Workers Self-Selection between Workplaces and Employment Protection Legislation

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Abstract

This model explains the self-selection of workers between workplaces which differ in the return to workers' skill and the level of job security. Incomplete information in the market enables the reallocation-process in which each worker chooses her best job. The model also clarifies the distortion of the efficiency as a result of providing workers with employment protection legislation (EPL). The simulation results provide some instruments to quantify the self-selection, to estimate the EPL damage in terms of labor productivity, to quantify the tradeoff between job security and the level of the wage, and to measure the effects of policy decisions.

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1. Introduction

This paper uses search theory tools, assigning and reallocation to create a partial equilibrium model and to test the effect of certain states on labor market outcomes. Unlike the majority of the literature, the model focuses on worker behavior and simplifies firm behavior as much as possible. The main objective of this note is to analyze the effect of employment protection legislation (EPL) on some market outcomes and especially on the workers'-skills-selection between workplaces. While previous researchers have accomplished this by focusing on firm behavior (particularly by job creation and job destruction), this paper concentrates on worker behavior.

Incomplete information creates unemployment and assignment problems during the entrance to the labor market and thus leads to the reallocation of workers, i.e. job-to-job transitions, over time. Firms determine wage levels in order to maximize profit given workers' bargaining power. The economy's output is a function of productivity and the quantity of labor. Each firm has an idiosyncratic technology which is a positive function of the workers' productivity.

Each worker has his own "best" job in which his comparative advantage is optimally utilized. Thus, each worker has a vector of continuous skills and workers are differentiated by their relative advantage, as in Teulings (1995, 2002 and 2005). A worker is not instantaneously allocated to his best job, where he earns the highest wage, since not all workers (firms) are able to observe all firms (workers) all the time, i.e. there is frictional matching. As time goes by, the matching improves, mainly through job-to-job transitions that increase the economy's average productivity. Eventually, the allocation becomes efficient, such that every worker is employed where his comparative advantage is best utilized given other workers' assignment and a steady state situation is achieved. Adding employment protection legislation *(EPL)* to the economy distorts this process, decreases average productivity and results in harmful worker self-selection between workplaces.

The next section describes the basic model without EPL, then another sector which provides its workers with better job security is plugged into the model. The analytical developments which are not in the focus of the model are detailed in the first part of the appendix; the second part of the appendix presents the simulation results and its main predictions.

The Basic-Model

Following are the main assumptions of the model. Assumptions 1-3 are crucial, while the fourth and fifth are standard; the rest are considered only for simplicity.

1. Incomplete information - it takes time for firms to find suitable workers and for workers to find suitable firms. This creates the search friction, the initial inefficiency of the workers' assignment, the positive surplus to firms from matching. Actually it is the source of the economy's growth.
2. There exists a multiplicative complementarity between workers employed at the same firm and in the same period.
3. Workers are heterogeneous in terms of productivity.
4. Firms are profit maximizers.
5. Wages are determined by negotiation between firms and workers – a bargaining game.
6. The level recruitment intensity among firms are fixed and determined exogenously and it determines the jobs' arrival rate in the economy. It means that workers (firms) are passive in terms of search (recruitment intensity).
7. There are no costs incurred to create a vacancy or to leave it unfilled and the number of firms as well as the number of the total workers are both fixed and determined exogenously[1].

The labor market consists of a unit mass of heterogeneous workers ($N = 1$) facing a continuum of firms and they are to be assigned to heterogeneous jobs, as in models such as Teulings (1995). There are J monopolistically competitive firms, which vary according to their level of technology, where J is a large fixed number that is determined exogenously and is much smaller than the number of workers N, $J << N$. Workers and firms are infinitely-lived and forward-looking and are either unemployed[2] or matched with a firm. Time is discrete. The market starts at an initial state and converges to steady state. The workers are the only factors of production in the economy.

There are $n_{jt} \in [0,1]$ employed workers in firm j at time t.

The output at firm j at time t *is*:

[1]This point is differenced from the standard search models in which the number of firms is determined in steady state where the vacancy cost is zero.

[2]In this model there is no difference between unemployed and inparticipant.

[1]

$$Y_{jt} = \left(\overline{a}_t \cdot e^{\varepsilon_t}\right)\left(\frac{\overline{a}_{jt}}{2}\int_{i \in n_{j,t}} a_{ij}^2 di\right)$$

$i \in n_{jt}$ e.g., *i* belongs to n_{jt}, the set of workers at firm *j* at time *t*.

a_{ij} is worker *i*'s level of skill in firm *j*, such that every worker has a vector of *J* skill levels $\tilde{a}_i = (a_{i1}, a_{i2}, ..., a_{iJ})$ with specific kind of continuous distribution. Each component represents her skill level at a different firm.

Alternatively, one could define a_{ij} as being the product of a characteristic of the worker and an interaction parameter between him and firm *j*, i.e. $x_{ij} = x_i \cdot a_{ij}$. In our model, x_i has the same value for all workers (though in the simulation x_i is differentiated according some worker types).

$\overline{a}_{jt}$ is the average of workers' skills at firm *j* at time *t*. $\overline{a}_t$ is the average level of skill for all employed workers in the economy at time *t*.

ε is a market shock and follows an auto regressive process *AR(1)*:

$$\varepsilon_t = \rho\varepsilon_{t-1} + \nu_t$$
$$\nu_t \ i.i.N. \sim \mathrm{N}\left(0, \sigma^2\right)$$
$$0 < \rho < 1$$

where ρ is the process' persistence.

The production function is built to create a multiplicative complementarity between workers in the same firm. Some sources of complementary among workers in the same firm are social interactions, kinds of pressures, common or uncommon languages, imitations, learning and well-known norms; some theories and empirical findings which support this complementary among workers are detailed in the appendix.

Aggregate output at time *t* is given by:

[2]

$$Y_t = \frac{1}{2}\overline{a}_t e^{\varepsilon_t} \int_{j \in J}\left(\overline{a}_{jt}\int_{i \in n_{jt}} a_{ij}^2 di\right) dj$$

The marginal productivity of worker *i* at firm *j* at time *t* is:

[3][3]

$$m_{ijt} = \frac{d\lfloor Y_{jt} \rfloor}{da_{ij}} = a_{ij}\overline{a}_{jt}\overline{a}_t e^{\varepsilon_t}$$

For simplicity, it is assumed that worker *i* does not affect the average skill level in the firm and he views it as given. Note that unlike Teuling (1995 and 2005), the marginal productivity of worker *i* in firm *j* is a function of the average skill level among workers employed in firm *j* and in the economy as a whole at time *t* (assumption 2).

Firms compete with each other to recruit workers. We assume that each firm tries to hire workers with the same intensity level λ, such that $0 \le \lambda \le 1$. It follows that the aggregate recruitment activity in the economy is given by $\Gamma = 1-(1-\lambda)^J$. In other words: a worker is examined by a given firm with probability λ, thus the probability that he is observed by at least one firm is Γ.

Accordingly, it implies that Γ times *N* is the job offer arrival rate. Here *N* is normalized to 1, thus that Γ is the job offer arrival rate. Note that even that $J << N$ there is a negligible probability that some workers would get some offers in one period, in that case the workers would decide each offer independently.[4]

Denote the expected value of worker *i* employed in firm *j* at time *t* by $V^e(e_{ijt})$, henceforth $V^e(e_j)$, and the expected value of being unemployed by $V^u(u_{it})$, henceforth $V^u(u)$. In order to simplify the model and avoid expanding it in directions that are not particularly relevant, assume that the arrival rate of job offers is Γ for both an employed and unemployed worker and that *r* is a constant real discount rate.

[3]Note that this result is a specific form of a more general setup in which a worker's marginal productivity is given by: $$m_{i,j,t} = (a_{i,j})^{\mu_1}(\overline{a}_{jt})^{\mu_2}(\overline{a}_t)^{\mu_3}(e^{\varepsilon_t})^{\mu_4} \Rightarrow$$ $$\log(m_{ijt}) = \mu_1\log(a_{ij}) + \mu_2\log(\overline{a}_{jt}) + \mu_3\log(\overline{a}_t) + \mu_4\log(\varepsilon_t)$$, here we assume that $\mu_1 = \mu_2 = \mu_3 = \mu_4 = 1$. This implies that the elasticity of individual productivity with respect to the individual, the firm, the whole economy and the shock components are identical and equal to one. One could think of parameter values that are more realistic.

[4]Many search models assume fixed search intensity, in particular, see Mortensen (1986) or Mortensen and Pissarides (1994), though the parameter can be endogenized without changing the main results. Fougere et al. (1998) estimates the classical job search model with endogenous search effort, with focus on the impact of a public employment service. This is usually done in order to analyze the effect of the unemployment benefit on the rate or duration of unemployment; for a survey, see van der Berg (1999), Mortensen and Pissarides (1999), Eckstein and van den Berg (2006) or Yashiv (2007).

$$V^e(e_j) \equiv (1-\tau)w_{ijt} + \frac{1}{1+r}\left[\ddot{P}\cdot V^e(e_{k\neq j,t+1}) + P(\omega_{ijt})V^e(e_{j,t+1}) + \overline{P}(\omega_{ijt})V^u(u_{t+1})\right]$$

$\ddot{P}\cdot V^e(e_{k\neq j,t+1})$ is the probability that worker *i* will be employed in the period at other firm $k \neq j$ times her expected value there. Following the assumption of equal jobs'-offers-rates wherever the worker is, we can ignore this expression and normalize the sum $P(\omega_{ijt}) + \overline{P}(\omega_{ijt}) = 1$. Now we can rewrite the value of worker *i* in firm *j* as:

$V^e(e_j) \equiv (1-\tau)w_{ijt} + \frac{1}{1+r}\left[P(\omega_{ijt})(1-\tau)w_{ijt} + \overline{P}(\omega_{ijt})V^u(u_{t+1})\right]$ and for simplify assuming that $V^u(u_{t+1}|t) = V^u(u_t)$;

The value of a worker becomes

[4]

$$V^e(e_j) \equiv (1-\tau)w_{ijt} + \frac{1}{1+r}\left[P(\omega_{ijt})(1-\tau)w_{ijt} + \overline{P}(\omega_{ijt})V^u(u_t)\right]$$

This equation much simplifies the model: it becomes a static model instead of dynamic one which the later requires solving a dynamic model, which it is not impossible but it makes the model too complicated in irrelevant directions. It is true that by simplifying the model we loss some precision; but given the objective of the model this loss is wealthy.

Recall that w_{ij} is the gross wage of worker *i* in firm *j* and τ is the rate of the tax on labor. Thus, the net wage is $(1-\tau)w_{ij}$.

The total revenue from taxes at time *t* is $\int_J \int_{i\in j} \tau w_{ijt}\, didj$ which is equivalent to:

[5]

$$Tax_t = N_{E,t}\tau\overline{w}_t$$

where $N_{E,t}$ is the level of employment in period *t*, $N_{E,t} \in [0,1]$, $\overline{w}_t$ is the average wage among all firms at time *t*

The unemployed job seeker's expected value is:

[6]

$$V^u(u) \equiv \alpha\cdot b_t + \frac{1}{1+r}\left[\tilde{\pi}_{i,t+1}(1-\tau)\overline{w}_t + (1-\tilde{\pi}_{i,t+1})V^u(u_t)\right]$$

where $\tilde{\pi}_{i,t+1}$ is the probability of being employed for worker *i* at time *t+1* and b_t is the unemployment benefit at time *t*. b_t is financed by tax revenue such that $b_t = \frac{N_{E,t}}{N_{Un,t}} \cdot \tau \cdot \overline{w_t}$, where $N_{Un,t}$ is the unemployment rate in period $t \forall_t \; N_{Un,t} + N_{E,t} = 1$.

α denotes the individual disutility from work plus the individual disutility from being unemployed, which is referred to as the "scar" of unemployment by Jahn and Wagner (2008). This scarring effect may depend on the worker's employment history, her age, her education, her family status, the local employment rate and norms (of the individual or of society).[5] Here we assume for simplicity that α is fix and equals for each.

Evaluating [4] at $w = w_R$ (where w_R is the worker's reservation wage), which implies $V^e(e_{j,t+1} | w = w_R) = V^u(u_{t+1})$, and then combining with [6];

$$V^e(e_j) = (1-\tau)w_{ijt} + \frac{V(e_{j,t+1})}{1+r}$$
$$V^u(u) = \alpha \cdot b_t + \frac{V(e_{j,t+1})}{1+r}$$

which implies:

$$V^e(e_j) > V^u(u) \Leftrightarrow (1-\tau)w_{ijt} > \alpha \cdot b_t \Rightarrow$$
$$w_{R,t} = \frac{\alpha \cdot b_t}{1-\tau}$$

It can be concluded therefore that when the natural arrival rate of offers is the same for employed and unemployed workers, a worker's reservation wage is equal to the product of the unemployment benefit and his leisure preference, which is not a surprising result[6].

When a firm *j* matches a candidate it offers her one period contract. A higher wage attracts more workers due to the competition between the firms (see Eckstein and Wolpin,1990).

[5] Clark and Oswald (1994), Clark et al. (2001), Bockerman (2002), Stutzer and Lalive (2004) and Layard (2005).

[6] In general, the reservation wage is lower if on-the-job search is possible and is lower than b if search intensity is higher when employed than when unemployed. See Burdet (1978) who was the first to extend the classical model. Wolpin (1992) extended his search model described in Wolpin (1987) and found that blacks have higher job-offer rates than whites, both while unemployed and employed; see also Wolinsky (1987) and Eckstein and Wolpin (1995).

Worker *i*'s wage at firm *j* at time *t* (w_{ijt}) is determined by firm and worker negotiation – a bargaining game.

[7]

$$
\begin{aligned}
&Arg\max_{w_{ij}} \left\{ \left(V\left(e_j\right) - V\left(e_k\right) \right)^{\beta} \left(m_{ij} - w_{ij} \right)^{1-\beta} \right\} = \\
&= Arg\max_{w_{ij}} \left\{ \left(w_{ij} - w_{ik} \right)^{\beta} \left(m_{ij} - w_{ij} \right)^{1-\beta} \right\}
\end{aligned}
$$

Where $0 \le \beta \le 1$ represents worker bargaining power. For the interpretation of this parameter, see Binmore et al. (1986). $m_{ij} - w_{ij}$ is the gap between worker's wage and her productivity.

The first-order maximization condition implies that a firm *j* would offer a worker *i*:

$$
\begin{aligned}
&w_{ij} - w_{ik} = \beta\left(m_{ij} - w_{ik}\right) \Rightarrow \\
&w_{ij} = \beta m_{ij} + (1-\beta) w_{ik}
\end{aligned}
$$

As many other search models, the wage offer at the new firm is a weighted average of the worker's productivity in the new firm and the worker's wage at her current firm (or worker's benefit if she is unemployed). The weights are function of workers bargaining power. For example, if $\beta = 1$ the worker would be offered a wage which equals to her marginal productivity at the new firm but if $\beta = 0$ - the wage offer would be equals to the worker's wage at her current firm.

When unemployed is hired:

$$
\begin{aligned}
&\left(w_{ij} - V^{u}(u) \right)^{\beta} \left(m_{ij} - w_{ij} \right)^{1-\beta} \\
&F.O.C \\
&w_{ij} - V^{u}(u) = \beta\left(m_{ij} - V^{u}(u) \right) \Rightarrow \\
&w_{ij} = \beta m_{ij} + (1-\beta) V^{u}(u)
\end{aligned}
$$

Turnover, Recruitment and Quitting

Firms fire unskilled workers and recruit skilled ones in order to increase their average productivity and their profit. As in Nash bargaining there is no agreement between worker and firm if there is no surplus from the agreement. Workers can also decide to quit and become unemployed.

Assume that in the economy there is a critical-productivity-threshold C_t. It may be that $C_t = K \cdot \overline{w_t}$, where K is a constant. This parameter in fact is a government policy tool, which, for example, one can considered it as a the ratio of the minimum wage to the average wage. Firms cannot pay their workers below C_t.

Firm *j* fires worker *i* at time *t* if and only if:

$$m_{ijt} < C_t$$

Define P_{ijt} as the probability of worker *i* *not* being fired by firm *j* at time *t*:

[8]

$$P_{ijt} = P(m_{ijt} > C_t) = P(\varepsilon_t > \log\left(\frac{C_t}{\overline{a}_{jt} a_{ij} \overline{a}_t}\right)) = P(\rho\varepsilon_{t-1} + \nu_t > \log\left(\frac{C_t}{\overline{a}_{jt} a_{ij} \overline{a}_t}\right)) =$$

$$= \overline{\Phi}\left(\frac{1}{\sigma}(\log\left(\frac{C_t}{\overline{a}_{jt} a_{ij} \overline{a}_t}\right) - \rho\varepsilon_{t-1})\right) \equiv P(\omega_{ijt})$$

σ determines the sensitivity between the worker's probability of being fired and the workers specific skill a_{ij} (see figure A.2 in the appendix).

Firms search for workers and the probability of a firm to find a worker *i* at time *t* and offering him a wage is:

[9]

$$\lambda \cdot P_{ijt} = \lambda \cdot P(m_{ijt} > C_t) = \lambda \cdot P(\omega_{ijt})$$

Where m_{ijt} is the worker's productivity in her new job. Note that $\lambda \cdot P(\omega_{ijt})$ increases with a_{ij}, the firm's idiosyncratic technology and the state of the economy.

A worker *i* at firm *j* at time *t* moves to another job *k* whenever a value-offer is greater than his current value asset:

$$\forall_{k \neq j}$$

$$V^e(e_k) > V^e(e_j)$$

$$\Leftrightarrow$$

$$(1-\tau)w_{ikt} + \frac{1}{1+r}\left[P(\omega_{ikt})(1-\tau)w_{ikt} + \overline{P}(\omega_{ikt})V^u(u_t)\right] >$$

$$(1-\tau)w_{ijt} + \frac{1}{1+r}\left[P(\omega_{jit})(1-\tau)w_{ijt} + \overline{P}(\omega_{ijt})V^u(u_t)\right]$$

Proposition:

A worker moves to a different firm if and only if he receives a higher wage.

Proof:

If $w_{ikt} > w_{ijt} \Rightarrow \overline{\Phi}\left(\frac{1}{\sigma}(\log\left(\frac{C_t}{\overline{a}_{kt}a_{ik}\overline{a}_t}\right) - \rho\varepsilon_{t-1})\right) > \overline{\Phi}\left(\frac{1}{\sigma}(\log\left(\frac{C_t}{\overline{a}_{jt}a_{ij}\overline{a}_t}\right) - \rho\varepsilon_{t-1})\right) \Rightarrow$

$P(\omega_{ik}) > P(\omega_{ij}) \Rightarrow V^e(e_k) > V^e(e_j)$.

If $V^e(e_k) > V^e(e_j)$ and $w_{ikt} < w_{ijt} \Rightarrow$

$\overline{\Phi}\left(\frac{1}{\sigma}(\log\left(\frac{C_t}{\overline{a}_{kt}a_{ik}\overline{a}_t}\right) - \rho\varepsilon_{t-1})\right) < \overline{\Phi}\left(\frac{1}{\sigma}(\log\left(\frac{C_t}{\overline{a}_{jt}a_{ij}\overline{a}_t}\right) - \rho\varepsilon_{t-1})\right) \Rightarrow P(\omega_{ik}) < P(\omega_{ij}) \Rightarrow$

$V^e(e_k) < V^e(e_j)$, a contradiction.

∘

Note that since all the firms have the same level of hiring intensity, a firm *J* would not suggest a worker *i* a wage which exceeds its current wage but if the marginal productivity of the candidate worker exceeds its current one. That is to that $w_{ikt} > w_{ijt} \Leftrightarrow m_{ikt} > m_{ijt}$ which means the steady state is *efficient* – i.e., after several interactions every worker is matched with a firm in which his comparative advantage is highest; highest, but it is highest given the other workers' allocation.

Job-to-Job Mobility

The probability of firm *j* hiring worker *i* away from firm *k* is defined as:

[10]

$$\tilde{\pi}_{ikj} = \lambda \cdot [P(\omega_{ijt}) \cdot I_{k,j}]$$
$$I_{k,j} = \begin{Bmatrix} 1 & if \ (V(e_j) > V(e_k)) \\ 0 & otherwise \end{Bmatrix}$$

Denote by $\tilde{\pi}_{i.Un,j,t} = \lambda \cdot [P(\omega_{ijt}) \cdot P_{i,Un,j,t}]$ the probability of firm *j* hiring unemployed worker *i* at time *t*, where $P_{i,Un,j,t}$ is the probability that a worker *i* will accept to be employed at firm *j* at time *t* instead of being unemployed

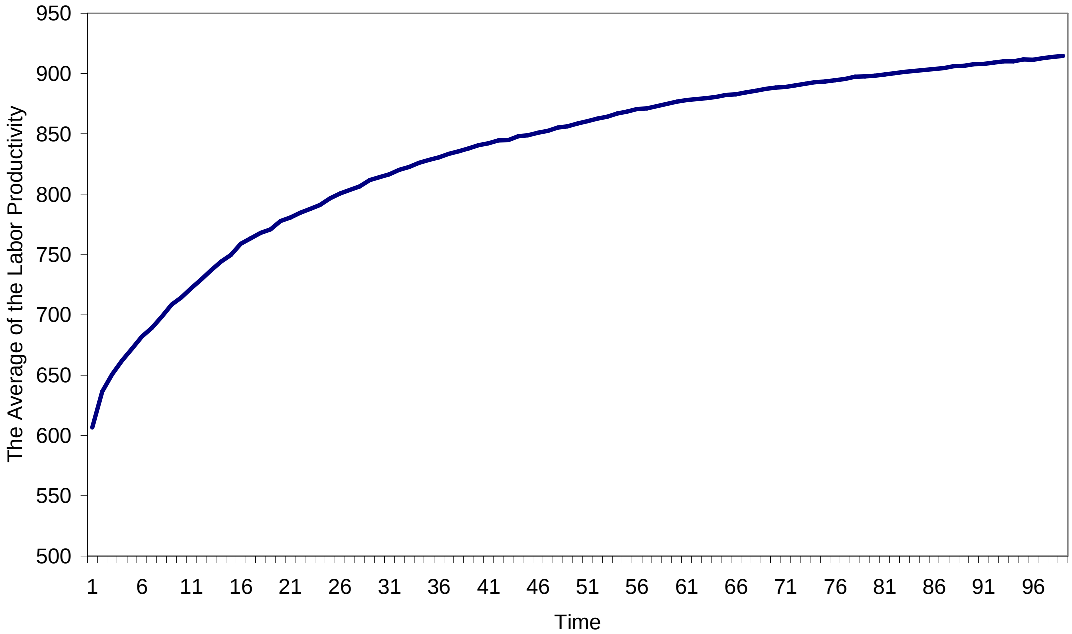
Note also that the probability that an unemployed person is contacted by at least one firm that would hire him:

$$\tilde{\pi}_{i,t} = 1 - \prod_{J,i\in N_{Ut}} \{1 - \lambda \cdot [P(\omega_{ijt}) \cdot I_{i,Un,j,t}]\} = 1 - \prod_{J,i\in N_{Ut}} (1 - \tilde{\pi}_{ijt})$$

Figure 1 depicts the convergence process of the economy to the steady state given the following parameters:

$$a_{i,j}[1] \approx U(0,1000); \alpha = 0.4, \tau = 0.25;\ UI = 0.02; \beta = 1;$$
$$\lambda = 0.03;\ \sigma^2 = 0.3;\ \rho = 0.95;\ C_t = 0.6(\overline{W}_t)$$

[Figure 1: Average labor productivity as a function of *t*]



UI is the ratio between the Unemployment Insurance and total tax revenue; which is usually about 6 percent. Without adding this parameter in the basic model the unemployment rate or other parameters in the model are not realistic.

Others sections which prove the existence of the steady state and that the economy converges to it with probability is detailed in the appendix.

<u>The public sector case and the workers self-selection between the sectors</u>

We now add a public sector in which firms are owned by the government[7] and define θ to be the share of such firms, where $\theta = \sum_{j=1\ if\ j\in\theta}^{J} \frac{j}{J}$.

[7]Casquel and Cunyat (2008) construct a simple theoretical labor market that incorporates differences in skill levels across workers (skilled and unskilled workers) in order to indentify the conditions under which temporary contracts are a way to achieve permanency (for additional examples, see Wasmer (1998).

A critical assumption here is that unlike firms in the private sector, the government does not observe the marginal product of each worker and therefore pays them all the same wage (or alternatively it does observe marginal productivity but is unable to pay each worker a different wage). The public sector pays its workers less return to their own skills and as compensates them with higher job security; this scheme impacts the workers self-selection between the sectors (Roy, 1951) and damages the productivity of the economy.

The Roy-model, in which the effects of self-selection into different occupations are discussed, is used here to analyze work transitions between sectors. According to this theory, workers choose jobs according to their expected wage. For this thesis, the expected wage level is a positive function of the workers' informal skills.

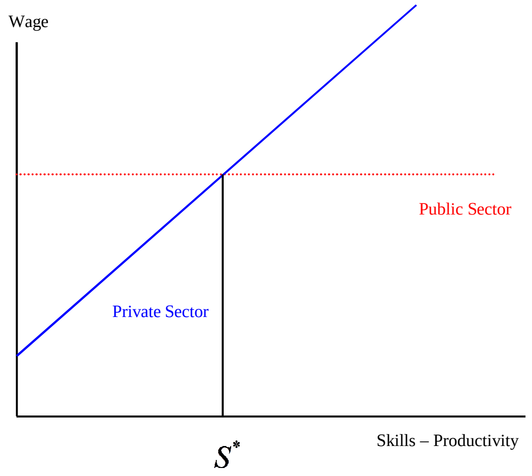
To illustrate assume that the market involves earning equality:

$$w_P = \alpha_P + \delta_P S$$
$$w_G = \alpha_G + \delta_G S$$

where w_i is the workers' wage, S is the level of skill, α_i is the intercept and δ_i is the return to skill; G for public sector and P for private sector. The trade-off is between the intercept level - one can assumes that it is caught by higher job security, and the return to skill. This situation leads workers with higher skills to work in the sector with a higher return to skills. Hence, a natural selection effect correlates between the workers' skill levels and the sector in which they work. This means positive selection in leaving the public sector and negative selection in leaving the private sector.

Figure 2 depicts schematically the model.

[Figure 2 - Roy Model – basic]



Here, all the workers who have higher skill than S^* would work in the private sector.

Roy's general framework has been applied to a variety of labor market settings[8]. In each application, the choice of occupation in Roy's original model is replaced with a choice of which market or sector to enter. Here, unlike the other researchers, I ask to explain the workers self-selection in mobility across sectors, where the initial decision in which sector to enter is not exogenous decision – unlike immigration between states.

The wage level in the public sector is a function Ψ of k current and past wage levels in the private sector: $W_{G,t} = \Psi(W_{E,t},...,W_{E,t-k})$ where $\Psi(k:1) \to (1:1)$. For example, Ψ could be an average times constant.

The government faces the following budget constraint:

[11]

$$N_{E,t} \cdot \tau \cdot \overline{w}_{E,t} + N_{G,t} \cdot \tau \cdot w_{G,t} = N_{U,t} \cdot b + N_{G,t} \cdot w_{G,t}$$
$$N_{E,t} \cdot \tau \cdot \overline{w}_{E,t} = (1 - N_{G,t} - N_{E,t})b + (1-\tau) N_{G,t} \cdot w_{G,t}$$

Where $N_{E,t}, N_{G,t}, \overline{w}_{E,t}$ are the number of employees in the private sector, the number of employees in the public sector and the average wage in the private sector. As can be seen from [4], $w_{G,t}$ is not the only variable that influences a worker's decision whether or not to accept a job offer. Thus, although the government pays the identical wage to all its workers, it is able to attract workers to the public sector by providing better employment protection, i.e. $\delta_{G,t} < \delta_t$.[9] Three of the variables ($\tau, N_{G,T}, \delta_{G,t}$) are given and the fourth, $W_{G,t}$, is determined endogenously according to [11].

The value of a worker employed in the public sector is given by:

[8] Including wage distribution (Heckman and Honoré. 1990), female labor force participation (Gronau,1974), union versus nonunion employment (Lee, 1978), choice of schooling (Willis and Rosen, 1979), internal and international migration (Nakosteen and Zimmer, 1980; Robinson and Tomes, 1982; Borjas,1987), training program participation (Ashenfelter and Card, 1985; Ham and LaLonde, 1996), choice of industry (Heckman and Sedlacek, 1990), and choice of one of 50 states to live in USA given the return to education variance across states (Dahl, 2002). In each application, the researchers replace the choice of "occupation" in Roy's original paper with a parallel choice of which market or sector to enter.

[9] Employment protection includes any set of regulations that limits the employer's ability to fire workers without delay or cost. Stringent layoff regulations increase the cost of firing workers, thereby reducing the productivity threshold at which firms are willing to lay workers off. For the effect on human capital accumulation, see Bertola (1994); for the effect on job creation. see Mortensen and Pissarides (1999); and for a survey, see Bassanini et al. (2008). The latter summarize the empirical evidence regarding the link between *EPL* and productivity. Using cross-section aggregate data for the OECD countries for the period 1982 to 2003, they conclude that about 40 percent of the variation in productivity is due to *EPL*.

[12]

$$V^e(e_G)\equiv U((1-\tau)w_{G,t})+\frac{1}{1+r}\left(P\cdot V^e(e_{i,t+1})+\overline{P}\cdot V^u(u_t)\right)=$$
$$=(1-\tau)w_{G,t}+\frac{1}{1+r}\left((1-\delta_{G,t})\cdot(1-\tau)w_{G,t}+\delta_{G,t}\cdot V^u(u_t)\right)$$

If worker *i* in firm *j* is employed in the public sector, he changes jobs if and only if:

[13]

$$\forall_{k\neq j}$$
$$V^e(e_j)\equiv(1-\tau)w_{ikt}+\frac{1}{1+r}\left[P(\omega_{ikt})(1-\tau)w_{ikt}+\overline{P}(\omega_{ikt})V^u(u_{i,t})\right]>$$
$$>(1-\tau)w_{G,t}+\frac{1}{1+r}\left((1-\delta_{G,t})(1-\tau)w_{G,t}+\delta_{G,t}V^u(u_{i,t})\right)\equiv V(e_G)$$

In the steady state, worker *i* will work in the public sector iff:

$$Max_j\left\{(1-\tau)w_{ij}+\frac{1}{1+r}\left[P(\omega_{ijt})(V^e(e_j))+\overline{P}(\omega_{ijt})V^u(u)\right]\right\}$$
$$<(1-\tau)w_G+\frac{1}{1+r}\left((1-\delta_{G,t})(1-\tau)w_G+\delta_{G,t}V^u(u)\right)$$

The number of workers employed in the public sector in the steady state is:

$$\int_i\left\{\begin{array}{l}(1-\tau)Max_j w_{ijt}+\frac{1}{1+r}\left[P^*(\omega_{ijt})(1-\tau)w_{ijt}+\overline{P}^*(\omega_{ijt})V^u(u)\right]<\\<(1-\tau)w_{G,t}+\frac{1}{1+r}\left[(1-\delta_{G,t})(1-\tau)w_{G,t}+\delta_{G,t}V^u(u)\right]\end{array}\right\}di$$
$$and$$
$$(1-\tau)w_G>\alpha\cdot b$$

There is no analytical solution to the model with a public sector and therefore conclusions can only be reached by way of simulations.

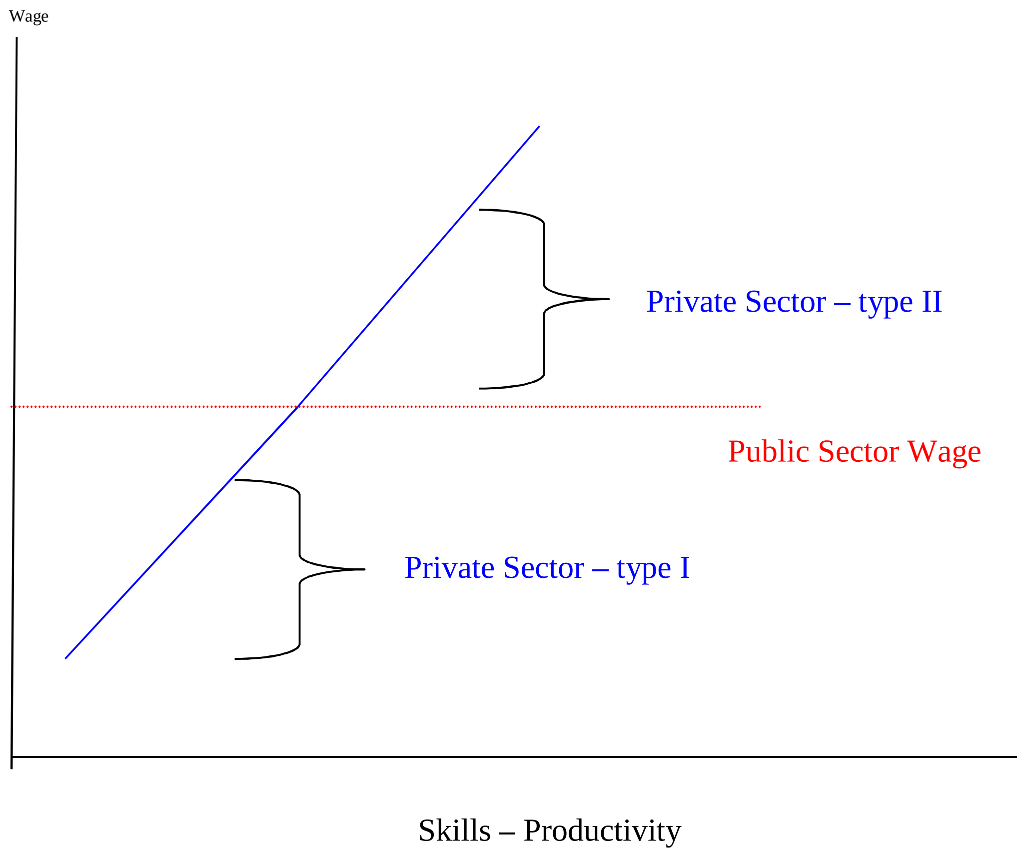
What can be said about worker transition between the public and private sectors? Without adding a difference among the workers related to preferences or average skills or something else – nothing will happen. This is to say that only the random assignment of the workers during the convergence to the steady state will determine which workers will be employed at the government sector, which workers will be unemployed and which workers will be employed at the private sector.

Alternatively, assuming that there are two groups of workers in the economy, each with a different distribution of informal skills (such as individual talent). The distributions have the same variance but different averages across firms, such that $x_1\bar{a}_J = \bar{a}_{1,J} < \bar{a}_{2,J} = x_2\bar{a}_J$. These informal skills are rewarded in the private sector (by means of the mechanism described above) but not in the public sector. Therefore, workers who leave the private sector for the public sector are negatively self-selected, i.e. they are free riders, while workers who leave the public sector for the private sector are positively self-selected.

Note that in the steady state the public sector, which provides its workers with better job security, cannot pay a wage equal to or exceeding the average wage in the private sector. Since workers in the private sector who are paid less than the average wage would leave for the public sector and this process would continue until all workers are employed in the public sector. Thus, in the steady state the public sector must pay a wage that is less than the average in the private sector, given the workers' formal skills, such as schooling.

Note that this is a particular case of a model in which workers are divided into a number of groups (*N*) according to level of formal skills – a number years of education - and each group is divided into a number of sub-groups (*K*) according to the average level of skills that are rewarded in the private sector but not in the public sector. In our case, *N* equals to 1 and *K* equals to 2 (see figure 3).

[Figure 3 - Roy model - two types of workers]



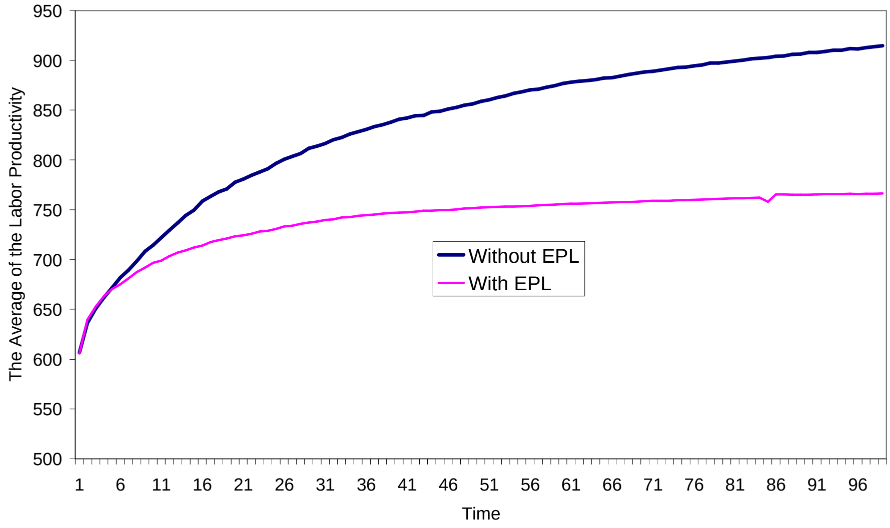
Figures 4 and 5 depict the effects of *EPL* on several economic outcomes assuming the following base line calibrations:

$\delta_{G,t} = 0, \tau = 0.25, \theta = 0.25, a_{ij} \approx U(0,1000); UI = 0.02, \alpha = 0.4, W_G = 0.9 \cdot \overline{W}_E$

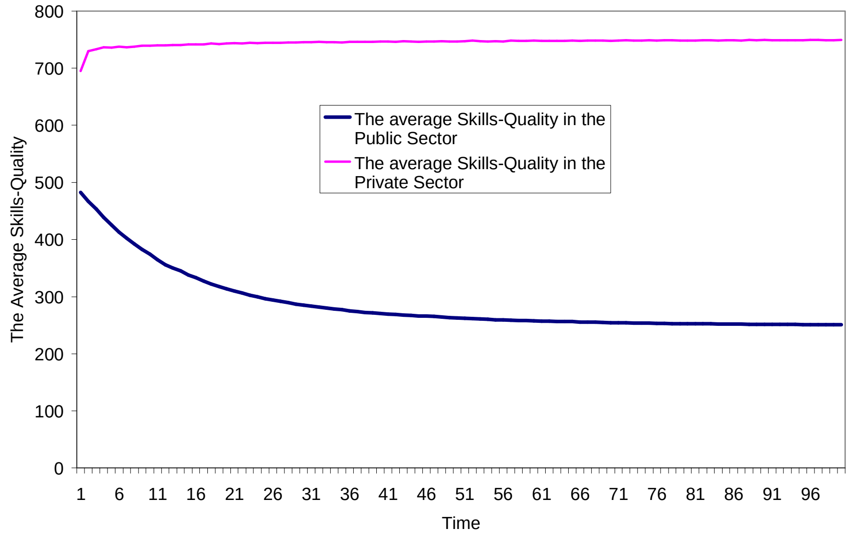
When there are two groups of workers (figure 5)

$a_{i,j}[1] \approx U(0,500);\ \ a_{i,j}[2] \approx U(500,1000)$

[Figure 4 - Average quality of workers at their current firms – the distortion of the productivity]



[Figure 5 - The quality of workers in all jobs]



Israel Case Study

In Israel there is no wide different in the (unconditional and conditional) average hourly wage between public and private sector. According to many specifications of multi-variables regressions the public sector premium estimation is between -4 to 4 percent. The estimation is changed across years and the methods of calculation. However, the similarity of the average wage between the two sectors is also characterized many other developed countries while a wide literature has been dealt with it for the last twenty years. Another stylized fact is that public sector employment share in developed countries is between 20 – 30 percent. In Israel is around 30 percent.

Considered that the employees' unobserved skills ($x_i a_{ij}$) are continuously distributed between firms; given other factors are equal (e.g. human capital). In addition considered that the workers are divided into two equal groups – weak and strong.

Following the mechanism detailed above, it is concluded that being employed in the public sector is *over supplied* – i.e. there are more workers that prefer to be employed at the public sector than the public sector can employ or finance (less than one third from the workers). To make it clear: if the middle wage is equal between private and public sectors then for half of the workers the wage in the public sector is higher compared to their alternative wage in the private sector, and given that there is less hazard in the public sector; therefore it is superior to be employed at the public sector for them (figure 3). As a consequence the public sector can afford to hire talented workers by making a kind of selection between them.

Thus, separating the workers into two equal groups, as it is done before, without letting the public sector to distinguish between them as well as determining exogenously quotas to the public sector workers (about 30 percent) – is not the most accurate way to describe the reality.

Alternatively, an elegant option to describe the reality is to divide the population for four types (or even more):

I. Workers with low skill (weak workers)

II. Workers with less than average skill

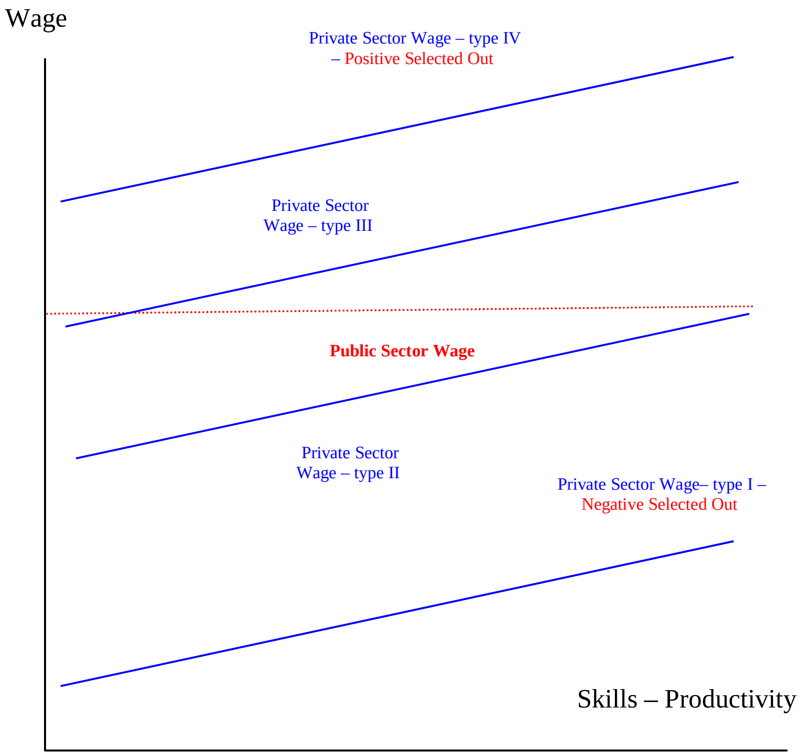
III. Workers with more than average skill

IV. Workers with high skill (brilliant workers)

The public sector suggests all of them the same wage (they are all have the same observed skills). However, after some periods, for example four years, the public sector employer distinguishes between the brilliant and the weak workers. Then the employer in the public sector dismisses the weak workers and provides the other workers with lower probability of being fired. The brilliant workers will be positively selected out following the transmission of the theoretical model and the public sector employer would remain with the middling workers. The proportions between the four groups as well as the discrepancy of workers' skill between firms within the groups determines whether the public sector workplace would be over supplied or not. In any case if the public sector is still over supplied it can be solved by changing these parameters or making the public sector selection tougher or determining exogenously quotas.

Following the division of the workers into four different groups we expect that the selection out from the public sector will be characterized by non-linear shape (a U-shape) respected to workers' informal skill. That means that respected to their informal skills (for example the residual wage) workers would be negative selected at the bottom but positively selected at the top (figure 6). That is exactly what we find in the empirical tests.

[Figure 6 – Roy model - Public Sector Selection]

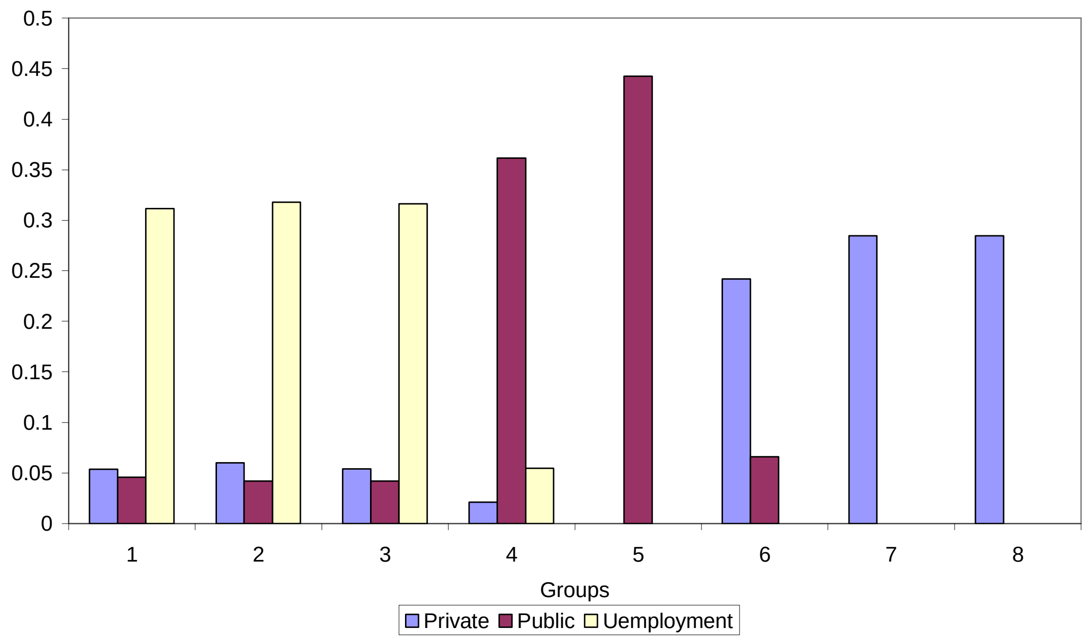


Actually, one can think about *K* different groups of workers where the groups are ordered respected to the workers relative average skill. Or even think about it as a continuous number of groups that has its own distribution; for example a normal distribution in which most of the workers are placed near or in the middle of the distribution. In that case the wage in the public sector relative to the average wage in the private sector as well as the parameters of the skill's distribution would determine the magnitude and the sign of the selection from and to the public sector.

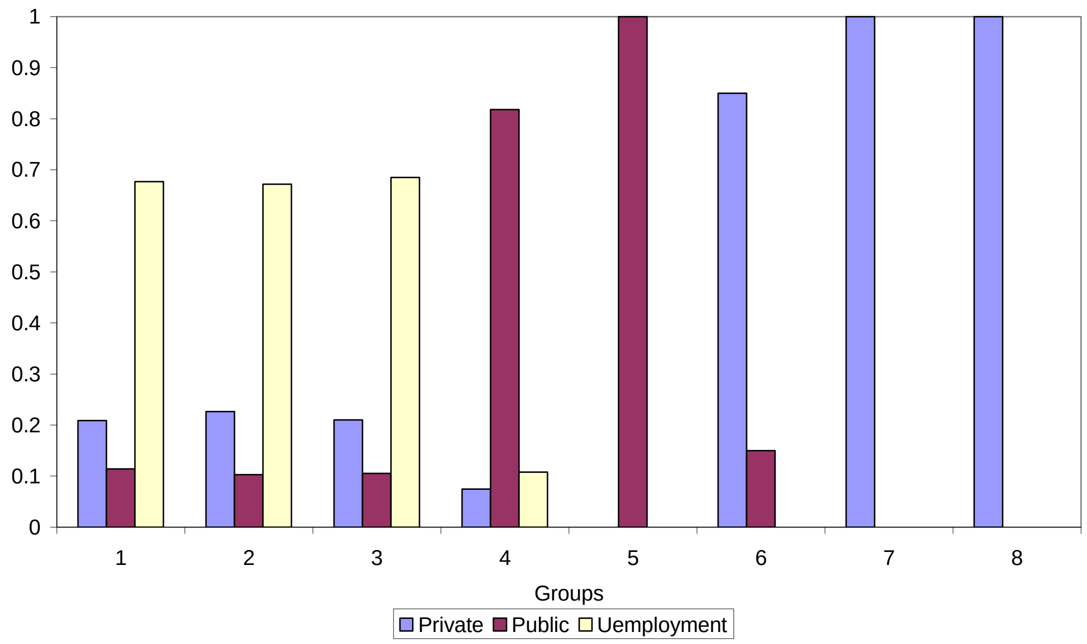
For example, I have submitted a simulation which includes eight ordinal groups of workers, from the lowest skilled to the highest. I assume that all the groups are at the same size (one over eight). I assume also that the public sector offers its workers a wage that equals the average wage in the private sector as well as higher protection legislation. As it was explained before this state would cause too many workers to be employed in the public sector. Thus– I have enabled the public sector to dismiss the less skilled workers after some periods (at this example the selection is done on workers that their skill is at the four lowest deciles and the number of periods is 4 – a kind of probation). The private sector suggests its workers an offer by the same mechanism as it was explained before. The results of the workers self-selection are described in figures 7 and 8. Figure 7 depicts the distribution among the sectors (this is to say that each color in that figure amounts to 1; while figure 8 depicts the distribution among the groups (this is to say that each group amounts to 1). The selection is conspicuous – the less skilled are mainly unemployed; the medium are employed at the public sector while the better skilled workers as well as some of the less skilled are employed in the private sector. The exact shape of these distributions is mainly depended on the assumption of the skills distribution among the workers but also depended on the other parameters in the model.

However, a potential claim is that the separation of the workers in terms of their skills among the sectors is not so sharp while, for example, there are also brilliant workers in the public sector. The answer for this claim is that the decision where to be employed is also affected by other factors, like risk aversion, leisure preference and many others which I don't take into account. In this simulation, I assume virtually that these factors are at the same level among the workers; substantially, it is the same as submitting a multi-variables-regression and holding these elements equal.

[Figure 8 – the skills distribution among the sectors]



[Figure 9 – the skills distribution among the groups]



The appendix depicts a sensitivity analysis which details how the outcomes are changed as a result of some changes in the policy or structural variables.

Summary

The model here lies in its focus on worker rather than firm behavior and it suggests that the introduction of *EPL* into an environment of incomplete information in the job market may help account for the self-selection of workers between workplaces and quantifies the distortion of this regularity in term of productivity.

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***Appendix* A.**

The production function

In spite of its significance, the literature on the interaction among workers within a firm is not extensive. Kandel and Lazear (1992) examine the theory of team production within the firm and focus on how workers as a team produce social pressure in order to solve the free rider problem. Kremer (1993) in his seminal work presents a production function – the O-Ring production function – in which, among other assumptions, there is a positive correlation between workers skill at the same firm. The O-Ring production function is a function of workers'-skills multiplication at the same firm.One of the implications of this assumption is that workers performing the same task earn higher wage in a high-skill firm than in a low-skill one. Ichino and Maggi (2000) examine shirking behavior within the firm and how changes in the workplace can affect it.[10] Winter (2004) shows, on a theoretical level, the optimality of offering differential incentive contracts in order to elicit worker effort, thus generating externalities for other workers. Gould and Winter (2005) find a positive peer effect between complementary baseball players and players who are competing for the same position and a negative peer effect between substitutable players.They explain their finding using the technological properties of the team production function without any reference to behavioral effects, such as peer pressure, norms or shame.Dur and Sul (2010) examine a wage-contract in a framework of principle-agent problem. In their model social interactions among workers affect each worker own effort and as a result the total output.

[10]Many authors have tried to empirically examine peer effects in a classroom environment, i.e. the effect of one student’s grades on another’s (see, for example, Lavy and Schlosser (2007),Hoxby (2000), Ammermueller et al. (2006) and many others)

Dynamic Growth – the stochastic process

Since we are ultimately interested in the behavior of the aggregate unemployment rate, in this section microeconomic behavior is aggregated up to the macroeconomic level. This exercise is non-trivial since firms differ from one another.

At the beginning all persons are unemployed. During the first period, the workers decide whether to accept a wage offer or become unemployed. The number of employees in firm *j* in the first period is given by:

[A.1]

$$n_{j,0} = \lambda \int_{i \in N_U} \pi_{i,Un,j,0} di = \lambda \cdot \{\mathrm{A} \cap \mathrm{B}\}$$

where A is defined as a group of workers whose productivity in firm *j* exceeds the critical productivity C_t. B is defined as a group of workers which would accept a job offer from firm *j* instead of being unemployed, which, as noted above, it is identical to a group of workers which are offered a wage which exceeds their reservation wage.

$$n_{j,0} = \lambda \cdot \{\mathrm{A} \cap \cdot \mathrm{B}\}$$

$$\begin{array}{lll} i \in A & IFF & \frac{1}{\sigma}\left(\log(\frac{C_0}{a_{ij}\bar{a}_{j0}\bar{a}_0}) - \rho\varepsilon_{t-1} \right) < \nu_t , \\ i \in B & IFF & (1-\tau)w_{ij} > b \end{array}$$

Given the large-number-of-workers assumption, a worker views $\{N_{E,t}, \bar{a}_{jt}, \bar{a}_t\}$ as given random variables for all *t* and *j*. The other variables $\{\tau, \lambda, \sigma, C(K)\}$ are exogenous.

Total employment after the first period solves the following equation:

[A.2]

$$N_{E,0} = \sum_{j,0} A \cap B = \int_{j \in J} n_{j,0} dj$$

A proof of one solution existence and convergence:

The R-H-S decreases with $N_{E,0}$ from $\lambda \int_{\substack{i \in N \\ j \in J}} \mathrm{B} \; didj$ to *0* while the L-H-S is the 45 degree line, $N_{E,0} \geq 0$ and $\lambda \int_{\substack{i \in N \\ j \in J}} \mathrm{B} \; didj \geq 0$. Thus, there is a unique intersection point and [13] has a unique solution $N^*_{E,0}$.

The result is depicted in figure 1 (its mirror image illustrates the result for the unemployment rate).

Denote the skill level CDF of the worker employed at firm j at time t by $G_{j,t}(\cdot)$ with corresponding density $g_{j,t}(\cdot)$.

The number of layoffs in firm j in period t is given by:

$$n_{F,j,t} = n_{j,t-1} \int_{\substack{\min a_{i,j,t} \\ i \in j}}^{\tilde{C}_t} g_j(a_{i,j})di = \begin{Bmatrix} 0 & if \ \min a_{ijt} > \tilde{C}_t \\ n_{j,t-1} \int_{\substack{\min a_{i,j,t} \\ i \in j}}^{\tilde{C}_t} g_j(a_{ij})di & if \ \min a_{ijt} < \tilde{C}_t \end{Bmatrix} =$$

$$= \Phi\left(\frac{1}{\sqrt{2}\sigma}\log(\frac{1}{\hbar_{jt}\hbar_t})\right) n_{j,t-1}\left(G_{\substack{jt \\ i\in j}}(\tilde{C}_t \middle| \tilde{C}_t > \tilde{C}_{t-1})\right)$$

where $\tilde{C}_t = \frac{C_t}{\bar{a}_t \bar{a}_{jt}}, \hbar_{j,t} = \frac{\bar{a}_{jt}}{\bar{a}_{jt-1}}, \hbar_t = \frac{\bar{a}_t}{\bar{a}_{t-1}}$.

$$n_{F,j,t} = n_{j,t-1} \int_{\substack{\min a_{i,j,t} \\ i \in j}}^{\tilde{C}_t} g_j(a_{i,j})di = \begin{Bmatrix} 0 & if \ \min a_{i,j,t} > \tilde{C}_t \\ n_{j,t-1} \int_{\substack{\min a_{i,j,t} \\ i \in j}}^{\tilde{C}_t} g_j(a_{i,j})di & if \ \min a_{i,j,t} < \tilde{C}_t \end{Bmatrix} =$$

$$= \begin{Bmatrix} 0 & if \ \tilde{C}_t < \tilde{C}_{t-1} \\ n_{j,t-1} \int_{\substack{\tilde{C}_{t-1} \\ i \in j}}^{\tilde{C}_t} g_j(a_{i,j})di & if \ \tilde{C}_t > \tilde{C}_{t-1} \end{Bmatrix} \Rightarrow$$

$$n_{F,j,t} = \left(P\left(\tilde{C}_t < \tilde{C}_{t-1}\right) \cdot 0 + P\left(\tilde{C}_t > \tilde{C}_{t-1}\right) \cdot n_{j,t-1} \int_{\substack{\tilde{C}_{t-1} \\ i \in j}}^{\tilde{C}_t} g_j(a_{i,j})di \right)$$

$$= \Phi\left(\frac{1}{\sqrt{2}\sigma}\log(\frac{1}{\rho_{j,t}\rho_t})\right) n_{j,t-1} \int_{\substack{\tilde{C}_{t-1} \\ i \in j}}^{\tilde{C}_t} g_j(a_{i,j})di =$$

$$= \Phi\left(\frac{1}{\sqrt{2}\sigma}\log(\frac{1}{\rho_{j,t}\rho_t})\right) n_{j,t-1}\left(G_{\substack{j,t \\ i\in j}}(\tilde{C}_t \middle| \tilde{C}_t > \tilde{C}_{t-1}) - G_{\substack{j,t \\ i\in j}}(\tilde{C}_{t-1} \middle| \tilde{C}_t > \tilde{C}_{t-1})\right) =$$

$$= \Phi\left(\frac{1}{\sqrt{2}\sigma}\log(\frac{1}{\rho_{j,t}\rho_t})\right) n_{j,t-1}\left(G_{\substack{j,t \\ i\in j}}(\tilde{C}_t \middle| \tilde{C}_t > \tilde{C}_{t-1})\right)$$

Thus, the number of layoffs in the economy as a whole at time t is given by:

$$N_{F,t} = \int_{j \in J} \Phi\left(\frac{1}{\sqrt{2}\sigma} \log(\frac{1}{\hbar_{j,t}\hbar_t}) \right) n_{j,t-1} \left(G_{\substack{jt \\ i \in j}} (\tilde{C}_t \middle| \tilde{C}_t > \tilde{C}_{t-1}) \right) dj$$

Denote the firing rate by $\delta_t = \dfrac{N_{F,t}}{N_{E,t}}$.

The number of workers who were hired (and had either been working at a different firm or unemployed) in period t is given by:

$$N_{P,t} = \int_{j \in J} \int_{\substack{k \neq j \\ i \in n_k}} \tilde{\pi}_{ikjt} didj + \int_{\substack{i \in N_{U,t} \\ j \in J}} \tilde{\pi}_{iUnjt} didj$$

$\int_{\substack{i \in N_{U,t} \\ j \in J}} \tilde{\pi}_{i,Un,j,t} didj$ is the flow of workers from unemployment to employment and

$\int_{j \in J} \int_{\substack{k \neq j \\ i \in n_k}} \tilde{\pi}_{ikjt} didj$ is the flow of job transitions.

Denote the hiring rate by $h_t = \dfrac{\int_{\substack{i \in N_{U,t} \\ j \in J}} \tilde{\pi}_{i,Un,j,t} didj}{N_{E,t}}$, the transitions rate by

$\gamma_t = \dfrac{\int_{j \in J} \int_{\substack{k \neq j \\ i \in n_k}} \tilde{\pi}_{i,k,j,t} didj}{N_{E,t}}$, the job finding rate by $\vartheta_t = \dfrac{\int_{\substack{i \in N_{U,t} \\ j \in J}} \tilde{\pi}_{i,Un,j,t} didj}{N_{U,t}}$ and the voluntary quitting rate by $q_t = \dfrac{\int_{\substack{i \in N_{U,t} \\ j \in J}} I_{i,j,Un,t} didj}{N_{E,t}}$.

The number of unemployed workers at time t is equal to the number of workers who were fired in the previous period, plus the number of workers who were unemployed in the previous period and didn't succeed in finding a job, plus the number of workers who quit their job voluntarily.

$$N_{U,1} = N_{F,0} + N_{Un,0} - N_{Un,0} \cdot find + N_{E,0} \cdot quit =$$
$$= N_{F,0} + \int_{\substack{i \in N_{U,0} \\ j \in J}} \left(1 - \tilde{\pi}_{i,Un,j,0}\right) didj + \int_{\substack{i \in N_{E,0} \\ j \in J}} I_{i,j,Un} didj$$

...

$$N_{U,t} = N_{F,t-1} + \int_{\substack{i \in N_{U,t-1} \\ j \in J}} \left(1 - \tilde{\pi}_{i,Un,j,t-1}\right) didj + \int_{\substack{i \in N_{E,t-1} \\ j \in J}} I_{i,j,Un} didj$$

The dynamic condition of the employment rate is:

$$N_{E,t-1} - N_{F,t-1} + N_{Un,J,t-1} - N_{J,Un,t-1} = N_{E,t}$$

Rearranging, we obtain the following dynamic equation:

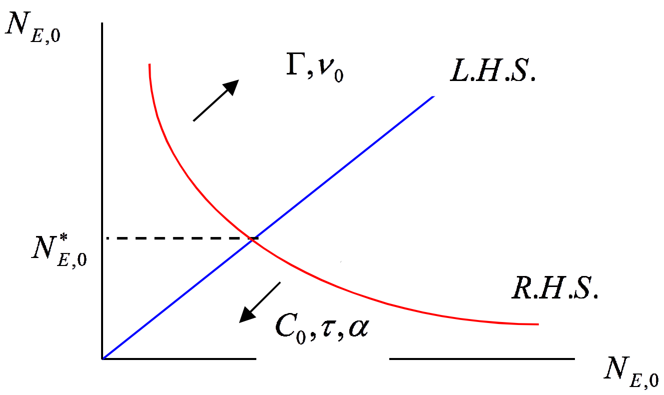
[A.3]

$$N_{E,t} = N_{E,t-1}(1 + h_{t-1} - \delta_{t-1} - q_{t-1})$$

To summarize the model, the exogenous parameters include the economy's structural parameters $\lambda, \sigma_{\upsilon}, \rho, G(a), \alpha$ and the policy parameters $\tau, C(K)$.

The outputs, including the flows and the steady state situation, are all determined endogenously.

[Figure A.1 - The evolution of employment in the first period]



Proposition 1

The expected average skill level of employed workers increases with t.(the proof is in the appendix)

$$E\left(\overline{a}_{ijt}\right) > E\left(\overline{a}_{ijt-1}\right)$$

<u>Proof of proposition 1</u>

The average skill level among the workers employed at firm *j* at time *t* is a weighted average of the workers who remained at the firm and those hired by the firm in period *t-1*.

$$\overline{a_{ijt}} = W_S\left(\overline{a_{i.j,t-1}}\middle| Stayed\right) + W_A\left(\overline{a_{i.j,t-1}}\middle| Arrived\right)$$
$$W_S + W_A = 1$$

A support claim

The expected average skill level of the workers who remained at the firm between *t-1* and *t* is greater than expected average skill of the workers who were already employed by the firm in *t-1*.

$$E\left[Ave_{i\in j}\left(a_{i,j,t-1}\middle|Stayed\right)\right]=$$
$$\Gamma\cdot E\left[Ave_{\substack{i\in j\\k\neq j}}\left(a_{i,j,t-1}\middle|I_{jk}=0\right)\right]+(1-\Gamma)\cdot E\left[Ave_{i\in j}\left(a_{i,j,t-1}\right)\right]+(1-\delta_j)\cdot E\left[Ave_{i\in j}\left(a_{i,j,t-1}\middle|L_{j,t-1}=0\right)\right]$$
$$> E\left[Ave_{i\in j}\left(a_{i,j,t-1}\right)\right]$$

Proof

The probability of form *j* to hire worker form firm *k* is

$$\tilde{\pi}_{ikj}=\lambda\cdot[P(\omega_{ijt})\cdot I_{kj}]$$
$$I_{k,j}=\begin{cases}1 & if\ \left(V(e_j)>V(e_k)\right)\\ 0 & otherwise\end{cases}$$
$$V(e_j)>V(e_k)\ \ IFF\ m_{ij}>m_{ik}$$
$$\Pr op(m_{ij}>m_{ik}):Function(\overset{+}{a}_{ij},\overset{-}{a}_{ik})$$
$$\Rightarrow$$
$$E\left[Ave_{\substack{i\in j\\k\neq j}}\left(a_{i,j}\middle|I_{jk}=1\right)\right]<E\left[Ave_{\substack{i\in j\\k\neq j}}\left(a_{ij}\right)\right]$$
$$E\left[Ave_{\substack{i\in j\\k\neq j}}\left(a_{i,j}\middle|I_{jk}=0\right)\right]>E\left[Ave_{\substack{i\in j\\k\neq j}}\left(a_{ij}\right)\right]$$

The probability of firm *j* to fire worker *i*:

$$\overline{P}\left(\omega_{ij}\right):Function(\overset{-}{a}_{ij})$$
$$\Rightarrow$$
$$E\left[Ave_{i\in j}\left(a_{i,j,t-1}\middle|L_{j,t-1}=0\right)\right]>E\left[Ave_{i\in j}\left(a_{i,j,t-1}\right)\right]$$

where $L_{j,t-1}$ is an indicator of layoffs.

2. The average skill level of the workers hired by firm *j* between *t-1* and *t* is greater than that of workers who were employed by the firm in *t-1*:

$$E\left[Ave_{i\notin j}\left(a_{ijt}\middle|Arrived\right)\right]=E\left[Ave_{\substack{i\in j\\k\neq j}}\left(a_{ijt}\middle|I_{kj}=1\right)\right]>E\left[Ave_{\substack{i\in j\\k\neq j}}\left(a_{i,j,t-1}\right)\right]$$

This is a result of the large-number-of-workers assumption according to which the skill of workers is equal across all firms ex-ante but in actuality develops differently due to the random process described above.

The new workers who were previously unemployed may decrease the average skill level of the workers at the firm. However, over time their skill level will increase and it can be assumed that their influence will become negligible. This is a result of the assumption that workers have the same average skill level across all firms.

Hence, the expected average skill level of workers at a firm increases with *t*.

∘

The Steady State

The steady state is defined as a situation in which there are no workers who can increase their utility by changing jobs or by quitting and becoming unemployed or by hiring and becoming employed instead unemployed. That means that in steady state $h_{steady\_state}, \gamma_{steady\_state}, q_{steady\_state} = 0$

Proposition 2

There exists a steady state and the economy concavely converges to it. This process is stochastic process but it convergence with probability.

Proof:

As time goes every worker has matched with a firm (think about unemployment as a kind of firm) such that his value is maximized. Denote this by $j = j^* \left| \hat{V}^e(i, j^*) = Max_{j \in J} V^e(e_j) \right.$. However, the chance that j^* varies with t is negligible. That is to say that there are no firms that can offer a worker $V^e{}_{k \neq j^*}(i,k,T+t) > V^e(i, j^*, T)$. Since $\forall_{i \in N} V(e_{j^*}) = V(e_{j^*,T}) = Max_{j \in J}\left(V(e_{j,T})\right)$, there are no workers who wish to change jobs. As a result, there are no further job transitions in the economy and therefore $\Pr ob\left\{\exists_{k \neq j^*} \left| \hat{V}_{k \neq j^*}(i,k,T+t) > \hat{V}(i, j^*, T)\right.\right\} = 0$.

If a worker is offered $V^e(e_{j^*})$, he will accept it by definition (since there are no transition costs in the model).

Thus, the probability that a worker will receive this offer is:

[A.4]

$$\tilde{\pi}_{ij^*t} = \lambda \cdot P\left(\omega_{ij^*t}\right) \cdot P\left[V^e(e_{j^*}) > V^e(e_j)\right] = \lambda \cdot P\left(\omega_{ij^*t}\right).$$

Since there are a large number of firms,

$P\left(\omega_{i,j^*,t}\right)=1-\Phi\left(\frac{1}{\sigma}(\log\left(\frac{C_t}{\overline{a}_{j^*t}a_{ij^*}\overline{a}_t}\right)-\rho\varepsilon_{t-1})\right)\approx 1$ and therefore $\tilde{\pi}_{i,j^*,t}=\lambda$.

The offers are not serially dependent and therefore the probability that a worker will not receive the offer until period *t* is $(1-\tilde{\pi}_{i,j^*,t})^t=(1-\lambda)^t$ which converges to *0* as *t* increases. Thus, $\tilde{\pi}_{i,j^*,t}\xrightarrow{t\to\infty}1$, and consequently $V^e(e_{j,T})=V^e(e_{j^*})$ where T is *t* after very much interactions, when $t\to\infty$.

This rule applies to every worker and to every firm.

In the steady state, a worker will be employed iff

$$\exists_j V^e(e_j)>V^u(u)\Rightarrow(1-\tau)\exists_j\left(w_{ijT}\right)>\alpha\cdot b_T\Rightarrow(1-\tau)\exists_j\left(w_{ijT}\right)>\alpha\frac{N_{E,T}}{N_{Un,T}}\tau\cdot\overline{w_T},$$

otherwise, he will decide to remain unemployed.

The source of the concavity of convergence is the concavity of the Negative-Geometric probability function.

Returning to [A.3] and setting $\begin{array}{l} h_{t-1},q_{t-1}\xrightarrow{t\to\infty}0\Rightarrow \\ N_{\mathrm{E,t}}\xrightarrow{t\to\infty}N_{E,t-1}\end{array}$, the steady state is achieved.

○

Note that if $\exists_j\left(\frac{w_{ijT}}{\overline{\overline{w_T}}}\right)>\alpha\frac{\tau}{1-\tau}\frac{N_{E,T}}{N_{Un,T}}$ for worker *i*, he will receive his highest offer before time *T* with a probability of $1-\prod_{t=t}^{T}\left(1-\tilde{\pi}_{i.Un,j,t}\right)$, which approaches 1 as *t* increases and he will work at firm *j*. But if $\frac{\forall_j\left(w_{ijT}\right)}{\overline{\overline{w_T}}}<\alpha\frac{\tau}{1-\tau}\frac{N_{E,T}}{N_{Un,T}}$ worker *i* will prefer to be unemployed.

This rule applies to every worker *i* and to every firm *j* and therefore in the steady state the unemployment rate fulfills the following conditions:

[A.5]

$$\forall_{i \in N_{E,T}} \frac{\exists_j \left(w_{i,j,T}\right)}{\overline{\overline{w_T}}} > \alpha \frac{\tau}{1-\tau} \frac{N_{E,T}}{N_{Un,T}}$$

and

$$\forall_{i \in N_{Un,T}} \frac{\forall_j \left(w_{i,j,T}\right)}{\overline{\overline{w_T}}} < \alpha \frac{\tau}{1-\tau} \frac{N_{E,T}}{N_{Un,T}}$$

Given that the worker does receive an offer, he would accept it only if $w_{ijT} > \alpha \frac{N_E}{N_{Un}} \frac{\tau}{1-\tau} \overline{w_T} \Rightarrow \frac{w_{ijT}}{\overline{\overline{w_T}}} > \alpha \cdot \frac{N_E}{N_{Un}} \frac{\tau}{1-\tau}$. By the definition of an average, it follows that $\exists_{i,j} \frac{w_{ijT}}{\overline{\overline{w_{ijT}}}} < 1, \exists_{i,j} \frac{w_{ijT}}{\overline{\overline{w_{ijT}}}} > 1$. Thus,

$$\exists_i \forall \alpha \cdot \frac{1-N_{Un}}{N_{Un}} \frac{\tau}{1-\tau} < 1 \rightarrow N_{Un} > \frac{\alpha \cdot \tau}{1-\tau+\alpha \cdot \tau} \rightarrow N_E < \frac{1-\tau}{1-\tau+\alpha \cdot \tau}$$

$$\exists_i \forall \alpha \cdot \frac{1-N_{Un}}{N_{Un}} \frac{\tau}{1-\tau} > 1 \rightarrow N_{Un} < \frac{\alpha \cdot \tau}{1-\tau+\alpha \cdot \tau} \rightarrow N_E > \frac{1-\tau}{1-\tau+\alpha \cdot \tau}$$

and the marginal worker which decides whether to be employed or unemployed defines the difference. Thus in steady state:

$$N_{Un} = \frac{\tau \cdot \alpha}{1-\tau+\tau \cdot \alpha}, N_E = \frac{1-\tau}{\tau \cdot \alpha + 1 - \tau}, \frac{N_E}{N_{Un}} = \frac{1-\tau}{\tau} \cdot \frac{1}{\alpha}$$

and the benefit-wage ratio in the steady state is given by:

$$b_T = \frac{N_E}{N_{Un}} \tau \cdot \overline{w_T} = \frac{1-\tau}{\alpha} \overline{w_T}$$

$$\Rightarrow \frac{b_T}{\overline{\overline{w_T}}} = \frac{1-\tau}{\alpha}$$

Substituting $N_E = \frac{1-\tau}{\tau \cdot \alpha + 1 - \tau}$ in [t] yields:

$$Tax_T = N_E \cdot \tau \cdot \overline{w_T} = \frac{1-\tau}{\tau \cdot \alpha + 1 - \tau} \cdot \tau \cdot \overline{w_T}$$

from which can be derived the tax rate that maximizes tax revenues:

$\tau \middle| Tax_{MAX} = \frac{1-\sqrt{\alpha}}{1-\alpha}, \alpha \neq 1$, which decreases with α. When $\alpha = 1$, $Tax = (1-\tau) \cdot \tau \cdot \overline{w_T}$ which derives $\tau | Tax_{MAX} = 0.5$

Proposition 3

During convergence to the steady state, the average productivity of workers and the total output grow.

Proof:

Following proposition 1, the average skill level among employed workers increases with t and therefore so does their average marginal productivity. Hence, the economy's total output grows during convergence.

$$\frac{d\overline{a}_j}{dt} > 0$$

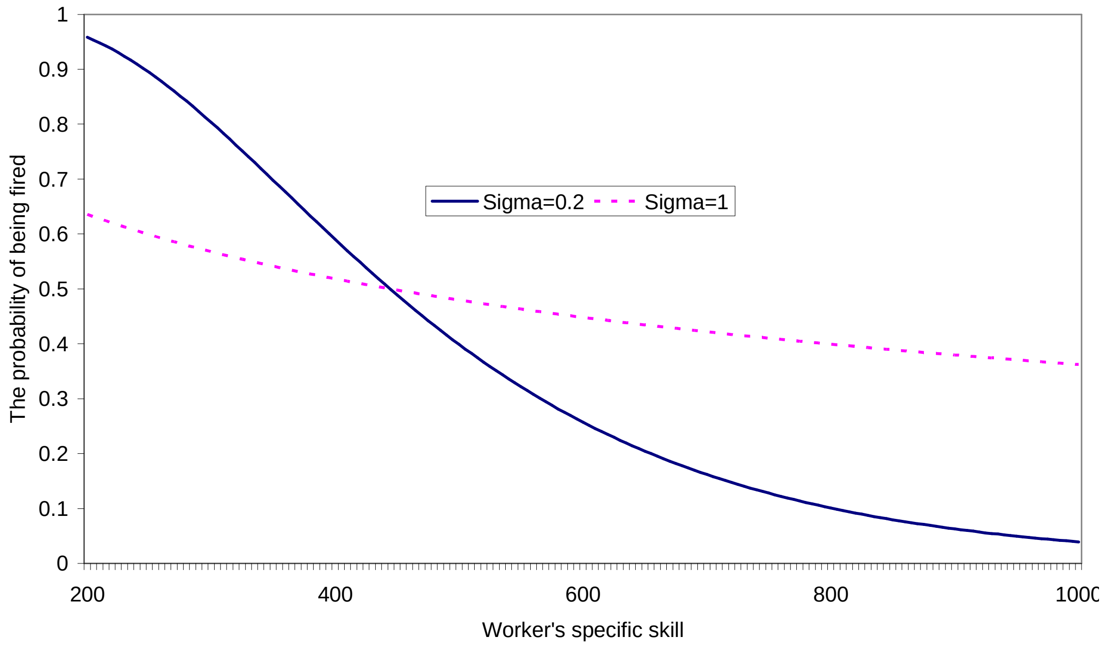
$$\frac{d\overline{a}_t \cdot \overline{a}_j e^{\varepsilon_t}}{d\overline{a}_j} = \overline{a}_t \cdot e^{\varepsilon_t} > 0 \Rightarrow \frac{dY_{jt}}{d\overline{a}_j} = \frac{1}{2}\overline{a}_t \cdot e^{\varepsilon_t} > 0$$

$$Y_t = \sum_{j \in J} Y_{j,t}$$

$$\frac{dm_{ij}}{d\overline{a}_j} = \frac{da_{ij}\overline{a}_j\overline{a}_t e^{\varepsilon_t}}{d\overline{a}_j} = a_{ij}\overline{a}_t e^{\varepsilon_t} > 0$$

$$\frac{dm_{i,j}}{dt}, \frac{dY_{j,t}}{dt}, \frac{dY_t}{dt} > 0$$

[Figure A.2 -The probability of being fired as a function of worker's specific skills]



The public sector and the economy efficiency

Proposition 4

The level of total output in the steady state decreases with θ.

$$\frac{dY_{SS}}{d\theta} \leq 0$$

Proof:

As already mentioned, the steady state is a situation in which every worker is employed where his value is the highest. However, in a situation where a share θ of firms provides its workers with greater employment protection and consequently raises their value, it becomes more common that workers with *EPL* reject higher-wage job offers from firms that do not provide *EPL* and workers without *EPL* accept lower-wage job offers from firms providing *EPL* whenever:

$$\begin{aligned}
&V^e(e_j) < V^e(e_G) \\
&(1-\tau)w_{ijt} + \frac{1}{1+r}\left[P(\omega_{ijt})w_{ijt} + \overline{P}(\omega_{ijt})V^u(u)\right] < \\
&< (1-\tau)w_{G,t} + \frac{1}{1+r}\left[(1-\delta_{G,t})w_{G,t} + \delta_{G,t}V^u(u)\right] \\
&\textit{and} \\
&w_{G,t} < w_{ijt}
\end{aligned}$$

Define the number of job offer rejections of this type in the economy as a whole as $R_{1,0}$ and the number of acceptances of this type as $A_{0,1}$. $R_{1,0}$ and $A_{0,1}$ are *positive functions* of θ:

$$\begin{aligned}
&R_{1,0} \equiv \Gamma \int_{i \in N_G} \left(\bar{I}_{G,k}(\theta) \middle| w_{i,k} > w_G)\right) di, \\
&A_{0,1} \equiv \Gamma \int_{i \in N_E} \left((I_{j,G}(\theta) \middle| w_{i,j} > w_G)\right) di \\
&\frac{dR_{1,0}}{d\theta} > 0, \frac{dA_{0,1}}{d\theta} > 0
\end{aligned}$$

where I ($\bar{I}$), as noted above, is an indicator that equals to *1* if a worker accepts (rejects) a wage offer and equal to *0* otherwise.

Note that:

$$\left(R_{1,0} = 0, A_{0,1} = 0\right) \Leftrightarrow \forall_i \left(w_{ijt} = Max(w_{ij})\right) \Leftrightarrow \forall_i \left(m_{ijt} = Max(m_{ijt})\right)$$

Consequently, *if a worker rejects a job offer with a higher wage she is also refusing to increase her individual productivity*. Alternatively, if a worker accepts a job offer with a lower wage she is also decreasing her productivity.

Thus, $\frac{d\overline{a}_{i,j,T}}{dR_{1,0}} \le 0$ and $\frac{d\overline{a}_{i,j,T}}{dA_{0,1}} \le 0$ and therefore $\frac{d\overline{m}_{i,j,T}}{dR_{1,0}} \le 0$ and $\frac{d\overline{m}_{i,j,T}}{dA_{0,1}} \le 0$.

Consequently, $\frac{dY_{ss}}{dA_{0,1}} \le 0$ and $\frac{dY_{ss}}{dR_{1,0}} \le 0$, i.e. total output decreases as well.

Formally:

$$\left.\begin{array}{l} \frac{dR_{1,0}}{d\theta} \ge 0, \frac{dA_{0,1}}{d\theta} \ge 0 \\ \frac{dY_{SS}}{dR_{1,0}} \le 0, \frac{dY_{SS}}{dA_{0,1}} \le 0 \end{array}\right\} \Rightarrow$$

$$\frac{\partial Y_{SS}}{\partial \theta} = \frac{dY_{SS}}{dR_{1,0}} \cdot \frac{dR_{1,0}}{d\theta} + \frac{dY_{SS}}{dA_{1,0}} \frac{dA_{0,1}}{d\theta} \le 0$$

∘

Intuitively, providing *EPL* distorts the efficient allocation of the factor of production, i.e. the workers, and the magnitude of the distortion increases with the share of firms that provide *EPL*.

Proposition 5

1. *In the steady state, providing some workers with EPL decreases the utility of the other workers in the economy.*
2. *The utilities of the workers who are being provided with EPL may be less than their utilities in a world without EPL and as θ increases the share of these workers increases.*

Proof:

The utility of the worker who does not have *EPL* is an increasing function of the wage and a decreasing function of the probability of being fired. Her wage is an increasing function of the firm's technology and that of the economy as a whole while the probability of being fired is a decreasing function of the firm's technology and that of the economy as a whole. A firm's level of technology is an increasing function of the average skill level of its workers while the economy's level of technology is an increasing function of the average skill level of all workers. In the steady state, the average skill level of the firm's workers is a decreasing function of θ(proposition 4). Thus, the utility of workers without *EPL* decreases with θ.

Formally*:*

$$\left.\begin{array}{l} \dfrac{dV^e(e_j)}{dw_{ij}} > 0, \dfrac{dV^e(e_j)}{d\overline{P}_{ij}} < 0 \\ \dfrac{dw_{ij}}{d\overline{a}_{jt}} => 0, \quad \dfrac{d\overline{P}_{ij}}{d\overline{a}_{jt}} < 0 \\ \dfrac{d\overline{a_{i,j}}}{d\theta} < 0 \ \ (proposition \ \ 4) \end{array}\right\} \Rightarrow$$

$$\frac{\partial V^e(e_j)}{\partial \theta} = \left\{ \left(\frac{dV^e(e_j)}{dw_{i,j}} \frac{dw_{i,j}}{d\overline{a}_{jt}} \right) + \left(\frac{dV^e(e_j)}{d\overline{P}_{ij}} \frac{d\overline{P}_{ij}}{d\overline{a}_{jt}} \right) \right\} \cdot \frac{d\overline{a}_{jt}}{d\theta} < 0$$

$$\frac{\partial V^e(e_j)}{\partial \theta} \equiv \eta_{ID}$$

where η_{ID} is the *indirect disutility* from the provision of *EPL*by some firms. Since it is always negative, the effect is also negative for workers in firms without *EPL*.

○

In order to examine the situation of workers who are provided with *EPL*, the direct positive effect of *EPL* needs to be taken into account. Thus,

$$\frac{\partial V^e(e_j)}{\partial EPL} = \eta_{ID} + \frac{dV^e(e_j)}{dEPL} =$$
$$= \eta_{ID} + \left[V^e(e_G) - \left(V^e(e_J^*) \middle| \theta > 0 \right) \right] \equiv \eta_{ID} + \eta_D$$

The first term represents the indirect effect of *EPL*, which decreases with θ, while the second is the direct effect, which increases with θ. Consequently, the net effect is unclear. What can be said is that the indirect effect of *EPL* is a kind of externality imposed through the inefficient allocation of the factors of production. In other words, the individual worker cannot change the situation through his behavior and takes it as given.

An alternative way of understanding the effect of *EPL* is the following. Workers with *EPL* change jobs only if they are offered a value higher than $V^e(e_G)$. Define the potential maximum value of a worker without *EPL* in the economy as a whole as:

$$\hat{V}^e(i, j^*, t \big| \theta = 0) \equiv Max_{j \in J} V_i^e(w_{ij}, a_{ij} \big| \theta = 0)$$

and with *EPL* as:

$$\hat{V}^e(i, j^*, t \big| \theta > 0) \equiv Max_{j \in J} V_i^e(w_{ij}, a_{ij} \big| \theta > 0).$$

Note that:

$$(w_{i,j}^{Max} \big| \theta = 0) \equiv Max_J(w_{ij} \big| \theta = 0) > Max(w_{ij} \big| \theta > 0) \equiv (w_{ij}^{Max} \big| \theta > 0)$$

since $\frac{\partial w_{ij}}{d\theta} = \frac{dw_{ij}}{d\overline{a_{ij}}} \cdot \frac{d\overline{a_{ij}}}{d\theta} < 0$.

As a result, it may occur that when θ firms provide their workers with *EPL*, then for worker *i* in firm *j* at time *T*:

$$\begin{aligned} &MaxV_i^e(w_{ij^*}, a_{ij} \big| \theta = 0) > MaxV_i^e(w_{ij}, a_{ij} \big| \theta > 0) \Rightarrow \\ &\left(V^e(e_j) \big| \theta = 0\right) > \left(V^e(e_j) \big| \theta > 0\right) \end{aligned}$$

Consequently some workers who have *EPL* (which increases their value given that other workers also have it) may be worse off compared to their potential value in an economy without *EPL*. This is an externality, due to the indirect distortion caused by *EPL*.

As θ increases, $(w_{ij}^{Max} \big| \theta > 0)$ decreases and consequently the probability of the situation in which $V_i^e\left((w_{ij}^{Max} \big| \theta = 0), a_{i,j}\right) > MaxV_i^e\left(w_{ij}, a_{ij} \big| \theta > 0\right)$ increases.

∘

Discussion:

The signs of the terms making up $\frac{\partial V^e(e_j)}{\partial \theta}$ can be reasonably assumed (as in proposition 5) and are generally agreed upon. The only assumption that needs to be examined is that of a *positive correlation* between the marginal productivity of one worker and the average skill level of his co-workers, which is directly derived from the production function [1]. This assumption leads to a *positive correlation* between the wage of one worker and the average skill level of his co-workers.

However, most of the models with exogenous growth assume that it is determined by exogenous technological progress, which enters the production function through the remainder *A*. In our model, as in others with endogenous growth, it is assumed that growth is driven by the increase in the average skill level of workers.

In a *standard matching model*, the effect of providing some workers with *EPL* is modeled as follows: firm *j* hires worker *i* iff $a_{ij}\overline{a}_{jt}\overline{a}_{t}e^{\varepsilon_t} > FV + C_t$. Due to the shortage in physical capital, the cost of creating a new vacancy is an increasing function of the number of workers in firm *j*: $FV = FV(\overset{+}{n})$. The shortage of capital may force the firm to fire lower-skilled workers and replace them with higher-skilled workers.

The firm chooses to fire a less-skilled worker *k* if it is more profitable than hiring the new worker *j* without firing the less skilled one or more formally when:

$$a_{ij}\overline{a}_{jt}\overline{a}_{t}e^{\varepsilon_t} - \left(FV(n+1) + C_t\right) < \left[a_{ij}\overline{a}_{jt}\overline{a}_{t}e^{\varepsilon_t} - \left(FV(n) - C_t\right)\right] - \left[a_{kj}\overline{a}_{jt}\overline{a}_{t}e^{\varepsilon_t} - \left(FV(n) + C_t\right)\right] - FC$$

$$\Leftrightarrow$$

$$FV(n+1) > a_{kj}\overline{a}_{jt}\overline{a}_{t}e^{\varepsilon_t} - C_t + FC$$

where FC is the cost of firing.[11] Note that it does not depend on the new worker's productivity.

If worker *k* has *FC* entitlement, and FC is very large, the firm chooses to hire the higher-skilled workers iff $a_{ij}\overline{a}_{jt}\overline{a}_{t}e^{\varepsilon_t} - \left(FV(n+1) + C_t\right) > 0$. Since FV increases with *n*, the firms that provide their workers with *FC* entitlement frequently give up and do not hire higher-skilled workers, i.e. they do not create a vacancy.

When a large number of firms provide their workers with *FC* entitlement the aggregate effect on growth and productivity becomes significant and according to the same logic used in the main model this may also reduce the utility even of workers who are provided with *FC* entitlement.

[11]One can view the equation $\begin{matrix} FV(n+1) > a_{kj}\overline{a}_{jt}\overline{a}_{t}e^{\varepsilon_t} - C_t + FC \\ a_{kj}\overline{a}_{jt}\overline{a}_{t}e^{\varepsilon_t} - C_t < -FC + FV(n+1) \end{matrix}$ as $\prod_e(\varepsilon) < -f + \prod_V$ which is identical to the equation on page 738 in Labor Economics.

Appendix B.

Simulation

The simulation is submitted in order to estimate the sensitivity of the results to the parameters' or variables' values.

At the beginning the calibration is described. Then the steady state outcomes are presented; afterwards the sensitivity analysis is described, at first by a macro outcomes' comparison and then by a micro comparison - the distributions of workers' skills across the sectors.

<u>Parameters calibration and data</u>

Number of Persons: 10,000; Number of Firms: 100; Number of period: 200.

<u>Policy Parameters</u>:

- The government sector asks to employ just workers whose skill's level is above the median worker – the worker's-selection by government sector=0.50;
- The probably of being fired in the government sector equals to zero - δ_G =0;
- Government wage relative to the average wage in the private sector=1;
- The ratio of the minimum wage to the average wage=0.45; (which is around the average ratio in developed countries.)
- Unemployment Insurance=1;
- The average level of the statutory income tax rate is 20 percent - τ =0.2; (in Israel the level is around 20 in the last four years).
- The share of the government's firms in the total firms - θ=0.2;

<u>Structural and behavior parameters</u>:

In the simulation a variation in two variables is added, the risk aversion and the leisure habit.

- The assumption is that the utilities functions are $CRRA\,U(w_{ijt}) = \frac{w_{ijt}^{1-\omega}}{1-\omega}$ in which the relative risk aversion parameter is distributed between 1 and 4 - $\omega \in U[1,4]$.
- The parameter of individual disutility from work plus the individual disutility from being unemployed is distributed between 0 and 0.5:

$\alpha \in U[0,.5]$

- In order to calibrate *Sigma* I have checked the standard deviation of a normalized (which its mean is around 20,000 which is in the same magnitude as the wage level) annual value of the level of the real total productivity if labor since 1965 in Israel (the data source is from the appendix of chapter 2 in the annual report of the bank of Israel).This parameter determines the correlation between the worker's own skill and the probability to be fired in the private sector; as it becomes higher the correlation becomes weaker.

σ =0.28;

- The aggregate return to skills in terms of wage in time t is normalized to 1. Half of the return is related to the worker own skill while the residual is equally divided between the specific firm's productivity which the worker is employed in and the productivity in the whole market:

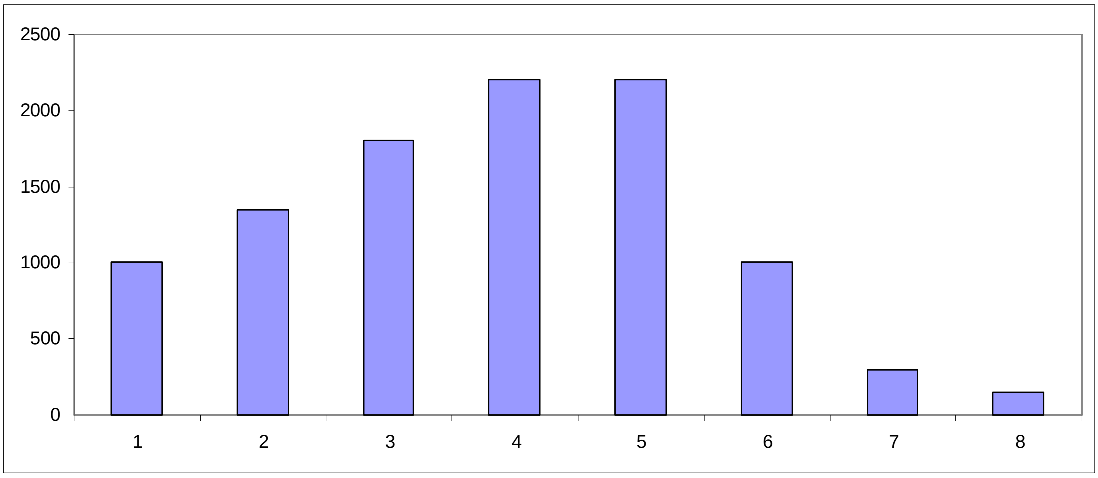
$\mu_1 = 1/2,\ \mu_2 = \mu_3 = (1-\mu_1)/2;$

- The surplus derived from job matching is equally divided between firms and workers (this is a standard value for this parameter in matching or search literature)

β =0.5;

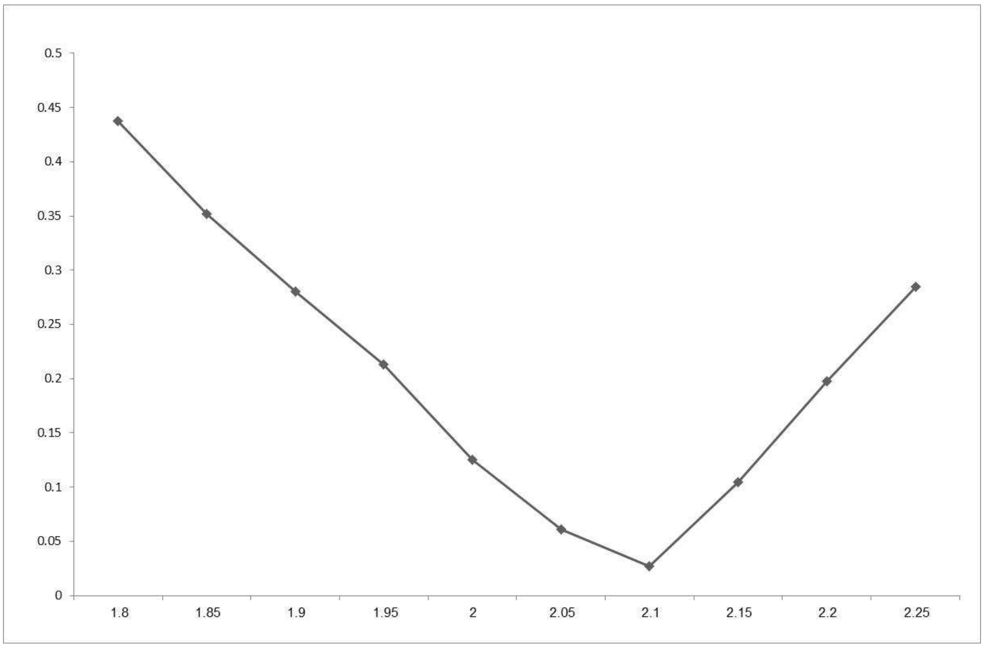
- The number of skills' order-levels among workers is eight and the levels are distributed with right tail as it is shown by the next figure.

Workers' skills distribution (8 levels)



- Given the number of skill's order-levels among workers the return to the workers' relative skills-order was calibrated to minimize a loss function. The loss function is a standard one. It contains two parameters derived from the empirical log-monthly-wage distribution in the private sector given the number of years of schooling of the workers – the average and the standard deviation. The function provides both parameters with the same weights. $Loss\_founction: Min\left(\left(\overline{w} - \hat{E}(w)\right) + \left(\sigma_w - \hat{\sigma}_w\right)\right)$ where $\overline{w}$ and σ_w are the average wage and the standard deviation wage in the simulation which are both function of the skill's return; $\hat{E}(w)$ and $\hat{\sigma}_w$ are empirical values of 15 years schooling workers at the private sector in Israel; their values are derived from CBS incomes surveys since 1995.

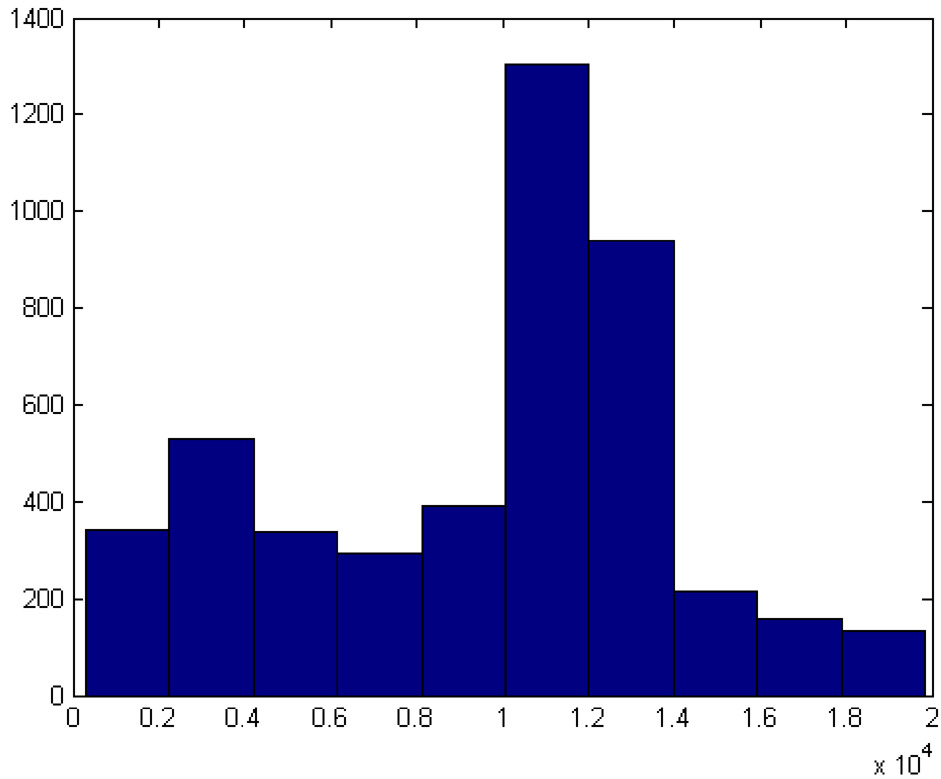
The loss function as a function of skills' return is described in the following figure:



The calibration result is that the Skills' Return across levels=2.08;

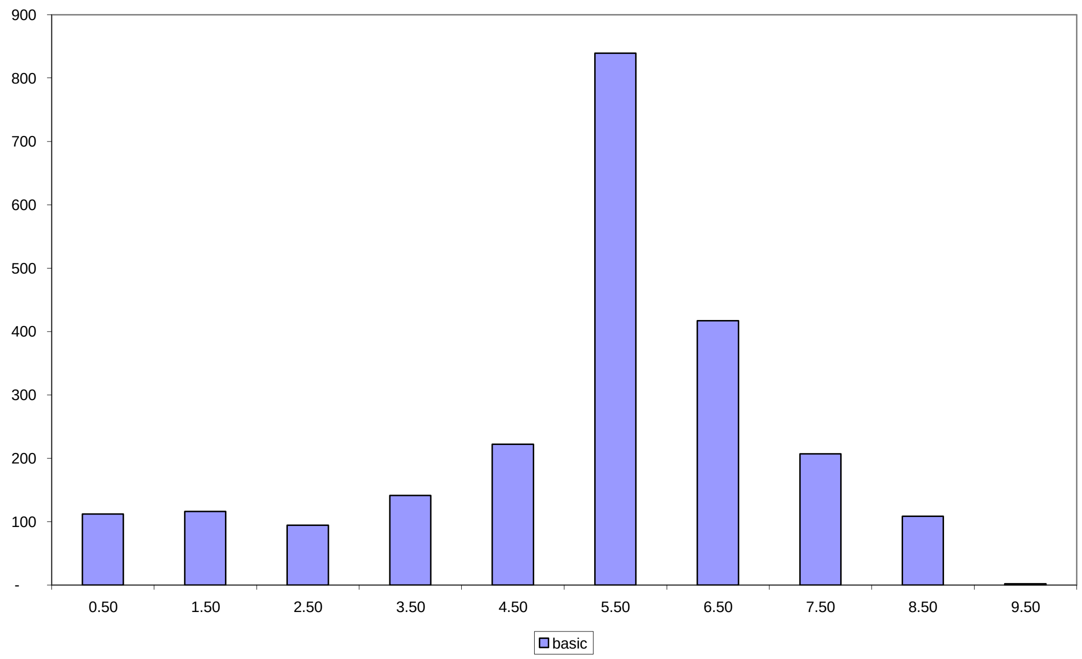
- Hiring intensity: the number of job's offers per firm per one iteration=15 which means that $\Gamma = 1-(1-\lambda)^J = 1-(1-\frac{15}{10{,}000})^{100} \cong 0.14$

The wage histogram in the private sector (created by the simulation)

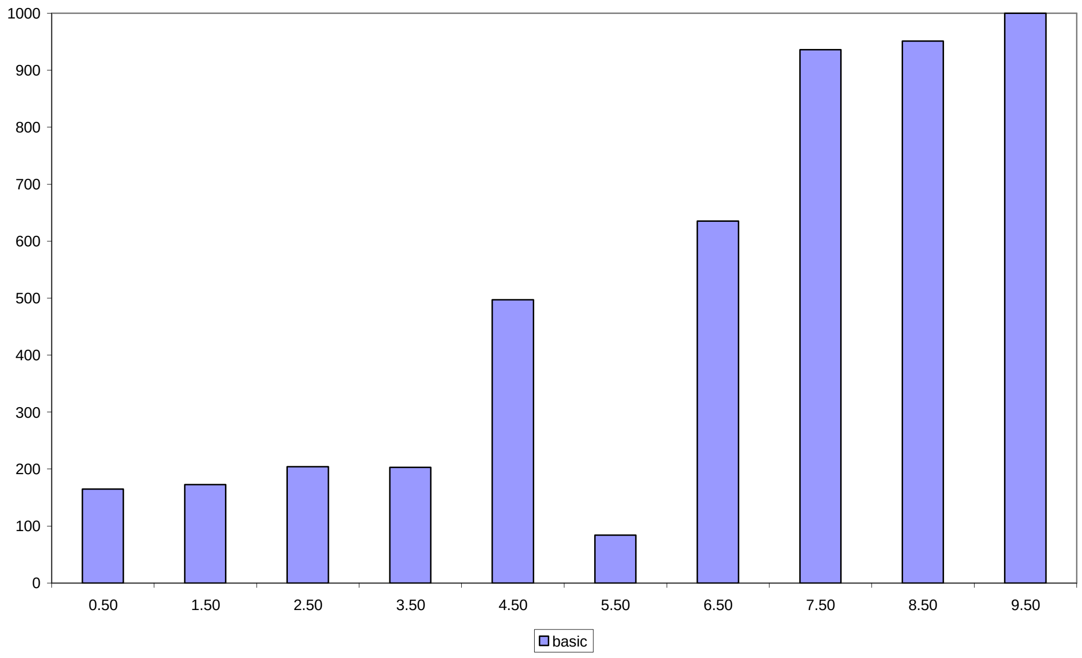


The self-selection of the workers during the convergence to the steady state (which is highly robust threw the values of the calibration) causes the final skills'-allocation to be shown as described in the next three figures. Workers characterized with medium skills are employed at the government sector; brilliant workers prefer to be employed at the private sector; and weak workers are unemployed. The workers selection form public to private sector respect to worker's skill is characterized by *U* shape, the weak skilled workers as well as the high skilled workers are selected out.

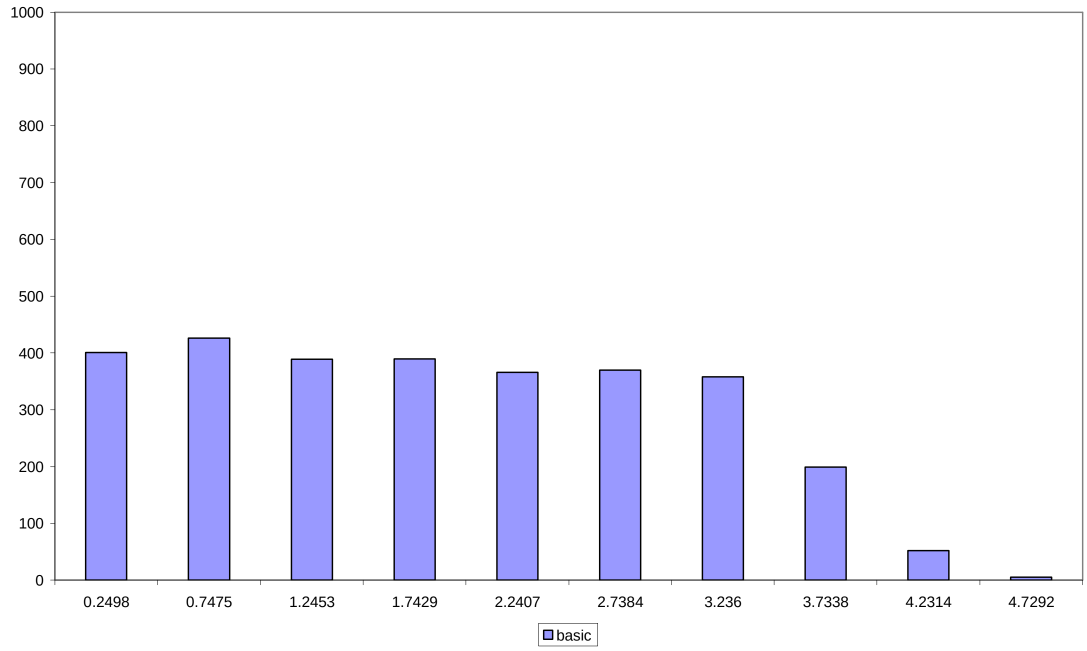
Government-workers-skills distribution



Private sector-workers-skills distribution



Unemployment skills distributions



Sensitivity Analysis

The following tables present the outcomes of a steady states' comparison respected to changes in some key variables. The variables are divided into two groups: policy variables and structural variables. Afterwards, the histograms of the workers' skills across the sector in the steady state, before and after the change in the variables, shed light on the transmission behind the results.

	Total Productivity	Unemployment	Government average skills	Private average skills	Government emloyment	Private Employment	Average Wage in the private sector	Benefit
Basic	17,238	35%	6,993	12,504	2,261	4,269	9,679	3,645
Policy Paramters								
No government	19,200	31%	-	10,438	-	6,941	8,842	4003.4
Average Wage 0.9	17,957	34%	6,075	12,281	1,780	4,849	9,794	3,869
Average Wage 1.1	16,091	33%	8,124	12,462	3,049	3,602	9,252	3,669
Average Wage 1.8	12,613	31%	10,654	9,678	4,835	2,068		
Delta=0.02	18,495	31%	5,237	11,358	978	5,907	9,370	4,133
UI=0.5	17,449	34%	7,007	12,282	2,296	4,327	9,681	1,895
Tax=0.4	17,397	38%	7,416	13,395	2,456	3,788	10,158	6,721
Minimum Wage=0.25	17,220	33%	6,797	12,329	2,189	4,544	9,534	3,956
Structural Parmeters								
leisure 0.5	17,286	39%	7,600	13,742	2,508	3,624	10,209	3,242
Ability Return 1		24%			1,024	6,597		
Umu=0.25	15,562	21%	6,439	10,148	985	6,893	7,681	5,700
Betta=1	17,415	34%	7,523	12,481	2,455	4,100	19,697	7,367
Sigma=0.5	17,283	34%	7,066	12,288	2,287	4,361	9,518	3,738
Search intensity	16,058	28%	6,040	11,699	2,408	4,771	8,884	4,587

	Productivity	Unemployment	Government average skills	Private average skills	Government emloyment	Private Employment	Average Wage in the private sector	Benefit
Basic	1	1	1	1	1	1		
Policy Paramters								
No government	11%	-4%		-17%		-	-9%	10%
Average Wage 0.9	4%	-1%	-13%	-2%	-21%	14%	1%	6%
Average Wage 1.1	-7%	-1%	16%	0%	35%	-16%	-4%	1%
Average Wage 1.8	-27%	-4%	52%	-23%	114%	-52%		
Delta=0.02	7%	-4%	-25%	-9%	-57%	38%	-3%	13%
UI=0.5	1%	-1%	0%	-2%	2%	1%	0%	-48%
Tax=0.4	1%	3%	6%	7%	9%	-11%	5%	84%
Minimum Wage=0.25	0%	-2%	-3%	-1%	-3%	6%	-1%	9%
Structural Parmeters								
leisure 0.5	0%	4%	9%	10%	11%	-15%	5%	-11%
Ability Return 1	-	-11%	-	-	-55%	55%		
Umu=0.25	-10%	-13%	-8%	-19%	-56%	61%	-21%	56%
Betta=1	1%	0%	8%	0%	9%	-4%	104%	102%
Sigma=0.5	0%	-1%	1%	-2%	1%	2%	-2%	3%
Search intensity	-7%	-6%	-14%	-6%	7%	12%	-8%	26%

The total productivity loss of a market with a government sector compared with a market without government is about 11 percent; the source of this productivity loss, as it is detailed in the theoretical model, is the inefficient allocation of the workers. However, further workers are now employed thus the average skill of the private sector workers as well as the average wage decrease.

A decrease in the government wage by 10 percent causes the employment in the public sector to decease by 13 percent which causes the total productivity in a steady state to increase by 4 percent. The average skills of the public sector decreases by 13 percent and the private sector employment increases by 14 percent. However, an increase in the government wage by the same percent causes inverted results but not at the same magnitude (ever larger impact). This is to show that the effect of changing the level of the wage in the government sector has no symmetry result.

The example in which the public sector wage is increased by 80 percent is made to demonstrate that if a work-place in the public sector would offer its workers a wage that great than the average wage in the private sector by 80 percent, it would attract most of the better skilled workers; except the last group of the workers (see details at the histograms).

An interesting finding is that reducing the employment protection legislation (*EPL*), which is done by increasing the probability of dismissals in the public sector by 2 percentage points (about fifty percent compared with the private sector average dismissals' probability), has very strong effect. Weakening *EPL*, which is the main relative advantage of working at the public sector, causes a decrease of 57 percent in the public sector employment.

Reducing the unemployment insurance (UI) has a very light impact. This hints that most of the unemployment share in the model is due to the workers' demand.

Given the calibration values, the results are not sensitive to the tax rate although the signs of its impact are conventional. The minimum wage has also weak effect. Reducing the ratio between the minimum wage and the average wage enables less skilled workers to be employed at the private sector instead of being unemployed or employed at the public sector. It also decreases the probability of being fired in the private sector which it makes the employment at the private sector to be more attractive compared with the public sector.

<u>Structural variables</u>

Raising the leisure preference has an interesting effect. Although it makes the immunity from dismissals in the public sector to be less attractive (because the option to be unemployed is now better off); it also increases the level of unemployment while less skilled workers now prefer to be unemployed. This channel causes the average wage in the private sector to increase by 5 percent and causes the option to be employed at the public sector to be wealthier. This is reflected by the histogram which depicts the skilled distribution at the public sector before and after the change.

Reducing the return to the order-level of the skills of the workers (the workers are divided into eight ordinary groups) as well as reducing the return to the worker own skills μ_1 has similar effect – they are both diminish the diverse in the wage distribution and enables less skilled to be employed at the private sector; this channel decreases sharply the average wage in the private sector which causes the public sector wage to decrease by the same rate. However, raising Beta, which represents the workers bargaining power when setting their wage, causes the average wage to increase which makes the public sector employment to be more attractive.

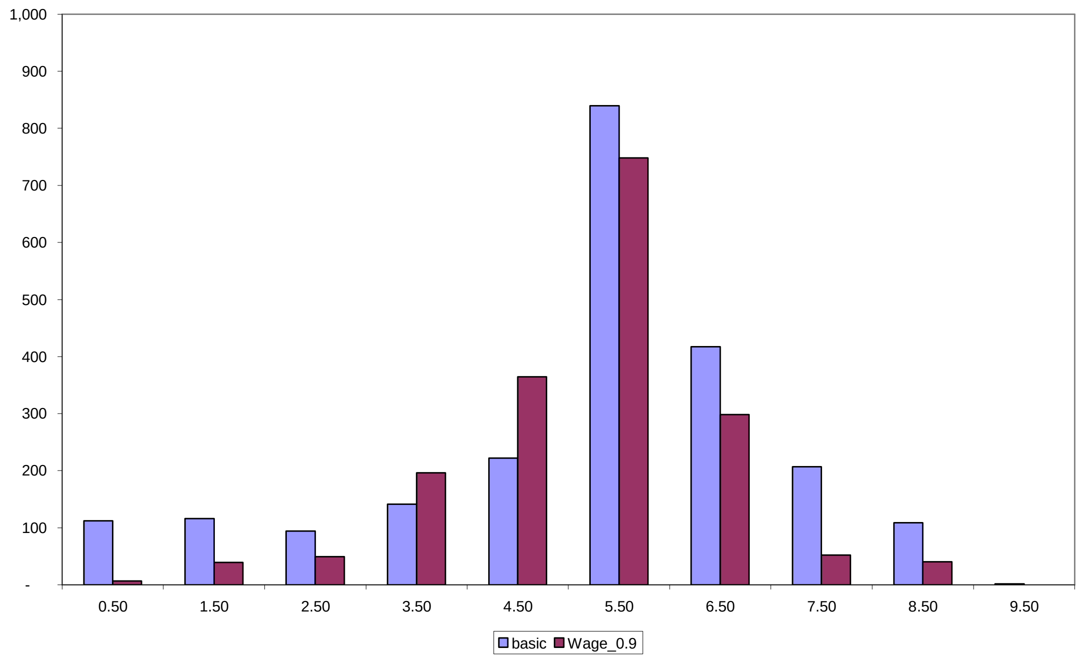
Raising Sigma, which it is the parameter that determines the probability of dismissals in the private sector as a function of the workers own skills (see figure A.2) reduces the difference between the average levels of the workers skills between public and the private sector. However, it does not have significant impact on the level of employment in each sector.

Finally, raising the hiring intensity parameter causes more workers and particularly weak skilled workers to find a job place early. This is reflected by a decrease in the unemployment rate, a decrease in the workers' average skills, and a decrease in the average wage.

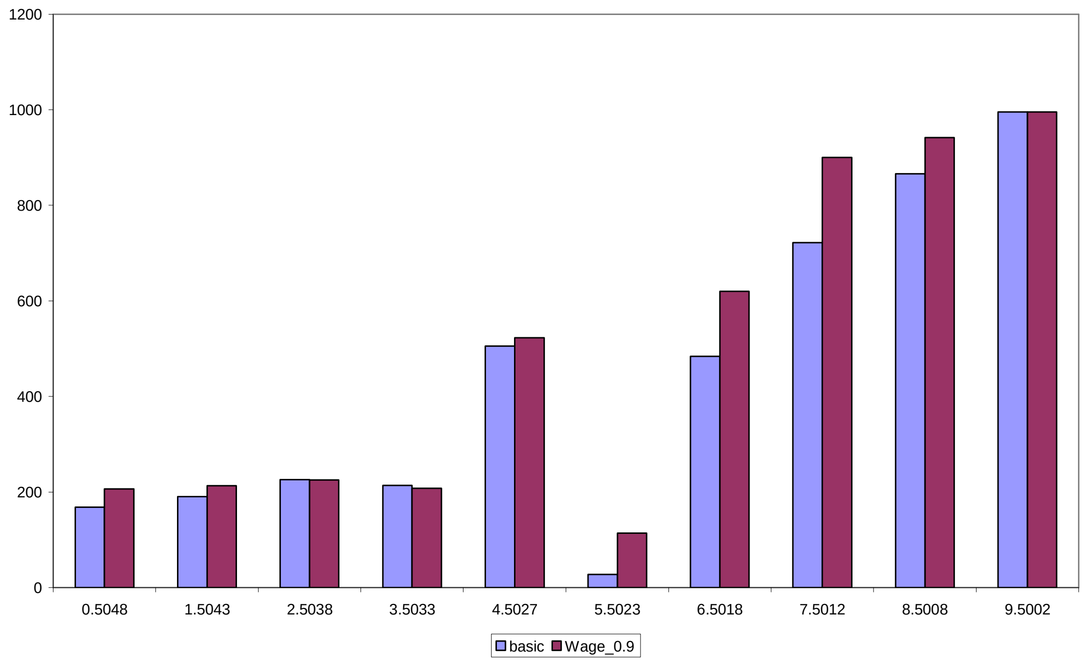
The following figures present the changes in the skills distributions due to changes in the policy variables or structural variables

<u>Government wage effect</u>

Government-workers-skills distribution

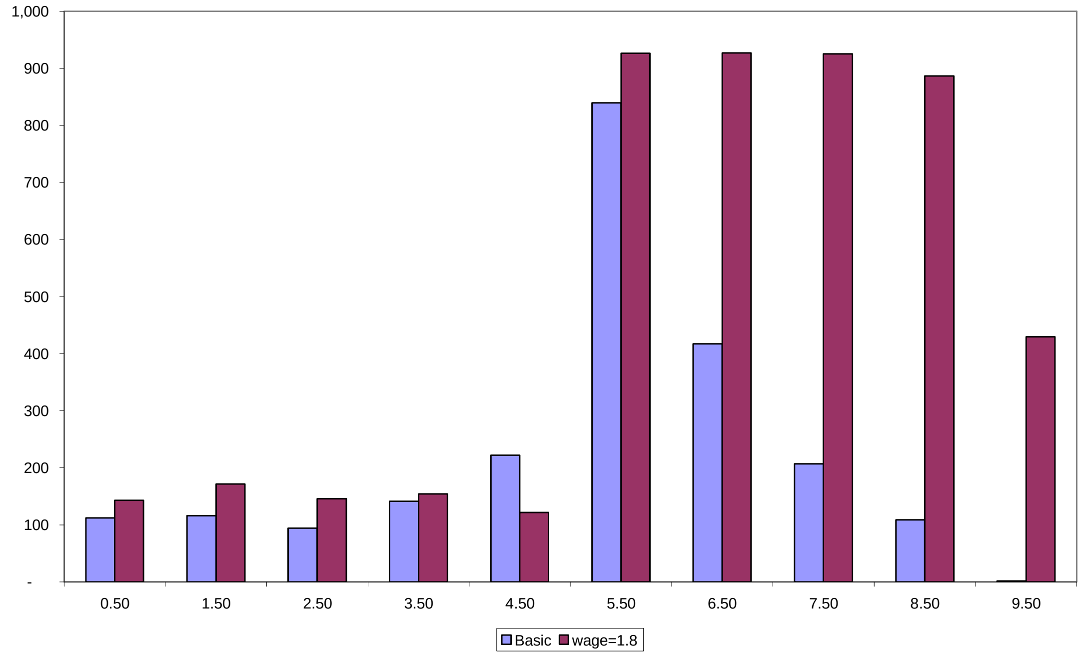


Private sector-workers-skills distribution

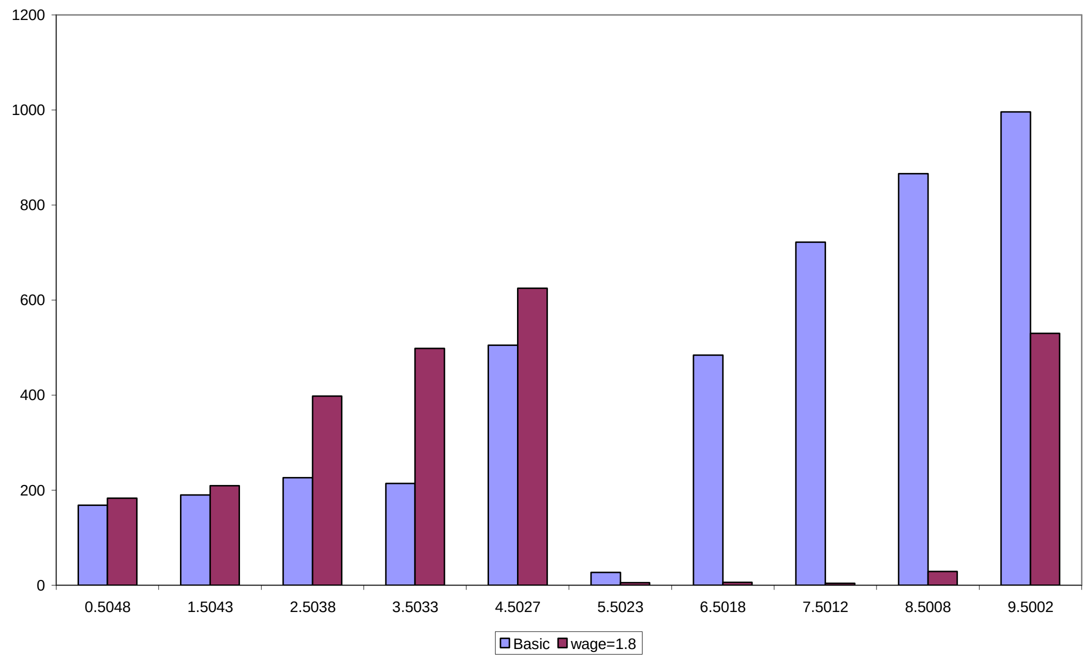


Government wage effect

Government-workers-skills distribution

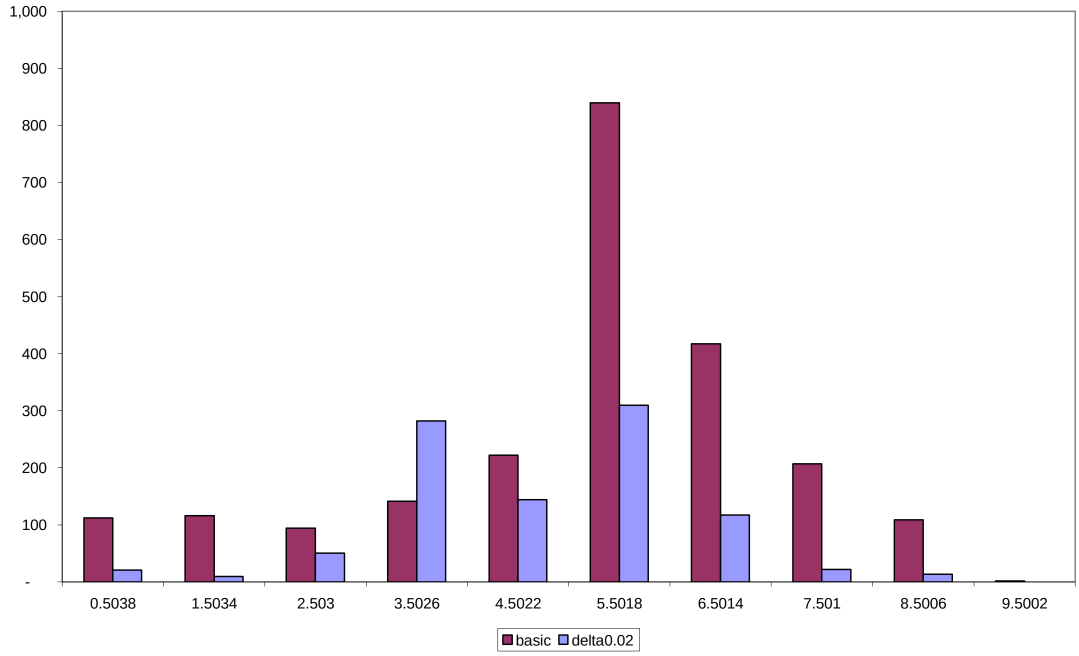


Private sector-workers-skills distribution

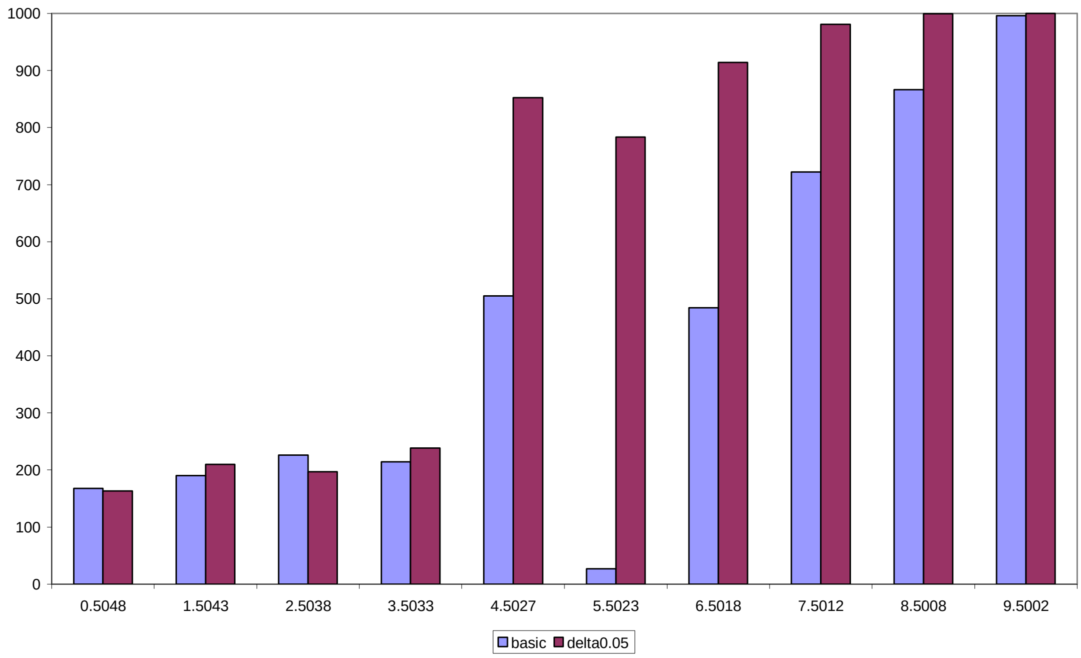


Probability of dismissals in the public sector effect

Government-workers-skills distribution

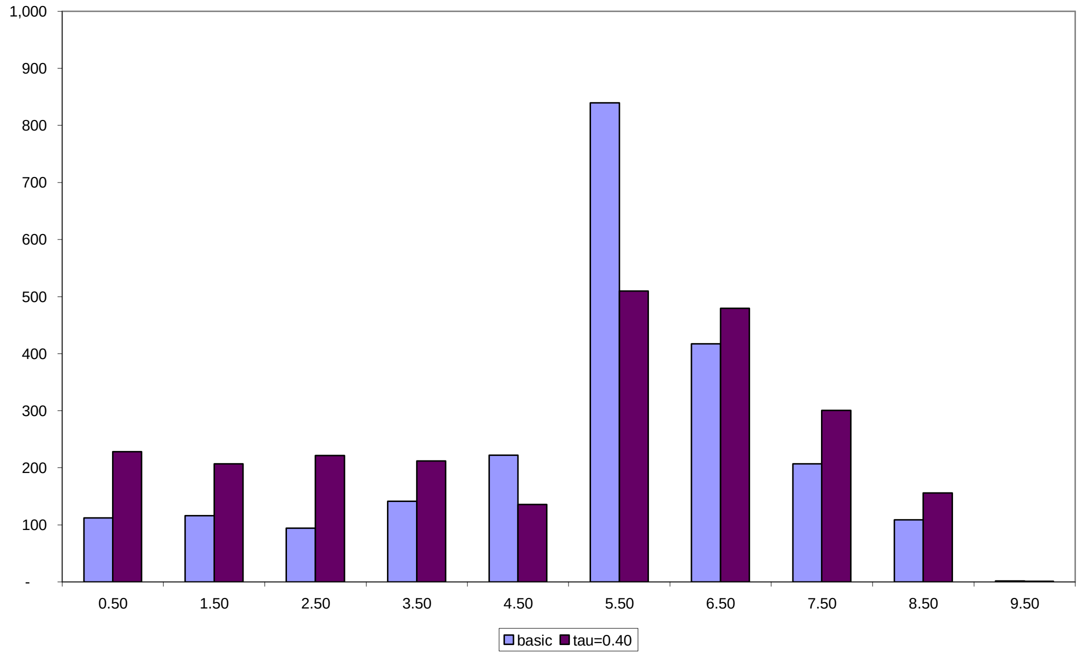


Private sector-workers-skills distribution

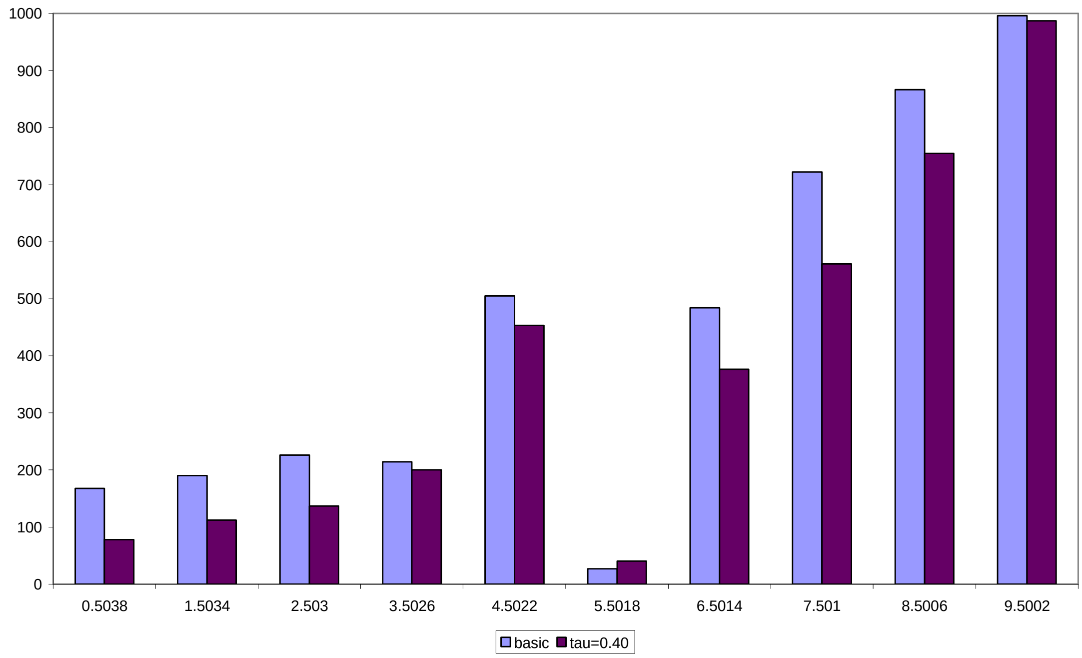


Tax level effect

Government-workers-skills distribution

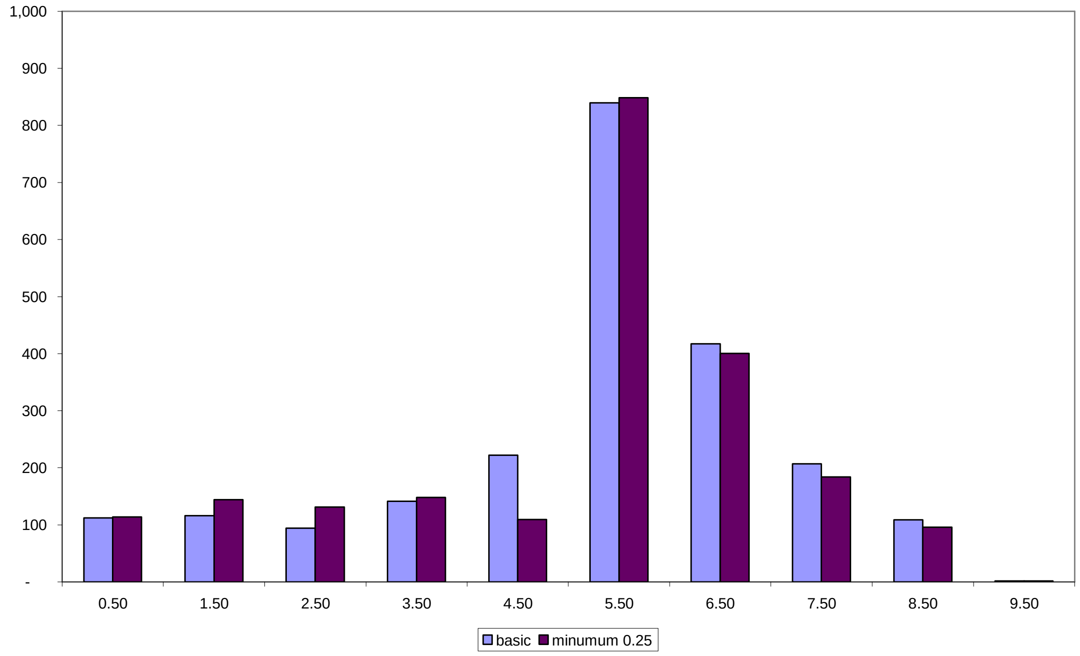


Private sector-workers-skills distribution

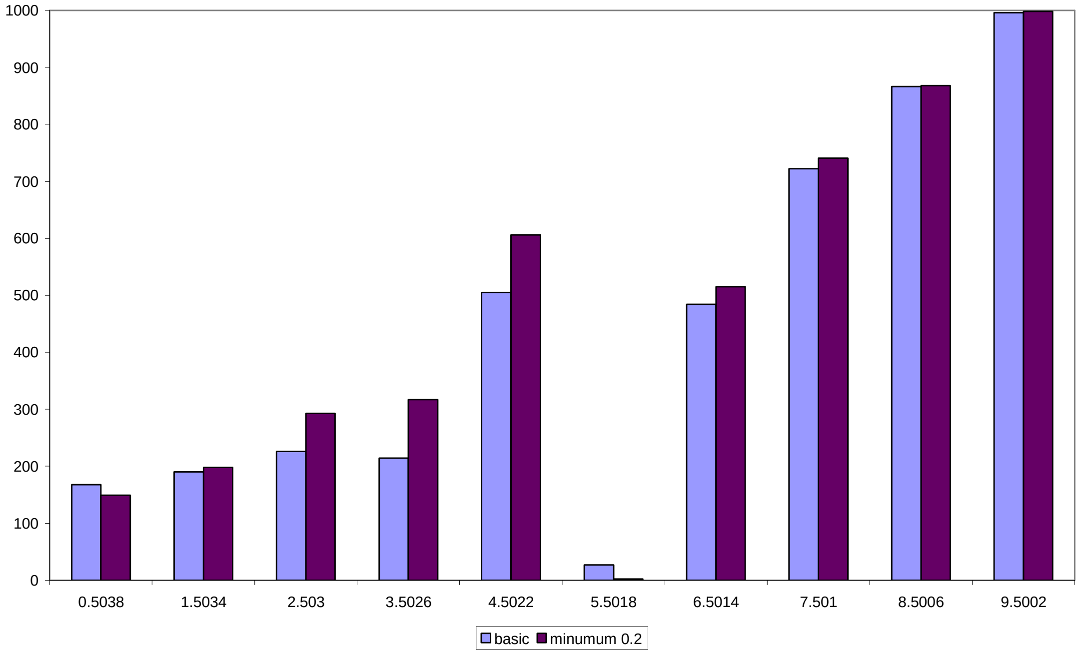


Minimum Wage effect

Government-workers-skills distribution

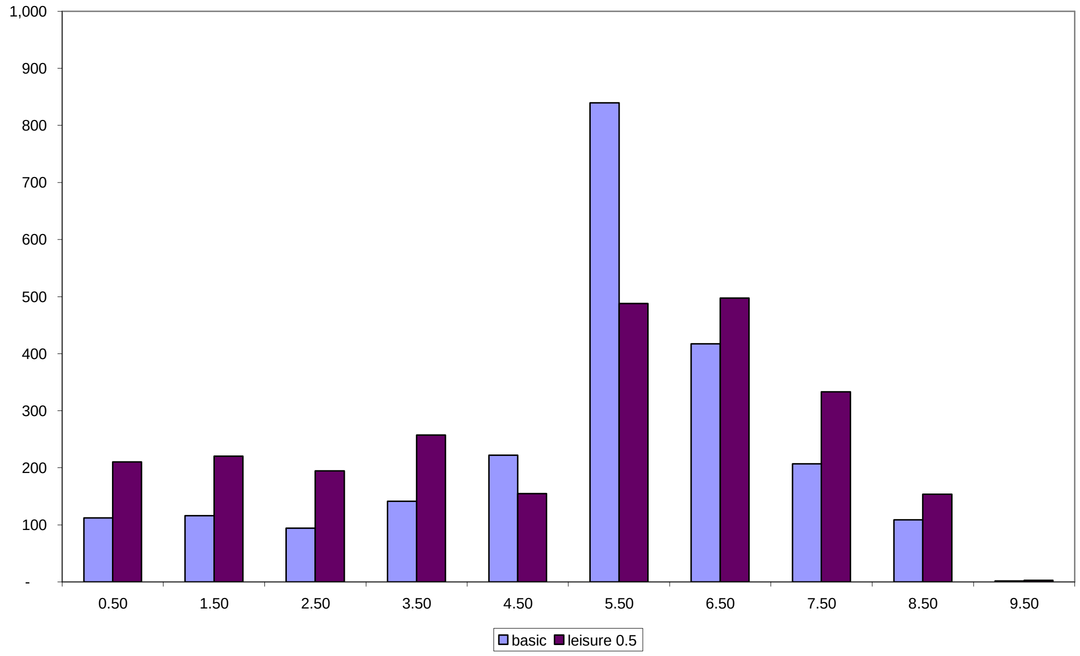


Private sector-workers-skills distribution

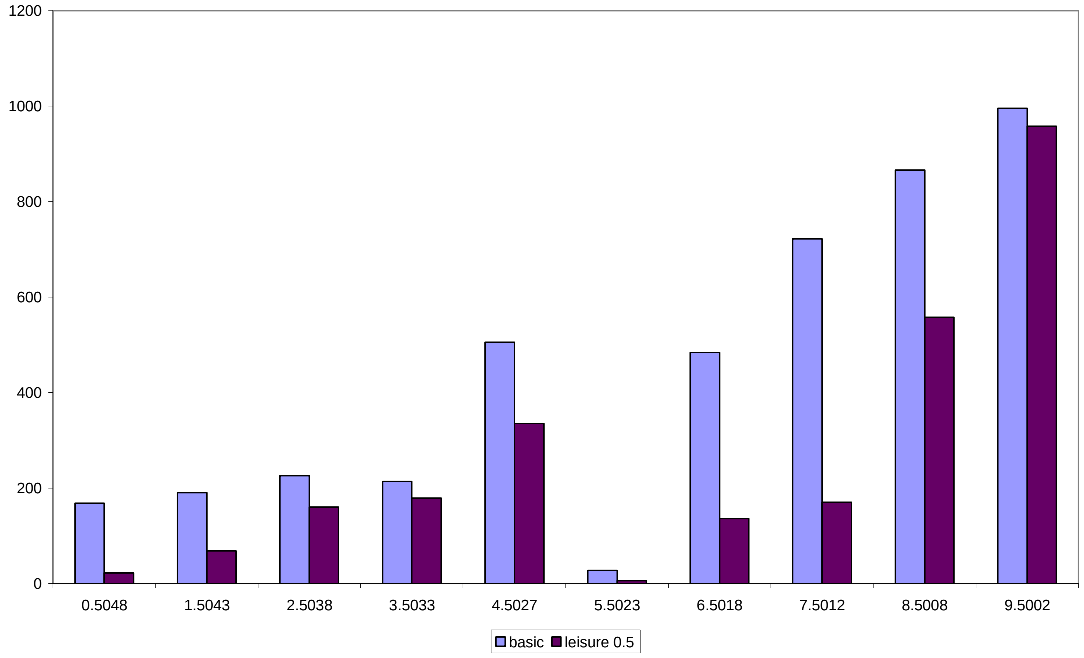


Leisure effect

Government-workers-skills distribution

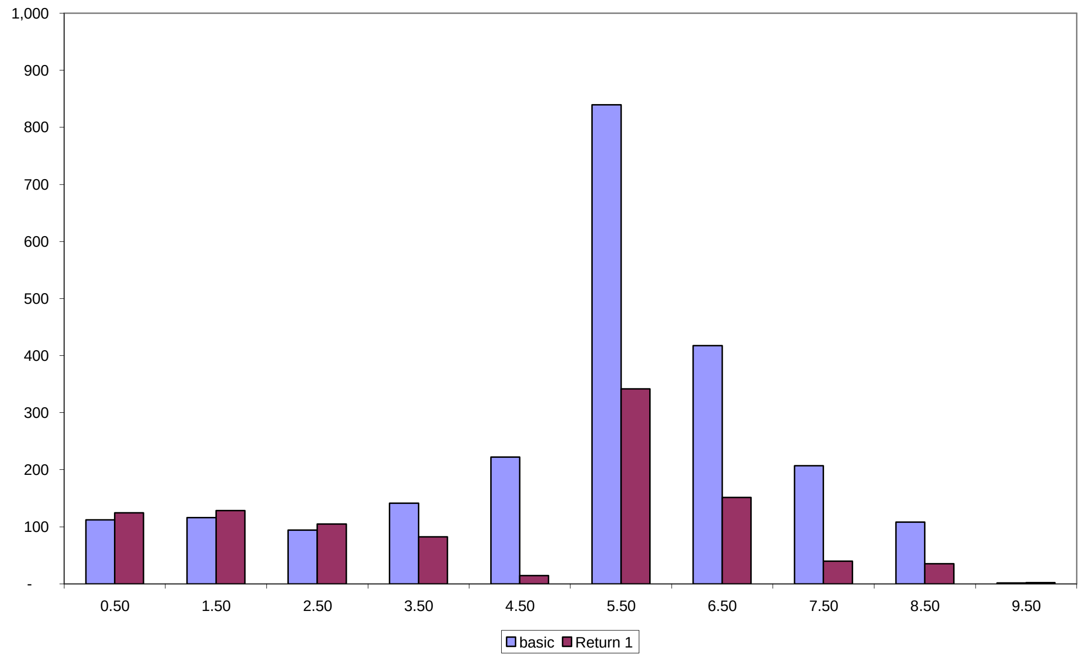


Private sector-workers-skills distribution

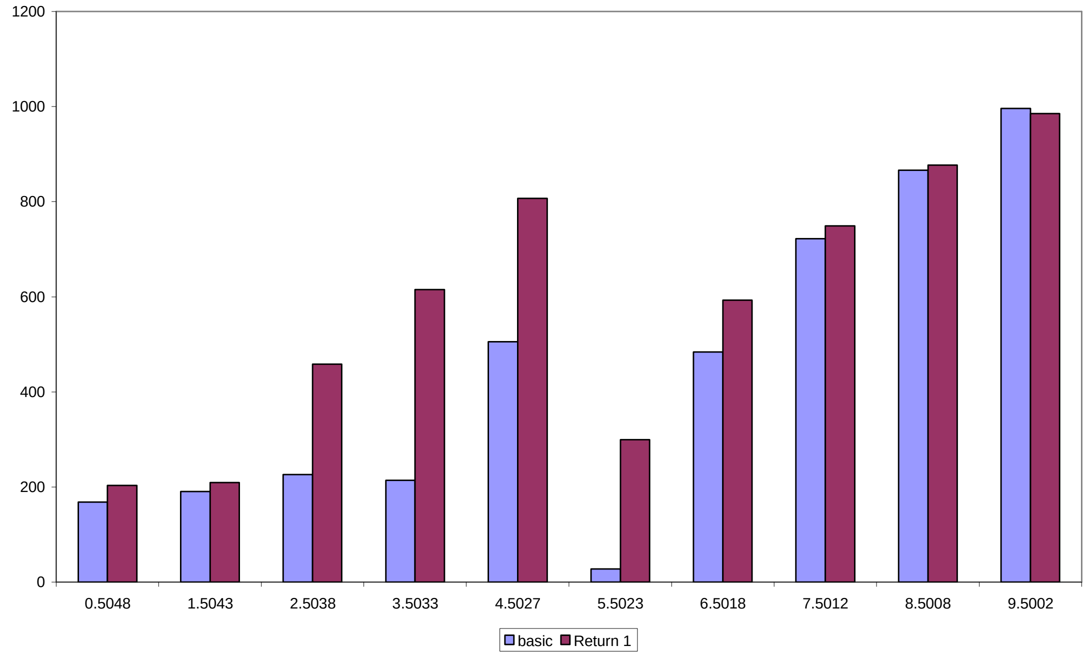


Ability Return effect

Government-workers-skills distribution

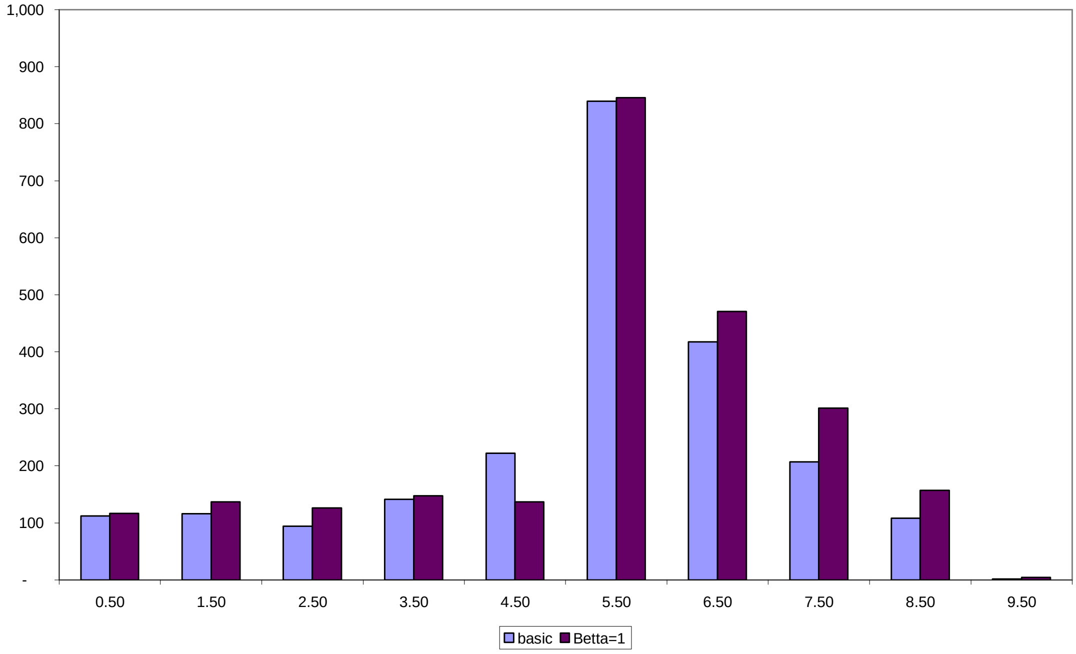


Private sector-workers-skills distribution

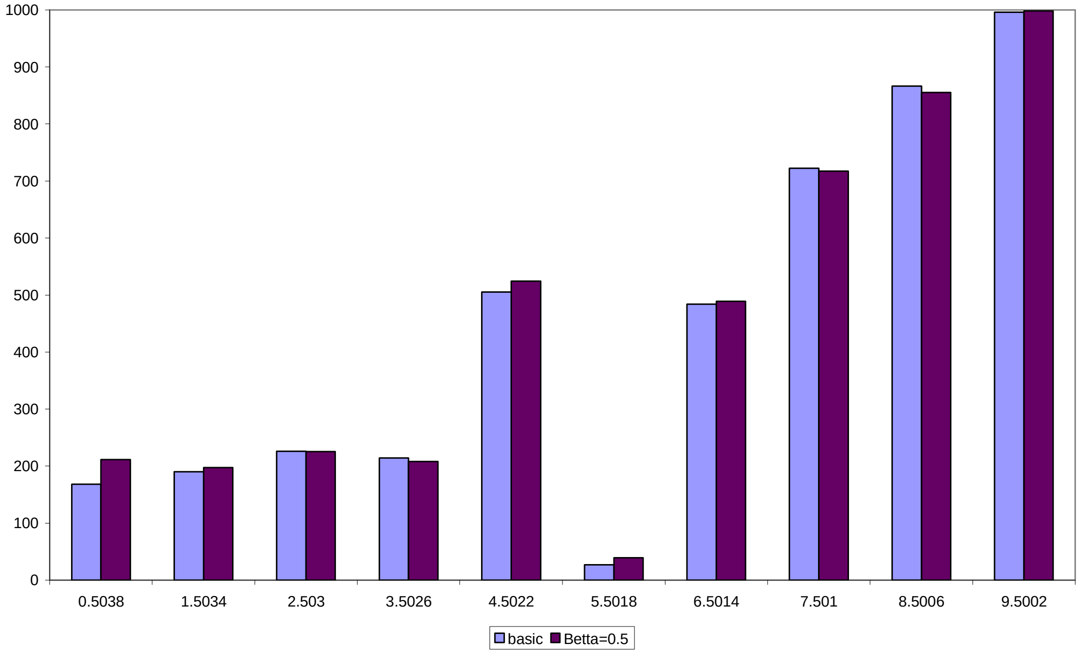


Beta effect

Government-workers-skills distribution

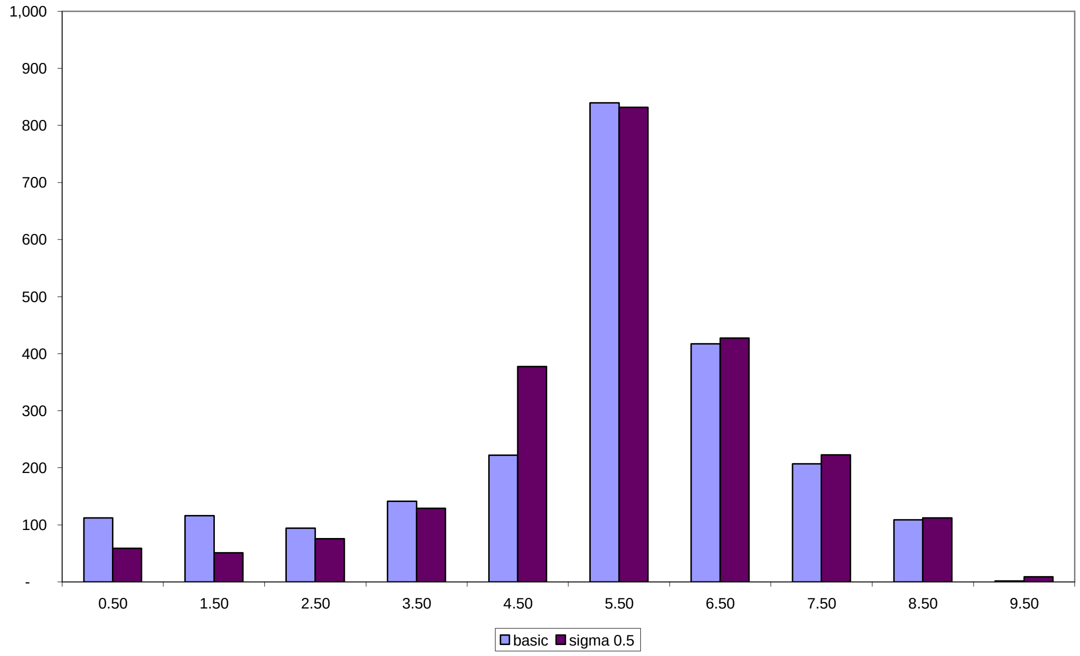


Private sector-workers-skills distribution

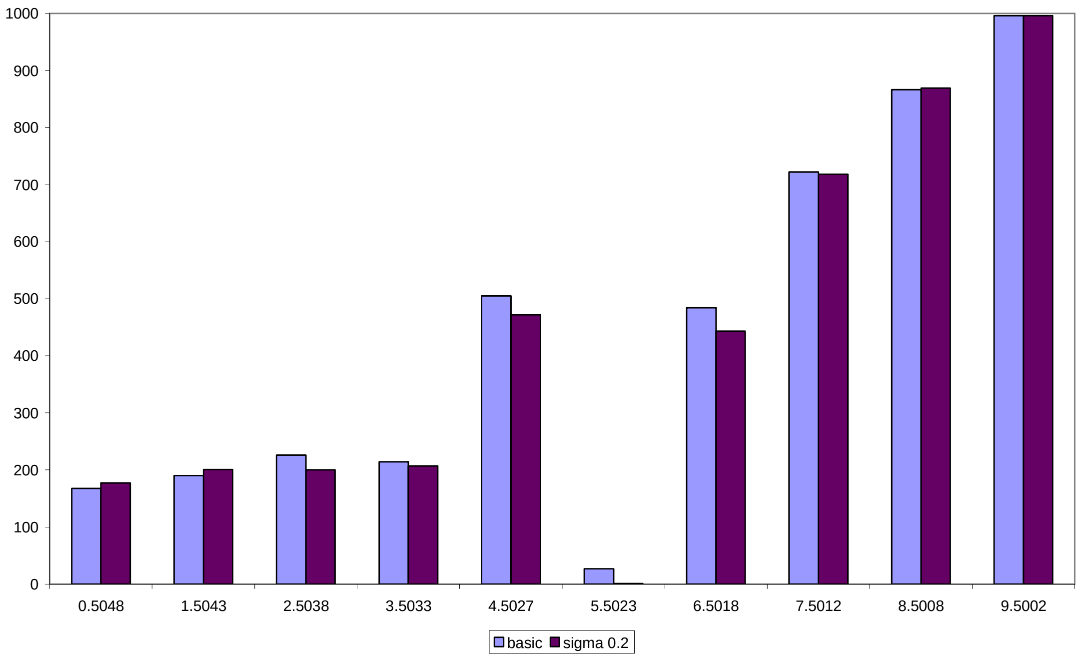


Sigma effect

Government-workers-skills distribution

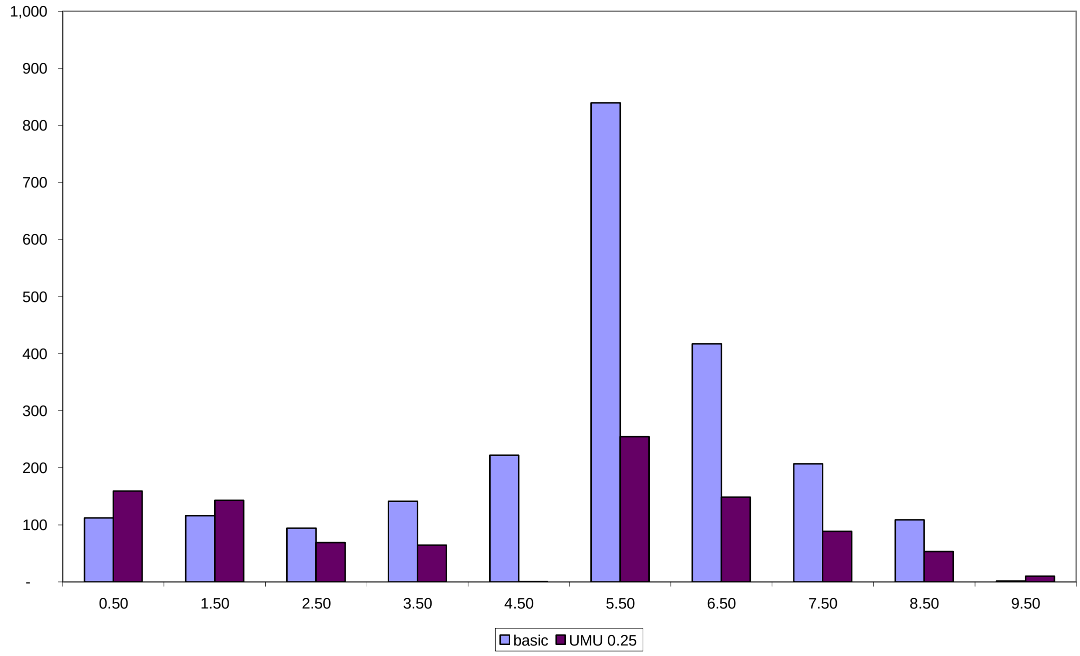


Private sector-workers-skills distribution

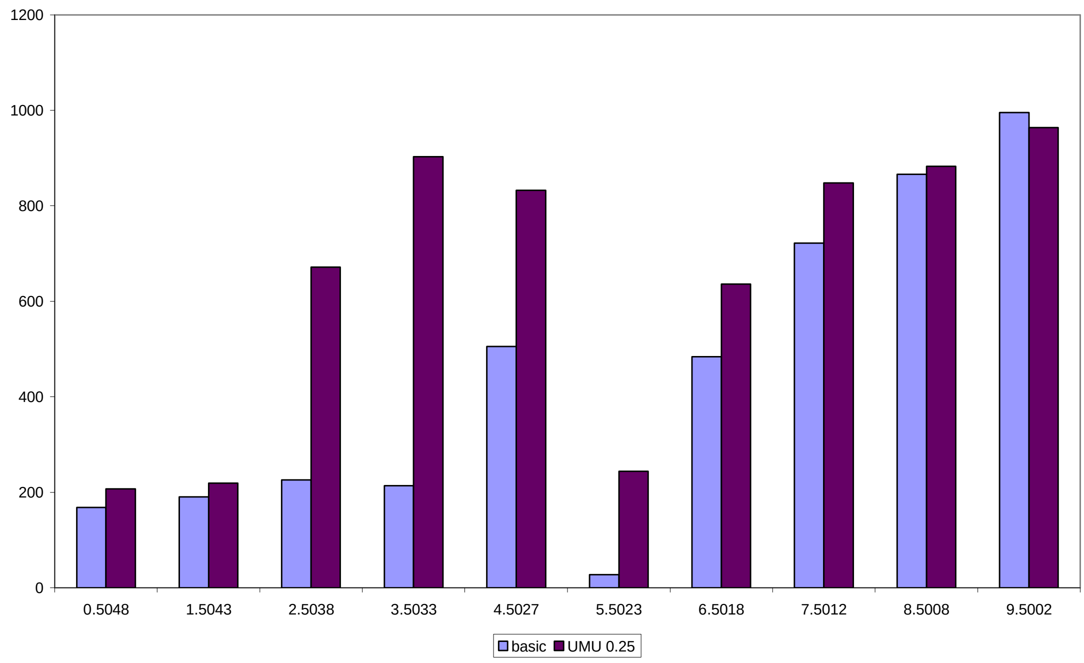


Umu effect

Government-workers-skills distribution



Private sector-workers-skills distribution



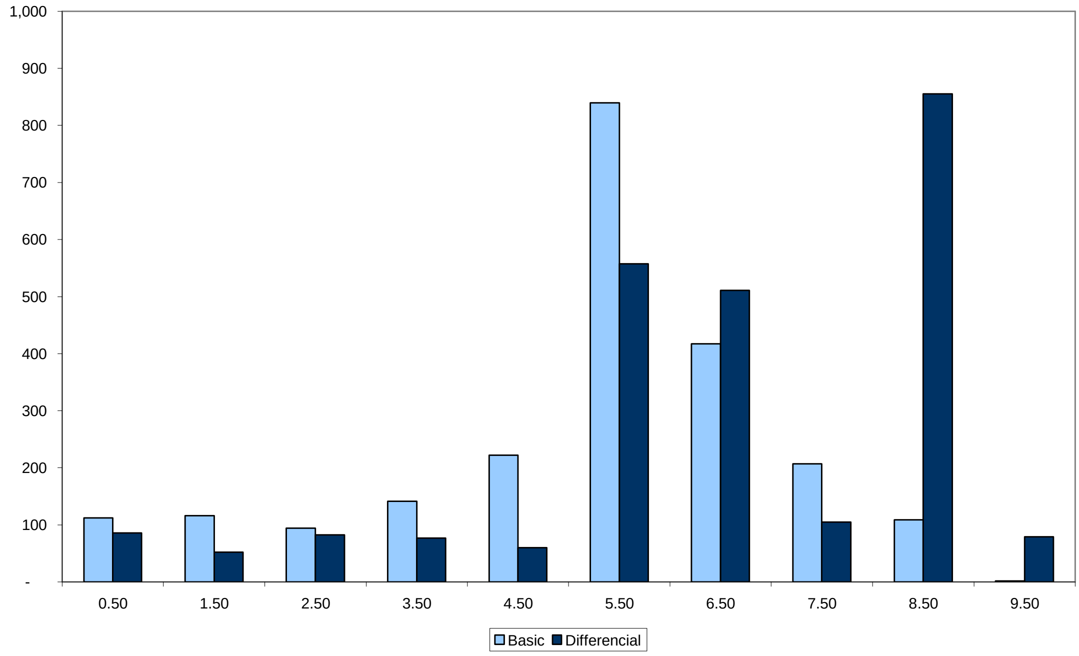
The difference between workers skills across the sectors is very prominent and it is easily observed by examining the histograms; the large gap in favor to the private sector is very robust.

One can ask what the public sector can do in order to reduce this gap. Given that the public sector is acting under defined budget it cannot raise the wage for all of its workers; it cannot even raise the wage and cut its employment so keeps its wage-bill stable because it is plausible to assume that the public sector is committed to a minimum production which cannot be provided below a crucial level of employment. (This is to say that a worker with given productivity in the public sector cannot replace completely two workers with half productivity compared with her.)

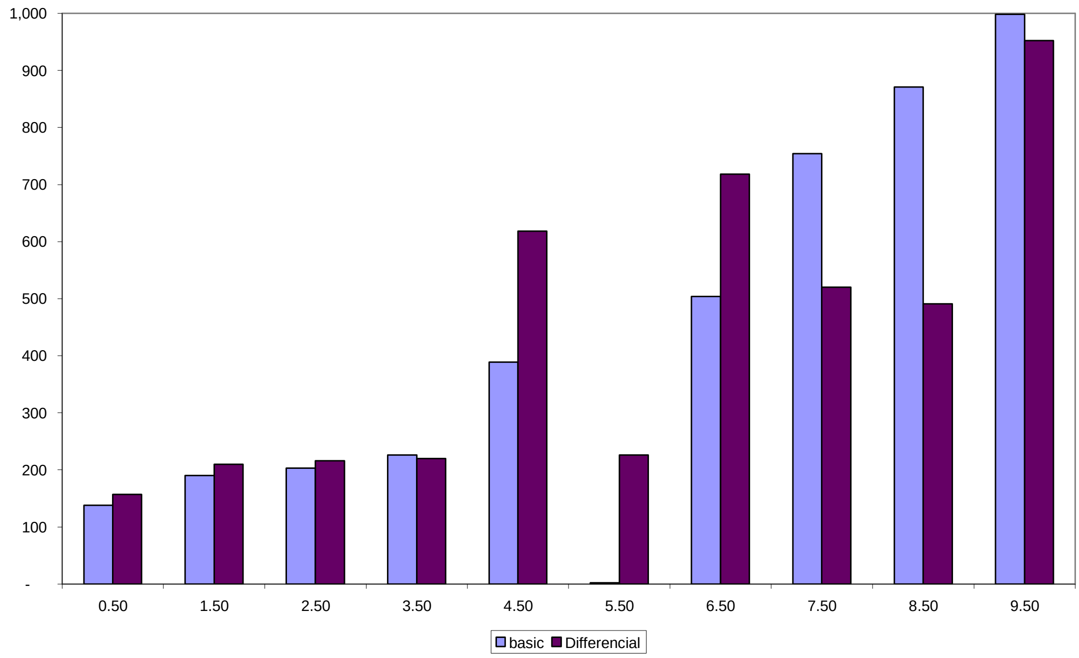
One option that the public sector can try is to suggest its workers two levels of wage; note that assuming three levels or more is not credible because it contradicts the main idea of the model in which the public sector cannot observe precisely its workers skills. In the following example the public sector suggests the top-quarter-skilled-workers a higher wage and to the third-quarter-skilled-workers a lower wage. (Remember that the public sector fires the first and the second bottom-quarters-skilled-workers). Note that the increase in the wage does not have to be equals to the decrease –in absolute terms, actually, given the wage increase the value of the decease is the value which causes the public sector wage-bill to be more or less as it was in the basic scenario. In that way the public sector can sharply raise its workers' skill and keeps its wage-bill stable. As the gap between the two levels of the wage becomes higher the workers' average skills in the government sector becomes higher, but the total productivity loss becomes higher as wall.

Differential Wages (Raising by 1.2, and reducing by 1.12)

Government-workers-skills distribution



Private sector-workers-skills distribution

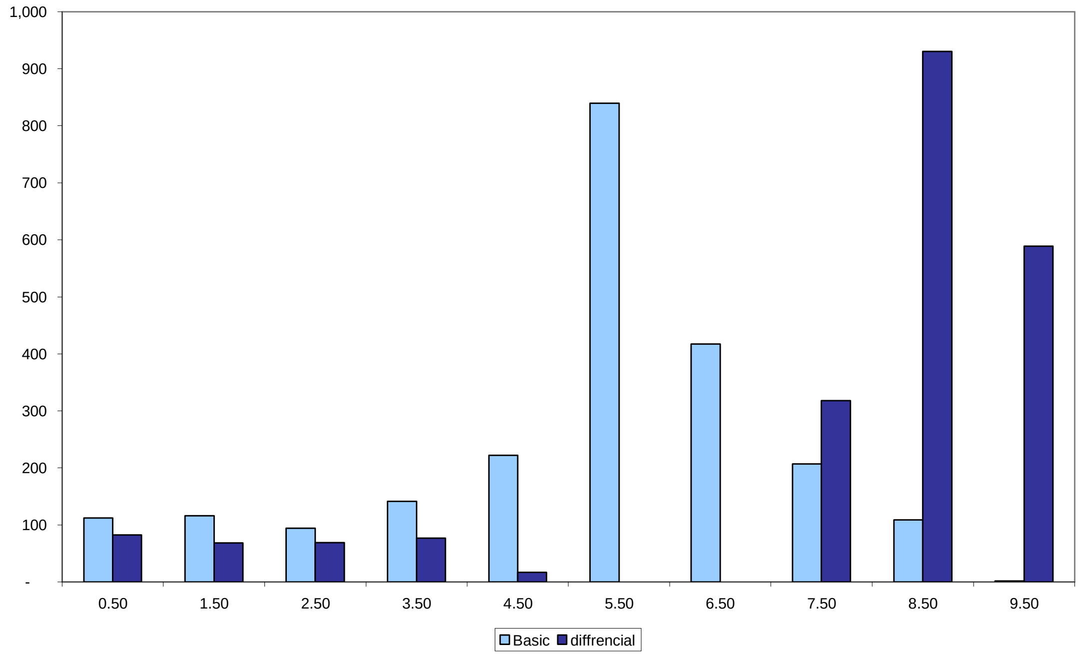


The rate of change compared with the basic scenario

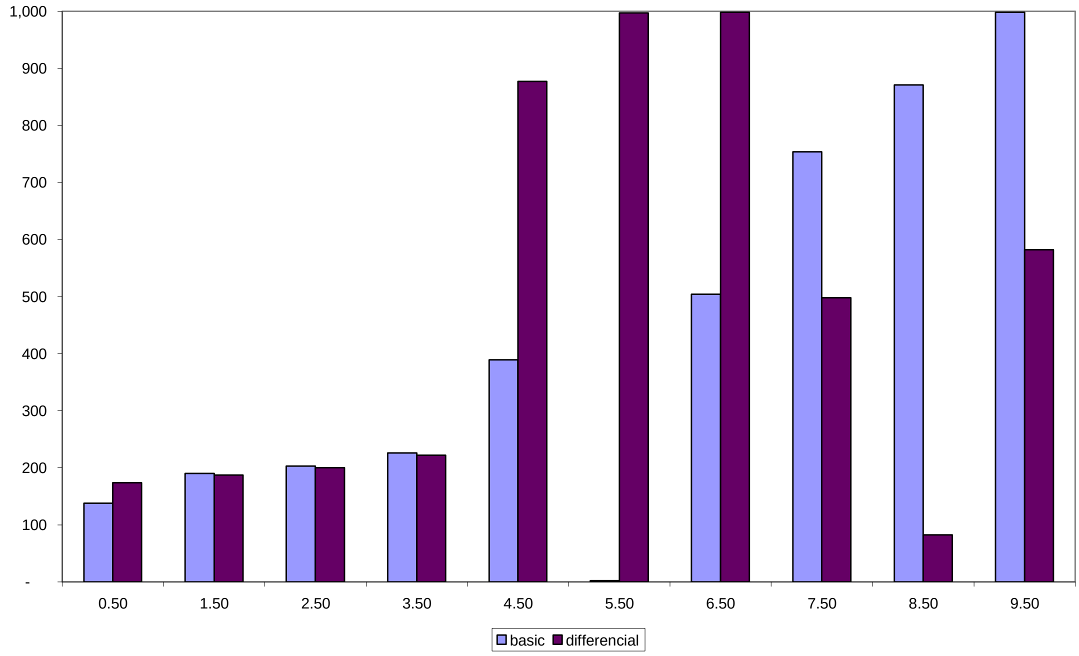
Total productivity	-4%
Government average skills	17%
Private avarage skills	-6%
Governent emoployment	9%
Private employment	1%
Unemployemet rate	-3%
Government avarage wage	-4%
Share of low wage	64%
Share of high wage	36%
Government wage bill	4%

Differential Wages (Raising by 1.6, and reducing by 1.4)

Government-workers-skills distribution



Private sector-workers-skills distribution



<u>The rate of change compared with the basic scenario</u>

Total productivity	-9%
Government average skills	78%
Private avarage skills	-24%
Goverment emoployment	-5%
Private employment	13%
Unemployemet rate	-4%
Government avarage wage	13%
Share of low wage	20%
Share of high wage	80%
Government wage bill	7%

Partial derivatives

The following tables present the partial derivatives of raising the government wage ratio respect to policy and structural variables. The outputs are the productivity in the market, the average skills among government workers, the average skills among private workers, the government employment and the private employment. The average wage ratio in the government sector increases two times by 10 percent, from 90 percent to 110 percent.

Some consultations are derived from the tables:

Policy variables

- The effect of the average wage (in the government sector) on productivity (efficiency) increases with the government average wage from 3.3 percent to 7.9 percent.
- The effect of the average wage on employment in the government sector and its workers' skill increases with the minimum wage.

- The effect of the average wage on employment in the private sector (in absolute terms) with the minimum wage.
- The effects of the average wage are not monotonous respect to the tax rate.
- The effect of the average wage on productivity decreases with the probability of being fired in the government sector.

Structural variables

- The effect of the average wage on productivity, on the employment in the government sector and on the employment in the private sector (in absolute terms) increases with UMU.
- The effect of the average wage on government sector is very large when Sigma is low.
- The effects of the average wage are not monotonous respect to Sigma.
- The effect of the average wage on the whole outcomes deceases in a weak way with the ability return in the private sector (with the discrepancy of the wage in the private sector).

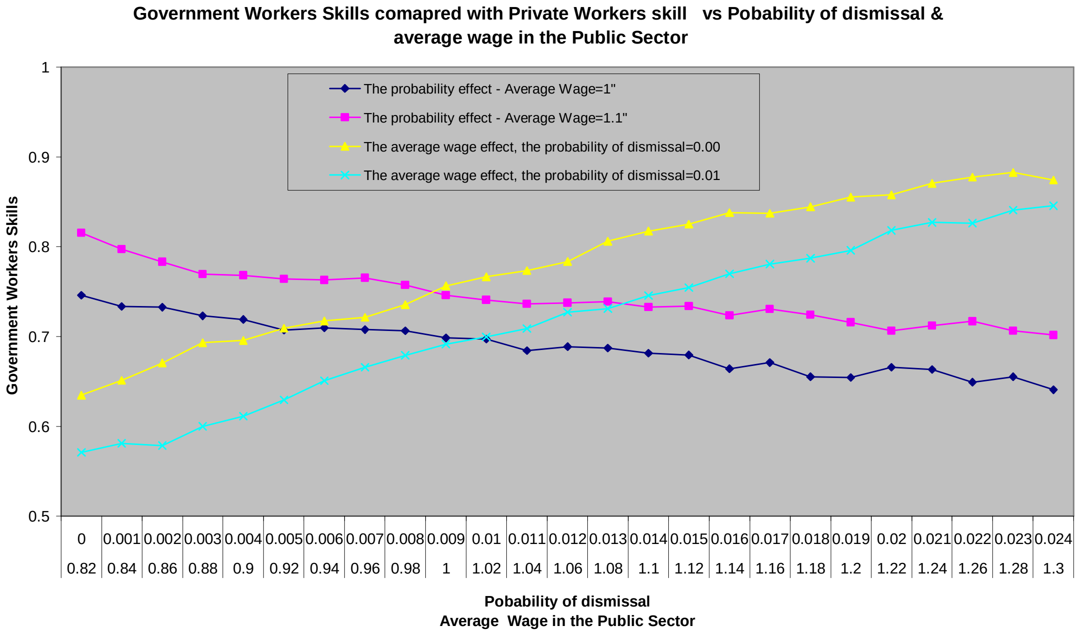
Interaction with policy variables

		Productivity	Average Skills among government workers	Average Skills among private workers	Governemnt Employment	Private Employment
Rasing average wage from 90 to 100 percent		-3.3	16.9	2.3	30.7	-13.2
Rasing average wage from 100 to 110 percent		-7.9	17.0	1.5	39.8	-19.3
Tax Rate						
	0.2	-4.8	16.4	1.8	32.5	-15.5
	0.25	**-5.6**	**17.0**	**1.9**	**35.2**	**-16.3**
	0.3	-5.3	16.9	1.8	34.0	-17.2
Minimum Wage						
	0.4	-5.1	16.7	1.5	31.3	-14.5
	0.45	**-5.6**	**17.0**	**1.9**	**35.2**	**-16.3**
	0.5	-5.5	17.7	2.9	36.3	-18.7
Prob. to be fired in the government sector						
	0	**-5.6**	**17.0**	**1.9**	**35.2**	**-16.3**
	0.02	-1.5	17.2	2.2	40.5	-6.9

Interaction with structural variables

		Productivity	Average Skills among government workers	Average Skills among private workers	Governemnt Employment	Private Employment
UMU1						
	0.25	-3.0	15.3	1.3	29.1	-13.8
	0.5	**-5.0**	**15.7**	**2.0**	**30.7**	**-14.2**
	0.75	-8.3	15.6	1.5	34.1	-15.4
Sigma						
	0.18	-4.5	25.7	4.9	125.2	-18.7
	0.28	**-5.0**	**15.7**	**2.0**	**30.7**	**-14.2**
	0.38	-4.9	18.9	3.9	39.6	-25.4
Ability Return						
	1.78	-5.2	16.2	2.9	30.7	-18.4
	2.08	**-5.0**	**15.7**	**2.0**	**30.7**	**-14.2**

A sum of the Total **derivatives** – the average wage and the probability of dismissals in the public sector



	Average Wage in the public sector *decrease* by 10 percent	**Probability of dismissal in the public sector *increase* by 1 percentage points**
Public Employment	-10.3%	-25.9%
Public Workers average Skill	-6.0%	-8.7%
Total Productivity	1.0%	3.6%
Private average skill	-0.5%	-2.4%
Private Employent	3.3%	13.5%
Total Employment	0.3%	0.8%