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A Leader-Followers Model of Power Transmission Capacity Expansion in a Market Driven Environment

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Abstract We introduce a model for analyzing the upgrade of the national transmission grid that explicitly accounts for responses given by the power producers in terms of generation unit expansion. The problem is modeled as a bilevel program with a mixed integer structure in both upper and lower level. The upper level is defined by the transmission company problem which has to decide on how to upgrade the network. The lower level models the reactions of both power producers, who take a decision on new facilities and power output, and Market Operator, which strikes a new balance between demand and supply, providing new Locational Marginal Prices. We illustrate our methodology by means of an example based on the Garver's 6-bus Network.

Keywords Power Network Expansion Planning · Generation Expansion · Bilevel Programming · k-th Best Algorithm

1 Introduction

The Emergence of liberalized systems for power generation is expected to increase competition and lower prices for consumers during the upcoming years.

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In many countries, the electricity sector has undergone substantial structural changes, due to a series of reforms which, during the last decade, have led to a process in the quest for efficiency and increase of social welfare. The reason why these reforms have been introduced may be mainly found on a list of key factors such as the poor performance of the state-run electricity sector in terms of high costs, inadequate expansion of access to electricity service for the load and/or unreliable supply, the inability of the state sector to finance needed expenditures on new investment and/or maintenance, the need to remove subsidies to the sector in order to release resources for other pressing public expenditure needs and the desire to raise immediate revenue for the government through the sale of assets from the sector (Bacon 1995; Int. Energy Agency 1999; World Energy Counc. 1998; Patterson 1999). See Hyman (2010) for considerations about how consumers can benefit to different extents from the deregulation.

With the liberalization, private companies started entering the power market, offering electricity at lower prices thereby introducing a competition pattern which should pass a larger share of benefits on to consumers.

In this respect, the evolution of electricity industry from the Vertically Integrated Utilities, which were the past standard structure for electricity generation and delivery, to the nowadays deregulated structure is introducing several challenges in planning and operating power systems. Particularly important are the aspects related to the coordination of generation systems with transmission lines. With no longer vertical structure ensuring automatic coordination of such activities it becomes necessary to develop mechanisms to align production and transmission expansion decisions of the market actors covering these two roles.

As a mention, in Italy, after the power market liberalization, most of the power generation system has been removed from the former monopolist and is now carried out by different GenCos (Generating Companies), even though competition is still really limited by an incumbent producer which alone holds about 50% of total generation capacity. The agency delegated to ensure the transmission system to operate in a secure way is the TransCo (Transmission Company), Terna (Genesi et al. 2008, TERNA, S.p.A 2007). Finally, tariffs are defined by the GME (Gestore del Mercato Elettrico), which is the Italian regulator, sometimes referred to as Market Operator (MO).

In Italy, investments in new transmission lines have been scarce during the past years, mainly due to low remunerative capacity of the transmission tariffs. This situation has driven the power system to face an augmented vulnerability, which has been highlighted by several incidents, such as the 2003 total black-out, when a tree flashover caused the tripping of a major tie-line between Italy and Switzerland (as reported in the Final Report of the Investigation Committee on the 28 September 2003 Blackout in Italy (2004)), which brought to an unbalancing of transmission on the other lines that eventually began tripping one after another. Over the course of several minutes the entire Italian power system collapsed, causing the worst black-out ever recorded in the national history.

These failures have called for a drastic redesign of the national transmission system, especially for what concerns the transmission expansion plans. Property and usage of the transmission grid, once up to different actors, have been unified according to a so-called TransCo model. Terna is nowadays the only company operating to secure the grid operations, with different producers ensuring power generation at the different nodes.

In this new environment, when planning the expansion of the transmission grid, it is important for the TransCo to take into account possible responses provided by GenCos. Not accounting for such reactions could most likely lead to potential over and under-investments beared by the TransCo across the different nodes. This proactive policy would result in a better exploitation of the new investments in transmission facilities.

We propose a deterministic multiperiod model for analysis of transmission grid expansion planning with competitive generation capacity planning in electricity markets. The purpose of the model is providing a tool to define the optimal grid upgrading program in a market driven environment. Interplay between TransCo and several independent GenCos is treated as a leader-followers Stackelberg game. Such game is expressed as the following sequential decision pattern: the TransCo decides on the best possible upgrades of transmission lines at time zero and the GenCos modify their production plans over time and potential capacity expansion at time zero accordingly, reaching an equilibrium together with the MO, which clears the market providing new Locational Marginal Prices (LMP). GenCos' reactions lead to a power production equilibrium that is properly taken into account within the TransCo decision problem, which receives back new LMPs and planned production used to determine the aggregated social costs, defined as a the weighted sum of costs paid by consumers and negative aggregated GenCo profits.

Coordination of generation and transmission expansion planning has become steadily more important with the increasing introduction of renewable production sources in the system, especially for what concerns wind technologies and the uncertainty that their introduction brings into the system. This has sparked the production of an increasing amount of literature investigating the effects of wind projects on the transmission network (see e.g. Baringo and Conejo 2012; Heejung and Baldick 2013; Maurovich et al. 2013).

Problem of coordination of transmission expansion planning with competitive generation capacity planning in electricity markets has been addressed by some recent literature (Ng, Zhong and Lee 2009; Roh and Shahidehpour 2007), and usually solved by means of decomposition techniques aiming at computing shadow prices for LMPs and introduce cuts to prevent GenCos and transmission companies expanding capacity in an uncoordinated manner. In this paper we solve the coordination problem using bilevel programming techniques. Our aim is to find a method to obtain a global optimal solution for the TransCo, given the responses provided by GenCos.

LMPs are obtained by including first order conditions of the market clearing problem into the TransCo problem. The resulting model is composed of two interdependent levels. The TransCo will take the role of leader by assuming the initial decision on how to upgrade the national transmission grid, while the GenCos are considered as followers who react to the TransCo decision by reviewing their facility expansion plan and related power production. At the same level of the GenCos is the MO, who clears the market and sets the LMP according to load and bids submitted by GenCos. Novelties introduced by our approach include:

1. The use of binary variables representing power producers' investment decisions in a bi-level programming framework applied to the management of power transmission expansion problems;
2. Development an algorithm for computing a Stackelberg equilibrium with mixed integer decision variables on both upper and lower level;
3. Performing a sensitivity analysis for determining the effects of changing budget availability up to the TransCo and the weighting factors expressing the relative importance of producers and consumers in the TransCo objective function.

We assume that the GenCos' bids will reflect the marginal production cost with a bid up.

The model can be used as a tool to analyze consequences of different focuses with respect to the relative importance of consumers or GenCos profitability on the structure of the national transmission grid, related profitability for the generation companies and changes in social costs. Namely, the model can explain how prices tend to converge when congestion is removed between market areas and how aggregated profitability is shared among the producers when the consumers assume steadily increasing importance for the Transmission Operator. As it can be expected, GenCos owning plants with low marginal production costs will erode market shares from the competitors. As a consequence, even though aggregated profits will decrease, the most efficient producers should be able to increase their profits.

In addition, the model could serve as a tool to investors to identify signals on the location of new generation and transmission facilities and help system planners and regulators to concur on the amounts and location of transmission capacity. Considerations about uncertainty related to load growth are not made in this work and shall be addressed in future research.

2 Literature Review

Due to its intrinsic complexity and multi-objective nature, the problem of coordinating power transmission and generation has been tackled using many different techniques. The first attempts to solve this problem date back to the late sixties and were based on a centralized approach (Galloway et al. 1966; Garver 1970). Linear programming was mainly used with the aim of minimizing the pooled costs for the system. The models were focused on the analysis of possible outages stemming from over-utilization of the network. No interplay was considered amongst the actors involved in the planning activity.

Interactions among power transmission and generation have gained more and more importance as game theoretic models have made their appearance in optimization theory. An example of application of such mixture of equilibrium concepts and optimization theory in analysis of electricity market equilibrium can be found in Hobbs (2001).

Recent literature on transmission-generation expansion may be classified according to the solution approach, heuristic or exact. First and foremost, models can be classified as heuristic approaches or exact solution methods. Other considerations may be made on the granularity used to model the actors, whether each player is considered as an active competitor or as an element of a competitive fringe. In the first case the actor has the power of influencing the prices through a fine tuning of its output, while in the latter case all actors are considered as price takers. Hierarchical structure of the model is another prominent aspect to be considered. Recent approaches encompass the use of bilevel programming (Bard 1998) where the transmission planner takes its decision with the first mover advantage, while generating companies have to decide accordingly, together with the MO. Yet an important point is given by the approach for modeling the equilibrium between actors. This can be done in several ways, leading to diverse considerations on the stability of the equilibrium and the possibility to use an exact solution method. Some authors use a Linear Complementarity Problem (LCP) approach (Ng et al. 2009; Shan and Ryan 2011), i.e. they bundle Karush-Kuhn-Tucker conditions of the players involved in the market game and solve the resulting joint system. Conversely, other authors have modeled market equilibrium using best response functions (Vespucci et al. 2010), i.e. starting from an initial point, they looped over the different actors solving the i-th actor problem while keeping the other actors' decisions as parameters. If convergence criteria are met, this would eventually lead to a stable Nash equilibrium.

Finally, it is important to distinguish how generation expansion is treated by the various authors. Many models, particularly the ones where equilibrium amongst actors is modeled through a LCP, do not consider the expansion planning problem of the generation side. This is due to complexity linked in modeling binary decisions within a Karush-Kuhn-Tucker framework. Interplay among generating companies is therefore defined as a Cournot-Nash game, where players compete through a wise choice of output quantities.

The seminal papers in Stackelberg leader-followers approach to power transmission expansion planning have been introduced by Sauma and Oren (2006, 2007). The authors define a three levels planning model considering both transmission and generation expansion decisions. The third level considers the operational problem for the GenCos and the market clearing problem given the decisions about generation and transmission capacity. The authors assume that the entire production in a given node is entirely covered by a single producer. This allows them to obtain a formulation of the demand function in a given node and related to a given producer. In this way each GenCo will decide at the same time price and quantity to be produced. The second level defines, for each GenCo, the generation capacity level, given the grid configuration provided in the first level by the Transmission Operator. Generation capacity is modelled as a continuous variable, meanwhile transmission lines are discrete variables. The problems defined in the second level are constrained by the equilibrium obtained by the first order conditions of the third level problems. Therefore the second level is defined as an Equilibrium Problem with Equilibrium Constraints (EPEC) whose solution cannot be obtained via plain use of first order conditions of each Genco Problem because of nonconvexity of each GenCo capacity expansion problem. The authors propose a solution approach based on best response functions in order to achieve an equilibrium between GenCos under different grid configurations. The first level problem is the problem of maximizing a social welfare function. The first level objective function is used to evaluate the quality of different predefined grid configuration proposals.

Pozo et al. (2013) extend the approach of the model proposed by Sauma and Oren (2006) by introducing additional assumptions that allow to reformulate the three-levels problem into a one-level MILP. The linearization is accomplished through the reformulation of the second level EPEC into a set of linear inequalities by discretizing the generation expansion variables and enumerating all the possible solutions of the Mathematical Program with Equilibrium Constraints defining the GenCos' expansion problems for all possible responses of the peer producers.

A work based on a bilevel programming approach is the one of Garcés et al. (2009), which considers the transmission expansion problem as a Stackelberg game with the TransCo assuming the role of the leader and deciding over the transmission network upgrade, while the GenCos are considered as operating under perfect competition. This implies that the lower level problem of the game is defined by the market clearing condition and no expansion planning is considered for the GenCos. A similar approach is taken on a subsequent article by Garcés et al. (2010) which add considerations on load shedding costs. Both works assume minimization of investment costs as the leader's objective.

Strategic behaviour of the GenCos in influencing market prices is also considered in Shan and Ryan (2011). Their approach is still based on a bilevel program, even though the two levels do not define decision problems up to two different decision makers, but rather two different decision stages - namely investment and operational stage - of a centralized system. The objective of such a system is to strike a balance between the minimization of the total cost up to consumers and the maximization of the aggregated profits of the GenCos.

A centralized approach is also taken by Aguado et al. (2012), which consider a Mixed Integer Linear Programming problem to obtain a pool based decision for transmission network expansion to be evaluated with appropriate physical and economic metrics. GenCos are not assumed to make any expansion plan.

A hierarchical approach is taken by Hesamzadeh et al. (2010). They consider the interaction between TransCo, GenCos and MO on three different levels. On the top level they consider the TransCo, which decides on transmission expansion while taking into account the response obtained by the equilibrium output of the GenCos. In addition, each GenCo problem is defined as a bilevel problem having as lower level the MO market clearing conditions. Each

GenCo submits a bid composed of the maximum output level and a bid-price and receives the LMP by the lower level. The equilibrium between GenCos is obtained by means of a best response function approach. These equilibria are computed with respect to different network expansion choices. The best expansion choice is then selected via a heuristic method: the TransCo starts with a dummy and uneconomical solution containing all the candidate lines; then lines are eliminated one by one starting from the least efficient one in terms of social welfare. The procedure iterates until budget constraints are satisfied.

A rather different approach is taken by Roh and Shahidehpour (2007), who consider a coordination model where the Independent System Operator (ISO), which takes care of system security, sends economical signals to TransCos and GenCos to incentivize a coordinated expansion of transmission and generation. In this framework, ISO obtains the expansion plan of GenCos and TransCos and performs a security check on the network. Should any infeasibility be detected, the ISO generates a Benders' cut which is then included into the GenCos and TransCos problems via Lagrangean Relaxation. LMP and Flowgate Marginal Prices are updated at each iteration with new dual variables of the load constraint and fed back to the GenCos and TransCos and the process starts anew. The last iteration is reached when there is no change in the social cost function. The authors also solve a stochastic version (Roh and Shahidehpour 2007) of the model with scenarios defined by possible outages deriving by malfunctioning of generation units or lines.

A similar approach to the one of Hobbs is taken by Ng et al. (2009). The authors consider the problems of TransCo, GenCos and MO as the solution of a LCP. The LCP is solved eliminating the constraints containing integer variables related to generation and tranmission expansion. The LCP is then reformulated into a quadratic optimization problem and the previously eliminated constraints are incorporated back into the problem.

3 General model

The model introduced in this article defines a sequential game between three players: the TransCo, a group of GenCos and the MO. The model is structured in two different, interrelated decision levels that represent the sequential nature of the decisions up to each player. Namely, the TransCo will take its decision on the transmission structure as first mover, then each GenCo will decide on power production levels and potential investments accordingly to the choice made by the TransCo. Bids provided by each GenCo are collected and sorted by a MO, which clears the market.

We assume that power is delivered to consumers spread over different nodes having different load and power production capacity, which in turn depend on the amount of existing and candidate generation units. The problem refers to a medium/long time horizon, which we discretize in years. The modeling framework considered in this paper is displayed in figure 1. The two-levels

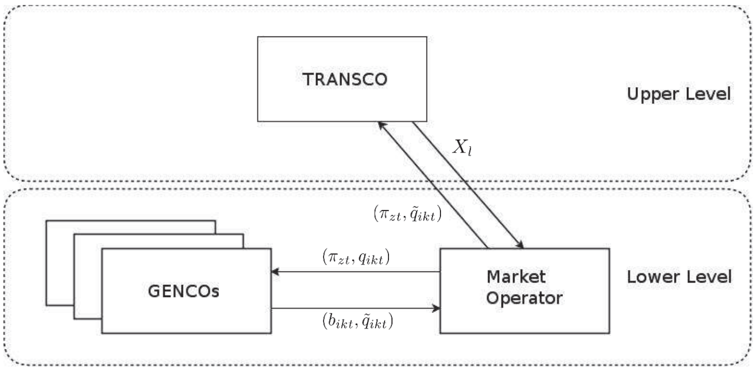

Fig. 1 Interdependancies between transmission company, generator companies and MO

Stackelberg game involves only the network planner as the upper level player and a group of power generating companies, whose bid-ask mechanism with the consumers is mediated by a MO, as the lower level. The TransCo aims at minimizing a social cost function, defined as the weighted sum of the total costs up to the consumers and the negative sum of the GenCos aggregated profit. The TransCo takes a decision on installation of new power transmission lines l which, together with the existing transmission lines, will define bounds for new capacities for the transmission corridors at time t. These bounds are used by the MO to define how much power can be transferred between nodes in order to cover the peak hour load of period t. Depending on the relative importance of consumers and GenCos, congestions might or might not occur, leading to several different LMPs or just one respectively.

Bids are sent to the MO by the GenCos in form of a pair $(b_{ikt}, \tilde{q}_{ikt})$ defining the price bid at time t from generator k belonging to GenCo i and the related quantity respectively. The MO, given the power flow bounds defined by the TransCo, will define the LMPs π_{zt} for node z at time t and the accepted quantity q_{ikt} of power generator i. GenCos aim at maximizing their profit by deciding how much power to supply and whether to open new generation units. We assume that GenCos do not influence LMPs through strategic bidding, so that their bids simply reflect a mark-up on their marginal costs.

As aforementioned, the modeling framework is composed of three decision problems where the TransCo problem and the GenCo problems together with the MO problem are sequentially interrelated. The model is complicated by the presence of binary variables both in the TransCo problem and in the GenCo problems. A dedicated algorithm has been developed in order to take care of binary decision variables featured in the equilibrium problem between GenCos and MO. Such algorithm falls within the family of the k-th best algorithms - see e.g. Bialas et al. (1982) - and it is tailored to the case encompassing integer variables in the lower level problem. We tackle the problem by solving a relaxation of the TransCo problem in which all binary decision variables are controlled by the TransCo as a tentative solution, while first order conditions are imposed for the continuous variables of the GenCo and MO problems. This returns a lower bound for the TransCo problem. One can normally expect that the response given by the GenCos in terms of binary variables will not coincide with the tentative solution provided by the optimal solution of the relaxed TransCo problem. In this case the algorithm creates appropriate cuts in order to prevent the TransCo to choose the same integer solution for the variables up to the GenCos and proceeds with a new iteration.

3.1 The Market Operator Problem

The main task of the MO is matching energy demand and supply at each time point and determine Locational Marginal Prices. Let us introduce the following notation

Sets:

- T: set of periods t; in every period we consider the peak hour load;
- I: set of producers i;
- Z: set of nodes z;
- L^E: set of existing transmission lines l;
- L^C: set of candidate transmission lines l;
- K^E_{iz}: set of existing technologies k belonging to producer i in node z;
- K^C_{iz}: set of candidate technologies k of producer i in node z.

Parameters:

- b_{ikt} [€/MWh]: price of sell bid of generation unit k belonging to GenCo i in period t;
- A_{zl}: incidence matrix of the transmission network;
- C_{zt} [MW]: load in node z in period t;
- B_l: subsceptance of line l;
- X_l: boolean parameter assuming value 1 if a line l is built;
- $\tilde{q}_{ikt}$ [MW]: power produced by generator k belonging to GenCo i at time t;
- $\overline{TR}_{lt}$ [MW]: maximum capacity of transmission line l in period t;
- $\underline{TR}_{lt}$ [MW]: minimum capacity of transmission line l in period t.

Decision variables:

- q_{ikt} [MW]: accepted bid for technology k of producer i in period t;
- TR_{lt} [MW]: power flow on transmission line l in period t;
- θ_{zt} voltage angle for terminal node in node z in period t.

Market clearing conditions for the perfect competitive system considered for a group of similar producers is given by the solution of the problem

$$\min_{q_{ikt}, TR_{lt}} \sum_{i \in I} \sum_{t \in T} \sum_{z \in Z} \sum_{k \in K^E_{iz} \cup K^C_{iz}} b_{ikt} q_{ikt} \tag{1}$$

subject to

$$\sum_{i \in I} \sum_{k \in K_{iz}^E \cup K_{iz}^C} q_{ikt} + \sum_{l \in L^E \cup L^C} A_{zl} TR_{lt} = C_{zt} \qquad z \in Z, \quad t \in T \tag{2}$$

$$0 \leq q_{ikt} \leq \tilde{q}_{ikt} \qquad z \in Z, \quad k \in K_{iz}^E \cup K_{iz}^C, \quad t \in T, \quad i \in I \tag{3}$$

$$TR_{lt} = B_l \left(\sum_{z \in Z} A_{zl} \theta_{zt} \right) X_l \qquad l \in L^C, \quad t \in T \tag{4}$$

$$TR_{lt} = B_l \left(\sum_{z \in Z} A_{zl} \theta_{zt} \right) \qquad l \in L^E, \quad t \in T \tag{5}$$

$$\underline{TR}_{lt} \leq TR_{lt} \leq \overline{TR}_{lt} \qquad l \in L^E \cup L^C, \quad t \in T \tag{6}$$

$$\underline{\theta}_z \leq \theta_{zt} \leq \overline{\theta}_z \qquad z \in Z, \quad t \in T \tag{7}$$

$$TR_{lt} \in \Re \qquad l \in L^E \cup L^C, \quad t \in T \tag{8}$$

$$\theta_{zt} \in \Re \qquad z \in Z, \quad t \in T \tag{9}$$

Solution of the introduced problem determines the accepted quantities minimizing the sum of the quantities times their bid prices. Constraint (2) ensures market balance between demand and supply in each zone and its dual variable represents the zonal LMP. Constraint (3) defines the bounds for the accepted production from each generator. The MO cannot accept more than what has been produced. Constraints (4) and (5) represent the power flow through candidate and existing lines. Finally $\underline{\theta}_z$ and $\overline{\theta}_z$ of the slack node z are set to zero. Notice that the power flow equation (4) for the candidate line is multiplied by a boolean parameter, which sets the transmission to zero when no line is built in the considered transmission corridor.

3.2 The GenCo Problem

At the same level as the MO is the set of GenCos. These actors aim at maximizing their own profit by submitting bids $(b_{ikt}, \tilde{q}_{ikt})$ to the MO and defining their optimal expansion plan according to the grid structure. The problem of the i-th GenCo involves the following notation

Parameters:

- π_{zt} [€/MWh]: Locational Marginal Price in node z at time t ;
- δ: discounting factor;
- c_{ik} [€/MWh]: generation cost of technology k for producer i;
- f_{ik}^G [€]: investment cost of technology k for producer i;
- Γ_{ik}^C [MW]: capacity of candidate technology k of producer i;
- Γ_{ik}^E [MW]: capacity of existing technology k of producer i;
- q_{ikt} [MW]: accepted bid for technology k of producer i in period t.

Decision variables:

– Y_{ik}: binary variable set to 1 if producer i activates candidate generation unit k;
– $\tilde{q}_{ikt}$ [MW]: power produced by generator k belonging to GenCo i at time t.

Each GenCo has a capacity limit for both existing and candidate generators. Such limits are expressed by the constraints

$$\tilde{q}_{ikt} \leq \Gamma^E_{ik} \qquad z \in Z, \quad k \in K^E_{iz}, \quad t \in T$$

and

$$\tilde{q}_{ikt} \leq \Gamma^C_{ik} Y_{ik} \qquad z \in Z, \quad k \in K^C_{iz}, \quad t \in T$$

respectively.

Finally, the quantity sold by each GenCo cannot be larger than the quantity accepted by the MO, i.e.

$$\tilde{q}_{ikt} \leq q_{ikt} \qquad z \in Z, \quad k \in K^E_{iz} \cup K^C_{iz}, \quad t \in T.$$

The decision problem of GenCo i is therefore the following

$$\max_{\tilde{q}_{ikt}, Y_{ik}} \sum_{t \in T} \delta^{-t} \sum_{z \in Z} \left(\pi_{zt} \sum_{k \in K^E_{iz} \cup K^C_{iz}} \tilde{q}_{ikt} - \sum_{k \in K^E_{iz} \cup K^C_{iz}} c_{ik} \tilde{q}_{ikt} \right) - \sum_{z \in Z} \sum_{k \in K^C_{iz}} f^G_{ik} Y_{ik} \quad (10)$$

subject to

$$\tilde{q}_{ikt} \leq \Gamma^C_{ik} Y_{ik} \qquad z \in Z, \quad k \in K^C_{iz}, \quad t \in T \quad (11)$$

$$\tilde{q}_{ikt} \leq \Gamma^E_{ik} \qquad z \in Z, \quad k \in K^E_{iz}, \quad t \in T \quad (12)$$

$$0 \leq \tilde{q}_{ikt} \leq q_{ikt} \qquad z \in Z, \quad k \in K^E_{iz} \cup K^C_{iz}, \quad t \in T \quad (13)$$

$$Y_{ik} \in \{0, 1\} \qquad k \in K^C_{iz} \quad (14)$$

In this problem, π_{zt} represents the LMP in node z in period t and is defined as the shadow price associated to constraint (2).

The solution of each GenCo problem changes accordingly to the bid level accepted by the MO in each node and the resulting Locational Marginal Price. The bounds on accepted bids are defined in constraint (13) and the upper bound q_{ikt} can be understood as a demand constraint at time t for power output up to generator k belonging to GenCo i. This latter in turn depends on the possibility of transferring electricity between different nodes. This mechanism establishes a hierarchical relation between the decision up to each GenCo in terms of MW to provide to the market and potential new installments and the decision taken by the TransCo in terms of opened transmission lines. According to this relation we will refer to the GenCo problem as $GP_i(\pi_{zt}, q_{ikt})$, where π_{zt} and q_{ikt} are defined by the MO as a sequential response to the decisions taken by the TransCo and as a concurrent response to the bids offered by the GenCos.

3.3 The TransCo Problem

TransCo aims at minimizing the weighted sum of aggregated costs up to consumers and negative GenCo profits given the expansion investment budget. If the consumers have a high importance in the aggregated social cost function the TransCo will add new lines to reduce congestion and let GenCos with low costs place a bid in different areas, lowering the value of the accepted bids and, consequently, the electricity prices. The problem is formalized as follows.

Parameters:

- C_{zt} [MW]: load in node z in period t;
- f_l^T [€]: investment cost for opening line l;
- H [€]: total budget for lines expansion;
- c_{ik} [€/MWh]: generation marginal cost of technology k for producer i;
- α: weighting factor ranging between 0 and 1, measuring the relative importance of consumers and producers in the social cost function minimized by the TransCo.

Decision variables:

- π_{zt} [€/MWh]: LMP in node z at time t;
- $\tilde{q}_{ikt}$ [MW]: power produced by GenCo i from generation unit k at time t;
- X_l: binary variable taking value 1 if line l is built.

$$\min_{\pi_{zt},\tilde{q}_{ikt},X_l} \alpha \sum_{t\in T}\sum_{z\in Z} C_{zt}\pi_{zt} - (1-\alpha)\sum_{t\in T}\delta^{-t}\sum_{z\in Z}\left(\pi_{zt}\sum_{k\in K_{iz}^E\cup K_{iz}^C}\tilde{q}_{ikt} - \sum_{k\in K_{iz}^E\cup K_{iz}^C} c_{ik}\tilde{q}_{ikt}\right) \tag{15}$$

subject to

$$\sum_{l\in L^C} f_l^T X_l \le H \tag{16}$$

$$(\pi_{zt},\tilde{q}_{ikt}) \in \Omega(X) \qquad z\in Z, \quad k\in K_{iz}^E\cup K_{iz}^C, \quad t\in T, \quad i\in I \tag{17}$$

$$X_l \in \{0,1\} \qquad l\in L^C \tag{18}$$

where (16) is the budget constraint to investment cost for lines expansion and $\Omega(X)$ represents the space of joint solutions of problems (1)-(9) and (10)-(14) parametrized by X, which represents the vector defined by variables X_l, $l \in L^C$. Such set contains the possible equilibria, defined in terms of decision variables $\tilde{q}_{ikt}, Y_{ik}$ for the involved GenCos and decision variables $q_{ikt}, TR_{lt}, \theta_{zt}$ for the MO. We have considered that the main objective of the TransCo is securing a viable power transmission by removing congestion between market areas and, more generally, reducing social costs. As such, investment costs have only been considered as a constraint, without modelling the effects in the objective function.

4 Model Reformulation

In the remainder of the article we introduce an iterative procedure in which the TransCo takes control of all binary variables at each iteration returning a tentative solution for the overall TransCo problem. Then a check is performed to ensure whether the solution found by the relaxation coincides with the solution of each GenCo with respect to the binary variables up to each of them. Shall this not happen, a cut and a penalty are introduced into the TransCo problem to prevent the same solution to be chosen again and the aformentioned relaxation is solved again with the new constraints.

As mentioned earlier, we assume that GenCos do not perform any sort of strategic bidding. The consequence is that bids will reflect marginal costs c_{ik} of the production from a given generation unit k multiplied by a GenCo specific bid up coefficient. This implies that the structure of the offer curve is completely determined by the marginal costs up to each generation unit of each GenCo and there will be no interplay to determine an equilibrium among offer prices. This simplifies the structure of our problem allowing us to reach an equilibrium in the system without directly recalling for a Stackelberg-Nash framework, as originally done in Hashimoto (1985), under Cournot assumption.

We show that the absence of an interplay between GenCos regarding prices allows us to completely define the quantity bids by means of a modified version of the MO Problem, as the sole lower level of the TransCo Problem. The modified MO Problem is given by[1]

$$\min_{q_{ikt}, TR_{lt}} \sum_{i \in I} \sum_{t \in T} \sum_{z \in Z} \sum_{k \in K_{iz}^E \cup K_{iz}^C} b_{ikt} q_{ikt} \tag{19}$$

subject to

$$\sum_{i \in I} \sum_{k \in K_{iz}^E \cup K_{iz}^C} q_{ikt} + \sum_{l \in L^E \cup L^C} A_{zl} TR_{lt} = C_{zt} \quad : \pi_{zt} \qquad z \in Z, \quad t \in T \tag{20}$$

$$q_{ikt} \leq \Gamma_{ik}^C Y_{ik} \quad : \sigma_{ikt}^C \qquad z \in Z, \quad k \in K_{iz}^C, \quad t \in T, \quad i \in I \tag{21}$$

$$q_{ikt} \leq \Gamma_{ik}^E \quad : \sigma_{ikt}^E \qquad z \in Z, \quad k \in K_{iz}^E, \quad t \in T, \quad i \in I \tag{22}$$

$$TR_{lt} = B_l \left(\sum_{z \in Z} A_{zl} \theta_{zt} \right) X_l \quad : \rho_{lt}^C \quad l \in L^C, \quad t \in T \tag{23}$$

$$TR_{lt} = B_l \left(\sum_{z \in Z} A_{zl} \theta_{zt} \right) \quad : \rho_{lt}^E \qquad l \in L^E, \quad t \in T \tag{24}$$

$$\underline{TR}_{lt} \leq TR_{lt} \leq \overline{TR}_{lt} \quad : \eta_{lt}^+, \eta_{lt}^- \qquad l \in L^E \cup L^C, \quad t \in T \tag{25}$$

$$\underline{\theta}_z \leq \theta_{zt} \leq \overline{\theta}_z \quad : \kappa_{zt}^+, \kappa_{zt}^- \qquad z \in Z, \quad t \in T \tag{26}$$

[1] For each constraint we report its dual variable after the colon.

$$q_{ikt} \geq 0 \qquad i \in I, \quad z \in Z, \quad k \in K^E_{iz} \cup K^C_{iz}, \quad t \in T \tag{27}$$

$$TR_{lt} \in \Re \qquad l \in L^E \cup L^C, \quad t \in T \tag{28}$$

$$\theta_{zt} \in \Re \qquad z \in Z, \quad t \in T \tag{29}$$

which represents the original problem (1)-(9) with constraint (3) replaced by constraints (11) and (12) with Y_{ik} controlled by the TransCo.

Even though a generic solution of problem composed of the TransCo and modified MO problem might return different results from the ones obtained as the optimal solution of (10)-(14) for each GenCo i, once we fix the value of Y_{ik} for each i and k then we have that $\tilde{q}_{ikt} = q_{ikt}$, as stated in the following

Proposition 1 *For a fixed choice of Y_{ik} the optimal power output of each GenCo i will be such that $\tilde{q}_{ikt} = q_{ikt}$.*

In correspondance of the choice Y_{ik} which is confirmed by the GenCos, also the values of $\tilde{q}_{ikt}$ and q_{ikt} will coincide. This allows us to only consider the modified MO problem as the lower level for the TransCo problem, shifting the focus on the definition of the rational response of the GenCos in terms of generation unit installments Y_{ik} to the choices made by the TransCo and the modified MO problem defining the upper and lower level of the bilevel problem respectively.

In principle, if GenCos had no possibility to expand capacity, it would be possible to compute the Stackelberg equilibrium simply by considering the TransCo problem as the upper level and the MO problem as the lower level. The reason for this is that bid prices are already given and the profit function for each GenCo is increasing with respect to the produced quantity. This implies that GenCos selected by the MO will be willing to offer power as long as their capacity limit is not hit.

The modified MO problem is therefore given by (1)-(9) with the inclusion of constraints (11) and (12) replacing constraint (3) and with $\tilde{q}_{ikt}$ replaced by q_{ikt}. We denote such problem as $MO(X)$ in order to stress out its dependancy on the upper and lower bounds on power transmission provided by TransCo. Let $dualMO\,(X)$ be the dual problem of the $MO(X)$ problem parametrized by the choice of all binary variables (X_l, Y_{ik}).

In (17) variables $(\pi_{zt}, \tilde{q}_{ikt})$ belong to the set $\Omega\,(X)$ if:

- π_{zt} is taken from the vector $\left(\pi_{zt}, \rho^C_{lt}, \rho^E_{lt}, \sigma^C_{ikt}, \sigma^E_{ikt}, \eta^+_{lt}, \eta^-_{lt}, \kappa^+_{zt}, \kappa^-_{zt}\right)$ defining the optimal solution to $dualMO\,(X)$;
- $\tilde{q}_{ikt}$ is taken from the vector $\left(\tilde{q}_{ikt}, \tilde{Y}_{ik}\right)$ which represents the optimal solution to $GP_i\,(\pi_{zt}, q_{ikt})$;
- q_{ikt} belongs to the vector $(q_{ikt}, TR_{lt}, \theta_{zt})$ which solves $MO\,(X)$ with $Y_{ik} = \tilde{Y}_{ik}$ where Y_{ik} is the tentative value chosen by the TransCo.

By Proposition 1 we have that $q_{ikt} = \tilde{q}_{ikt}$ for any solution $(\pi_{zt}, \tilde{q}_{ikt}) \in \Omega\,(X)$, therefore we can solve the TransCo problem constrained only by the optimality conditions for the modified MO problem for different tentative values of the binary decision Y_{ik} until it coincides with the one of the GenCos.

In order to obtain correspondance between the decisions Y_{ik} defined by the bilevel problem TransCo-$MO(X)$ and the ones up to the GenCos, Y_{ik} is a parameter defined by the TransCo as a tentative value and corrected or confirmed by GenCos after the solution of their decision problem.

The i-th GenCo is given by (10)-(14). We stress out that the GenCos are willing to provide as much power as they can since their profit increases with supplied power.

The problem to be solved by the TransCo is given by (15)-(18) and defines a mathematical problem with bilevel structure entailing one leader (TransCo) and multiple followers (GenCos and MO). We have included a constraint on the total budget H that the TransCo is allowed to use for transmission expansion. As explained earlier, the lack of interplay in the definition of the price bids allows us to consider only the modified MO problem as the lower level. The dual of the modified MO Problem is

$$\begin{aligned}
\max \quad & -\sum_{z\in Z}\sum_{t\in T} C_{zt}\pi_{zt} + \sum_{i\in I}\sum_{z\in Z}\sum_{k\in K_{iz}^C}\sum_{t\in T} \Gamma_{ik}^C Y_{ik}\sigma_{ikt}^C + \sum_{i\in I}\sum_{z\in Z}\sum_{k\in K_{iz}^E}\sum_{t\in T} \Gamma_{ik}^E \sigma_{ikt}^E \\
& + \sum_{l\in L^E\cup L^C}\sum_{t\in T} \overline{TR}_{lt}\eta_{lt}^+ - \sum_{l\in L^E\cup L^C}\sum_{t\in T} \underline{TR}_{lt}\eta_{lt}^- + \sum_{z\in Z}\sum_{t\in T}\overline{\theta}_z\kappa_{zt}^+ - \sum_{z\in Z}\sum_{t\in T}\underline{\theta}_z\kappa_{zt}^- \\
\text{s.c.} \quad & \pi_{zt} - \sigma_{ikt}^E \le b_{ikt} \qquad i\in I,\quad t\in T,\quad z\in Z,\quad k\in K_{iz}^E \\
& \pi_{zt} - \sigma_{ikt}^C \le b_{ikt} \qquad i\in I,\quad t\in T,\quad z\in Z,\quad k\in K_{iz}^C \\
& \sum_{z\in Z} A_{zl}\pi_{zt} + \rho_{lt}^E - \eta_{lt}^+ + \eta_{lt}^- = 0 \qquad l\in L^E,\quad t\in T \\
& \sum_{z\in Z} A_{zl}\pi_{zt} + \rho_{lt}^C - \eta_{lt}^+ + \eta_{lt}^- = 0 \qquad l\in L^C,\quad t\in T \\
& \sum_{l\in L^C} B_l A_{zl} X_l \rho_{lt}^C + \sum_{l\in L^E} B_l A_{zl}\rho_{lt}^E - \kappa_{zt}^+ + \kappa_{zt}^- = 0 \qquad z\in Z,\quad t\in T \\
& \sigma_{ikt}^C \ge 0 \qquad i\in I,\quad k\in K_{iz}^C,\quad z\in Z,\quad t\in T \\
& \sigma_{ikt}^E \ge 0 \qquad i\in I,\quad k\in K_{iz}^E,\quad z\in Z,\quad t\in T \\
& \eta_{lt}^+, \eta_{lt}^- \ge 0 \qquad l\in L^E\cup L^C,\quad t\in T \\
& \rho_{lt}^C \in \Re \qquad l\in L^C,\quad t\in T \\
& \rho_{lt}^E \in \Re \qquad l\in L^E,\quad t\in T \\
& \kappa_{zt}^+, \kappa_{zt}^- \ge 0,\quad \pi_{zt}\in\Re \qquad z\in Z,\quad t\in T
\end{aligned} \tag{30}$$

In order to provide a viable relaxation for the TransCo problem we replace the set $\Omega\,(X)$ by the primal-dual optimality conditions of the modified MO Problem. We obtain the following mixed integer nonlinear problem.

$$\min \quad \alpha\sum_{t\in T}\sum_{z\in Z} C_{zt}\pi_{zt} - (1-\alpha)\sum_{t\in T}\delta^{-t}\sum_{z\in Z}\left(\pi_{zt}\sum_{k\in K_{iz}^E\cup K_{iz}^C} q_{ikt} - \sum_{k\in K_{iz}^E\cup K_{iz}^C} c_{ik}q_{ikt}\right) \tag{31}$$

subject to

$$\sum_{l\in L^C} f_l^T X_l \le H \tag{32}$$

$$\pi_{zt} - \sigma^E_{ikt} \le b_{ikt} \qquad i\in I, \quad t\in T, \quad z\in Z, \quad k\in K^E_{iz} \tag{33}$$

$$\pi_{zt} - \sigma^C_{ikt} \le b_{ikt} \qquad i\in I, \quad t\in T, \quad z\in Z, \quad k\in K^C_{iz} \tag{34}$$

$$\sum_{z\in Z} A_{zl}\pi_{zt} + \rho^E_{lt} - \eta^+_{lt} + \eta^-_{lt} = 0 \qquad l\in L^E, \quad t\in T \tag{35}$$

$$\sum_{z\in Z} A_{zl}\pi_{zt} + \rho^C_{lt} - \eta^+_{lt} + \eta^-_{lt} = 0 \qquad l\in L^C, \quad t\in T \tag{36}$$

$$\sum_{l\in L^C} B_l A_{zl} X_l \rho^C_{lt} + \sum_{l\in L^E} B_l A_{zl}\rho^E_{lt} - \kappa^+_{zt} + \kappa^-_{zt} = 0 \qquad z\in Z, \quad t\in T \tag{37}$$

$$\sum_{i\in I}\sum_{k\in K^E_{iz}\cup K^C_{iz}} q_{ikt} + \sum_{l\in L^E\cup L^C} A_{zl} TR_{lt} = C_{zt} \qquad z\in Z, \quad t\in T \tag{38}$$

$$q_{ikt} \le \Gamma^C_{ik} Y_{ik} \qquad i\in I, \quad z\in Z, \quad k\in K^C_{iz}, \quad t\in T \tag{39}$$

$$q_{ikt} \le \Gamma^E_{ik} \qquad i\in I, \quad z\in Z, \quad k\in K^E_{iz}, \quad t\in T \tag{40}$$

$$TR_{lt} = B_l\left(\sum_{z\in Z} A_{zl}\theta_{zt}\right) X_l \qquad l\in L^C, \quad t\in T \tag{41}$$

$$TR_{lt} = B_l\left(\sum_{z\in Z} A_{zl}\theta_{zt}\right) \qquad l\in L^E, \quad t\in T \tag{42}$$

$$\underline{TR}_{lt} \le TR_{lt} \le \overline{TR}_{lt} \qquad l\in L^E\cup L^C, \quad t\in T \tag{43}$$

$$\underline{\theta}_z \le \theta_{zt} \le \overline{\theta}_z \qquad z\in Z, \quad t\in T \tag{44}$$

$$\begin{aligned}&\sum_{i\in I}\sum_{t\in T}\sum_{z\in Z}\sum_{k\in K^E_{iz}\cup K^C_{iz}} b_{ikt}q_{ikt} - \sum_{z\in Z}\sum_{t\in T} C_{zt}\pi_{zt} + \sum_{i\in I}\sum_{z\in Z}\sum_{k\in K^C_{iz}}\sum_{t\in T}\Gamma^C_{ik}Y_{ik}\sigma^C_{ikt} + \\ &+\sum_{i\in I}\sum_{z\in Z}\sum_{k\in K^E_{iz}}\sum_{t\in T}\Gamma^E_{ik}\sigma^E_{ikt} + \sum_{l\in L^E\cup L^C}\sum_{t\in T}\overline{TR}_{lt}\eta^+_{lt} - \sum_{l\in L^E\cup L^C}\sum_{t\in T}\underline{TR}_{lt}\eta^-_{lt} \\ &+\sum_{z\in Z}\sum_{t\in T}\overline{\theta}_z\kappa^+_{zt} - \sum_{z\in Z}\sum_{t\in T}\underline{\theta}_z\kappa^-_{zt} = 0\end{aligned} \tag{45}$$

$$q_{ikt} \ge 0 \qquad i\in I, \quad k\in K^E_{iz}\cup K^C_{iz}, \quad z\in Z, \quad t\in T \tag{46}$$

$$\sigma^C_{ikt} \ge 0 \qquad i\in I, \quad k\in K^C_{iz}, \quad z\in Z, \quad t\in T \tag{47}$$

$$\sigma^E_{ikt} \ge 0 \qquad i\in I, \quad k\in K^E_{iz}, \quad z\in Z, \quad t\in T \tag{48}$$

$$\eta^+_{lt}, \eta^-_{lt} \ge 0, \quad TR_{lt}\in\Re \qquad l\in L^E\cup L^C, \quad t\in T \tag{49}$$

$$\rho^C_{lt}\in\Re \qquad l\in L^C, \quad t\in T \tag{50}$$

$$\rho^E_{lt}\in\Re \qquad l\in L^E, \quad t\in T \tag{51}$$

$$\kappa^+_{zt}, \kappa^-_{zt} \ge 0, \quad \theta_{zt}, \pi_{zt}\in\Re \qquad z\in Z, \quad t\in T \tag{52}$$

$$X_l\in\{0,1\} \qquad l\in L^C \tag{53}$$

$$Y_{ik}\in\{0,1\} \qquad i\in I, \quad k\in K^C_{iz}, \quad z\in Z \tag{54}$$

where (32) represents the upper level constraint, (33)-(37) represent the constraints for problem $dualMO(X)$, (38)-(44) represent the $MO(X)$ constraints and (45) requires the duality gap between the solutions of $MO(X)$ and of $dualMO(X)$ to be zero.

The objective function of problem (31)-(54) entails nonlinearities given by the products between LMP and produced power, while constraints (37), (41) and (45) contain products between continuous and integer variables.

It is anyway possible to provide an equivalent linear formulation for problem (31)-(54) by the following considerations. The complementarity system of the first order conditions for the modified MO problem implies that

$$\pi_{zt} q_{ikt} = b_{ikt} q_{ikt} + \sigma^E_{ikt} q_{ikt} \qquad i \in I, \quad t \in T, \quad z \in Z, \quad k \in K^E_{iz} \tag{55}$$

$$\pi_{zt} q_{ikt} = b_{ikt} q_{ikt} + \sigma^C_{ikt} q_{ikt} \qquad i \in I, \quad t \in T, \quad z \in Z, \quad k \in K^C_{iz} \tag{56}$$

$$\sigma^E_{ikt} q_{ikt} = \Gamma^E_{ik} \sigma^E_{ikt} \qquad i \in I, \quad t \in T, \quad z \in Z, \quad k \in K^E_{iz} \tag{57}$$

$$\sigma^C_{ikt} q_{ikt} = \Gamma^C_{ik} Y_{ik} \sigma^C_{ikt} \qquad i \in I, \quad t \in T, \quad z \in Z, \quad k \in K^C_{iz} \tag{58}$$

These equalities imply that

$$\pi_{zt} \sum_{k \in K^E_{iz} \cup K^C_{iz}} q_{ikt} = \sum_{k \in K^E_{iz} \cup K^C_{iz}} b_{ikt} q_{ikt} + \sum_{k \in K^E_{iz}} \Gamma^E_{ik} \sigma^E_{ikt} + \sum_{k \in K^C_{iz}} \Gamma^C_{ik} Y_{ik} \sigma^C_{ikt}$$

which allows us to substitute the quadratic term in the objective function by the equivalent expression containing products of continuous and binary variables, as in constraints (37), (41) and (45).

Let us now consider how the product Xy of the continuous variable y times the binary variable X can be reformulated as a linear expression. Let M denote a large positive parameter and replace the product Xy by a continuous auxiliary variable $\tilde{y}$ such that

$$\tilde{y} - y \leq M(1-X), \qquad y - \tilde{y} \leq M(1-X), \qquad \tilde{y} \leq MX, \qquad -\tilde{y} \leq MX \tag{59}$$

if $y \in \Re$, while

$$\tilde{y} - y \leq M(1-X), \qquad y - \tilde{y} \leq M(1-X), \qquad \tilde{y} \leq MX \tag{60}$$

if $y \geq 0$. We can provide the following mixed integer linear equivalent formulation of problem (31)-(54), which we call Integer Leader Relaxation (ILR).

$$\begin{aligned} \min \quad & \alpha \sum_{t \in T} \sum_{z \in Z} C_{zt} \pi_{zt} - (1-\alpha) \sum_{t \in T} \delta^{-t} \sum_{z \in Z} \left(\sum_{k \in K^E_{iz} \cup K^C_{iz}} b_{ikt} q_{ikt} + \right. \\ & \left. + \sum_{k \in K^E_{iz}} \Gamma^E_{ik} \sigma^E_{ikt} + \sum_{k \in K^C_{iz}} \Gamma^C_{ik} \tilde{\sigma}^C_{ikt} - \sum_{k \in K^E_{iz} \cup K^C_{iz}} c_{ik} q_{ikt} \right) \end{aligned} \tag{61}$$

subject to

$$\sum_{l \in L^C} f^T_l X_l \leq H \tag{62}$$

$$\pi_{zt} - \sigma^E_{ikt} \leq b_{ikt} \qquad i \in I, \quad t \in T, \quad z \in Z, \quad k \in K^E_{iz} \tag{63}$$

$$\pi_{zt} - \sigma^C_{ikt} \leq b_{ikt} \qquad i \in I, \quad t \in T, \quad z \in Z, \quad k \in K^C_{iz} \tag{64}$$

$$\sum_{z \in Z} A_{zl}\pi_{zt} + \rho^E_{lt} - \eta^+_{lt} + \eta^-_{lt} = 0 \qquad l \in L^E, \quad t \in T \tag{65}$$

$$\sum_{z \in Z} A_{zl}\pi_{zt} + \rho^C_{lt} - \eta^+_{lt} + \eta^-_{lt} = 0 \qquad l \in L^C, \quad t \in T \tag{66}$$

$$\sum_{l \in L^C} B_l A_{zl}\tilde{\rho}^C_{lt} + \sum_{l \in L^E} B_l A_{zl}\rho^E_{lt} - \kappa^+_{zt} + \kappa^-_{zt} = 0 \qquad z \in Z, \quad t \in T \tag{67}$$

$$\sum_{i \in I} \sum_{k \in K^E_{iz} \cup K^C_{iz}} q_{ikt} + \sum_{l \in L^E \cup L^C} A_{zl} TR_{lt} = C_{zt} \qquad z \in Z, \quad t \in T \tag{68}$$

$$q_{ikt} \leq \Gamma^C_{ik} Y_{ik} \qquad i \in I, \quad z \in Z, \quad k \in K^C_{iz}, \quad t \in T \tag{69}$$

$$q_{ikt} \leq \Gamma^E_{ik} \qquad i \in I, \quad z \in Z, \quad k \in K^E_{iz}, \quad t \in T \tag{70}$$

$$TR_{lt} = B_l \left(\sum_{z \in Z} A_{zl}\tilde{\theta}_{zlt} \right) \qquad l \in L^C, \quad t \in T \tag{71}$$

$$TR_{lt} = B_l \left(\sum_{z \in Z} A_{zl}\theta_{zt} \right) \qquad l \in L^E, \quad t \in T \tag{72}$$

$$\underline{TR}_{lt} \leq TR_{lt} \leq \overline{TR}_{lt} \qquad l \in L^E \cup L^C, \quad t \in T \tag{73}$$

$$\underline{\theta}_z \leq \theta_{zt} \leq \overline{\theta}_z \qquad z \in Z, \quad t \in T \tag{74}$$

$$\sum_{z \in Z} A_{zl}\tilde{\theta}_{zlt} - \sum_{z \in Z} A_{zl}\theta_{zt} \leq M(1 - X_l) \qquad l \in L^C, \quad t \in T \tag{75}$$

$$\sum_{z \in Z} A_{zl}\theta_{zlt} - \sum_{z \in Z} A_{zl}\tilde{\theta}_{zt} \leq M(1 - X_l) \qquad l \in L^C, \quad t \in T \tag{76}$$

$$\sum_{z \in Z} A_{zl}\tilde{\theta}_{zlt} \leq M X_l \qquad l \in L^C, \quad t \in T \tag{77}$$

$$-\sum_{z \in Z} A_{zl}\tilde{\theta}_{zlt} \leq M X_l \qquad l \in L^C, \quad t \in T \tag{78}$$

$$\tilde{\rho}^C_{lt} - \rho^C_{lt} \leq M(1 - X_l) \qquad l \in L^C, \quad t \in T \tag{79}$$

$$\rho^C_{lt} - \tilde{\rho}^C_{lt} \leq M(1 - X_l) \qquad l \in L^C, \quad t \in T \tag{80}$$

$$\tilde{\rho}^C_{lt} \leq M X_l \qquad l \in L^C, \quad t \in T \tag{81}$$

$$-\tilde{\rho}^C_{lt} \leq M X_l \qquad l \in L^C, \quad t \in T \tag{82}$$

$$\tilde{\sigma}^C_{ikt} - \sigma^C_{ikt} \leq M(1 - Y_{ik}) \qquad i \in I, \quad t \in T, \quad z \in Z, \quad k \in K^C_{iz} \tag{83}$$

$$\sigma^C_{ikt} - \tilde{\sigma}^C_{ikt} \leq M(1 - Y_{ik}) \qquad i \in I, \quad t \in T, \quad z \in Z, \quad k \in K^C_{iz} \tag{84}$$

$$\tilde{\sigma}^C_{ikt} \leq M Y_{ik} \qquad i \in I, \quad t \in T, \quad z \in Z, \quad k \in K^C_{iz} \tag{85}$$

$$\begin{aligned} &\sum_{i \in I} \sum_{t \in T} \sum_{z \in Z} \sum_{k \in K^E_{iz} \cup K^C_{iz}} b_{ikt} q_{ikt} - \sum_{z \in Z} \sum_{t \in T} C_{zt}\pi_{zt} + \sum_{i \in I} \sum_{z \in Z} \sum_{k \in K^C_{iz}} \sum_{t \in T} \Gamma^C_{ik}\tilde{\sigma}^C_{ikt} + \\ &+ \sum_{i \in I} \sum_{z \in Z} \sum_{k \in K^E_{iz}} \sum_{t \in T} \Gamma^E_{ik}\sigma^E_{ikt} + \sum_{l \in L^E \cup L^C} \sum_{t \in T} \overline{TR}_{lt}\eta^+_{lt} - \sum_{l \in L^E \cup L^C} \sum_{t \in T} \underline{TR}_{lt}\eta^-_{lt} \\ &+ \sum_{z \in Z} \sum_{t \in T} \overline{\theta}_z \kappa^+_{zt} - \sum_{z \in Z} \sum_{t \in T} \underline{\theta}_z \kappa^-_{zt} = 0 \end{aligned} \tag{86}$$

$$q_{ikt} \geq 0 \qquad i \in I, \quad k \in K^E_{iz} \cup K^C_{iz}, \quad z \in Z, \quad t \in T \tag{87}$$

$$\sigma_{ikt}^C, \tilde{\sigma}_{ikt}^C \geq 0 \qquad i \in I, \quad k \in K_{iz}^C, \quad z \in Z, \quad t \in T \tag{88}$$

$$\sigma_{ikt}^E \geq 0 \qquad i \in I, \quad k \in K_{iz}^E, \quad z \in Z, \quad t \in T \tag{89}$$

$$\eta_{lt}^+, \eta_{lt}^- \geq 0, \quad TR_{lt} \in \Re \qquad l \in L^E \cup L^C, \quad t \in T \tag{90}$$

$$\rho_{lt}^C, \tilde{\rho}_{lt}^C \in \Re \qquad l \in L^C, \quad t \in T \tag{91}$$

$$\rho_{lt}^E \in \Re \qquad l \in L^E, \quad t \in T \tag{92}$$

$$\kappa_{zt}^+, \kappa_{zt}^- \geq 0 \quad \theta_{zt}, \pi_{zt} \in \Re \qquad z \in Z, \quad t \in T \tag{93}$$

$$\tilde{\theta}_{zlt} \in \Re \qquad z \in Z, \quad l \in L^C, \quad t \in T \tag{94}$$

$$X_l \in \{0,1\} \qquad l \in L^C \tag{95}$$

$$Y_{ik} \in \{0,1\} \qquad i \in I, \quad k \in K_{iz}^C, \quad z \in Z \tag{96}$$

When this problem is solved, we obtain the optimal values q_{ikt}^* and π_{zt}^*, and the candidate value for the decision on candidate generation units Y_{ik}^*. Moreover, a candidate value for the i-th GenCo profit v_i^* can be computed with the canditate values for the LMPs, power production and candidate decision on generation units installments. Such values can be used to perform a comparison with the GenCo problems. Namely, once one obtains the solution of problem $GP_i\left(\pi_{zt}^*, q_{ikt}^*\right)$ it will be possible to compare the decision on opening of candidate generation units stemming from the solution of $GP_i\left(\pi_{zt}^*, q_{ikt}^*\right)$ which we denote by $\bar{Y}_{ik}$ with the candidate value Y_{ik}^*. In addition one will compare the GenCos' candidate profit value obtained by solving (61)-(96) and denoted by $\bar{v}_i$ with the actual optimal profit v_i^* obtained solving problems $GP_i\left(\pi_{zt}^*, q_{ikt}^*\right)$ for each GenCo i. If the comparison returns that $\bar{Y}_{ik} \neq Y_{ik}^*$ and $\mid v_i^* - \bar{v}_i \mid \geq \epsilon$ with $\epsilon > 0$ and small, then a cut and penalty is inserted in problem (61)-(96) and a new iteration is performed. The details of such procedure shall be explained in more detail in the remainder of the article.

5 Solution Approach

The solution of the ILR problem (61)-(96) will not be in general the solution of the original problem (15)-(18). In fact, given a solution (X_l^*, Y_{ik}^*) of the ILR problem, this coincides with the solution of (15)-(18) iff

$$Y_{ik}^* = \bar{Y}_{ik} \qquad i \in \mathcal{I}, \quad k \in K_{iz}^C. \tag{97}$$

If the solution of i-th GenCo problem does not coincide with the solution of ILR, it is necessary to devise a procedure to prevent the TransCo from choosing the same integer solution when solving the ILR. Generally speaking, the TransCo is forced to compute the optimal solution of a sequence of relaxed problems with an increasing number of constraints until the optimal solution also satisfies the equilibrium problem between the GenCos (10)-(14). This is done by inserting an appropriate cut and a penalty term in the objective function. Namely, at iteration r we define the following auxiliary variable as a function of the variable Y_{ik}

$$\tilde{Y}_{ik}^{(r)} = a_{ik}^{(r)} Y_{ik} + b_{ik}^{(r)} \tag{98}$$

with

$$\begin{aligned} a_{ik}^{(r)} &= \begin{cases} 1 \ \text{ if } Y^*_{ik[r-1]} = 1 \\ -1 \text{ if } Y^*_{ik[r-1]} = 0 \end{cases} \\ b_{ik}^{(r)} &= \begin{cases} 0 \text{ if } Y^*_{ik[r-1]} = 1 \\ 1 \text{ if } Y^*_{ik[r-1]} = 0 \end{cases} \end{aligned} \tag{99}$$

where $Y^*_{ik[r-1]}$ defines the integer values of the candidate optimal solution of the ILR problem at iteration $r-1$, taken among all the $\left|K^C\right|$ variables controlled by the upper level decision maker through the ILR problem. This auxiliary variable is then included in the term

$$\frac{\sum_{i\in\mathcal{I}} \sum_{k\in K_i^C} \tilde{Y}_{ik}^{(r)}}{|K^C|}$$

which sums up to unit when the computed solution is the same as the candidate optimal ILR solution. In order to avoid that any of the optimal solutions for iterations from 1 to $r-1$ is picked up in the r-th iteration, we define the binary variable u_r, which takes the value one and triggers a penalty term in the leader objective function when a previous integer solution is chosen. Formally, we need to build a condition such that u_r turns to unit when $\frac{\sum_{i\in\mathcal{I}} \sum_{k\in K_i^C} \tilde{Y}_{ik}^{(r)}}{|K^C|} = 1$. Such a cut is defined as

$$1 + u_r \geq \frac{\sum_{i\in\mathcal{I}} \sum_{k\in K_i^C} \tilde{Y}_{ik}^{(r)}}{|K^C|} + \epsilon \tag{100}$$

where $\epsilon < \frac{1}{|K^C|}$.

When the right hand side of (100) is greater than one, binary variable u_r takes value 1 to satisfy the constraint. At the same time, a penalty term Mu_r is added to the ILR objective function, with M being a large positive scalar.

An explicit reformulation of (100) in terms of the involved binary decision variables is needed in order to be used as a constraint to penalize the ILR problem. By multiplying each side of (100) by $\left|K^C\right|$ and rearranging the result we obtain

$$\sum_{i\in\mathcal{I}} \sum_{k\in K_i^C} \tilde{Y}_{ik}^{(r)} - \left|K^C\right| u_r \leq \left|K^C\right| (1-\epsilon). \tag{101}$$

Posing $\epsilon = \frac{1}{|K^C|} - \frac{1}{|K^C|^2}$ we can express the cut as

$$\sum_{i\in\mathcal{I}} \sum_{k\in K_i^C} \tilde{Y}_{ik}^{(r)} - \left|K^C\right| u_r \leq \frac{\left|K^C\right|^2 - \left|K^C\right| + 1}{|K^C|} \tag{102}$$

and substituting the expression (98) in (102) we finally obtain the reformulation

$$\sum_{i\in\mathcal{I}}\sum_{k\in K_i^C} a_{ik}^{(r)} Y_{ik} - \left|K^C\right| u_r \le \frac{\left|K^C\right|^2 - \left|K^C\right| + 1}{\left|K^C\right|} - \sum_{i\in\mathcal{I}}\sum_{k\in K_i^C} b_{ik}^{(r)} \qquad (103)$$

At iteration n the problem of the TransCo, denoted by $PILR_{[n]}$, is composed of constraints (62)-(96) including cuts (103) with index $r = 1, ..., n$ and with the following objective function

$$\alpha \sum_{t\in T}\sum_{z\in Z} C_{zt}\pi_{zt} - (1-\alpha)\sum_{t\in T}\delta^{-t}\sum_{z\in Z}\left(\sum_{k\in K_{iz}^E\cup K_{iz}^C} b_{ikt}q_{ikt} + \right.$$
$$\left. + \sum_{k\in K_{iz}^E} \Gamma_{ik}^E \sigma_{ikt}^E + \sum_{k\in K_{iz}^C} \Gamma_{ik}^C \tilde{\sigma}_{ikt}^C - \sum_{k\in K_{iz}^E\cup K_{iz}^C} c_{ik}q_{ikt}\right) + M\sum_{r=1}^{n} u_r. \qquad (104)$$

As for the other approaches in the family of k-th best algorithms, this method assumes that the rational reaction of the lower level is a singleton. Conversely, such problem could return a result which is not the same found by the GenCos in terms of profitability and choice of binary variables only because of multiple alternative optimal solutions. This can generate sub-optimal solutions for the problem (15)-(18).

The algorithm is described below, where $\overline{v}_{i[n]}$ denotes the optimal profit of the i-th GenCo at iteration n, $v^*_{i[n]}$ denotes the candidate profit for GenCo i computed by the TransCo at iteration n and ϵ is a small positive real number.

The algorithm is composed of:

- a master problem, named $PILR_{[n]}$;
- a group of sub problems named $GP_i\left(\pi^*_{zt}, q^*_{ikt}\right)$ for each GenCo, exchanging information about feasibility (and therefore overall optimality) of the candidate solution.

$PILR_{[n]}$ proposes a lower bound for the solution to the problem, together with the candidate values for new generation unit installments up to each GenCo and related optimal profit value $(Y^*_{ik[n]}, v^*_{i[n]})$. Such solution is feasible for the TransCo problem (15)-(18) only if the involved GenCos actually confirm the candidate values when they solve their decision problem. Solution of problem $GP_i\left(\pi^*_{zt}, q^*_{ikt}\right)$ for each GenCo will provide the vector $(\bar{Y}_{ik[n]}, \overline{v}_{i[n]})$, where the first element denotes the optimal value for the variables defining installments of new generation units for GenCo i and $\overline{v}_{i[n]}$ denotes the optimal profit for GenCo i when TransCo computes solution X^*_l.

The TransCo problem (15)-(18) is feasible and candidate solution is optimal if either $\bar{Y}_{ik[n]}$ and $Y^*_{ik[n]}$ coincide, or $\overline{v}_{i[n]}$ and $v^*_{i[n]}$ coincide. If feasibility requirements are not met, a new cut and a penalty term are introduced into

Algorithm 1 Progressive Penalization Algorithm

(Step 1)
$n \leftarrow 0$
solve $ILR \rightarrow (\pi^*_{zt[n]}, q^*_{ikt[n]}, Y^*_{ik[n]}, X^*_{l[n]}, v^*_{i[n]})$
if ILR is infeasible **then**
  STOP. The problem is infeasible
else
  (Step 2)
  for $i = 1 \rightarrow |\mathcal{I}|$ **do**
    solve $GP_i\left(\pi^*_{zt[n]}, q^*_{ikt[n]}\right) \rightarrow (\bar{Y}_{ik[n]}, \overline{v}_{i[n]})$
  end for
  if $\bar{Y}_{ik[n]} = Y^*_{ik[n]}$ **or** $\mid \overline{v}_{i[n]} - v^*_{i[n]} \mid < \epsilon$ **then**
    $X^*_{l[n]}$ is the optimal solution of (15)-(18)
  else
    (Step 3)
    while $\bar{Y}_{ik[n]} \neq Y^*_{ik[n]}$ **and** $\mid \overline{v}_{i[n]} - v^*_{i[n]} \mid \geq \epsilon$ **do**
      $n \leftarrow n + 1$
      solve $PILR_{[n]} \rightarrow (\pi^*_{zt[n]}, q^*_{ikt[n]}, Y^*_{ik[n]}, X^*_{l[n]}, v^*_{i[n]})$
      if $PILR_{[n]}$ is infeasible **then**
        STOP. The problem is infeasible
      else
        for $i = 1 \rightarrow |\mathcal{I}|$ **do**
          solve $GP_i\left(\pi^*_{zt[n]}, q^*_{ikt[n]}\right) \rightarrow (\bar{Y}_{ik[n]}, \overline{v}_{i[n]})$
        end for
      end if
    end while
    $X^*_{l[n]}$ is the optimal solution of (15)-(18)
  end if
end if

the PILR to force the next iteration to avoid picking the previous candidate solution $Y^*_{ik[n]}$.

Step 1 computes the lowest possible bound, Step 2 checks whether the solution obtained in Step 1 is feasible for the overall problem (15)-(18) and while the program is not feasible a new iteration will add a cut and a new penalty term into the PILR problem. This is done by defining a loop in Step 3. The algorithm terminates when the first feasible solution is found or concludes that no feasible solution exists.

6 Numerical Experiments

We tested our approach using the classical 6-bus example from Garver, under a 5 periods time horizon. The initial structure of the grid for our test case is depicted in figure 2. Data for the implementation are presented in tables 1-3.

We assume that the grid has 9 initial connections between nodes, 5 loads and 10 initial generation units. We assume to have four GenCos, respectively denoted as A1, A2, A3 and A4. Load is mostly concentrated in nodes 2, 4 and 5, as shown in table 3, while the main node for production is number 6.

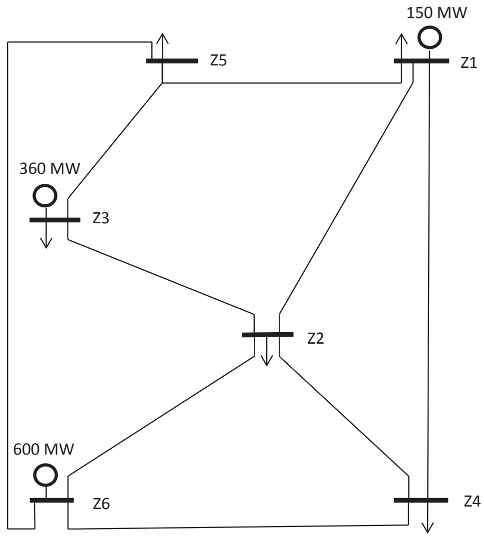

Fig. 2 Initial configuration of the 6-bus Garver grid in our Case Study (load changes over periods and is displayed in table 3)

Generation units are labelled from G1 to G15 with units G11-G15 considered as candidate generation units. Details on distribution of the generation units on the grid are displayed in table 2, where the bid up coefficient defines a term to be multiplied to the marginal cost to define the price bid. It is assumed that at most three lines are admitted in each transmission corridor.

Table 1 Line data for Garver's 6-bus example

from	to	Reactance (p.u.)	Capacity (MW)	Investment Cost (M€)
1	2	0.40	100	7.7232
1	3	0.38	100	7.33704
1	4	0.60	80	11.5848
1	5	0.20	100	3.8616
1	6	0.68	70	13.12944
2	3	0.20	100	3.8616
2	4	0.40	100	7.7232
2	5	0.31	100	5.98548
2	6	0.30	100	5.7924
3	4	0.59	82	11.39172
3	5	0.20	100	3.8616
3	6	0.48	100	9.26784
4	5	0.63	75	12.16404
4	6	0.30	100	5.7924
5	6	0.61	78	11.77788

We have performed two set of analyses on the model. First, we have set the focus on how different budget availability up to the TransCo can influence the

Table 2 Data on Generation units

Unit	marginal cost (€)	Capacity (MW)	Investment cost (M€)	owner	node	bid up coefficient
G1	10	150	-	A1	Z1	1.0072
G2	20	120	-	A3	Z3	1.06
G3	22	120	-	A4	Z3	1.069
G4	25	120	-	A4	Z3	1.069
G5	8	100	-	A4	Z6	1.069
G6	12	100	-	A4	Z6	1.069
G7	15	100	-	A4	Z6	1.069
G8	17	100	-	A4	Z6	1.069
G9	19	100	-	A4	Z6	1.069
G10	21	100	-	A4	Z6	1.069
G11	11	162	4.5171	A2	Z3	1.0074
G12	10	100	6.441	A3	Z6	1.06
G13	9	150	2.7746	A4	Z6	1.069
G14	9.4	157	2.9796	A4	Z6	1.069
G15	12.2	102	2.026	A1	Z1	1.0072

Table 3 Load (MW) over time periods

node/Period	1	2	3	4	5
Z1	80	81.07	82.26	83.51	84.18
Z2	240	242	242.91	243.55	244.12
Z3	50	51.03	52.11	53.02	54.60
Z4	180	181.76	182.04	183.75	184.51
Z5	240	240.22	241.13	242.07	243.85
Z6	0	0	0	0	0

components of the social cost function, namely profitability of the GenCos and social costs up to consumers. Then we have moved our attention on how the profitability of GenCos and social costs change when we shift the weighting factor α, which measures the relative importance of consumers and producers in the aggregate social cost function.

Concerning the first set of analyses, results are displayed in figure 3. We have set the weighting factor parameter α to 0.5 and performed a parametric analysis of social costs and aggregated GenCo profits as the TransCo has a steadily increasing budget availability for investments on transmission lines. Figure 6 (left) shows us two distinct effects of a budget increase. For smaller increases the effect is a convergence of average (over periods) Locational Marginal Prices from different nodes, whilst for larger increases the converged prices follow a common path downwards. This has an impact on GenCos' profits as explained later on.

As it can be seen from the tables, GenCo A4, who has the largest availability of generation units and who can benefit of low marginal production costs tends to increase the production, eroding market shares from its competitors. The installation of new lines in the grid makes relatively easy to improve profitability for the GenCos by selling power produced from generation units with lower marginal costs. The TransCo is basically allowing GenCos to reach new market nodes by selling energy where there is shortage but retaining some congestion on the lines which does prevent the prices to change too much. In particular, GenCo A4 opens a new generation unit and starts selling cross nodes with generation units having very low marginal costs, even though revenues for A4 increase due to the lower marginal costs, its production does not manage to cover the entire load in all nodes. This means that generation units belonging to other GenCos will place higher bids in order to cover the remaining load and prices will not decrease to their possible lower limit. Some GenCos will experience a decrease of power sold as consumers can buy from cheaper producers, while other GenCos will have an increase in sales and some of them (A2 and A4) will install new generation units with lower marginal costs compared to the competition. As a consequence prices tend to converge to a common point as it is shown in figure 6 (left). Also the consumers will benefit of such upgrade as they can buy energy for lower prices.

When no more benefits can be added to GenCos' profitability by removing congestion, the TransCo will completely move its focus on social costs up to consumers and keep removing congestion to allow companies willing to bid at lower prices to sell cross nodes. This will decrease the overall GenCos' profits, as it can be seen on the second portion of the curves in figure 3 (left), but it will lower to a greater extent social costs, as it can be seen in figure 3 (right).

In addition, we report the different configurations of the grid with respect to the minimum and the maximum budget amount considered for the TransCo. Such configurations are depicted in figure 4. The figures show the production capacities and the transmission levels between nodes. Transmission between two nodes with increasing indices has been labelled with positive sign, whilst transmission between two nodes with decreasing indices has been labelled with negative sign. The comparison between the two configurations shows how TransCo manages to lower down costs up to consumer by opening lines between different nodes. New lines are shown as dashed lines. We particularly notice that node $Z2$ in figure 4 (left), which do not feature any production unit, has to buy power from node $Z3$ since there is a congested line between node $Z2$ and $Z6$. Generation units with the lowest marginal costs are located in node $Z6$. As a new line is added between nodes $Z2$ and $Z6$ the amount of power purchased by $Z2$ from generators located in node $Z3$ drastically decreases, while transmission from $Z6$ to $Z2$ almost doubles. The new configuration (figure 4 right) allows consumers to buy power from nodes different from $Z3$, whose generation units exhibit a higher production cost compared to the generation units placed in different nodes. As a result, the generation units expansion planning depicted in figure 4 (left) is rather different compared to the one shown in figure 4 (right). In the left figure there is generation expansion in nodes $Z1$ and $Z3$ - namely GenCo A1 opens candidate generation unit G15 in node $Z1$ and GenCo A2 opens candidate generation unit G11 in node $Z3$ - whilst in figure 4 (right) there is generation expansion in nodes $Z1$ and $Z6$ with the same generation unit opened in node $Z1$ and GenCo A4 opening candidate generation unit G14 in node $Z6$.

The effects of varying the weighting factor are displayed in figures 5 and 6 (right). We have performed the analyses setting a value of 30 Million Euros

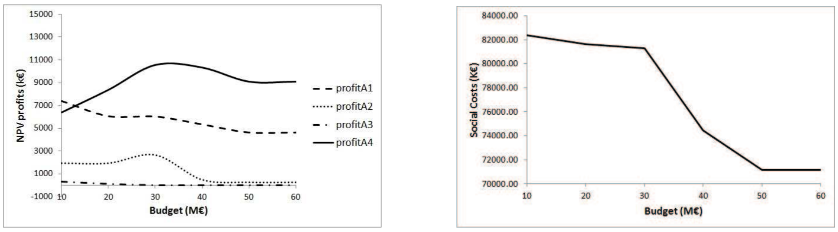

Fig. 3 Net Present Value of profits and total social costs for different levels of Budget (weighting factor set to 0.5)

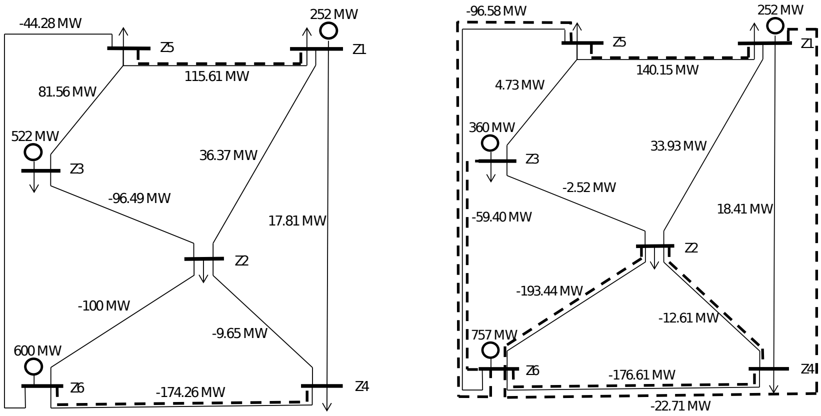

Fig. 4 Grid configuration related to budget level 10 M€ (a) and 60 M€ (b)

of the TransCo budget and assigning α the values $0, 0.2, 0.4, 0.6, 0.8, 1$. For increasing values of α the grid upgrading plan has an increasing focus on congestion removal. This implies an initial convergence of prices over different nodes, as shown in figure 6 (right) which, in turn allows more efficient GenCos (in this case A2 opens new generation units in node $Z3$) to increase their market share by opening new generation units with lower marginal production costs. As a consequence some companies will erode market share to others. When the weighting factor approaches the unit value (i.e. only consumers are important in the social cost function) the TransCo will select a grid expansion plan which further penalizes GenCos as some zones will have lower LMPs. This entails a profit decrease for some GenCos.

All computations have been carried out using GAMS/CPLEX 12.1.0 using four parallel threads on a Intel Core i7 machine running at 2.67 GHz with 8 GB of RAM. Computational time is heavily influenced by the data, ranging between little over one minute for cases where GenCos respond to transmission investments by opening new generation units to several minutes, when no GenCo finds it profitable to open a new power generation unit.

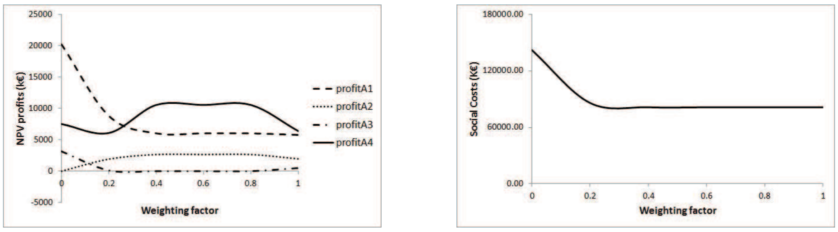

Fig. 5 Net Present Value of profits and total social costs for different choices of the weighting factor (budget set to 30 M€)

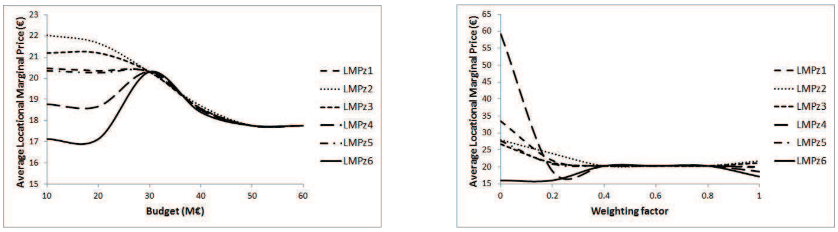

Fig. 6 Locational Marginal Prices for different choices of budget (weighting factor set to 0.5) and weighting factor (budget set to 30 M€)

7 Conclusion

We have developed a model for analyzing and supporting the decisions on transmission grid upgrades within a market environment. We have taken explicit account of possible reaction of GenCos and MO to grid upgrade decisions both under an operational and an investment viewpoint. The problem has been modeled as a Bilevel Programming Problem with binary decision variables in both upper and lower level equilibrium problem. A dedicated algorithm, extending the idea of the k-th best algorithm to include mixed binary structure in the lower level problem has been developed to solve the problem.

The model has been analyzed using the 6-bus Garver test case considering the possible outcomes stemming from different budget availability for the TransCo and shifts in level of relative importance assigned to consumers and producers by the TransCo. Results show how grid upgrades can help the system keeping the power prices low or allow particular GenCos to exploit their efficiency to increase profits.

There are several possible directions for future research. One of them is to include strategic bidding into the framework. This will seriously increase the computational burden as the modeling approach will entail a three level hierarchical formulation where GenCos and MO are respectively in the intermediate and lower level. As a consequence, new solution approaches must be considered. Another possible developement could be to include the effects of stochasticity and risk attitude on decisions of each player both on upper and lower levels. Finally, it would be beneficial to consider possible collaborative patterns between TransCo and GenCos, where a vector of side payments could complement a coordinated effort to provide electricity under the optimization of a system performance.

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Appendix

Proof (Proposition 1) The proposition can be proved by making considerations on the structure of the Karush-Kuhn-Tucker conditions for the modified MO problem. We report the complementarity conditions of such system

$$
\begin{aligned}
&\left(\pi_{zt} - \sigma^E_{ikt} - b_{ikt}\right) q_{ikt} = 0 && i \in I, t \in T, z \in Z, k \in K^E_{iz} \\
&\left(\pi_{zt} - \sigma^C_{ikt} - b_{ikt}\right) q_{ikt} = 0 && i \in I, t \in T, z \in Z, k \in K^C_{iz} \\
&\left(q_{ikt} - \Gamma^E_{ik}\right) \sigma^E_{ikt} = 0 && i \in I, t \in T, z \in Z, k \in K^E_{iz} \\
&\left(q_{ikt} - \Gamma^C_{ik} Y_{ik}\right) \sigma^C_{ikt} = 0 && i \in I, t \in T, z \in Z, k \in K^C_{iz} \\
&\left(\overline{TR}_{lt} - TR_{lt}\right) \eta^+_{lt} = 0 && l \in L^E \cup L^C, t \in T \\
&\left(TR_{lt} - \underline{TR}_{lt}\right) \eta^-_{lt} = 0 && l \in L^E \cup L^C, t \in T \\
&\left(\overline{\theta}_z - \theta_{zt}\right) \kappa^+_{zt} = 0 && z \in Z, t \in T \\
&\left(\theta_{zt} - \underline{\theta}_z\right) \kappa^-_{zt} = 0 && z \in Z, t \in T
\end{aligned}
\tag{105}
$$

Let us first prove that the margin of every GenCo is non negative. From the first two equations of the complementarity system (105) one has

$$\pi_{zt} q_{ikt} = \sigma^E_{ikt} q_{ikt} + b_{ikt} q_{ikt}$$

$$\pi_{zt} q_{ikt} = \sigma^C_{ikts} q_{ikt} + b_{ikt} q_{ikt}$$

with $\sigma^E_{ikt} q_{ikt}, \sigma^C_{ikt} q_{ikt} \geq 0$.
This implies that

$$\pi_{zt} q_{ikt} \geq b_{ikt} q_{ikt}$$

and since by assumption $b_{ikt} \geq c_{ik}$, one has that

$$\pi_{zt} q_{ikt} \geq c_{ik} q_{ikt}$$

therefore the margin is non negative, which means that each GenCo is willing to increase its production until either the maximum accepted bid or a capacity bound is reached.
Now we show the main claim of the proposition. The power produced by each GenCo cannot be such that $\tilde{q}_{ikt} > q_{ikt}$ and it can be $\tilde{q}_{ikt} < q_{ikt}$ if and only if $\tilde{q}_{ikt} = \Gamma^E_{ik}$ for existing generation units or $\tilde{q}_{ikt} = \Gamma^C_{ik} Y_{ik}$ for candidate generation units. But then it would be $q_{ikt} > \Gamma^E_{ik}$ or $q_{ikt} > \Gamma^C_{ik} Y_{ik}$ which is not feasible for the modified MO Problem. Therefore, once the MO solves the aforementioned problem it must be $\tilde{q}_{ikt} = q_{ikt}$ for each GenCo. □