the goods market equilibrium condition from (2) and (9) as:

$$Y = C + \dot{K}_p + \delta K_p + I. \tag{13}$$

Before deriving the equilibrium dynamics, let define the following stationary variables: $y = Y/H; c = C/H; k_g = K_g/H; k_p = K_p/H$. Then, recalling the equations in (4), (8), (10), (11) and (12), we can derive the following dynamic system in b, k_g, k_p, c:

[6] Proof runs like in Greiner (2008) and it immediately follows from rewriting the intertemporal constraint as: $\dot{B} = [(1-\tau_k)r - (2-\alpha-\beta)\phi] B - \left[(1-\alpha)\tau - i - (2-\alpha-\beta)\phi\bar{b} \right] Y + \tau\delta K_p$.

$$\dot{b} = b\left[(1-\tau)\,(\beta\frac{y}{k_p} - \delta) - (g_e\xi y)^{\kappa_1}\,k_g^{\kappa_2}\right] - (2-\alpha-\beta)\,\phi\left(b - \bar{b}y\right) + \\ - y\,[\tau\,(1-\alpha) - i] + \tau\delta k_p; \tag{14a}$$

$$\dot{k}_g = g_i\xi y - \delta k_g - (g_e\xi y)^{\kappa_1}\,k_g^{1+\kappa_2}; \tag{14b}$$

$$\frac{\dot{k}_p}{k_p} = \frac{y}{k_p} - \frac{c}{k_p} - \xi\frac{y}{k_p} - \delta - (g_e\xi y)^{\kappa_1}\,k_g^{\kappa_2}; \tag{14c}$$

$$\frac{\dot{c}}{c} = (1-\tau)\,(\beta\frac{y}{k_p} - \delta) - \rho - (g_e\xi y)^{\kappa_1}\,k_g^{\kappa_2}; \tag{14d}$$

$$y = k_g^{\alpha}k_p^{\beta}, \tag{14e}$$

where $\xi = i - \phi\left(\frac{b}{k_g^{\alpha}k_p^{\beta}} - \bar{b}\right)$. These equations, together with the initial values of b, k_g, k_p, and the transversality condition (3b) characterize the dynamics of the economy.

Thus we can define a competitive equilibrium as follows:

Definition 1 *A competitive equilibrium is a set of infinite sequences of the quantities* $\{b(t), k_g(t), k_p(t), c(t)\}$*, prices* $\{r(t), w(t)\}$*, and fiscal policy parameters* $\{i, \tau, \bar{b}, \phi\}$*, such that, given prices and fiscal policy, individuals maximize utility subject to* (4)*, firms maximize profits subject to* (7)*,* (8) *and* (10)*, transversality condition holds, the government balance constraint is fulfilled and markets clear in every period.*

3.1 Steady state

Our objective is now to analyze the dynamics of the economy around a balanced growth path, to be defined as:

Definition 2 *A balanced growth path is a competitive equilibrium, such that for some initial conditions* $b(0) = b_0, k_g(0) = k_{g0}, k_p(0) = k_{p0}$*, consumption and the stocks of public debt and human capital, as well as the stocks of private and public capital grow at constant rates.*

It follows that along such an equilibrium $\dot{Y}/Y = \dot{B}/B = \dot{C}/C = \dot{K}_p/K_p = \dot{K}_g/K_g = \dot{H}/H = \gamma$.

Then imposing the conditions $\dot{c} = \dot{b} = \dot{k}_g = \dot{k}_p = 0$ on the equations (14), we can summarize the steady state[7] of the economy as:

[7] A $\tilde{}$ is introduced to mark the steady state values.

$$\tilde{b} = \frac{\tilde{k_g}^{\alpha}\tilde{k_p}^{\beta}\left[\tau\left(1-\alpha\right)-i-\left(2-\alpha-\beta\right)\phi\bar{b}\right]+\tau\delta k_p}{\left(1-\tau\right)\left(\beta\tilde{k_g}^{\alpha}\tilde{k_p}^{\beta-1}-\delta\right)-\left(g_e\xi\right)^{\kappa_1}k_g^{\theta}k_p^{\beta\kappa_1}-\left(2-\alpha-\beta\right)\phi}; \tag{15a}$$

$$g_i\xi\tilde{k_g}^{\alpha}\tilde{k_p}^{\beta} = \left(g_e\xi\right)^{\kappa_1}\tilde{k_g}^{1+\theta}\tilde{k_p}^{\beta\kappa_1}; \tag{15b}$$

$$\tilde{k_g}^{\alpha}\tilde{k_p}^{\beta-1} - \frac{\tilde{c}}{\tilde{k_p}} - \xi\tilde{k_g}^{\alpha}\tilde{k_p}^{\beta-1} = \left(g_e\xi\right)^{\kappa_1}\tilde{k_g}^{\theta}\tilde{k_p}^{\beta\kappa_1}; \tag{15c}$$

$$\left(1-\tau\right)\left(\beta\tilde{k_g}^{\alpha}\tilde{k_p}^{\beta-1}-\delta\right)-\rho = \left(g_e\xi\right)^{\kappa_1}\tilde{k_g}^{\theta}\tilde{k_p}^{\beta\kappa_1}, \tag{15d}$$

where $\theta = \alpha\kappa_1 + \kappa_2$. Substituting (15d) into (15a), and assuming $\delta = 0$ for the sake of analytical tractability, we get the following expression for the debt to GDP ratio:

$$\frac{\tilde{b}}{\tilde{y}} = \frac{\tau(1-\alpha)-i-\left(2-\alpha-\beta\right)\phi\bar{b}}{\rho-\left(2-\alpha-\beta\right)\phi}. \tag{16}$$

The steady state debt to GDP ratio thus may either be positive or negative. For the former to be true, a strong enough reaction to increasing debt is in general required[8]. Otherwise, notably for very low values of ϕ or if a high reaction coefficient is coupled with large primary surplus and a debt target close to zero, the government must be a creditor in order to pay for its spending.

By using this result in (15b) and solving for k_g[9], we have:

$$\tilde{k_g} = \left(\frac{g_i\xi^{1-\kappa_1}}{g_e^{\kappa_1}}\tilde{k_p}^{\beta(1-\kappa_1)}\right)^{\dfrac{1}{1+\theta-\alpha}}. \tag{17}$$

It is clear from this result that $\xi > 0$ is a necessary condition for the existence of an economically meaningful equilibrium[10], that means that both $\frac{\rho}{2-\alpha-\beta} < \phi < \frac{\rho i}{(1-\alpha)\tau+(1-\alpha-\beta)i-\rho\bar{b}}$ and the reverse are to be ruled out. Given the results for b/y and k_g, substituting (15c) into (15d), we can now reduce the steady state to the following system of two equations in k_p and c:

[8]If ϕ is close to zero, sustained growth may still be compatible with the government being a debtor, provided it is running a large primary surplus. In that respect notice that, if $\phi = 0$, the numerator in the RHS of (16) is equal to the primary surplus as a share of GDP

[9]Recall that ξ depends on b/y and we need to eliminate it before solving for k_g. Thus ξ is equal to: $\frac{\rho\left(i+\phi\bar{b}\right)-\phi\left[\left(1-\alpha-\beta\right)i+\left(1-\alpha\right)\tau\right]}{\rho-\left(2-\alpha-\beta\right)\phi}$.

[10]That is one with positive values of c, k_g, k_p.

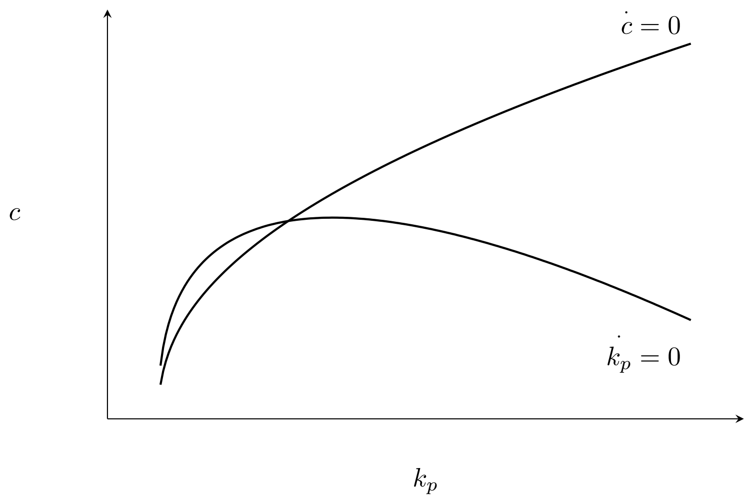

Figure 1: Steady state

$$
\begin{aligned}
\tilde{c} = {} & (1-\xi)\left(\frac{g_i\xi^{1-\kappa_1}}{g_e^{\kappa_1}}\right)^{\frac{\alpha}{1+\theta-\alpha}} \tilde{k_p}^{\frac{\beta(1+\kappa_2)}{1+\theta-\alpha}} \\
& - (g_e\xi)^{\kappa_1}\left(\frac{g_i\xi^{1-\kappa_1}}{g_e^{\kappa_1}}\right)^{\frac{\theta}{1+\theta-\alpha}} k_p^{\frac{\beta(\kappa_1+\kappa_2)}{1+\theta-\alpha}+1};
\end{aligned}
\tag{18a}
$$

$$
\tilde{c} = \left[1-(1-\tau)\beta-\xi\right]\left(\frac{g_i\xi^{1-\kappa_1}}{g_e^{\kappa_1}}\right)^{\frac{\alpha}{1+\theta-\alpha}} \tilde{k_p}^{\frac{\beta(1+\kappa_2)}{1+\theta-\alpha}} + \rho k_p. \tag{18b}
$$

It is clear from (18a) that if $\xi > 1$, the only possible equilibrium would be one with negative consumption, so we must rule out this case. We can also see that for $1-(1-\tau)\beta-\xi \geq 0$, (18b) is always positive, so that the system is like the one shown in Figure 1[11].

This result can therefore be summarized in the following proposition:

Proposition 1 *For a given fiscal policy, $\xi < 1-(1-\tau)\beta$ is a sufficient condition for the existence and uniqueness of an equilibrium with positive real values of k_g, k_p, c.*

At the same time, nothing can be said on the matter when $(1-\tau)\beta < \xi < 1$. In order to analyze the dynamic properties of our model we should resort to simulation, but we can still get insight into the matter by looking at a simplified version of the model.

3.2 The model with arbitrarily set public spending

We now rewrite our model assuming that the feedback effect of the debt deviation from its target only applies to labor income taxes, that is condition (11b) still holds, while (11a) is replaced with the following:

[11] Details about the derivation of the graph are shown in Appendix.

$$I = iY. \tag{10c}$$

All other assumption are kept equal and again $\delta = 0$ is assumed for the sake of analytical tractability. Then proceeding like in the previous subsection we can derive the following condition to summarize the steady state of the economy:

$$\frac{\dot{b}}{b} = 0 \Rightarrow \frac{\tilde{b}}{\tilde{y}} = \frac{\tau(1-\alpha) - i - (1-\alpha-\beta)\phi\bar{b}}{\rho - (1-\alpha-\beta)\phi}; \tag{19a}$$

$$\frac{\dot{k_g}}{k_g} = 0 \Rightarrow \tilde{k_g} = \left[\frac{g_i i}{(g_e i)^{\kappa_1}} \tilde{k_p}^{\beta(1-\kappa 1)}\right]^{\frac{1}{1+\theta-\alpha}}; \tag{19b}$$

$$\begin{aligned}\frac{\dot{k_p}}{k_p} = 0 \Rightarrow \tilde{c} = (1-i)\left[\frac{g_i i}{(g_e i)^{\kappa_1}}\right]^{\frac{\alpha}{1+\theta-\alpha}} \tilde{k_p}^{\frac{\beta(1+\kappa_2)}{1+\theta-\alpha}} \\ -(g_e i)^{\kappa_1}\left[\frac{g_i i}{(g_e i)^{\kappa_1}}\right]^{\frac{\theta}{1+\theta-\alpha}} \tilde{k_p}^{\frac{\beta(\kappa_1+\kappa_2)}{1+\theta-\alpha}+1};\end{aligned} \tag{19c}$$

$$\frac{\dot{c}}{c} = 0 \Rightarrow \tilde{c} = [1-(1-\tau)\beta]\left[\frac{g_i i}{(g_e i)^{\kappa_1}}\right]^{\frac{\alpha}{1+\theta-\alpha}} \tilde{k_p}^{\frac{\beta(1+\kappa_2)}{1+\theta-\alpha}}. \tag{19d}$$

Equations in (19) are very similar to the ones in (16), (17) and (18), the main difference being i in place of ξ so that what has been said in the previous subsection about the latter is now true with regard to the former[12]:

Proposition 2 *For a given tax rate, $i < 1-(1-\tau)\beta$ is a sufficient condition for the existence of a unique equilibrium with positive real values of k_g, k_p, c in this simplified version of the model.*

3.3 Stability

To investigate the dynamics of the system in the vicinity of the steady state, we linearize around $\tilde{b}, \tilde{k_g}, \tilde{k_p}, \tilde{c}$, to have[13]:

$$\begin{pmatrix} \dot{b} \\ \dot{k_g} \\ \dot{k_p} \\ \dot{c} \end{pmatrix} = \begin{pmatrix} \rho-(1-\alpha-\beta)\phi & j_{12} & j_{13} & 0 \\ 0 & j_{22} & j_{23} & 0 \\ 0 & j_{32} & j_{33} & -1 \\ 0 & j_{42} & j_{43} & 0 \end{pmatrix} \begin{pmatrix} b-\tilde{b} \\ k_g-\tilde{k_g} \\ k_p-\tilde{k_p} \\ c-\tilde{c} \end{pmatrix}, \tag{20}$$

where

[12]Except for i being lower than 1, which is true by assumption.

[13]In deriving the Jacobian matrix some steady state relations have been used. The original matrix and the following simplifications are shown in Appendix.

$$j_{22} = -\left(1+\theta-\alpha\right)\left(g_e i\right)^{\kappa_1} \tilde{k_g}^{\theta} \tilde{k_p}^{\beta\kappa_1},$$

$$j_{23} = \left[\beta\left(1-\kappa_1\right)\right]\left(g_e i\right)^{\kappa_1} \tilde{k_g}^{\theta+1} \tilde{k_p}^{\beta\kappa_1-1},$$

$$j_{32} = \left(1-i\right)\alpha \tilde{k_g}^{\alpha-1} \tilde{k_p}^{\beta} - \theta\left(g_e i\right)^{\kappa_1} \tilde{k_g}^{\theta-1} \tilde{k_p}^{\beta\kappa_1+1},$$

$$j_{33} = \left(1-i\right)\beta \tilde{k_g}^{\alpha} \tilde{k_p}^{\beta-1} - \left(1+\beta\kappa_1\right)\left(g_e i\right)^{\kappa_1} \tilde{k_g}^{\theta} \tilde{k_p}^{\beta\kappa_1},$$

$$j_{42} = \alpha\left(1-\tau\right)\beta \tilde{k_g}^{\alpha-1} \tilde{k_p}^{\beta-1} - \theta\left(g_e i\right)^{\kappa_1} \tilde{k_g}^{\theta-1} \tilde{k_p}^{\beta\kappa_1},$$

$$j_{43} = \beta\left(1-\tau\right)\left(\beta-1\right) \tilde{k_g}^{\alpha} \tilde{k_p}^{\beta-2} - \beta\kappa_1\left(g_e i\right)^{\kappa_1} \tilde{k_g}^{\theta} \tilde{k_p}^{\beta\kappa_1-1}.$$

The element $j_{11} = \rho - (1-\alpha-\beta)\,\phi$ is an eigenvalue of the Jacobian matrix (that will be denoted by J), while the others can be found as the eigenvalues of the submatrix complementary to j_{11}. The determinant of the submatrix is equal to:

$$|J_{11}| = j_{22}\times j_{43} - j_{42}\times j_{23} = \frac{\left(g_e i\right)^{2\kappa_1} \tilde{k_g}^{2\theta} \tilde{k_p}^{2\beta\kappa_1} \beta\kappa_1\left(1+\theta-\alpha+\beta-\beta\kappa_1\right)}{\tilde{k_p}}$$
$$- \left(g_e i\right)^{\kappa_1} \tilde{k_g}^{\alpha+\theta} \tilde{k_p}^{\beta(1+\kappa_1)-2} \left(1-\tau\right)\beta\left(\beta+\alpha+\beta\kappa_2-\theta-1\right) > 0. \tag{21}$$

Therefore J_{11} has either one or three positive eigenvalues. In order to know which of the two cases is true, we focus on the signs of the coefficients of the characteristic polynomial, that in our case is given by:

$$p(\lambda) = -\lambda^3 + \lambda^2\left(j_{22}+j_{33}\right) - \lambda\left(j_{22}\times j_{33} + j_{43} - j_{32}\times j_{23}\right) - j_{33}\times j_{42} + j_{22}\times j_{43}$$

According to the Descarte's rule of signs, unless we have three sign differences in $p(\lambda)$, we can rule out J_{11} having three positive roots, that means if the signs are not like $(-+-+)$, the hypothesis of only one positive eigenvalue must be true. The coefficients of λ_2 and λ, after some manipulations, are accordingly:

$$j_{22}+j_{33} = \left(1-i\right)\beta \tilde{k_g}^{\alpha} \tilde{k_p}^{\beta-1} - \left(g_e i\right)^{\kappa_1} \tilde{k_g}^{\theta} \tilde{k_p}^{\beta\kappa_1}\left(2+\theta+\beta\kappa_1-\alpha\right); \tag{22}$$

$$j_{22}\times j_{33} + j_{43} - j_{32}\times j_{23} = \frac{\left(1-\tau\right)\beta \tilde{k_g}^{\alpha} \tilde{k_p}^{\beta-1}\left(\beta-1\right) - \beta\kappa_1\left(g_e i\right)^{\kappa_1} \tilde{k_g}^{\theta} \tilde{k_p}^{\beta\kappa_1}}{\tilde{k_p}}$$
$$+ \left(g_e i\right)^{\kappa_1} \tilde{k_g}^{\theta} \tilde{k_p}^{\beta\kappa_1} \times$$
$$\left\{\left[1+\beta\left(\kappa_1+\kappa_2\right)+\theta-\alpha\right]\left(g_e i\right)^{\kappa_1} \tilde{k_g}^{\theta} \tilde{k_p}^{\beta\kappa_1} - \left(1-i\right) \tilde{k_g}^{\alpha} \tilde{k_p}^{\beta-1} \beta\left(1+\kappa_2\right)\right\}. \tag{23}$$

The signs of (22) and (23) are ambiguous, but we can already state that if the former is negative, $p(\lambda)$ has one positive and 2 negative roots. Thus let investigate the opposite case. Notice that $\left(1-i\right)\beta \tilde{k_g}^{\alpha} \tilde{k_p}^{\beta-1}$ in (22) is lower than $\left(1-i\right) \tilde{k_g}^{\alpha} \tilde{k_p}^{\beta-1} \beta\left(1+\kappa_2\right)$ in (23), while $\left(g_e i\right)^{\kappa_1} \tilde{k_g}^{\theta} \tilde{k_p}^{\beta\kappa_1}\left(2+\theta+\beta\kappa_1-\alpha\right)$ in

(22) is higher than $[1+\beta(\kappa_1+\kappa_2)+\theta-\alpha](g_e i)^{\kappa_1}\tilde{k_g}^{\theta}\tilde{k_p}^{\beta\kappa_1}$ in (23). Therefore a positive coefficient for λ^2 would imply (23) < 0 and this in turn entails the coefficient of λ being positive too and the matrix J_{11} having one positive and two negative eigenvalues. Since this is always true, the steady state's stability properties crucially depend on the sign of the eigenvalue we initially found. Thus the following proposition is straightforward:

Proposition 3 *If a steady state exists, $\rho < (1-\alpha-\beta)\phi$ is a necessary and sufficient condition for the economy to exhibit saddle-path stability in its neighborhood.*

We could expect that a similar condition holds for the full model as well; moreover, when the reaction coefficient also applies to the public spending rate, a lower value of ϕ may be needed in order to ensure the existence of a three-dimensional stable manifold. Although we are not able to derive any such condition, in all of the simulations carried out over a wide range of reasonable parameter values, a minimum ϕ was always found such that saddle-path stability emerged.

4 Numerical analysis

The complexity of the model prevents us from analytically exploring its transitional dynamics, so insights into the effects of fiscal policy will be obtained by resorting to numerical methods. In this section, after calibrating the model to characterize a benchmark economy, several policy experiments are carried out by using the relaxation algorithm proposed by Trimborn et al. (2008).

Calibration Our benchmark economy will be a developed one with high levels of tax rates and public spending, and a huge stock of human capital, which in our standard formulation will be as large as around ten times the GDP. The elasticity of production with respect to public capital is set equal to 0.15: it is far lower than the known result found by Aschauer (1989) and lies within the range of the commonly accepted values in literature. In the next section, we will also investigate the consequences of setting α equal to 0.035. This is to follow Baxter and King (1993) who assume this parameter to be equal to the output share of public investment. The value of β is set so that 35% of output accrues to private capital and the remaining 50% to human capital. The same value is used by Turnovsky (2004), and it is also close to the estimate of 0.37 found by Cole and Neumayer (2006) for a panel of 52 countries. The rate of time preference is set equal to 0.05, that is a quite common value for this kind of literature. Since we are describing a developed economy, the base tax and spending rates are set to high levels, accordingly 0.35 and 0,38. After accounting for the debt correction, these values lead to an equilibrium public spending share on output close to 28% and to a labor tax rate around 46%: both are not far from pre-crisis values observed in several OECD countries. We also assume the spending shares on public capital and education services to be respectively equal to 0.1 and 0.2 in the benchmark case: this is to reflect that in developed countries most of the public expenditures accrue to consumption as well as the fact that OECD average spending shares on infrastructure and education lied steadily around 3% and below 6% of GDP respectively along the

Table 1: Steady state, base formulation

b/y	k_g/y	k_p/y	c/y	growth rate
0.81	0.42	2.29	0.58	2.01%

2000s. As to the human capital technology, the values to be assigned to the elasticities with respect to public spending on education services (κ_1) and infrastructure (κ_2) are uncertain. Most of the estimates for κ_1 are between 0.12 (Card and Krueger, 1992) and 0 (Coleman, 1966). Consistently with this findings, Rioja (2005) sets this parameter equal to 0.1 and Glomm and Ravikumar (1998) use values between 0 and 0.15. According to Betts (1996), the estimates of this parameter are higher the older the dataset used, suggesting that education spending might be more effective in earlier stages of the development path. However, using U.S. data, Aghion et al. (2005) find that investments in "high brow" education[14] are more growth-enhancing for states that are close to the technological frontier while "low brow" education raises growth the most in states that are far from the frontier. Thus we could also expect the elasticity of human capital to education spending not to be lower in developed countries. In support of this, Blankenau et al. (2007) find an estimate of 0.18 for a panel of 23 rich countries. Consistently with this result, a value of 0.2 is chosen by Chen (2005) in a model calibrated with U.S. data and Agenor (2011) with regard to a developing economy, while Annabi et al. (2011), in assessing the effects of increasing public spending on education in Canada, set the parameter equal to 0.18. In our benchmark parametrization we pick an intermediate value of 0.15, and then we perform sensitivity analysis along the range of values found in literature. An appropriate value of κ_2 is harder to pin down since it is seldom found in this kind of literature, thus we will follow Agenor (2011) in setting it equal to 0.1 and refer to the sensitivity analysis for testing alternative values between 0.05 and 0.2. The debt correction coefficient (ϕ) is set equal to 0.5 in our main specification and then we will assess the consequences of choosing alternative values between 0.25 and 0.75. Of course, the higher this coefficient the more effective would be the debt correction rule in pursuing the targeted debt-GDP ratio: the value we choose here entails the government not actually being able to reach the target. Since the value placed on ϕ is crucial to the dynamic properties of the model, it is worth noting that all of the values within this range are consistent with saddle-path stability in our benchmark parametrization. The debt to GDP target is set to 0.6, as it is ruled by the Maastricht Treaty for E.U. countries. Finally, setting a value of 0.045 for the depreciation rate of physical capital and introducing appropriate multiplicative constants in the production functions for goods (0.2689) and human capital (0.0603) leads to the baseline equilibrium summarized in Table 1.

[14] That is education oriented toward research at the frontier of technology. They model productivity growth as resulting from both innovation of technology and imitation and posit the former to make intensive use of highly educated workers while imitation relies more on combining physical capital with less educated labor.

4.1 Transitional dynamics

In this section we perform several policy experiments, our main purpose being to assess the growth effects of alternative debt-reducing policies. After plotting the transitional dynamics associated with policies aimed at cutting the debt-GDP ratio, we will simulate the short and long run effects of fiscal changes which do not affect the equilibrium debt to GDP ratio.

Debt-reducing policies We first simulate a fiscal policy tightening which may be achieved either through a 2.5 percentage points increase in the base tax rate or an equal expenditure rate cut. A comparison is shown in figure 2: the measure that is the most effective in reducing the debt to GDP ratio, i.e. the spending cut, is associated with a higher equilibrium output-human capital ratio and a lower long run growth rate. Both the spending cut and the tax increase cause consumption to jump upward on impact. The crowding-in effect from the former enables households to consume and (slightly) invest more, while the tax increase, by reducing the net return on capital, affects the investment decisions, thus leading to a drop in the growth rate. In the subsequent periods the lowering debt, because of the combined effects of reduced interest payments and the feedback rule, frees resources for government spending so that both public capital and education investment increase. In the spending cut case however, this effect is too scarce to compensate for the initial expenditure reduction and the new steady state will be characterized by reduced accumulation rates for both public and human capital. On the contrary, if the government had raised the base tax rate, the reduced debt stock leads to higher public investments (with respect to the initial steady state): since the resulting effect on the marginal productivity of private capital overwhelms the discouraging effect on private investment[15] from the higher capital tax rate, the economy will end up with a higher long-run growth rate. Finally the dotted line in figure 2 shows that the negative impact effect from increasing the tax rate may be weakened by delaying the tax reform (by one period, in our simulation). Now the adjustment process is non-monotonic: the consumption-GDP ratio increases on impact (the jump being smaller than in the previous case) and keeps increasing till the tax reform is effective, whereas the investment rate and the growth rate fall accordingly[16], before converging to the new steady state.

Figure 3 shows the effect of reducing the targeted debt-GDP ratio to 55%. Both a labor tax rate increase and a spending cut would immediately follow once the new target is effective. Since the non-distortionary tax is the one reacting to the target shift, the households' consumption-investment choice is not affected, while the spending cut has crowding-in effects on private investment. Moreover, tax revenues increase by a factor $(1-\alpha)\phi y$, while public spending decreases by ϕy, so that the crowding-in effects dominate the additional tax levy and both consumption and private investment rise on impact. As a consequence the growth rate also jumps upward on impact, before sharply decreasing as both the human and public capital accumulation processes slow down. As the debt to GDP ratio approaches its new steady state value, public spending starts increasing again, thus enhancing the private capital marginal productivity and

[15] It could be useful reminding that, whereas the capital tax increase is permanent, most of the labor tax raise will be canceled out because of the reducing debt-GDP ratio.

[16] Again, the drop is less pronounced than the one arising from a sudden tax increase.

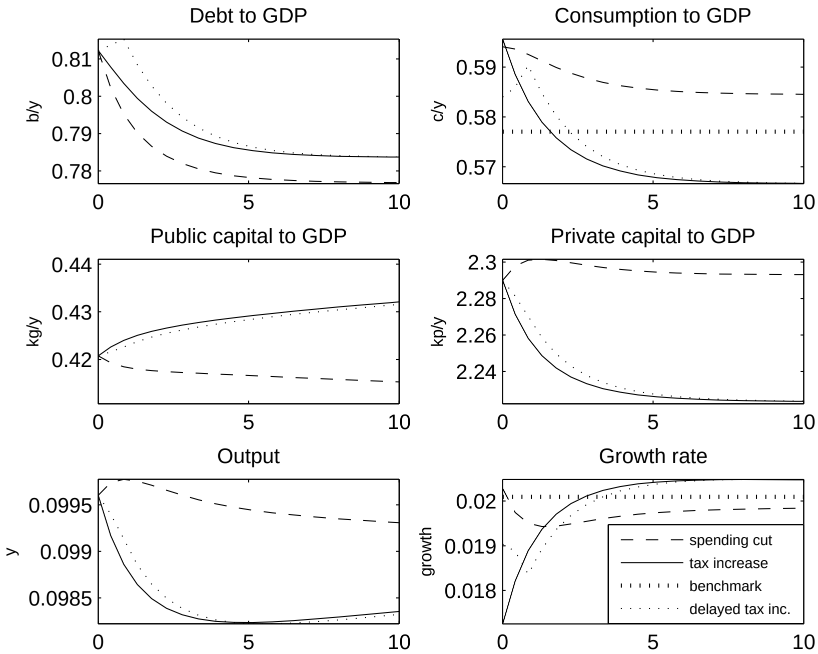

Figure 2: Tax-based versus spending-based fiscal consolidations

pushing the growth rate up toward its new long-run value. Along the new BGP both the tax and the spending rates are almost unchanged, the debt-GDP ratio is 6.5% lower than in the initial steady state, while the physical capital stocks to human capital ratios and the output-human capital ratio as well as the equilibrium growth rate are slightly higher.

Budget-neutral fiscal changes Now we focus on some compensated fiscal reforms, meaning that such changes are intended not to affect the current budget balance or the equilibrium debt-GDP ratio. First we consider the transitional dynamics associated with a shift in the composition of public productive spending. Since we are only moving resources from education spending to public capital accumulation and vice-versa, the budget balance on impact is not affected, while the long-run public debt to GDP ratio (as it is clear from (15)) will change together with the private capital stock as a consequence of the tax deduction on capital depreciation. However, while the long-run effects of such policy reforms are arguably consistent with the simulations carried out in Agenor 2009, we expect the transitional dynamics to be different since in our model the feedback effect of debt affects both the spending and tax rates along the transition.

When a 2 percentage points increase in public capital investment is coupled with an equal decrease in education spending (that is, we set $g_e = 0.18$ and $g_i = 0.12$), consumption drops on impact as households anticipate the future path of private capital marginal productivity and invest more at time 0. The raise in private investment boosts growth and this in turn leads to higher consumption

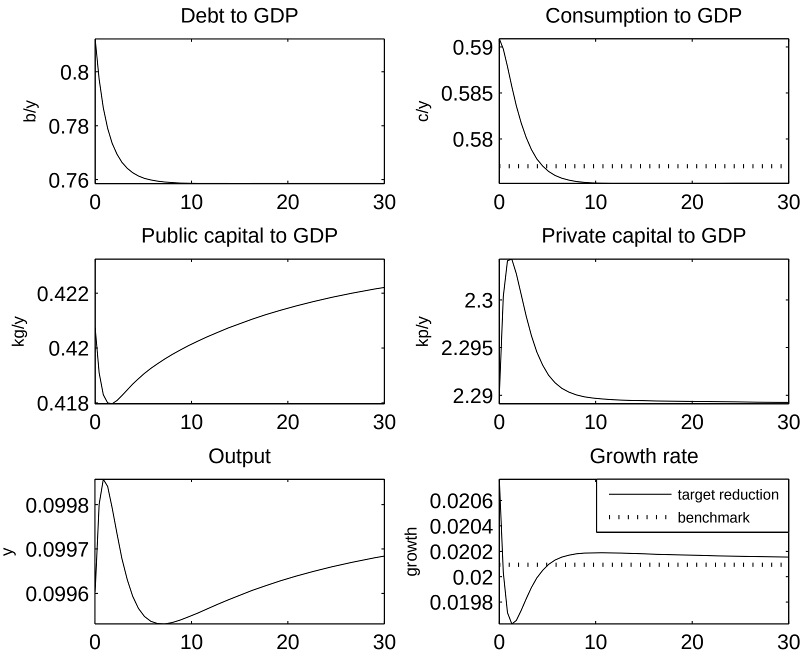

Figure 3: Debt-GDP target reduction

and lower debt-GDP ratio. However, the shift in government spending results in a higher stock of public infrastructure and a lower stock of human capital, the former effect enhancing the marginal productivity of private capital, while the latter hampers it. As the labor tax and the spending rates react to the lower stock of debt, the combined outcome of fiscal revenues relying relatively more on capital taxes and the crowding-out effect of public spending make private investment less attractive. This adds to the overwhelming effect of lower human capital accumulation to slow down economic growth. As a result, the growth rate slowly drops back to a value close to the initial one, while consumption, private capital and public debt converge toward their new long-run levels, which, as shares of GDP, are close to the initial steady state. Of course, transitional dynamics are the other way around if we switch resources from public capital accumulation to education spending, as it is shown in figure 4. However, now public infrastructure are too poorly funded and the additional education investment can't balance the slack in public capital accumulation; thus the economy turns out growing at a slower pace than it did in the initial steady state.

Finally we focus on the government dimensions. In figure 5 the transitional dynamics associated with scaling down the government size are compared with those triggered by expanding it. The former is realized by cutting the spending rate to 35.5% and the tax rate to 31.9% and is represented by the solid line in figure 5, while the dashed line corresponds to an equivalent increase in both the policy rates. The new tax rate is set taking into account the tax allowance on the

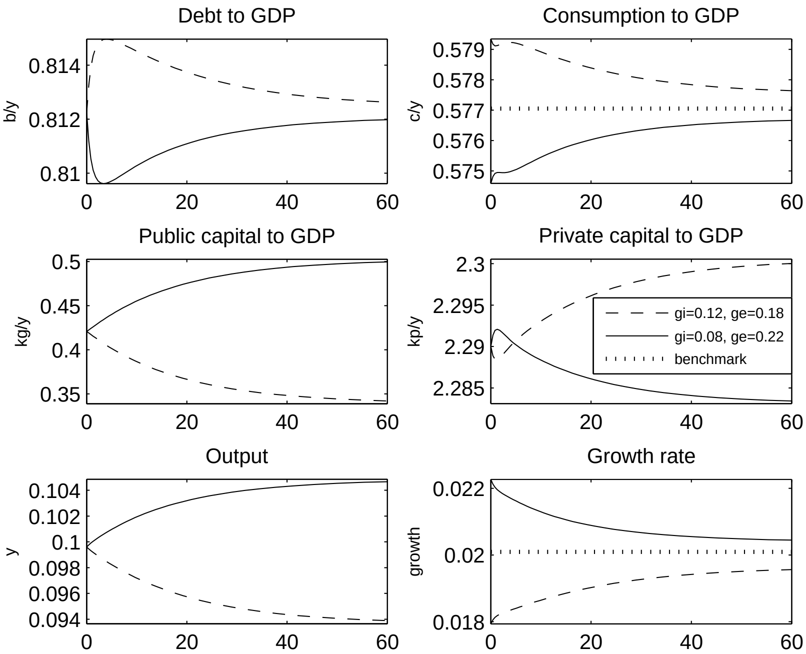

Figure 4: Shift in the public spending composition

new steady state capital stock so not to affect the equilibrium debt-GDP ratio[17]. Therefore, unlike the previous cases where the debt correction rule mostly offset the initial changes, now the policy reforms are completely effective.

Since the changes in the spending share affect the accumulation rate of public and human capital, it takes time for the consequences to fully arise. On the contrary, the tax reforms instantaneously affect individual decisions. Thus in the expansionary case, the crowding out effect from higher public spending and the income and substitution effects from raising the tax rate cause the private investment rate and, in turn, the growth rate of the economy to drop on impact. Then, as public and human capital accumulate at a faster pace, private investment becomes more attractive and goods production starts increasing faster too, although the output to human capital ratio decreases because of the diminishing returns to scale in human capital. After a few periods, the growth rate converges to its new steady state value, that is higher with respect both to its initial level and to the smaller government case. Again, a short/long run growth trade-off has emerged which, as in our first simulation, may turn weaker if the tax adjustment were to be delayed, say by one period. This would lead the households to instantly cut their consumption, thus partially offsetting the crowding out effect on private investment. After this, the growing debt-GDP ratio and the consequent decrease in public spending would push private consumption upward until the tax rate is actually raised. As the debt decreases

[17]As a consequence, the fiscal reform is not cost neutral on impact and the current budget balance worsens (improves) as the government size increases (decreases).

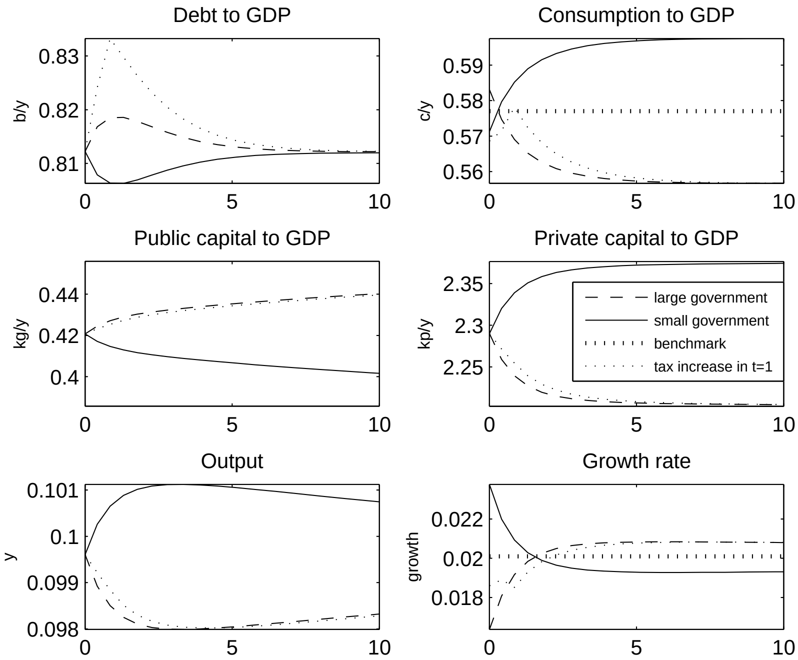

Figure 5: Changes in the government size

and public investment starts growing again the transitional dynamics would replicate the one described above.

5 Sensitivity analysis

The values to be assigned to the coefficient ϕ in the debt-correction rule and to the parameters describing the human capital technology are both crucial to delineate the transitional dynamics and the most questionable. As to the strength of the response to a change in the debt-GDP ratio, we tested alternative values in a range between 0.25 and 0.75. Higher values of ϕ imply lower equilibrium debt to GDP ratios and faster adjustment to the new steady state; all the rest goes the way described above. As we have already seen, appropriately describing the human capital formation is a more demanding task. Thus, we replicate the simulations under two alternative scenarios: one characterized by a higher contribution of public investment to human capital formation (where $\kappa_1 = \kappa_2 = 0.2$), and a low contribution case, where the elasticities with respect to government spending are both set equal to 0.05. Figure 6 shows how different human capital production technologies affect the transitional dynamics associated with reducing the government size. Since no clear difference emerges with respect to the baseline scenario along the transition, only the adjustment path of private capital and the growth rate are plotted. Both private investment and the growth rate jump on impact as the lower tax rate instantly enhances the marginal productivity of capital. But this effect is only temporary since the marginal productivity growth is also expected to be hampered as a consequence of the lower level of public investment. Since this last effect is obviously lower in the low contribution scenario, the jump in the private capital accumulation will be higher in this case, and so it will be the reduction in the debt to GDP ratio as well. However, the higher the contribution of public spending to human capital formation the stronger the transitional dynamics triggered by changes in the fiscal policy. Thus, while the adjustment process takes longer in the high contribution case, on the other side the decline in the long run growth is also higher in this case, and symmetrically the long run growth would be more sustained if the government size were expanded.

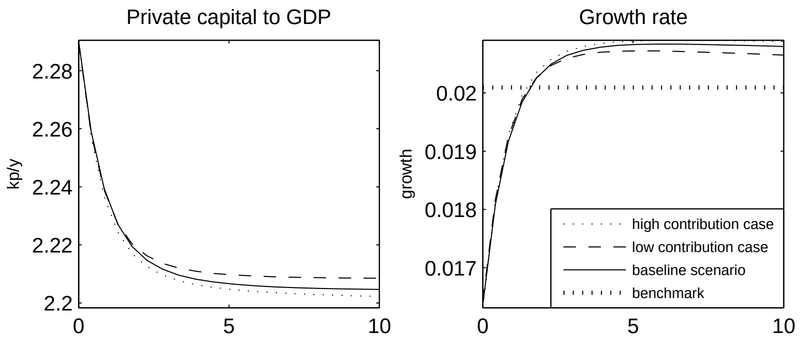

Figure 6: Changes in the government size with different human capital technologies

In general, implementing other fiscal reforms yields similar results, with both the short and long run effects of fiscal policy changes being more pronounced and the respective transitional dynamics taking longer when public investment has a bigger role in human capital production. Moreover, as can be expected, the differences between the three scenarios are more distinct when the spending rate or its composition are affected. When only the tax rate is concerned, although the choice of the parameters affects the transition path, the long run effects of fiscal changes are not sensibly different in the three cases. The transitional dynamics associated with reducing the debt to GDP ratio and changing the composition of public expenditure under the three scenarios are reported in appendix. In the following (figure 7) we compare the effects of cutting the base spending rate to 0.355 for the three choices of (κ_1, κ_2). When such a change occurs, the crowding-in effect on private spending is offset by the agents anticipating the adverse effect of spending cut on human capital accumulation. In the normal case and, more noticeably, in the high contribution scenario, the first effect prevails on impact and then it gives way to the adverse consequences on marginal capital productivity of human and public capital accumulating at a slower pace.

A similar, although less pronounced, dynamics is observed when public contribution to human capital formation is low.

Finally, we want to check if the results are robust with respect to changes in the output elasticity of public capital. Therefore, the simulations are replicated assuming $\alpha = 0.035$. This is to follow Baxter and King (1993), who pick a value

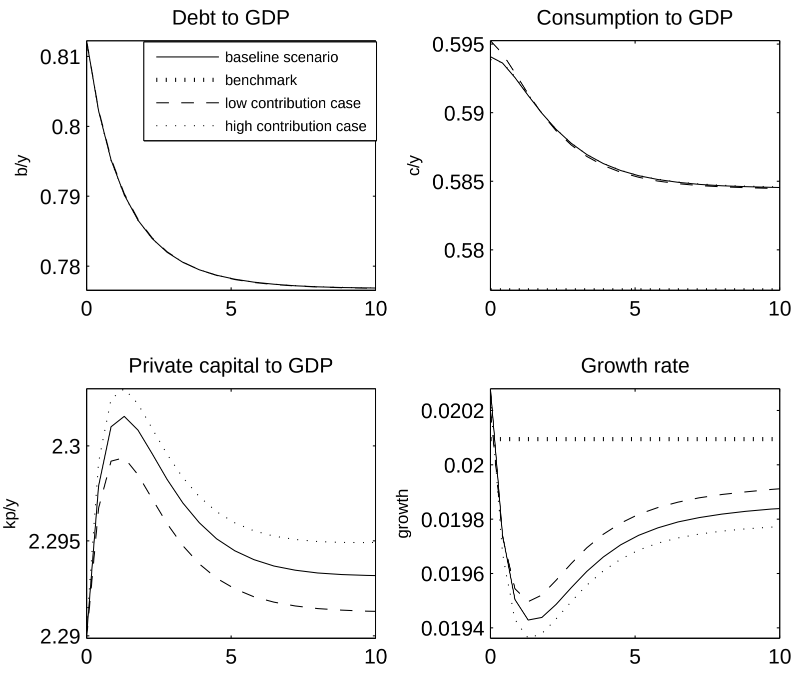

Figure 7: Spending cut with different human capital technologies

of 0.05 to match the U.S. post-war share of public investment in national income. We set this value in order to be consistent with the average general government gross fixed capital formation in the Euro Area throughout the first decade of the 2000s[18]. In Appendix D we show the transitional dynamics associated with the usual fiscal policy shocks. Now, the marginal productivity of capital is less sensitive to changes in the stock of public capital. Thus, in all the simulations that have been carried out, the long-run ratios as well as the economy's growth rate are in general less responsive to the shocks and converge faster to the new steady state values. However, the qualitative results are unaffected.

6 Conclusions

In this paper we have analyzed the effects of fiscal policies in an endogenous growth model with public capital, representing the existing stock of infrastructure, and government financed human capital accumulation. Infrastructure, other than entering the production function, are assumed to be essential, together with the provision of education services, to human capital formation. We also assumed that the government may issue bonds in order to finance its spending, but it sticks to a budgetary rule requiring both the tax and the expenditure rates to automatically adjust in response to the debt-GDP ratio deviating from a targeted value. After deriving a sufficient condition for the existence of

[18]Source: AMECO series database.

a unique equilibrium, we resorted to a simplified version of the model to get insights into its dynamic behavior: we found that a strong enough reaction to a debt increase should ensure the existence of a saddle-path stable equilibrium.

Most of our attention, however, has been devoted to simulating the impact of fiscal changes on an economy calibrated to approximate an average Euro Area country. The purpose of the simulations was twofold. First we compared the consequences of alternative debt-reducing policies. We found that both a spending cut and a tax raise are effective in reducing the debt-GDP ratio, the former being associated with a lower balanced growth rate. As the debt to GDP ratio decreases, the income tax rate decreases too and the public spending share rises, as an effect of the budgetary rule applied by the government. Thus, when fiscal consolidation is pursued through higher taxes, in the long run the initial tax rate raise is partially offset while public investment increases. By prompting productivity, the public capital stock accumulating at a faster pace encourages private investment; thus, the reduction in disposable income only affects the consumption level and the economy ends up growing at a higher rate. Conversely, when public spending is cut, the productivity growth slows down together with the accumulation rate of infrastructure. As a consequence, the crowding-in effect from reducing public spending is offset by investment being less attractive and economic growth will not benefit from the debt reduction. We also found that reducing the debt target from 60% of GDP to 55% induces a 6.5% reduction in the debt-GDP ratio. In that case, since the capital tax rate is not affected, the initial drop in public spending is partially offset by the crowding-in effect on private investment, so that in the short time the growth rate decreases by far less than in the tax-based consolidation case. And, since the debt reduction frees resources for both increasing public spending and cutting income taxes, the new long-run growth rate is also higher than in our benchmark. Then we examined the effects of two fiscal policy changes which are meant to come at zero cost for the government. First, it was shown that reallocating public funds from education spending to infrastructure may boost growth both in the short and in the long run. Finally, we tested the effects of changing the government size. Now a trade-off emerges between short and long-run growth. An equal raise of fiscal revenues and public spending causes the growth rate to shrink on impact, as a consequence of the higher taxes. Then, as the public and human capital accumulation rate rises, the growth rate recovers up to a slightly higher value with respect to the previous steady state. The opposite holds true when the government reduces its presence into the economy.

These results may be conditioned by the labor tax being non-distortionary. If we allow for labor-leisure choice, raising the labor income tax, by affecting the labor supply, may reduce the growth rate. It may also cause public consumption to have real effects on output and capital. Thus introducing elastic labor supply is an extension that is worth to be considered, although, by generating serious non linearities, this would complicate the analysis and cast doubts on the existence of a balanced growth path. Moreover, while public spending accounts for almost all of primary and secondary education funding on average in the OECD group, 30% of spending on tertiary education is provided by private agents. Since university degree education is a key determinant of growth for developed economies, another interesting extension would be to introduce a mixed (private-public) education system.

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Appendix

A Steady state diagram

Let start from equation (18a): here we have $c = 0$ when $k_p = 0$ but for positive values of k_p the sign of c is ambiguous, and so is the sign of the first derivative too.

$$c'_{(18a)} = \frac{(1-\xi)\left(\frac{g_i\xi^{1-\kappa_1}}{g_e^{\kappa_1}}\right)^{\frac{\alpha}{1+\theta-\alpha}} k_p^{\frac{\beta(1+\kappa_2)}{1+\theta-\alpha}-1}\beta(1+\kappa_2)}{1+\theta-\alpha}$$
$$-(g_e\xi)^{\kappa_1}\left(\frac{g_i\xi^{1-\kappa_1}}{g_e^{\kappa_1}}\right)^{\frac{\theta}{1+\theta-\alpha}} k_p^{\frac{\beta(\kappa_1+\kappa_2)}{1+\theta-\alpha}}\left(\frac{\beta(\kappa_1+\kappa_2)}{1+\theta-\alpha}+1\right).$$

It follows from $\frac{\beta(1+\kappa_2)}{1+\theta-\alpha} < 1$ that $\lim_{k_p\to 0^+} c' = \infty$, so that $c' > 0$ is positive at least for k_p close to zero and then, as k_p increases, it decreases, eventually turning negative, as it is clear from the second derivative being negative:

$$c''_{(18a)} = \frac{(1-\xi)\left(\frac{g_i\xi^{1-\kappa_1}}{g_e^{\kappa_1}}\right)^{\frac{\alpha}{1+\theta-\alpha}} k_p^{\frac{\beta(1+\kappa_2)}{1+\theta-\alpha}-2}\beta(1+\kappa_2)}{1+\theta-\alpha}\left[\frac{\beta(1+\kappa_2)}{1+\theta-\alpha}-1\right]$$
$$-(g_e\xi)^{\kappa_1}\left(\frac{g_i\xi^{1-\kappa_1}}{g_e^{\kappa_1}}\right)^{\frac{\theta}{1+\theta-\alpha}} k_p^{\frac{\beta(\kappa_1+\kappa_2)}{1+\theta-\alpha}-1}\left(\frac{\beta(\kappa_1+\kappa_2)}{1+\theta-\alpha}\right).$$

By taking the first derivative in (18b), we have:

$$c'_{(18b)} = \rho + [1-(1-\tau)\beta-\xi]\left(\frac{g_i\xi^{1-\kappa_1}}{g_e^{\kappa_1}}\right)^{\frac{\alpha}{1+\theta-\alpha}} k_p^{\frac{\beta(1+\kappa_2)}{1+\theta-\alpha}-1}\left[\frac{\beta(1+\kappa_2)}{1+\theta-\alpha}\right],$$

so that $c'(0) = 0$, $\lim_{k_p\to 0^+} c' = \infty$ and $\lim_{k_p\to\infty} c' = \rho$. Since c in (18a) is inverted-u-shaped, while in (18b) it is positive[19] and increasing, at least one equilibrium exists. By comparing the two first derivatives in a right neighborhood of 0, we can conclude the first curve to be steeper than the second one, so that the equilibrium is a unique one, indeed:

$$\lim_{k_p\to 0^+} c'_{(18a)} - c'_{(18b)} =$$
$$\lim_{k_p\to 0^+}(1-\tau)\beta\left(\frac{g_i\xi^{1-\kappa_1}}{g_e^{\kappa_1}}\right)^{\frac{\alpha}{1+\theta-\alpha}} k_p^{\frac{\beta(1+\kappa_2)}{1+\theta-\alpha}-1}\left[\frac{\beta(1+\kappa_2)}{1+\theta-\alpha}\right]-\rho.$$
$$-(g_e\xi)^{\kappa_1}\left(\frac{g_i\xi^{1-\kappa_1}}{g_e^{\kappa_1}}\right)^{\frac{\theta}{1+\theta-\alpha}} k_p^{\frac{\beta(\kappa_1+\kappa_2)}{1+\theta-\alpha}-1}\left(\frac{\beta(\kappa_1+\kappa_2)}{1+\theta-\alpha}\right) = \infty.$$

[19] Recall we are restricting our analysis to the case $1-(1-\tau)\beta \geq \xi$.

B Jacobian matrix of the simplified model

The dynamic system for the simplified model can be summarized as follows:

$$\dot{b} = b\left[(1-\tau)\,\beta\frac{y}{k_p} - (g_e iy)^{\kappa_1}\,k_g^{\kappa_2}\right] - (1-\alpha-\beta)\,\phi\left(b - \bar{b}y\right) - y\,[\tau\,(1-\alpha) - i]\,;$$

$$\dot{k_g} = g_i iy - (g_e iy)^{\kappa_1}\,k_g^{1+\kappa_2};$$

$$\frac{\dot{k_p}}{k_p} = \frac{y}{k_p} - \frac{c}{k_p} - i\frac{y}{k_p} - (g_e iy)^{\kappa_1}\,k_g^{\kappa_2};$$

$$\frac{\dot{c}}{c} = (1-\tau)\,\beta\frac{y}{k_p} - \delta - \rho - (g_e iy)^{\kappa_1}\,k_g^{\kappa_2};$$

$$y = k_g^{\alpha} k_p^{\beta},$$

The Jacobian matrix for this system is: $\begin{pmatrix} j_{11} & j_{12} & j_{13} & 0 \\ 0 & j_{22} & j_{23} & 0 \\ 0 & j_{32} & j_{33} & -1 \\ 0 & j_{42} & j_{43} & 0 \end{pmatrix}$, where

$$\begin{aligned}
j_{11} &= (1-\tau)\beta k_g^{\alpha} k_p^{\beta-1} - (g_e i)^{\kappa_1}\,k_g^{\theta} k_p^{\beta\kappa_1} - (1-\alpha-\beta)\phi, \\
j_{12} &= b\left[(1-\tau)\beta k_g^{\alpha-1} k_p^{\beta-1} - \theta\,(g_e i)^{\kappa_1}\,k_g^{\theta-1} k_p^{\beta\kappa_1}\right] \\
&\quad - \alpha k_g^{\alpha-1} k_p^{\beta}\left[(1-\alpha)\tau - i - (1-\alpha-\beta)\phi\bar{b}\right], \\
j_{13} &= b\left[(\beta-1)\,(1-\tau)\,\beta k_g^{\alpha} k_p^{\beta-2} - \beta\kappa_1\,(g_e i)^{\kappa_1}\,k_g^{\theta} k_p^{\beta\kappa_1-1}\right] \\
&\quad - \beta k_g^{\alpha} k_p^{\beta-1}\left[(1-\alpha)\tau - i - (1-\alpha-\beta)\phi\bar{b}\right], \\
j_{22} &= \alpha g_i i k_g^{\alpha-1} k_p^{\beta-1} - (1+\theta)\,(g_e i)^{\kappa_1}\,k_g^{\theta} k_p^{\beta\kappa_1}, \\
j_{23} &= \beta g_i i k_g^{\alpha} k_p^{\beta-1} - \beta\kappa_1\,(g_e i)^{\kappa_1}\,k_g^{\theta+1} k_p^{\beta\kappa_1-1}, \\
j_{32} &= (1-i)\,\alpha k_g^{\alpha-1} k_p^{\beta} - \theta\,(g_e i)^{\kappa_1}\,k_g^{\theta-1} k_p^{\beta\kappa_1+1}, \\
j_{33} &= (1-i)\,\beta k_g^{\alpha} k_p^{\beta-1} - (1+\beta\kappa_1)\,(g_e i)^{\kappa_1}\,k_g^{\theta} k_p^{\beta\kappa_1}, \\
j_{42} &= \alpha\,(1-\tau)\,\beta k_g^{\alpha-1} k_p^{\beta-1} - \theta\,(g_e i)^{\kappa_1}\,k_g^{\theta-1} k_p^{\beta\kappa_1}, \\
j_{43} &= \beta\,(1-\tau)\,(\beta-1)\,k_g^{\alpha} k_p^{\beta-2} - \beta\kappa_1\,(g_e i)^{\kappa_1}\,k_g^{\theta} k_p^{\beta\kappa_1-1}.
\end{aligned}$$

We can exploit $(1-\tau)\,\beta\frac{y}{k_p} - \delta - \rho - (g_e iy)^{\kappa_1}\,k_g^{\kappa_2}$ from $\frac{\dot{c}}{c} = 0$ to get:

$$j_{11} = \rho - (1-\alpha-\beta)\phi;$$

and $g_i iy = (g_e i)^{\kappa_1}\,k_g^{\theta+1} k_p^{\beta\kappa_1}$ from $\frac{\dot{k_g}}{k_g} = 0$ to get:

$$j_{22} = -\left(g_e i\right)^{\kappa_1} k_g^{\theta} k_p^{\beta\kappa_1}(1+\theta-\alpha),$$
$$j_{23} = \left(g_e i\right)^{\kappa_1} k_g^{\theta+1} k_p^{\beta\kappa_1-1}(\beta(\kappa_1)).$$

C Effects of fiscal reforms under different human capital production technologies

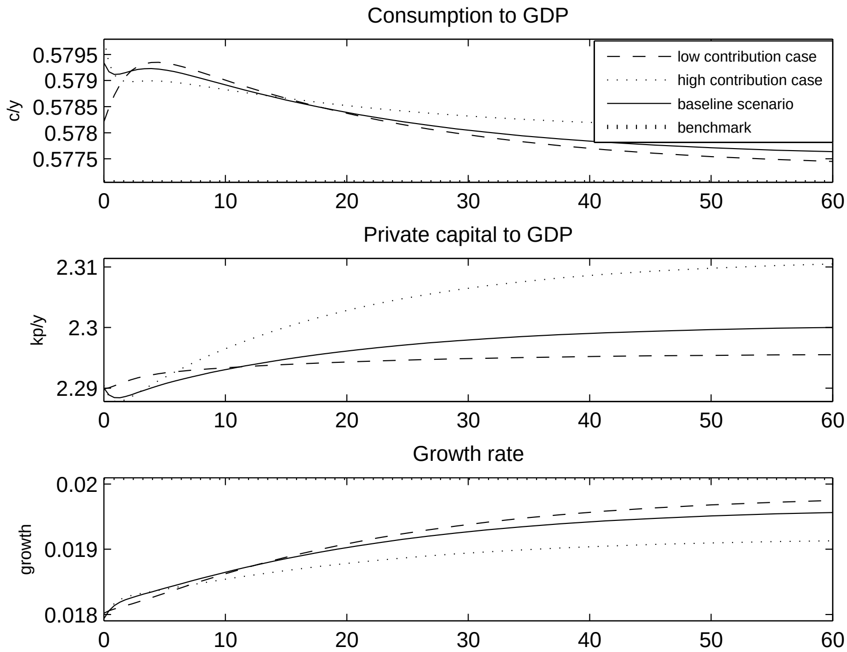

Figure 8: Spending shift from public capital accumulation to education

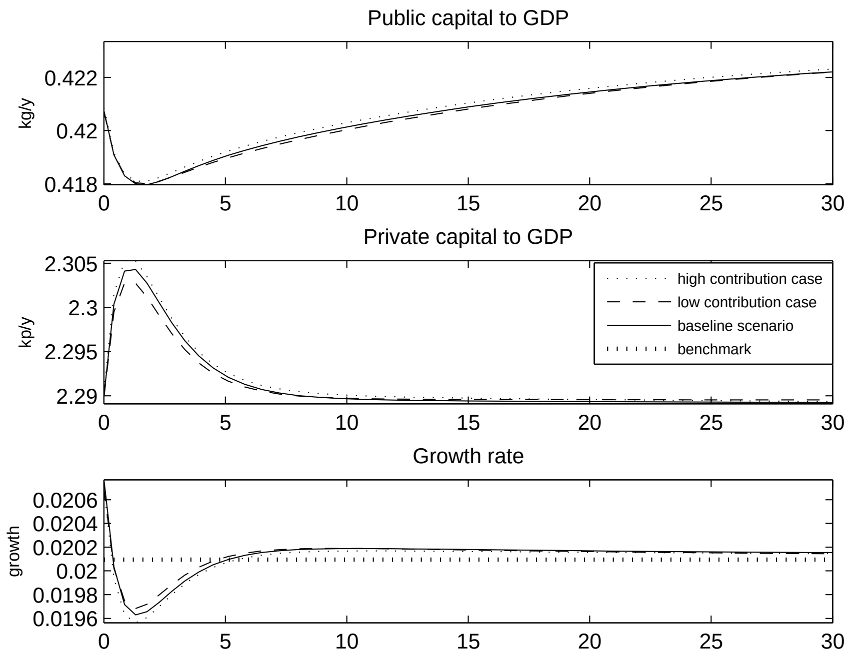

Figure 9: Debt-GDP target reduction to 55%

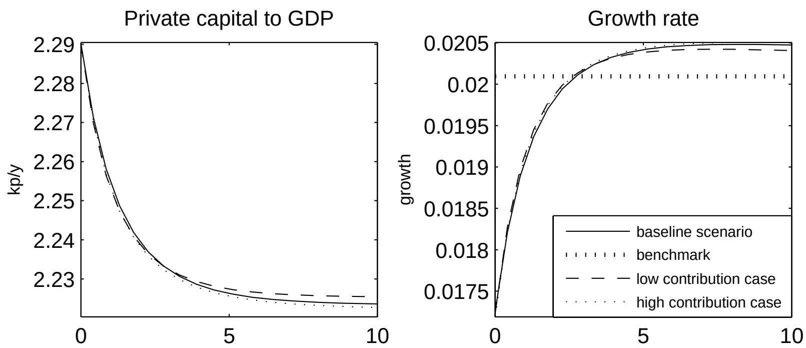

Figure 10: Tax increase from $\tau = 0.35$ to $\tau = 0.75$

D Fiscal policy effects with low output elasticity with respect to infrastructure

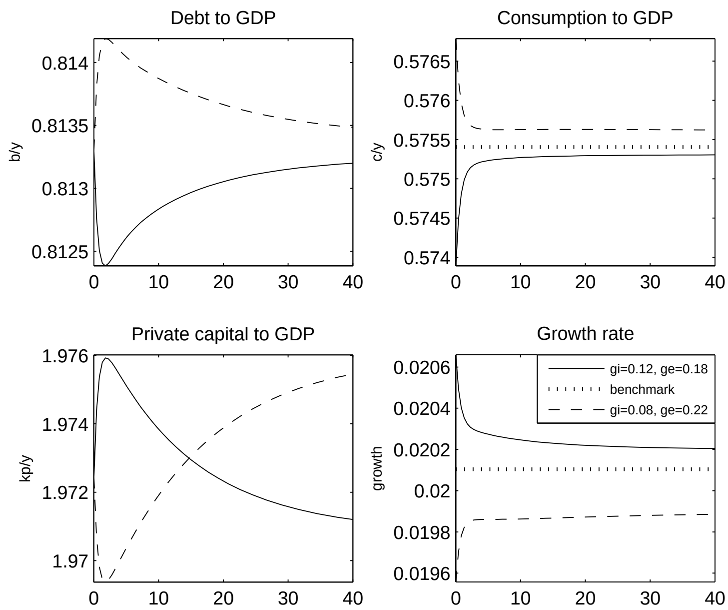

Figure 11: Changes in the composition of productive public spending

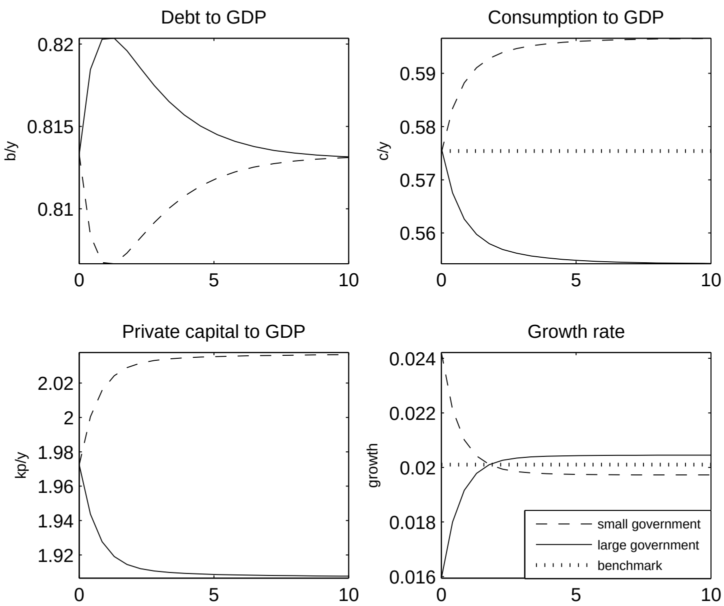

Figure 12: Changes in the government size

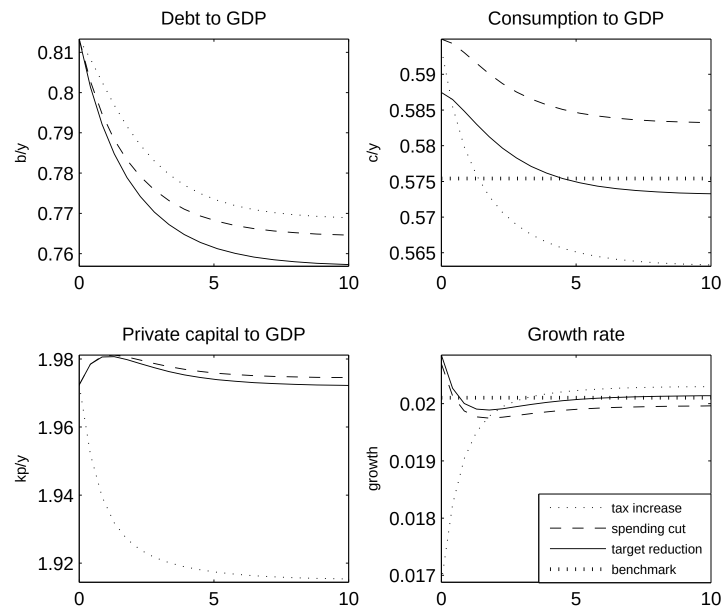

Figure 13: Tax and spending-based debt reductions