Inflation and Taxes in a New Keynesian Model with Underground Economy

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September 30, 2014

Abstract

This paper studies the underground economy using a two-sector monetary dynamic stochastic general equilibrium model. The informal sector exists as a mean to avoid taxation and labor market regulation. We find that downward nominal wage rigidity in the regular labor market strongly affects the reallocation of hours worked between the two sectors when the economy is hit by exogenous shocks. We also present budget neutral monetary and fiscal policies that aim to reduce the size of the underground economy. Our model firmly advises against the use of inflation to tax the cash-intensive irregular sector. Instead, inflationary policies in which seigniorage revenues are channeled into the decrease of the social security tax rate are able to reduce the size of the underground economy and to yield welfare gains, both in the short and in the long run.

JEL Classification: E26; E63.
Keywords: Underground Economy; DSGE; Money; Downward Nominal Wage Rigidity; Fiscal Reforms.

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1 Introduction

After the second world war the size of the underground economy around the world has increased to currently reach significant levels, both in developing and high-income countries.[1] There are several reasons for governments and policy makers to be concerned about this phaenomenon: (i) underground activities elude taxation and produce negative effects on government revenues; (ii) workers in the informal sector lack any form of social protection; (iii) in presence of a large shadow sector official indicators like those on unemployment, labor force, consumption and income turn out to be unreliable; (iv) resorting to informal output distorts competition among firms.[2]

Despite the consequences of a sizeable underground sector have been largely experienced by many countries, policy makers have not been doing much to address this problem. A first step toward this direction would be to understand clearly the determinants of the shadow economy.

Fortunately, these have been explored by many empirical and theoretical works. For instance, some authors (Allingham and Sandmo, 1972; Andreoni et al., 1998; Blackwell, 2010) point out that the strength of deterrence measures affects tax evasion. Despite the latter and the shadow economy are different concepts, we know that almost all the activities in the informal sector elude taxation. Therefore, one should reckon the underground sector and tax evasion to be affected by common factors - deterrence measures in this case. However, researchers have reached so far little consensus on the negative effect deterrence is able to exert on the shadow economy.[3]

Differently from deterrence, most economists concur that the tax and social security contribution burdens are like to be among the main determinants of the size of the underground economy.[4] Moreover, Feld and Schneider (2010) conclude that tax and social security contribution burdens are far the most influent cause for the size of the German shadow economy.

Another strand of the literature puts the emphasis on the burden of regulation and the quality of institutions as the main determinants of the shadow economy.[5] Entrepreneurs might resort to irregular workers to avoid minimum wage constraints and restrictions on dismissal (Packard et al., 2012). Similarly, onerous bureaucracies and weak legal systems might drive firms toward informal activities.

This paper presents a two-sector dynamic stochastic general equilibrium (DSGE) model that allows us to study how taxation and labor market regulation affect the size of the informal economy. In fact, the model explicitly includes an underground sector that flanks the regular one such that total final output is the sum of legal and informal production. To be precise, we define the underground sector as the one including "*all market-based legal production of goods and services that are deliberately concealed from public authorities*" to avoid the payment of taxes and social

[1]Schneider (2000) estimates the size of the shadow economy for 18 OECD countries using the currency demand approach. He finds a strong increase over the period 1960 to 1998.

[2]See Schneider and Enste (2000); Tanzi (2002).

[3]Feld et al. (2007) conduct a study for Germany finding no consistent effect of deterrence on the size of the shadow economy. Feld and Larsen (2005, 2008); Feld and Larsen find no effects of fines and punishments on the shadow economy using individual survey data for Germany over the period 2004 to 2007. Instead, they find a significative negative impact of the subjectively perceived risk of detection on informal activities for the same years.

[4]See, for instance, Johnson et al. (1998); Giles and Caragata (2001); Tanzi (1999); Schneider and Enste (2000); Dell'Anno (2003).

[5]For instance, Schneider and Enste (2000) discuss this point in a wider analysis of the shadow economy. Using data for 69 countries, Friedman et al. (2000) point out that firms do not go underground to avoid taxes but corruption and regulations. Also, Loayza et al. (2005) conclude that heavier regulation, especially in product and labor markets, lower growth and increase informal activities. Their same conclusions are drawn by Enste (2010), who analyzes data for 25 OECD countries from 1999 to 2005.

security contributions or to escape certain legal market standards and administrative obligations (Schneider, 2012, p. 6). Thus, illegal activities are not part of the analysis.

The structure of our model is similar to Busato and Chiarini (2004), who are among the first to study the underground sector by embedding it in a DSGE model. More recently, Orsi et al. (2013) extended this framework by describing a richer set of tax rates and exogenous shocks, while Busato et al. (2012) investigated the equilibrium implications of fiscal policies with tax evasion. However, none of these works either include money (and monetary policy) or frictions in goods and labor markets in the analysis of the shadow economy.

The main aim of this paper is to analyze the dynamics of the underground economy by developing a monetary New Keynesian model which features monopolistic competition in the goods market and rigidities in the response of prices and wages to exogenous shocks. While most studies try to evaluate only the impact of fiscal reforms on the share of the informal economy, the introduction of money endows the policy maker with an additional instrument of economic policy. Since cash-intensity is a natural feature of transactions that take place in the underground sector, a monetary DSGE model is better suited for the analysis of the shadow economy, especially if one is interested in policies aimed at reducing its size.[6] In fact, the monetary authorities could explore the possibility of accepting a higher inflation rate in order to tax the informal economy.

In addition, we differentiate the formal from the informal sector by imposing that nominal regular wages are downward rigid.[7] This puts a constraint on the ability of firms to cut costs when negative shocks hit the economy and creates incentives to substitute regular workers with informal ones. Thus, downward nominal wage rigidity (DNWR) is an important determinant of the shadow economy, at least in the short run.

With this setup, we first study the dynamic properties of the model by computing impulse response functions for a series of exogenous shocks. Then, we perform deterministic simulations to assess the effectiveness of different budget neutral policies in reducing the size of the shadow economy. To this end, two classes of policies are considered: (1) cold turkey inflationary policies in which seigniorage revenues are used to lower lump-sum taxes; (2) cold turkey inflationary policies in which seigniorage revenues are used to decrease the burden of distortionary taxation.

Our main findings are can be summarized as follows. First, when exogenous shocks hit the economy, DNWR strongly affects the reallocation of resources between the regular and the underground sector. Second, the inflation tax is not a good instrument for reducing the size of the underground economy. While the informal sector barely shrinks as the cold turkey inflationary policy gets stronger, agents in the economy bear welfare losses at any time horizon. Finally, the largest decrease in the share of informal activities occurs when seigniorage revenues from the inflationary policy are channeled into the reduction of the social security tax rate. Moreover, this policy is welfare improving both in the short and in the long run.

The paper is organized as follows. Section 2 presents the model. In Section 3 we describe the calibration procedure and the computation of the welfare measure. Section 4 shows and discusses the dynamic properties of the model. Section 5 presents and evaluates different policies to reduce the size of the underground economy. Section 6 concludes.

[6] Cash-intensity is a well-known feature of the underground sector. It is also the linchpin of the approach, first developed by Gutmann (1977) and Feige (1979) and then statistically refined by Tanzi (1982), of estimating the size of the shadow economy by using data on currency demand.

[7] This assumption is well supported by theoretical works and empirical evidence. See Section 2.4 for further details.

2 The Model

We consider a two-sector economy with five agents. A representative household takes decisions on consumption, investments and labor effort. Final output is manufactured by perfectly competitive producers. A continuum $i \in [0,1]$ of intermediate goods produced by monopolistic competitors are used as inputs in the final good production. The intermediate good producers rent capital and hire labor from households and face quadratic adjustment costs à la Rotemberg when changing their prices. The government levies taxes on households and firms and issues free-risk bonds to finance the final good purchases. Finally, the central bank sets the one period nominal interest rate according to a Taylor-type rule.

As in Orsi et al. (2013) the economy is divided in two sectors: the regular sector, where all transactions are recorded by the fiscal authorities, and the irregular one, which allows to hide profits from taxation (all the transactions that occur in this sector are kept secret). Each intermediate firm produces its output by using both regular and irregular technologies, where the latter is used to increase profits by tax evasion. However, every period the fiscal authorities have a positive probability p of detecting illegal transactions. If detected, firms are surcharged by a factor s over the total amount they evaded. Households may also conceal part of their personal income by spending a fraction of their time working in the irregular sector.

In addition to nominal rigidities due to Rotemberg adjustment costs on price changes, nominal wages in the regular sector are downward rigid. A last source of inefficiency is introduced along with money through cash-in-advance (CIA) constrained households and firms as in Schmitt-Grohé and Uribe (2007).

2.1 Households

A representative infinitely living household maximizes his lifetime utility function which is separable in consumption and labor:

$$U_t^h = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \Gamma_0 \frac{(H_t^m + H_t^u)^{1+\xi}}{1+\xi} - \Gamma_1 \frac{(H_t^u)^{1+\varphi}}{1+\varphi} \right\}, \tag{1}$$

where $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution, $\beta \in (0,1)$ is the discount factor, $\Gamma_0 \geq 0$ is a scale parameter regulating the overall disutility of working, $\Gamma_1 \geq 0$ is a scale parameter indicating the disutility of working in the irregular sector, ξ is the inverse of the Frisch elasticity of aggregate labor supply while φ is that of underground labor supply.[8]

Households can smooth their consumption by means of a complete set of nominal contingent claims. Purchases of consumption goods are subject to a CIA constraint of the following form:

$$\eta^h P_t C_t \leq M_t^h, \tag{2}$$

where M_t^h is the quantity of money held by the representative household at time t and $\eta^h \in [0,1]$ is a parameter indicating the share of cash-constrained expenditures. Households are subject to a period-by-period budget constraint of the form

$$P_t C_t + E_t Q_{t,t+1} X_{t+1} + P_t I_t + M_t^h + P_t T_t^{LS} = X_t + (1-\tau_t^h) W_t^m H_t^m + (1-\tau_t^k) R_t^k K_t + W_t^u H_t^u + D_t + M_{t-1}^h, \tag{3}$$

[8] We assume as in Busato and Chiarini (2004), Busato et al. (2012) and Orsi et al. (2013), that individuals face an idiosyncratic cost of working in the underground sector. This specific cost can be justified by the lack of any social and health insurance, or by job insecurity.

where $Q_{t,t+1}$ is the stochastic discount factor between period t and period $t+1$ such that $E_t Q_{t,t+1} X_{t+1}$ is the value at time t of a random nominal payment in $t+1$. The variable I_t denotes investments, $P_t T_t^{LS}$ is nominal lump-sum taxation, H_t^m and H_t^u are total hours worked in the official and in the underground sector respectively. Nominal wages in the regular sector are denoted by W_t^m and are subject to the income tax τ_t^h, while W_t^u are those earned working irregularly. The variable K_t is the stock of capital rented by households to firms in exchange for the capital interest rate R_t^k. This return is taxed at the rate τ_t^k. Finally, D_t denotes profits received by the representative household for the ownership of intermediate firms: as it will become clear later, this amount is net of corporate and social security taxes.

Capital evolves according to the rule

$$K_{t+1} = (1-\delta) u_t^k K_t + \gamma_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] I_t, \tag{4}$$

where δ is the constant depreciation rate of capital and

$$S\left(\frac{I_t}{I_{t-1}} \right) = \frac{\phi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2$$

are convex investment adjustment costs, the intensity of which is regulated by the parameter $\phi_I > 0$. Capital accumulation in equation (4) can be hit by two different shocks. As in Justiniano et al. (2010), investments are subject to a shock γ_t in the efficiency with which the final good can be transformed into physical capital. This is defined by the process $\gamma_t = e^{z_t^\gamma}$, where z_t^γ follows a stationary AR(1) process with an IID normal error term $z_t^\gamma = \rho_{z^\gamma} z_{t-1}^\gamma + \epsilon_{z^\gamma,t}$. In addition, we introduce a shock on the quality of capital u_t^k as in Furlanetto and Seneca (2013). This shock evolves according to the exogenous process $u_t^k = e^{z_t^{u^k}}$, where $z_t^{u^k} = \rho_{z^{u^k}} z_{t-1}^{u^k} + \epsilon_{z^{u^k},t}$ and $\epsilon_{z^{u^k},t}$ is an IID normal innovation.[9]

The households' problem consists in choosing optimal contingent plans $\{C_t, H_t^u, H_t^m, X_{t+1}, K_{t+1}, I_t, M_t\}$ so as to maximize lifetime utility (1) subject to (2), (3) and (4). We characterize the solution to this problem by the following set of first order conditions:

$$C_t^{-\sigma} = \lambda_t^h \left(1 + \zeta_t^h \eta^h \right) \tag{5}$$

$$H_t^m + H_t^u = \left(\lambda_t^h \right)^{\frac{1}{\xi}} \left[\frac{(1-\tau_t^h) w_t^m}{\Gamma_0} \right]^{\frac{1}{\xi}} \tag{6}$$

$$H_t^u = \begin{cases} (\lambda_t^h)^{\frac{1}{\varphi}} \left(\frac{w_t^u - (1-\tau_t^h) w_t^m}{\Gamma_1} \right)^{\frac{1}{\varphi}}, & \text{if} \quad W_t^u \geq (1-\tau_t^h) W_t^m \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

$$E_t Q_{t,t+1} = \beta E_t \frac{\lambda_{t+1}^h}{\lambda_t^h} \frac{P_t}{P_{t+1}} \tag{8}$$

[9]Some studies have recently placed emphasis on capital quality shocks as an important source of aggregate fluctuations. Notably, Gertler and Karadi (2011) present a quantitative monetary DSGE model with financial intermediaries where negative capital quality shocks are used to simulate the recent Great Recession. Their model generates comovements of hours, output, consumption and investments (i.e. a business cycle) in response to changes in the quality of capital. Furlanetto and Seneca (2013) draw the same conclusion regarding the ability of a capital quality shock to trigger business cycles, provided that the shock is persistent. In particular, they look closer at capital depreciation shocks and compare them to capital quality and investment specific technology shocks.

$$q_t = \beta E_t \frac{\lambda_{t+1}^h}{\lambda_t^h} \left[(1-\tau_{t+1}^k) r_{t+1}^k + q_{t+1} u_{t+1}^k (1-\delta) \right] \tag{9}$$

$$1 = q_t \gamma_t \left[1 - \phi_I \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} - \frac{\phi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] + \beta \mathbb{E}_t q_{t+1} \frac{\lambda_{t+1}^h}{\lambda_t^h} \gamma_{t+1} \phi_I \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \tag{10}$$

$$\beta E_t \frac{\lambda_{t+1}^h}{\lambda_t^h} \frac{P_t}{P_{t+1}} = (1 - \zeta_t^h), \tag{11}$$

where lower case letters denote variables in real terms, λ_t^h is the Lagrange multiplier associated to the flow budget constraint (3) and can be interpreted as the marginal utility of income with cash-constrained expenditures, ζ_t^h is the marginal utility of money and q_t is the real Tobin's q.

Equation (6) represents the total labor supply schedule. Equation (7) describes the households' decision of spending part of their working time in the underground sector. This choice depends on the net-of-taxes wage differential between the underground sector and the market: if positive, individuals will spend part of their time working irregularly according to the value of $1/\varphi$, that is the Frisch elasticity of irregular labor supply.

Equations (5) and (8) can be combined together to obtain the Euler equation

$$\frac{C_t^{-\sigma}}{P_t(1+\zeta_t^h \eta^u)} = \beta E_t \frac{C_{t+1}^{-\sigma}}{P_{t+1}(1+\zeta_{t+1}^h \eta^u)} R_t,$$

where we have used the fact that $E_t Q_{t,t+1} = R_t^{-1}$. [10]

Equation (9) defines the Tobin's q as the marginal value of an additional unit of installed capital, measured in terms of the consumption goods. It is the average discounted value of the sum between the (net-of-taxes) interest rate on capital and the next period marginal value of capital (net of depreciation). Equation (10) represents the marginal value of investments. Finally, equation (11) defines the marginal value of money holdings.

2.2 Final good producer

The final good is a composite good that is produced combining a continuum of intermediate goods via the Dixit-Stiglitz aggregator function

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}. \tag{12}$$

The parameter $\epsilon > 1$ denotes the intratemporal elasticity of substitution across different varieties of intermediate goods. Each period the final good producer solves a profit maximization problem, subject to the aggregator (12), given the market price P_t and the price of each variety $P_{i,t}$. The resulting demand for variety i is then

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t. \tag{13}$$

[10] R_t denotes the risk-free nominal interest rate from period t to $t+1$. By a no-arbitrage condition, R_t must equal the inverse of the price in t of a portfolio that pays one dollar in every state of the world in period $t+1$.

Finally, the zero profit condition for the competitive final producer delivers the price index P_t

$$P_t = \left[\int_0^1 P_{i,t}^{1-\epsilon} di \right]^{\frac{1}{\epsilon}}. \tag{14}$$

2.3 Intermediate Good Firms

Firm i can produce intermediate output in the formal sector by combining capital, $K_{i,t}$, and regular labor, $H_{i,t}^m$, via a constant returns to scale Cobb-Douglas production function

$$Y_{i,t}^m = A_t^m \left(H_{i,t}^m\right)^\alpha K_{i,t}^{1-\alpha}. \tag{15}$$

The variable A_t^m denotes total factor productivity (TFP) in the regular market at time t and it is defined as $A_t^m = A^m e^{z_t^m}$, where A^m is the steady state value of regular technology and z_t^m follows a stationary AR(1) process with an IID normal error term $z_t^m = \rho_{z^m} z_{t-1}^m + \epsilon_{z^m,t}$.

Underground output is produced by hiring irregular workers, $H_{i,t}^u$, and combining them with capital K_t via a constant returns to scale Cobb-Douglas technology

$$Y_{i,t}^u = A_t^u \left(H_{i,t}^u\right)^{\alpha_u} K_{i,t}^{1-\alpha_u}, \tag{16}$$

where A_t^u is the underground sector TFP at time t. This variable evolves according to the expression $A_t^u = A^u e^{z_t^u}$, where A_t^u is the steady state value of irregular technology, $z_t^u = \rho_{z^u} z_{t-1}^u + \epsilon_{z^u,t}$ and $\epsilon_{z^u,t}$ ia an IID normal error. Thus, regular and irregular technologies differ in terms of the output elasticities of capital and labor, α and α_u, and in TFP. In particular, the assumption of sector specific TFP is strongly connected with the source of the labor in equation (16), coherently with the empirical evidence that shows a clear relationship between the workers' level of education and their participation in the irregular labor market.[11]

Following Busato and Chiarini (2004) and, more recently, Orsi et al. (2013), we assume that the goods produced in the two sectors are homogeneous and that firm i intermediate output at time t is given by the sum of its official and unofficial production

$$Y_{i,t} = Y_{i,t}^m + Y_{i,t}^u. \tag{17}$$

Using equation (17), each period we can compute firm i share of underground to total output as

$$S_{i,t}^u = \frac{Y_{i,t}^u}{Y_{i,t}}.$$

Since goods $Y_{i,t}^m$ and $Y_{i,t}^u$ in equation (17) are indistinguishable, the following identity holds at all t:

$$P_{i,t}^m = P_{i,t}^u = P_{i,t}.$$

[11] For instance, Gallaway and Bernasek (2002) and Marcelli et al. (1999) show that, at least in urban settings, low skilled workers are more likely to work in the irregular sector compared to those workers who attained a higher level of education. Amaral and Quintin (2006) link the presence of low-skill workers in the underground sector to the fact that informal managers have access to less outside finance than their formal counterpart, and they substitute part of physical capital with low-skill workers. Also, Reyneri (2003) points out that a large share of workers employed in the irregular sector are migrants. This happens because in many advanced countries, although in presence of high unemployment rates, there is still unfilled demand for low-skilled, low-paid and less productive workers. Migrants are then attracted by these opportunities, but many of them fail to obtain a residence visa and are forced to work irregularly.

Also, equation (17) states that the decision on the share (if any) of irregular production on total intermediate output is up to firm i.

The fiscal authorities fight tax evasion by inspecting firms. When irregularities are detected, they apply a surcharge s over the total amount that has been evaded. Defining $p \in (0,1)$ as the non-zero probability of being inspected, a necessary condition for the existence of unofficial production is that, at all t, $1 - ps\tau_t^c \geq 0$, where $\tau_t^c \leq 1$ is the corporate tax rate.[12]

Money is demanded by firms because intermediate producers are cash-constrained with respect to wage payments as in Schmitt-Grohé and Uribe (2007):

$$M_{i,t}^f \geq \nu^m (1 + \tau_t^s) H_{i,t}^m W_t^m + \nu^u H_{i,t}^u W_t^u, \tag{18}$$

where $\tau_t^s \leq 1$ is the deterministic social security tax rate at time t, while ν^m and ν^u are parameters indicating the share of nominal wages that firms must pay cash in the regular and in the underground sector respectively. Because informal workers do not sign a regular contract, we assume that the share of wages to be paid cash is larger in the underground sector than in the official one, that is $\nu^u > \nu^m$.[13]

In addition to this inefficiency, monopolistic producers face quadratic adjustment costs à la Rotemberg (1982) when changing prices

$$\text{Price adjustment cost} = \frac{\phi_P}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2 Y_t,$$

where $\phi_P > 0$ is a parameter measuring the degree of price stickiness. These costs are proportional to the magnitude of the adjustment and to the level of economic activity.

Each period firm i faces the following budget constraint:

$$\begin{aligned} M_{i,t}^f + B_{i,t}^f = {} & M_{i,t-1}^f + R_{t-1} B_{i,t-1}^f + P_{i,t} \left[(1 - \tau_t^c) Y_{i,t}^m + (1 - ps\tau_t^c) Y_{i,t}^u \right] \\ & - (1 + \tau_t^s) W_t^m H_{i,t}^m - W_t^u H_{i,t}^u - R_t^k K_{i,t} - \frac{\phi_P}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2 P_t Y_t - D_{i,t}, \end{aligned} \tag{19}$$

where $B_{i,t}^f$ denotes bond holdings of firm i in period t. Following again Schmitt-Grohé and Uribe (2007), we assume that the firm's initial wealth is nil. In our notation it means that $M_{i,-1}^f + R_{-1} B_{i,-1}^f = 0$. Moreover, firms conduct a profit distribution policy such that their financial wealth at the beginning of each period is zero, that is $M_{i,t-1}^f + R_{t-1} B_{i,t-1}^f = 0$ for all t. Thus, we derive firm i nominal profits from equation (19):

$$\begin{aligned} D_{i,t} = {} & P_{i,t} \left[(1 - \tau_t^c) Y_{i,t}^m + (1 - ps\tau_t^c) Y_{i,t}^u \right] - (1 + \tau_t^s) W_t^m H_{i,t}^m - W_t^u H_{i,t}^u - R_t^k K_{i,t} \\ & - \frac{\phi_P}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2 P_t Y_t - \left(1 - R_t^{-1} \right) M_{i,t}^f. \end{aligned} \tag{20}$$

The objective of the firm is to choose optimal contingent plans $\{H_{i,t}^m,\ H_{i,t}^u,\ K_{i,t},\ M_{i,t},\ P_{i,t}\}$ in order to maximize the present discounted value of the future stream of profits given by

$$E_0 \sum_{t=0}^{\infty} Q_{0,t} D_{i,t},$$

[12] Each period, firm i total after-tax revenues are given by $(1 - \tau_t^c) Y_{i,t}^m + (1 - ps\tau_t^c) Y_{i,t}^u$. Thus, the above condition ensures that revenues from illegal production are positive.

[13] We believe this assumption to be plausible especially if the objective of the firm is to conceal this illegal practice from fiscal authorities.

subject to (13), (15), (16) and (17), given the vector of prices $\{P_t, W_t^u, W_t^m, R_t\}$.[14]

Since the Rotemberg's model of sluggish price adjustment allow firms to change their price in each period and, in doing so, they all face an identical problem, we can anticipate the result that firms are symmetric, meaning that $P_{i,t} = P_t$. Then, letting nominal marginal costs MC_t be the Lagrange multiplier associated with the constraint (17), and ζ_t^f that associated with the CIA constraint, profit maximizing intermediate producers behave according to the following equations:

$$\frac{\alpha Y_t^m}{H_t^m} = \frac{(1+\tau_t^s)(1+\zeta_t^f \nu^m) w_t^m}{mc_t} \tag{21}$$

$$\begin{cases} \frac{\alpha_u Y_t^u}{H_t^u} = \frac{(1+\zeta_t^f \nu^u) w_t^u}{mc_t} & \text{if } 1 - ps\tau^c \geq 0 \\ H_t^u = 0 & \text{otherwise} \end{cases} \tag{22}$$

$$\frac{(1-\alpha)Y_t^m + (1-\alpha_u)Y_t^u}{K_t} = \frac{r_t^k}{mc_t} \tag{23}$$

$$R_t = \frac{1}{1-\zeta_t^f} \tag{24}$$

$$(\epsilon-1)(1-\tau_t^c) - \tau_t^c(1-ps)S_t^u + \phi_p(\Pi_t - 1)\Pi_t - \beta E_t \frac{\lambda_{t+1}^h}{\lambda_t^h} \phi_p(\Pi_{t+1}-1)\Pi_{t+1}\frac{Y_{t+1}}{Y_t} - \epsilon mc_t = 0, \tag{25}$$

where lower case letters denote variables expressed in real terms and $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate.

Equations (21) and (22) describe the optimal demand for labor in the regular and in the underground sector respectively. They state that firms hire workers until the marginal productivity of labor equals its marginal cost. Equation (23) regulates the demand for capital, which is rented from households up to the equality of its marginal productivity with its marginal cost. Equation (24) states that the opportunity cost of holding money equals the gross nominal interest rate. Finally, equation (25) is the New Keynesian Phillips curve (NKPC) in presence of the underground sector. Note that setting S_t^u and τ_t^c to zero, this relation simply becomes the standard NKPC.[15] Besides, equation (25) can be manipulated to obtain an expression for the markup that monopolistic producers charge over their marginal costs when setting the price of intermediate goods:

$$markup_t = \frac{\epsilon}{(\epsilon-1)(1-\tau_t^c) - \tau_t^c(1-ps)S_t^u + \phi_p(\Pi_t-1)\Pi_t - \beta E_t \frac{\lambda_{t+1}^h}{\lambda_t^h}\phi_p(\Pi_{t+1}-1)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}}. \tag{26}$$

Interestingly, expression (26) depends on the the share of the underground sector on total output. In fact, firms are able to reduce marginal costs by keeping a share of their transactions unrecorded. This reduction is measured in terms of unities of output subtracted to corporate taxation, discounted by the probability of detection times the surcharge they are forced to pay if caught. When $\phi_P = 0$ (prices are fully flexible) firms face lower costs and their ideal markup is then found to be

$$markup_t = \frac{\epsilon}{(\epsilon-1)(1-\tau_t^c) - \tau_t^c(1-ps)\frac{Y_t^u}{Y_t}}.$$

[14]Note that, given the previous profits policy rule, firms do not decide on bond purchases because they implicitly do so when choosing money holdings.

[15]Because of firms symmetry, S_t^u represents the size of the underground sector in the whole economy.

2.4 Downward Nominal Wage Rigidity

We introduce DNWR in the regular sector by imposing that

$$W_t^m = max(W_{t-1}^m, P_t mrs_t), \tag{27}$$

where mrs_t is the marginal rate of substitution between consumption and labor at time t.

Equation (27) states that time t nominal wages cannot be lower than their previous level in $t-1$, thus reflecting the inability or unwillingness of employers to reduce workers' nominal salary.[16] There are reasons to add this equation to the model as a large number of empirical works confirm the presence of substantial DNWR in many developed countries.[17]

Another argument for having equation (27) is that DNWR seems to play an important role in explaining the high level of unemployment in the Euro area during the recent crises, as argued by Schmitt-Grohé and Uribe (2013). However, differently from their approach, DNWR is not a source of involuntary unemployment in our model because we do not assume, as they do, that individuals always supply a fixed amount of labor $\bar{H}$.

Instead, we introduce DNWR in our economy to unveil the role they play when firms can explicitly resort to irregular workforce. In a seminal paper, Friedman et al. (2000) find that the size of the underground sector is mainly affected by certain features of the business and legal environment in which firms operate rather than by the burden of taxes. Still in this strand, Anderberg (2003) investigates whether labor market institutions play a role in determining the size of the underground sector.

We argue that DNWR represent a strong friction in the regular labor market, especially during economic downturns. In fact firms will move production away from the regular sector whenever they are unable to set wages at the efficient level. Therefore the existence of a shadow sector provides firms with more flexibility and helps them to stabilize production when negative shocks hit the economy. We are going to discuss these dynamics further in Section 4.

2.5 The Government

Each period the consolidated government faces an exogenous stream of expenditures defined as $G_t = Ge^{z_t^G}$, where $z_t^G = \rho_z^G z_{t-1}^G + \epsilon_{z^G,t}$ is an AR(1) process with an IID normal innovation $\epsilon_{z^G,t}$. In order to finance these expenditures the government prints money, M_t, issues one period risk-free bonds with nominal value B_t, and collects taxes in the amount $P_t T_t$. Thus, the government budget constraint in period t can be written as

$$P_t T_t + B_t + M_t - M_{t-1} = R_{t-1} B_{t-1} + P_t G_t. \tag{28}$$

Equation (28) can be easily rearranged to obtain an expression which defines real public debt at time t:

$$b_t = \frac{R_{t-1} b_{t-1}}{\Pi_t} + \frac{m_{t-1}}{\Pi_t} - m_t + G_t - T_t. \tag{29}$$

[16]The inability of reducing nominal wages can be easily justified by the existence of regular job contracts that arise from negotiation between unions and firms. Employers can also take the decision of leaving wages unchanged during an economic downturn to avoid further morale and productivity reduction in their workforce, as discussed by Bewley (1999) and Kahneman et al. (1986).

[17]Among them, Du Caju et al. (2013) examine the reasons for why firms avoid cutting wages; Behr and Pötter (2010) assess the existence of DNWR using a flexible parametric approach; Babecký et al. (2010) investigate downward nominal and real wage rigidity in Europe; Holden and Wulfsberg (2008) reject the hypothesis of no DNWR using data for hourly nominal wages at industry level in 19 OECD countries.

Equation (29) states that real debt increases with interest payments and primary deficit, but can be reduced through monetization.

Because of the rich structure of tax rates, total tax revenues are given by

$$T_t = \tau_t^h w_t^m H_t^m + \tau_t^k r_t^k K_t + \tau_t^c \left(Y_t^m + psY_t^u\right) + \tau_t^s w_t^m H_t^m + T_t^{LS}, \tag{30}$$

where T_t^{LS} denotes lump-sum taxation. Each period the latter is set according to the following rule:

$$T_t^{LS} = T_{t-1}^{LS} + \phi_{TT}\left(T^{LS} - T_{t-1}^{LS}\right) + \phi_{Tb}\frac{(b_{t-1} - b)}{Y^m}, \tag{31}$$

where ϕ_{TT} and ϕ_{Tb} are parameters, while T^{LS} and b/Y^m are the steady state value of lump-sum taxes and the target level of public debt to GDP ratio, respectively. Fiscal rules of the form of (31) can be found in Berg et al. (2012): in our case, its use is supported by the purpose of limiting government revenues volatility when monetary and fiscal policies are implemented, as well as by that of debt sustainability.[18]

Finally, a monetary authority sets the one-period nominal interest rate according to a feedback rule of the form

$$ln\left(\frac{R_t}{R}\right) = \alpha_R ln\left(\frac{R_{t-1}}{R}\right) + (1-\alpha_R)\left[\alpha_\pi ln\left(\frac{\Pi_t}{\Pi^\star}\right) + \alpha_y ln\left(\frac{Y_t^m}{Y_{t-1}^m}\right)\right] + \epsilon_{R,t}, \tag{32}$$

where $\Pi^\star$ is the target level of inflation, R is the steady state value of gross nominal interest rate which directly depends on the monetary authority's target of inflation, α_R is a parameter and $\epsilon_{R,t}$ is an IID normal shock to monetary policy.

3 Welfare Measure and Calibration

The purpose of this work is twofold: first, we want to show the dynamic properties (i.e., the response to exogenous shocks) of a DSGE model which explicitly includes an informal sector used by agents to avoid taxation; secondly, we study and rank combinations of monetary and fiscal policies aimed at reducing the share of irregular production. For the first goal to be reached, a sensible calibration of the model is required. For the second one, we need to specify a utility-based measure which has the ability to unambiguously evaluate the outcomes of the different policies.

3.1 The Welfare Measure

Policy makers are often interested in the welfare costs or gains of implementing a specific policy. Welfare is commonly defined as the present discounted value of consumers' lifetime utility. Depending on the setting of the policy experiment (i.e., stochastic vs deterministic), this definition entails different computational procedures one should be aware of in order to obtain non-spurious welfare rankings.[19] As Section 5 explains, our policy experiments are completely deterministic: no infor-

[18]See Section 5.

[19]Kim and Kim (2003) clarify this question using a two country business cycle model. They show how a linear approximation of the first order conditions around a deterministic steady state (i.e., standard log-linearization) underestimates welfare gains from risk-sharing. They even show that, under certain parameter values, incomplete markets deliver higher welfare compared to complete risk-sharing. This happens because linear approximation ignores second (and higher) order terms, that is uncertainty.

mative or imperfect credibility issues are considered.[20] Therefore, we can rely on a simple measure of welfare. Since Lucas (1987), it is usual to rank alternative policies using consumption-equivalent metric. The latter is defined as the constant fraction of consumption that the households have to give up in each period in order to be indifferent between two policies. Consider, for instance, two alternative policies that could be implemented by the government, a and b. Then, the present discounted value of the utility flow under policy a is:

$$V^a = \sum_{t=1}^{\infty} \beta^t U(C_t^a, H_t^a) \ ,$$

where C_t^a and H_t^a denote consumption and total hours under policy a. Similarly,

$$V^b = \sum_{t=1}^{\infty} \beta^t U(C_t^b, H_t^b)$$

is the present discounted value of the utility flow under policy b.

We define ω as the welfare cost, in consumption equivalents units, of adopting a instead of b. Formally, ω solves the following equation:

$$V^a = \sum_{t=1}^{\infty} \beta^t U((1-\omega)C_t^b, H_t^b). \tag{33}$$

When $\omega > 0$, the welfare level achieved under policy b exceeds that one under policy a. Therefore, the ω's which result from comparing different couples of policies can be easily used to assess the impact of certain measures on the agents in the economy and, finally, to rank them.

3.2 Calibration

The model is calibrated at quarterly frequency. We set the value of the subjective discount factor β to 0.99. The capital depreciation rate δ is 0.025 and the parameter regulating the bulk of investment adjustment costs, ϕ_I, is set to 4. The intertemporal elasticity of substitution σ is set to one, which means that the utility function is logarithmic in consumption. We give a value of one to the inverse of the Frisch elasticity ξ of labor supply and a value of 0.06 to that of the underground labor supply, φ, as in Orsi et al. (2013). The structural parameters indicating the disutility of labor, Γ_0, and the additional disutility stemming from being employed irregularly, Γ_1, have values 6.8719 and 0.0564 respectively. These values have been endogenously derived to deliver a steady state value of the share of the underground economy S^u equal to 0.25 and a steady state value of the share of irregular hours (H^u/H) equal to 1/3.

In the production functions, the labor share in the regular sector α is set to 0.65, while the share in the underground sector α_u is set to 0.75, meaning that the underground sector is labor intensive compared to the regular one. The intratemporal elasticity of substitution among varieties, ϵ, is equal to 6 and implies a steady state markup of about 33%, in line with the assumptions in the literature.[21] Intermediate firms face price adjustment costs, with a degree of price stickiness measured by the parameter ϕ_P, which is set to 60.

[20] In these features, our setting follows that one of Forni et al. (2010), who evaluate the welfare effect of fiscal consolidation under different fiscal policies.

[21] The assumptions on the steady state mark-up level range from 10% to 40%. See, for instance, Keen and Wang (2007).

Parameters in the cash-in-advance constraints are set as in Schmitt-Grohé and Uribe (2007). Precisely, the fraction of consumption held in money, η^h, is 0.35, while the fraction of wage payments that firms must hold in money, ν^m, is 0.63. In addition, since our model includes irregular labor, we assume that wages in the underground sector must be completely paid cash, that is ν^u equals one.

The government decides the target level of some key macroeconomic variable. It sets the target annual level of debt-to-output ratio, (b/Y^m), to 0.6, which implies a steady state level of debt, b, equal to 1.8.[22] Fiscal reaction parameters in the closure rule (31) are set according to Berg et al. (2012). Precisely, ϕ_{TT} is equal to 0.25 and ϕ_{Tb} to 0.02. The monetary authority determines the target level of gross inflation, $\Pi^\star$: its value is set to 1.005, which implies an annual value close to 2%. Still looking at the Taylor rule, the interest rate smoothing parameter, α_R, has value 0.8 as in Christiano et al. (2009). The remaining parameters, α_π and α_y, are set to 2.04 and 0.08 respectively as in Smets and Wouters (2007).

We now turn to exogenous processes. As described in the previous Section, these all follow AR(1) processes the persistence of which is regulated by specific parameters (i.e., parameters $\rho_{z^\gamma}, \rho_{z_{u^k}}, \rho_{z^m}, \rho_{z^u}, \rho_{z^G}$). These are all set according to Smets and Wouters (2007) estimates except for the persistence in the capital quality process, $\rho_{z_{u^k}}$, which is set as in Lambertini and Uysal (2013). The same sources are used to calibrate the standard deviations of the shocks to these AR(1) processes. We report these values in Table 1.

Finally, the tax system is described by four average tax rates levied on households (τ^h), firms (τ^c and τ^s) and capital owners (τ^k). The tax rate oh households income is set to 0.15; τ^c to 0.1; τ^s and τ^k to 0.05. Firms evading taxes face a probability p of being caught by fiscal authorities. If this happens, they are forced to pay a surcharge s over the total amount due. This probability and the subsequent surcharge are calibrated as in Orsi et al. (2013) to 0.03 and 1.3 respectively.

Table 1: Exogenous processes

Parameter	Interpretation	Value
ρ_{z^γ}	Persistence of investment specific technology shock	0.71
$\rho_{z_{u^k}}$	Persistence of capital quality shock	0.88
ρ_{z^m}	Persistence of regular sector technology shock	0.95
ρ_{z^u}	Persistence of underground sector technology shock	0.95
ρ_{z^G}	Persistence of government spending shock	0.97
$\sigma_{\epsilon_{z^\gamma}}$	Standard deviation of investment specific technology shock	0.0045
$\sigma_{\epsilon_{z_{u^k}}}$	Standard deviation of capital quality technology shock	0.002
$\sigma_{\epsilon_{z^m}}$	Standard deviation of regular sector technology shock	0.0045
$\sigma_{\epsilon_{z^u}}$	Standard deviation of underground sector technology shock	0.0045
$\sigma_{\epsilon_{z^G}}$	Standard deviation of government spending shock	0.0045
σ_{ϵ_R}	Standard deviation of monetary policy shock	0.0024

[22]This number follows from the normalization of total output to one and from the fact that we are dividing a stock by a flow.

4 Impulse Response Analysis

Monetary Policy shock

Figure 1 plots the response of several variables to both a positive (solid line) and a negative (dotted line) 0.5% shock to the nominal interest rate. For the sake of clarity, we refer to a positive (negative) monetary policy shock as an increase (decrease) of the nominal interest rate.

First of all, some variables show clear asymmetric responses to positive and negative shocks. We argue this is mainly due to the presence of DNWR in the regular sector. The mechanism is the following:[23] a fall in the nominal interest rate fosters aggregate demand as revealed by the increase in output, consumption, hours and, to a lesser extent, investments. Inflation goes up as rising demand puts pressure on the price of goods. However, the positive response of output and hours after the first period is driven by an expansion of the informal sector at the expense of regular sector activities. This is due to downward rigidities in the regular job market. In fact, the effect of a negative shock to the nominal interest rate is to boost hours and output in both sectors on impact. The sharp increase in the marginal rate of substitution between consumption and leisure drives wages in the regular sector upward. Because the shock is temporary, the nominal interest rate returns gradually to its steady state slowing down the initial economic boom. Firms would like to cut real wages, but those producing regularly are constrained by DNWR which forces them to pay salaries above their efficiency level. For this reason, firms in the official sector decide to substitute part of regular workforce with informal workers.[24] This substitution effect is also buoyed by the reduction in the opportunity cost of holding cash-paid wages. Thus, output and hours in the regular sector fall below their steady state level while those in the informal sector increase. As a result, the share of underground production on total output rises.[25]

In the light of this reasoning, it is fair to expect that the combination of DWNR and the existence of a modeled underground sector will affect further the response of the economy to a reduction in the nominal interest rate the more expansionary the policy is. Figure 2, which plots the Impulse Response Functions (IRFs) of six variables for different magnitudes of the negative monetary shock - precisely 0.25%, 0.5% and 1% - explores this possibility. When monetary authorities try to stimulate the economy by cutting the quarterly policy rate by 1%, the model predicts a negative effect on consumption, output, hours and investments in the medium run. Monetary policy, which was intended to be expansionary, produces instead a negative business cycle. This apparently counterintuitive result arises because the initial surge in labor costs is relevant and the DNWR constraint is binding for a considerable period of time.[26] As previously discussed, low-cost money and excessively high wages in the official sector push firms to reallocate a large share of regular production toward the shadow economy. However, differently from milder monetary policies

[23]We consider the case of a negative shock to the nominal interest rate to show how the effects of expansionary monetary policy can be distorted by DNWR and irregular production.

[24]We assume that workers can be fired, or that their working hours can be reduced, without costs. In this sense, the only protection regular workers receive is about their wage level.

[25]The response of the economy to a positive monetary shock is quite similar to that one triggered by a negative shock, except for the lack of the initial increase in real regular wages. A rise in the nominal interest rate depresses demand. Firms operating in the regular sector cannot cut down costs sharply because wages are downward rigid. Again, there is a shift from regular to informal production.

[26]When the policy rate is reduced by 1% the constraint is binding from period 2 to 9. This means two years in which regular wages are away from their efficiency level. This time lapse decreases with the magnitude of the shock: the constraint is binding for 4 (1) periods after a negative 0.5% (0.25%) shock.

(i.e., 0.25% and 0.5% interest rate cuts), when the nominal interest rate is abruptly cut down by 1%, this shift does not compensate the reduction in regular production and the overall economic activity stumbles. Thus, DNWR is an important source of frictions in the transmission of monetary policy. Expansionary policies might have negative implications for the business cycle and might help informal activities to flourish.

Capital Quality shock

Figure 3 displays the effects of both positive and negative 1% capital quality shock. Coherently with the work of Gertler and Karadi (2011) and Furlanetto and Seneca (2013), shocks to capital quality are able to generate co-movement in consumption, output and investments, providing a high degree of persistency. We focus again on negative shocks which are more interesting as the DNWR constraint binds. Following the shock, output and consumption start to decline reaching a trough after some quarters. The high degree of persistence implies a very slow recovery: both of the variables remain well below their steady state level after 30 quarters. Because the shock destroys the existent stock of capital, investments peak up after a large initial drop as K_t is below its steady state level. Inflation falls as aggregate demand weakens.

As the shock directly hits capital producers, the marginal rate of substitution between consumption and labor, and then the efficient wage level, go down. Because of pre-existing agreements, regular wages remain above their efficiency level for some quarters. Such rigidity in the regular labor market implies a larger fall in official output and a substitution of the latter with that one produced irregularly.[27] This effect reflects in the response of hours: those spent working in the underground sector move in the opposite direction with respect to the sign of the shock.

Investment-specific Technology shock

Figure 4 shows the IRFs generated by a shock to investment-specific technology. The relevance of this shock as an important source of fluctuations in the main economic aggregates has been recently debated in the literature.[28] We focus on a positive 1% shock to investment-specific technology. The increase in the efficiency with which investments are transformed into physical capital induces households to postpone consumption and exploit the investment opportunity. Lower consumption levels increase the marginal utility of working and enhance agents' willingness to work. However, this crowding-out effect on consumption is relatively small and the effect on regular wages, even if positive, is negligible. Inflation and the real interest interest slightly move up while the share of underground production barely shrinks. The IRFs generated from this model are very similar to those in Justiniano et al. (2010). Instead, they differ from those shown by Furlanetto and Seneca (2013) especially in the response of output and hours when the shock is highly persistent.

[27] In addition to DNWR, another reason for moving production away from the regular sector is that it is capital intensive if compared with the informal production. Thus, a negative shock which destroys the existent capital stock gives incentives to move production where the capital itself has minor impact.

[28] Among the most prominent examples, Justiniano et al. (2010) find that shocks to the marginal efficiency of investments are the main source of aggregate fluctuations in hours, output and investments over the cycle. They obtain this result by breaking the neoclassical equivalence between the marginal rate of substitution of leisure for consumption and the marginal productivity of labor through monopolistic competition in goods and labor markets.

Regular Sector Technology shock

Figure 5 plots the response of our economy following a 3% TFP shock in the regular sector. Contrary to what predicted by the standard real business cycle model without nominal rigidities, total hours worked move countercyclically. This result is in line with the empirical results in the literature.[29] However, the existence of irregular production gives reasons to further explore the mechanism underlying the response of hours. To this end, we consider a positive technology shock. Output, consumption and investments rise. Because of the lower marginal utility of income and consumption, individuals who are employed in the regular sector decide to work less. In the meantime, a large share of firms move production away from the underground sector to exploit the productivity shock in the regular one. Hours worked in the informal economy hugely drop, while those in the official sector rise. The shadow sector shrinks, suggesting that irregular activities are countercyclical.[30] Overall, hours decline as the income effect dominates. Because monetary policy does not react strong enough to lower inflation, the real interest rate increases on impact before it falls below its steady state.

Impulse responses which follow a technology shock can also be used as a tool to test whether the shadow sector plays the role of device for mitigating recessions and smoothing consumption. To have this property, irregular production has to be counter-cyclical. Literature on the underground economy has not yet reached a consensus over the cyclical properties of informal activities. [31] We contribute to this debate by showing the simulated series from our model for regular (solid line) and irregular (dotted line) output variations in Figure 6. The two series clearly display a strong negative correlation, thus giving evidence of two distinct business cycles. Also, we compare the responses of output, consumption, investments, real interest rate and inflation to a 3% negative TFP shock that hits our economy (solid line) with those produced by a model which differs from the previous one only by the absence of underground production (dotted line). Results are displayed in Figure 7: when both the legitimate and the underground sector are modeled, the negative TFP shock is partially absorbed by a reallocation of resources toward informal activities. Because the effect of a negative productivity shock is worsened by the existence of DNWR in the regular sector, firms would be forced to reduce production massively if the underground alternative is not available. Thus, the shadow sector is a sort of buffer for the economy when negative shocks hit.

Government Spending shock

The last shock affects government spending (Figure 8). Being the response to positive and negative shocks symmetric, we focus on the first one since the reasoning can be easily reversed for the negative case. An unanticipated increase in government spending produce a crowding-out effect on consumption and investments. Because of rationality and market completeness, forward looking agents expect higher future taxes. Therefore, they reduce consumption and increase their willingness to work. Real regular wages increase on impact driven by a strong labor demand, but the effect is suddenly reversed as labor supplied by rational agents peaks. The shock has a minor, but still positive, effect on inflation as the increase in the demand of public goods is counterbalanced

[29] Notably, Galí (1999) and Galí and Rabanal (2005) estimate the IRFs of identified productivity shocks in the US economy. He finds the fall in employment to be consistent with data.

[30] We are going to tackle this issue further below.

[31] To have an idea of such conundrum, Bajada (2003) and Giles (1997) give evidence of procyclical relations between the sectors, while Busato and Chiarini (2004), Russo (2008) and, recently, Orsi et al. (2013) support the *double business cycle* hypothesis.

by a decrease in private demand. The share of irregular to total output slightly increases, mainly because agents expect higher future taxes to service the increase in real debt.

Overall, the economy is only marginally affected by the shock because the nominal interest rate is governed by the Taylor rule. In fact, the latter prescribes to increase the nominal interest rate as the expansionary fiscal policy shock puts upward pressure on output and inflation in equation (32). As a result, the rule stabilizes the real interest rate and attenuates the effects of government spending shocks.

5 Inflation, Taxes and the Size of the Underground Sector

In this section we discuss how monetary and fiscal authorities could implement different policies aimed at reducing the size of the underground sector. To be precise, since the agents resort to irregular production to avoid tax payments, we study budget neutral inflationary policies in which seigniorage revenues can be used to lighten the burden of taxation.

The economic literature on tax evasion generally recognizes two motives for studying inflation. The first one is the well known *public finance motive* for inflation put forward by Phelps (1973). In a nutshell, the idea is that, as long as tax evasion constrains the governments' ability of collecting funds, seigniorage might be an important source of revenues in those economies where the share of underground activities is large.[32]

The second rationale, much less spread in the literature but strongly related to the previous one, is connected with cash-intensiveness of transactions in the underground sector. By creating an asymmetry on the effects of the inflation tax between the two sectors - firms must pay cash the entire amount of irregular workers' salary, while only a fraction of regular wages is regulated by cash - we try to understand whether increases in the inflation rate can be used to reduce the share of informal activities.[33]

The following analysis does not tackle the optimality of monetary and fiscal policies. Rather, we first show the macroeconomic effects of cold-turkey policies that permanently raise the target of inflation. Then, we assess how the increase in seigniorage revenues could be combined with a reduction of distortionary taxation in order to decrease the size of the informal sector while leaving medium run budget revenues unchanged.

5.1 The effects of cold-turkey inflation

We now show our model results when the monetary authorities implement a sharp, permanent, increase in the target of inflation. Figure 9 displays the path of the selected variables toward the new steady state of the economy. The transition is measured as percentage deviations from the initial steady state, except for inflation which is expressed as annual rate. The annual inflation target is

[32] Cukierman et al. (1992) provide both theoretical and empirical support to this thesis. Their model implies that unstable and polarized political systems generate inefficiencies in the equilibrium tax structure and more reliance on seigniorage revenues. In the same year Cukierman et al. (1992) demonstrates that the optimal inflation tax depends positively on tax collection costs. More recently, Koreshkova (2006) describes and quantifies the public-finance motive for inflation when tax evasion is integrated in a general equilibrium framework.

[33] The idea that individuals shift their decisions away from cash-intensive activities as inflation rises can be found, for instance, in Nicolini (1998) and Ennis (2009). Also, Yesin (2004) studies the optimal interest rate when there are tax evasion and tax collection costs. Despite he does not explicitly refer to cash-intensity, he shows how optimal inflation depends on the elasticity of substitution between regular and irregular production.

shifted from $\Pi^{\star}_{old}$ to $\Pi^{\star}_{new}$, where $\Pi^{\star}_{old} = 2\%$, that is the steady state value we choose to calibrate the model. We consider three possible values for the new target level of inflation: $\Pi^{\star}_{new} = \{4\%, 6\%, 8\%\}$. These policies are budget neutral in the medium run as they leave government total revenues unchanged. This is possible because the government does not intervene on the target level of the debt to GDP ratio, and seigniorage revenues generated by the inflationary policy can be used to reduce the fixed amount of lump-sum taxes in equation (31).

Irrespectively from the magnitude of the intervention, the inflation rate always overshoots its new target level, especially in the first year after the implementation. This effect is triggered by a sharp increase in investments (not reported in the figure) which boosts aggregate demand: assuming that the policy is fully credible, agents perfectly anticipates the increase in inflation and their expectations drive down the value of the ex-ante real interest rate. However, the increase in output in the first periods is mainly prompted by the increase in irregular production, as the path of S^u clearly shows. In this sense, the effect of cold turkey inflation is similar to an expansionary monetary policy shock: the real rate of interest largely decreases on impact (it even becomes negative when $\Pi^{\star}_{new} = \{8\%\}$), stimulating economic activity. Wages go up in both sectors. When the real interest rate starts reverting to its initial level, firms in the formal sector cannot bring salaries down to the desired level because of the presence of DNWR. This friction in the labor market raises the convenience of resorting to irregular work. As the economy stabilizes, the described effect vanishes and the medium run incidence of the inflation tax is such that the new steady state value of S^u is slightly lower. The increases in inflation and in the nominal interest rate have negative effects on the demand of consumption goods from the cash-constrained households. The new steady state level of consumption is lower than the initial one for all the values of $\Pi^{\star}_{new}$. The long run effect on total output is small but positive because of the increase in investments and capital accumulation in the first periods when the ex-ante real interest rate is low.

Overall, these policies do not seem to be suited to reduce the size of the underground sector. The tiny contraction in the new steady state level of S^u comes at the expense of a sensible fall in consumption, while the effects of DNWR overcome those of the inflation tax in the short run causing S^u to rise.[34] Also, output jumps up in the first year after the policies are implemented, but the new steady state level is only marginally affected by the permanent increase in inflation. The next Table reports steady state comparisons of the three cold-turkey inflationary policies against the benchmark steady state of the model calibrated at $\Pi^{\star}_{old} = 2\%$:

Table 2: Steady State comparisons (% changes)

	From 2% to 4% (1)	From 2% to 6% (2)	From 2% to 8% (3)
Output (Y)	0.0228	0.1525	0.3913
Consumption (C)	-0.3485	-0.7947	-1.3389
Investments (I)	0.1325	0.3717	0.7202
Share of underground economy (S^u)	-0.0957	-0.1886	-0.2790
Real regular wages (w^m)	-0.2455	-0.4839	-0.7152
Total hours worked (H)	-0.0663	-0.0245	0.1271

[34] The same conclusion on this feeble effect of higher inflation on the size of the informal sector is also reported in Nicolini (1998), despite he develops a very different framework.

Results displayed in Table 2 give evidence of the poor effect these policies produce on S^u. Nevertheless, we want a measure to better assess the impact of the monetary intervention. To this end, the next table reports the welfare cost, valued in consumption equivalent units as detailed in section 3, of adopting cold-turkey inflationary policies vs. keeping $\Pi^{\star}_{old} = 2\%$:

Table 3: Welfare cost of cold-turkey inflation

	From 2% to 4% (1)	From 2% to 6% (2)	From 2% to 8% (3)
Steady state	-0.0031	-0.0078	-0.0143
0-5 quarters	-0.0042	-0.0125	-0.0259
0-10 quarters	-0.0041	-0.0113	-0.0222
0-20 quarters	-0.0037	-0.0102	-0.0198

In addition to steady state comparisons, Table 3 displays welfare costs along the transition for 5, 10 and 15 quarters after the new inflation target has been implemented.[35] All of the considered policies deliver welfare losses at any time horizon. Moreover, the shorter the transition period, the bigger is the loss because the inflation rate initially overshoots its target level. As we expected, welfare lowers as the policy gets stronger because cash constrained households are forced to reduce their consumption. Looking back at equation (1), individuals draw utility from consumption and disutility from work. Also, being employed in the underground sector is a source of additional disutility as workers lack any form of social protection. Therefore, welfare gains from inflationary policies could be theoretically obtained if the contraction of S^u is large enough and the decrease in time spent working irregularly compensates the individuals for consuming less. As we did not observe such a reaction of the share of informal activities to changes in the inflation target, we conclude that monetary policy alone is not a good instrument for reducing the size of the underground sector.

5.2 The effects of seigniorage financed tax reductions

As cold turkey inflation fails to bring the desired effects on S^u, we now want to assess whether a fiscal intervention could be more effective. To remain in the framework of budget neutral policies, we perform deterministic policy simulations in which seigniorage revenues originating from the increase in the inflation target are used to decrease the burden of distortionary taxation.

We begin by showing the long run benefits of combining fiscal and monetary measures instead of resorting to monetary policy only:

[35] Again, the benchmark is to leave the inflation rate at the original steady state level of 2%.

Table 4: Steady State comparisons (% changes)

	$[\Pi^{\star}_{new},\, \tau^s]$ (1)	$[\Pi^{\star}_{new},\, \tau^k]$ (2)	$[\Pi^{\star}_{new},\, \tau^c]$ (3)
Output (Y)	0.9621	1.3618	0.8105
Consumption (C)	1.0866	1.0905	0.7977
Investments (I)	1.3956	3.5557	1.5914
Share of underground economy (S^u)	-0.9856	-0.3182	-0.1117
Real regular wages (w^m)	1.5381	1.1587	1.1465
Total hours worked (H)	0.4472	0.0679	0.3465

For the selected variables, Table 4 shows the percentage changes between our benchmark steady state - the one obtained by increasing the inflation target from 2% to 4%, with seigniorage revenues used to reduce lump-sum taxes - and the steady state obtained when the resources freed by monetary policy are used to reduce τ^s (column 1), τ^k (column 2) and τ^c (column 3).[36] Reducing distortionary, instead of lump-sum, taxes brings about long run benefits: output, consumption, investments, regular wages and employment are higher, regardless of the tax rate that is being reduced. Also, the share of underground economy is lower, suggesting that distortionary taxes have a major impact on S^u than the economy's inflation rate. A further analysis of columns (1)-(3) points out that cuts to τ^k are the most wholesome for the main economic aggregates (output, consumption and investments), at least in the long run. The reason is that, compared to the other policies, the economy ends up with the largest stock of capital.

Instead, we find that the largest reduction in the share of informal activities is achieved by targeting the social security tax rate τ^s. When the latter is reduced in place of lump-sum taxes, the steady state value of S^u shrinks by almost 1%. This result stems as firms' choice to evade taxes every period is essentially a decision on the proportion of irregular workforce to employ and a cut in τ^s directly affects the demand side of the labor market by lowering the tax wedge on regular labor.

Steady states comparison is a good exercise to understand where the economy would end up if a specific policy were implemented. However, it fails to disclose important pieces of information. Comparing the variables' terminal values neither say anything about the time needed for convergence, nor about the transition to the new steady state. Policies characterized by high long run returns might be costly if shorter time horizons are considered. We tackle this issue by focusing on the transition path of some variables to their new steady state in the first 20 quarters after the intervention. Also, we compute the welfare cost associated with any of the considered policies at different time spans. Because welfare is measured in consumption equivalent units, the following discussion will be mainly centered on understanding differences in consumption responses.

Figure 10 plots the response of the selected variables to a 2% increase in the annual inflation target when the relative revenues are used to decrease the value of τ^s (solid line), τ^k (dashed line)

[36]We do not show results for the same intervention when the reduced tax rate is τ^h. The reason is that this policy is equivalent to a seignorage-financed decrease in τ^s. Both the policies affect the agents' labor decisions in the regular market: a lower value of τ^s acts as a positive permanent labor demand shock, while a reduction in τ^h as a positive supply shock. As expected, the equilibrium wage w^m is higher in the first case. However, after the deduction of income taxes, both measures have the same impact on the households budget constraint.

and τ^c (dash-dotted line). [37] We also add the response to the previously discussed policy (dotted line), that is when seigniorage revenues are used to lower lump-sum taxes, to give visible evidence of the advantage of combining monetary and fiscal measures. As we commented on Table 4, long run effects on output and consumption are always positive. The dynamics of the adjustment, instead, are very different. When the policy targets τ^k and τ^c, consumption stay below the initial steady state level for a prolonged period of time. In the first case, the marginal value of a unit of installed capital, measured by the Tobin's q, rises. The optimal households behavior is then to reduce consumption and to increase their investment activity according to equation (10). This crowding effect on consumption adds to the reduction in purchasing power coming from higher inflation. The fall in consumption is lower when the policy is meant to reduce τ^c. In this case, the transition during the first quarters is very similar to that one observed when reducing lump-sum taxes. However, once the inflation rate stabilizes, households, who own firms, benefit from the corporate tax cut.

Differently, a seigniorage financed decrease in τ^s delivers higher levels of consumption both in the long term and along the transition. We argue this result arises because a reduction in the social security tax rate tempers the short term frictions coming from DNWR. In fact, the time t cost of a unit of regular labor for the representative intermediate producer is $(1+\tau_t^s)w_t^m$. From the previous analysis of cold-turkey inflationary policies, we know that regular wages jump up suddenly after the shift in the inflation target because the low real rate of interest stimulates aggregate demand. While this effect vanishes, DNWR prevents firms from curbing regular wages to the desired level. As we discussed in Section 4, the existence of this friction in the regular sector adds incentive for substituting formal with informal activities during economic downturns, when wage cuts become necessary to preserve employment. From this perspective, a reduction in τ_t^s mitigates the negative effects, and the above-mentioned substitution, that arise from the divergence between w_t^m and its efficient level. Thus, this is the only policy to produce a short run decrease of S^u.[38] Given the authorities' aim of fighting the underground economy, this is a relevant result.

For the purpose of completeness, we briefly comment on the path of the other variables in Figure 10. As regards output, all the policies produce a very similar behaviour in the short run: the initial peak follows the real interest rate decline, while the subsequent adjustment mainly depends on DNWR and the targeted tax rate. When wage rigidities bind in the first periods after the policy is implemented, the best output response is the one associated with a reduction in τ^s. Reasons are the same backing the previous discussion on the response of consumption. After roughly three years, instead, the growth in output prompted by a cut in τ^k dominates as the demand for investments overcome the fall in consumption.

All these policies leave government revenues unchanged in the medium run but make them highly volatile in the first two quarters after the enforcement: the first period rise is driven by large initial seigniorage revenues, while the subsequent slump is mainly due to lower tax revenues. Yet, all the three measures unleash resources as real debt is free to move toward a higher steady state level. In fact, the long run debt to GDP target ratio is still satisfied because of the increase in measured output.

We complete our analysis by comparing the welfare implications of adopting different combinations of monetary and fiscal policies:[39]

[37] We assume that seigniorage revenues can be used to decrease only one tax rate at a time.

[38] As we already claimed, the short term expansion of informal production is due to the interaction between the monetary shock and DNWR.

[39] Because we focus on seigniorage financed tax rates reductions which have no impact on long term government

Table 5: Welfare cost of mixing monetary and fiscal policies

	$[\Pi^{\star}_{new},\, \tau^s]$ (1)	$[\Pi^{\star}_{new},\, \tau^k]$ (2)	$[\Pi^{\star}_{new},\, \tau^c]$ (3)
Steady state	0.0082	0.0104	0.0058
0-5 quarters	0.0019	-0.0055	-0.0014
0-10 quarters	0.0015	-0.0067	-0.0019
0-20 quarters	0.0018	-0.0059	-0.0016

When welfare is computed with respect to steady state values of consumption and hours worked, the highest value is obtained by using seigniorage revenues to lower τ^k. We already expected this result from the previous analysis of Table 4. What is striking in Table 5, instead, is the welfare ranking along the transition. Column (1) shows welfare results when the policy targets τ^s: agents in the economy are better off both in the short and in the long run as they can consume more. Instead, long run benefits of reducing distortionary taxes in columns (2) and (3) turn to be costly along the transition. When revenues from monetary policy are used to reduce τ^k or τ^c, welfare losses occur at any time lapse. The drop in consumption is large and prolonged in the first case because of the crowding-out effect of investments, while a lower corporate tax rate τ^c generates an increase in total hours worked in the economy which overcomes the positive effect on consumption. Also, policies in columns (2) and (3) still produce losses when welfare is computed in a transition period of five years. This is a too long time lapse that strongly suggest not to implement them.

This last finding, along with the evidence of the relatively poor effect of reducing τ^c or τ^k on S^u, leads us to conclude that policy makers should use seigniorage revenues to decrease the social security tax rate τ^s. Because going underground is essentially a decision on the proportion of irregular workers to employ, the most relevant reduction in the size of the underground sector occurs when the policy targets τ^s as the cost difference between regular and informal labor is suddenly cut down. Moreover, because this cost difference is strongly affected by DNWR, decreasing τ^s is the only way to achieve both a lower S^u and welfare gains in the short run.

6 Conclusion

In this paper we develop a two sector DSGE model that explicitly accounts for the existence of underground production in the economy. Our goal is twofold: first, we want to simulate the model under various exogenous shocks in order to understand the features and the determinants of the shadow economy; second, we study monetary and fiscal policies designed to reduce the size of the underground sector.

We find that, despite cash-intensity is a natural feature of transactions that take place in the informal sector, the use of inflation to tax them is unappealing as the shadow economy barely shrinks, while individuals bear welfare losses. Also, impulse response analysis emphasizes the importance of DNWR in the reallocation of labor between the formal and the underground sector when the economy is hit by exogenous shocks. Policy makers should pay attention to this friction in the regular labor market: our simulations clearly show that the effects of monetary policy

revenues, welfare is computed using seigniorage financed reductions in lump-sum taxes as benchmark policy.

can be highly distorted by the existence of DNWR. Finally, we conclude that, among the budget neutral policies we study, the largest reduction in the share of irregular production is achieved when seigniorage revenues from higher inflation are used to lower the social security tax rate. Moreover, this is the only policy able to yield welfare gains both in the short and in the long run.

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Appendix

Complete set of equilibrium conditions

$$C_t^{-\sigma} = \lambda_t^h (1 + \zeta_t^h \eta^h)$$

$$H_t^m + H_t^u = \left(\lambda_t^h\right)^{\frac{1}{\xi}} \left[\frac{(1-\tau_t^h) w_t^m}{\Gamma_0}\right]^{\frac{1}{\xi}}$$

$$H_t^u = \begin{cases} (\lambda_t^h)^{\frac{1}{\varphi}} \left(\frac{w_t^u - (1-\tau_t^h) w_t^m}{\Gamma_1}\right)^{\frac{1}{\varphi}}, & \text{if } \; W_t^u \geq (1-\tau_t^h) W_t^m \\ 0, & \text{otherwise} \end{cases}$$

$$E_t Q_{t,t+1} = \beta E_t \frac{\lambda_{t+1}^h}{\lambda_t^h} \frac{1}{\Pi_{t+1}}$$

$$q_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}^h}{\lambda_t^h} \left[(1-\tau_{t+1}^k) r_{t+1}^k + q_{t+1} u_{t+1}^k (1-\delta)\right]$$

$$1 = q_t \gamma_t \left[1 - \phi_I \left(\frac{I_t}{I_{t-1}} - 1\right) \frac{I_t}{I_{t-1}} - \frac{\phi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] + \beta \mathbb{E}_t q_{t+1} \frac{\lambda_{t+1}^h}{\lambda_t^h} \gamma_{t+1} \phi_I \left(\frac{I_{t+1}}{I_t} - 1\right) \left(\frac{I_{t+1}}{I_t}\right)^2$$

$$\beta E_t \frac{\lambda_{t+1}^h}{\lambda_t^h} \frac{1}{\Pi_{t+1}} = (1 - \zeta_t^h)$$

$$K_{t+1} = (1-\delta) K_t + \gamma_t \left[1 - \frac{\phi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t$$

$$\frac{\alpha Y_t^m}{H_t^m} = \frac{(1+\tau_t^s)(1+\zeta_t^f \nu^m) w_t^m}{mc_t}$$

$$\begin{cases} \frac{\alpha_u Y_t^u}{H_t^u} = \frac{(1+\zeta_t^f \nu^u) w_t^u}{mc_t} & \text{if } \; 1 - ps\tau^c \geq 0 \\ H_t^u = 0 & \text{otherwise} \end{cases}$$

$$\frac{(1-\alpha) Y_t^m + (1-\alpha_u) Y_t^u}{K_t} = \frac{r_t^k}{mc_t}$$

$$R_t = \frac{1}{1 - \zeta_t^f}$$

$$(\epsilon - 1)(1 - \tau_t^c) - \tau_t^c (1 - ps) \frac{Y_t^u}{Y_t} + \phi_p (\Pi_t - 1) \Pi_t - \beta E_t \frac{\lambda_{t+1}^h}{\lambda_t^h} \phi_p (\Pi_{t+1} - 1) \Pi_{t+1} \frac{Y_{t+1}}{Y_t} - \epsilon mc_t = 0$$

$$m_t = \nu^m (1 + \tau_t^s) H_t^m w_t^m + \nu^u H_t^u w_t^u + \eta^h C_t$$

$$b_t = \frac{R_{t-1} b_{t-1}}{\Pi_t} + \frac{m_{t-1}}{\Pi_t} - m_t + G_t - T_t$$

$$T_t = \tau_t^h w_t^m H_t^m + \tau_t^k r_t^k K_t + \tau_t^c \left(Y_t^m + psY_t^u\right) + \tau_t^s w_t^m H_t^m + T_t^{LS}$$

$$T_t^{LS} = T_{t-1}^{LS} + \phi_{TT} \left(T^{LS} - T_{t-1}^{LS}\right) + \phi_{Tb} \frac{(b_{t-1} - b)}{Y^m}$$

$$ln\left(\frac{R_t}{R}\right) = \alpha_R ln\left(\frac{R_{t-1}}{R}\right) + (1 - \alpha_R)\left[\alpha_\pi ln\left(\frac{\Pi_t}{\Pi^\star}\right) + \alpha_y ln\left(\frac{Y_t^m}{Y_{t-1}^m}\right)\right] + \epsilon_{R,t}$$

$$Y_t = Y_t^u + Y_t^m$$

$$Y_t^m = A_t^m \left(H_t^m\right)^\alpha \left(K_t\right)^{(1-\alpha)}$$

$$Y_t^u = A_t^u \left(H_t^u\right)^{\alpha_u} \left(K_t\right)^{(1-\alpha_u)}$$

$$Y_t = C_t + I_t + G_t + \frac{\phi_P}{2} \left(\Pi_t - 1\right)^2 Y_t$$

$$\gamma_t = \gamma e^{z_t^\gamma}$$

$$u_t^k = e^{z_t^{u^k}}$$

$$A_t^m = A_m e^{z_t^m}$$

$$A_t^u = A_u e^{z_t^u}$$

$$G_t = G e^{z_t^G}$$

$$z_t^\gamma = \rho_z^\gamma z_{t-1}^\gamma + \epsilon_{z^\gamma,t}$$

$$z_t^{u^k} = \rho_{z^{u^k}} z_{t-1}^{u^k} + \epsilon_{z^{u^k},t}$$

$$z_t^m = \rho_z^m z_{t-1}^m + \epsilon_{z^m,t}$$

$$z_t^u = \rho_z^u z_{t-1}^u + \epsilon_{z^u,t}$$

$$z_t^G = \rho_z^G z_{t-1}^G + \epsilon_{z^G,t}$$

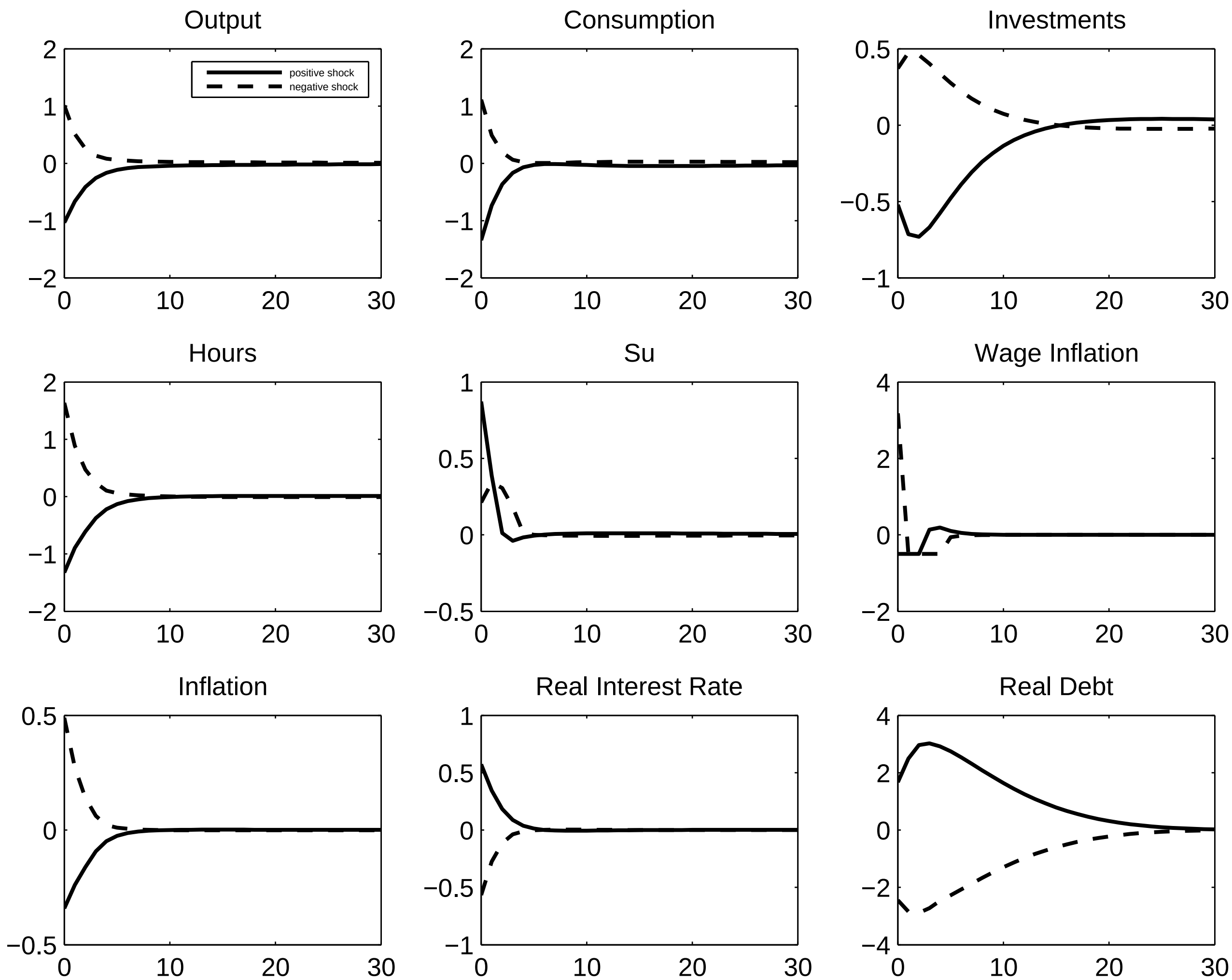

Figure 1: 0.5% Monetary Policy shock

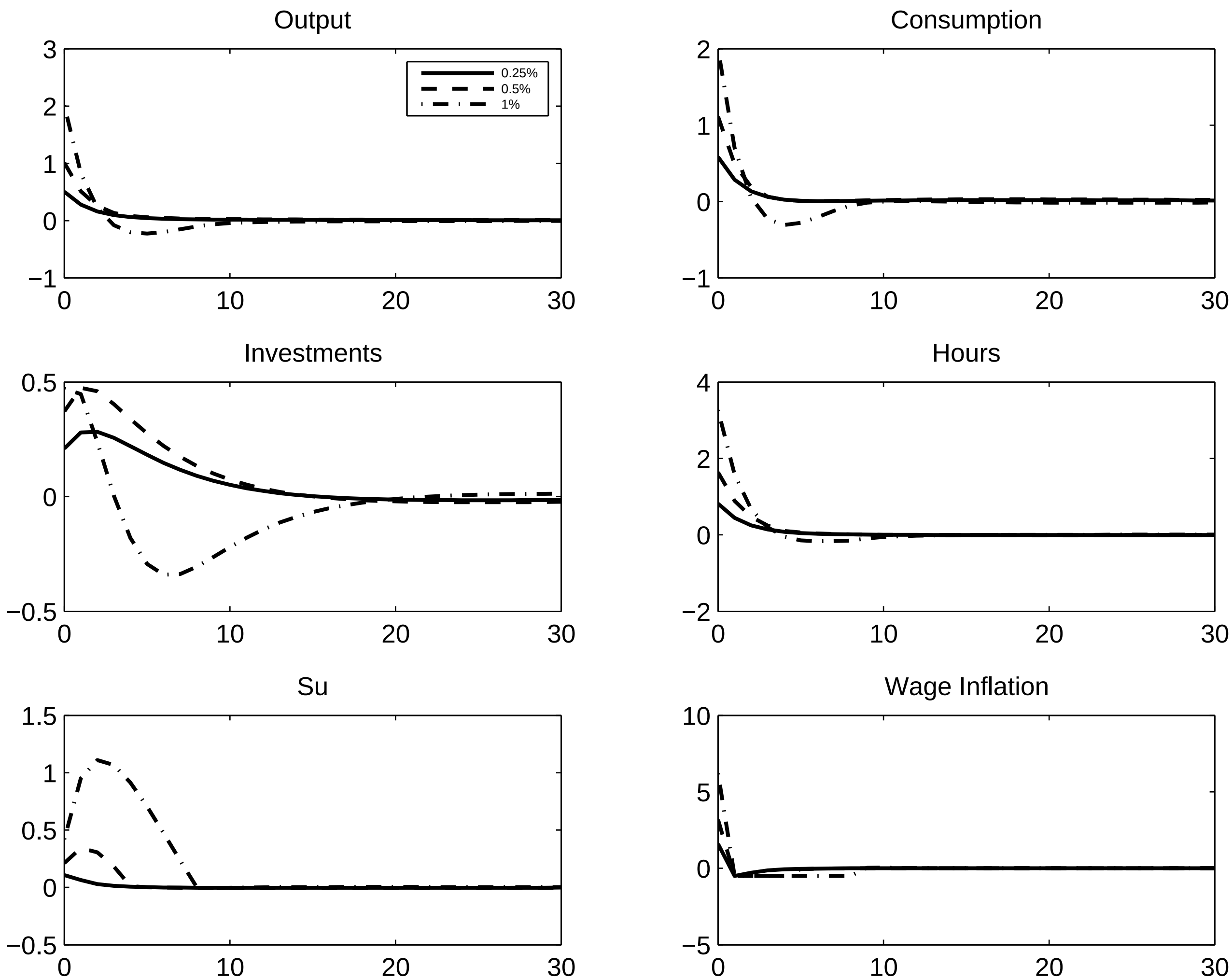

Figure 2: Negative Monetary Policy shock, different magnitude

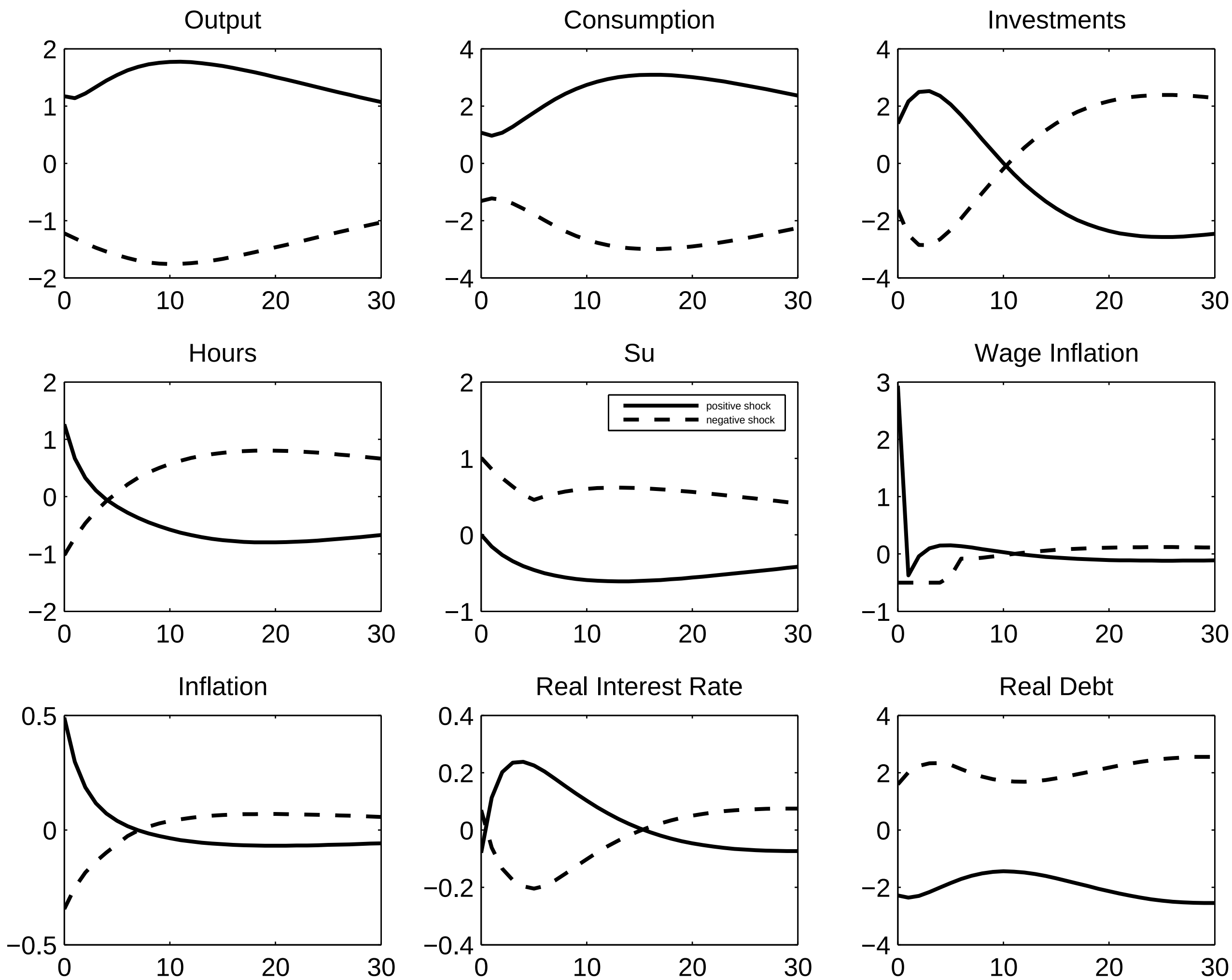

Figure 3: 1% Capital Quality shock

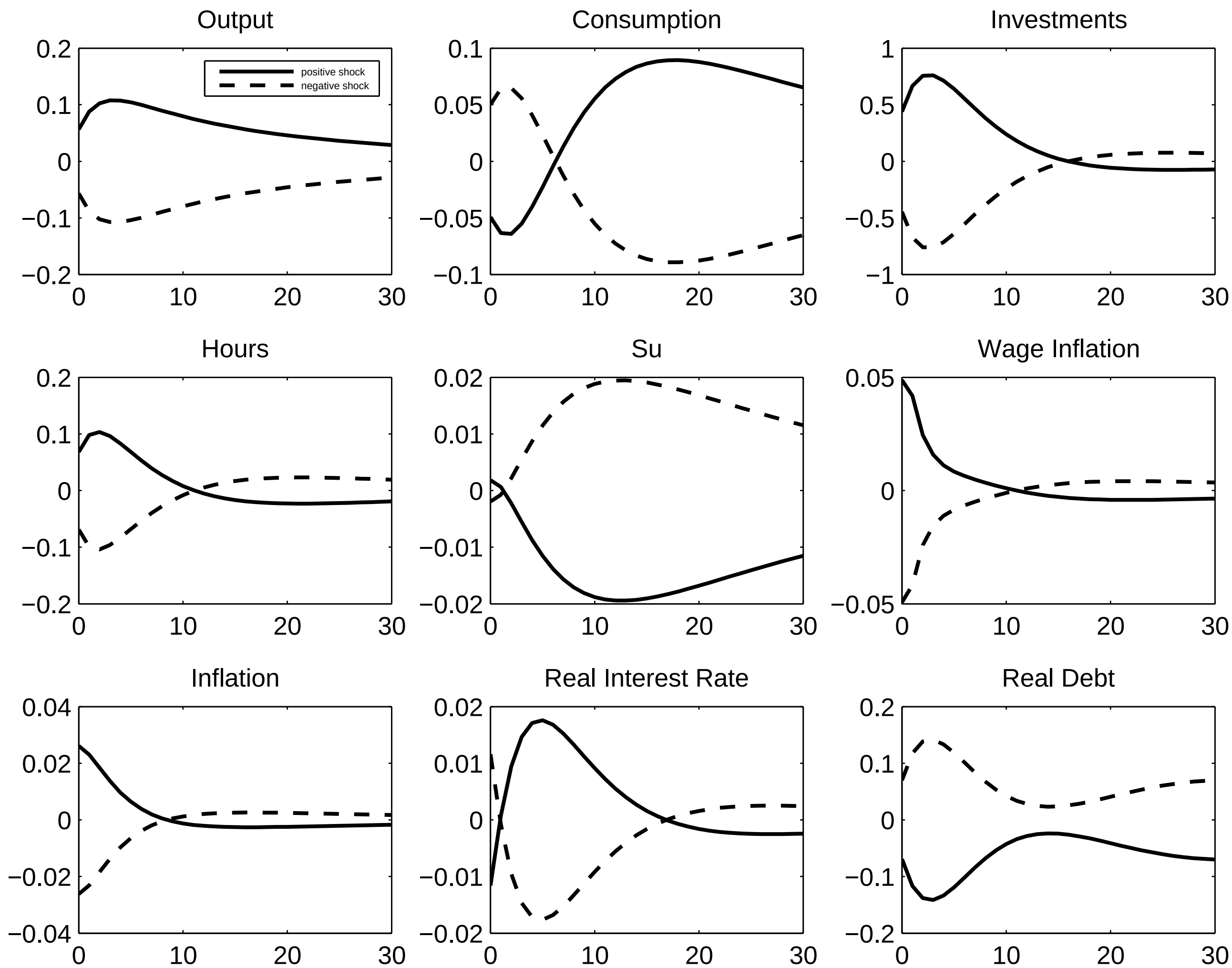

Figure 4: 1% Investment-specific Technology shock

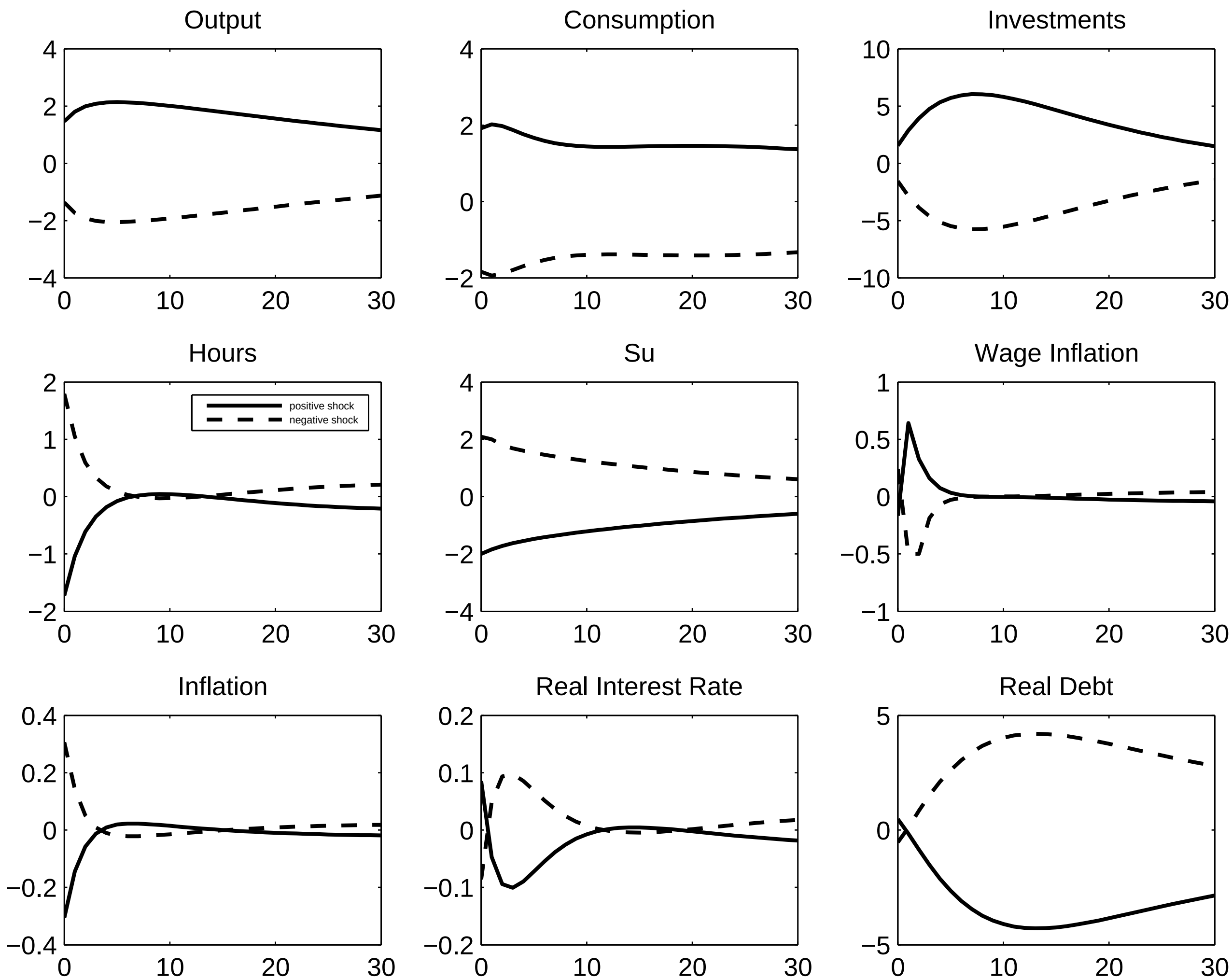

Figure 5: 3% Regular Sector Technology shock

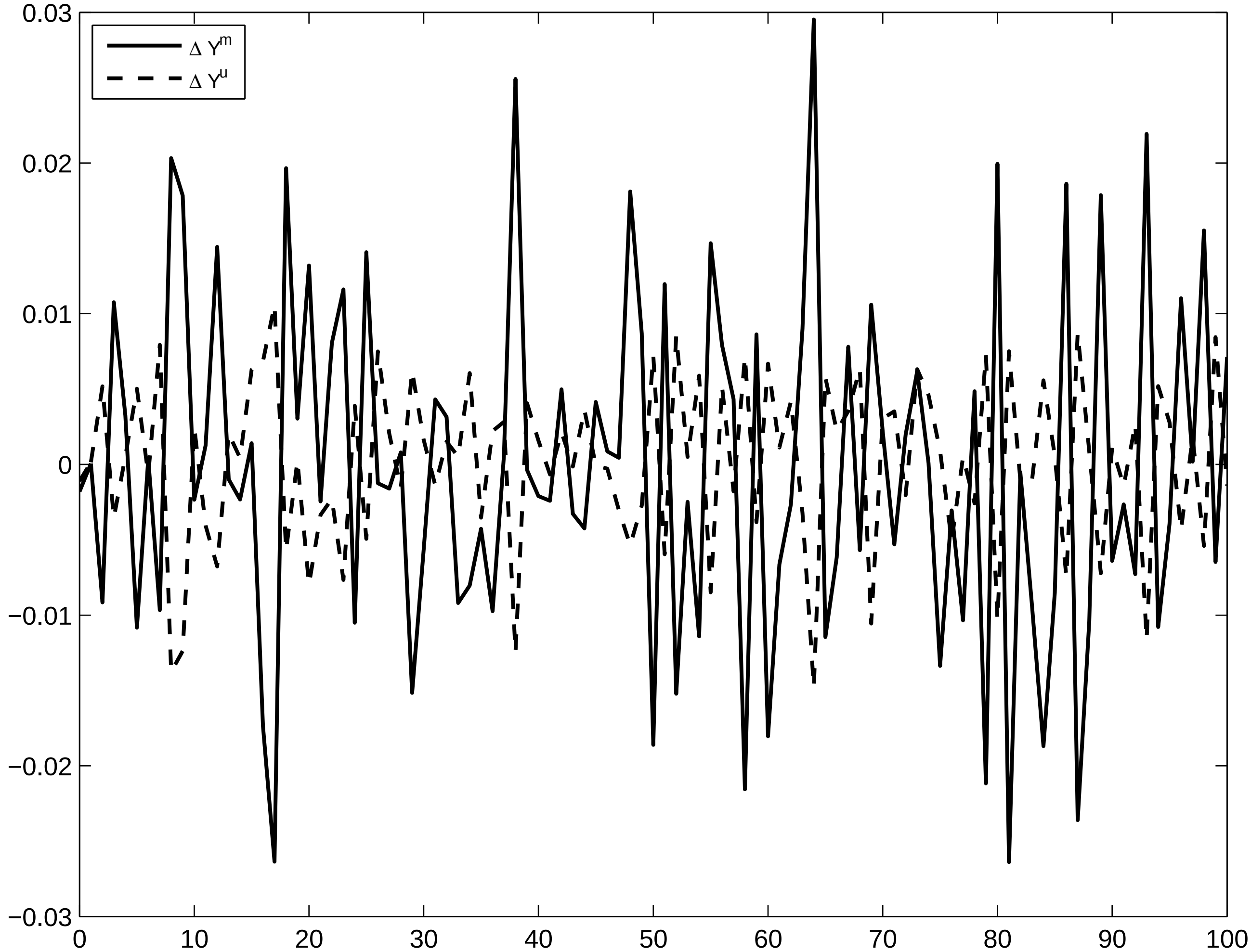

Figure 6: Simulated Output Variations

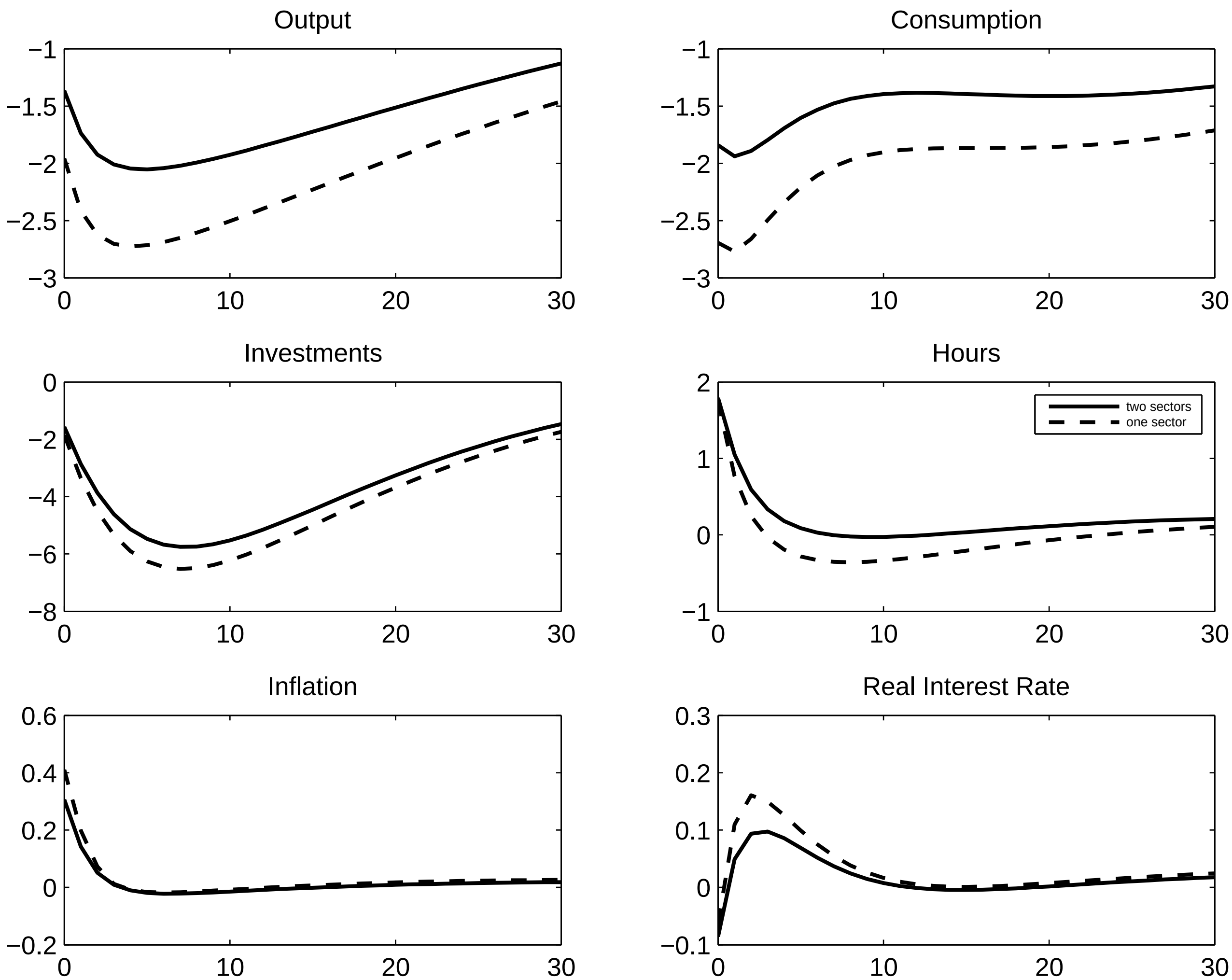

Figure 7: 3% Regular Sector negative Technology shock

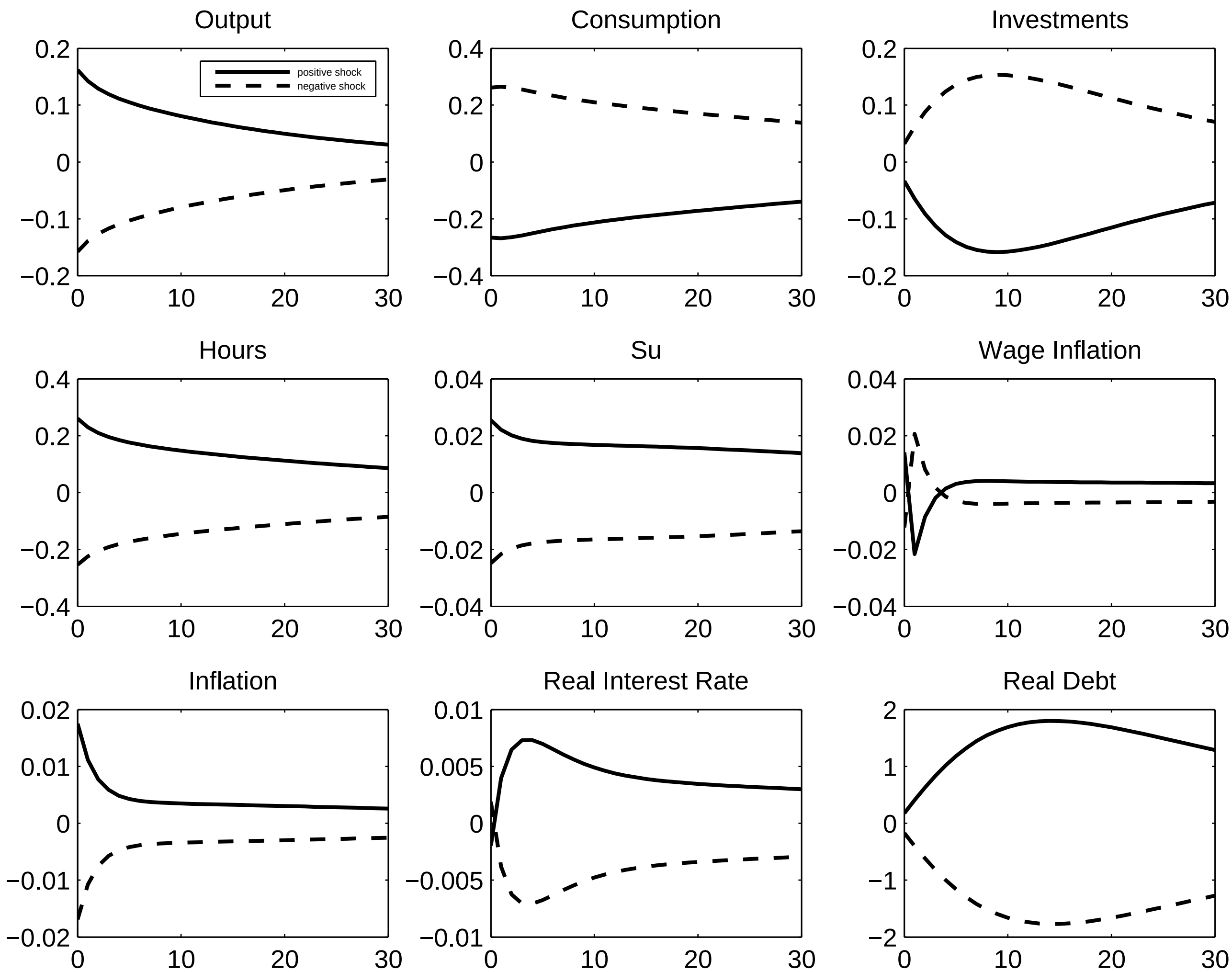

Figure 8: 2% Government Spending shock

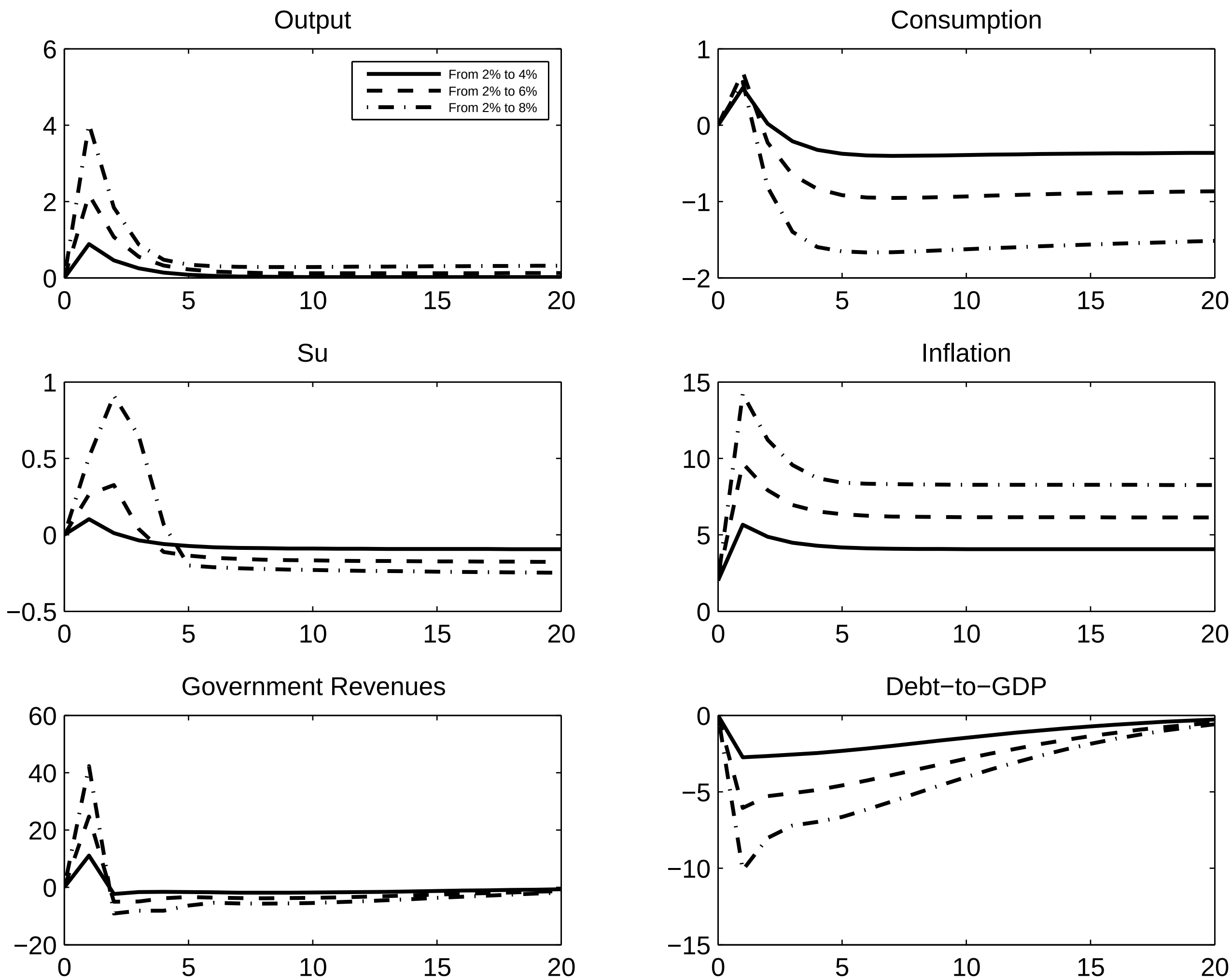

Figure 9: Cold Turkey inflation

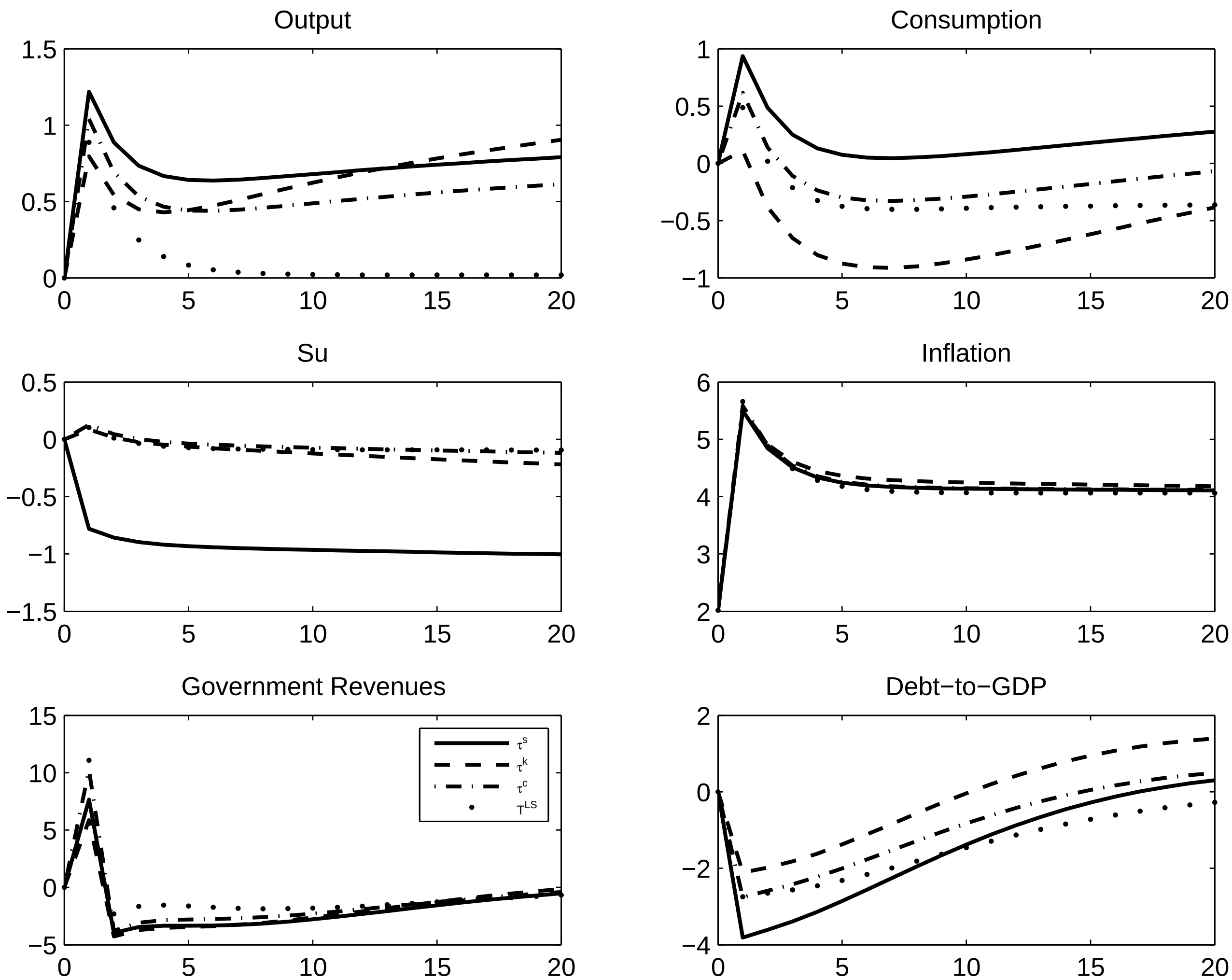

Figure 10: Seigniorage-financed tax reduction