Higher order beliefs and the dynamics of exchange rates

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Abstract

This paper investigates the role of higher order beliefs in the formation of exchange rates. Our model combines a standard macroeconomic dynamics for the exchange rates with a microeconomic specification of agents' heterogeneity and their interactions. The empirical analysis relies on a state space model estimated through Bayesian methods. We exploit data on macroeconomic fundamentals in a panel of subjective forecasts on the euro/dollar exchange rate. The equilibrium strategy on the optimization process of the predictors shows that higher order beliefs is the relevant factor in performing individual forecasting. Moreover public information, namely past exchange rates and fundamentals, plays a crucial role as a coordination device to generate expectations among agents on the basis of their forecasting abilities.

Keywords: beauty contest, higher order beliefs, exchange rates, economic fundamentals, survey data

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In the last decades, modelling the behavior of exchange rate has become one of the most challenging tasks demanding the attention of economists. Meese and Rogoff (1983) were the first to point out the impossibility of structural and time series exchange rate models to outperform a random walk in terms of forecasting ability. This discouraging result seemed partially overcome with the introduction of cointegration analysis at least in the long run (1 to 3 years forecasts).[1] Over short horizons the introduction of Taylor rule based- fundamentals[2] or the informational value of order flow[3] were apparently the only possible solutions to improve the quality of the analysis.

The crucial element that allowed a better forecasting performance on shorter horizons was the so-called *micro based* approach. The heterogeneity of participants and the bargaining process of private informations led directly to the conclusion that understanding the interactions among agents and the consequent volatility in the market is peculiar to capture the effective exchange rate dynamics.[4] The utility-based new open economy macroeconomic framework by Devereux and Engel (2002) or the rational expectations present value model of Engel and West (2005) were proposed as different alternatives motivating the use of heterogeneous information of agents in asset pricing models.[5]

Our analysis builds on the micro-structure of foreign exchange markets looking at an environment of dispersed information about fundamentals which may influence traders' preferences and expectations. The information that traders optimally choose to collect is determined by their interest to align their actions with the fundamentals as well as with other agents' predictions in the spirit of Morris and Shin (2002). A beauty contest framework in analogy to Keynes'[6] is introduced as a game with continuous actions, higher order beliefs and dispersed information.[7]

[1] Mark (1995) and Chinn and Meese (1995) found a substantial improvement in the success in forecasting exchange rates compared to the random walk model using an error correction term structure over forecasting horizons of 2-3 years, in a monetary model structure. These results were nonetheless criticized as time dependent by Faust et al. (2003) and Cheung et al. (2005). An increase in the performance in forecasting ability of the monetary models was then registered thanks to the implementation of panel cointegration methods. Husted and MacDonald (1998) used Pedroni (1999) panel cointegration test, while Groen (2005) showed outperformance of a random walk in a panel of three currencies for short and long run. Nonetheless, Mark and Sul (2001) used a panel cointegration version of Mark's paper over 17 currencies with no statistical evidence of outperformance and a smaller forecasting error. Cerra and Saxena (2010) discovered a rise in the predictive success in the long run using a large panel of currencies.

[2] See Molodtsova et al. (2008, 2011) and Molodtsova and Papell (2009).

[3] See Evans and Lyons (2005).

[4] O'Hara (1995) claims that market micro-structure is the study of the aggregate process of exchanging assets under explicit trading rules. The regulation of trading in exchange markets has important implications for the process of price formation and more generally for all unobservable relevant characteristics.

[5] A recent discussion of microeconomic structures on exchange rates was developed by Bacchetta and Van Wincoop (2012).

[6] Keynes (1936).

[7] Throughout the paper, we use the terms 'beauty contest' or 'higher order beliefs' interchangeably. The term beauty contest is due to Keynes' famous claim that in order to form their demand for an asset, investors behave as individuals in a

Our basic macroeconomic framework is a standard monetary model with two countries and heterogeneous information structure. We develop a structural model-based approach for the analysis of the micro-strategic behavior of forecasters and test its relevance on the aggregate macro dynamics of the eur/usd currency for the period ranging from 2006 to 2012. A dynamic game of incomplete information is modeled to evaluate the role of higher order beliefs in exchange rate settings.[8] In particular the social learning process in place interchanges the amount of private and public information and relates the discrepancy between the two to the role that higher-order beliefs play in determining outcomes and the way information is used in equilibrium. We search whether a potential nexus exists among the choices of market predictors.[9] Then we identify the precise link guiding this micro-strategic behavior to the aggregate macro dynamics. The choice of professional forecasters in the currency is consequently tested at micro and macro levels. The empirical test is based on one-period ahead survey predictions provided by heterogeneous professional forecasters. The estimation of the structural parameters of the model provides two main findings. First, individual predictions about future values of the currency rely heavily on the role of higher order beliefs. We find that the value of beauty contest accounts for about 82 per cent in the optimal rule investors apply to decide their next periods forecasts. Secondly, public information plays the most important role in determining forecasts, i.e., around more than 75 per cent, compared to to private information.

Our contribution relates to the whole fast-growing literature on heterogeneity of expectations and higher order beliefs. The most important contributions belong to a series of papers proposed by Bacchetta and van Wincoop who have explored the implication of heterogeneity in expectations on different theoretical models. Bacchetta and Van Wincoop (2004) propose the so-called scapegoat theory of the disconnect between exchange rates and fundamentals. They suppose the existence of uncertainty in the market about the true source of exchange rate fluctuations. When agents observe currency movements

beauty contest: the beauty contest was an experiment where people were asked to guess not the prettiest girl among those presented in the newspaper contest, but the girl they thought that the majority would consider the prettiest. He suggests an analogy to financial markets claiming that agents do not only try to generate forecasts to predict the future behavior of assets but also try to guess other market participants' forecasts and the forecasts of the forecasts of the others, just like if they were guessing not the best asset but the asset they believe the others will buy. Thus investors are said to have 'higher order beliefs': they have beliefs that try to take into account forecasts at increasing orders, related to the orders of subjects taken into account in their calculation according to this scheme. See also footnote 7.

[8]Since the classical contribution of Harsanyi (1967) the rational behavior in such environments depends not only on economic agents' belief about fundamentals, but is also a function of beliefs of higher order, i.e., player's beliefs about other players' beliefs about other players' belief and so on.

[9]See Evans and Rime (2012) for a survey.

that are inconsistent with their expectations, they search for an explanation to these unexpected changes. A weight, higher than average, is assigned to some fundamentals taken as *scapegoats*. The heterogeneity is modelled looking at investors who receive private signals about the persistence of shocks. They are not able to capture whether the fluctuation on exchange rate is motivated by unobserved factors or simply by a weight, larger than expected one, assigned to macro fundamentals. The latter could be interpreted as a scapegoat, so that the weights attributed to them systematically change over time, thus determining parameter instability. Fratzscher et al. (2012) develop an empirical test of this theory using as a proxy of scapegoat fundamentals, Consensus Economics of London surveys of predictors. The predictors of this panel are asked to rate on a quantitative scale the importance of six key variables (short-term interest rates, long-term interest rates, growth, inflation, current account, equity flows) as drivers of the dynamics of the exchange rate. The authors find that the inclusion of these expectations in the model of exchange rate determination improves the power of the fundamentals in explaining currency movements.[10] Bacchetta and Van Wincoop (2006) focus instead on the role of order flow rather than macroeconomic characteristics of the market and introduce a possible explanation for the empirical results verified by Evans and Lyons (2002), Payne (2003), and Froot and Ramadorai (2005).[11] Assuming that agents are risk averse with heterogenous information on the fundamental values, the authors show that due to the imperfect correlated signals among investors, transitory shocks may continuously influence the dynamics of exchange rate. These results originate from a counterbalance effect of risk aversion and uncertainty of information. On one side, the risk-sharing impact is justified since traders must be compensated for the *extra* risk assumed as a consequence of their actions. On the other side, the uncertainty of information matters since investors may confuse an appreciation or depreciation of the exchange rate caused by a liquidity shock with that induced by information on fundamentals. Therefore, if this mix-up concerns information that becomes public in the relative distant future, the impact of order flow on exchange

[10] In the short run the heterogeneity in the individual evaluation may lead to overrate the random macroeconomic fundamental.

[11] Evans and Lyons (2002) exploit data pertaining to bilateral transactions among FX dealers via Reuters Dealing 2000-1 electronic trading system. They follow Meese and Rogoff (1983)'s methodology to investigate the out of sample forecasting ability of their linear model. Unfortunately, they do not take into account potential issue of simultaneity bias emerging when exchange rate movements cause order flow. In order to evaluate the possible feed-back effects of exchange rates on order flow, an alternative methodology was suggested by Payne (2003). He elaborates a VAR model estimating information on the size of transactions. This methodology allows for a more precise estimation of the information provided by the order flow. Froot and Ramadorai (2005) extends the framework of Payne (2003) considering inflation and interest rate differentials alongside order flow and excess returns. They also estimate long-run effects of international flows on exchange rates and their relation to fundamentals proposing a decomposition of permanent and transitory components of asset returns.

rates is amplified by the *infinite regress* of investors' individual beliefs. Given the observed prices and quantities, they learn, not only the fundamental values of foreign currencies, but also the other investors' forecasts. The social learning process further disorientates about the informational role that liquidity and fundamental shocks may assume, thus extending the impact of order flow on exchange rates.[12] In particular Bacchetta and Van Wincoop (2013) hypothesize a heterogeneous information structure with public and private signals received by forecast market predictors. Each agent needs to keep into account the average expectation of other market participants about the next $t-$ periods exchange rates, when forming her expectation. The difference existing between the expectations about the others' expectations and the actual fundamental, difficult to perceive in the short run, generates the observed wedge between the actual dynamics of the rate and the fundamentals in the long run.

As regards instead the social learning structure of our model, our attention is devoted to a recent literature which tries to understand whether public information improves the effectiveness of policies and is beneficial to markets since it reduces asymmetric information. This framework was mainly popularized by Morris and Shin (2002) and perfectly explains the dynamics of coordination among agents. When coordination incentives are not very strong in the society, higher precision on public information could be in principle welfare-improving. When agents have higher incentives to coordinate, more weight is given to the public signal relative to the private one in the choice of equilibrium actions. Thus it is possible that a potential overreaction to the public signal cancels out the impact of private information. Under certain conditions, this means that a public announcement may destabilize markets, reduce efficiency due to their impacts on higher-order beliefs and can be detrimental for the welfare of agents. This approach has been used as a static representation of many settings with incomplete information and strategic interaction including financial markets (Allen et al., 2006), business cycle models (Angeletos and La'O, 2009), investment decisions (Angeletos et al., 2012), price adjustment with monopolistic competition (Woodford, 2002).

Finally for what involves the heterogeneity of survey predictions, there exists an extensive empirical literature on rationality and inefficiency of predictions. The seminal paper in this field is Ito (1990) which tests individual biases and idiosyncratic effects for a set of disaggregate expectations about the 1-, 3- and 6-month-ahead JY/US rate from the JCIF survey over the period 1985-1987 finding substantial

[12]Note that if the private signal involves imminent shifts in fundamentals, this effect is practically thinned down. Intuitively, when private signal concerns next period realizations of fundamentals, there is no need to extract any information from other investors' forecasts since they will share information on fundamentals in a short time.

heterogeneity among predictors.[13] Similar results can be confirmed by Elliott and Ito (1999) and Benassy-Quere et al. (2003). MacDonald and Ian (1996) replicate Ito's test for 3- and 12-month- ahead estimates of the BP/usd, DM/usd and JY/usd rates from the 1989-1992 and find significant evidence of heterogeneous expectations. Extending the Consensus data set to 1995, Chionis and MacDonald (1997) confirm the presence of individual effects for predictors. Mitchell and Pearce (2007) conducted a thorough analysis of unbiasedness and success rate of predictions along with tests for heterogeneity and strategic forecasting, finding systematic heterogeneity in predictions.[14] The remainder of this paper is as follows: Section 1 introduces the theoretical setup and the equilibrium solutions both at micro and macro levels. Section 2 presents the empirical estimation of the structural model, while some comments are proposed on Section 3 about the posterior estimates and policy implications. Section 4 concludes.

1. Theoretical Model

The macroeconomic model is a standard two-country monetary model identified by the following basic relationships. Define E_t as the nominal exchange rate between home and foreign country, where P_t is the level of home prices and P_t^* is the level of prices in the foreign country.[15] We assume that Purchasing Power Parity holds,

$$E_t = \frac{P_t}{P_t^*}, \tag{1}$$

implying that the exchange rate at which two currencies trade equals the price levels of the two countries. Rearranging this expression in log terms we get:

$$p_t = p_t^* + s_t \tag{2}$$

where p_t and p_t^* are, respectively, the log of home and foreign price level, while s_t is the log of real exchange rate between home and foreign country, i.e., $s_t = log(E_t)$. The second building block of the

[13]This literature stems from the recent availability of individual survey-predictions of exchange rates. Previously, a long strand of literature has studied inefficiency and irrationality of exchange rates forecasts. Dominguez (1986) tests the efficiency of foreign exchange market showing that predictors systematically fail in forecasting in the magnitude and the direction of exchange rates movements. Avraham et al. (1987) test the same hypothesis in a high inflationary Israel of the eighties rejecting the notion of rationality of exchange rate expectations. Cavaglia et al. (1993) find that exchange rates forecasts in the EMS are biased. Chinn and Frankel (1994) propose a test rejecting the hypothesis of efficiency and unbiasedness of exchange rate predictions.

[14]For a complete review of the tests of heterogeneity hypothesis using disaggregated survey expectations of professional forecasters, see Jongen et al. (2008).

[15]The starred superscripts usually indicate the variables for foreign country.

model is determined by the money-demand functions for the two countries:[16]

$$\frac{M^d}{P} = YL(i) \tag{3}$$
$$\frac{M^{*d}}{P^*} = Y^*L(i^*) \tag{4}$$

Equations (3) and (4) state that real money demand in each country is proportional to nominal income. The money demand increases in order to hold money for transactional purposes and is negatively related to interest rate. L is a negative function of i because of the negative opportunity cost of holding money instead of an interest-earning asset paying i. Long run equilibrium in the money market is achieved when the real money supply, determined by the central bank and indicated with M, is equal to the real demand for money balances, i.e., when $M = M^d$. The money market equilibrium relationship for the two countries is then:

$$\frac{M}{P} = YL(i) \tag{5}$$
$$\frac{M^*}{P^*} = Y^*L(i^*). \tag{6}$$

Rewriting both expressions in log terms, we get:

$$m_t - p_t = \phi y_t - \alpha i_t \tag{7}$$
$$m_t^* - p_t^* = \phi y_t^* - \alpha i_t^* \tag{8}$$

where y_t is the log of real output. We assume that parameters α and ϕ are the same for both countries.[17] Finally, the uncovered interest rate parity (UIP) holds. Returns in both countries are equal after controlling for the expected depreciation of home country, namely

$$(1 + i_t) = (1 + i_t^*)\,\frac{{E^e}_{t+1}}{E_t} \tag{9}$$

[16]We assume for simplicity the same functional forms for the demand functions of the countries.
[17]We do not impose any a-priori difference in the structure of the two countries.

which rewritten in the approximate form reads

$$1 + i_t = 1 + i_t^* + \frac{E^e{}_{t+1} - E_t}{E_t} \tag{10}$$

and in log terms becomes

$$\mathbb{E}_t(s_{t+1} - s_t) = i_t - i_t^*. \tag{11}$$

with $log(E^e{}_{t+1} - E_t) = E_t(s_{t+1} - s_t)$. We augment the standard UIP condition with a term ψ,

$$\mathbb{E}_t(s_{t+1} - s_t) = i_t - i_t^* + \psi_t \tag{12}$$

which stands for observed deviations from UIP. The UIP deviation has been tested in the literature by a series of empirical contributions.[18] Since UIP condition is based on rational expectations and risk neutrality, deviations from UIP can be interpreted either as an expectational error or a risk premium associated with liquidity or hedge trade.[19] We can rewrite the variables on fundamentals as $f_t = m_t - m_t^* - \phi(y_t - y_t^*)$ and define $\lambda = \alpha/(1+\alpha)$. By putting together conditions (7) and (8) and using PPP (Eq. 2) and UIP (Eq. 12), we derive a general expression for the dynamics the exchange rate:

$$s_t = (1-\lambda)[f_t + \mathbb{E}_t \sum_{j=1}^{\infty} \lambda^j f_{t+j}] - \lambda[\psi_t + \mathbb{E}_t \sum_{j=1}^{\infty} \lambda^j \psi_{t+j}], \tag{13}$$

where the exchange rate depends on the path of the current and expected fundamentals and UIP deviations ψ. A general solution of this equation under the hypothesis that agents share the public information and perfectly know the model of the economy implies that expectations about the future values of fundamentals and deviations are homogeneous.[20]

A more sophisticated and realistic information structure generalizes the common expectations framework to include information heterogeneity as proposed by Bacchetta and Van Wincoop (2006, 2013). In

[18]See James et al. (2012) for a complete review.

[19]This factor is necessary because information heterogeneity necessarily generates a deviation from UIP. This hypothesis differentiates this framework from the standard rational expectations structure where the perfect knowledge of investor's behavior eliminates any deviation from UIP thanks to a non-arbitrage condition. Note that the deviation from UIP can also be explained with the presence of information on fundamentals enclosed in order flow as in Evans (2010) and Chinn and Moore (2011). We will abstract from a specific modelling of this factor in this work and hypothesize in the empirical analysis that this factor is an unknown stochastic process. We leave the introduction of the role of order flow in our model for future research.

[20]For the functional form of the solution in case of rational expectations see Bacchetta and Van Wincoop (2012).

this setting, agents are heterogeneous due to their private signals on the future level of fundamentals. This hypothesis implies that the expected value of the future exchange rate is computed as the average expectation among agents. Further heterogeneity implies that agents may have private trading needs due to liquidity or hedge trade that can generate demand for foreign bond that affect the exchange rate as indicated by the term $\mathbb{E}_t \sum_{j=1}^{\infty} \lambda^j \psi_{t+j}$. We hypothesize that trading needs expressed by ψ are unrelated to expectations about future fundamental macroeconomic variables and then make the simplifying assumption that ψ_t is an *i.i.d.* process with variance σ_ψ^2.

Finally we assume that f_t is a linear combination of two random walk processes f_1 and f_2, and then the general process for the fundamental is a random walk as well, $f_{t+1} = f_t + \epsilon_{t+1}^f$ with the variance of ϵ_{t+1}^f being σ_f^2. Substituting equations 2, 7, 8 into 12 we obtain a new equation for the dynamics of the exchange rate:

$$s_t = \lambda \bar{\mathbb{E}}_t s_{t+1} + (1 - \lambda) f_t - \lambda \psi_t, \tag{14}$$

where the role played by heterogeneous expectations is clearly identified by the term $\bar{\mathbb{E}}_t s_{t+1}$. The current value of the rate will thus depend on next period expectations of the other predictors. When *news* about fundamentals alter expectations on the future value of the exchange rate, the solution will be affected and depend on the average of all expectations in the next period. Without loss of generality, we assume that each investor at time t captures a private signal about the fundamental at time $t + 1$. In a more general perspective, private signals may involve infinite periods ahead for n predictors. In this case we would observe an infinite dimensional system where the exchange rate at time t depends on the fundamental at time t and the average expectation at t of the fundamental at time $t + 1$ and so on (Bacchetta and Van Wincoop, 2006).[21] In our model, however, heterogeneous one step ahead expectation for each time period is the appropriate model to perform an empirical analysis based on actual data.

Our framework is compatible with the study of the strategic mechanism that drives heterogeneous predictions that ultimately influences the market. A structural approach for the analysis of the microeconomic strategic behavior of professional forecasters is consequently proposed. In the rest of this section, first we describe a social learning model about the choices of professional forecasters in the exchange rates market. Then we integrate it in the standard macro monetary economic model described above.

[21] See Townsend (1983) on this point.

1.1. Social learning on exchange rates

Let us suppose a two-period economy populated by a finite series of predictors, $n = \{1, ..., N\}$. In period t, each agent i observes noisy private and public signals about the exchange rate s_t which belongs to a set $\Psi : s \in \Psi$ and evolves according to the stochastic process:

$$s_t = s_{t-1} + \gamma_t \quad \text{where} \quad \gamma_t \sim N(0, \sigma_\gamma^2) \qquad \text{and precision} \quad \rho_s \equiv \sigma_\gamma^{-2} \tag{15}$$

where the shock γ_t occurring at the beginning of period t is normally distributed with mean 0, variance σ_γ^2, and precision $\rho_s \equiv \sigma_\gamma^{-2}$. After the realization of a shock, each agent i receives a common public signal about the fundamental f_t as a function of the exchange rate:

$$f_t = s_t + \eta_t \qquad \eta_t \sim N(0, \sigma_\eta^2) \qquad \text{and precision} \quad \rho_f \equiv 1/\sigma_\eta^2 \tag{16}$$

and a private personal signal:

$$x_{it} = s_t + \epsilon_{it} \qquad \epsilon_{it} \sim N(0, \sigma_\epsilon^2) \qquad \text{and precision} \quad \rho_{x_i} \equiv 1/\sigma_\epsilon^2. \tag{17}$$

So while information on the fundamental f_t is common knowledge among agents, the private signal x_{it} is idiosyncratic to agent i and not observed by the other predictors. The common posterior about s_t, taking into account public information, is therefore normally distributed with mean $\mathbb{E}[s_t|f_t] = \frac{\rho_s s_{t-1} + \rho_f f_t}{\rho_s + \rho_f}$ and precision $\rho[s_t|f_t] = \rho_s + \rho_f$. For notational simplicity, we denote the mean and the precision of the posterior distribution, respectively as $\tilde{y}_t = \mathbb{E}[s_t|f_t]$ and $\rho_{\tilde{y}} = \rho[s_t|f_t]$.

Private posteriors are defined by mean $\mathbb{E}[s_t|f_t, x_{it}] = \frac{\rho_{\tilde{y}} \tilde{y}_t + \rho_{x_i} x_{it}}{\rho_{\tilde{y}} + \rho_{x_i}}$ and precision $\rho[s_t|f_t, x_{it}] = \rho_{\tilde{y}} + \rho_{x_i}$. The weight of public signal in the Bayesian projection of s on the information set $H_i(t) = \{f_t; x_{it}\}$ is $\alpha_{\tilde{y}} = \frac{\rho_{\tilde{y}}}{\rho_{\tilde{y}} + \rho_{x_i}}$, while the weight of the private one is $\alpha_{x_i} = \frac{\rho_{x_i}}{\rho_{\tilde{y}} + \rho_{x_i}}$. The posterior mean for each agent i is then derived as: $\mathbb{E}_i(s_t) = \alpha_{x_i} x_{it} + \alpha_{\tilde{y}} \tilde{y}_t$.

Let $e_{it} = \mathbb{E}_{it}(s_{t+1}) \in \mathbb{R}$ denote the predictor i's expected evaluation on exchange rates, while $\bar{e}_t \equiv \int_j e_{jt} dj$ and $\sigma_e^2 \equiv \int_j [e_{it} - \bar{e}_t]^2 dj$ are respectively the mean and the dispersion of investor's expected evaluations in the economy. Each predictor's preferences are explicitly characterized by the following concave increasing function:

$$U(e_{it}, \bar{e}_t, \sigma_e^2, s_t). \tag{18}$$

As general as possible, we assume that the dispersion σ_e has only a second-order non strategic effect, i.e., $U_{e\sigma} = U_{K\sigma} = U_{s\sigma} = 0$ while $U_\sigma(e_{it}, \bar{e}_t, 0, s_t) = 0$, $\forall e_{it}; \bar{e}_t; s_t$. Under perfect information on the exchange rate s_t, due to symmetry ($e_{it} = \bar{e}_t = s_t$, $\forall i$), the best response is given by the unique equilibrium characteristics where predictors' choice exactly coincides with her expectation. In the case of imperfect information instead optimality requires that for any (x_{it}, f_t), the predictor's choice $e_{it} = e_{it}(x_{it}; f_t; \rho_{\tilde{y}}; \rho_{x_i})$ is such that:

$$\mathbb{E}[U_e(e_i, \bar{e}, \sigma_e^2, s)|x_{it}; f_t; \rho_{\tilde{y}}; \rho_{x_i}] = 0, \qquad \forall i, t. \tag{19}$$

In the case of a finite number of investors (as in Marinovic et al., 2011), individual's expected utility assumes the following form:[22]

$$U(e_{it}, \bar{e}_t, \sigma_e^2, s_t) = -(1-\delta)(e_{it} - s_t)^2 - \delta(e_{it} - \bar{e}_t)^2. \tag{20}$$

The first component is a quadratic loss in the distance between the optimal choice e_i and the fundamental s_t, while the second component is a quadratic loss in the distance between the choice e_{it} and the average $\bar{e}_t$. Each predictor wants to minimize the expected distance between their evaluation and the average. The parameter $\delta \in (0,1)$ is a scalar that parametrizes the intensity of the coordination motive, i.e., the importance that agent i assigns to the expectations of the other predictors of the market.

More intuitively, Eq. (20) describes the predictor's process in terms of the decision between two incentives, that constitute the *reward* rule that judges agent's forecast success. The first incentive induces the agent to anchor his/her predictions on the fundamentals. It relies on the distance between the actual spot and the action of the agent and represents the cost of the forecast error, i.e. the cost of making a mistake with respect to the realized fundamental. The second incentive instead induces predictors to guess the opponents' beliefs because it expresses the cost of distancing from the prediction of the consensus. This is the factor associated with the presence of higher order beliefs that we also call beauty contest factor and whose weight is expressed by the parameter δ. The quadratic specification of the utility function ensures the linearity of the predictors' best responses and of the efficient allocations. Solving for e_{it}, we obtain that:

$$e_{it}(x_i; f; \rho_{\tilde{y}}; \rho_{x_i}) = (1-\delta)\mathbb{E}_i[s_t|x_{it}; f_t; \rho_{\tilde{y}}; \rho_{x_i}] + \delta\mathbb{E}[\bar{e}_t|x_{it}; f_t; \rho_{\tilde{y}}; \rho_{x_i}] \tag{21}$$

[22] See appendix AppendixA for the case of a continuum of investors.

which can be rewritten as:

$$e_{it}(x_i; f; \rho_{\tilde{y}}; \rho_{x_i}) = (1-\delta)\mathbb{E}_i[s_t|x_{it}; f_t; \rho_{\tilde{y}}; \rho_{x_i}] + \delta\frac{e_{it}}{n} + \delta\mathbb{E}_i[e_{-it}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_{x_i}] \tag{22}$$

where $\mathbb{E}_i[e_{-it}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_{x_i}] = \mathbb{E}[(\frac{e_{1t}+...+e_{i-1t}+e_{i+1t}+...e_{nt}}{n})|x_{it}; f_t; \rho_{\tilde{y}}; \rho_{x_i}]$. In the unique equilibrium with heterogeneous information, each individual $i \neq j$ at time t follows a linear strategy $e_{it} = e_t(x_t; f_t; \rho_{\tilde{y}}; \rho_{x_i})$ with:

$$e_t(x; f; \rho_{\tilde{y}}; \rho_{x_i}) = \varphi_x x_t + \varphi_{\tilde{y}}\tilde{y}_t \tag{23}$$

where $\tilde{y}_t = \mathbb{E}[s_t|f_t] = \frac{\rho_s s_{t-1}+\rho_f f_t}{\rho_s+\rho_f}$. According to this strategy, the predictor's expectation about the other $(n-1)$ agents is linear in $(s_t; f_t)$ and is given by:

$$\begin{aligned}
\mathbb{E}_i[e_{-it}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_{x_i}] &= \mathbb{E}_i[\varphi_x x_{-it} + \varphi_{\tilde{y}}\tilde{y}_t] \\
&= \varphi_x\mathbb{E}_i[s_t + \epsilon_{-it}] + \varphi_{\tilde{y}}\tilde{y}_t \\
&= \varphi_x\mathbb{E}_i[s_t] + \varphi_{\tilde{y}}\tilde{y}_t.
\end{aligned} \tag{24}$$

Plugging this expression into predictor's i best response,

$$e_{it} = (1-\delta)\mathbb{E}_i[s_t|x_{it}; f_t; \rho_{\tilde{y}}; \rho_{x_i}] + \delta\frac{e_{it}}{n} + \delta\frac{n-1}{n}\mathbb{E}_i[e_{-it}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_{x_i}], \tag{25}$$

and substituting the posterior mean of s_t, i.e., $\mathbb{E}_i(s_t) = \alpha_{x_i}x_{it} + \alpha_{\tilde{y}}\tilde{y}_t$, it follows that:

$$e_{it} = (1-\varrho+\varrho\varphi_x)\left[\frac{\rho_{x_i}}{\rho_{\tilde{y}}+\rho_{x_i}}x_{it} + \frac{\rho_{\tilde{y}}}{\rho_{\tilde{y}}+\rho_{x_i}}\tilde{y}_t\right] + \varrho\varphi_{\tilde{y}}\tilde{y}_t. \tag{26}$$

The coefficients $(\varphi_x; \varphi_{\tilde{y}})$ for the optimal linear strategy must satisfy:

$$\varphi_x = \frac{(1-\varrho)\rho_{x_i}}{(1-\varrho)\rho_{x_i}+\rho_{\tilde{y}}} \quad \text{and} \quad \varphi_{\tilde{y}} = \frac{\rho_{\tilde{y}}}{(1-\varrho)\rho_{x_i}+\rho_{\tilde{y}}} \tag{27}$$

as the unique solution of the system, with $\varrho = \frac{n\delta-\delta}{n-\delta}$. The solution of the social learning game relies on the individual expectation about next period exchange rate such that:

$$\mathbb{E}_{it}(s_{t+1}) = \varphi_x x_{it} + \varphi_y\tilde{y}_t, \tag{28}$$

where the sensitivity of the predictor's expectations to exchange rates is driven by two factors. First, as discussed above, the weight of the beauty contest factor, i.e., δ, identifies the importance assigned to the expectations of other predictors. Note that when $\delta = 0$, the best response is given by $e_{it} = \mathbb{E}[s_t|x_{it}; \tilde{m}_t; \rho_{\tilde{y}}; \rho_{x_i}]$ so that a predictor's optimal choice coincides with personal expectation. Higher values of δ induces the agent to take mainly into account public sources of information when making own prediction. Second, the sensitivity of predictor's expectations to the exchange rate depends on the quality of private and public signal in terms of precision. We learn that higher order beliefs places a greater weight to the public signal compared to the private one (Morris and Shin, 2002). Agents put less weight on their own private signal because the public signal acts as a coordinating device in order to predict the actions of others. Using the individual solution of the social learning game as from Eq. 28, we aggregate individual predictions among the n investors, such that:

$$\bar{\mathbb{E}}_t s_{t+1} = \varphi_x \bar{x}_t + \varphi_y \tilde{y}_t, \tag{29}$$

by substituting it in Eq.14, we obtain:

$$s_t = \lambda(\varphi_x \bar{x}_t + \varphi_{\tilde{y}} \tilde{y}) + (1 - \lambda) f_t - \lambda \psi_t. \tag{30}$$

Knowing that $\tilde{y}_t = \mathbb{E}[s_t|f_t] = \frac{\rho_s s_{t-1} + \rho_f f_t}{\rho_s + \rho_f}$ and rearranging, we get:

$$s_t = (1 - \lambda + \lambda \tau_2 \varphi_{\tilde{y}}) f_t + \lambda \varphi_x \bar{x}_t + \lambda \tau_1 \varphi_{\tilde{y}} s_{t-1} - \lambda \psi_t, \tag{31}$$

with $\tau_1 = \frac{\rho_s}{\rho_s + \rho_f}$ and $\tau_2 = \frac{\rho_f}{\rho_s + \rho_f}$ that can be reduced to

$$s_t = \beta_1 f_t + \beta_2 \bar{x}_t + \beta_3 s_{t-1} + \beta_4 \psi_t, \tag{32}$$

where $\beta_1 = (1 - \lambda + \lambda \tau_2 \varphi_{\tilde{y}})$, $\beta_2 = \lambda \varphi_x$, $\beta_3 = \lambda \tau_1 \varphi_{\tilde{y}}$ and $\beta_4 = -\lambda$. The first term indicates the role of fundamentals in the determination of exchange rate, the second term indicates the role of the strategic interaction of higher order beliefs, the third term represents the role of persistence, the fourth term finally figures out the role of liquidity trade.

2. Empirical Model

2.1. Methods and Data

According to the exchange rate model *à la* Bacchetta and Van Wincoop (2006) and introducing a strategic mechanism of interaction as in Ottaviani and Sørensen (2006), in Section 1.1 we built our economic framework to describe how expectations on exchange rates are formed in the context of a monetary macro model.

We can frame the structural link between the micro and the macro components of our setting as follows:

$$s_t = \beta_1 f_t + \beta_2 \bar{x}_t + \beta_3 s_{t-1} + \beta_4 \psi_t + \epsilon_{s,t} \tag{33}$$

$$f_t = \alpha_0 + \alpha_1 f_{1,t} + \alpha_2 f_{2,t} + \epsilon_{f,t} \tag{34}$$

$$f_{1,t} = \phi_{01} + \phi_{11} f_{1,t-1} + \phi_{12} f_{2,t-1} + \epsilon_{f_1,t} \tag{35}$$

$$f_{2,t} = \phi_{02} + \phi_{21} f_{1,t-1} + \phi_{22} f_{2,t-1} + \epsilon_{f_2,t} \tag{36}$$

$$\tilde{y}_t = \frac{\rho_f}{\rho_f + \rho_s} f_t + \left(1 - \frac{\rho_f}{\rho_f + \rho_s}\right) s_{t-1} \tag{37}$$

$$\psi_t = \rho_\psi \psi_{t-1} + \epsilon_{\psi,t} \tag{38}$$

$$\mathbb{E}_{it}[s_{t+1}] = \varphi_{\tilde{y}} \tilde{y}_t + \varphi_x x_{i,t}, \qquad i = 1, \ldots, N \tag{39}$$

$$x_{i,t} = x_{i,t-1} + \epsilon_{x_i,t}, \qquad i = 1, \ldots, N \tag{40}$$

where both the macro dynamics of the exchange rate and the structural parameters of the micro behavior are described. The shocks $\epsilon_t = (\epsilon_{s,t}, \epsilon_{f,t}, \epsilon_{f_1,t}, \epsilon_{f_2,t}, \epsilon_{\psi,t}, \epsilon_{x_i,t})$, $i = 1, \ldots, N$ are all Gaussian with mean zero and standard deviation respectively $\sigma_s, \sigma_f, \sigma_{f_1}, \sigma_{f_2}, \sigma_\psi$ and, $\sigma_{x_i,t}$, whereas N is the number of agents that make predictions on exchange rates.

It is worth noting that eq. (33) corresponds to the structural definition of the exchange rates given in (32) where the coefficients β_i are functions of the structural parameters. Equations (37) and (39) mimic the learning process defined in Section 1.1. In particular, eq. (37) describes the set of public informations in the market due to a combination of past observations on exchange rates and actual fundamentals. In turn, eq. (39) identifies the mechanism forming individual expectation as the mixed effect of private and public informations weighted respectively by $\varphi_x = \frac{(1-\varrho)\rho_x}{(1-\varrho)\rho_x + (\rho_s + \rho_f)}$ and $\varphi_{\tilde{y}} = \frac{\rho_f + \rho_s}{(1-\varrho)\rho_x + (\rho_s + \rho_f)}$ with $\varrho = \frac{\delta(N-1)}{N-\delta}$.

We define the dynamics of the fundamentals f_t, $f_{1,t}$, $f_{2,t}$ and the private information flows $x_{i,t}$. The fundamental f_t is assumed to be a linear combination of two observable factors $f_{i,t}$ with $i = 1, 2$, plus an error term. In particular, $f_{i,t}$ are random walks[23] and influence directly s_t and $\tilde{y}_t$, while, ψ_t is a sequence of IID shocks, i.e., we set $\rho_\psi = 0$ to be consistent with the theoretical setup defined in Section 1. The individual sets of private information $x_{i,t}$ are unobservable variables with a random walks structure. This assumption could be in principle relaxed although it is reasonable in this setup due to the non-stationary nature of the exchange rates, their expectations and most of the determinants (Engel and West, 2005). The model defined in (33-40) can be rewritten in compact form as

$$\Gamma_0 \mathbf{x}_t = c_x + \Gamma_1 \mathbf{x}_{t-1} + \Gamma_\epsilon \boldsymbol{\epsilon}_t \tag{41}$$

and in particular $\mathbf{x}_t = (s_t, f_t, f_{1,t}, f_{2,t}, \tilde{y}_t, \psi_t, \mathbb{E}_{it}, x_{i,t})$, $i = 1, \ldots, N$, while Γ_0, Γ_1 and Γ_ϵ are appropriate square matrices of parameters that define the system (33-40). By pre-multiplying eq. (41) with Γ_0^{-1} we get

$$\mathbf{x}_t = \Theta_c + \Theta_x \mathbf{x}_{t-1} + \Theta_\epsilon \boldsymbol{\epsilon}_t. \tag{42}$$

Some of the variables described through eqs. (41) are potentially unobservable. For our empirical analysis, we consider as observables the expectations $\mathbb{E}_{it}[s_{t+1}]$ which are represented by our dataset on heterogeneous survey forecasts concerning the actual exchange rates and the two fundamentals $f_{i,t}$, $i = 1, 2$, that is, $\hat{\mathbf{y}}_t = (\hat{s}_t, \hat{f}_{j,t}, \hat{\mathbb{E}}_{it}[s_{t+1}])$, with $j = 1, 2$ and $i = 1, \ldots, N$.[24] In the empirical analysis we evaluate the expectation for $N = 15$ institutions which represent the most influential companies providing predictions for exchange rates in the whole market.[25]

Data on expectations have been obtained from Foreign Exchange Consensus Forecasts ($FECF$). It is a a survey, namely, *Consensus Forecast of London*, in which, on the second Monday of every month, panelists are asked to forecast spot rates for the use against the euro over a range of time horizons. They are almost 250; and around 40 on average per each publication are identified individually with their names.

We refer to *individual forecast* as the forecast within the panel components whose identity is explicitly

[23] In our theoretical framework, this hypothesis is necessary to derive eq.(14). For this reason, we set $\phi_{11} = \phi_{22} = 1$ and $\phi_{21} = \phi_{12} = 0$.

[24] We use the symbol $\hat{}$ to distinguish *observed* from *theoretical* variables.

[25] For details about the exchange rate dealing and how this affects the market concentration in the foreign exchange predictions, see AppendixC.

indicated in the publication. In particular we focus on one-month-ahead forecasts from January 2006 to June 2012. Since the number and the identity of forecasters is not constant, we manage a database implementing a *conservative* approach. First we collect the predictions of the institutions that appeared at least once among the individual forecasts in the time span considered. Then, we assemble all available forecasts, while recording a missing value when the predictor was either absent in the panel for that month publication or when the company prediction for the month was indicated as '*na*'. Consequently we obtain individual predictions for 15 institutions as described in AppendixC.

Macroeconomic variables have been downloaded from Datastream. Following Fratzscher et al. (2012), monthly spot exchange rates on *usd* to euro have been computed by averaging over the month daily observations from December 2005 to August 2012.

Related to the macroeconomic model in Section 1, fundamentals are $f_{1,t} = m_t - m_t^*$ and $f_{2,t} = y_t - y_t^*$. In particular $f_{1,t}$ is the difference of the logarithms of the money supply measured by the variable $M2$ for *usd* and euro at a monthly frequency, whereas $f_{2,t}$ is the difference of the logarithm of the GDP of USA and euro area. Quarterly data on GDP have been disaggregated to a monthly frequency using the methodology described by Proietti (2006). Following Golinelli and Parigi (2008), we used as leading indicators, long term interest rates (per cent per annum), harmonized unemployment rate, retail trade and industrial production.[26]

We thus consider the following measurement equations to link our theoretical model to the real-world economy:

$$\hat{\mathbf{y}}_t = S\mathbf{x}_t \tag{43}$$

where S is a selection matrix that links the actual data set to the macroeconomic model. Equations (42-43) represent a linear and Gaussian *state-space* system for which the likelihood can be computed in closed form through the Kalman filter. In particular, eq. (42) represents the latent structure of the model, or transition equation, while (43) is the so called measurement equation. It is worth noting that our database on subjective forecasts is affected by missing values. However this is not a relevant problem, since the Kalman filter predicts missing data and allows for the computation of the likelihood function in a natural way (see Koopman et al., 1999 for a treatment on this point).

In order to deal with non-stationary observed data on exchange rates, it is appropriate to use first

[26]See AppendixC for details on data for macroeconomic variables.

differences. To take into account this transformation, it is easy to redefine the model as follows

$$\Delta\hat{\mathbf{y}}_t \equiv \begin{bmatrix} \Delta\hat{s}_t \\ \Delta\hat{f}_{1,t} \\ \Delta\hat{f}_{2,t} \\ \Delta\hat{\mathbb{E}}_{it}[s_{t+1}] \end{bmatrix} + \begin{bmatrix} s_t - s_{t-1} \\ f_{1,t} - f_{1,t-1} \\ f_{2,t} - f_{2,t-1} \\ \mathbb{E}_{it}[s_{t+1}] - \mathbb{E}_{it-1}[s_t] \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \gamma_i \end{bmatrix}, \ i = 1, \ldots, N \tag{44}$$

where γ_i are Gaussian measurement errors with standard deviation σ_{E_i} that might affect the observables. Notice that in Eq.44 we included also the lags of the observed variables which are not specified in the vector $\mathbf{x}_t$. To fix the problem it is possible to generalize the vector of the variables of the model as follows

$$\tilde{\boldsymbol{x}}_t = (s_t, f_t, f_{1,t}, f_{2,t}, \tilde{y}_t, \psi_t, \mathbb{E}_{it}[s_{t+1}], x_{i,t}, s_{t-1}, f_{1,t-1}, f_{2,t-1}, \mathbb{E}_{it-1}[s_t]), \quad i = 1, \ldots, 15 \tag{45}$$

to finally obtain the reduced form which reads

$$\begin{aligned} \Delta\hat{\mathbf{y}}_t &= \tilde{S}\tilde{\boldsymbol{x}}_t + \tilde{\boldsymbol{\gamma}}_t \\ \tilde{\boldsymbol{x}}_t &= \tilde{\Theta}_c + \tilde{\Theta}_x\tilde{\boldsymbol{x}}_{t-1} + \tilde{\Theta}_\epsilon\tilde{\boldsymbol{\epsilon}}_t. \end{aligned} \tag{46}$$

2.2. Prior distributions and inferential methods

Our main goal is to jointly estimate the structural and the reduced form parameters of the model. Our first interest is to capture the effect of higher order beliefs on the dynamics of the rate. This is identified by the weight δ in the decision process of the individual predictor. Our second task is to measure the role of private and public informations to determine the actual expectation. In order to do that we need to explore the coefficients φ_x and $\varphi_{\tilde{y}}$ obtained from (27).

In the theoretical model of Section 1, we show that the coefficient φ_x measures the relevance of private information in the formation process of expectations, while, $\varphi_{\tilde{y}}$ indicates the relevance of public information. There is also an influence of the value of δ on the dimensions of φ_x and $\varphi_{\tilde{y}}$. The higher the value of δ, the greater the weight associated to the public signal with respect to the private one.

We recur to Bayesian estimation methods here, and in particular to Markov chain Monte Carlo algorithms ($MCMC$), which have proved to be successful in the empirical macroeconomic literature (Kim and Pagan, 1995; Canova, 2007). This task can be easily handled using a Random Walk Metropolis

Hastings algorithm.[27] All of the calculations in this paper are based on software written using the Ox© 7.0 language of Doornik (2001) combined with the state space library `ssfpack` of Koopman et al. (1999).

Our prior choices on the parameters are summarized in Tables 1 to 3. Overall, we considered prior densities that match the domain of the structural parameters. In particular, we select a prior distribution for the delta parameter equal to 0.5 (and standard deviation 0.1), consequently assigning an equal weight to the two incentives present in the decision function of our predictors (eq. 20).

A priori, we assume that public and private informations play the same role when agents form their own expectations, that is to say without forcing the model to privilege some sources of information. This guess is consistent with the hypothesis that φ_x and $\varphi_{\tilde{y}}$ are equal. Since these weights depend on the precision coefficients ρ_f, ρ_s and ρ_x, we need to find prior distributions for them and at least on average, set $\varphi_x = \varphi_{\tilde{y}} = 0.5$. To obtain this result, we set the prior distributions for ρ_f and ρ_s as Gamma with mean 1 and standard deviation 0.1 whereas ρ_x is still Gamma, but with larger expected value, namely, 4 and standard deviation 0.4.[28]

The discount factor λ is a Beta variable with mean 0.5 and standard deviation 0.1. Furthermore, we assume a weakly informative prior for α_1 and α_2 that are both Gaussian with mean 0 and rather large variance with respect to the mean, i.e., 1. Finally the standard deviations of the shocks, including standard deviations of the measurement errors, are rather dispersed, and in particular their standard deviations are quite large with respect to the corresponding expected values. They are Inverse Gamma with mean 0.6 and standard deviation 0.2.

3. Posterior estimates and Policy Implications

Posterior estimates have been obtained by running again the $MCMC$ algorithm for 200.000 iterations with a burn-in of 10.000. That is an adequate choice to remove the dependence on the initial conditions. As usual in macro-econometrics (see An and Schorfheide, 2007), initial conditions have been obtained by maximizing the posterior mode for the parameters. Results are reported in Table 1 to 3.

In particular, Table 1 and Figure 1 include posterior estimates of the structural relevant parameters, namely posterior averages and credibility intervals, whereas Figure 1 displays prior versus posterior comparisons.

[27] See Robert and Casella (1999, ch. 6-7) for a general treatment on MCMC algorithms and Monte Carlo methods in general

[28] An extensive sensitivity analysis suggests that posterior estimates of φ_x and $\varphi_{\tilde{y}}$ are robust with respect to this choice.

Table 1: **Posterior computation (MCMC) - Structural parameters**

	Posterior distribution			Prior information		
	Parameter value	Mean	95% Cred. Int.	Mean	S.E.	Type
$p(\beta_1 \mid \hat{\boldsymbol{y}})$	$\beta_1 = 1 - \lambda + \lambda\varphi_{\tilde{y}} \frac{\rho_f}{\rho_f + \rho_s}$	0.2302	[0.170, 0.301]			
$p(\beta_2 \mid \hat{\boldsymbol{y}})$	$\beta_2 = \lambda\varphi_x$	0.6489	[0.559, 0.724]			
$p(\beta_3 \mid \hat{\boldsymbol{y}})$	$\beta_3 = \lambda\varphi_{\tilde{y}} \frac{\rho_s}{\rho_f + \rho_s}$	0.1209	[0.070, 0.177]			
$p(\beta_4 \mid \hat{\boldsymbol{y}})$	$\beta_4 = -\lambda$	-0.8561	[-0.916, -0.779]			
$p(\alpha_1 \mid \hat{\boldsymbol{y}})$		0.0842	[-0.716, 0.851]	0	1	Normal
$p(\alpha_2 \mid \hat{\boldsymbol{y}})$		-2.5237	[-3.724, -1.289]	0	1	Normal
$p(\rho_f \mid \hat{\boldsymbol{y}})$		0.8764	[0.706, 1.070]	1	0.1	Gamma
$p(\rho_s \mid \hat{\boldsymbol{y}})$		1.2342	[1.029, 1.454]	1	0.1	Gamma
$p(\rho_x \mid \hat{\boldsymbol{y}})$		3.618	[2.910, 4.394]	4	0.4	Gamma
$p(\varphi_x \mid \hat{\boldsymbol{y}})$	$\varphi_x = \frac{(1-\varrho)\rho_x}{(1-\varrho)\rho_x + (\rho_s + \rho_f)}$	0.2412	[0.148, 0.352]			
$p(\varphi_{\tilde{y}} \mid \hat{\boldsymbol{y}})$	$\varphi_{\tilde{y}} = \frac{\rho_f + \rho_s}{(1-\varrho)\rho_x + (\rho_s + \rho_f)}$	0.7588	[0.648, 0.852]			
$p(\sigma_s \mid \hat{\boldsymbol{y}})$		1.019	[0.468, 1.455]	0.6	0.16	Inv. Gamma
$p(\sigma_f \mid \hat{\boldsymbol{y}})$		9.2763	[6.986, 12.625]	0.6	0.16	Inv. Gamma
$p(\sigma_{f1} \mid \hat{\boldsymbol{y}})$		0.5854	[0.503, 0.683]	0.6	0.16	Inv. Gamma
$p(\sigma_{f2} \mid \hat{\boldsymbol{y}})$		0.9472	[0.812, 1.116]	0.6	0.16	Inv. Gamma
$p(\sigma_\phi \mid \hat{\boldsymbol{y}})$		0.8621	[0.408, 1.588]	0.6	0.16	Inv. Gamma
$p(\lambda \mid \hat{\boldsymbol{y}})$		0.8561	[0.779, 0.916]	0.5	0.1	Beta
$p(\delta \mid \hat{\boldsymbol{y}})$		0.8202	[0.694, 0.902]	0.5	0.1	Beta

The first interesting result relates to the value of the coefficient δ. A sensitive shift to the right of the posterior is observed in the comparison with its prior distribution confirming the important role of the beauty contest in the predictor's evaluation process. Individuals assign more weight than expected (82%) of interpreting correctly the other predictors' beliefs and a smaller weight (18%) to the cost of making forecast error with respect to the fundamentals. The rational incentives of predictors are therefore distorted. The value worth of the consensus is definitely higher than the option of making the right choice on the basis of their own private information.

The result is relevant, not only for the correct comprehension of the micro-behavior of predictors, but also because it helps to explain the persistent presence of inefficiencies in predictions markets. This is confirmed by a long strand of literature on the inefficiency of survey expectations for exchange rates (Cavaglia et al., 1993; Ito, 1990; Frankel and Froot, 1990; MacDonald and Ian, 1996; Mitchell and Pearce, 2007; Pancotto et al., 2014).

Higher cost in the wrong choice on the consensus's evaluation leads to an optimization process with apparently odd conclusions. Predictors may in principle persist in incorrect predictions compared to

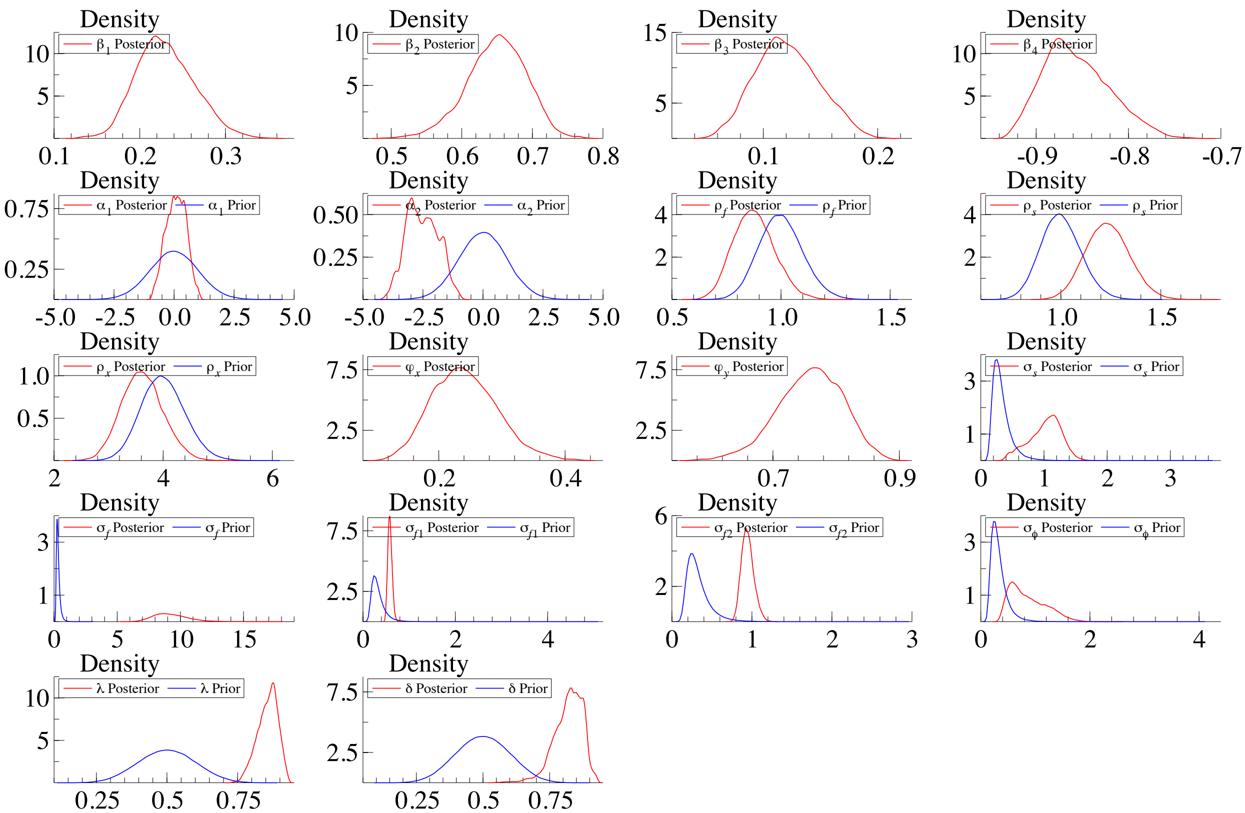

Figure 1: Prior vs. Posterior distributions of the structural parameters

the actual market realization since their choice is in line with what they expect the consensus will do. When this incentive prevails, a potential bias at the aggregate level could be observed in forecasting the exchange rates. A second relevant point is the role that public and private information have on individual forecast. Our analysis is based on the coefficients φ_x and $\varphi_{\tilde{y}}$. Results suggest that public informations accounts for about 75 percent of the predictions, whereas just 25 percent depends on private informations. This is coherent with the previous result related to the weight associated with higher order beliefs. The combination between higher order beliefs and information structure ensures rather a rational behavior in the decision problem. When agents care more about the consensus prediction rather than their own personal assessment, they reduce implicitly the importance of their private signal while considering more the public one. Robustness checks suggest that this phenomenon is always verified despite ex-ante values of parameters φ_x and $\varphi_{\tilde{y}}$ are assumed to be equal and independent by the precision of both signals. Although the precision of the private signal ρ_x is higher than the precisions of the public one, i.e., ρ_f and ρ_s, the role of higher order beliefs is largely confirmed. Public information in a beauty contest environment, therefore, acts as a coordinating device. This is a central result, firstly, proposed by Morris and Shin (2002), that we have intentionally integrated in our framework to test its presence and intensity in the context of the exchange rate market. Furthermore, it also furnishes a complementary result to the empirical test of the scapegoat model of Bacchetta and Van Wincoop (2004) implemented by Fratzscher et al. (2012). They discover that using survey predictions on fundamentals as proxies for scapegoat effects increases the ability to explain exchange rate movements. Public information is, therefore, capable of capturing changes in the actual dynamics of the rate. Our result is closely related to these points. First, we estimate the intensity of beauty contests factor and, as theoretically predicted, we discover that the importance of the role of higher order beliefs is associated to a greater weight assigned on public information. Second, using the result of Fratzscher et al. (2012), we infer that public information that agents overrate can be a distorted information. The predictors are rationally searching for *fundamental information*, but then they end up in overweighting a public information that is not informative. This is clearly due on one side to the presence of higher order beliefs and on the other side to the uncertainty that the heterogeneity of expectations conveys. The fundamental is therefore transformed into a scapegoat in case of uncertainty about the structural parameters. In particular, the high value of the variance of the fundamental, $p(\sigma_f|\hat{\boldsymbol{y}})$ (around 9) in table 1 suggests a coherence between the short term uncertainty suggested by Fratzscher et al. (2012) and the one derived by the fundamental movements. This exactly generates the scapegoat effect discussed by Bacchetta and Van Wincoop (2004).

3.1. Robustness checks and goodness-of-fit

In this section we evaluate the performance of our model against the data. First we compare our model with a rational expectation dynamics. As a benchmark, we consider the rational expectation model that closely mimics the dynamics defined in eq. 14. In particular we consider

$$s_t = \lambda \mathbb{E}_t[s_{t+1}] + (1-\lambda) f_t - \lambda \psi_t + \epsilon_t, \tag{47}$$

in which f_t is described in 34, while ψ_t is an independent and identically distributed sequence. Furthermore, rational expectations are defined such that $\mathbb{E}_t[s_{t+1}] = s_t + \eta_t$, where η_t is a gaussian shock with mean zero and constant variance. Rational expectation dynamics on exchange rates has been estimated through MCMC. In particular for each posterior draw of the parameters, we solved the rational expectation system using Sims (2002) and implementing it with the Ox package `LiRE` of Mavroeidis and Zwols (2007). Then for each parameter, we simulated the one-step-ahead prediction of the rational expectation dynamics. Figure 2 compares the average rational expectations trajectory and the average forecast from

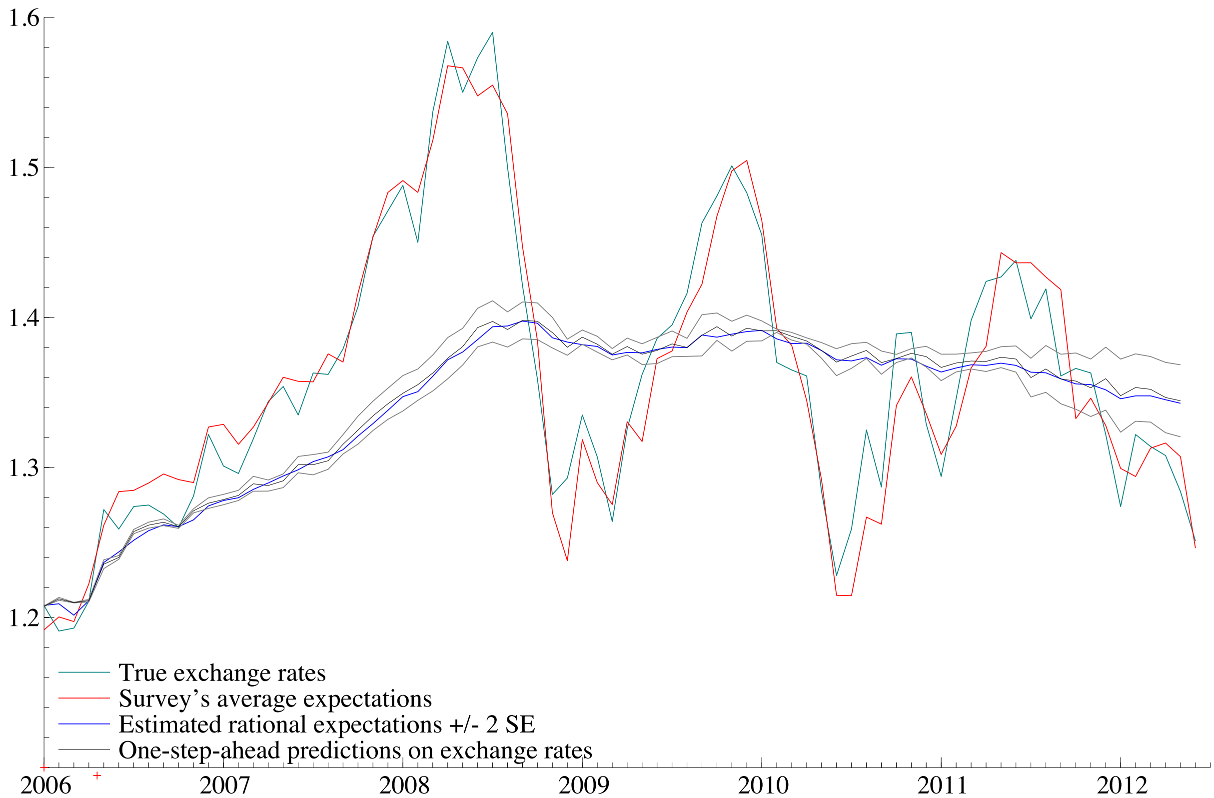

Figure 2: Actual exchange rates (green line), average form the survey (red line) together with an estimate of the rational expectation on exchange rates $\mathrm{E}_t[s_{t+1}]$ (blue line) and the one-step-ahead prediction on s_t (black line)
.

the survey[29]. Figure 2 points out the predicted average expectations. They differ substantially from the observed ones and it emerges that the predicted exchange rates do not replicate at all the dynamics of the actual exchange rates.

[29]Here rational expectations has been computed as an average trajectory compared to the posterior draws of the model's parameters.

This empirical evidence can be considered as a symptom that rational expectations fail in providing a good fit to actual data. This is the reason why using survey data appears to be a viable strategy to provide a better description of actual exchange rates. As a second exercise we thus computed the predicted exchange rates and exchange rate returns. Figure 3 displays one-step-ahead prediction for exchange rates and for exchange rates returns.

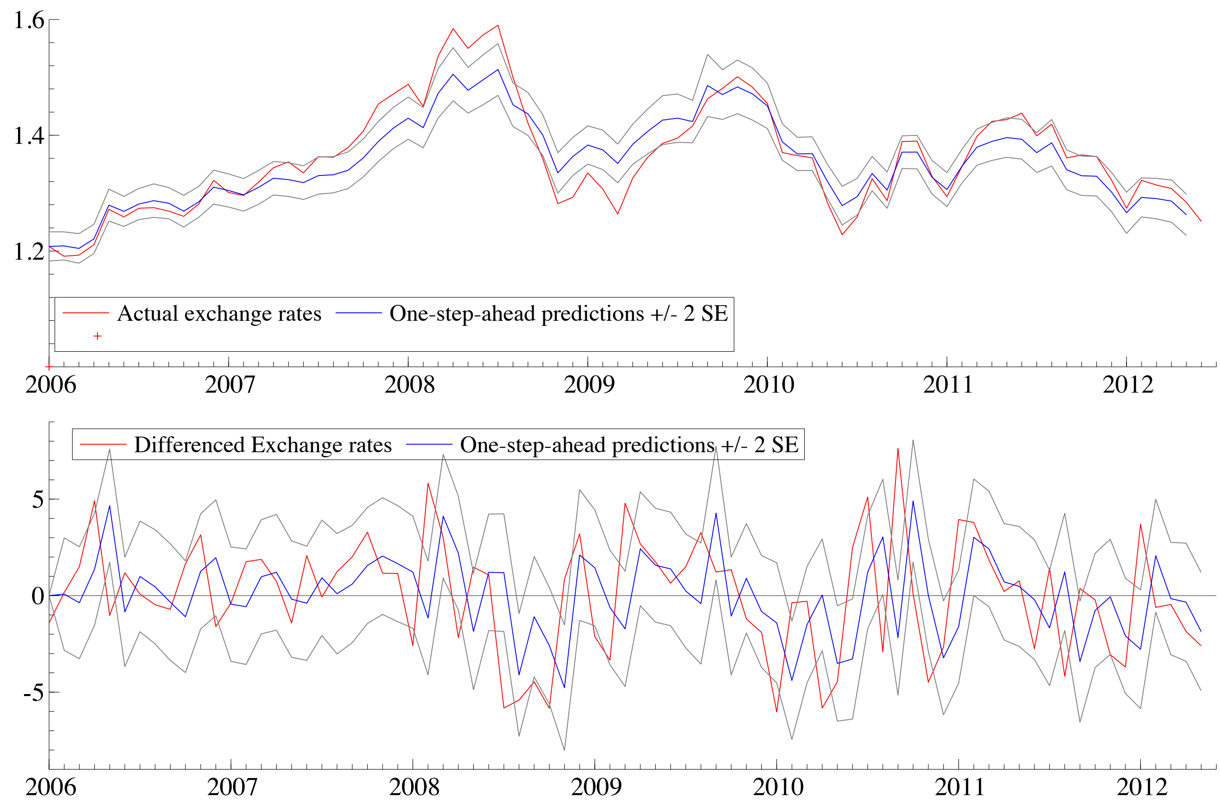

Figure 3: Upper panel: Actual exchange rates (red line) vs. predicted exchange rates (blue line) together with 95% credibility bands. Lower panel: Actual exchange rates returns (red line) vs. predicted exchange rates returns (blue line) together with 95% credibility bands.

In this case, actual data are replicated quite accurately by the model. In fact more than 70% of actual exchange rates fall in the 95% credibility interval. In particular predicted estimates provide a good proxy for the trending behavior of the true data. Finally, we investigate the role of measurement errors in the model. We thus compare the model with measurement errors together with the estimated model. We evaluate the goodness-of-fit through the marginal likelihood estimated as the harmonic means of the likelihood function evaluated for each posterior draw of the parameter vector (see An and Schorfheide, 2007 for this point). The log-marginal likelihoods for the model with and without measurement errors are respectively -2349.2 and the -2359.7, which imply a Bayes factor of about e^{10} in favor of the former model. This evidence suggests a strong rejection of the model with no measurement errors.

As a further exercise, we evaluate the Forecast Error Variance Decomposition (FEVD) for the observable variables, and in particular for the exchange rates, to evaluate the relevance of the measurement errors on the variability of the dependent variable.

TO BE ADDED

4. Conclusions

The paper has investigated the dynamics of the exchange rate according to a social learning game with incomplete information and heterogeneous expectations. The baseline specifications enclose an empirical test of higher order beliefs theory (Bacchetta and Van Wincoop (2006)) using survey data from Consensus Economics of London on exchange rates forecasts.

Our goal was twofold: first, to verify whether a mechanism of higher order beliefs exists in the performance of the exchange rate expectations trying to identify the optimization process of each forecaster in the market. Secondly, exploiting the scapegoat theory of exchange rates (Bacchetta and Van Wincoop (2004)), we discussed the generating process of higher order beliefs (or beauty contest mechanism) and how this is related to public and private information on the basis of Morris and Shin (2002).

The framework has required a standard two countries macro model encompassing a social learning structure of the forecasts. The empirical analysis implements a state space model estimated through Bayesian methods. The formation of each forecast's expectations is generated by taking into account the expectations of the other participants in the market. This phenomenon is entirely dependent on the uncertainty of information. Public information in particular coordinates heterogeneous expectations of predictors overweighting signals from specific fundamentals. Predictors perceive them as the most prevalent factors to form their forecasts. The role of their own private assessments instead slowly fades away and this result is still robust in the case of higher precision of private information. Therefore, we have shown that rational behavior of predictors can lead to systematically biased predictions when uncertainty of fundamentals is present and heterogeneity of predictions cannot be denied.

Table 2: **Posterior computation (MCMC) - Other parameters**

	Posterior distribution		Prior information		
	Mean	95% Cred. Int.	Mean	S.E.	Type
$p(\sigma_{x_1}\|\hat{\boldsymbol{y}})$	0.6314	[0.395, 1.081]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_2}\|\hat{\boldsymbol{y}})$	0.6096	[0.393, 0.972]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_3}\|\hat{\boldsymbol{y}})$	0.6044	[0.375, 0.975]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_4}\|\hat{\boldsymbol{y}})$	0.6098	[0.392, 0.977]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_5}\|\hat{\boldsymbol{y}})$	0.6467	[0.393, 1.104]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_6}\|\hat{\boldsymbol{y}})$	0.5871	[0.376, 0.958]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_7}\|\hat{\boldsymbol{y}})$	0.5902	[0.372, 1.125]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_8}\|\hat{\boldsymbol{y}})$	0.6077	[0.371, 1.104]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_9}\|\hat{\boldsymbol{y}})$	0.5806	[0.372, 0.934]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{10}}\|\hat{\boldsymbol{y}})$	0.6039	[0.386, 0.963]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{11}}\|\hat{\boldsymbol{y}})$	0.6247	[0.381, 1.017]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{12}}\|\hat{\boldsymbol{y}})$	0.6142	[0.380, 1.056]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{13}}\|\hat{\boldsymbol{y}})$	0.5919	[0.391, 0.913]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{14}}\|\hat{\boldsymbol{y}})$	0.6243	[0.404, 0.983]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{15}}\|\hat{\boldsymbol{y}})$	0.6564	[0.406, 1.183]	0.6	0.16	Inv. Gamma

Table 3: **Posterior computation (MCMC) - Measurement Errors**

	Posterior distribution		Prior information		
	Mean	95% Cred. Int.	Mean	S.E.	Type
$p(\sigma_{E_1}\|\hat{\boldsymbol{y}})$	1.4055	[1.170, 1.686]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_2}\|\hat{\boldsymbol{y}})$	1.7511	[1.386, 2.204]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_3}\|\hat{\boldsymbol{y}})$	2.5709	[2.162, 3.071]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_4}\|\hat{\boldsymbol{y}})$	3.9357	[3.343, 4.640]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_5}\|\hat{\boldsymbol{y}})$	0.867	[0.604, 1.218]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_6}\|\hat{\boldsymbol{y}})$	3.3454	[2.844, 3.961]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_7}\|\hat{\boldsymbol{y}})$	2.8154	[2.346, 3.380]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_8}\|\hat{\boldsymbol{y}})$	1.5442	[1.293, 1.841]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_9}\|\hat{\boldsymbol{y}})$	2.6808	[2.277, 3.157]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{10}}\|\hat{\boldsymbol{y}})$	1.9843	[1.693, 2.337]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{11}}\|\hat{\boldsymbol{y}})$	2.9299	[2.456, 3.525]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{12}}\|\hat{\boldsymbol{y}})$	2.9091	[2.450, 3.444]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{13}}\|\hat{\boldsymbol{y}})$	2.5205	[2.148, 2.969]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{14}}\|\hat{\boldsymbol{y}})$	2.9644	[2.542, 3.491]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{15}}\|\hat{\boldsymbol{y}})$	1.687	[1.410, 2.014]	0.6	0.16	Inv. Gamma

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AppendixA. Continuous case

Let us assume now a continuum of agents in the society. As before each predictor's preferences are generically characterized by:

$$U(e_{it}, \bar{e}_t, \sigma_e^2, s_t) \tag{A.1}$$

We have verified the existence of a (unique) linear equilibrium strategy expressed by eq. 27 in case of a finite numbers of investors. Here we show that the linear equilibrium is also unique in the continuous case. The best response of investor$-i$ is:

$$e_{it}(x_i; f; \rho_{\tilde{y}}; \rho_{x_i}) = (1-\delta)\mathbb{E}_i[s_t|x_{it}; f_t; \rho_{\tilde{y}}; \rho_{x_i}] + \delta\mathbb{E}_i \int e_j dj | x_{it}; f_t; \rho_{\tilde{y}}; \rho_{x_i}] \tag{A.2}$$

To make the notation as simple as possible, let us denote $\mathbb{E}_i[.|x_{it}; f_t; \rho_{\tilde{y}}; \rho_{x_i}] = \mathbb{E}_i[.]$. We can start iterating forward such that:

$$\begin{aligned}
e_{it} &= (1-\delta)\mathbb{E}_i[s_t] + \delta\mathbb{E}_i\left[(1-\delta)\int \mathbb{E}_j[s_{t+1}]dj + \delta\int \mathbb{E}_j[\bar{e}_t]dj\right] \\
&= (1-\delta)\mathbb{E}_i[s_t] + \delta(1-\delta)\mathbb{E}_i\bar{\mathbb{E}}[s_{t+1}] + \delta^2\mathbb{E}_{it}\int \mathbb{E}_j[\bar{e}_t]dj \\
&= \vdots \\
&= (1-\delta)\mathbb{E}_i[s_t] + \delta(1-\delta)\mathbb{E}_i\bar{\mathbb{E}}_t s_{t+1} + \delta^2(1-\delta)\mathbb{E}_i\bar{\mathbb{E}}_t^{(2)} s_{t+1} + \ldots
\end{aligned}$$

$\bar{\mathbb{E}}_t s_{t+1}$ is the average expectations operator given by:

$$\bar{\mathbb{E}}[\cdot] \equiv \int \mathbb{E}_j[\cdot]dj$$

$$\bar{\mathbb{E}}_t^{(k)}[\cdot] \equiv \int \mathbb{E}_j\bar{\mathbb{E}}^{(k-1)}[\cdot]dj = \bar{\mathbb{E}}_t\bar{\mathbb{E}}_t{}^{(k-1)}[\cdot]$$

where $\bar{\mathbb{E}}^{(k)}[\cdot]$ denote the average expectation of order k. We get the optimal action of any agent as a simple geometric sum of higher order beliefs:

$$e_{it} = (1-\delta)\sum\nolimits_{k=0}^{\infty} \delta^k \mathbb{E}_i[\bar{\mathbb{E}}_t^{(k)} s_{t+1}] \tag{A.3}$$

Thus higher level of δ implicitly reveals a greater importance of the weight assigned to the higher order beliefs. To check if the infinite sum is bounded, we need to solve for $\mathbb{E}_i[\bar{\mathbb{E}}^{(k)} s_{t+1}]$. Since the investor$-i$'s expected value of s_t is given by eq. 28, i.e.,

$$\mathbb{E}_{it}(s_{t+1}) = \varphi_x x_{it} + \varphi_y \tilde{y}_t$$

while the average expectation across predictors is equal to 29, i.e.,

$$\bar{\mathbb{E}}_t s_{t+1} = \varphi_x \bar{x}_t + \varphi_y \tilde{y}_t$$

We can rewrite the expectation of $\bar{\mathbb{E}}_t s_{t+1}$ for each investor$-i$ as:

$$\begin{aligned} \mathbb{E}_{it}\bar{\mathbb{E}}_t s_{t+1} &= \varphi_x \mathbb{E}_{it}[\bar{x}_{t+1}] + \varphi_y \tilde{y}_t \\ &= \varphi_x \mathbb{E}_{it}[s_{t+1} + \epsilon_{it+1}] + \varphi_y \tilde{y}_t \end{aligned}$$

Then since $\epsilon_{it+1} \sim N(0, \sigma_\epsilon^2)$ and due to symmetry ($\bar{s}_{t+1} = s_{t+1}$, $\forall i$), we easily rewrite:

$$\begin{aligned} \mathbb{E}_{it}\bar{\mathbb{E}}_t s_{t+1} &= \varphi_x \mathbb{E}_{it}[s_{t+1}] + \varphi_y \tilde{y}_t \\ &= \varphi_x \left(\varphi_x x_{it} + \varphi_y \tilde{y}_t\right) + \varphi_y \tilde{y}_t = \\ &= \varphi_x^2 x_{it} + \varphi_y^2 \tilde{y}_t \end{aligned}$$

this implies that the average expectation of $\bar{\mathbb{E}}_t s_{t+1}$ is given by:

$$\bar{\mathbb{E}}_t^{(2)} s_{t+1} = \bar{\mathbb{E}}_t \bar{\mathbb{E}}_t s_{t+1} = \varphi_x^2 \bar{x}_t + \varphi_y^2 \tilde{y}_t$$

Iterating the procedure for any k,

$$\bar{\mathbb{E}}_t^{(k)} s_{t+1} = \varphi_x^k \bar{x}_t + \varphi_y^k \tilde{y}_t$$

$$\mathbb{E}_{it}\bar{\mathbb{E}}_t^{(k)} s_{t+1} = \varphi_x^{k+1} x_{it} + \varphi_y^{k+1} \tilde{y}_t \tag{A.4}$$

Now substituting eq. (A.4) to eq. (A.3), we obtain that:

$$
\begin{aligned}
e_{it} &= (1-\delta)\sum_{k=0}^{\infty}\delta^k\left(\varphi_x^{k+1}x_{it}+\varphi_y^{k+1}\tilde{y}_t\right)\\
&= \frac{(1-\delta)}{1-\delta\varphi_x}\varphi_x x_{it}+\left(1-\frac{(1-\delta)}{1-\delta\varphi_x}\right)\varphi_y\tilde{y}_t
\end{aligned}
$$

while simplyfing,

$$
e_{it}=\frac{(1-\varrho)\rho_{x_i}}{(1-\varrho)\rho_{x_i}+\rho_{\tilde{y}}}x_{it}+\frac{\rho_{\tilde{y}}}{(1-\varrho)\rho_{x_i}+\rho_{\tilde{y}}}\tilde{y}_t
$$

which is exactly the linear equilibrium in a continuum of agents.

AppendixB. MCMC algorithm

The intuition behind MCMC is to build a Markov chain transition kernel starting from a given initial point and with limiting invariant distribution equal to the posterior distribution of the interested quantities. Under suitable conditions (seeRobert and Casella, 1999, chap. 6-7), such a transition kernel converges in distribution to the target posterior density $p(\boldsymbol{\theta}|\mathbf{y})$. This Markov chain trajectories are obtained through simulations on the basis of two-steps procedure. First, a new movement is proposed by simulating the new position from a proposal distribution, and second, this move is accepted or rejected according to some suitable probabilities that depend on the likelihood function and on the prior distribution of the parameters $p(\boldsymbol{\theta})$. In a nutshell, given a starting value for the parameter's vector $\boldsymbol{\theta}^{(0)}$, we simulate trajectories of the Markov chain $\{\boldsymbol{\theta}^{(j)},\ j=1,\ldots,n\}$ whose draws converge to the posterior distribution. Once convergence is achieved, inference can be based on the generated serially dependent sample simulated from the posterior. More precisely, estimates of the posterior means $\mathrm{E}_{p(\boldsymbol{\theta}|\mathbf{y})}[\boldsymbol{\theta}]$ are obtained by averaging over the realization of the chains, i.e., $\hat{\boldsymbol{\theta}}=n^{-1}\sum_{j=1}^{n}\boldsymbol{\theta}^{(j)}$. To account for serial correlation induced by the markovian nature of this procedure, we estimate the numerical standard error of the sample posterior mean using the approach implemented, for instance, by Kim et al. (1998). In this application, $\boldsymbol{\theta}$ are the structural parameters of the model including, amongst others λ, δ, ρ_{x_i}, ρ_f and ρ_s.

In the MCMC literature, there are many different ways to make a step from the Markov chain. Our inferential procedure is based on a Random Walk Metropolis-Hastings algorithm, that has been proved to be effective in the DSGE framework (An and Schorfheide, 2007), and in which the proposal distribution depends uniquely on the current state of the chain at time j, i.e., $q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(j)})$. Once parameters are updated, the exact likelihood function is evaluated through the Kalman filter to eqs. (42) -(44)

We thus propose through the random-walk step new values for these parameters and then we compute the reduced form to evaluate the exact likelihood.

The procedure can be summarized as follows

`MCMC algorithm`

- Initialize the chain at $\boldsymbol{\theta}^{(0)}$
- At step $j = 1, \ldots, n$
  - Update $\boldsymbol{\theta}$ in block through a random walk Metropolis-Hastings scheme

$$\boldsymbol{\theta}^* \sim q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(j-1)});$$

  - Compute $L(\mathbf{y}|\boldsymbol{\theta}^*)$ through Kalman filter;
  - Compute the acceptance probability $\alpha(\boldsymbol{\theta}^{(j-1)}, \boldsymbol{\theta}^*)$ defined as

$$\alpha(\boldsymbol{\theta}^{(j-1)}, \boldsymbol{\theta}^*) = \frac{p(\boldsymbol{\theta}^*)L(\mathbf{y}|\boldsymbol{\theta}^*)q(\boldsymbol{\theta}^{(\boldsymbol{j-1})}|\boldsymbol{\theta}^*)}{p(\boldsymbol{\theta}^{(j-1)})L(\mathbf{y}|\boldsymbol{\theta}^{(j-1)})q(\boldsymbol{\theta}^{\boldsymbol{*}}|\boldsymbol{\theta}^{(j-1)})}$$

  - Draw u from an $U(0,1)$ random variable. If $\alpha(\boldsymbol{\theta}^{(j-1)}, \boldsymbol{\theta}^*) \leq u$
    - Then $\boldsymbol{\theta}^{(j)} = \boldsymbol{\theta}^*$;
    - Else $\boldsymbol{\theta}^{(j)} = \boldsymbol{\theta}^{(j-1)}$;
- $j = j + 1$

AppendixC. Foreign exchange Consensus Survey data

On the second Monday of every month, Foreign Exchange Consensus Forecasts (FECF) asks their panelists to forecast spot rates for the use against the euro over a range of time horizons. Each panelist provides a series of forecasts with different maturities: one that refers to the next month, one to the next quarter, one to the next year and one to the next two years. Panelists are almost 250, and around 40 per each publication on average are identified individually with their names. Nonetheless, FECF reports in the publication the *Consensus forecast* which is the mean of all forecasts received by the company, including those panelists whose names are not explicitly indicated. The forecasts of the panelists whose names are not indicated explicitly are polled into an average forecast that is reported as well and is defined in the publication as *Other Forecasters*. Let us refer to the *individual forecasts* as the forecasts of the components of the panel whose identity is explicitly indicated in the publication.[30]

The companies in the poll are indicated with their names in the publication and listed in descending order of their 1 year percentage change estimates. Consequently, the order in which the panelists appear in the list may change every month. Moreover, the composition of the panelists whose names appear explicitly is not always the same. There are institutions that provide predictions in almost every publication in the time span considered, while some others provide a lower frequency. There are also cases in which the company is present in the list but the forecast is not provided. This is indicated with the label *na* in the value for the forecast.[31]. The names of the companies included in our database are presented in Table C.4.

Forex trading has shown a very fast modernization in the last years fundamentally based on electronic trading that has led to a complete restructuring of forex market. Large banks had the necessary resources to develop sophisticated proprietary trading platforms through which satisfy their own customers and also provide trading service to smaller banks, which have mostly withdrawn from the market due to the high costs associated with the investment in these platforms. Small banks have nonetheless maintained their ability to provide liquidity to their customers using the proprietary platforms of the large banks but under their names. This procedure is called *white labeling* and is certainly efficient for the market functioning but has also driven a substantial concentration of market information. In this case large banks can observe directly small banks' trading flows and extract from these data possibly relevant information. The concentration driven by the phenomenon of white labeling is striking since the three larger bank trading platforms account for 70% of the total market share.[32]

Other variables are reported in the same panel where forecasts are indicated: the annual percentage change of the consensus rate and the discount (or premium) of the survey consensus forecast with respect

[30]We refer to *Consensus forecasts* and *Other forecasters* indicating the same group of forecasters in the publication as previously described.

[31]When we created our database we considered both the lack of a prediction of the company for that month and of the presence of the company in the list as *NA*

[32]See James et al. (2012), page 30, about the Euromoney FX Survey, where the list of all the institutions with a trading platform is reported.

Table C.4: Predictions of individual forecasters: missing observations in absolute value and percentage on the total number of observations in the sample. Last column: companies with proprietary trading platforms (Euromoney FX survey).

	n missing	% on total	white label
Bank.of.Tokio.Mitsubushi	1	1	
Barclays.Capital	31	40	yes
BNP.Paribas	7	9	
BoA.Merril.Linch	4	5	
Citygroup	47	60	yes
Commerzbank	4	5	
Deutsche.Bank.Research	18	23	yes
General.Motors	0	0	
HSBC	1	1	yes
IHS.Global.Insight	0	0	
JPMorgan	10	13	yes
Oxford.Economics	7	9	
Royal.Bank.of.Canada	0	0	
UBS	3	4	yes
WestLB	3	4	

to the spot rate at the date of the forecast. In the same page, the FECF presents data on Current Account Balances in usd and consensus forecast on current account balances for major countries of the euro area.[33] Country risk indicators are also reported for information to the subscribers: current account and budget balance (in % of GDP) for the current year and the prediction for the next year; public debt of the preceding year (in % of GDP) and sovereign debt ratings of both Moody's and S&P. Two figures are presented to the readers: in the first, the usd per euro exchange rate together with usd less euro 3 Month interest rate futures differential contracts(as of the end of the following quarter). In the second, the Yen/euro cross rate to explore currency linkages with the japanese currency. The yen per usd exchange rate is the only other currency for which the prediction of the panelists appears in detail in the publication every month.

AppendixC.1. Data availability and database construction

We study monthly forecasts of the usd against the euro exchange rate for a period ranging from January 2006 to June 2012, which amounts to 78 monthly predictions over this time span. As previously mentioned, the number and the identity of forecasters is not constant so we had to build a database using the information of the individual forecast in a constructive way. We implemented a conservative approach.

[33]In detail: Germany, France, Italy, Austria, Belgium, Finland, Greece, Ireland, Netherlands, Portugal, Spain, and Euro zone.

We collected the predictions of the institutions that appeared at least once among the *individual forecasts* in the time span considered. Then, we collected all available forecasts and recorded a missing value the occurrence when the predictor was either non present in the panel for that month publication or when the company prediction for the month was indicated as 'na'. Consequently we obtained individual predictions for 15 institutions:

AppendixC.2. Data on fundamentals

The data source for all macro data is Datastream.

Spot exchange rate data:

We collected data to euro exchange rate at daily frequency from december 2005 to august 2012. To obtain a monthly series for the exchange rate we implemented the choices of Fratzscher et al. (2012). We use nominal bilateral exchange rate changes vis-a-vis the reference currency, in the benchmark specification using changes over the past month. Since we know the exact day when the surveys were conducted, these exchange rate changes are calculated relative to the exchange rate of the previous business day.

GDP: data on GDP are at quarterly frequency. We need to disaggregate them in order to have a monthly frequency that matches the survey frequency. To obtain the monthly data we use the following macroeconomic indicators following Golinelli and Parigi: Long term interest rate (per cent per annum), harmonized unemployment rate; Retail trade; Industrial production. For the money supply the difference of the log of M2 for US $ and euro (frequency).