Life-Cycle Portfolio Choice with Liquid and Illiquid Financial Assets *

Claudio Campanale
University of Alicante

Carolina Fugazza
CeRP

Francisco Gomes
London Business School

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*Claudio Campanale, Departamento de Fundamentos del Análisis Económico, Universidad de Alicante, Campus San Vicente del Raspeig, 03690, Alicante, Spain. Phone: +34 965903614 ext. 3262. Fax: +34 965903898. E-mail: claudio@ua.es.

Carolina Fugazza, Center for Research on Pensions and Welfare Policies, Via Real Collegio 30, 10024, Moncalieri (TO), Italy. Phone: +39 011 6705048. Fax: +39 011 6705042. E-mail: fugazza@cerp.unito.it.

Francisco Gomes, London Business School, Regent's Park, London NW1 4SA. United Kingdom. Phone: +44 (0) 207 0008215. Fax: +44 (0) 207 7243317. E-mail: fgomes@london.edu.

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Abstract

Traditionally quantitative models that have studied households' portfolio choice have focused exclusively on the different risk properties of alternative financial assets. We introduce differences in liquidity across assets in the standard life-cycle model of portfolio choice. More precisely, in our model, stocks are subject to transaction costs, as considered in recent macro literature. We show that, when these costs are calibrated to match the observed infrequency of households' trading, the model is able to generate patterns of portfolio stock allocation over age and wealth that are constant or moderately increasing, thus more in line with the existing empirical evidence.

Keywords: household portfolio choice, self-insurance, cash-in-advance, transaction cost.

JEL codes: G11, D91, H55

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and suggestions.

Any remaining errors or inconsistencies are entirely our responsibility.

1 Introduction

The last decade has witnessed a substantial surge of academic interest in the problem of households' financial decisions. A number of empirical facts have been documented regarding in particular the stockholding behavior of households. These include the fact that participation rates, even though increasing over the years, are still at about half of the population and the moderate share allocated to stocks by participants. It has also been documented that the share of financial wealth allocated to stocks is increasing in wealth and roughly constant or moderately increasing in age.[1] Equally important has been the development of theoretical models that, based on a workhorse of modern macroeconomics, that is, the precautionary savings model, have tried to explore the same issue. These models generate a puzzle that is the quantitative equivalent of the equity premium puzzle: given the historical equity premium households should invest most of their financial wealth in stocks, something that is at odds with the empirical evidence. In the context of asset allocation decisions this puzzle is further compounded with the fact that the patterns of stock holdings by wealth and age are also inconsistent with the data.

The current paper adds to this latter line of research by exploring the role played by differences in the liquidity of different classes of financial assets. In order to do this we merge the basic framework as presented for example by Haliassos and Michaelides (2003) in an infinite horizon framework and by Cocco,

[1] Among the papers that have uncovered the patterns of household financial behavior are Ameriks and Zeldes (2004), Bertaut and Starr-McCluer (2000) and Heaton and Lucas (2000) for the US. The book by Guiso et al. (2001) documented the same facts for a number of other industrialized countries as well and the work by Calvet, Campbell and Sodini (2007) has gone in much greater details to document stock-holding behavior among Swedish households.

Gomes and Maenhout (2005) in a life-cycle context with the monetary model in Alvarez et al. (2002).

This is accomplished in the following way. We assume that agents receive a stochastic stream of earnings that are uninsurable during working life and then a fixed pension benefit during retirement. As is standard practice we also assume a no borrowing and no short sale constraint. Agents thus save to self-insure their consumption level against earnings fluctuations. They also save to finance consumption in retirement. Agents have access to two assets. One asset pays a safe return while the second asset pays a risky return but offers a premium in expectation over the safe asset. As in Alvarez et al. (2002) we assume that the two assets are held in separate accounts and that transactions between the two accounts require payment of a fixed cost. In what follows we will often call stock account or simply stock the one where the risky asset is held . We will often call monetary or liquid account, or simply money, the one where the risk-free asset is held. Households receive their wages in the monetary account and a cash-in-advance constraint holds, so that consumption goods can only be purchased with the available money. Payment of the transaction cost allows the agent to relax the constraint. This gives the liquid asset an advantage as an asset to insure consumption levels. The reason is that an agent that faced a negative earnings shock and needed to use savings to maintain consumption levels would need to pay the transaction cost to move assets from the stock account to the monetary account. This advantage is stronger the greater the transaction cost. Similarly a retired agent who is using accumulated wealth to supplement her pension income would like to hold a certain balance in the liquid account rather than paying the fixed cost in every period.

Once the investor optimization problem is solved, interesting patterns emerge. The standard model with no transaction costs can only generate the well known policy functions for the stock share that start at 100 percent when the agent has very little wealth and then monotonically decline as wealth increases.[2] In the model presented here the current share of stocks becomes a state variable. The optimal stock share decision depends on the current stock share — as well as current wealth and earnings — and displays more complex shapes that include patterns that are increasing in wealth especially when both wealth and current earnings are small. When the policy functions are used to simulate the households' life-cycle decisions, this leads also to important changes in portfolio stock allocations as a function of wealth and age. With respect to age the model generates a stock share profile that is either hump-shaped or moderately increasing, depending on the parametrization used. With respect to wealth the simulated data show portfolio allocations to stocks that are increasing over the bottom to mid quartiles of the distribution and then level off or moderately decline at the top. This occurs also when the behavior of stock shares over wealth is conditioned on age. While still not a perfect match with the data these patterns improve significantly over those produced by conventional models.

A critical issue in the present model is the level of the fixed transaction cost. In the model it is assumed that this cost is a monetary one that is subtracted from available resources in the budget constraint. At first sight one may think that this cost is small, based on casual empirical evidence. On the other hand the cost includes also the time and information processing cost that is involved

[2]This holds under the assumption of no or small correlation between earnings and risky returns. More discussion on this issue will be given later.

in making the associated financial plan. This cost is reflected in the frequency of transactions that we observe among households. The empirical evidence in this respect shows that transactions in stock accounts are rare for a large fraction of households, suggesting that once the planning costs are factored in the overall cost is non-trivial.[3]

The present paper is related to two different strands of literature. The first one is the literature on portfolio allocation in precautionary savings models. This literature was first explored by Heaton and Lucas (1997 and 2000) and Haliassos and Michaelides (2003) in an infinite horizon setting and by Campbell et al. (2001), Cocco, Gomes and Maenhout (2005) and Gomes and Michaelides (2005) in a life-cycle setting. These papers documented the basic properties of this type of model and pointed out the difficulties it has to explain the low participation rates and conditional stock shares observed in the data, in some cases proposing possible solutions. They have also shown that some positive contemporaneous correlation between earnings and stock market returns help reducing the stock demand at low wealth levels but rejected this explanation as lacking empirical support.

More recently a number of papers and in particular the ones by Benzoni et al. (2007), Gomes and Michaelides (2003), Lynch and Tan (2011), Polkovnichenko (2007) and Wachter and Yogo (2010) have looked for explanations of patterns of household stock market investment over the life-cycle and over wealth levels. Benzoni et al. (2007) and Lynch and Tan (2011) consider alternative specifications of the labor income process which can also deliver portfolio shares

[3] See Bilias et al. (2010) and the Investment Company Institute report "Equity Ownership in America" (2005).

that are increasing in wealth, conditional on age. However, in Benzoni et al. (2007) this effect only takes place early in life, since it is driven by the low-frequency correlation between stock return and labor income. Naturally, as the agent approaches retirement this correlation becomes irrelevant. The objective of their paper is to match the unconditional share as a function of age, so it is only necessary to generate this effect early in life. Likewise, in Lynch and Tan (2011) the result is driven by business cycle fluctuations in the conditional distribution of income shocks, and therefore the effect is again only present for young households. Gomes and Michaelides (2003), Polkovnichenko (2007) and Wachter and Yogo (2010) show that it is also possible to generate equity portfolio shares that increase with wealth, conditional on age, by considering alternative specifications for the utility function. Both Gomes and Michaelides (2003) and Polkovnichenko (2007) use habit formation preferences, however they point out that, in order to get strong effects within this model, the importance of the habit must be very high, and therefore it implies counter-factually high levels of wealth accumulation. Wachter and Yogo (2010) assume multiple goods. Their model generates an increasing relationship between wealth and the portfolio share of risky assets conditional on age. However the average life-cycle profile is declining, hence does not match the data very well, unless a process for labor earnings with the risk of zero income realizations is assumed, something that they view as extreme. We see our theory as complementary to the ones mentioned above, but more general, since it allows us to match the weakly increasing pattern of the portfolio share both over the life-cycle and over wealth, conditional on age without the need to resort to any form of correlation between labor earnings and market returns, something that is absent during retirement and is likely to be weak at the end of the working life.

The other strand of literature includes models of monetary economics that assume a portfolio choice between money and other assets, like capital or bonds, and some frictions. Examples are the papers by Alvarez et al. (2002), Akyol (2004) and Khan and Thomas (2011). Alvarez et al. (2002) construct a model that is similar to the current one in the assumption about the cash-in-advance constraint on consumption purchases; their model is focused on studying the effects of money injections on interest rates and exchange rates. Their framework though is different from the incomplete market model used here. Akyol (2004) uses the incomplete market model to study the optimality of the Friedman rule when agents have access to two assets, money and a bond. In his model a friction is introduced by assuming that trading in the bond market can be performed only before the uncertainty about labor earnings is resolved. Khan and Thomas (2011) consider a model with endogenous market segmentation and show that it can generate sluggish and persistent adjustments of prices and interest rates to a monetary shock in an endowment economy as well as a hump shaped response of employment and output to productivity shocks.

There is a growing body of literature in finance that studies the role of inaction in household behavior in assets markets. Our model generates infrequent portfolio adjustments by assuming a fixed transaction cost. An alternative approach that is often used in the continuous time literature is to assume observation costs. One example in this line of research is the paper by Abel et al. (2007) who construct a model where agents choose optimally the timing of portfolio observation under the assumption that the observation is costly. Portfolio adjustments occur at the time of observation. Alvarez et al. (2012) construct a model where both observations and transactions are costly. Using a unique Italian dataset they test the implications of their model and find that transaction costs seem to be quantitatively larger and more important to rationalize households' trade. This lends support to our choice to study the behavior of conditional portfolio shares under infrequent portfolio adjustment by assuming a fixed transaction rather than an observation cost.

In the literature about household portfolio choice transaction costs have been traditionally considered on housing transactions rather than on the risky financial asset. [4] More recently Ang et al. (2011) study the portfolio holdings in a model with two risky assets, one tradable only at random instants of time and meant to represent private equity. While their framework and objects of study are different from the current ones some of their results are consistent with those reported in this paper. In particular they find that an increase in the expected time between transactions, hence a reduction in the liquidity of the risky asset, reduces its portfolio share. A similar result is obtained here by increasing the cost of performing a transaction in the stock market.

In the field of asset pricing Chien et al. (2012) show that a model with a small fraction of households that re-balance their portfolio in every period and a large fraction of infrequent traders improves substantially the ability of the theory to explain the large counter-cyclical volatility of aggregate risk compensation. Our work complements their by showing that a plausible mechanism to generate infrequent re-balancing is also consistent with the observed households' portfolio allocation. This is especially interesting because other approaches like internal habit formation that have been successfully tried to improve our understanding

[4] Examples are the work of Cocco (2005) and Yao and Zhang (2005).

of asset pricing have proven less successful on the asset allocation dimension as it was shown by Gomes and Michaelides (2003).

Finally this research is also related to some recent papers that have tried to estimate the relationship between wealth changes and the share invested in risky assets using a panel data approach on individual household data. These include the works of Brunnermeier and Nagel (2008) and Chiappori and Paiella (2011). These papers find only a weak relationship between wealth and households' risky investment. The current paper by generating a non-monotone relationship between the stock share of market participants and wealth may help rationalize those findings.

The rest of the paper is organized as follows. In section 2 we present the description of the model, in section 3 we report the choice of parameters, in section 4 we report the main findings of the analysis and finally in section 5 some short conclusions are outlined. The paper is completed by one appendix providing a short description of the numerical methods used to solve the model and one with a brief description of data construction.

2 The Model

2.1 Preferences

The model is partial equilibrium and is formulated in a life-cycle framework. Time is divided into discrete periods of one year length. Agents enter the model at age 20 and live up to a maximum of 100 years, that is, 80 model periods. We denote with T the maximum number of periods an agent can live in the model. We assume that the agent works the first 45 years and retires afterwards and that all along her life she faces a time varying probability of surviving from age t to age $t+1$ denoted with π_{t+1}. Preferences are defined on consumption only and are represented by a standard expected utility with iso-elastic utility index. The agent's goal is thus to maximize the following objective:

$$E_1 \sum_{t=1}^{T} \beta^{t-1} \Big(\prod_{j=1}^{t} \pi_j \Big) u(c_t) \tag{1}$$

where β is a standard discount factor. The agent is endowed with one unit of time. He does not value leisure so that the time endowment is entirely spent to work in the market.

2.2 Labor income process

The agent efficiency as a worker is age dependent according to the function $G(t)$. This function is meant to capture the hump-shaped profile of earnings over the working life. The deterministic component of labor efficiency units is hit by a stochastic shock represented by a first order autoregressive process in logarithms. Denoting the stochastic component of income with z_t this will then evolve according to the law of motion:

$$\ln z_t = \rho \ln z_{t-1} + \varepsilon_t \tag{2}$$

where ε_t is a normal i.i.d. shock. We normalize wages to one so that labor income can be written simply as $y_t = G(t) z_t$. After retirement the agent receives a fixed pension benefit $y_R^{z_R}$ related to her earnings in the last working period, so that her nonfinancial income is $y_t = y_R^{z_R}$.

2.3 Assets

Earnings shocks cannot be insured due to missing markets. The agent then uses savings to smooth consumption in the face of earnings fluctuations. In doing so he has access to two assets. The first asset is a risk-free, liquid financial asset. This asset is meant to represent cash, checking and savings accounts, certificates of deposits and money market mutual funds, that is, all assets that are typically classified as liquid financial assets - as opposed to bonds and stocks - in the empirical literature. Wages are paid in the form of this asset which on top is the only asset that can be used to purchase consumption. We denote with m_t the amount of this asset that the agent holds at the beginning of period t and with R^m_{t+1} the return on holding the asset from time t to time $t+1$. The second asset is a less liquid financial asset that we call stock for convenience. This asset is risky and provides a positive expected return premium above the liquid asset. This asset cannot be used directly to purchase consumption goods. We denote the amount of stock held at the beginning of period t with s_t and the return on holding stock from t to $t+1$ with $\tilde{R}^s_{t+1}$. A no borrowing and no short-sale constraints are assumed.

The two assets are held in separate accounts and a fixed cost must be paid to make a transaction between the two accounts. This cost is fixed in the sense that it is independent of the amount of the risky asset that is traded. We make it proportional to earnings though, so as to capture the idea that the cost includes the monetary equivalent of the time spent to make financial decisions.[5] We denote the transaction cost with TC in the model. This is the key assumption

[5] It is customary in the literature that uses entry costs to make them proportional to income; see for example Gomes and Michaelides (2005).

in the model since it makes money more valuable as an asset to insure against consumption fluctuations.

In order to highlight the difference between the economic importance/results of the transaction cost and that of a participation cost (as considered by some previous papers), we write down a general formulation of the model with both. However, as it will become clear, the participation cost has very different implications from the transaction cost. In particular it does not affect the shape of the portfolio decision rule and therefore it cannot help to match the observed portfolio allocations patterns as a function of wealth. For this reason, for most of the paper, we will focus on a version of the model with the participation cost set to zero, and only at the end we present the results for a positive value.

Finally we omit an explicit modeling of housing wealth given that this is not the focus of the model and would further complicate the numerical solution of the model. However given the importance that housing has in households' economic decisions we decide to model it following the approach in Gomes and Michaelides (2005) who introduce in their model a flow of expenditures on housing services that does not give utility and that must be subtracted from income. We denote the fraction of income that is spent on housing with $h(t)$ to capture its dependence on the household's age.

2.4 Household's optimal program

Given the informal description of the individual problem stated above it is possible to write the household's optimization problem in dynamic programming form.

In describing the value function we first write the indirect utility in the case when the household decides to make a transaction between the two accounts. This will read:

$$V_t^{tr}(s_t, m_t, z_t) = \max_{c_t, s_{t+1}^o, m_{t+1}^o} \Big\{ u(c_t) + \beta \mathbf{E} V_{t+1}(s_{t+1}, m_{t+1}, z_{t+1}) \Big\} \tag{3}$$

under the following constraints:

$$c_t + s_{t+1}^o + m_{t+1}^o \leq y_t(1 - h(t)) + m_t + s_t - y_t(TC + I_{s_t>0}PC) \tag{4}$$

$$s_{t+1} = \tilde{R}_{t+1}^s s_{t+1}^o, \quad m_{t+1} = R_{t+1}^m m_{t+1}^o \tag{5}$$

$$m_{t+1}^o \geq 0, \quad s_{t+1}^o \geq 0 \tag{6}$$

and the law of motion of z_t in equation 2. In this case the maximization of the right-hand side of the value function is taken with respect to consumption and both assets. Equation 4 is the budget constraint. The agent pays the fixed cost $y_t TC$ which allows him to buy or sell stocks, hence the amount of resources potentially available for consumption and asset purchases subtracts this cost from the sum of current earnings net of housing expenditures, money and stocks. The agent can then use these resources without further restrictions to buy consumption and the two assets. The variable I_{s_t} is an indicator function. If the agent started the period with a strictly positive amount of stocks then $I_{s_t} = 1$ and the agent must pay the participation cost PC as well. Equation 5 shows the laws of motion of stock and liquid holdings: It gives us the amount of resources in the monetary and stock accounts that the agent will have at the beginning of the next period, given the optimal choices of the two assets m_{t+1}^o and s_{t+1}^o. The last equation is the non-negativity constraint that applies to the holdings of the two assets. It simply says that the agent cannot short-sell either asset. We use a separate notation for the control variables m_{t+1}^o and s_{t+1}^o and their corresponding state variables m_{t+1} and s_{t+1} because the return earned on the two assets makes the value of the control and state different.

Next we write the indirect utility in the case the agent decides not to perform any transaction between the money and stock account:

$$V_t^{ntr}(s_t, m_t, z_t) = \max_{c_t, m_{t+1}^o} \Big\{ u(c_t) + \beta \mathbf{E} V_{t+1}(s_{t+1}, m_{t+1}, z_{t+1}) \Big\} \tag{7}$$

subject to the following constraints:

$$c_t + m_{t+1}^o \leq y_t(1 - h(t)) + m_t - y_t I_{s_t>0} PC \tag{8}$$

$$s_{t+1} = \tilde{R}_{t+1}^s s_t \tag{9}$$

$$m_{t+1} = R_{t+1}^m m_{t+1}^o \tag{10}$$

$$m_{t+1}^o \geq 0 \tag{11}$$

and equation 2. In the value function equation m_{t+1}^o denotes the amount of the liquid asset to carry into the next period. Equation 8 is the budget constraint. It reflects the fact that if no transaction between the two accounts is made the agent does not pay any fixed transaction cost but she will only be able to use her current earnings and the initial amount of money to purchase consumption. At the same time the balance on the monetary account carried over to the next period cannot exceed the sum of earnings net of housing expenditures and current money balances minus consumption. Again, if the agent started the period with positive stock holdings $I_{s_t} = 1$ and the agent must pay the participation cost which is then paid regardless of whether a transaction is made or not. Equation 9 describes the fact that in the no transaction case the amount of stock carried to the next period is simply the gross return on the current amount. For this same reason in the equation defining the value function, maximization is taken only with respect to consumption and the liquid asset. Finally the last equation represents the usual no borrowing constraint.

As the laws of motion of stocks, equations (4) and (9) suggest, an implicit assumption is that either all the return on the stock takes the form of price appreciations or that dividends are immediately reinvested in the stock account at no cost. In reality part of the return on equity comes from dividends that are paid in the monetary account. Contrary to the standard model, with fixed transaction costs the way the return is split between capital gains and dividends is relevant for the investor's decision problem. For this reason in the result section we will also consider sensitivity analysis using an alternative version of the model where part of the return is paid in the form of a dividend.

The optimal value function and the optimal decision about whether to make a transaction or not is obtained by comparing the indirect utility in the two cases. This is summarized by the equation:

$$V_t(s_t, m_t, z_t) = max\{V_t^{tr}(s_t, m_t, z_t), V_t^{ntr}(s_t, m_t, z_t)\}. \tag{12}$$

The model does not admit analytical solutions and is then solved numerically. The solution to the model is especially difficult in this case for two reasons. First, once the fixed transaction cost is introduced the holdings of the two assets enter separately as a state variable, hence the model has two continuous state variables which are also the two continuous controls.[6] Second, the fixed transaction cost breaks the concavity of the objective function forcing the use of slow direct search methods for the optimization at each state space point.[7] Details of the

[6]Models without a fixed transaction cost only have one continuous state variable, that is, the sum of all financial assets.

[7]See Corbae (1993) on this point.

solution algorithm are provided in the Appendix.

3 Parameter Calibration

The utility index takes the form: $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$ and a value of 5 is chosen for σ, the coefficient of relative risk aversion. The subjective discount factor β is set equal to 0.94. The deterministic component of labor earnings $G(t)$ is represented by a third order polynomial. The coefficients of the polynomial are taken from the profile estimated by Cocco, Gomes and Maenhout (2005) for high-school graduates; when aggregated over five year periods the profile is also consistent with the one estimated by Hansen (1993) for the general population. As far as the idiosyncratic shock is concerned we assume that it can be represented by an AR(1) process in logarithms, that is, we assume $ln(z_{t+1}) = \rho ln(z_t) + \varepsilon_{t+1}$ where ε is a normal random variable $N(0, \sigma_\varepsilon^2)$ and is i.i.d. We assume that the autocorrelation coefficient is 0.95 and that the standard deviation of the innovation is 0.158, in line with the values used by Hugget and Ventura (2000).

In calibrating the pension benefit we follow a procedure that is similar to the one in Huggett and Ventura (2000) albeit in a simplified way. Social security benefits in the U.S. can be divided into a fixed hospital and medical component and an old age component which is related to average working age earnings. In order to calibrate the old age component we exploit the high persistence of the earnings process and compute the average life-time earnings conditional on the shock in the last year of work. We then apply the formula used by the U.S. social security system to compute the benefit. This implies a replacement ratio of 90 percent up to 0.2 times average earnings, a marginal replacement ratio of 32 percent from 0.2 to 1.24 times average earnings and a marginal replacement ratio of 15 percent above 1.24 times average earnings. No further benefit is credited above 2.47 times average earnings. In order to proxy for the hospital and medical component of the social security benefit, which is not related to past earnings, we then then add a fixed term to the benefit computed with the above formula. We thus stop short of fully linking benefits to working age average earnings — as in Huggett and Ventura (2000) — since this would require the addition of a further state variable. Still our calibration gives some progressive features that help matching wealth-to-income ratios across the wealth distribution.

As for the housing expenditure process we assume that it is described by a third order polynomial and take the values of the coefficients from the estimates presented in Gomes and Michaelides (2005).

As far as the asset returns process are concerned, it is assumed that the real return on the liquid asset is 2 percent and that the expected real return on the stock is 6 percent. Following a tradition in this literature, the implied premium is lower than the historical one.[8] The process for the stock return is assumed to be normal and i.i.d. over time with a standard deviation of 18 percent, in line with the historical evidence about the US Standard and Poor's 500 index. The participation cost is set to zero in the baseline calibration so that we can focus on the effects of having a transaction cost in the model.

The most critical parameter to calibrate for the purpose of this model is the size of the transaction cost. This transaction cost includes both the monetary cost and non-monetary costs.[9] Quantifying the non-monetary component of the

[8] See Cocco, Gomes and Maenhout (2005) for the reasons behind this choice.

[9] Non-monetary costs include the time cost of gathering the information about the different assets and to make the decision about how much to invest in each, and "psychological" costs such as those required to overcome status quo biases or inertia.

cost is very difficult if not impossible. For this reason we follow an alternative strategy. Clearly the size of the cost will affect the frequency of transactions. We thus calibrate the cost so that once we simulate the model, the fraction of households that do not make a transaction in any given period matches the one in the data. To our knowledge there are two sources of data about households's transactions in the stock market. One are the reports "Equity Ownership in America"compiled by the Investment Company Institute and based on interviews of a sample of stock holding households. The second is the paper by Bilias et al. (2010) which reports data based on the PSID. The two sources report quite different figures. According to the report "Equity Ownership in America"about 40 percent of stockholding households make a transaction in a given year. According to Bilias et al. (2010) between 25 and 30 percent of the general population make a transaction over a 5 years period. Because of this wide difference we run the model with two different levels of the fixed cost calibrated to match the empirical counterparts from the two data sources. In what follows these two different choices will be referred to as the low and high transaction cost case. As we will see the main qualitative features of the model results that we want to highlight are common to the two levels of the cost, even though quantitatively the results will differ across the two experiments. We will provide a discussion of the magnitude of the cost needed to match the transaction frequencies that we use for calibration in a section below, after the presentation of the simulated results.

Finally we assume that the transaction cost is the same both for stock purchases and for stock sales. One might argue that the planning cost in the case of sales is much lower since an agent who faces the need to liquidate the asset in the face of negative earnings shocks or to supplement retirement income may simply do that with no planning. If this was true though, we should observe a greater frequency of sales compared to purchases which is not the case neither in the Investment Company Institute (2002) report nor in the paper by Bilias et al. (2010). In fact sales seem to occur less frequently both during market upswings and during market downswings. Since the difference is not big we think it is a reasonable first approximation to make the cost symmetric across the two different types of transaction.

4 Results

In this section we describe the results of the model. The section is divided into three subsections. In the first one sample decisions rules are reported. In the second subsection we report the results of the simulation of the baseline parametrization and in the final subsection we report results of some further simulations.

4.1 Decision rules

We report the optimal share invested in stock and the decisions to make a transaction in the high transaction cost parametrization for an agent who is 45 years old. We do that for an agent with the lowest earnings shock and for an agent with the highest earnings shock. These are representative of the overall patterns of stock holding and transaction decision rules that can be found at other ages, labor earnings shocks and parameterizations. Figure 1 reports the transaction decision for the agent with the lowest earnings shock. On the two horizontal axis we report the state variables, that is, current wealth and the

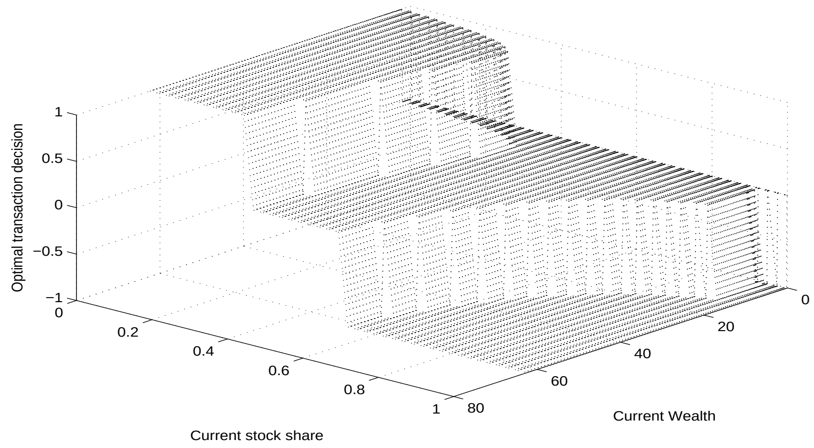

Figure 1: Transaction decision rule: Low earnings shock.

share of this wealth invested in stocks prior to making the decision.[10] On the vertical axis we report the decision to make a transaction. This decision is a discrete one and we make the convention that a 0 means that no transaction is made, a +1 that the agent buys stocks and a -1 that the agent sells stocks. The figure shows that for any level of wealth the agent will buy stocks when the current share is low, she will sell stocks when the current share is high and will not make any transaction in an intermediate range of the current share. Also the inaction region is very wide for low levels of wealth and then narrows as wealth increases. Another property of the transaction decision rule can be seen by looking jointly at figure 1 and figure 2. This reveals that the projection

[10]In the section describing the model the two state variables were the quantities of the two assets. The reasons for this change of variables are related to the numerical method used to solve the model and are highlighted in the appendix. This said we think it is also more instructive for the purpose of understanding the mechanics of the model to redefine the state variables.

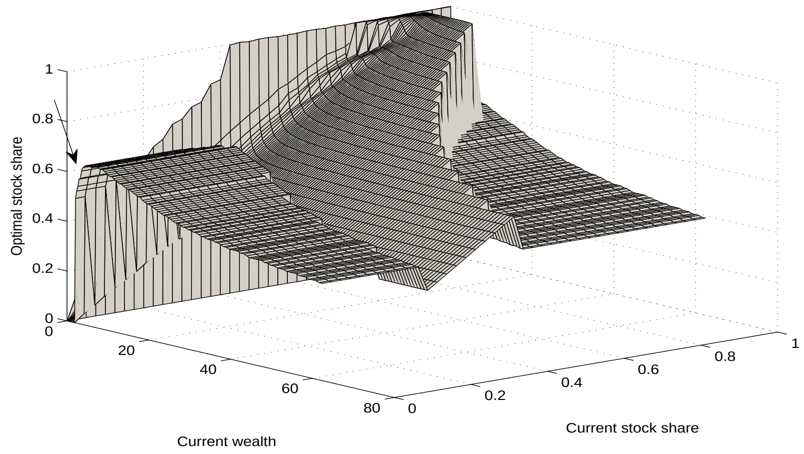

Figure 2: Stock share decision rule: Low earnings shock.

of the no transaction region on the horizontal plane in figure 1 forms a band around the projection of the share decision in figure 2 on the vertical plane. This can be easily interpreted by observing that an agent will not make any transaction when his current share is close to the optimal one. This band is large when wealth is small and narrows down when wealth increases since with more wealth a smaller deviation from the optimal share will make it convenient for the agent to pay the fixed transaction cost and readjust her portfolio.

Turning now to figure 2 we can examine the optimal stock share decision rule. There are three main patterns that we want to highlight. First, for low values of wealth the stock share is increasing in wealth provided that we are in a region where the agent finds it optimal to make a transaction.[11] Second,

[11] We have added an arrow to the graph to highlight that feature which may not be immediately seen given the complexity of the 3-dimensional graph. Also notice that the angle under which the surfaces are seen in the two figures is different. Once again this is done in order to make the graphical representation more clear.

once wealth passes a certain threshold the optimal share starts to decrease with further increases in wealth. Third, there is the region where no transaction is made, corresponding to the band in the middle of the graph. In this band the optimal share is declining in wealth for a given current share and is increasing in the current share for a given wealth. Notice that this may give rise to a complicated relationship between wealth and the optimal share: for low values of the current stock share, the optimal stock share will first decline, then increase and finally decline again as wealth increases.

The interpretation of these patterns is the following. Given persistence, a low earnings agent will want to hold some of his wealth in the form of the liquid asset in order to supplement her consumption beyond her earnings without having to incur the fixed transaction cost. Given the amount of the liquid asset that is needed to accomplish this task, its share will decline with total wealth, hence the optimal stock share will increase. Past a certain level of wealth though, the optimal stock share will start to decline for the usual diversification reasons well highlighted for example in Cocco, Gomes and Meanhout (2005). In the no transaction region the forces at play are different. In this region in fact the optimal share is entirely determined by the total amount of stock at the beginning of the period and the optimal saving decision. It can be seen that in this particular case the optimal share in this area is decreasing in wealth for a constant current share of stocks and increasing in the current share of stock for a given level of wealth.

In figure 3 we report the optimal decision rule for a 45 year old agent endowed with the highest earnings shock.[12] In this case the graph can be divided in two

[12]We omit the corresponding graph for the optimal transaction decision since it does not

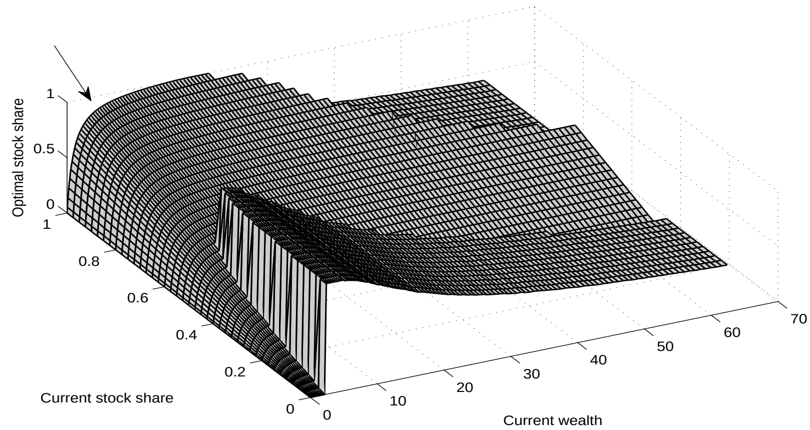

Figure 3: stock share decision rule: High earnings shock.

broad areas. The first one corresponds to the no transaction region and it is the band highlighted by the arrow. The forces that drive its shape are the same as the ones in the previous case, that is, the current amount of equities and the optimal saving decision. The difference is that for the high earners these two forces generate an optimal stock share decision that is increasing in wealth for a given current share of stocks in the portfolio. The second area corresponds to the region where the agent finds it optimal to pay the transaction cost. In this area the optimal stock share is equal to 100 percent at low levels of current wealth and then declines. This pattern is similar to the one observed in standard models without transaction costs.

Summarizing, while in the standard models with no transaction costs the decision rules for the optimal stock share are monotonically declining in wealth, once fixed transaction costs are considered a more diverse picture emerges.[13]

add any new insight with respect to the graph for the lowest earnings shock agent.

[13]This statement about the basic model with no transaction costs is true under the assump-

In particular we can see that for low earnings agents the relationship between wealth and the optimal share of wealth invested in stocks is increasing in a range of low wealth levels, those presumably experienced by low earners. On the other hand, provided they are in the no transaction region, a similar relationship between wealth and the optimal stock share can be observed also for the decision rules of high earnings agents. Whether this is sufficient to generate a positive cross-sectional relationship between wealth and the stock share of market participants depends then on the path of wealth accumulation through the different regions experienced by agents with different earnings history. This can be discovered by simulating the model.

4.2 Simulation results

We simulate a cohort of 2000 agents across their 80 period long life-cycle. Since the realized path of stock returns may affect the observed pattern of stockholding we repeat the simulation 50 times to smooth out these fluctuations. The main focus of the results will be the behavior of the stock share conditional on participation by wealth and age. We omit the analogous results concerning participation rates since it is already known that fixed costs can generate the patterns observed in the data. We still report the average participation rate as a further check on the reasonableness of the size of the chosen transaction cost.

tion that labor earnings are not correlated with the stock return. Under a sufficiently large positive correlation a different result would hold, however positive and high correlation is not supported empirically. See Cocco et al. (2005) and Haliassos and Michaelides (2003) on this point.

Table 1: Average statistics

	Tc (low)	Tc (high)	Data
Participation rate	73.4	51.6	51.1
Stock shares (P)	84.0	69.4	60.1

4.2.1 Baseline model

In this subsection we describe results for the baseline set of parameters. Table 1 reports the aggregate participation rate and stock share for participants in the low and high transaction cost cases, together with their empirical counterpart. [14] What we see in the first row of the table is that in the low cost scenario the participation rate is 73.4 percent, far higher than in the data. When the transaction cost is raised to match the level of inactivity reported in Bilias et al. (2010), the participation rates plunges to a value of 51.6 percent, very close to the 51.1 percent observed in the data.

Looking at the second row of table 1 we can see that in the low cost case the share of wealth invested in stocks by households that participate in the stock market is 84.0 percent. In the high transaction cost case we observe a substantial decline to 69.4 percent, still somewhat higher but much closer to the data. This decline reflects the liquidity motive for holding the risk-free asset. When it is costly to make and carry out stock market investment decisions, households will want to hold a larger percentage of their wealth in the form of the risk-free, liquid asset to smooth their consumption in the face of time-varying and uncertain earnings.

[14]The participation rate and conditional stock share are taken from the Survey of Consumer Finances, 2007.

We next move to the the simulated conditional stock shares by wealth levels. This is done in table 2 which reports the average share of the financial portfolio held in stocks, conditional on participation, by wealth quartiles and separately for the top 5 percent wealthiest households. For comparison we also report the corresponding figures taken from the 2007 Survey of Consumer Finances. As it can be seen the model generates a relationship that is positive at low to intermediate levels of wealth independently of the size of the cost. In the low cost scenario the conditional share moves from 34.3 percent for the bottom wealth quartile to 93.7 percent for the third quartile and then declines to 69.2 percent for the top 5 percent of the wealth distribution. The model thus cannot reproduce a monotonically increasing profile, although it can explain why the poorest households hold a smaller share of stocks than those in the next richer quartiles of the wealth distribution. This result is quite important since it has been particularly difficult to explain this fact so far. The main explanation in fact relied on a strong and positive correlation between earnings shocks and stock market return which has little empirical support.[15] In the high cost scenario results further improve. The conditional share is increasing over the whole range of quartiles, moving from 44.1 to 71.3 percent from the bottom to the top one. It then only modestly declines to 67 percent for the top 5 percent of the wealth distribution.

In the data, when we condition on age, the relationship that exists between net worth and the share of financial wealth invested in stocks becomes weak.

[15] Wachter and Yogo (2010) propose an alternative theory based on non-homotetic preferences. That theory is able to generate shares of risky assets that are increasing in wealth within most age groups. However they do not report the relationship between wealth and the stock share for the whole population, that is, without conditioning on age.

Table 2: Conditional shares by wealth percentiles (Baseline)

	0-25	25-50	50-75	75-100	Top 5%
Data	55.9	59.7	59.3	61.7	62.5
Transaction cost (L)	34.3	77.0	93.7	81.8	69.2
Transaction cost (H)	44.1	63.1	68.3	71.3	67.0

In table 3 we thus report the share of wealth invested in stocks by stockholders conditional on wealth by ten year age groups. The table is organized in three panels.[16] The top one reports data from the Survey of Consumer Finances. The other two panels report the simulated results of the model with transaction costs in the low and high cost scenarios.[17] As it can be seen results are broadly similar to those that do not condition on age. In the low transaction cost scenario in the second panel we see that in the first two age groups, that is, the one from age 20 to 30 and from age 30 to 40 the relationship is increasing over all the wealth classes. For later age groups the relationship has again the inverted U shape that can be found for the general population. Once again this represents an improvement over the standard model where at all ages the relationship between wealth and the stock share is negative unless positive contemporaneous correlation between earnings shocks and stock returns is assumed, something not supported by the empirical evidence. Notice that even more recent models that exploit some more sophisticated form of correlation between earnings and stock

[16] The model simulates the life-cycle over 80 periods meant to represent age 21 to age 100. In the table we do not report the statistics for the two oldest age groups to economize on space. The patterns of stock holding by wealth observed within these two age groups do not differ from those for the other groups.

[17] Some entries in the table show an n.a. This reflects the fact that in the corresponding wealth-age cell the participation rate is 0.

Table 3: Conditional shares by wealth percentiles and age (Baseline)

	Quart. I	Quart. II	Quart. III	Quart. IV	Top 5%
Data					
20-30	44.7	55.4	41.3	56.0	65.6
30-40	58.6	60.5	59.7	56.7	54.9
40-50	59.8	65.4	65.4	64.7	61.6
50-60	72.3	66.6	68.5	67.6	60.6
60-70	62.1	48.6	54.4	63.7	59.3
70-80	40.3	44.6	45.7	62.1	68.9
TC low					
20-30	n.a	53.2	43.8	83.8	91.8
30-40	20.0	74.8	93.4	94.6	95.5
40-50	72.2	92.5	95.1	92.0	89.5
50-60	82.6	94.4	91.6	80.1	74.8
60-70	85.8	91.6	78.3	62.9	57.9
70-80	51.0	90.5	88.4	69.6	60.6
TC high					
20-30	n.a	n.a	n.a	46.3	46.2
30-40	n.a	43.2	63.6	72.4	68.7
40-50	57.9	55.6	76.5	73.7	74.5
50-60	59.1	68.3	77.8	70.2	75.3
60-70	63.5	69.2	71.6	63.5	61.4
70-80	50.9	70.3	71.5	68.7	59.7

market performance like the one of Lynch and Tan (2011) still would run into trouble for agents close to or past the retirement age when there is no or very little wage uncertainty remaining, hence little or no room for any pattern of correlation between nonfinancial income and the stock return. Once again the high transaction cost scenario further improves results. As it can be seen in the last panel of the table, under this scenario, the share of wealth held in stocks is broadly increasing in wealth for all age groups up to the 50 to 60 group. For the remaining two groups we can still see the inverted U-shaped pattern, however the declining leg is milder and only confined to the top quartile and top 5 percent of the distribution.

Finally in figure 4 we report the allocation to stocks along the life-cycle for stock market participants. The continuous line shows the empirical profile which exhibits an hump-shaped pattern. The dashed and dashed dotted lines represent the life-cycle profiles for the models with fixed transaction costs in the high and low cost scenarios. In both cases the pattern of stock shares is increasing in age in the first part of the working life. The share then declines to give rise to a hump-shaped trajectory in the low cost scenario, while it remains roughly constant in the high cost scenario. Overall the life-cycle profile for the high cost scenario follows quite closely its empirical counterpart, while the one in the low cost scenario is somewhat higher, a feature that could be already foretold from the life-cycle averages reported in table 1. One caveat is in order concerning these profiles. The empirical one is obtained as the cross-section of the observed stock share for stockholders. Estimation work conducted by Ameriks and Zeldes (2004) has shown though that the actual profile depends on

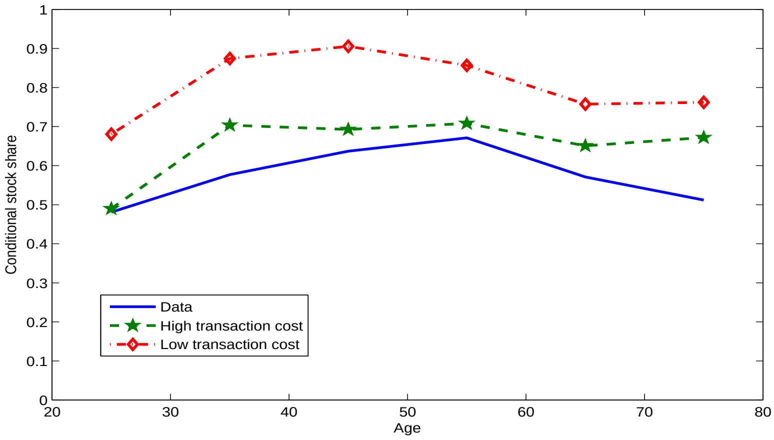

Figure 4: Life-cycle stock share for participants.

the underlying identifying assumptions.[18] The profile can be either increasing or mildly hump-shaped depending on the identifying assumption. In light of this observation it is clear that while models without transaction costs gives rise to counterfactual results, since the higher share invested by young households is not observed in the data regardless of the controls in the estimation, the model with transaction costs can match the mildly increasing profile during working age that is observed in the data, again independently of the identifying assumptions.

4.2.2 The size of the transaction cost

In this section we discuss the size of the transaction cost and compare it with similar empirical figures and with other calibrated models in the literature. In the low cost scenario, in order to obtain the frequency of transactions reported

[18]The issue arises because age, time and cohort are linearly dependent so that when constructing age profiles it is impossible to simultaneously identify time and cohort effects.

in the ICI survey we set the fixed cost at a level of 0.6 percent of the household's non-financial income. The simulated data show that in this scenario households perform on average one transaction every 2.15 years, so that the transaction cost paid per year of participation in the stock market turns out to be 0.31 percent of average income. In the high cost scenario, in order to match the frequency of transactions observed in the PSID and reported in Bilias et al. (2010) we need to set the cost to 7 percent of the household's non financial income. However, in this case the frequency of transactions in the simulated data drops to one every 7.1 years of participation in the stock market. Consequently the transaction cost per year of participation is in fact only 1.33 percent of average income. For comparison we may look at the empirical estimates presented in Vissing-Jørgensen (2002). Those estimates refer indeed to participation costs, however we believe the comparison is still useful to get a sense of whether our values are in the correct order of magnitude. Using the 1989 and 1994 waves of the PSID Vissing-Jørgensen (2002) estimates that a per period participation cost of 260 dollars (in 2000 prices) is needed to rationalize the non-participation of three quarters of the households. Compared to average earnings of about 40000 dollars this is about 0.6 percent.[19] As we can see the per period amount of the transaction cost paid in the low cost scenario is below the figure reported by Vissing-Jørensen. In the high cost scenario it is somewhat larger but still in the same order of magnitude.

In the macro literature, Khan and Thomas (2011) consider a model with two assets, money and bonds, to study the impact of monetary shocks. They calibrate the cost of making a transaction between the two accounts to match

[19] The source for this figure is Díaz-Giménez et al. (1997)

aggregate money velocity. In order to do that they assume a uniform distribution of the cost with support between 0 and 25 percent of the average endowment. Our values, as a percentage of income, fall well within that range.

Overall we believe it is reassuring that our model matches the observed frequency of transaction with a cost that is comparable both with the values found in the empirical literature and with those used in quantitative models that follow similar calibration procedures.

4.2.3 Utility cost of transactions

In the baseline model the cost of making a transaction was defined as a monetary cost that is subtracted from the available resources in the household's budget constraint. This cost was meant to represent both monetary costs and time and psychological costs to make financial decisions. To check the robustness of our results, we now simulate an alternative version of the model where the cost is modeled as a utility penalty that is suffered by the agent only in case she performs a transaction. The penalty enters as a multiplicative factor in the utility function and its size is chosen so that in the simulated data the fraction of households that make a transaction is consistent with the empirical evidence. Results are reported in tables 4 and 5. What can be seen from the two tables is that results are similar to the ones of the previous section. The first row of table 4 shows that in the low cost scenario the relationship between wealth and the conditional stock share describes an inverted U. The second row shows that the relationship in the high cost scenario is increasing over the first three quartiles of the wealth distribution and then flattens as we move from the third to the top quartile.

Table 4: Conditional shares by wealth percentiles (Utility cost of transaction)

	0-25	25-50	50-75	75-100	Top 5%
Low cost	35.1	80.7	93.8	81.5	68.6
High cost	57.2	64.4	74.4	74.3	67.4

Table 5 reports the share invested in stocks by wealth for different age groups. The top panel shows that in the low cost scenario the relationship is increasing in the first two age groups and has the usual inverted U pattern for the remaining age groups. In the high cost scenario, portrayed in the bottom panel, we can see more complicated patterns of stock shares. Within the 60-70 and 70-80 age groups we can observe the familiar inverted U relationship between the stock share and wealth. In the 40-50 and 50-60 age groups the stock share is broadly increasing in the sense that the fraction of the portfolio invested in stocks in the top quartile and in the top 5 percent of the distribution is larger than the one in the bottom two quartiles. The profile though is non-monotone. The stock share is instead increasing over the entire wealth distribution in the 30 to 40 year group.

Finally in figure 5 we report the life-cycle profile of the stock share for stock market participants when the cost of transaction is modeled as a utility cost. The dashed line represents the low cost scenario and displays a hump-shaped profile as the one in the data although the peak of the stock share is reached earlier in life. Moreover like in the baseline model the overall share is quite higher than in the data. The continuous line represents the life-cycle profile of the stock share in the high cost scenario. In this case the profile is roughly constant during working life and then modestly declines afterwards.

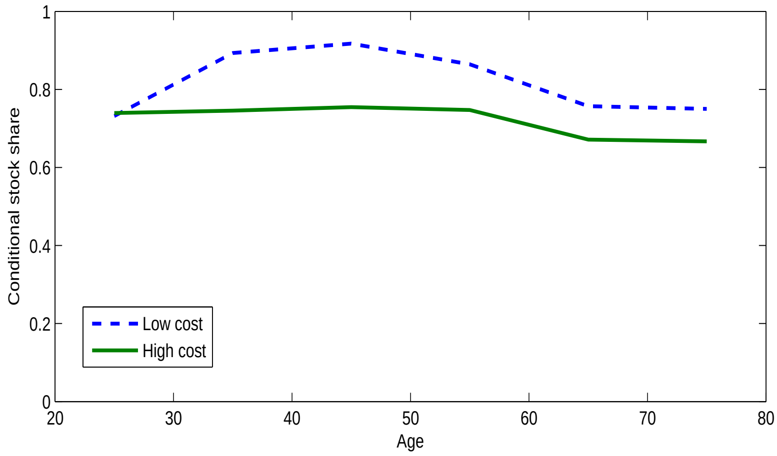

Figure 5: Life-cycle stock share for participants.

Aggregate statistics for the two choices of parameters are the following. The participation rate is 73.9 percent in the low cost scenario and is 56.9 percent in the high cost scenario. The share of wealth invested in stocks by households who hold stocks is 85.0 percent in the low cost scenario and is 73.1 percent in the high cost scenario. Both values are very similar to the ones of the model with monetary cost of transactions.

Overall this allows us to conclude that the choice about how to model the cost of making a transaction does not have an important influence on the model results once the cost is sized so that the model reproduces the empirical infrequency of the households' stock market transactions. Given the difficulty of providing a monetary equivalent to the non monetary component of the cost we see the consistency of results across the two alternative formulations as an important sign of the robustness of our findings.

Table 5: Conditional shares by wealth percentiles and age (Utility cost of transaction)

	Quart. I	Quart. II	Quart. III	Quart. IV	Top 5%
Low cost					
20-30	n.a	48.7	54.2	89.7	92.1
30-40	26.2	78.3	94.9	95.4	95.7
40-50	78.2	93.0	95.9	92.2	88.7
50-60	85.6	95.0	91.5	79.7	74.7
60-70	85.5	91.9	77.8	61.9	57.9
70-80	52.9	86.5	86.4	69.6	61.3
High cost					
20-30	n.a	n.a	n.a.	73.5	62.3
30-40	n.a.	50.5	65.9	81.7	87.4
40-50	68.4	65.7	81.4	78.5	74.9
50-60	59.6	76.3	81.2	74.5	77.2
60-70	68.1	73.4	72.8	62.1	57.5
70-80	61.9	70.3	71.4	66.5	61.9

4.3 Sensitivity Analysis

In this subsection we will perform some sensitivity analysis on a few parameters. We take as a reference point the baseline model with the transaction cost formulated in monetary terms and the high cost scenario. We select the high cost scenario because it generates a participation rate that is very close to the one in the data, while the low cost scenario generates participation rates that are more than 20 percentage points above the empirical counterpart. Moreover the average portfolio allocation to stocks in the population of stockholders is also closer to the data. We first re-simulate the baseline scenario assuming that agents start life with positive instead of zero wealth. Then using this as a starting point we consider three more extensions, one where the correlation between the earnings shock and the stock return shock is positive, one where we admit a disastrous labor income shock and finally one where there is a separate dividend and capital appreciation component to the stock return. We also report the results of the simulation of the version of the model where fixed participation costs are active. We consider them both in isolation and jointly with the transaction cost. The latter case corresponds to the full model. Finally we briefly report the wealth-to-nonfinancial income ratios at key percentiles of the wealth distribution.

4.3.1 Non zero initial wealth

It is customary in the literature to assume that agents start life with zero wealth. In the above simulations we conformed to that tradition. However in the high transaction cost case this leads to extremely low participation rates in the first decade of life since most households will not have accumulated enough wealth to justify paying this cost. Since this is not the case in the data, we try to amend the model, although in a crude way. In this section we report results for the model when we assume that agents start life with some positive wealth. Initial wealth is calibrated so that its ratio with earnings is in line with the same ratio that is observed in the data for the 20 to 30 years of age group. This initial wealth distribution is assumed to be log-normal with standard deviation also equal to its empirical counterpart in the youngest age group. For simplicity we assume that the agent starts out life with all its wealth in the liquid asset.

The average participation rate and conditional share are 53.5 percent and 69.7 percent, almost identical to the 51.6 and 69.4 percent found in the baseline model where all households start life with zero initial wealth. Conditional shares by wealth percentiles are reported in table 6. As we can see in the second line, the model still generates an increasing profile of the allocation of wealth to stocks over all of the wealth quartiles followed by a small decline for the top 5 percent of the distribution. The values are also very close to the ones in the baseline model except for an increase in the share held by the bottom quartile, which is 51.0 percent in the experiment with non zero-initial wealth. Stock shares by wealth conditional on age are reported in the top panel of table 7. The patterns of stock shares are also consistent with the baseline model. The stock share is increasing when moving from the bottom to the top quartile of the wealth distribution within each age group. Within this broad pattern we can see that for the two youngest groups the share is monotonically increasing while for the remaining age groups the pattern is either hump-shaped or it has other spells of non-monotonicity within the general increasing pattern. Finally the life-cycle profile of stock shares for market participants is reported in figure

6. The figure reports the baseline case as a continuous line while the case with non zero initial wealth is represented by the dashed line. Two remarks can be done. First the life-cycle profile of stock shares is virtually flat in the model with positive initial wealth. Second this profile exceeds by a few percentage points and then coincides with the baseline case starting from the second decade of life. This is not surprising since in the model many agents are liquidity constrained early in life, hence they will quickly consume their initial endowment of wealth. After a decade of life the pattern of wealth accumulation reverts back to what it would have been without the initial endowment of wealth. This is reflected in the coincidence of the two life-cycle profiles.

One caveat must be made at this point. With fixed transaction costs the choice to assume that all initial wealth is liquid in not neutral. The fixed transaction cost in fact introduces substantial inertia in the portfolio allocation. For this reason we simulated the model by also assuming that a fraction of 20 year old agents receive the wealth transfer in the form of stocks and calibrated this case to match the participation rate and the stock share observed in the data for the 20 to 30 year old group. In this case the conditional share would converge from the initial condition to the one in the baseline model in two decades. We decided not to report this case to economize on space.

4.3.2 Positive correlation between earnings and stock return

In this paragraph we present results for a version of the model where we assume that the shock to earnings shows positive contemporaneous correlation with the stock return. We set the value of the correlation to 0.15, the number estimated and used by Campbell et al. (2001). As expected the fraction of the portfolio

Table 6: Conditional shares by wealth percentiles (Sensitivity analysis)

	0-25	25-50	50-75	75-100	Top 5%
Baseline	44.1	63.1	68.3	71.3	67.0
Non zero initial wealth	51.0	62.0	69.5	71.1	66.2
Positive correlation	58.4	57.2	69.7	65.9	62.0
Disastrous earnings	58.4	58.9	69.0	71.5	66.2
Dividend	65.3	60.5	71.1	69.1	62.2

allocated to stock in the population of stock holders goes down, falling from 69.7 percent to 66.1 percent. The behavior of the stock shares by wealth levels is reported in table 6 and that by wealth levels within 10 years age cells in the second panel of table 7. Table 6 shows that under the assumption of positive correlation between stock returns and labor earnings shocks, the pattern of stock shares over the wealth distribution becomes more markedly hump-shaped, with an increase in the share from 58.4 percent to 69.7 percent between the bottom and the second quartile followed by a decline to 62 percent for the top 5 percent of the distribution. When we look at table 7 we also see a more markedly inverted U shaped behavior of conditional stock shares, which in this case can be found for all age groups except the youngest one. The dashed dot line in figure 6 reports the life-cycle profile of the conditional stock share in this case. This profile is virtually flat and except for the first decade of life it lies slightly below the one for the baseline case as one would expect given that the positive correlation between earnings and the stock return reduces the benefit of stock investing.

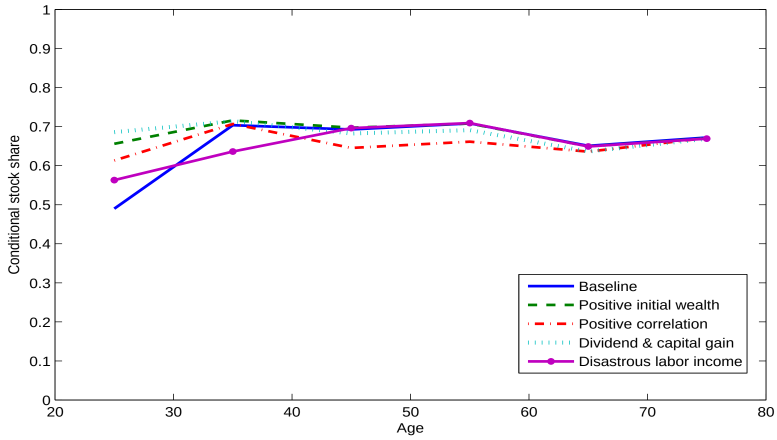

Figure 6: Life-cycle stock share for participants: sensitivity analysis

4.3.3 Disastrous labor earnings shock

Following Cocco et al. (2005) we now consider the possibility of a disastrous earnings realization. We assume that during working life the household receives with a 0.5 percent probability a one period near zero earnings shock. This amounts to a substantial increase in background risk. As a consequence of higher wealth accumulation, the participation rate increases from 53.3 to 64.3 percent. At the same time the higher background risk leads to a decline in the conditional share from 69.7 percent to 67.7 percent. The behavior of the conditional stock shares is shown in the fourth row of table 6. As it can be seen it is moderately increasing over the four quartiles of the wealth distribution. The conditional stock share in the top 5 percent of the distribution is slightly smaller than the one of the top quartile but still larger than the one of the bottom half of the distribution. The third panel of table 7 reports stock shares by wealth conditional on age. As it can be seen the pattern is broadly constant when moving from the bottom to the top of the distribution. In the three older age groups this masks a hump shaped profile, while in the remaining age groups the profile is either irregular or U-shaped. Finally, the continuous line with a dot marker in figure 6 shows that the stock share for participants is increasing over the first three decades of life and then constant afterwards, a pattern not dissimilar from the one of the baseline case.

4.3.4 Separate dividend and capital gain

Most life-cycle asset allocation models do not make a distinction between the capital gain and the dividend component of the return on public equity and for this reason we decided to follow this tradition in the baseline model. Absent fixed transaction costs this practice is irrelevant for the results. In the presence of fixed transaction costs though this is not any more true. If stocks pay a dividend on the liquid account they become themselves a source of money that can relax the cash-in-advance constraint. This would make stocks more attractive compared to the case where all the return is in the form of price appreciation. On the other hand to the extent that the transaction cost makes transactions from the liquid to the stock account infrequent, the dividend might sit for several periods in the liquid account yielding a lower return than the one it would earn on average in the stock market. This in turn would make stock less attractive. To check the implications of a separate dividend and capital gain component of the return we now assume that stock holdings pay a constant 2 percent dividend and that price growth is stochastic and averages 4 percent. The standard deviation of the price appreciation is set at 18 percent. This calibration is meant to leave both the expected return and its volatility unchanged from the baseline model. This choice overestimates the contribution of the dividend to the overall stock return. For example, Dammon, Spatt and Zhang (2004) use a 2 percent nominal dividend yield in a model where the capital gain is set at 9 percent. The average participation rate is in this case 56.7 percent and the average share of the portfolio allocated to stocks by stock market participants is 68.1 percent, both values are very close to the ones in the baseline model and indeed the stock share is within one percentage point from the baseline. Results for the pattern of stock shares by wealth are reported in table 6. The pattern is now somewhat less increasing than in the baseline parametrization. While it is still true that the share increases between the bottom and top quartile of the distribution, the pattern is non monotone with the share held by the bottom quartile being larger than the one of the next poorest quartile. The bottom panel of table 7 shows that when we condition on age the relationship between wealth and the stock share is similar to the baseline case: it is increasing for the 30 to 40 age group and hump shaped for the older age groups. The dotted line in figure 6 reports the conditional stock share by age. As it can be seen the line is virtually constant and overlapping with the one corresponding to the model with non zero initial wealth. Summarizing we can say that the two potential effects of separating the dividend yield from the capital gain component of the return offset each other leaving investors' behavior almost unchanged even if the assumed dividend yield somewhat overstates the empirical one.[20]

[20]We also simulated the model under the assumption of a 1 percent dividend yield. This assumption would make the contribution of the dividend yield to the total return on stocks equal to the one in Dammon et al. (2004). Results were even closer to the ones of the baseline parametrization. For this reason we do not report them in the paper.

Table 7: Conditional shares by wealth percentiles and age (Sensitivity analysis)

	Quart. I	Quart. II	Quart. III	Quart. IV	Top 5%
Non-zero initial wealth					
20-30	n.a.	n.a.	62.5	63.9	71.6
30-40	44.8	54.2	68.9	72.0	77.2
40-50	61.0	57.1	76.6	72.3	69.7
50-60	58.4	68.9	77.8	70.2	72.2
60-70	63.6	69.4	71.5	63.5	62.0
70-80	43.4	71.1	71.4	67.8	58.1
Positive correlation					
20-30	n.a	n.a	62.2	62.0	68.9
30-40	n.a	70.3	73.9	68.4	64.5
40-50	51.7	51.5	74.7	57.8	56.4
50-60	53.4	65.5	74.9	62.5	62.3
60-70	64.3	70.2	70.3	63.1	59.7
70-80	59.9	72.6	73.1	67.3	61.5
Disastrous earnings shock					
20-30	65.8	63.7	44.7	54.9	70.4
30-40	75.7	44.3	56.9	73.3	76.0
40-50	69.8	59.5	76.1	71.6	68.6
50-60	66.9	69.2	78.2	70.2	70.9
60-70	64.2	69.7	72.0	64.1	62.6
70-80	61.1	69.5	71.9	68.5	58.7
Dividend					
20-30	n.a	76.1	44.2	69.3	73.9
30-40	51.1	68.9	74.3	72.1	74.5
40-50	63.8	56.1	74.9	69.1	62.5
50-60	56.7	69.1	75.8	65.3	62.6
60-70	63.1	70.4	70.4	61.8	59.3
70-80	59.6	71.0	73.9	67.5	60.6

4.3.5 The model with participation costs

Models that rely on fixed costs to trade in the stock market may use the alternative formulation of fixed per-period participation costs. In this section we will report the results of a version of the baseline model where we switch off the fixed transaction cost and allow the cost to take the form of a participation cost. We will also simulate the general model that allows for both the transaction cost and a participation cost at the same time. In both cases the participation cost is set at 0.6 percent of annual non-financial income. Results are presented in tables 8 and 9 and in figure 7.

Starting with the model featuring the participation cost only we notice in the first row of table 8 that the portfolio stock share for market participants is declining in wealth from 100 percent in the bottom quartile of the wealth distribution down to 68.1 percent at the top 5 percent. The top panel of table 9 shows that the same relationship between wealth and the stock share occurs within each age group. When we look at life-cycle profiles in figure 7 we notice that in the model with participation costs the share starts near 100 percent in the first decade of working life and then declines until retirement age. After retirement age it increases slightly. In the data, depending on the estimation procedure the life-cycle profile is either hump-shaped or mildly increasing. Hence the model with participation costs generates counterfactual results in all dimensions. [21]

We next move to the model with both costs. Following the other sensitivity analysis we examine the high cost scenario for the transaction cost. Compared with the baseline model, the average participation rate declines from 53.5 per-

[21] Similar results are obtained in the absence of any cost or have been obtained in the literature with one-time entry costs.

Table 8: Conditional shares by wealth percentiles: Participation cost

	0-25	25-50	50-75	75-100	Top 5%
Participation cost	100.0	99.9	98.5	82.5	68.1
Both costs	58.1	65.9	73.3	71.5	66.5

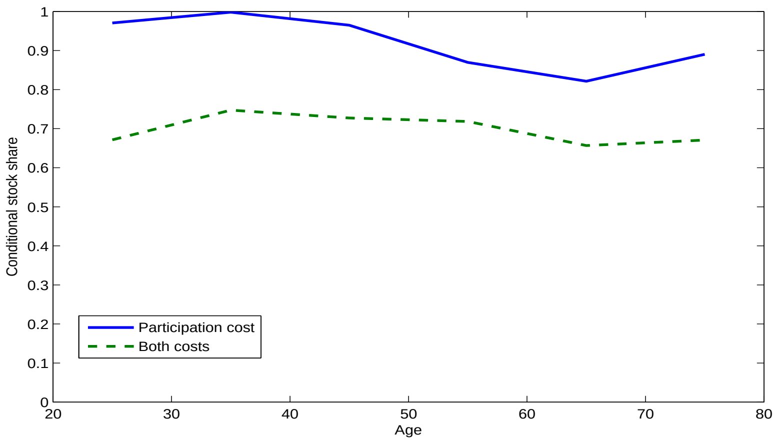

Figure 7: Life-cycle stock share for participants.

cent to 51.8 percent. The decline is minor and reflects the small size of the participation cost compared to the transaction cost. The average share invested in stocks by households who do hold stocks slightly increases to 71.7 percent. The relationship between the conditional share and wealth is reported in table 8. The conditional share starts at 58.1 percent for the bottom quartile of the distribution, increases to 73.3 percent for the third quartile and then tapers off at the top quartile. It also shows a modest decline to 66.5 percent for the top 5 percent of the distribution. Once we condition on age, the share invested in stock as a function of wealth shows the usual co-existence of moderately increasing or hump-shaped profiles found in the previous cases. It is increasing for the

Table 9: Conditional shares by wealth percentiles and age (Sensitivity analysis)

	Quart. I	Quart. II	Quart. III	Quart. IV	Top 5%
Participation cost					
20-30	n.a	n.a	100.0	100.0	100.0
30-40	n.a	100.0	100.0	100.0	100.0
40-50	100.0	100.0	99.9	94.3	87.9
50-60	100.0	99.7	89.1	70.4	68.0
60-70	99.8	93.1	79.1	66.4	64.0
70-80	100.0	99.9	94.4	80.5	73.6
Both costs					
20-30	n.a	n.a	60.8	66.9	69.2
30-40	n.a	55.4	75.3	74.3	76.2
40-50	56.9	65.9	78.2	74.1	72.1
50-60	64.5	72.5	78.1	71.0	74.3
60-70	68.6	71.3	71.2	63.0	60.9
70-80	53.8	72.3	72.4	68.6	59.5

two youngest group. It increases over the first three wealth groups and then tapers off with further increases in wealth for the middle aged groups and it is hump-shaped for the oldest age groups. The life-cycle profile of the conditional stock share is reported in figure 7. As the dashed line shows the pattern is very similar to the baseline case except for the first decade of life. The share is virtually constant across the life-cycle with only a minor increase between the first and second decade of life and a minor decline in the retirement decade of life.

Overall these results show two things. First the model with fixed participation costs produces counterfactual results both along the life-cycle and the wealth dimension. Analyzing a model with a more complex structure of trading

Table 10: Wealth-to-nonfinancial income at key percentiles

Percentile	25	50	75	95
Data (Financial wealth)	0.07	0.45	1.51	5.38
Model	0.08	0.82	2.62	5.59

costs that features both participation and transaction costs does not lead to a fundamental change in the results we obtain when only transaction costs are considered. This justifies the choice to focus on the latter case for most of the analysis and highlights the key contribution that fixed transaction costs can make to understand households' portfolio choices.

4.3.6 Wealth distribution

In this section we report the wealth-to-nonfinancial income ratios at the percentiles of the wealth distribution that we have used to describe the patterns of conditional stock shares. Since an important focus of the present paper is to show that the introduction of fixed transaction costs improves the ability of the model to explain the relationship between stock shares and wealth, it is important that the wealth accumulation pattern generated by the model is broadly consistent with the one that is found in actual data.

We perform the comparison using the baseline model simulated assuming that agents start life with positive wealth calibrated to match the first two moments of the empirical wealth distribution among the youngest age group. Results are reported in table 10. The empirical figures are constructed using financial wealth as a measure of the household's wealth. Since in the present paper there is no explicit housing wealth and housing expenditures reduce the amount of income available for consumption and savings, this is the more appropriate data counterpart to the model. As we can see the model provides a good approximation to the data. The wealth-income ratio at the 25th and 95th percentile of the distribution virtually coincides in the model and in the data. The model still provides a reasonable approximation at the other two percentiles although both at the 75th percentile and especially at the median it somewhat overestimates the wealth-income ratio that we find in the data.

5 Summary and Conclusions

In the current paper we have constructed a life-cycle portfolio choice model with uninsurable labor earnings risk. There is by now an important literature in this area. The current paper departs from that literature in that it re-interprets the risk-free asset as a liquid financial asset. Consumption can be purchased only with the liquid asset and a cash-in-advance constraint that can be relaxed by paying a fixed-cost to make transactions between the stock and the liquid account is assumed.

The assumptions made allow it to produce some improvements over conventional models that assume entry or participation costs. These improvements are in the area of the allocation to stocks over age and wealth for households who participate in equity markets. These results are obtained using parameterizations that are also consistent with empirical population averages. In particular the model generates average participation rates and conditional stock shares that are only a few percentage points above their data counterpart. This is not surprising though. The model abstracts from certain sources of background risk like marital and health risk that would help reduce the share allocated to stocks.[22]

Integrating these features into the model represents an interesting and promising avenue for future research. Another interesting development of the current model would be represented by a model with a more complete menu of financial assets including bonds beside the liquid account and stocks. Such an extension even though interesting would definitely be very challenging numerically.

[22]The impact of demographic shocks on asset allocation decisions has been studied by Love (2009).

Appendix A. Numerical solution.

In this Appendix we describe the numerical solution method. Since the simulation is standard we will focus our attention on the dynamic programming problem where the fixed transaction cost introduces certain complications that make the numerical solution harder and more time consuming. The household's maximization problem can be described by a finite horizon dynamic program that can be solved by the well known backward iteration method. The assumption of a fixed transaction cost in the stock account introduces two complications that must be addressed. The first one is the need to keep track separately of the amount of the two assets held. The second one is that a fixed transaction cost makes the value function non concave thus making fast optimization algorithms — like Newton methods — unsuitable. This latter point was made in a two period model by Corbae (1993).

To solve the dynamic programming problem we first make a variable transformation. To introduce this transformation, let first denote with $\bar{s}$ and $\bar{m}$ the upper bound of the stock and bond interval where the numerical value function is defined. With no borrowing and no short sale constraint the value function will thus be defined over the set $[0, \bar{s}] \times [0, \bar{m}]$. Let $M = \{m_1, m_2, ..., m_n\}$ and $S = \{s_1, s_2, ..., s_m\}$ be the grid points for the liquid and illiquid asset respectively with $m_n = \bar{m}$ and $s_m = \bar{s}$. At each iteration on the value function the problem defined by the RHS of the Bellman equation must be solved for all pairs (s_i, m_j) with $s_i \in S$ and $m_j \in M$. Clearly for $s_i = \bar{s}$ and $m_j = \bar{m}$ or pair of sufficiently high values for the two state variables the constraint set includes choices for m^o and s^o that far exceed $\bar{m}$ and $\bar{s}$ imposing the evaluation of the value function by extrapolation far from the rectangle where it is defined.

This may introduce severe approximation errors and for this reason we decided to redefine the value function over an alternative but equivalent set of state variables that include the current wealth W and the current share invested in stock that we denote with α. Under no borrowing and no short sale constraints α is bounded between 0 and 1 and no extrapolation of the value function is ever needed. Clearly extrapolation may be needed along the wealth dimension but in a way that is no different than in the standard consumption-saving model and hence to a much more limited degree than without the variable transformation.

To address the second problem, that is, the non concavity of the value function, we decided to use a two step direct search method. In the first step we define an action grid that is denser than the state space grid and search for the maximum over the action grid. That is, if $\alpha \in [0, 1]$ takes values $\{\alpha_1, \alpha_2, ..., \alpha_n\}$ in the state space grid, for each interval $[\alpha_i, \alpha_{i+1}]$ we lay $n_\alpha - 1$ equally spaced points. Similarly, given $W \in [0, \bar{W}]$ for each interval defined by two adjacent points in the state space for wealth $[W_i, W_{i+1}]$ we lay $m_w - 1$ equally spaced points. The maximization of the RHS of the Bellman equation then is performed by directly searching over the whole set of $((n-1) * n_\alpha + 1) \times ((m-1) * m_w + 1)$ points defined by the finer grid and not just on the $n*m$ points of the state space grid. This gives a first approximation to the solution, say the pair (α_{i^*}, w_{j^*}). Next we refine the solution along the wealth dimension. We fix α_{i^*}, consider the two-sided interval around w_{j^*}, that is the interval $[w_{j^*-1}, w_{j^*+1}]$ and lay n_* points between the two extremes.[23] We then search over this new grid for the maximum. If we call this new maximum w_{j^*,j^*}, the solution to the maximization

[23]Clearly when w_{j^*} falls at the edge of the domain of the numerical value function along the wealth dimension the interval around w_{j^*} will be one-sided.

problem will be the pair $(\alpha_{i^*}, w_{j^*,j^*})$.

The state space grid contains 251 points along the wealth dimension and 41 along the current stock share dimensions. Points along the latter dimension are equally spaced, while those along the former are set so that the grid is finer close to the origin and becomes progressively coarser. Within each interval determined by the state space points we set 4 points along the conditional share dimension and 2 along the wealth dimension, that is, each interval is divided into 5 and 3 subintervals respectively. In the refined search along the wealth dimension we use 200 points in each $[w_{j^*-1}, w_{j^*+1}]$ subinterval. In order to evaluate the value function at points in the choice space that do not coincide with points in the state space we interpolate by using bi-cubic spline approximation of the value function. The chosen grid search optimization methods makes the problem effectively discrete. Using a choice space that is finer than the state space and the two step search allow us to reduce the number of function evaluations while ensuring more accuracy in the solutions. To give an idea of the accuracy of the method, observe that the grid implies that the optimal choice of α is done over steps of 0.5 percentage points. The grid over wealth is non-uniform, hence a single number cannot be given. Around average earnings the step corresponds in economic terms to 0.02 percent of that average.[24] We tried to double the number of points along both dimension and did not find that changed the results in any significant way.

[24] In dollar terms, if we assume an average wage of between 40000$ and 50000$ this corresponds to between 15 and 20 $.

Appendix B. Data Construction.

In this appendix we briefly describe the procedure used to construct the empirical stock shares. Data come from the 2007 issue of the Survey of Consumer Finance. The Survey of Consumer Finance is a survey conducted every three years by the Board of Governors of the Federal Reserve System. It is widely used as a source for data about households' balance sheet since its non-random design offers more reliable information about the asset holdings at the top of the wealth distribution.

In order to construct portfolio stock shares for stock market participants by wealth we classify households based on quartiles of net worth. Net worth is defined as the sum of financial and non-financial assets minus all debt. Financial assets include all liquid accounts, certificates of deposits, stocks and bonds held both directly and indirectly, retirement accounts, the cash value of life-insurance and equity interest in trusts, annuities and managed investment accounts. Non financial assets include the primary residence, vehicles, investment real estate and business equity. Debt includes mortgage and home equity loans for primary residence and investment real estate, credit card balances and other loans.

The stock shares are defined as the fraction of financial wealth invested in stock. Financial wealth includes:

- checking and savings accounts, money market deposits and mutual funds, saving funds not invested in stocks, certificates of deposits, call /cash accounts.

- Bonds of all type: savings, government, tax exempt, mortgage backed, corporate and foreign bonds.

- Directly owned stocks.
- Investment funds: stock mutual funds, combination of mutual funds, other mutual funds.
- Retirement accounts: IRA and Keoghs, job based 401k accounts, thrift savings accounts.
- cash value life insurance.
- Other managed assets like personal annuities and trusts and a miscellaneous group of other financial assets.

Stocks include all stocks held directly and indirectly through mutual funds, IRAs, Keoghs and thrift type retirement accounts, or in other managed accounts like trusts and annuities. The Survey of Consumer Finance provides only a qualitative answer with regard to the fraction of a mutual fund, retirement account or managed account that is invested in stocks. We thus had to make an imputation to reach a quantitative figure. For mutual funds we imputed a fraction of 1 if the fund is defined as a stock mutual fund and of $\frac{1}{2}$ if it is a combination fund. For IRAs and Keoghs we attribute a fraction to stocks of 1 if the fund is mostly invested in stocks, of $\frac{1}{2}$ if it is split between stocks and either bonds or money market assets and of $\frac{1}{3}$ if it is invested in all the three categories of assets. For stocks in the remaining categories of funds and managed accounts we attribute the full value to stock if it is described as mostly invested in stocks and $\frac{1}{2}$ if it is described as split.

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