In Medio Stat Virtus: Towards a More Balanced Economy*

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Abstract

We consider an economy made up by two industrial sectors. Each industry is characterized by a linear city where two firms are located at the extremes and compete over prices. Sector A produces a private good. Sector B supplies a social service. We compare two different environments. A market economy where all firms are for-profit. A mixed economy where both firms are for-profit in Sector A, whilst in Sector B one firm maximizes the surplus of its customers. We find that coverage of Sector B is nonlower under the mixed economy. For relatively large values of transport costs in the city, the coverage is strictly larger and also the utilitarian welfare is enhanced. Our results are robust when ideological rather than transport costs are considered.

JEL classification: L33 (Comparison of Public and Private Enterprises and Nonprofit Institutions); L13 (Oligopoly and Other Imperfect Markets).

Keywords: market economy; mixed economy; market coverage; utilitarian welfare.

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1 Introduction

In a contribution about the economic crisis, Stiglitz (2009) observes that the "financial markets have mismanaged themselves, and they are so risky that people have lost confidence. That is what has happened in September 2008: the lack of confidence in financial markets led to the pull-out of money, so that even what had been viewed to be strong corporations could simply not get finance". Restoring confidence first in the financial markets and then in the real economy may thus have crucial welfare-enhancing effects in periods of crisis. An important contribution in this respect has been traditionally given by government regulation. Stiglitz believes that also cooperative and socially-oriented enterprises may play a key role since "they are less inclined to exploit those with whom they interact: their workers, their customers, and their suppliers", with the effect of increasing confidence among their stakeholders. Accordingly, Stiglitz argues that an economy is more likely to be successful if it is able to "find a balance between markets, government, and other institutions, including not-for-profits and cooperatives".

In conclusion, Stiglitz argues that cooperative and nonprofit enterprises may help enhance an economy by increasing welfare. The aim of this paper is to provide a microeconomic foundation of this idea by studying whether and how utilitarian welfare is affected by the presence of socially-oriented firms in the economy.

Economists have paid great attention to the topic of firms pursuing something more than the mere maximization of profits. Three strands of literature are worth mentioning. First, the literature on mixed oligopoly mainly focuses on competition between state-owned welfare-maximizing public firms and profit-maximizing private firms: see De Fraja and Del Bono (1990) for a survey. More recently, studies on Corporate Social Responsibility (CSR) have become mainstream. CSR is a form of corporate self-regulation, according to which firms commit to a behavior that takes into account not only the shareholder interests (profit), but also the utility of agents dealing with the firm (stakeholders), such as employees, business partners, consumers and environment. See Kitzmueller and Shimshack (2012) for a recent survey. Finally and quite naturally, the literature on social enterprises: see, e.g., Bonatti et al. (2005) and Borzaga et al. (2010).

Even though each of the three aforementioned strands focuses on different types of "alternative organizations" (publicly owned firms, CSR firms and social enterprises), a somewhat common way to model their objective function is adopted. Firms are assumed to care both about profits and the stakeholders' benefit, where the latter is often identified with consumer welfare (see e.g., Brekke et al., 2012).

Accordingly, we consider an economy made up by two industrial sectors. Each industry is characterized by a Hotelling-type linear city where two firms are located at the extremes and compete over prices. One sector, say A, produces a private good (e.g., cars). The other sector, say B, supplies a social service (e.g., health care).

We compare two different environments. An economy where all firms are for-profit. This environment is defined as a *market economy*. Another economy where both firms are for-profit in industry A, whilst sector B is made up by a mixed duopoly. A nonprofit firm which maximizes the surplus of its consumers and a standard profit-maximizer competitor. This environment is defined as a mixed economy.

We first find that for relatively high value of transport costs in the Hotelling city the social service market is fully covered, i.e., all individuals have access to the social service, only if the economy is mixed. In this respect the mixed economy turns out to be more effective than the market economy. We then show that larger coverage is a necessary condition for the utilitarian welfare, defined as the sum of the utility of all firms and consumers, to be enhanced by the mixed economy. In that case the mixed economy turns out to be more efficient than the market economy. Finally, we consider an alternative linear city for the social service sector B, where the individuals bear ideological rather than transport costs. We demonstrate that our findings are robust to this new specification.

The remainder of the paper is organized as follows. Section 2 describes the setup of the model. In Section 3 (4) we study the market (mixed) economy equilibrium properties. The welfare analysis is provided in Section 5. In Section 6 we consider an alternative form of mixed economy, where the nonprofit firm in Sector B is allowed to sell below cost. Section 7 contains the analysis with the ideological costs. Finally, Section 8 concludes.

2 Setup

Consider an economy made up of two industrial sectors, denoted by A and B. Each industry $j = A, B$ is characterized by a Hotelling-type linear city of length 1 where two firms, indexed by i, j with $i = 0, 1$, are located at the extremes, firm $0, j$ is at $x = 0$ and firm $1, j$ at $x = 1$: see Figure 1. Potential buyers of mass one are uniformly distributed along the linear city. The two industrial sectors differ in that Sector A produces a private good, *e.g.,* cars, whilst Sector B supplies a social service, *e.g.,* health care. Each individual demands at most one unit of the commodity.

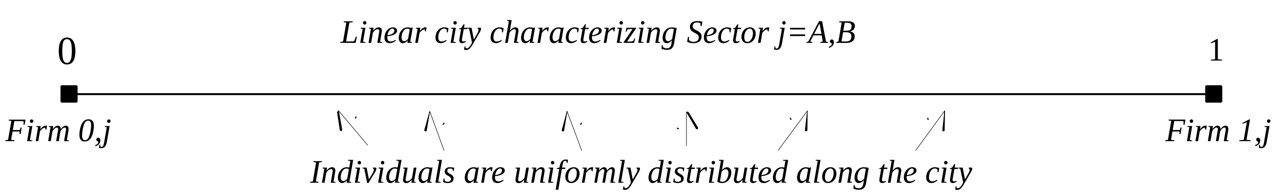

Figure 1: the Hotelling-type linear city

To compute the demand of each firm i, j, we denote with s_j the utility from the consumption of one unit of the commodity produced in Sector $j = A, B$ and we define surplus of a consumer as the difference between the consumption utility and total purchase costs. In symbols, surplus of an individual located at point $x \in [0,1]$ is given by

$$\begin{array}{lll} s_j - p_{0,j} - tx & \text{when buying the commodity from firm } 0,j, & (a) \\ s_j - p_{1,j} - t(1-x) & \text{when buying the commodity from firm } 1,j, & (b) \\ 0 & \text{when not buying,} & (c) \end{array} \tag{1}$$

where $p_{i,j}$ is the unit price charged by firm i,j, whilst tx $(t(1-x))$ is the cost borne to go to firm $0,j$ $(1,j)$, $t > 0$ being the unit cost of transportation: see Figure 2 for a graphical representation of (1-a) and (1-b) as a function of x. Solving (1-a) = (1-b) by x gives the location of an individual who is indifferent between purchasing the commodity from firm $0,j$ or firm $1,j$:

$$x_{I,j} = \frac{1}{2} + \frac{p_{1,j} - p_{0,j}}{2t}. \tag{2}$$

We denote with $D_{0,j} = [0, x_{0,j}]$ and $D_{1,j} = [x_{1,j}, 1]$ the demand shares of firms $0,j$ and $1,j$, with $x_{0,j} \leq x_{I,j} \leq x_{1,j}$.[1]

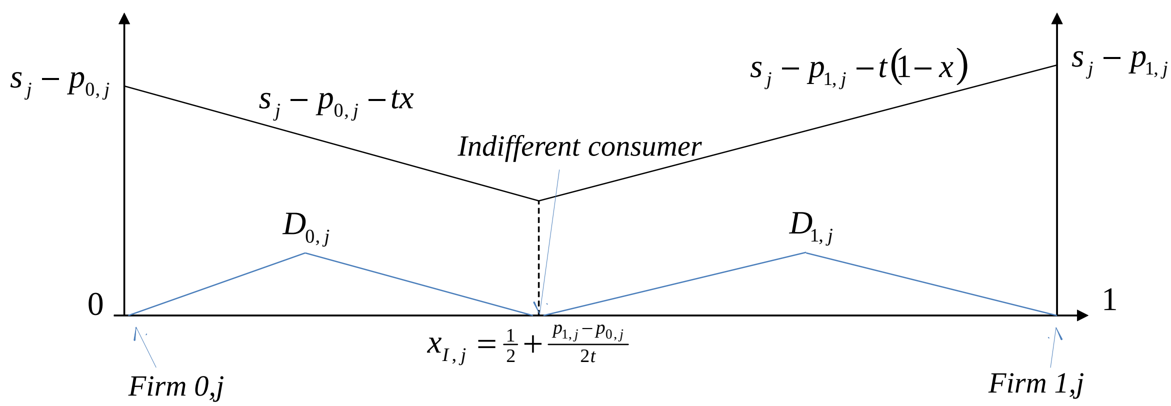

FIGURE 2: MARKET FULL COVERAGE IN SECTOR $j = A, B$

Firm i,j is assumed to incur constant unit production costs $c_j \geq 0$. Accordingly, its profit function is

$$\Pi_{i,j} = (p_{i,j} - c_j) D_{i,j}. \tag{3}$$

In addition, we compute surplus of firm $0,j$'s customers,

$$CS_{0,j} = \int_0^{x_{0,j}} (s - p_{0,j} - ty)\, dy = D_{0,j} \left(s - p_{0,j} - \frac{t}{2} D_{0,j} \right). \tag{4}$$

Similarly, surplus of firm $1,j$'s customers is

$$CS_{1,j} = \int_{x_{1,j}}^1 [s - p_{1,j} - t(1-y)]\, dy = D_{1,j} \left(s - p_{1,j} - \frac{t}{2} D_{1,j} \right). \tag{5}$$

Finally, surplus of individuals who do not buy is obviously zero,

$$CS_{H,j} = \int_{x_{0,j}}^{x_{1,j}} 0 dy = 0. \tag{6}$$

The analysis proceeds by comparing two different economic environments.

[1] Note that Figure 2 depicts the case $x_{0,j} = x_{I,j} = x_{1,j}$. This scenario is referred to as market full coverage, as we will explain shortly in detail.

(i) An economy where all firms i, j, *i.e.*, each firm $i = 0, 1$ in each sector $j = A, B$, are for-profit. By definition, a for-profit firm i, j aims at maximizing profit $\Pi_{i,j}$.

(ii) An economy where both firms are for-profit in industry A, whilst sector B is made up by the following mixed duopoly: firm $0, B$ maximizes the surplus of its customers $CS_{0,B}$ and it is referred to as a nonprofit firm; firm $1, B$ is instead a standard profit-maximizer, which targets profit $\Pi_{1,B}$.

We introduce the following

Definition 1 *An economy where all firms are profit-maximizer is defined as a market economy. An economy where both firms are profit-maximizer in sector A, while one firm is nonprofit in sector B is defined as a mixed economy.*

The timing of events in our framework is as follows.

- At $t = 0$ in each sector $j = A, B$ of each economy, either market or mixed, firms $0, j$ and $1, j$ choose simultaneously $p_{0,j}$ and $p_{1,j}$ to maximize their objective functions. We assume no strategic interaction among firms across industrial sectors given the very different nature of, and thus the very different demand for, the two commodities supplied.
- At $t = 1$ profits accrue to the firms.

Finally, we let

$$s_j > c_j, \tag{7}$$

according to which the unit consumption utility s_j is higher than the unit production cost c_j in both industries. This is a necessary condition for trade between consumers and firms to occur.

The analysis proceeds as follows. Section 3 studies the Nash equilibrium of the competition game taking place at $t = 0$ in the market economy. In Section 4 we move to the analysis of the mixed economy.

3 Market Economy

In this section, we are interested in computing the Nash equilibrium of the market economy and studying how it is affected by different values of the unit transport cost t. Since all firms maximize profit $\Pi_{i,j}$ and are symmetric within each sector $j = A, B$, we consider firm $0, j$ as the representative one.

Firm $0, j$ solves the following program

$$\begin{gathered} \max_{p_{0,j}, x_{0,j}} (p_{0,j} - c_j) D_{0,j} \text{ with } D_{0,j} = [0, x_{0,j}] \\ \text{s.t. } x_{0,j} \leq x_{I,j} \text{ and } s_j - p_{0,j} - t x_{0,j} \geq 0, \end{gathered} \tag{8}$$

The objective function of firm $0, j$ is its profit. Recalling that firm $0, j$ lies at the leftmost point of the unit segment, the first constraint in (8) ensures the furthest customer of firm $0, j$ is at most the indifferent individual located at $x_{I,j}$, see (2). When such a constraint is binding, all individuals buy either from firm $0, j$ or firm $1, j$, as depicted in Figure 2. In symbols, $x_{0,j} = x_{I,j}$ (and $x_{I,j} = x_{1,j}$ for symmetry of firms) and the market is said to be fully covered. Instead, when the constraint is not active, $x_{0,j} < x_{I,j}$ (and $x_{I,j} < x_{1,j}$ for symmetry of firms), there is no full coverage, *i.e.*, individuals located in $(x_{0,j}, x_{1,j})$ do not buy, as depicted in Figure 3. The second constraint in (8) requires that all consumers of firm $0, j$ get a nonnegative surplus.

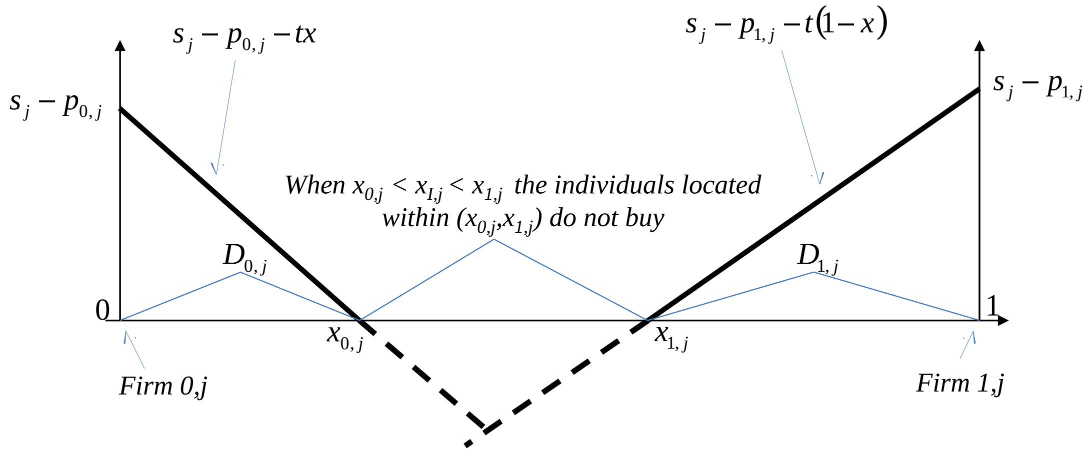

FIGURE 3: MARKET PARTIAL COVERAGE IN SECTOR $j = A, B$

Lemma 1 *In the market economy the symmetric Nash equilibrium of each industrial Sector $j = A, B$ takes the following features. The equilibrium prices are $p^*_{0,j} = p^*_{1,j} = p^*_j$, where*

$$p^*_j = \begin{cases} t + c_j & \text{if } t < \frac{2}{3}(s_j - c_j), \\ s_j - \frac{t}{2} & \text{if } \frac{2}{3}(s_j - c_j) \leq t \leq s_j - c_j, \\ \frac{s_j + c_j}{2} & \text{if } t > s_j - c_j, \end{cases} \tag{9}$$

The equilibrium demands are instead $D^*_{0,j} = D^*_{1,j} = D^*_j$, where

$$D^*_j = \begin{cases} \frac{1}{2} & \text{if } t \leq s_j - c_j, \\ \frac{s_j - c_j}{2t} & \text{if } t > s_j - c_j. \end{cases} \tag{10}$$

Proof. See Appendix A. ■

Before explaining the results of Lemma 1, we compute the equilibrium values of firms' profits and consumer surplus. Plugging (9) and (10) into (3) gives the symmetric equilibrium profits $\Pi^*_{0,j} = \Pi^*_{1,j} = \Pi^*_j$ of each firm in the two sectors,

$$\Pi^*_j = \begin{cases} \frac{t}{2} & \text{if } t < \frac{2}{3}(s_j - c_j), \\ \frac{2s_j - t - 2c_j}{4} & \text{if } \frac{2}{3}(s_j - c_j) \leq t \leq s_j - c_j, \\ \frac{(s_j - c_j)^2}{4t} & \text{if } t > s_j - c_j. \end{cases} \tag{11}$$

Similarly, substituting (9) and (10) into (4) and (5) yields the symmetric equilibrium consumer surplus

$CS^*_{0,j} = CS^*_{1,j} = CS^*_j$ in the two sectors,

$$CS^*_j = \begin{cases} \frac{4s_j - 5t - 4c_j}{8} & \text{if } t < \frac{2}{3}(s_j - c_j), \\ \frac{t}{8} & \text{if } \frac{2}{3}(s_j - c_j) \leq t \leq s_j - c_j, \\ \frac{(s_j - c_j)^2}{8t} & \text{if } t > s_j - c_j. \end{cases} \tag{12}$$

We discuss the results of Lemma 1 by studying the three relevant intervals of t separately. When the transport costs are relatively low, $t < \frac{2}{3}(s_j - c_j)$, only the first constraint in (8) is binding. This means that all individuals purchase and that the indifferent consumer gets positive surplus. In that case, the equilibrium price $p^*_j = t + c_j$ is increasing in t but the equilibrium demand $D^*_j = \frac{1}{2}$ is unaffected because larger transportation costs make the consumers more captive, giving firms larger market power. As a result, firms' profits $\Pi^*_j = \frac{t}{2}$ (consumer surplus $CS^*_j = \frac{4s_j - 5t - 4c_j}{8}$) are positively (is negatively) affected by t.

When transport costs are larger, $\frac{2(s_j - c_j)}{3} \leq t \leq s_j - c_j$, both constraints of program (8) are binding, which implies that there is still full coverage, but the indifferent consumer gets zero surplus. In that case, the equilibrium price $p^*_j = s_j - \frac{t}{2}$ becomes decreasing in t. The intuition is as follows. Plugging $x_{0,j} = x_{I,j}$ into $s_j - p_{0,j} - tx_{0,j} = 0$ with $p^*_{0,j} = p^*_{1,j} = p^*_j$ yields the zero-surplus condition of the indifferent consumer,

$$s_j - \frac{p^*_{0,j} + p^*_{1,j}}{2} - \frac{t}{2} = s_j - p^*_j - \frac{t}{2} = 0. \tag{13}$$

As t increases, p^*_j must decrease in order for (13) to be fulfilled. Since the equilibrium demand $D^*_j = \frac{1}{2}$ is instead unaffected, firms' profits $\Pi^*_j = \frac{2s_j - t - 2c_j}{4}$ (consumer surplus $CS^*_j = \frac{t}{8}$) are negatively (is positively) affected by t.

Finally, for relatively high transport costs, $t > s_j - c_j$, only the second constraint in (8) is active. In that case, sector j is not fully covered, $D^*_{0,j} + D^*_{1,j} = \frac{s_j - c_j}{t} < 1$. Put differently, a fraction $1 - \frac{s_j - c_j}{t}$ of individuals is not served. Note that the equilibrium price $p^*_j = \frac{s_j + c_j}{2}$ does not depend on t. Yet both $\Pi^*_j = \frac{(s_j - c_j)^2}{4t}$ and $CS^*_j = \frac{(s - c_j)^2}{8t}$ decrease with t because the demand $D^*_j = \frac{s_j - c_j}{2t}$ is negatively affected.

The above reasoning illustrates the role of transport costs in the current framework. Higher t makes it more difficult to serve all the potential customer since, *ceteris paribus*, their surplus is negatively affected. Indeed, expressions (1-a) and (1-b) are decreasing in t.

4 Mixed Economy

In this section we turn our focus on the mixed economy. According to Definition 1, sector A is made up by two for-profit firms, $0, A$ and $1, A$, whose equilibrium choices has been investigated in Lemma 1. Equilibrium profits and consumer surplus in Sector A are thus given by (11) and (12), respectively, with $j = A$.

By contrast, in sector B firm $0, B$ is consumer-surplus-maximizer rather than profit-maximizer. Accordingly, it solves the following problem:

$$\begin{array}{c}\max_{p_{0,B},x_{0,B}} D_{0,B}\left(s_B - p_{0,B} - \frac{t}{2}D_{0,B}\right) \text{ with } D_{0,B} = [0, x_{0,B}] \\ \text{s.t.} \\ x_{0,B} \leq x_{I,B}, \\ s_B - p_{0,B} - t x_{0,B} \geq 0, \\ \Pi_{0,B} = (p_{0,B} - c_j) D_{0,B} \geq 0.\end{array} \tag{14}$$

The first two constraints are as in (8), with $j = B$, while the third constraint ensures that firm $0, B$ profits are nonnegative. The objective function consists in the surplus of firm $0, B$ customers, $CS_{0,B}$.

Lemma 2 *In the mixed economy the symmetric Nash equilibrium of Sector A is as in Lemma 1. By contrast, the Nash equilibrium of Sector B takes the following features. The equilibrium prices are*

$$p^{**}_{0,B} = c_B;\ p^{**}_{1,B} = \begin{cases} \frac{t}{2} + c_B & \text{if } t < \frac{4}{3}(s_B - c_B), \\ 2s_B - c_B - t & \text{if } \frac{4}{3}(s_B - c_B) \leq t \leq \frac{3}{2}(s_B - c_B), \\ \frac{s_B + c_B}{2} & \text{if } t > \frac{3}{2}(s_B - c_B). \end{cases} \tag{15}$$

The equilibrium demands are

$$D^{**}_{0,B} = \begin{cases} \frac{3}{4} & \text{if } t < \frac{4(s_B - c_B)}{3}, \\ \frac{s_B - c_B}{t} & \text{if } t \geq \frac{4(s_B - c_B)}{3}. \end{cases}\quad D^{**}_{1,B} = \begin{cases} \frac{1}{4} & \text{if } t < \frac{4(s_B - c_B)}{3}, \\ 1 - \frac{s_B - c_B}{t} & \text{if } \frac{4(s_B - c_B)}{3} \leq t \leq \frac{3(s_B - c_B)}{2}, \\ \frac{s_B - c_B}{2t} & \text{if } t > \frac{3(s_B - c_B)}{2}. \end{cases} \tag{16}$$

Proof. See Appendix B. ■

Before commenting on the results of Lemma 2, we calculate equilibrium firms' profits in Sector B. To this aim, we plug (15) and (16) into (3) with $j = B$:

$$\begin{array}{l}\Pi^{**}_{0,B} = 0; \\ \Pi^{**}_{1,B} = \begin{cases} \frac{t}{8} & \text{if } t < \frac{4(s_B - c_B)}{3}, \\ \frac{(2s_B - 2c_B - t)(t + c_B - s_B)}{t} & \text{if } \frac{4(s_B - c_B)}{3} \leq t \leq \frac{3(s_B - c_B)}{2}, \\ \frac{(s_B - c_B)^2}{4t} & \text{if } t > \frac{3(s_B - c_B)}{2}. \end{cases}\end{array} \tag{17}$$

Similarly, we plug (15) and (16) into (4) and (5) to compute the equilibrium consumer surplus,

$$\begin{aligned} CS^{**}_{0,B} &= \begin{cases} \frac{3(8s_B - 3t - 8c_B)}{32} & \text{if } t < \frac{4(s_B - c_B)}{3}, \\ \frac{(s_B - c_B)^2}{2t} & \text{if } t \geq \frac{4(s_B - c_B)}{3}. \end{cases} \\ CS^{**}_{1,B} &= \begin{cases} \frac{8s_B - 5t - 8c_B}{32} & \text{if } t < \frac{4(s_B - c_B)}{3}, \\ \frac{(s_B - c_B - t)^2}{2t} & \text{if } \frac{4(s_B - c_B)}{3} \leq t \leq \frac{3(s_B - c_B)}{2}, \\ \frac{(s_B - c_B)^2}{8t} & \text{if } t > \frac{3(s_B - c_B)}{2}. \end{cases} \end{aligned}$$

We have all the elements to explain the results of Lemma 2. First remark that the consumer-surplus-maximizer firm $0, B$ charges $p^{**}_{0,B} = c_B$ and ends up with zero profits. This is because customer surplus $CS_{0,B}$ is negatively affected by the price, as one can check by inspecting the objective function of program (14). On the contrary, the for-profit firm $1, B$ sets a larger price, $p^{**}_{1,B} > p^{**}_{0,B}$ for any t. As a result, the demand share of firm $0, B$ is greater.

More precisely, when $t < \frac{4(s_B - c_B)}{3}$ only the first constraint of program (14) is binding, *i.e.*, full coverage occurs and the indifferent consumer gets positive surplus. The equilibrium price charged by the for-profit firm $1, B$, $p_{1,B}^{**} = \frac{t}{2} + c_B$, is increasing in t as in Section 3. Notwithstanding, its demand $D_{1,B}^{**} = \frac{1}{4}$ does not decrease because the consumers become more captive as t increases. As a result, firm $1, B$'s profits, $\Pi_{1,B}^{**} = \frac{t}{8}$, are positively affected by t, while consumer surplus, $CS_{1,B}^{**} = \frac{8s - 5t - 8c_B}{32}$, is negatively affected. Price and demand share of the nonprofit firm, $p_{0,B}^{**} = c_B$ and $D_{0,B}^{**} = \frac{3}{4}$, respectively, do not depend on t. Yet surplus of its consumers, $CS_{0,B}^{**} = \frac{3(8s_B - 3t - 8c_B)}{32}$, is negatively affected by t simply because moving along the segment becomes more costly, see formula (4) with $j = B$.

When $t \in \left[\frac{4(s_B - c_B)}{3}, \frac{3(s_B - c_B)}{2}\right]$, there is full coverage, while the indifferent consumer gets zero surplus. The price charged by the for-profit firm, $p_{1,B}^{**} = 2s_B - c_B - t$, is decreasing in t as in Section 3. Accordingly, its demand $D_{1,B}^{**} = 1 - \frac{s_B - c_B}{t}$ increases with t. Firm $1, B$'s profit, $\Pi_{1,B}^{**} = \frac{(2s_B - 2c_B - t)(t + c_B - s_B)}{t}$, is first increasing and then decreasing in t, while surplus of its consumers, $CS_{1,B}^{**} = \frac{(s_B - c_B - t)^2}{2t}$, is positively affected by t. Conversely, the demand share of the nonprofit firm, $D_{0,B}^{**} = \frac{s_B - c_B}{t}$, decreases with t and so does the surplus of its consumers, $CS_{1,B}^{**} = \frac{(s_B - c_B)^2}{2t}$.

Finally, when $t > \frac{3(s_B - c_B)}{2}$ there is no full coverage. The equilibrium prices, $p_{0,B}^{**} = c_B$ and $p_{1,B}^{**} = \frac{s_B + c_B}{2}$ are unaffected by t, while the demand shares, $D_{0,B}^{**} = \frac{s_B - c_B}{t}$ and $D_{1,B}^{**} = \frac{s_B - c_B}{2t}$, decrease with t. We can conclude that: (i) firm $1, B$ profits, $\Pi_{1,B}^{**} = \frac{(s_B - c_B)^2}{4t}$, decreases with t; (i) surplus of consumers of both firms are negatively affected by t.

An important result emerges from the above analysis. A positive fraction of consumers do not enjoy the social service when the transport cost exceeds a given threshold. In symbols,

$$D_{0,B}^{**} + D_{1,B}^{**} < 1 \text{ iff } t > \frac{3(s_B - c_B)}{2}. \tag{18}$$

Interestingly, the same holds true in Industry B of the market economy if and only if $t > s_B - c_B$. Since inequality $\frac{3(s_B - c_B)}{2} > s_B - c_B$ holds, we can write the following

Proposition 1 *(i) When transport cost $t \leq s_B - c_B$, all individuals have access to the social service under both types of economy. (ii) When transport cost $t \in \left(s_B - c_B, \frac{3(s_B - c_B)}{2}\right]$, all individuals have access to the social service only if the economy is mixed. (iii) When transport cost $t > \frac{3(s_B - c_B)}{2}$, there is no full coverage and more individuals have access to the social service under the mixed economy, $D_{0,B}^{**} + D_{1,B}^{**} = \frac{3}{2}\frac{s_B - c_B}{t} > 2D_B^* = \frac{s_B - c_B}{t}$.*

The above result is due to the fact that the nonprofit firm sets the price equal to the marginal cost, thus easing the purchase also for individuals who live far away. More exactly, plugging $p_{0,B}^{**}$ and $p_{1,B}^{**}$ into the LHS of (13) with $j = B$ yields surplus of the indifferent consumer located at $x_{I.B}$:

$$s_B - \frac{p_{0,B}^{**} + p_{1,B}^{**}}{2} - \frac{t}{2}. \tag{19}$$

The above value is decreasing in $p_{0,B}^{**}+p_{1,B}^{**}$ and t. Social service market coverage is non-lower under the mixed economy because the sum of equilibrium prices is lower, $p_{0,B}^{**}+p_{1,B}^{**}<p_{0,B}^{*}+p_{1,B}^{*}$ for any t.

5 Welfare Analysis

The aim of this section is to compare welfare arising in the market economy, where competition occurs between for-profit firms, to welfare when the economy is mixed in that firm 0 in sector B is consumer-surplus-maximizer. We adopt a utilitarian approach by defining welfare W as the sum of firms' profits and individuals' surplus in the two sectors.

Let us first consider the market economy. The overall profits made by the firms are

$$\sum_i \sum_j \Pi_{i,j}^{*} = 2\Pi_A^{*} + 2\Pi_B^{*}. \tag{20}$$

The total surplus of individuals is

$$\sum_i \sum_j CS_{i,j}^{*} + CS_{H,j} = 2CS_A^{*} + 2CS_B^{*} + 0. \tag{21}$$

Summing up the two above values yields the equilibrium welfare in the market economy, denoted by W_{MA}:

$$W_{MA} = 2\Pi_A^{*} + 2\Pi_B^{*} + 2CS_A^{*} + 2CS_B^{*}. \tag{22}$$

Let us turn our attention to the mixed economy. The overall profits made by the firms are

$$\sum_i \sum_j \Pi_{i,j}^{**} = 2\Pi_A^{*} + \Pi_{0,B}^{**} + \Pi_{1,B}^{**}. \tag{23}$$

Instead, the total surplus of individuals is

$$\sum_i \sum_j CS_{i,j}^{**} + CS_{H,j} = 2CS_A^{*} + CS_{0,B}^{**} + CS_{1,B}^{**} + 0. \tag{24}$$

The equilibrium welfare in the mixed economy, denoted with W_{MI}, is thus given by

$$W_{MI} = 2\Pi_A^{*} + \Pi_{0,B}^{**} + \Pi_{1,B}^{**} + 2CS_A^{*} + CS_{0,B}^{**} + CS_{1,B}^{**}. \tag{25}$$

As mentioned, we aim at comparing (22) with (25). Recalling that the nonprofit firm make zero profits, $\Pi_{0,B}^{**}=0$, we denote with ΔW the difference in welfare between the market and the mixed economy, $W_{MA}-W_{MI}$,

$$\Delta W = \left(2\Pi_B^{*} + 2CS_B^{*}\right) - \left(\Pi_{1,B}^{**} + CS_{0,B}^{**} + CS_{1,B}^{**}\right). \tag{26}$$

Note that ΔW does not depend on the equilibrium values in Sector A, $2\Pi_A^{*}$ and $2CS_A^{*}$. These values are indeed equal across both economic environments, market and mixed. As a consequence, we can disregard Sector A and focus our attention on what happens in Sector B. As we will see in the next section, where we allow the nonprofit firm to sell below cost, the role played by Sector A is to produce profits which contribute to the survival of the nonprofit firm.

We now have all the elements to derive our main result. We study the sign of (26) to determine under which conditions the market economy either enhances or worsen the utilitarian welfare. Taking into account (11) and (12) with $j = B$, we can write

$$2\Pi_B^* + 2CS_B^* = \begin{cases} \frac{4(s_B - c_B) - t}{4} & \text{if } t \le s_B - c_B, \\ \frac{3}{4}\frac{(s_B - c_B)^2}{t} & \text{if } t > s_B - c_B. \end{cases} \tag{27}$$

Note that the sum of firms' profits and consumer welfare in Sector B of the market economy is negatively affected by t. The intuition is as follows. According to Lemma 1 firms' profits $\Pi_B^* = \frac{t}{2}$ (consumer surplus $CS_B^* = \frac{4s_B - 5t - 4c_B}{8}$) are positively (is negatively) affected by t when $t \le \frac{2}{3}(s_B - c_B)$, while firms' profits $\Pi_B^* = \frac{2s_B - t - 2c_B}{4}$ (consumer surplus $CS_B^* = \frac{t}{8}$) are negatively (is positively) affected by t. The negative effect on consumer surplus in the first case and on profits in the second case is shown to prevail. By contrast, the market is not fully covered when $t > s_B - c_B$ and welfare decreases with t because total demand $2D_B^* = \frac{s_B - c_B}{t}$ is negatively affected, as shown in Section 3.

Similarly, we can write the sum of firms' profits and consumer welfare in Sector B of the mixed economy

$$\Pi_{1,B}^{**} + CS_{0,B}^{**} + CS_{1,B}^{**} = \begin{cases} \frac{16(s_B - c_B) - 5t}{16} & \text{if } t < \frac{4(s_B - c_B)}{3}, \\ \frac{4(s_B - c_B) - t}{2} - \frac{(s_B - c_B)^2}{t} & \text{if } \frac{4(s_B - c_B)}{3} \le t \le \frac{3(s_B - c_B)}{2}, \\ \frac{7}{8}\frac{(s_B - c_B)^2}{t} & \text{if } t > \frac{3(s_B - c_B)}{2}. \end{cases} \tag{28}$$

When $t < \frac{4(s_B - c_B)}{3}$, summation (28) is decreasing t because the negative effect of t on $CS_{0,B}^{**} = \frac{3(8s_B - 3t - 8c_B)}{32}$ and $CS_{1,B}^{**} = \frac{8s_B - 5t - 8c_B}{32}$ outdoes the positive one on $\Pi_{1,B}^{**} = \frac{t}{8}$. When $\frac{4(s_B - c_B)}{3} \le t \le \frac{3(s_B - c_B)}{2}$, summation (28) is first increasing and then decreasing in t and reaches its maximum at $t = \sqrt{2}(s_B - c_B)$. This is due to the fact that $\Pi_{1,B}^{**} = \frac{(2s_B - 2c_B - t)(t + c_B - s_B)}{t}$ reaches its maximum at $t = \sqrt{2}(s_B - c_B)$ too. Finally, the market is not fully covered when $t > \frac{3(s_B - c_B)}{2}$ and welfare decreases with t because total demand $D_{0,B}^{**} + D_{1,B}^{**} = \frac{3(s_B - c_B)}{2t}$ is negatively affected.

The sign of $\Delta W = W_{MA} - W_{MI}$ is studied in the following

Proposition 2 *(i) When* $t \le \frac{6(s_B - c_B)}{5}$ *welfare is larger under the market economy, i.e.,* $\Delta W \ge 0$. *(ii) When* $t > \frac{6(s_B - c_B)}{5}$ *welfare is larger under the mixed economy, i.e.,* $\Delta W < 0$.

Proof. See Appendix C. ■

Before commenting on Proposition 2, we recall that the sum of equilibrium prices in Sector B is lower under the mixed economy. In symbols, $p_{0,B}^{**} + p_{1,B}^{**} < p_{0,B}^* + p_{1,B}^*$ for any t, which means that price competition is tougher. The common wisdom is that competition improves welfare. Yet welfare is larger under the more competitive mixed economy only for relatively large values of transport cost t. To understand why, we discuss the results of Proposition 2 by considering the relevant intervals of t separately.

When $t \leq s_B - c_B$ there is full coverage of Sector B under both types of economy. We know that summations (27) and (28) are decreasing in t. More interestingly, moving from a market to a mixed environment firms' profits are negatively affected, while consumer surplus is positively affected in Sector B. In symbols, $2\Pi^*_B > \Pi^{**}_{1,B}$ and $2CS^*_B > CS^{**}_{0,B} + CS^{**}_{1,B}$. This trade-off is standard. Think, *e.g.*, of a symmetric Cournot n-oligopoly. Increasing the number of competitors yields the same trade-off because the price falls, total demand increases but each firm sells a lower quantity. Our framework is different in that each firm's demand is unaffected by the price reduction. This is shown to make the surplus gain lower than the profit loss (in absolute value). In symbols, $\left(CS^{**}_{0,B} + CS^{**}_{1,B}\right) - 2CS^*_B < 2\Pi^*_B - \Pi^{**}_{1,B}$, or equivalently, $\Delta W > 0$. As a result, welfare is greater under the market economy.

When $(s_B - c_B) < t < \frac{4(s_B - c_B)}{3}$, full coverage of Sector B occurs only in the mixed environment. Summations (27) and (28) are decreasing in t, as shown above. Yet the negative effect on (27), *i.e.*, on welfare in Sector B of the market economy, is larger than that on (28) due to the reduction in demand, $2D^*_B = \frac{s_B - c_B}{t}$. As a result, welfare becomes greater under the mixed economy at $t = \frac{6(s_B - c_B)}{5}$.

Also when $\frac{4(s_B - c_B)}{3} \leq t \leq \frac{3(s_B - c_B)}{2}$, full coverage of Sector B occurs only in the mixed environment. Expression (27), still decreasing in t, is lower than in interval $(s_B - c_B) < t < \frac{4(s_B - c_B)}{3}$. By contrast, there is an inverted U-shaped relationship between (28) and t, as explained above, and welfare is greater under the mixed economy.

Finally, when $t > \frac{3(s_B - c_B)}{2}$ there is partial coverage under both environments, but the demand of firm $0, B$ is greater in the mixed economy because of the lower price, $p^{**}_{0,B} = c_B < p^*_{0,B} = \frac{s_B + c_B}{2}$. This is shown to make the surplus gain enjoyed by consumers of the nonprofit firm larger than the profit loss (in absolute value), $\left(CS^{**}_{0,B} + CS^{**}_{1,B}\right) - 2CS^*_B > 2\Pi^*_B - \Pi^{**}_{1,B}$. As a consequence, welfare is enhanced under the mixed economy, *i.e.*, $\Delta W < 0$. This result is in line with the abovementioned Cournot example, according to which competition enhances welfare.

Summing up, full coverage of the social service market is more likely to occur in the mixed economy because price competition is tougher. If this is the case also under the market economy, *i.e.*, if t is relatively low, welfare is enhanced when all firms are for-profit because the profit gain enjoyed by the firms is larger than the surplus loss suffered by the consumers. As t increases, instead, welfare becomes larger in the mixed economy because a greater fraction of consumers have access to the social service.

6 Extension I: Mixed Economy with Transfers

In this section we enrich our analysis by considering an alternative form of mixed economy, where the nonprofit firm in Sector B is allowed to sell below cost. Accordingly, we modify the timing of events in our framework by assuming that at $t = 1$ the nonprofit firm receives a lump-sum transfer d on top of the profits realized. For this reason, we refer to this environment as *mixed economy with transfers.* The amount

d is taken from both external donors, whose wealth is denoted by R, and from profits of firms operating in Industry A, $\sum_i \Pi_{i,A}$.[2] In symbols, $d = r + k$, where r is the donors' contribution and k is that of firms $0, A$ and $1, A$.[3]

The strategic behavior of the two for-profit firms in Sector A is not affected since the transfer k is lump-sum. Equilibrium profits in Sector A are thus given by (11), with $j = A$, minus the transfer paid by each firm and the equilibrium consumer surplus is as in (12), with $j = A$.

In sector B the consumer-surplus-maximizer firm $0, B$ solves a new problem, where the break-even constraint $\Pi_{0,B} \geq 0$ of program (14) is removed and substituted with the price non-negativity constraint, $p_{0,B} \geq 0$:

$$\begin{array}{c} \max_{p_{0,B},x_{0,B}} D_{0,B}\left(s_B - p_{0,B} - \frac{t}{2}D_{0,B}\right) \text{ with } D_{0,B} = [0, x_{0,B}] \\ \text{s.t.} \\ x_{0,B} \leq x_{I,B}, \\ s_B - p_{0,B} - t x_{0,B} \geq 0, \\ p_{0,B} \geq 0. \end{array} \tag{29}$$

Lemma 3 *In the mixed economy with transfers the symmetric Nash equilibrium of Sector A is as in Lemma 1. By contrast, the Nash equilibrium of Sector B takes the following features. The equilibrium prices are*

$$\overset{\circ}{p}_{0,B} = 0;\ \overset{\circ}{p}_{1,B} = \begin{cases} c_B & \text{if } t \leq c_B, \\ \frac{t+c_B}{2} & \text{if } c_B < t < \frac{4s_B - c_B}{3}, \\ 2s_B - t & \text{if } \frac{4s_B-c_B}{3} \leq t \leq \frac{3s_B-c_B}{2}, \\ \frac{s_B+c_B}{2} & \text{if } t > \frac{3s_B-c_B}{2}. \end{cases} \tag{30}$$

The equilibrium demands are

$$\overset{\circ}{D}_{0,B} = \begin{cases} 1 & \text{if } t \leq c_B, \\ \frac{3t+c_B}{4t} & \text{if } c_B < t < \frac{4s_B-c_B}{3}, \\ \frac{s_B}{t} & \text{if } t \geq \frac{4s_B-c_B}{3}; \end{cases} \quad \overset{\circ}{D}_{1,B} = \begin{cases} 0 & \text{if } t \leq c_B, \\ \frac{t-c_B}{4t} & \text{if } c_B < t < \frac{4s_B-c_B}{3}, \\ \frac{t-s_B}{t} & \text{if } \frac{4s_B-c_B}{3} \leq t \leq \frac{3s_B-c_B}{2}, \\ \frac{s_B-c_B}{2t} & \text{if } t > \frac{3s_B-c_B}{2}. \end{cases} \tag{31}$$

Proof. As in Appendix B, with $\overset{\circ}{p}_{0,B} = 0$ rather than $p^{**}_{0,B} = c_B$. ■

Before commenting on the results of Lemma 3, we calculate equilibrium firms' profits and consumer surplus in Sector B of the mixed economy with transfers. To this aim, we plug (30) and (31) into (3) with $j = B$:

$$\begin{array}{l} \overset{\circ}{\Pi}_{0,B} + d = \begin{cases} -c_B + d & \text{if } t \leq c_B, \\ -c_B\frac{3t+c_B}{4t} + d & \text{if } c_B < t < \frac{4s_B-c_B}{3}, \\ -c_B\frac{s_B}{t} + d & \text{if } t \geq \frac{4s_B-c_B}{3}; \end{cases} \\ \overset{\circ}{\Pi}_{1,B} = \begin{cases} 0 & \text{if } t \leq c_B, \\ \frac{(t-c_B)^2}{8t} & \text{if } c_B < t < \frac{4s_B-c_B}{3}, \\ \frac{(t-s_B)(2s_B-c_B-t)}{t} & \text{if } \frac{4s_B-c_B}{3} \leq t \leq \frac{3s_B-c_B}{2}, \\ \frac{(s_B-c_B)^2}{4t} & \text{if } t > \frac{3s_B-c_B}{2}, \end{cases} \end{array} \tag{32}$$

[2] Including firm $1, B$'s profits as a source of transfers to firm $0, B$ would complicate the computations without adding any additional insight.

[3] The amount k can be thought of as a voluntary contribution and/or a non-distortionary lump-sum tax paid by the for-profits and transferred to the nonprofit. Such a tax may represent a fiscal regime that benefits the nonprofit.

where $d = r + k$ is the amount transferred to the firm $0, B$ at $t = 1$. Observe that firm $0, B$ incurs losses for any t when $d = 0$, *i.e.*, $\Pi^{\circ}_{0,B} < 0$. We assume that the two sources of funding, donors' wealth plus total profits of Sector A, are sufficient to recover such losses, thus enabling the nonprofit firm to break-even for any t,

$$R + \sum_i \Pi^{*}_{i,A} \geq d = \left| \Pi^{\circ}_{0,B} \right| . \tag{33}$$

Put differently,

$$\Pi^{\circ}_{0,B} + d = 0. \tag{34}$$

To compute the equilibrium consumer surplus we plug (30) and (31) into (4) and (5),

$$\begin{aligned} CS^{\circ}_{0,B} &= \begin{cases} s_B - \frac{t}{2} & \text{if } t \leq c_B, \\ \frac{(3t+c_B)(8s_B-3t-c_B)}{32t} & \text{if } c_B < t < \frac{4s_B-c_B}{3}, \\ \frac{s_B^2}{2t} & \text{if } t \geq \frac{4s_B-c_B}{3}; \end{cases} \\ CS^{\circ}_{1,B} &= \begin{cases} 0 & \text{if } t \leq c_B, \\ \frac{(t-c_B)(8s_B-5t-3c_B)}{32t} & \text{if } c_B < t < \frac{4s_B-c_B}{3}, \\ \frac{(s_B-t)^2}{2t} & \text{if } \frac{4s_B-c_B}{3} \leq t \leq \frac{3s_B-c_B}{2}, \\ \frac{(s_B-c_B)^2}{8t} & \text{if } t > \frac{3s_B-c_B}{2}. \end{cases} \end{aligned} \tag{35}$$

We have all the elements to explain the results of Lemma 3. First remark that the consumer-surplus-maximizer firm $0, B$ charges zero price. This is because customer surplus $CS_{0,B}$ is negatively affected by the price, as one can check by inspecting the objective function of program (14). On the contrary, the for-profit firm $1, B$ sets at least $p^{\circ}_{1,B} = c_B$ in order not to incur losses. As a result, the demand share of firm $0, B$ is greater, $D^{\circ}_{0,B} > D^{\circ}_{1,B}$ for any t.

More precisely, when $t \leq c_B$ the individuals can move easily along the segment. In symbols, (1-a) and (1-b) depend mainly on $p_{i,B}$ when $j = B$, *i.e.*, the consumers care about the price when they decide which firm they buy from. Since $p^{\circ}_{0,B} = 0$ and $p^{\circ}_{1,B} = c_B$, the individual located at point 1 ends up with $s_B - t$ when buying from the faraway nonprofit firm and with $s_B - c_B$ when resorting to the nearby for-profit firm. The former value is non-lower given $t \leq c_B$. As a result, all individuals resort to the non-profit firm, $D^{\circ}_{0,B} = 1$ and $D^{\circ}_{1,B} = 0$. Note that even if both price $p^{\circ}_{0,B} = 0$ and demand $D^{\circ}_{0,B} = 1$ are unaffected by t, the equilibrium surplus of clients of firm $0, B$, $CS^{\circ}_{0,B} = s_B - \frac{t}{2}$, is decreasing in t simply because they bear greater transportation costs.

At $t > c_B$ those consumers located far from firm $0, B$ prefer to resort to the for-profit rival, *i.e.*, $D^{\circ}_{1,B} > 0$, despite the price differential.

When $t \in \left(c_B, \frac{4s_B-c_B}{3}\right)$ full coverage occurs and the indifferent consumer gets positive surplus. The equilibrium price charged by the for-profit firm $1, B$, $p^{\circ}_{1,B} = \frac{t+c_B}{2}$, is increasing in t as in Section 3. Notwithstanding, its demand $D^{\circ}_{1,B} = \frac{t-c_B}{4t}$ is increasing as well because the consumers become more captive. As a result, firm $1, B$'s profits, $\Pi^{\circ}_{1,B} = \frac{(t-c_B)^2}{8t}$, are positively affected by t, while the effect on $CS^{\circ}_{1,B} = \frac{(t-c_B)(8s_B-5t-3c_B)}{32t}$ is ambiguous. Conversely, the demand share of the nonprofit firm, $D^{\circ}_{0,B} = \frac{3t+c_B}{4t}$, decreases with t. As a consequence its losses, $\left|\Pi^{\circ}_{0,B}\right| = c_B\frac{3t+c_B}{4t}$, as well as surplus of its consumers, $CS^{\circ}_{0,B} = \frac{(3t+c_B)(8s_B-3t-c_B)}{32t}$, decrease with t.

When $t \in \left[\frac{4s_B-c_B}{3}, \frac{3s_B-c_B}{2}\right]$, there is full coverage, while the indifferent consumer gets zero surplus. The price charged by the for-profit firm, $p^{\circ}_{1,B} = 2s_B - t$, is decreasing in t as in Section 3. Accordingly, its demand $D^{\circ}_{1,B} = \frac{t-s_B}{t}$ increases with t. The effect of t on firm $1,B$'s profit, $\Pi^{\circ}_{1,B} = \frac{(t-s_B)(2s_B-c_B-t)}{t}$, is ambiguous, while surplus of its consumers, $CS^{\circ}_{1,B} = \frac{(s_B-t)^2}{2t}$, is positively affected by t. Conversely, the demand share of the nonprofit firm, $D^{\circ}_{0,B} = \frac{s_B}{t}$, decreases with t. As a consequence its losses, $\left|\Pi^{\circ}_{0,B}\right| = c_B\frac{s_B}{t}$, as well as surplus of its consumers, $CS^{\circ}_{1,B} = \frac{s_B^2}{2t}$, decrease with t.

Finally, when $t > \frac{3s_B-c_B}{3}$ there is no full coverage. The equilibrium prices, $p^{\circ}_{0,B} = 0$ and $p^{\circ}_{1,B} = \frac{s_B+c_B}{2}$ are unaffected by t, while the demand shares, $D^{\circ}_{0,B} = \frac{s_B}{t}$ and $D^{\circ}_{1,B} = \frac{s_B-c_B}{2t}$, decrease with t. We can conclude that: (i) firm $1,B$ profits and firm $0,B$ losses, $\Pi^{\circ}_{1,B} = \frac{(s_B-c_B)^2}{4t}$ and $\left|\Pi^{\circ}_{0,B}\right| = c_B\frac{s_B}{t}$, respectively, decreases with t; (i) surplus of consumers of both firms are negatively affected by t.

Following the analysis in Section 4, we are interested in comparing the market coverage in Sector B of the mixed economy with and without transfers. If the nonprofit firm is allowed to make losses, full coverage does not occur only when $t > \frac{3s_B-c_B}{2}$, according to Lemma 3. Recall that the corresponding interval in the case of no transfer is $t > \frac{3(s_B-c_B)}{2}$, see (18), where $\frac{3s_B-c_B}{2} > \frac{3(s_B-c_B)}{2}$ for any $c_B > 0$. We can write the following

Proposition 3 *(i) When transport cost* $t \leq \frac{3(s_B-c_B)}{2}$, *all individuals have access to the social service under both types of mixed economy. (ii) When transport cost* $t \in \left(\frac{3(s_B-c_B)}{2}, \frac{3s_B-c_B}{2}\right]$, *all individuals have access to the social service only under the mixed economy with transfers. (iii) When transport cost* $t > \frac{3s_B-c_B}{2}$, *there is no full coverage but more individuals have access to the social service under the mixed economy with transfers,* $D^{\circ}_{0,B} + D^{\circ}_{1,B} = \frac{3s_B-c_B}{2t} > D^{**}_{0,B} + D^{**}_{1,B} = \frac{3s_B-3c_B}{2t}$.

This result is due to the fact that the nonprofit firm sets an even lower price when transfers are allowed, 0 instead of c_B, thus easing furtherly the purchase for individuals who live far away.

As in the previous section we proceed by running a welfare analysis. It is worth noting that we have to include the donors' wealth R in the computation of welfare. We first compare welfare arising in the mixed economy with transfers to those in the mixed economy (without transfers). We then move to the comparison between welfare in the market economy and in the mixed economy with transfers.

The equilibrium welfare in the mixed economy (without transfers) is given by (25) plus R. In the mixed economy with transfers the overall profits made by the firms plus the donors' wealth are

$$(2\Pi^*_A - k) + \left(\Pi^{\circ}_{0,B} + d\right) + \Pi^{\circ}_{1,B} + (R - r)\,. \tag{36}$$

Recall that the amount $d = r + k$ denotes the lump-sum transfer from donors and for-profit firms i, A to the nonprofit firm $1, B$. Instead, the total surplus of individuals is

$$2CS_A^* + CS_{0,B}^{\circ} + CS_{1,B}^{\circ} + 0. \tag{37}$$

The equilibrium welfare in the mixed economy with transfers, denoted with W_{MI}°, is thus given by

$$W_{MI}^{\circ} = 2\Pi_A^* + \Pi_{0,B}^{\circ} + \Pi_{1,B}^{\circ} + R + 2CS_A^* + CS_{0,B}^{\circ} + CS_{1,B}^{\circ}. \tag{38}$$

We denote with ΔW° the difference $W_{MI}^{\circ} - W_{MI}$,

$$\Delta W^{\circ} = W_{MI}^{\circ} - W_{MI} = \left(\Pi_{0,B}^{\circ} + \Pi_{1,B}^{\circ} + CS_{0,B}^{\circ} + CS_{1,B}^{\circ}\right) - \left(\Pi_{1,B}^{**} + CS_{0,B}^{**} + CS_{1,B}^{**}\right), \tag{39}$$

and study its sign in the following

Proposition 4 *Welfare in the mixed economy (without transfers) is larger than welfare in the mixed economy with transfers, i.e.,* $\Delta W^{\circ} < 0$ *for any* t.

Proof. See Appendix D. ■

Allowing for transfers to the nonprofit firm makes price competition even tougher and the market coverage increases compared with that under the mixed economy (without transfers). On the one hand, the consumers benefit from this. On the other hand, the nonprofit firm incurs losses net of the transfers and, overall, firms' total profits are negatively affected. Such negative effect is shown to outdo the consumer surplus gain. We conclude that welfare is larger when no transfers are allowed.

The last step of our analysis in this section consists of comparing welfare in the market economy to welfare in the mixed economy with transfers. In symbols, we study the sign of $\Delta W' = W_{MA} - W_{MI}^{\circ}$, where

$$W_{MA} - W_{MI}^{\circ} = (2\Pi_B^* + 2CS_B^*) - \left(\Pi_{0,B}^{\circ} + \Pi_{1,B}^{\circ} + CS_{0,B}^{\circ} + CS_{1,B}^{\circ}\right).$$

First notice that $\Delta W'$ is necessarily positive when $t \leq \frac{6(s_B - c_B)}{5}$ according to Propositions 2 and 4. To simplify the reading of our results we introduce the following notation

$$t^{\circ} = \begin{cases} \frac{4s_B - c_B}{3} & \text{if} \quad c_B \in \left[0, \frac{3\sqrt{2}-1}{17} s_B\right] \\ \frac{4s_B - 2c_B - \sqrt{2s_B^2 - 4s_B c_B - 2c_B^2}}{2} & \text{if} \quad c_B \in \left(\frac{3\sqrt{2}-1}{17} s_B, \frac{s_B}{3}\right) \end{cases}$$

where $t^{\circ} > \frac{6(s_B - c_B)}{5}$.

Proposition 5 *(i) Suppose* $c_B \in \left[0, \frac{s_B}{3}\right)$*: the welfare is larger (lower) in the market economy than in the mixed economy with transfers, i.e.,* $\Delta W' \geq (<)\, 0$*, iff* $t \leq (>)\, t^{\circ}$*. (i) Suppose* $c_B \in \left[\frac{s_B}{3}, s_B\right]$*: the welfare is larger in the market economy, i.e.,* $\Delta W' > 0$ *for any* t.

Proof. See Appendix ■

We depict t° in plane (c_B, t) with $c_B \in [0, s_B]$ to discuss the results of Proposition 2: see Figure 4.

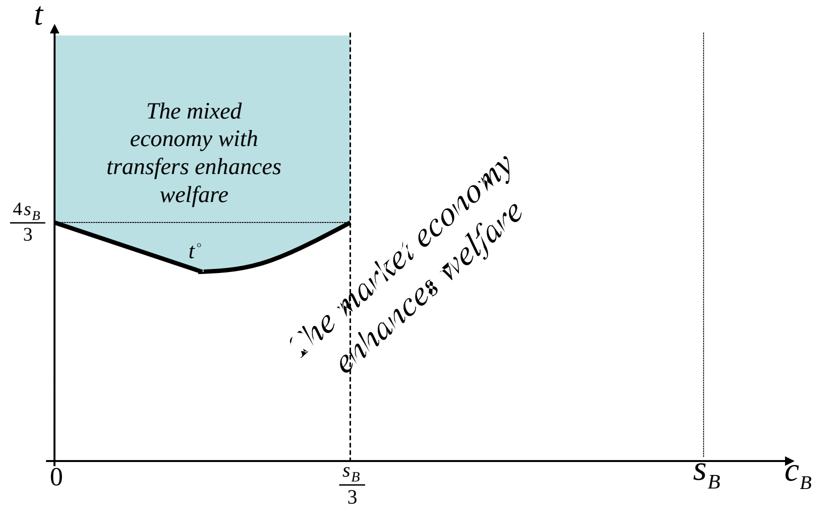

FIGURE 4: MARKET ECONOMY VERSUS MIXED ECONOMY WITH TRANSFERS IN TERMS OF WELFARE

Consider the portion of plane $t > t^{\circ} \cup c_B \in \left[0, \frac{s_B}{3}\right)$, where the mixed economy with transfers enhances welfare compared to the market economy. First note that $t^{\circ} \in \left[\frac{4s_B - c_B}{3}, \frac{3s_B - c_B}{2}\right]$. According to Propositions 1 and 3, this implies that market coverage is strictly greater under the mixed economy with transfers when $t > t^{\circ}$. In turn, larger coverage is shown to enhance welfare, provided that c_B is low relatively to s_B. To explain why we focus on interval $t > \frac{3s_B - c_B}{2} \geq t^{\circ}$, where

$$\Delta W' = W_{MA} - W_{MI}^{\circ} = (2\Pi_B^* + 2CS_B^*) - \left(\Pi_{0,B}^{\circ} + \Pi_{1,B}^{\circ} + CS_{0,B}^{\circ} + CS_{1,B}^{\circ}\right) =$$

$$\left(2\frac{(s_B - c_B)^2}{4t} + 2\frac{(s_B - c_B)^2}{8t}\right) - \left(-c_B\frac{s_B}{t} + \frac{(s_B - c_B)^2}{4t} + \frac{s_B^2}{2t} + \frac{(s_B - c_B)^2}{8t}\right) = -\frac{(s_B + c_B)(s_B - 3c_B)}{8t}.$$

It is worth observing that both $(2\Pi_B^* + 2CS_B^*)$ and $\left(\Pi_{0,B}^{\circ} + \Pi_{1,B}^{\circ} + CS_{0,B}^{\circ} + CS_{1,B}^{\circ}\right)$ are decreasing in c_B. When $c_B < \frac{s_B}{3}$, $W_{MA} < W_{MI}^{\circ}$ because the losses incurred by the nonprofit firm are relatively low. More precisely, the surplus gain enjoyed by the consumers of the nonprofit firm outdoes the profit loss incurred by firm $0, B$ when moving from the market to the mixed economy with transfers. As c_B increases, instead, the losses of the nonprofit firm becomes significant, with the effect that $W_{MA} > W_{MI}^{\circ}$. In symbols, the negative effect of c_B on $\left(\Pi_{0,B}^{\circ} + \Pi_{1,B}^{\circ} + CS_{0,B}^{\circ} + CS_{1,B}^{\circ}\right)$ is larger than that on $(2\Pi_B^* + 2CS_B^*)$,

$$\left|\frac{\partial\left(\Pi_{0,B}^{\circ} + \Pi_{1,B}^{\circ} + CS_{0,B}^{\circ} + CS_{1,B}^{\circ}\right)}{\partial c_B}\right| = \frac{7s_B - 3c_B}{4t} > \left|\frac{\partial\left(2\Pi_B^* + 2CS_B^*\right)}{\partial c_B}\right| = \frac{3(s_B - c_B)}{2t}.$$

A similar reasoning can be applied to explain why welfare is larger in the mixed economy with transfers when $t^{\circ} < t \leq \frac{3s_B - c_B}{2} \cup c_B \in \left[0, \frac{s_B}{3}\right)$.

We sum up the results of this section. Allowing the nonprofit firm in Sector B to sell below cost enhances the coverage of the social service market (Proposition 3) but worsens welfare (Proposition 4) compared with the mixed economy (without transfers). This occurs mainly because the nonprofit firm incurs losses net of the transfer.

When comparing welfare in the market economy and in the mixed economy with transfers, we find that larger coverage is still a necessary condition for welfare to be greater under the latter economy. Such a condition becomes sufficient when c_B is low relatively to s_B, in which case the before-transfer losses incurred by the nonprofit firm are relatively low (Proposition 5).

7 Extension II: t as ideological cost

Throughout the above analysis the linear city denoted a physical space and parameter t a transportation cost. This is in line with the original Hotelling (1929) framework. In this section we check the robustness of our findings in the previous sections by proposing an alternative interpretation.

We assume that Sector A is still represented by a physical space. Instead, we disregard transportation costs in the social service sector B and assume that the unit segment represents a space of firm types. More precisely, a hypothetical firm located in $x \in [0, 1]$ is assumed to maximize a convex combination of its profits and surplus of its consumers, where x is the weight attached to profits and $1 - x$ to consumer surplus. Accordingly, a nonprofit firm, which attaches maximum weight to consumer surplus is located at the extreme left of the unit segment, $x = 0$. By contrast, a for-profit firm lies on the extreme right, $x = 1$, because it puts weight 1 on its profits.

Similarly, location of the individuals along the segment denotes their ideological position towards the type of the firms. The ideal type for an individual located in $x \in [0, 1]$ consists in a firm attaching weight x to its profits and weight $1 - x$ to surplus of its consumers. The x- individual incurs thus the ideological cost tx ($t(1 - x)$) when buying from a nonprofit firm located at 0 (a for-profit firm located at 1), t denoting the unit cost to fill the ideological distance between an individual's ideal type of firm and the actual types. This is an example of single-peaked preferences in the spirit of the median voter framework.

This alternative framework does not affect the strategic interaction in Sector B of the mixed economy. Indeed, the two rivals are still located at the extremes of the city, the nonprofit firm $0, B$ at $x = 0$ and the for-profit firm $1, B$ at $x = 1$. Accordingly, the Nash equilibria of the mixed economy and of the mixed economy with transfers are as in Lemmata 2 and 3, respectively.

By contrast, the strategic interaction in Sector B of the market economy is affected, in that the two for-profit rivals are both located at $x = 1$, rather than lying at the extremes of the segment. As a result, the results of Lemma 1 are not valid anymore. One can easily check that, given the same extreme-right location of the two firms, their strategic behavior boils down to Bertrand undercutting, where both firms set the price equal to marginal cost, $p_B^I = c_B$. This implies that both firms make zero profit. The symmetric equilibrium demand of each firm is instead $D_B^I = \min\left\{\frac{1}{2}, \frac{s_B - c_B}{2t}\right\}$, which means that full coverage occurs if and only if $t \leq s_B - c_B$. In that case the equilibrium consumer surplus of each firm's clients is

$$CS_B^I = \frac{1}{2}\left(s_B - c_B - \frac{t}{2}\frac{1}{2}\right) = \frac{4s_B - 4c_B - t}{8}, \tag{40}$$

obtained after plugging $p_B^I = c_B$ and $D_B^I = \frac{1}{2}$ into (5). The market is not fully covered if $t > s_B - c_B$, in which case the equilibrium consumer surplus of each firm's clients is

$$CS_B^I = \frac{s_B - c_B}{2t}\left(s_B - c_B - \frac{t}{2}\frac{s_B - c_B}{2t}\right) = \frac{3}{8}\frac{(s_B - c_B)^2}{t}, \tag{41}$$

obtained after plugging $p_B^I = c_B$ and $D_B^I = \frac{s_B - c_B}{2t}$ into (5). Since the equilibrium profits are zero, welfare in Sector B is simply given by $2CS_B^I$. Substituting (40) and (41) into $2CS_B^I$ yields exactly (27). This means that welfare in Sector B of the market economy is not affected when t denotes an ideological cost rather than a transportation one. The intuition is as follows. The two-for profit firms charge lower price, $p_B^I = c_B$ instead of p_B^* in (9) with $j = B$. This affects negatively their profits and positively the consumer surplus. These two opposite effects are shown to be equal in magnitude.

We can conclude that our findings in the previous sections are robust to this alternative specification of parameter t.

8 Conclusion

In this paper we provided a comparison in terms of utilitarian welfare between two different economic environments. A market economy where all firms are for-profit. A mixed economy where one firm maximizes the surplus of its customers. We found that for relatively large value of transport costs in the Hotelling city coverage of the social service market is larger under the mixed economy. We then showed that larger coverage is a necessary condition for the utilitarian welfare to be enhanced by the mixed economy. Finally, we demonstrated that our findings are robust when ideological rather than transport costs are considered.

A Lemma 1

Recalling that $D_{0,j} = [0, x_{0,j}]$, firm $0, j$ problem (8) can be rewritten as

$$\begin{array}{c} \max_{p_{0,j}, x_{0,j}} \Pi_{0,j} = (p_{0,j} - c_j)\, x_{0,j} \\ \text{s.t. } x_{0,j} \leq \frac{p_{1,j} - p_{0,j} + t}{2t} \text{ and } x_{0,j} \leq \frac{s_j - p_{0,j}}{t}. \end{array} \tag{42}$$

The Lagrangean is

$$\mathcal{L} = (p_{0,j} - c_j)\, x_{0,j} - \lambda \left(x_{0,j} - \frac{p_{1,j} - p_{0,j} + t}{2t} \right) - \mu \left(x_{0,j} - \frac{s_j - p_{0,j}}{t} \right). \tag{43}$$

FOCs are

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial p_{0,j}} = x_{0,j} - \frac{\lambda}{2t} - \frac{\mu}{t} = 0, \\ \frac{\partial \mathcal{L}}{\partial x_{0,j}} = p_{0,j} - c_j - \lambda - \mu = 0. \end{cases} \tag{44}$$

Since $\Pi_{0,j}$ increases with $x_{0,j}$ at least one of the two constraints must be binding at a solution to (42). We thus study three alternative scenarios.

1) $\mu = 0$ and $\lambda > 0$, then only the first constraint in (42) is binding, *i.e.*,

$$\frac{p_{1,j} - p_{0,j} + t}{2t} < \frac{s_j - p_{0,j}}{t}. \tag{45}$$

This means that the market is fully covered and that the indifferent consumer gets positive surplus. Plugging $\mu = 0$ into the FOCs yields

$$\begin{cases} x_{0,j} = \frac{\lambda}{2t}, \\ p_{0,j} - c_j = \lambda, \end{cases} \tag{46}$$

whereby $x_{0,j} = \frac{p_{0,j} - c_j}{2t}$. Solving

$$\begin{cases} x_{0,j} = \frac{p_{0,j} - c_j}{2t}, \\ x_{0,j} = \frac{p_{1,j} - p_{0,j} + t}{2t}, \end{cases} \tag{47}$$

by $p_{0,j}$ and $x_{0,j}$ yields

$$p_{0,j} = \frac{p_{1,j} + c_j + t}{2} \text{ and } x_{0,j} = \frac{p_{1,j} + t - c_j}{4t}. \tag{48}$$

Plugging $p_{0,j} = \frac{p_{1,j} + c_j + t}{2}$ into (45) yields

$$p_{1,j} < \frac{4s_j - 3t - c_j}{3}. \tag{49}$$

Finally, note that λ is positive iff $p_{0,j} - c_j > 0$. Substituting $p_{0,j} = \frac{p_{1,j} + c_j + t}{2}$ yields

$$p_{1,j} > c_j - t. \tag{50}$$

2) $\mu \geq 0$ and $\lambda \geq 0$, then both constraints are binding, *i.e.*, $\frac{p_{1,j} - p_{0,j} + t}{2t} = \frac{s_j - p_{0,j}}{t}$ or

$$p_{0,j} = 2s_j - t - p_{1,j} \text{ and } x_{0,j} = \frac{t + p_{1,j} - s_j}{t}. \tag{51}$$

This means that the market is fully covered and that the indifferent consumer gets zero surplus. FOCs are as in (44), whereby

$$\begin{cases} \mu = 2t\frac{t + p_{1,j} - s_j}{t} - (2s_j - t - p_{1,j}) + c_j, \\ \lambda = 2\,(2s_j - t - p_{1,j}) - 2c_j - 2t\frac{t + p_{1,j} - s_j}{t}. \end{cases} \tag{52}$$

after plugging (51). Both μ and λ must be nonnegative:

$$\frac{4s_j - c_j - 3t}{3} \leq p_{1,j} \leq \frac{3s_j - c_j - 2t}{2}. \tag{53}$$

3) $\mu > 0$ and $\lambda = 0$, then only the second constraint in (42) is binding, *i.e.*,

$$\frac{p_{1,j} - p_{0,j} + t}{2t} > \frac{s_j - p_{0,j}}{t}. \tag{54}$$

This implies that firm 0 is local monopolist. Plugging $\mu > 0$ and $\lambda = 0$ into the FOCs yields

$$\begin{cases} x_{0,j} - \frac{\mu}{t} = 0, \\ p_{0,j} - c_j - \mu = 0, \end{cases} \tag{55}$$

whereby $\mu = p_{0,j} - c_j = tx_{0,j}$. Taking into account the constraint $x_{0,j} = \frac{s_j - p_{0,j}}{t}$ we have

$$p_{0,j} = \frac{c_j + s_j}{2} \text{ and } x_{0,j} = \frac{\frac{c_j+s_j}{2} - c_j}{t} = \frac{s_j - c_j}{2t}. \tag{56}$$

Substituting $p_{0,j} = \frac{c_j+s_j}{2}$ into (54) yields

$$p_{1,j} > \frac{3s_j - 2t - c_j}{2}. \tag{57}$$

Finally, we check that $\mu > 0$, *i.e.*, $p_{0,j} - c_j = tx_{0,j} > 0$: $\frac{s_j - c_j}{2} > 0$, which holds true given (7).

Summing up the three scenarios yields firm 0 best response:

$$p_{0,j} = \begin{cases} \frac{p_{1,j}+c_j+t}{2} & \text{if } c_j - t \le p_{1,j} < \frac{4s_j-3t-c_j}{3}, \\ 2s_j - t - p_{1,j} & \text{if } \frac{4s_j-c_j-3t}{3} \le p_{1,j} \le \frac{3s_j-c_j-2t}{2}, \\ \frac{c_j+s_j}{2} & \text{if } p_{1,j} > \frac{3s_j-2t-c_j}{2}, \end{cases} \tag{58}$$

and

$$x_{0,j} = \begin{cases} \frac{p_{1,j}+t-c_j}{4t} & \text{if } c_j - t \le p_{1,j} < \frac{4s_j-3t-c_j}{3}, \\ \frac{t+p_{1,j}-s_j}{t} & \text{if } \frac{4s_j-c_j-3t}{3} \le p_{1,j} \le \frac{3s_j-c_j-2t}{2}, \\ \frac{s_j-c_j}{2t} & \text{if } p_{1,j} > \frac{3s_j-2t-c_j}{2}. \end{cases} \tag{59}$$

The symmetric Nash equilibrium is hence as follows.

a) $p_j^* = \frac{p_j^*+c_j+t}{2}$, hence $p_j^* = c_j + t$ and $x_{0,j}^* = \frac{c_j+t+t-c_j}{4t} = \frac{1}{2}$. The corresponding interval becomes $c_j - t < c_j + t < \frac{4s_j-3t-c_j}{3}$ or

$$t < \frac{2}{3}(s_j - c_j). \tag{60}$$

b) $p_j^* = 2s_j - t - p_j^*$, hence $p_j^* = s_j - \frac{t}{2}$ and $x_{0,j}^* = \frac{-s_j+t+s_j-\frac{t}{2}}{t} = \frac{1}{2}$. The corresponding interval becomes $\frac{4s_j-c_j-3t}{3} \le \frac{2s_j-t}{2} \le \frac{3s_j-c_j-2t}{2}$ or

$$\frac{2s_j - 2c_j}{3} \le t \le s_j - c_j. \tag{61}$$

c) If $p_{1,j} > \frac{3s_j-2t-c_j}{2}$ we have local monopolies. Since $p_j^* = \frac{c_j+s_j}{2}$ and $x_j^* = \frac{s_j-c_j}{2t}$, the relevant interval is $\frac{c_j+s_j}{2} > \frac{3s_j-2t-c_j}{2}$ or $t > s_j - c_j$.

B Lemma 2

For-profit firm $1, B$ best response functions are given by (58) and (59), *mutatis mutandis*:

$$p_{1,B} = \begin{cases} \frac{p_{0,B}+c_B+t}{2} & \text{if } c_B - t \le p_{0,B} < \frac{4s_B-3t-c_B}{3}, \\ 2s_B - t - p_{0,B} & \text{if } \frac{4s_B-c_B-3t}{3} \le p_{0,B} \le \frac{3s_B-c_B-2t}{2}, \\ \frac{c_B+s_B}{2} & \text{if } p_{0,B} > \frac{3s_B-2t-c_B}{2}, \end{cases} \tag{62}$$

and

$$D_{1,B}=\begin{cases}\frac{p_{0,B}+t-c_B}{4t} & \text{if } c_B-t\leq p_{0,B}<\frac{4s_B-3t-c_B}{3},\\ \frac{t+p_{0,B}-s_B}{t} & \text{if } \frac{4s_B-c_B-3t}{3}\leq p_{0,B}\leq\frac{3s_B-c_B-2t}{2},\\ \frac{s_B-c_B}{2t} & \text{if } p_{0,B}>\frac{3s_B-2t-c_B}{2}.\end{cases} \tag{63}$$

Recalling that $D_{0,B}=[0,x_{0,B}]$ and formula (4), nonprofit firm 0 solves the following problem:

$$\begin{array}{c}\max_{p_{0,B},x_{0,B}} x_{0,B}\left(s-p_{0,B}-\frac{t}{2}x_{0,B}\right),\\ \text{s.t. } x_{0,B}\leq\frac{p_{1,B}-p_{0,B}+t}{2t},\ x_{0,B}\leq\frac{s_B-p_{0,B}}{t},\text{ and } p_{0,B}\geq c_B\end{array} \tag{64}$$

First notice that the objective function is decreasing in $p_{0,B}$, hence $p_{0,B}=c_B$ is optimal. In such a case the two constraints can be rewritten as

$$x_{0,B}\leq\frac{p_{1,B}-c_B+t}{2t}\text{ and } x_{0,B}\leq\frac{s_B-c_B}{t} \tag{65}$$

and the objective function as $x_{0,B}\left(s_B-c_B-\frac{t}{2}x_{0,B}\right)$. F.O.C. is

$$s_B-c_B-tx_{0,B}=0 \tag{66}$$

and S.O.C. is verified since $-t<0$. As a consequence, solution to (64) is

$$x_{0,B}=\frac{s_B-c_B}{t}\text{ if }\frac{p_{1,B}-c_B+t}{2t}\geq\frac{s_B-c_B}{t}, \tag{67}$$

in which case only the second constraint in (64) is binding. This means that firms are local monopolists. By contrast, solution to (64) is

$$x_{0,B}=\frac{p_{1,B}-c_B+t}{2t}\text{ if }\frac{p_{1,B}-c_B+t}{2t}<\frac{s_B-c_B}{t}, \tag{68}$$

in which case only the first constraint in (64) is binding. This means that the market is fully covered and the indifferent individual gets nonnegative surplus. Summing up yields nonprofit firm 0 best response:

$$p_{0,B}=c_B;\ D_{0,B}=\begin{cases}\frac{p_{1,B}-c_B+t}{2t} & \text{if } p_{1,B}<2s_B-c_B-t,\\ \frac{s_B-c_B}{t} & \text{if } p_{1,B}\geq 2s_B-c_B-t.\end{cases} \tag{69}$$

The Nash equilibrium is hence as follows.

a) If $t<\frac{4}{3}(s_B-c_B)$, $p^{**}_{1,B}=\frac{2c_B+t}{2}$, $D^{**}_{1,B}=\frac{1}{4}$, $p^{**}_{0,B}=c_B$ and $D^{**}_{0,B}=\frac{\frac{2c_B+t}{2}-c_B+t}{2t}=\frac{3}{4}$.Notice that both $p^{**}_{1,B}<2s_B-c_B-t$ and $D^{**}_{0,B}<\frac{s_B-c_B}{t}$ are equivalent to $t<\frac{4}{3}(s_B-c_B)$, which is true.

c) If $\frac{4}{3}(s_B-c_B)\leq t\leq\frac{3}{2}(s_B-c_B)$, then $p^{**}_{1,B}=2s_B-t-c_B$, $D^{**}_{1,B}=1-\frac{s_B-c_B}{t}$, $p^{**}_{0,B}=c_B$ and $D^{**}_{0,B}=\frac{2s_B-t-c_B-c_B+t}{2t}=\frac{s_B-c_B}{t}$. Notice that $\frac{p^{**}_{1,B}-c_B+t}{2t}=\frac{s_B-c_B}{t}$.

d) If $t>\frac{3}{2}(s_B-c_B)$, then $p^{**}_{1,B}=\frac{c_B+s_B}{2}$, $D^{**}_{1,B}=\frac{s_B-c_B}{2t}$, $p^{**}_{0,B}=c_B$ and $D^{**}_{0,B}=\frac{s_B-c_B}{t}$. Notice that $p^{**}_{1,B}>2s_B-t-c$ and $\frac{s_B-c_B}{t}+\frac{s_B-c_B}{2t}<1$ are equivalent to $t>\frac{3}{2}(s_B-c_B)$, which is true.

C Proposition 2

First note that the ordering of t-cutoffs in Lemmas 1 and 2 is

$$\frac{2(s_B-c_B)}{3}<s_B-c_B<\frac{4(s_B-c_B)}{3}<\frac{3(s_B-c_B)}{2}$$

1. If $t \leq s_B - c_B$, $\Delta W = \frac{4(s_B-c_B)-t}{4} - \frac{16(s_B-c_B)-5t}{16} = \frac{t}{16} > 0$.

2. If $s_B - c_B < t < \frac{4(s_B-c_B)}{3}$, $\Delta W = \frac{3}{4}\frac{(s_B-c_B)^2}{t} - \frac{16(s_B-c_B)-5t}{16} = \frac{[2(s_B-c_B)-t][6(s_B-c_B)-5t]}{16t}$, where $2(s_B - c_B) - t > 0$ and $6(s_B - c_B) - 5t \geq 0$ iff $t \leq \frac{6(s_B-c_B)}{5}$.

3. If $\frac{4(s_B-c_B)}{3} \leq t \leq \frac{3(s_B-c_B)}{2}$, $\Delta W = \frac{3}{4}\frac{(s_B-c_B)^2}{t} - \left(\frac{4(s_B-c_B)-t}{2} - \frac{(s_B-c_B)^2}{t}\right) = \frac{8tc_B-8ts_B-14c_Bs_B+2t^2+7c_B^2+7s_B^2}{4t} > 0$ iff $\left(2-\frac{\sqrt{2}}{2}\right)(s_B - c_B) < t < \left(2+\frac{\sqrt{2}}{2}\right)(s_B - c_B)$. Note that $\left(2-\frac{\sqrt{2}}{2}\right)(s_B - c_B) < \frac{4(s_B-c_B)}{3}$ and $\left(2+\frac{\sqrt{2}}{2}\right)(s_B - c_B) > \frac{3(s_B-c_B)}{2}$.

4. If $t > \frac{3(s_B-c_B)}{2}$, $\Delta W = \frac{3}{4}\frac{(s_B-c_B)^2}{t} - \left(\frac{(s_B-c_B)^2}{4t} + \frac{(s_B-c_B)^2}{2t} + \frac{(s_B-c_B)^2}{8t}\right) = \frac{1}{8}\frac{(s_B-c_B)^2}{t} > 0$.

D Proposition 4

First note that the ordering of t-cutoffs in Lemmas 2 and 3 depends on c_B and s_B:

a. if $c_B < \frac{s_B}{7}$, $c_B < \frac{4}{3}(s_B - c_B) < \frac{4s_B-c_B}{3} < \frac{3}{2}(s_B - c_B) < \frac{3s_B-c_B}{2}$;

b. if $\frac{s_B}{7} < c_B < \frac{4s_B}{7}$, $c_B < \frac{4}{3}(s_B - c_B) < \frac{3}{2}(s_B - c_B) < \frac{4s_B-c_B}{3} < \frac{3s_B-c_B}{2}$;

c. if $\frac{4s_B}{7} < c_B < \frac{3s_B}{5}$, $\frac{4}{3}(s_B - c_B) < c_B < \frac{3}{2}(s_B - c_B) < \frac{4s_B-c_B}{3} < \frac{3s_B-c_B}{2}$;

d. if $\frac{3s_B}{5} < c_B < s_B$, $\frac{4}{3}(s_B - c_B) < \frac{3}{2}(s_B - c_B) < c_B < \frac{4s_B-c_B}{3} < \frac{3s_B-c_B}{2}$.

a. Let us begin with ordering a.

1. $t \leq c_B$, $\Pi^{**}_{0,B} + \Pi^{**}_{1,B} + CS^{**}_{0,B} + CS^{**}_{1,B} - \left(\Pi'^{**}_{1,B} + CS'^{**}_{0,B} + C'S^{**}_{1,B}\right) = -c_B + 0 + s_B - \frac{t}{2} + 0 - \left(\frac{t}{8} + \frac{3(8s_B-3t-8c_B)}{32} + \frac{8s_B-5t-8c_B}{32}\right) = -\frac{3}{16}t < 0$.

2. $c_B < t < \frac{4}{3}(s_B - c_B)$,
$-c_B\frac{3t+c_B}{4t} + \frac{(t-c_B)^2}{8t} + \frac{(3t+c_B)(8s_B-3t-c_B)}{32t} + \frac{(t-c_B)(8s_B-5t-3c_B)}{32t} - \left(\frac{t}{8} + \frac{3(8s_B-3t-8c_B)}{32} + \frac{8s_B-5t-8c_B}{32}\right) = -\frac{(2t+c_B)c_B}{16t} < 0$.

3. $\frac{4}{3}(s_B - c_B) < t < \frac{4s_B-c_B}{3}$,
$-c_B\frac{3t+c_B}{4t} + \frac{(t-c_B)^2}{8t} + \frac{(3t+c_B)(8s_B-3t-c_B)}{32t} + \frac{(t-c_B)(8s_B-5t-3c_B)}{32t} - \left(\frac{(2s_B-2c_B-t)(t+c_B-s_B)}{t} + \frac{(s_B-c_B)^2}{2t} + \frac{(s_B-c_B-t)^2}{2t}\right) = \frac{(4s_B-3c_B-t)(4s_B-5c_B-3t)}{16t}$ where $4s_B - 5c_B - 3t < 0$ and $4s_B - 3c_B - t > 0$, $4s_B - 3c_B > t$.

4. $\frac{4s_B-c_B}{3} < t < \frac{3}{2}(s_B - c_B)$,
$-c_B\frac{s_B}{t} + \frac{(t-s_B)(2s_B-c_B-t)}{t} + \frac{s_B^2}{2t} + \frac{(s_B-t)^2}{2t} - \left(\frac{(2s_B-2c_B-t)(t+c_B-s_B)}{t} + \frac{(s_B-c_B)^2}{2t} + \frac{(s_B-c_B-t)^2}{2t}\right) = \frac{(t+c_B-2s_B)c_B}{t}$, where $t + c_B - 2s_B < 0$.

5. $\frac{3}{2}(s_B - c_B) < t < \frac{3s_B-c_B}{2}$,
$-c_B\frac{s_B}{t} + \frac{(t-s_B)(2s_B-c_B-t)}{t} + \frac{s_B^2}{2t} + \frac{(s_B-t)^2}{2t} - \left(\frac{(s_B-c_B)^2}{4t} + \frac{(s_B-c_B)^2}{2t} + \frac{(s_B-c_B)^2}{8t}\right) = \frac{-\left(8tc_B-16ts_B-14c_Bs_B+4t^2+7c_B^2+15s_B^2\right)}{8t}$, which is negative in the interval under analysis.

6. $t > \frac{3s_B-c_B}{2}$, $-c_B\frac{s_B}{t} + \frac{(s_B-c_B)^2}{4t} + \frac{s_B^2}{2t} + \frac{(s_B-c_B)^2}{8t} - \left(\frac{(s_B-c_B)^2}{4t} + \frac{(s_B-c_B)^2}{2t} + \frac{(s_B-c_B)^2}{8t}\right) = -\frac{c_B^2}{2t} < 0$

b. We now focus on ordering b.

1. $t \leq c_B$, $-c_B + 0 + s_B - \frac{t}{2} + 0 - \left(\frac{t}{8} + \frac{3(8s_B-3t-8c_B)}{32} + \frac{8s_B-5t-8c_B}{32}\right) = -\frac{3}{16}t$

2. $c_B < t < \frac{4}{3}(s_B - c_B)$,
$-c_B\frac{3t+c_B}{4t} + \frac{(t-c_B)^2}{8t} + \frac{(3t+c_B)(8s_B-3t-c_B)}{32t} + \frac{(t-c_B)(8s_B-5t-3c_B)}{32t} - \left(\frac{t}{8} + \frac{3(8s_B-3t-8c_B)}{32} + \frac{8s_B-5t-8c_B}{32}\right) = -\frac{(2t+c_B)c_B}{16t} < 0$

3. $\frac{4}{3}(s_B - c_B) < t < \frac{3}{2}(s_B - c_B)$,
$-c_B\frac{3t+c_B}{4t} + \frac{(t-c_B)^2}{8t} + \frac{(3t+c_B)(8s_B-3t-c_B)}{32t} + \frac{(t-c_B)(8s_B-5t-3c_B)}{32t} - \left(\frac{(2s_B-2c_B-t)(t+c_B-s_B)}{t} + \frac{(s_B-c_B)^2}{2t} + \frac{(s_B-c_B-t)^2}{2t}\right) = \frac{(4s_B-3c_B-t)(4s_B-5c_B-3t)}{16t}$, where $4s_B - 5c_B - 3t < 0$ and $4s_B - 3c_B - t > 0$.

4. $\frac{3}{2}(s_B - c_B) < t < \frac{4s_B-c_B}{3}$,
$-c_B\frac{3t+c_B}{4t} + \frac{(t-c_B)^2}{8t} + \frac{(3t+c_B)(8s_B-3t-c_B)}{32t} + \frac{(t-c_B)(8s_B-5t-3c_B)}{32t} - \left(\frac{(s_B-c_B)^2}{4t} + \frac{(s_B-c_B)^2}{2t} + \frac{(s_B-c_B)^2}{8t}\right) = -\frac{\left(18tc_B-16ts_B-28c_Bs_B+5t^2+15c_B^2+14s_B^2\right)}{16t}$, which is negative.

5. $\frac{4s_B-c_B}{3} < t < \frac{3s_B-c_B}{2}$,
$-c_B\frac{s_B}{t} + \frac{(t-s_B)(2s_B-c_B-t)}{t} + \frac{s_B^2}{2t} + \frac{(s_B-t)^2}{2t} - \left(\frac{(s_B-c_B)^2}{4t} + \frac{(s_B-c_B)^2}{2t} + \frac{(s_B-c_B)^2}{8t}\right) = \frac{-\left(8tc_B-16ts_B-14c_Bs_B+4t^2+7c_B^2+15s_B^2\right)}{8t}$, which is negative in the relevant interval.

6. $t > \frac{3s_B-c_B}{2}$, $-c_B\frac{s_B}{t} + \frac{(s_B-c_B)^2}{4t} + \frac{s_B^2}{2t} + \frac{(s_B-c_B)^2}{8t} - \left(\frac{(s_B-c_B)^2}{4t} + \frac{(s_B-c_B)^2}{2t} + \frac{(s_B-c_B)^2}{8t}\right) = -\frac{c_B^2}{2t} < 0$

c. Ordering c.

1. $t \leq \frac{4}{3}(s_B - c_B)$,
$-c_B + 0 + s_B - \frac{t}{2} + 0 - \left(\frac{t}{8} + \frac{3(8s_B-3t-8c_B)}{32} + \frac{8s_B-5t-8c_B}{32}\right) = -\frac{3}{16}t < 0.$

2. $\frac{4}{3}(s_B - c_B) < t < c_B$,
$-c_B + 0 + s_B - \frac{t}{2} + 0 - \left(\frac{(2s_B-2c_B-t)(t+c_B-s_B)}{t} + \frac{(s_B-c_B)^2}{2t} + \frac{(s_B-c_B-t)^2}{2t}\right) = \frac{(t+c_B-s_B)(c_B-s_B)}{t}$, where $(c_B - s_B) < 0$ and $t + c_B - s_B > 0$.

3. $c_B < t < \frac{3}{2}(s_B - c_B)$,
$-c_B\frac{3t+c_B}{4t} + \frac{(t-c_B)^2}{8t} + \frac{(3t+c_B)(8s_B-3t-c_B)}{32t} + \frac{(t-c_B)(8s_B-5t-3c_B)}{32t} - \left(\frac{(2s_B-2c_B-t)(t+c_B-s_B)}{t} + \frac{(s_B-c_B)^2}{2t} + \frac{(s_B-c_B-t)^2}{2t}\right) = \frac{(4s_B-3c_B-t)(4s_B-5c_B-3t)}{16t}$, where $4s_B - 5c_B - 3t < 0$ and $4s_B - 3c_B - t > 0$.

4. $\frac{3}{2}(s_B - c_B) < t < \frac{4s_B-c_B}{3}$,
$-c_B\frac{3t+c_B}{4t} + \frac{(t-c_B)^2}{8t} + \frac{(3t+c_B)(8s_B-3t-c_B)}{32t} + \frac{(t-c_B)(8s_B-5t-3c_B)}{32t} - \left(\frac{(s_B-c_B)^2}{4t} + \frac{(s_B-c_B)^2}{2t} + \frac{(s_B-c_B)^2}{8t}\right) = -\frac{18tc_B-16ts_B-28c_Bs_B+5t^2+15c_B^2+14s_B^2}{16t} < 0.$

5. $\frac{4s_B-c_B}{3} < t < \frac{3s_B-c_B}{2}$,
$-c_B\frac{s_B}{t} + \frac{(t-s_B)(2s_B-c_B-t)}{t} + \frac{s_B^2}{2t} + \frac{(s_B-t)^2}{2t} - \left(\frac{(s_B-c_B)^2}{4t} + \frac{(s_B-c_B)^2}{2t} + \frac{(s_B-c_B)^2}{8t}\right) = \frac{-\left(8tc_B-16ts_B-14c_Bs_B+4t^2+7c_B^2+15s_B^2\right)}{8t}$, which is negative.

6. $t > \frac{3s_B-c_B}{2}$, $-c_B\frac{s_B}{t} + \frac{(s_B-c_B)^2}{4t} + \frac{s_B^2}{2t} + \frac{(s_B-c_B)^2}{8t} - \left(\frac{(s_B-c_B)^2}{4t} + \frac{(s_B-c_B)^2}{2t} + \frac{(s_B-c_B)^2}{8t}\right) = -\frac{c_B^2}{2t} < 0.$

d. Finally, ordering d.

1. $t \leq \frac{4}{3}(s_B - c_B)$, $-c_B + 0 + s_B - \frac{t}{2} + 0 - \left(\frac{t}{8} + \frac{3(8s_B - 3t - 8c_B)}{32} + \frac{8s_B - 5t - 8c_B}{32}\right) = -\frac{3}{16}y < 0$

2. $\frac{4}{3}(s_B - c_B) < t < \frac{3}{2}(s_B - c_B)$,
$-c_B + 0 + s_B - \frac{t}{2} + 0 - \left(\frac{(2s_B - 2c_B - t)(t + c_B - s_B)}{t} + \frac{(s_B - c_B)^2}{2t} + \frac{(s_B - c_B - t)^2}{2t}\right) = \frac{(t + c_B - s_B)(c_B - s_B)}{t}$. where $c_B - s_B < 0$ and $t + c_B - s_B > 0$.

3. $\frac{3}{2}(s_B - c_B) < t < c_B$,
$-c_B + 0 + s_B - \frac{t}{2} + 0 - \left(\frac{(s_B - c_B)^2}{4t} + \frac{(s_B - c_B)^2}{2t} + \frac{(s_B - c_B)^2}{8t}\right) = -\frac{8tc_B - 8ts_B - 14c_B s_B + 4t^2 + 7c_B^2 + 7s_B^2}{8t} < 0.$

4. $c_B < t < \frac{4s_B - c_B}{3}$,
$-c_B \frac{3t + c_B}{4t} + \frac{(t - c_B)^2}{8t} + \frac{(3t + c_B)(8s_B - 3t - c_B)}{32t} + \frac{(t - c_B)(8s_B - 5t - 3c_B)}{32t} - \left(\frac{(s_B - c_B)^2}{4t} + \frac{(s_B - c_B)^2}{2t} + \frac{(s_B - c_B)^2}{8t}\right) = -\frac{\left(18tc_B - 16ts_B - 28c_B s_B + 5t^2 + 15c_B^2 + 14s_B^2\right)}{16t} < 0.$

5. $\frac{4s_B - c_B}{3} < t < \frac{3s_B - c_B}{2}$,
$-c_B \frac{s_B}{t} + \frac{(t - s_B)(2s_B - c_B - t)}{t} + \frac{s_B^2}{2t} + \frac{(s_B - t)^2}{2t} - \left(\frac{(s_B - c_B)^2}{4t} + \frac{(s_B - c_B)^2}{2t} + \frac{(s_B - c_B)^2}{8t}\right) = \frac{-\left(8tc_B - 16ts_B - 14c_B s_B + 4t^2 + 7c_B^2 + 15s_B^2\right)}{8t}$, which is negative.

6. $t > \frac{3s_B - c_B}{2}$, $-c_B \frac{s_B}{t} + \frac{(s_B - c_B)^2}{4t} + \frac{s_B^2}{2t} + \frac{(s_B - c_B)^2}{8t} - \left(\frac{(s_B - c_B)^2}{4t} + \frac{(s_B - c_B)^2}{2t} + \frac{(s_B - c_B)^2}{8t}\right) = -\frac{c_B^2}{2t} < 0.$

E Proposition 5

First note that the ordering of t-cutoffs in Lemmas 1 and 2 depends on c_B and s_B:

a. if $c < \frac{2}{5}s_B$, $c_B < \frac{2}{3}(s_B - c_B) < s_B - c_B < \frac{4s_B - c_B}{3} < \frac{3s_B - c_B}{2}$;

b. if $\frac{2}{5}s_B < c_B < \frac{s_B}{2}$, $\frac{2}{3}(s_B - c_B) < c_B < s_B - c_B < \frac{4s_B - c_B}{3} < \frac{3s_B - c_B}{2}$;

c. if $\frac{1}{2}s_B < c_B < s_B$, $\frac{2}{3}(s_B - c_B) < s_B - c_B < c_B < \frac{4s_B - c_B}{3} < \frac{3s_B - c_B}{2}$;

a.

1. If $t \leq c_B$, $\Delta W' = \frac{t}{4} > 0$.

2. If $c_B < t < \frac{2}{3}(s_B - c_B)$, $\Delta W' = \frac{(c_B + t)^2}{16t}$, which is positive.

3. If $\frac{2}{3}(s_B - c_B) < t < s_B - c_B$, $\Delta W' = \frac{(c_B + t)^2}{16t}$.

4. If $s_B - c_B < t < \frac{4s_B - c_B}{3}$, $\Delta W' = \frac{18c_B t - 16s_B t - 24s_B c_B + 12s_B^2 + 13c_B^2 + 5t^2}{16t}$. If $\frac{3-\sqrt{5}}{4}s_B < c_B < \frac{2}{5}s_B$, $\Delta W' > 0$ because the discriminant of the numerator, $s_B^2 - 6s_B c_B + 4c_B^2$, is negative. If $c_B \leq \frac{3-\sqrt{5}}{4}s_B$, $\Delta W' > 0$ if
$$t < \frac{8}{5}s_B - \frac{9}{5}c_B - \frac{2}{5}\sqrt{s_B^2 - 6s_B c_B + 4c_B^2}. \tag{70}$$
$\frac{8}{5}s_B - \frac{9}{5}c_B - \frac{2}{5}\sqrt{s_B^2 - 6s_B c_B + 4c_B^2} > \frac{4s_B - c_B}{3}$ for $c_B \leq \frac{3-\sqrt{5}}{4}s_B$. It follows that $\Delta W' > 0$.

5. If $\frac{4s_B-c_B}{3} \le t \le \frac{3s_B-c_B}{2}$, $\Delta W' = \frac{4c_Bt-8s_Bt-6s_Bc_B+7s_B^2+3c_B^2+2t^2}{4t}$, which is negative iff

$$2s_B - c_B - \frac{\sqrt{2s_B^2 - 4s_Bc_B - 2c_B^2}}{2} < t < 2s_B - c_B + \frac{\sqrt{2s_B^2 - 4s_Bc_B - 2c_B^2}}{2}. \tag{71}$$

$2s_B - c_B - \frac{\sqrt{2s_B^2-4s_Bc_B-2c_B^2}}{2} < \frac{4s_B-c_B}{3}$ iff $c_B < \frac{3\sqrt{2}-1}{17}s_B$, $2s_B - c_B - \frac{\sqrt{2s_B^2-4s_Bc_B-2c_B^2}}{2} < \frac{3s_B-c_B}{2}$ iff $c_B > \frac{s_B}{3}$, and $2s_B - c_B + \frac{\sqrt{2s_B^2-4s_Bc_B-2c_B^2}}{2} > \frac{3s_B-c_B}{2}$. Summing up: $\Delta W' < 0$ if $c_B < \frac{3\sqrt{2}-1}{17}s_B$ or $2s_B - c_B - \frac{\sqrt{2s_B^2-4s_Bc_B-2c_B^2}}{2} < t \le \frac{3s_B-c_B}{2} \cup \frac{3\sqrt{2}-1}{17}s_B < c_B < \frac{s_B}{3}$; $\Delta W' > 0$ if $\frac{4s_B-c_B}{3} < t < 2s_B - c_B - \frac{\sqrt{2s_B^2-4s_Bc_B-2c_B^2}}{2} \cup \frac{3\sqrt{2}-1}{17}s_B < c_B < \frac{s_B}{3}$ or $\frac{s_B}{3} \le c_B \le \frac{2}{5}s_B$.

6. If $t > \frac{3s_B-c_B}{2}$, $\Delta W' = \frac{(s_B+c_B)(3c_B-s_B)}{8t}$, which is negative iff $c_B < \frac{s_B}{3}$ and positive iff $\frac{s_B}{3} < c_B < \frac{2}{5}s_B$.

b.

1. If $t < \frac{2}{3}(s_B - c_B)$, $\Delta W' = \frac{t}{4} > 0$.
2. If $\frac{2}{3}(s_B - c_B) \le t \le c_B$, $\Delta W' = \frac{t}{4} > 0$.
3. If $c_B < t < s_B - c_B$, $\Delta W' = \frac{(t+c_B)^2}{16t} > 0$.
4. If $s_B - c_B < t < \frac{4s_B-c_B}{3}$, $\Delta W' = \frac{18c_Bt-16s_Bt-24s_Bc_B+12s_B^2+13c_B^2+5t^2}{16t} > 0$ because the discriminant of the numerator, $s_B^2 - 6s_Bc_B + 4c_B^2$, see (71), is negative when $\frac{3-\sqrt{5}}{4}s_B < c_B$ which is implied by $\frac{2}{5}s_B < c_B < \frac{1}{2}s_B$.
5. If $\frac{4s_B-c_B}{3} \le t \le \frac{3s_B-c_B}{2}$, $\Delta W' = \frac{4c_Bt-8s_Bt-6s_Bc_B+7s_B^2+3c_B^2+2t^2}{4t} > 0$ because $\frac{2}{5}s_B < c_B < \frac{1}{2}s_B$ implies $c_B \ge \frac{s_B}{3}$.
6. If $t > \frac{3s_B-c_B}{2}$, $\Delta W' = \frac{(s_B+c_B)(3c_B-s_B)}{8t} > 0$ given $\frac{2}{5}s_B < c_B < \frac{1}{2}s_B$.

c.

1. If $t < \frac{2}{3}(s_B - c_B)$, $\Delta W' = \frac{t}{4} > 0$.
2. If $\frac{2}{3}(s_B - c_B) \le t \le s_B - c_B$, $\Delta W' = \frac{t}{4} > 0$.
3. If $s_B - c_B < t \le c_B$, $\Delta W' = \frac{2t^2-4t(s_B-c_B)+\left(3s_B^2-6s_Bc_B+3c_B^2\right)}{4t}$, which is positive since the discriminant of the polynomial is negative.
4. If $c_B < t < \frac{4s_B-c_B}{3}$, $\Delta W' = \frac{12s_B^2-20s_Bc_B+11c_B^2+16c_Bt+5t^2-16ts_B}{16t}$, which is positive since the discriminant of the polynomial is negative.
5. If $\frac{4s_B-c_B}{3} \le t \le \frac{3s_B-c_B}{2}$, $\Delta W' = \frac{4c_Bt-8s_Bt-6s_Bc_B+7s_B^2+3c_B^2+2t^2}{4t} > 0$ because $\frac{1}{2}s_B < c_B < s_B$ implies $c_B \ge \frac{s_B}{3}$.
6. If $t > \frac{3s_B-c_B}{2}$, $\Delta W' = \frac{(s_B+c_B)(3c_B-s_B)}{8t} > 0$ given $\frac{1}{2}s_B < c_B < s_B$.

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