Sovereign Default in a Multi-Country Economy

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Abstract

This paper presents a model of endogenous sovereign default in a multi-country economy, where international investors' risk aversion affects the dynamic interactions between country-specific fundamentals, default risk, and bond prices. Risk averse investors perceive government bonds as imperfect substitutes, so that they seek to diversify their sovereign debt portfolios according to a CES demand function. The model suits extensive empirical evidence on the euro-area sovereign debt market, pointing at investors' sentiments as a major determinant of the shift in yield dynamics occurred in the wake of the euro sovereign debt crisis. In a quantitative assessment, the model is calibrated to Greece, and simulations generate dynamics of government bond yields and macroeconomic fundamentals consistent with empirical evidence on the pre-crisis and the crisis periods.

Keywords: endogenous sovereign default, multi-country economy, imperfect substitution, government bond yields, DSGE model, euro-area sovereign debt crisis
JEL Classification: E21, E43, F34, F44

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1 Introduction

Since the inception of the euro sovereign debt crisis in late 2009, soaring government debt levels, as well as increasing volatility and widening cross-country differentials of government bond yields, have raised relevant concerns about the possibility for sovereigns across the euro area to roll over their debts and avoid default. Sovereign default, a phenomenon previously thought as mostly confined to emerging countries, has become a concrete risk in one of the most developed economic regions as well. When hefty haircuts on Greek government debt were imposed on international investors in early 2012, sovereign default in developed economies left the domain of speculation to become a real event.[1]

This paper is motivated by this observation and its main objective is to lay out a theoretical framework aimed at understanding and quantifying the empirical regularities observed in the recent euro-area sovereign debt crisis. This analysis is useful to assess the suitability of policies that seek to moderate financial turmoil by stabilizing the volatility and cross-country dispersion of government bond yields in a large financially integrated economic region.

Empirical evidence on the dynamics of euro-area government bond yields has reached a general consensus on two clear-cut facts. First, macroeconomic fundamentals account for a large share of the dynamics of sovereign debt yields. More specifically, a deterioration in a country's macroeconomic conditions (e.g. worsening budget imbalances and trade deficits) increases its sovereign default risk and leads to a rise in its government bond yields.

Second, international investors' sentiments (i) are a major driver of sovereign debt yields, by affecting the pricing paradigm applied to country-specific characteristics; and (ii) vary over time. In periods of extreme financial distress, risk averse investors tend to react more strongly to movements in variables proxying default risk, implying a lower financial integration in the aggregate sovereign debt market and a higher sensitivity of government bond yields to changes in country-specific fundamentals. In this sense, a rising risk aversion can explain the increasing volatility and the widening differentials of sovereign yields during the recent euro-area sovereign debt crisis. In particular, two features of investors' risk aversion have been emphasized: (i) an international common risk factor, driving the volatility of interest rates, and (ii) a perceived imperfect substitutability between bonds issued by different sovereigns, affecting the cross-country convergence of yields.

[1] Zettelmeyer, Trebesch and Gulati (2013) provide a detailed account of the Greek sovereign default, gone down to history as the largest debt restructuring ever.

To account for these empirical regularities, this paper describes a general equilibrium model with endogenous sovereign default for a multi-country economy populated by risk averse international lenders. In this theoretical framework, there are three key modifications to the standard specification of sovereign default models. First, an infinite number of countries interacts on an international market for government bonds in a large economic region (or *union*). The extension of the standard single-country model to a multi-country framework is essential in order to describe a union of financially integrated economies such as the European monetary union. Second, international investors' preferences are explicitly modeled and affect the domestic sovereign's debt and default policies. Also, this modification implies that countries interact in the union-wide market for sovereign debt, thus creating a large economic region similar to the euro area, where the foremost participants in the international government bond market are financial institutions of countries belonging to the monetary union itself. Third, investors are risk averse along the intratemporal dimension: since bonds issued by different governments are perceived as imperfect substitutes, lenders seek to optimally diversify their sovereign debt portfolios.

As emerges from the model, in every country the domestic sovereign's decision to repay its debt depends on the state of the whole economic region, thus determining interesting dynamics for sovereign debt yields. In fact, the probability of the domestic sovereign defaulting next period increases with high levels of outstanding debt (i.e. low government bond prices), and low productivity shocks this period. In turn, the level of domestic outstanding sovereign debt is related to domestic prices as well as aggregate outstanding debts and interest rates. The results show that, with different calibrations of the model parameters, a higher degree of intratemporal risk aversion determines larger volatility of government bond yields, due to different sensitivities in movements of country-specific fundamentals. Finally, I interpret these results as promising in sight of an empirical assessment of the model.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 discusses some stylized empirical regularities on government bonds as imperfect substitutes and interprets them as a proxy for perceived financial integration in the euro area. Section 4 presents and analyses the theoretical features of the model. Section 5 contains a quantitative assessment of the model. Section 6 concludes.

2 Related Literature

This paper is directly related to two different strands of the recent research in international macroeconomics and finance: the literature on (i) endogenous sovereign default and international lending and (ii) the determinants of government bond yields in the euro area.

As regards the first strand of related research, this paper contributes to the literature on sovereign default and international lending, and extends the theoretical framework first described in the seminal works by Eaton and Gersovitz (1981) and, more recently, Aguiar and Gopinath (2006) and Arellano (2008). As in the standard framework, my model studies the interactions between sovereign default risk, output and foreign debt in a small open economy, where a benevolent government borrows from risk-neutral international lenders in order to smooth the domestic representative household's consumption. By reputational arguments, these models assume that the fear of losing access to international capital markets is a sufficient incentive for sovereigns to repay their foreign debt. Further, notice that default triggers an exogenous decline in output, which shrinks by a constant fraction. A positive feature of this class of models is their fair ability to replicate several dynamics and moments of macroeconomic variables, notably in emerging countries.[2]

However, a first limitation of this sort of models of sovereign default stems from the motivation for the existence of international lending, since debt contracts lack enforceability and governments do not feature any commitment technology. Bulow and Rogoff (1989*a*) and, more recently, Yue (2010) argue that reputational arguments are not sufficient to explain international lending to small countries, which instead depends on the direct sanctions available to creditors. Likewise, according to Schumacher, Trebesch and Enderlein (2013), recent empirical evidence on the rising costs from creditor litigation against sovereigns supports legal sanctions as an explanation for repayment of sovereign debt. From a different standpoint, Sandleris (2008) argues that neither sanctions nor reputation considerations are necessary to enforce repayment, as information revelation is the essential mechanism for repayment of non-enforceable foreign debt.

A second limitation of standard models of sovereign default lies in the assumption of exogenous output costs following the debtor country's loss of access to international capital markets. Bulow and Rogoff (1989*b*), Borensztein and Panizza (2009) and Yeyati and Panizza (2011) argue that this modeling choice is too simplistic, since costs of a sovereign default stem from the loss of reputation of the domestic government as well as the exclusion of domestic firms from international trade, the disruption of the domestic

[2] Besides the aforementioned references, see Neumeyer and Perri (2005).

financial system, the seizure by foreign creditors of domestic assets held abroad, and the political instability affecting domestic authorities.

Recent models take a step farther by incorporating endogenous output costs of default. In the works by Mendoza and Yue (2012), Engler and Große Steffen (2013), and Sosa-Padilla (2013), a sovereign default triggers an inefficient substitution of imported inputs, binding collateral constraints on the interbank market, and a contraction of credit to the corporate sector, respectively, which determine a contraction in domestic output. A crucial assumption of these models is the link between the financial and the real economy through working capital financing.

Although the model in this paper does not include endogenous output costs of default, it adds realistic features to the standard framework by incorporating risk-averse international investors and considering a multi-country integrated economy. As argued by Lizarazo (2009, 2013), increases in the level of financial wealth of risk-averse international investors, the debtor sovereign becomes less credit constrained, and subsequently the debtor's default risk is lower, whereas its bond prices and capital inflows are higher. Additionally, the risk premium in the asset prices of the sovereign country can be decomposed into two components: a base premium that compensates the investors for the probability of default (as in the risk-neutral case) and an "excess" premium that compensates them for taking the risk of default.

Further, the model herein presented suits an environment with several financially integrated borrowing sovereigns as it extends the standard framework to a multi-country economy, hence relating to a number of recent works. In the model by Lizarazo (2009), the existence of common risk-averse investors generates financial links between the emerging economies: through a wealth effect, if a negative shock forces a country to default, investors' wealth and tolerance to risk decrease and constrain the borrowing capacity of other ailing countries; through a substitution effect, a negative shock in one country forces investors to rebalance their portfolio towards countries with relatively solid fundamentals, generating a so-called "flight-to-quality" phenomenon. When debt contracts can be renegotiated, as in Arellano and Bai (2013), if one country experiences negative output shocks, the borrowing conditions for the other countries worsen as well, generating a possible contagion across countries; foreseeing these events, the common international lenders accepts higher haircuts from the first defaulting country. Contagious sovereign debt crises occur in financially integrated economies in the finite-horizon model by Bolton and Jeanne (2011): the authors show that fiscal integration should accompany financial integration to reach an efficient equilibrium supply of government debt, although it might produce relevant welfare losses for the "safe-haven" country. Finally, Guerrieri, Iacoviello and Minetti (2012) develop a two-region business cycle model calibrated on the euro-area periphery and core blocks, where contractionary shocks can generate a sovereign default and, subsequently, either a repudiation effect (which causes additional output contraction in the presence of binding capital requirements on banks) or a punishment effect (which may have opposite effects on output in the two regions of the economy).

However, to the best of my knowledge, the literature on endogenous sovereign default in a multi-country framework has never addressed the problem of an inefficient allocation of capitals due to investors' perceived diversity in the sovereign debt market.

As regards the second strand of research, the determinants of government bond yields in the euro area have been the focus of an extensive literature, especially since the start of the crisis in late 2009. According to the standard view, euro-area government bond yields should reflect liquidity and default risk. On the one hand, evidence on the role of liquidity risk is mixed. For instance, Manganelli and Wolswijk (2009) and Favero, Pagano and von Thadden (2010) find that spreads between euro-area government bond yields are related to market liquidity, cyclical conditions, and an aggregate risk factor. Yet, Favero and Missale (2012), and Oliveira, Dias Curto and Nunes (2012) observe that liquidity differentials play at most a minor role in explaining yield spreads, both before and during the financial crisis period. Therefore, this model does not explicitly account for the role of liquidity risk in the determination of bond yield dynamics.

On the other hand, a broad consensus view argues that default risk, proxied by deteriorating macroeconomic fundamentals (e.g. subdued economic activity, fiscal imbalances and current account deficits), explains a large share of the dynamics of euro-area government bond yields. On the basis of a semiparametric time-varying coefficient model and non-linear GVAR approach, respectively, Bernoth and Erdogan (2012) and Favero (2012) argue that a country's relative fiscal position has a significant impact on interest rate differentials, with the sensitivity of spreads to fundamentals decreasing until the mid-2000's and increasing in the wake of the financial crisis; among others, Sgherri and Zoli (2009) interpret this phenomenon as a clear shift in market sentiments and global investors' risk aversion. Further, as showed in the work by Oliveira, Dias Curto and Nunes (2012), since the inception of the financial crisis, a sharp rise in the sensitivity of financial markets to fundamentals has been a relevant determinant of the rise in sovereign yield spreads, and Bernoth, von Hagen and Schuknecht (2012) suggest that the cost of loose fiscal policy and deteriorating fundamentals has increased considerably. Finally, Favero and Missale (2012) and Beirne and Fratzscher (2013) observe that investors' pricing paradigm varies as their perception of fundamentals and the implied default risk changes over time.

3 Government Bonds as Imperfect Substitutes

On the basis of empirical evidence on the euro-area sovereign debt market, this paper presents a model where international investors' risk aversion affects the equilibrium dynamics of government bond yields, portfolio allocations and macroeconomic fundamentals. More specifically, creditors feature an intratemporal form of risk aversion: lenders perceive bonds issued by borrowing sovereigns as imperfect substitutes, and allocate their capitals so as to optimally diversify their portfolios (so that they face a constant elasticity of substitution between each pair of government bonds). Therefore, risk averse investors perceive investments (i.e. wealth) in different government bonds (along the intratemporal dimension) as inherently different and, thus, with a different value (i.e. a different marginal utility).[3] In this section, I seek to investigate the rationale behind the crucial assumption in this model, i.e. the perceived imperfect substitutability between government bonds, with a focus on the characteristics of the euro-area sovereign debt market.[4]

3.1 Financial Integration in the Euro Area

The perceived diversity of sovereign debt essentially depends on the presence of frictions that hinder the efficient allocation of capitals among different investment alternatives, and is hence related to the degree of financial integration of the union-wide sovereign debt market. Theoretically, the market for a given class of financial instruments in a specific

[3]As Christiano (2006) observes, "[...] in reality assets are only imperfectly substitutable. Even assets that generate the same expected return are not the same. Different assets have different risks associated with them. This is why people like to diversify their holdings. They don't want their portfolio to contain too much of one asset. To compensate them for holding a disproportionate amount of some asset, its rate of return has to be high" (pp. 2-3).

[4]As Lizarazo (2013) argues, both individual and institutional investors feature risk aversion due to several factors, which can be classified into two categories: regulations over the composition of investments portfolios and the characteristics of the institutions' management. As regards the first source of risk aversion, banks face capital adequacy ratios, mutual funds face asset leverage restrictions, and pension funds and insurance companies face strict limits on their exposure to risk. Regarding the second source, for each class of institutional investor, managers ultimately make the portfolio allocation decisions. These managers can also be treated as risk averse agents, since they might face mandates from the ultimate providers of funds and their compensation depends on the performance of the portfolio that they manage.

region can be defined as integrated if (i) all (or most) potential market participants (with the same relevant characteristics) across the region face a single set of rules and receive equal treatment in their operations on the market for such financial instruments, and (ii) there exist no (or few) transaction costs for investments on the basis of their location within the region.[5] In other words, in a financially integrated region no barriers discriminate economic agents and their investments in a particular set of financial instruments because of locational differences. This definition implies that financial instruments with identical cash flows should feature the same price, and that no systematic differences should emerge in the portfolio allocation and sources of funding of individual economic agents (after controlling for their specific characteristics) inside the region.[6]

A detailed account of the process of integration in euro-area financial markets is reported in a number of studies, and is beyond the scope of this study.[7] According to the typical narrative, during the first decade of the European monetary union, decreasing barriers to exchange have boosted the process of convergence and gradually reduced financial segmentation. The presence of a single currency, a unique monetary policy, and a union-wide wholesale payment system have been the essential conditions for the creation of deeply integrated continental financial markets. However, even during the process of convergence, financial markets have never been fully integrated, because of cross-country differences in the rules governing trades and transaction costs affecting several classes of financial instruments. Eventually, in the wake of the euro-area sovereign debt crisis, although financial markets have stabilized across the board in the euro area thanks to the interventions by the European Central Bank, conditions have remained fragile.

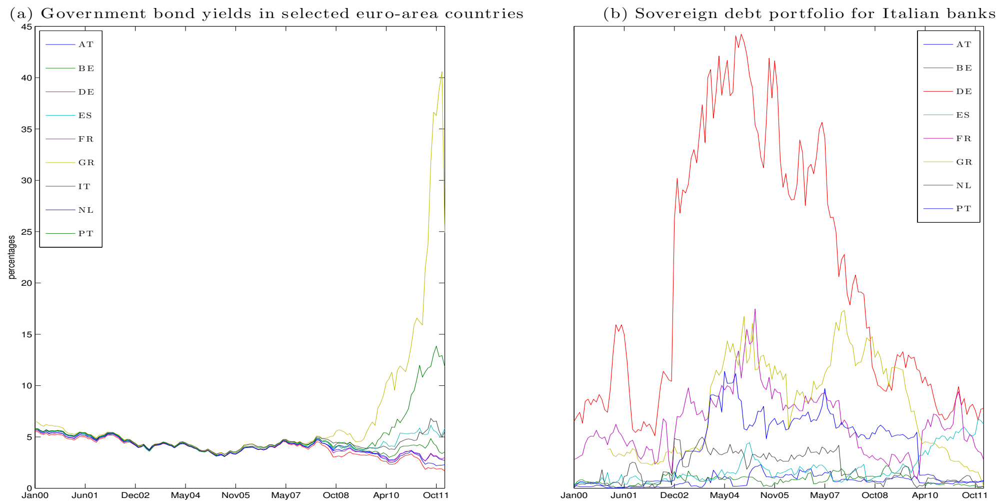
Importantly, a number of studies focuses on how convergence patterns in the euro-area sovereign debt market have affected investors' preferences. Oliveira, Dias Curto and Nunes (2012) document the increasing financial integration in the euro-area sovereign debt market in the wake of the monetary unification, signaled by the dramatic convergence of euro-area government bond yields. Hartmann, Maddaloni and Manganelli (2003) argue that such convergence occurred since investors perceived euro-area government bonds as an ever more homogeneous class of financial products; Pagano and von Thadden (2004)

[5]See Baele et al. (2004).

[6]See Hartmann, Maddaloni and Manganelli (2003).

[7]In what follows, I focus on the rationale behind the perceived diversity of government bonds for investors in the European monetary union from the observation of some stylized facts. For evidence on the process of convergence in euro-area financial markets, see the references in Section 2, as well as Adam et al. (2002), Baele et al. (2004), Baele and Inghelbrecht (2008), Abad, Chuliá and Gómez-Puig (2010), Wagenvoort, Ebner and Morgese Borys (2011), ECB (2013) and Christiansen (2014).

Figure 1: Evolution of prices and quantities in the euro-area sovereign debt market



Sources: Thomson Reuters Datastream and Banca d'Italia.
Notes: Panel (a) reports 10-year yields on general government bonds. Panel (b) reports Italian banks' holdings of general government securities other than shares (total maturity). Axis ticks for Panel (b) are not reported due to confidentiality of data.

reach the same conclusion and note that, even in the transition to monetary unification, small but sizable yield differentials suggested that euro-area bonds were still not considered as perfect substitutes, due to modest but persistent differentials in fundamental risk across countries. As reported by ECB (2013), with the start of the financial crisis, a reversal in the convergence process occurred, as country-specific risk led market participants to perceive bonds issued by different sovereigns as inherently different. Battistini, Pagano and Simonelli (2014) connect this phenomenon to the apparent increase in euro-area banks' holdings of domestic sovereign debt, and investigate the relation between the rising home bias in banks' sovereign debt portfolios and the cross-country dispersion of yield differentials.

In the context of the euro-area sovereign debt market, such trends in the process of convergence emerge from both price-based and quantity-based indicators of financial integration. Figure 1 illustrates developments in government bond yields and Italian banks' holdings of foreign sovereign debt from ianuary 2000 to December 2011. As showed in Panel (a), following the convergence process in the transition to monetary unification and the period of relative stability thereafter, the Greek budgetary crisis in October 2009 triggered a sharp rise in the volatility and dispersion of euro-area government bond yields:

these dynamics hint at a reversal in the process of financial integration due to a shift in the risk pricing of country-specific fundamentals and an increase in investors' risk aversion. Panel (b) also suggests an upward trend in financial segmentation during the recent euro-area sovereign debt crisis, as Italian banks progressively reduced their exposures to foreign sovereign debt and increased the home bias of their portfolios;[8] further, the lower volatility in holdings during the most recent period might signal an increase in barriers and transaction costs in trades of euro-area government bonds faced by Italian banks.

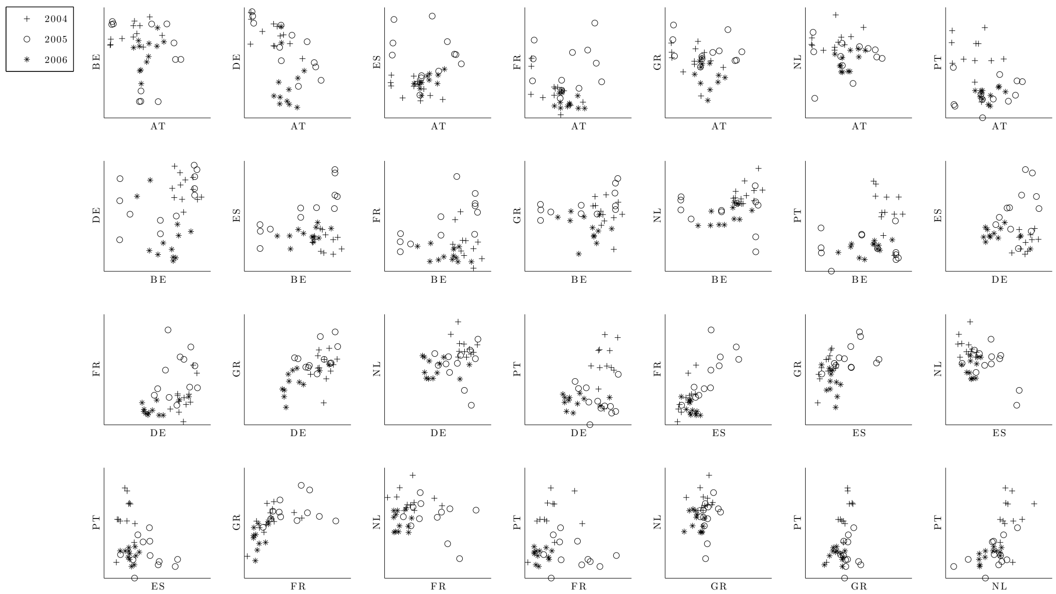
Several factors might have contributed to this persistent segmentation. In particular, trading costs arising from bid-ask spreads, brokerage commissions, or transaction fees, as well as clearing and settlement fees might have hindered the efficient allocation of funds between different government bonds; further, cross-country differences in taxation and legal arrangements (e.g. different procedures for debt restructurings in the event of a default) might have shifted investors' preferences over different sovereign debt contracts. Especially the latter determinants have likely played a maior role in informing investors' perceptions of government bonds issued by different sovereigns as imperfect substitutes.

3.2 Modeling Government Bonds as Imperfect Substitutes

A theoretical challenge lies in the choice of how to model imperfect substitution between government bonds. Considering the simple framework in which an agent with standard (i.e. increasing and concave) preferences chooses the quantities of a pair of ordinary goods in order to maximize her utility, economic theory holds that the optimal composition of the basket of goods depends on (i) the relative price and (ii) the agent's income: as the relative price of the first good (in terms of the second good) increases, the agent shifts to a basket composed of a greater (smaller) quantity of the second (first) good; as her income increases, the agent moves to a basket with a larger quantity of both goods. Then, if several changes in relative price and income solely determine a substitution effect, the sequence of equilibria (i.e. optimal allocations) would lie on a unique indifference curve. In this model, investors are assumed to have standard preferences, which generate downward-sloping and convex indifference curves, and government bonds are considered ordinary goods. Hence, investors' holdings of pairs of government bonds at different points in time would represent a sequence of optimal sovereign debt portfolios. If the income effect were small, movements

[8] Italian banks' exposures to domestic sovereign debt are not showed as they would dwarf foreign holdings of government debt. The same pattern in the relative composition of bank sovereign portfolios emerges in a sample of ten euro-area countries, as reported in Battistini, Pagano and Simonelli (2014).

Figure 2: Sovereign debt portfolio diversification in Italian banks before the crisis



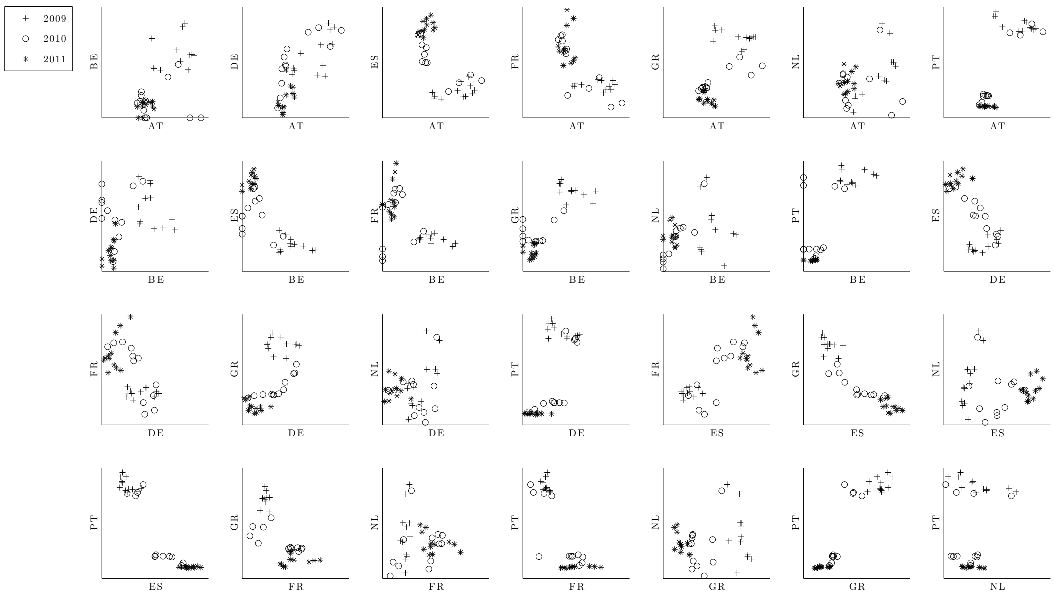
Source: Banca d'Italia.

Notes: Each data point refers to Italian banks' holdings (expressed in EUR millions) of general government securities other than shares (total maturity) issued by a single pair of countries in a given month. Axis ticks are not reported due to confidentiality of data.

in relative prices (as proxied by inverse yields) and income (as proxied by total assets) would generate a sequence of equilibria lying on the same indifference curve.

In this regard, some key stylized facts offer insights into investors' decisions over the optimal composition of their sovereign debt portfolios. Figures 2 and 3 show the composition of Italian banks' sovereign debt portfolios in the periods 2004-2006 (henceforth, pre-crisis period) and 2009-2011 (henceforth, crisis period), respectively; each panel depicts the holdings (in million euros) of government bonds issued by a pair of sovereigns (indicated on the two axes); each point corresponds to a particular month. As shown in Figure 2, no particular pattern emerges in the pre-crisis period: only the allocation of funds between Spanish and Portuguese sovereign debt seems to lie on a unique indifference curve for investors with standard preferences across the whole period (except for short periods of some months). However, as emerges from Figure 3, optimal sovereign debt portfolios seem to lie on downward-sloping and convex indifference curves for the whole crisis period: this is the case for several pairs of sovereigns, notably Austria with Spain and France, Belgium with Spain and France, Spain with Greece and Portugal, France with Greece and

Figure 3: Sovereign debt portfolio diversification in Italian banks during the crisis



Source: Banca d'Italia.
Notes: See Figure 2.

Portugal and, to a lesser extent, Germany with France. Again, the same pattern emerges for numerous sub-periods of shorter duration. The difference in the patterns emerging from the two periods is not surprising. First, in the pre-crisis period income effects might occur due to large changes in total assets, so that the equilibrium points actually lie on different indifference curves: in fact, the average (absolute) annual growth rate of total assets for Italian banks in the pre-crisis period is 7.54%, almost twice the same figure during the crisis period (4.18%), where incidentally total assets remain roughly constant (indicating the presence of positive as well as negative growth rates). Second, since a marked convexity of the indifference curves is related to a high degree of imperfect substitutability, a clearer pattern can be expected to emerge in times of greater financial turmoil such as the crisis period (relative to the pre-crisis period), when investors' risk aversion rises.[9]

Hence, assuming that investors perceive government bonds as imperfect substitutes seems an appropriate modeling choice. Finally, notice that the particular specification of investors' preferences, which feature a constant elasticity of substitution between government bonds, is preferred because it is flexible (in fact, the elasticity parameter can

[9]Notice that, *ex post*, this observation provides evidence for the shift in market sentiments reported in the literature mentioned in Section 2.

be calibrated to different conditions in market sentiments) and facilitates the aggregation in union-wide composite indexes for total sovereign debt portfolios and government bond prices necessary in the multi-country framework studied in this model.

4 The Model

Consider a large region (or *union*) composed of an infinite number of small open economies. The size of the union is normalized to unity. Every economy receives a stochastic stream of income; endowments are heterogeneous across countries. In each country, the sovereign trades one-period discount bonds with risk averse competitive international investors, in order to insure domestic households against negative productivity shocks and smooth their consumption patterns over time. Since each debt security is a non-contingent claim to one unit of output, union-wide financial markets are incomplete.

Two types of financial frictions affect the economic region's market for government bonds. First, debt contracts are not enforceable and governments lacks any commitment technology. Thus, a sovereign may default on the total amount of outstanding debt at any time. Second, government bonds represent claims on differentiated goods, so that they are perceived as imperfect substitutes. As investors are risk averse, they want to hold a positive quantity of all the available securities, and thus seek to diversify their sovereign debt portfolios, allocating their intratemporal expenditures according to downward-sloping demand functions for government bonds. Hence, risk aversion affects lenders' behavior along the intratemporal dimension, whereas along the intertemporal dimension investors are assumed to be risk neutral (i.e. there are no consumption/profit smoothing motives driving investors' optimal decisions).[10]

In this section, I first lay out the timing of events; then, I describe the optimal decisions for international lenders in the union-wide sovereign debt market and for the domestic household and government in every country; finally, I define the equilibrium of the union economy.

In order to have a clear picture of the model, it is convenient to consider the timing of events, which is illustrated in Figure 4. Importantly, notice that events simultaneously occur in every country at each stage. At the beginning of period $t \in [1, \infty)$, the state of the economy in country $i \in [0, 1]$ is characterized by the sovereign debt market conditions (i.e. bond prices and quantities) determined at $t - 1$. Following the endowment shock, country

[10] A model with intertemporal risk aversion in investors' preferences is presented by Lizarazo (2013).

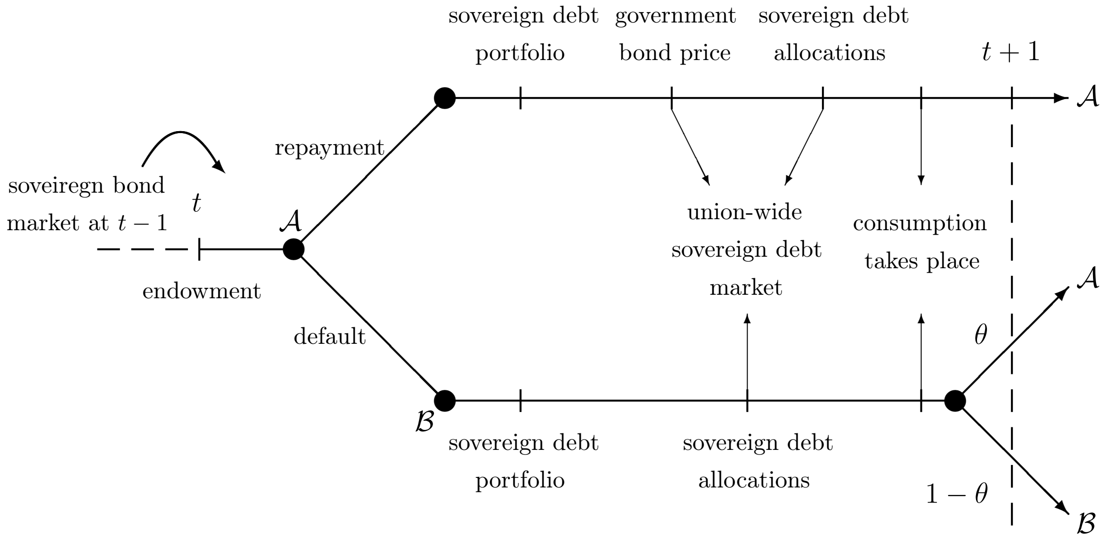


Figure 4: Timing of events in country i at period t.

i's sovereign stands at node $\mathcal{A}$ and decides whether to repay or default on its outstanding debt.[11] If the sovereign opts for repayment, the cross-border market for government bonds opens: first, international lenders chooses the optimal amount of aggregate investments in government bonds (i.e. their sovereign debt portfolios); second, taking investors' aggregate investments as given, the sovereign determines the optimal bond price; finally, a set of bilateral demand functions for bonds issued by every sovereign determines pairwise (i.e. investors-to-sovereign) allocations and clears the union-wide sovereign debt market. Subsequently, the domestic household consumes the current endowment plus net capital inflows (i.e. net proceedings of the domestic sovereign's trades in bonds with international creditors). Eventually, agents reach node $\mathcal{A}$ next period and face the same problems.

If the sovereign opts for default, then it reaches node $\mathcal{B}$ and does not participate in the union-wide sovereign debt market; hence, the trades of domestic government bonds do not occur at time t and international investors purchase bonds issued by every other sovereign. Once transfers of bonds have realized, the domestic household consumes country i's endowment (net of exogenous costs). Eventually, agents in the defaulting country face the same problems at period $t+1$ as at period t (i.e. they reach node $\mathcal{A}$) with constant

[11] As recent episodes witness, sovereign credit events usually entail defaults on both foreign and domestic debt. However, a complete interruption of financial relationships between the government and domestic investors is not frequent: domestic investors usually suffer write-offs in old contracts and underwrite new contracts with their sovereign. Such feature of the relationship between domestic investors and government often implies dramatic consequences in terms of production and welfare. In the model presented herein, output costs of default are modeled as exogenous (Arellano, 2008, Aguiar and Gopinath, 2006).

probability θ, or they return to node $\mathcal{B}$ with probability $1-\theta$.

In this sense, the union-wide sovereign debt market features a Stackelberg game between governments and investors. Due to the cross-country heterogeneity of endowments and investors' appetite for diversification, sovereigns feature market power, thus acting as *leaders* and determining the optimal prices, taking into account their effects on investors' demand for government bonds. Given government bond prices, creditors can finally choose the amount of bonds purchased from each sovereign on the basis of bilateral demand functions. In what follows, I characterize the equilibrium strategic incentives implied by the framework *à la* Stackelberg by describing the problems of international investors, the household and the sovereign government.

4.1 International Investors

At time t, international investors purchase one-period discount bonds $b^i_{t+1} \geq 0$ issued by sovereign in country $i \in [0,1]$ at price $q^i_t \in (0,1]$. In equilibrium, investors follow a two-stage budgeting approach: first, they determine the optimal intertemporal choice on the aggregate amount of government bonds; second, they choose the optimal intratemporal allocation of expenditures among different government bonds. The two stages of the investor's problem are presented proceeding backwards.

International investors perceive bonds issued by different governments as imperfect substitutes, and diversify their sovereign debt portfolios according to the following CES preferences:

$$b^*_{t+1} = \left[\left(\frac{1}{\rho^*_t} \right)^{\frac{1}{\eta}} \int_0^1 (1-d^i_t) h^i_t (b^i_{t+1})^{\frac{\eta-1}{\eta}} \, \mathrm{d}i \right]^{\frac{\eta}{\eta-1}}, \tag{1}$$

where b^*_{t+1} is an aggregate index representing creditors' sovereign debt portfolios, which combines their holdings of bonds issued by every sovereign i, whereas d^i_t and h^i_t are two indicator functions taking on value one if sovereign i defaults and has a good credit history, respectively, and zero otherwise. Formally,

$$h^i_t = \begin{cases} 1 & \text{if either (i) } d^i_{t-1} = 0 \text{ or (ii) } d^i_{t-1} = 1 \text{ with probability } \theta, \\ 0 & \text{if } d^i_{t-1} = 1 \text{ with probability } 1-\theta. \end{cases}$$

Hence, the investor accounts for the possibility of exclusion of country i's government from the union-wide sovereign debt market, which occurs if the product $(1 - d_t^i)h_t^i$ is equal to zero. Finally, $\rho_t^* \equiv \int_0^1 (1-d_t^i)h_t^i \, \mathrm{d}i$ is the tally of participating sovereigns and thus measures the size of the union's capital markets.

In order to determine their optimal allocation of expenditures, i.e. $\int_0^1 (1-d_t^i)h_t^i q_t^i b_{t+1}^i \, \mathrm{d}i$, international lenders compose their equilibrium sovereign debt portfolios according to the demand function below:

$$b_{t+1}^i = \left(\frac{q_t^i}{q_t^*}\right)^{-\eta} \frac{b_{t+1}^*}{\rho_t^*}, \tag{2}$$

where

$$q_t^* = \left[\frac{1}{\rho_t^*} \int_0^1 (1 - d_t^i)h_t^i (q_t^i)^{1-\eta} \, \mathrm{d}i\right]^{\frac{1}{1-\eta}} \tag{3}$$

is the union-wide aggregate bond price index.

Three considerations about investors' preferences are appropriate. First, any pair of bonds features the same constant elasticity of substitution $\eta > 1$. Parameter η, as a measure for the degree of substitutability between bonds, represents a gauge of international lenders' risk aversion and, thus, perceived segmentation of union-wide financial markets: the lower the η, the higher imperfect substitutability, risk aversion, and financial segmentation. Second, since income distribution problems among investors are neglected, b_{t+1}^* can be regarded (assuming the appropriate aggregation conditions to be fulfilled) as a multiple of a representative investor's portfolio. Sovereign debt diversity can then be interpreted either as different investors purchasing different bonds, or as diversification on the part of each investor. Third, since imperfect substitutability between government bonds implies a downward-sloping demand function, an increase in the price of a government bond determines only a sizable change in its demand. In contrast, a raise in the price of one of two bonds that were perfect substitutes (for instance, due to perfect correlation) would result in the demand shifting entirely to the security whose price has not changed (as the demand function for each bond would be perfectly elastic).

In order to determine their optimal intertemporal choice on the aggregate amount of bonds b_{t+1}^*, international investors solve the recursive maximization problem below:

$$\Pi(s_t^*) = \max_{b_{t+1}^*} \left\{\pi_t + \frac{1}{1+r} E_t[\Pi(s_{t+1}^*)]\right\} \tag{4}$$

where π_t denotes profits from trades in government bonds and, combining equations (1)-(3), can be written as

$$\begin{aligned}\pi_t &= \int_0^1 (1-d_t^i)[(1-d_{t-1}^i)h_{t-1}^i b_t^i - h_t^i q_t^i b_{t+1}^i]\ \mathrm{d}i \\ &= r_t^* b_t^* - q_t^* b_{t+1}^*, \end{aligned} \tag{5}$$

with $r_t^* \equiv (1/\rho_{t-1}^*) \int_0^1 (1-d_t^i)(1-d_{t-1}^i)h_{t-1}^i(q_{t-1}^i/q_{t-1}^*)^{-\eta}\ \mathrm{d}i > 1$ denoting the union-wide aggregate index for bond returns, and r the (net) risk-free interest rate. Notice that r_t^* moves inversely with government bond prices, i.e. higher bond prices determine a higher bond price index and a lower bond returns index. Further, s_t^* denotes the state of the domestic economy at the beginning of period t. Since lenders are risk neutral along the intertemporal dimension, the first-order condition for problem (4) is the following zero expected-profit condition:

$$q_t^* = \frac{E_t\, r_{t+1}^*}{1+r},$$

which equates the aggregate bond price index to the aggregate discounted expected return on union-wide government bonds.

4.2 The Domestic Economy

In country i and at period t, domestic households are identical and risk averse, and share the same preferences, given by

$$E_0 \sum_{t=0}^{\infty} \beta_i^t u_i(c_t^i), \tag{6}$$

where E_0 denotes the expectations operator, β_i the household's subjective discount factor, $u_i(\cdot)$ the strictly increasing and concave current-period utility function, and c_t^i consumption.

The government is benevolent and its objective is to maximize the utility of households. The sovereign determines its optimal policies by deciding (i) whether to repay or default on the total amount of its outstanding debt and (ii) if repayment is optimal, the bond price. Let y_t^i denote a stochastic endowment, which follows a Markov process with transition probability function $p(y_t^i|y_{t-1}^i)$, and s_t^i the state of the economy when the country i's credit history is in good standing (i.e. $(1-d_t^i)h_t^i = 1$). If country i can access union-wide capital markets, the domestic government determines the optimal bond price by solving the following problem:

$$U_i^R(s_t^i) = \max_{q_t^i}\{u_i(c_t^i) + \beta_i E_t[U_i(s_{t+1}^i)]\} \tag{7}$$

subject to the flow budget constraint

$$\begin{aligned} c_t^i &= y_t^i + q_t^i b_{t+1}^i - b_t^i \\ &= y_t^i + q_t^i \left(\frac{q_t^i}{q_t^*}\right)^{-\eta} \frac{b_{t+1}^*}{\rho_t^*} - \left(\frac{q_{t-1}^i}{q_{t-1}^*}\right)^{-\eta} \frac{b_t^*}{\rho_{t-1}^*}, \end{aligned} \tag{8}$$

where the second equality is obtained by substituting for b_{t+1}^i as given by equation (2). The government also faces a non-negativity constraint for consumption, $c_t^i \geq 0$, and two boundary conditions for the bond price, $0 < q_t^i \leq 1$. An upper bound on debt, i.e. $b_{t+1}^i \leq B^i$, prevents the government from engaging in Ponzi schemes but does not bind in equilibrium. Using equation (2), the latter condition translates into a constraint on the minimum value of the domestic bond price, i.e. $q_t^i \geq Q^i$, where Q^i is a function of the steady state values of the aggregate variables and B^i.

If sovereign i defaults on its outstanding debt and cannot access capital markets (i.e. $(1-d_t^i)h_t^i = 0$), its continuation value after default is given by:

$$U_i^D(y_t^i) = u_i(c_{Dt}^i) + \beta_i E_t\left[\theta U_i(s_{0,t+1}^i) + (1-\theta)U_i^D(y_{t+1}^i)\right], \tag{9}$$

where $c_{Dt}^i = (1-\omega_i)y_t^i$ denotes household consumption under default, $\omega_i \in (0,1)$ exogenous output costs of default, and s_{0t}^i the subset of states in s_t^i such that the domestic government's outstanding debt at period t equals zero. In this sense, the domestic sovereign starts period t fresh and need not repay any foreign creditor.

Completing the description of the model, at the beginning of time t, the domestic sovereign chooses whether to exert the default option. Hence, the sovereign solves the problem below:

$$U_i(s_t^i) = \max_{d_t^i}\{(1-d_t^i)U_i^R(s_t^i) + d_t^i U_i^D(s_{Dt}^i)\}, \tag{10}$$

which denotes the general continuation value of the household before the determination of a default policy by the government.

4.3 Default Sets, Default Incentives and Recursive Equilibrium

As emerges from the model described above, the state of international investors (s_t^*) differs from the state of the domestic economy, whether in good credit standing (s_t^i), financial autarky (y_t^i) or fresh access to capital markets after default (s_{0t}^i). Formal definitions of the states for the different agents are provided below.

Definition 1. *(i) Define the state of the union economy (i.e. the relevant state for international investors' problem* (4)*) s_t^* as the collection of the collection of every country's optimal decisions on government bond prices and default options at $t-1$, as well as credit histories at $t-1$ and t, i.e.$s_t^* = \{(q_{t-1}^i, d_{t-1}^i, h_{t-1}^i, h_t^i)_{i\in[0,1]}\}$.*

(ii) Moreover, define the state of the domestic economy in good credit standing (i.e. the relevant state for the domestic sovereign's problem (7)*) s_t^i as the collection of country i's income realization, government bond price as well as the aggregate sovereign debt portfolios, the aggregate bond price index, and the size of the union sovereign debt market, i.e. $s_t^i = \{y_t^i, q_{t-1}^i, s_t^*\}$.*

(iii) Finally, define the state of the domestic economy with access to capital markets after default (i.e. the relevant state for the domestic sovereign's problem (9)*) s_{0t}^i as the the subset of states in s_t^i such that the domestic government's share of union-wide portfolios at the beginning of period t equals zero, i.e. $s_{0t}^i = \{y_t^i, \infty, s_t^*\}$.*

From Definition 1, it is clear that the problems of international investors and domestic sovereigns ultimately depend on every country's default policies, credit histories, and bond prices and quantities. However, international investors' problem does not depend on any country-specific variable, but only on aggregate conditions; thus, aggregation significantly simplifies the analysis, as the state of the union economy actually depends on union-wide indexes only.[12] Conversely, the state of each domestic economy depends on domestic variables and accounts for the effects of every foreign country's variables through aggregate variables. As regards Definition 1*(i)*, including credit histories at $t-1$ and t is necessary because, given a default at $t-1$, a country's access to capital markets at t is the realization of a time-invariant stochastic process. Thus, the set of default options and credit histories at time $t-1$ is not sufficient to well define the state of the union economy. As regards Definition 1*(iii)*, fixing the domestic bond price equal to infinity is equivalent to setting the domestic amount of sovereign debt equal to zero.

[12] More specifically, the relevant state variables for investors are the aggregate bond returns index and the sovereign debt portfolio index; country i's problem instead depends on the aggregate bond price index, the sovereign debt portfolio index, and the size of the union-wide sovereign debt market.

After the definition of the state of the union and domestic economies, it is possible to identify default sets conditional on the state at time t in country i. The following definitions are consistent with the sovereign debt literature (see, for instance, Arellano, 2008, Mendoza and Yue, 2012, Lizarazo, 2013).

Definition 2. *Define country i's default set $\mathcal{D}_i(q^i_{t-1}, s^*_t)$ as the set of endowments y^i_t for which contracts in the union economy's sovereign debt market make it optimal to default for sovereign i, i.e.*

$$\mathcal{D}_i(q^i_{t-1}, s^*_t) = \{y^i_t : U^R_i(s^i_t) < U^D_i(s^i_{Dt})\}. \tag{11}$$

Hence, the default set at $t+1$, $\mathcal{D}_i(q^i_{t-1}, s^*_t)$, and the transition probability function of endowments, $p(y^i_t|y^i_{t-1})$, can be used to compute the expected value of any variable x^i_{t+1} dependent of country i's endowment only.

The following propositions determine the dynamic interactions between default incentives and income realizations and government bond prices within the domestic economy.

Proposition 1. *Let $\hat{q}^i_{t-1} > \bar{q}^i_{t-1}$, $\hat{s}^i_t = (y^i_t, \hat{q}^i_{t-1}, s^*_t)$, and $\bar{s}_t = (y^i_t, \bar{q}^i_{t-1}, s^*_t)$. If default is optimal in state $\hat{s}^i_t$, then it will be optimal in state $\bar{s}^i_t$, i.e. $\mathcal{D}_i(\hat{q}^i_{t-1}, s^*_t) \subseteq \mathcal{D}_i(\bar{q}^i_{t-1}, s^*_t)$.*

Proof. See Appendix A. □

Proposition 1 states that a lower government bond price increases the probability of the domestic sovereign defaulting on its outstanding debt. Analogous conclusions were reached by Eaton and Gersovitz (1981), Chatterjee et al. (2007) and Arellano (2008), who observed that the value of staying in the contract decreases with the level of outstanding debt. The same result occurs in this model since the value of repayment (default) is increasing in (independent of) the government bond price: through the CES demand function for domestic government bonds, a higher bond price generates a lower level of debt, hence improving the fiscal stance (i.e. the wealth) of the domestic sovereign under repayment relative to default. As a consequence, proposition 1 can be extended so as to analyze the effect of aggregate variables on the domestic sovereign's default incentives. Since aggregate indexes b^*_t and q^*_{t-1} (ρ^*_{t-1}) are positively (negatively) related to the amount of domestic outstanding debt b^i_t, it then follows that an increase in b^*_t or q^*_{t-1} (ρ^*_{t-1}) induces a higher (lower) probability of default.[13]

[13]Further, assuming that (i) the domestic stochastic endowment is an i.i.d. process, (ii) there exist no exogenous output costs of default (i.e. $\omega_i = 0$), and (iii) the sovereign can never re-access international

Given state $s_t^i = \{y_t^i, q_{t-1}^i, s_t^*\}$ and union-wide variables $\{q_t^*, r_t^*, b_{t+1}^*, \rho_t^*\}$ at period t, the recursive equilibrium of country i is defined as the set of policy functions for (i) consumption c_t^i; (ii) international investors' sovereign debt portfolio allocations b_{t+1}^i; (iii) the sovereign's bond price q_t^i, default option d_t^i and default sets $\mathcal{D}_i(q_{t-1}^i, s_t^*)$; and (iv) value functions $\{U_i(s_t^i), U_i^R(s_t^i), U_i^D(y_t^i)\}$, such that:

- Taking as given the domestic government's default and bond price policies as well as international investors' sovereign debt portfolio allocations, c_t^i satisfies the domestic households' budget constraint (8);
- Taking as given the domestic government's default and bond price policies, b_{t+1}^i satisfies international investors' demand function for government bonds (2);
- Taking as given the domestic government default policy, q_t^i and d_t^i are consistent with the domestic sovereign's maximization problems, and $\mathcal{D}_i(q_{t-1}^i, s_t^*)$ is given by condition (11).
- Value functions $\{U_i(s_t^i), U_i^R(s_t^i), U_i^D(y_t^i)\}$ solve problems (10), (7), and (9).
- The union-wide market for sovereign debt clears, so that union-wide demand (left-hand side) equals supply (right-hand side) of government bonds; by Walras law, union-wide consumption equals union-wide endowments at each point in time.

Given the definition of equilibrium in the domestic economy above, it is now appropriate to highlight some analytical features of the model, by focusing first on the bond price schedule and then on some debt and default implications.

4.4 The Bond Price Schedule

When country i's sovereign can access union-wide capital markets in order to smooth household consumption, the domestic government bond price is determined as follows:

$$q_t^i = \mu\beta_i \frac{E_t[(1 - d_{t+1}^i)\lambda_{t+1}^i]}{\lambda_t^i}, \qquad (12)$$

capital markets after default (i.e. $\theta = 0$), a necessary condition for default is that the absence of contract that allow the borrowing government to roll over its debt. In other words, sovereign default arises only when all contracts available generate fiscal deficits. This result, together with the monotonicity and concavity of the utility function $u_i(\cdot)$, implies that default incentives decrease in present wealth. Thus, in times of adverse output shocks, the available contracts cannot offset the weak state of the economy or provide insurance for a highly indebted sovereign because none can increase consumption relative to wealth. For a proof of these results, see Arellano (2008).

where $\mu \equiv \frac{\eta}{\eta-1}$ is the union-wide mark-up over the (perfect competition) marginal cost of debt and λ_t^i is the Lagrange multiplier associated with household budget constraint, i.e. the marginal utility of consumption. Condition (12) is the Euler equation for the domestic sovereign's problem (7) subject to (8) and defines the optimal bond price schedule conditional on repayment by sovereign i at time t.[14] The bond price schedule has a standard interpretation: from the sovereign's perspective, the marginal benefit of borrowing today side, i.e. the bond price, is equal to the marginal cost of repayment tomorrow, i.e. the mark-up-augmented stochastic discount factor net of the probability of sovereign default next period. Hence, the bond price schedule accounts for sovereign default risk: an increase in the probability of default next period reduces the stochastic discount factor and thus the face value of the bond.

However, the assumption of a monopolistically competitive sovereign debt market introduces novel features in the bond price schedule compared to related models of endogenous sovereign default. The term in expectations on the right-hand side of equation (12) can be manipulated so as to identify the different components of the bond-pricing kernel:

$$\begin{aligned} q_t^i &= \mu \left\{ \beta_i \frac{E_t[1-d_{t+1}^i]E_t[\lambda_{t+1}^i]}{\lambda_t^i} + \beta_i \frac{Cov_t[(1-d_{t+1}^i), \lambda_{t+1}^i]}{\lambda_t^i} \right\} \\ &= \mu \, (q_{RNt}^i + q_{RAt}^i). \end{aligned} \tag{13}$$

The first component, μ, represents a constant upward shift on the overall price, determined from investors' intratemporal risk aversion (i.e. aimed at portfolio diversification). The second component, q_{RNt}^i, compensates the lenders for the expected loss from default. Finally, the third component,q_{RAt}^i, represents the "negative" risk premium and adds upward pressure on the bond price, due to the borrowing government's intertemporal risk aversion (aimed at consumption smoothing). The main determinant of q_{RAt}^i is the covariance term, which is non-negative, since the government internalizes the detrimental effects of default next period. The following proposition formalizes this result.

Proposition 2. *Let the exogenous output costs of default be smaller than the debt-to-GDP ratio, i.e.* $\omega_i < b_{t+1}^i / y_{t+1}^i$*, and the debt level be strcitly positive, i.e.* $b_{t+1}^i > 0$*. Then, default is inversely related to the marginal utility of consumption next period, i.e.*

[14] In order to focus on the main determinants of the bond price schedule, the first-order condition for the domestic sovereign's problem disregards the upper boundary on the bond price and, thus, the Lagrange multiplier of the associated constraint. The numerical evaluation of the model makes sure that such constraint is neither violated nor binding.

$Cov_t[(1 - d^i_{t+1}), \lambda^i_{t+1}] \geq 0$.

Proof. See Appendix A. □

Proposition 2 states that a higher probability of default is associated with a lower marginal utility of consumption.[15] Then, everything else being equal, the borrower's anticipation of the benefits of repayment reduces the wedge between the price under no commitment and the price that would prevail if the sovereign could trade a risk-free asset. In other words, given the same probability of default, the domestic economy's bond prices in this model are higher than the prices that would be observed in a model with risk neutral investors.

This result contrasts with previous models focusing on investors' risk aversion. In particular, Lizarazo (2013) shows that an economy owes an "excess" risk premium to risk averse investors compared to an economy borrowing from risk neutral investors. The results in the present model are ultimately driven by the assumption on international investors' intratemporal risk aversion (as opposed to the intertemporal type of risk aversion in Lizarazo, 2013): the perceived imperfect substitutability between bonds gives substantial leeway to borrowing sovereigns, so that the latter can determine optimal prices by imposing a mark-up and internalizing the effects of their default option.

5 Quantitative Assessment of the Model

5.1 Data

In March 2012, after a lengthy bargaining process, the majority of private investors of Greek sovereign debt agreed to a bond swap, thus accepting a reduction in the face value of their bonds by more than half, an extension of the maturity of their holdings, and a decrease in the interest rate paid by the government.[16] This event marked the largest

[15] The numerical evaluation of the model ensures that the first condition, i.e. $\omega_i < b^i_{t+1}/y_{t+1}$, holds. The second condition, i.e. $b^i_{t+1} > 0$, is always rue when the probability of default is strictly positive.

[16] In February 2012, in order to prevent a disorderly default and a possible exit of Greece from the euro area, an IMF/ECB bailout package included a debt restructure agreement with private investors affecting more than €200 billion of Greek government bonds (see *Reuters*, February 29th 2012, "Insight: How the Greek debt puzzle was solved"). Following the use of collective action clauses by the Greek government to settle its obligations with private investors, the International Swaps and Derivatives Association (ISDA) declared the debt restructuring as a credit event (i.e. a "technical" default), hence triggering swaps (i.e. payouts on the underlying CDS contracts) on about $3 billion of default insurance at an auction held on March 19th 2012 (see *Bloomberg*, March 9th 2012, "Greek Credit Swaps Payouts to Be Expedited After Trigger Ruling").

sovereign-debt restructuring in history, as it allowed Greece to write off about €100 billion ($130 billion), accounting for 29 percent of its outstanding debt and 48 percent of its 2011 GDP.[17] Greece also experienced a severe economic crisis, with a cumulative output loss of about 11 percent in the two quarters before the default.[18] This section documents this default event and the business cycle features of the Greek economy.

The series for real GDP, consumption, and the current account-to-GDP ratio, drawn from the European Central Bank's database, are seasonally adjusted. The interest rate spread (obtained from DataStream) is computed as the difference between each country and the Eurozone's (changing composition) euro spot interest rate on 1-year benchmark government bonds.[19] After these adjustments, all series are detrended with an HP filter.

Table 3 reports business cycle statistics for Greece between the first quarter of 2002 and the first quarter of 2012. The first column reports the standard deviations and correlations (with output and spread) of the relevant variables in the full sample, whereas the second and the third columns report the same statistics for the pre-crisis (2002.1-2008.2) and the crisis periods (2008.3-2012.1), respectively.[20] Finally, the fourth column shows the p-values of the Levene's test for the standard deviations of the pre-crisis and crisis periods. This test is a useful alternative to Bartlett's test when the sample distributions are not normal, and especially when they are prone to outliers.

The sample statistics for Greece are consistent with well-known results reported in the literature on sovereign default in emerging economies (see, for instance, Neumeyer and Perri, 2005, Aguiar and Gopinath, 2006, Arellano, 2008): consumption has a higher volatility than output and both the current account and the yield spread are countercyclical (albeit the negative correlation with output of the spread is not statistically significant). However, unlike a typical emerging economy, Greece does not display positively correlated current account and spread in any considered period. Interestingly, according to Levene's

[17] *The Economist*, March 17th 2012, "Greece's default: The wait is over".

[18] The decline in output is computed as the sum of the deviations of (log) real GDP from its HP filter trend in the last quarter of 2011 and the first quarter of 2012.

[19] Data on Greek treasury notes with shorter maturities are available from several sources, but all series are fraught with missing values.

[20] The separation between pre-crisis and crisis period aims at increasing the number of observations in each period as much as possible, in order to maximize the power of the sample statistics. Although the narrative in the paper identifies the revision of the deficit expectations by the Greek government in the fourth quarter of 2009 as the clear start of the shift in government bond yield dynamics, an early signal was the collapse of Lehman Brothers in the third quarter of 2008, which arguably affected investors in euro-area financial markets, as well (see, for instance, Bernoth, von Hagen and Schuknecht, 2012, Christiansen, 2014). This timing choice is thus consistent with the main focus of the paper, i.e. the effects of investors' sentiments on sovereign yield dynamics.

Table 1: Greece Business Cycle Statistics, 2002.1-2012.1

	Full	Pre-Crisis	Crisis	Levene's test
	[1]	[2]	[3]	[4]
$\sigma(y)$	2.199	1.145	1.733	0.455
$\sigma(c)/\sigma(y)$	1.296	0.505	1.752	0.552
$\sigma(ca/y)$	1.635	1.490	0.810	0.045
$\sigma(r)$	9.603	0.075	15.932	0.096
$\rho(c, y)$	0.754	-0.091	0.894	–
	(0.000)	(0.66)	(0.000)	–
$\rho(ca/y, y)$	-0.580	0.009	-0.439	–
	(0.000)	(0.966)	(0.133)	–
$\rho(r, y)$	-0.185	-0.208	-0.091	–
	(0.26)	(0.309)	(0.769)	–
$\rho(ca/y, r)$	0.095	-0.240	-0.051	–
	(0.564)	(0.237)	(0.869)	–

Notes: The series for real GDP (y), consumption (c), and the current account-to-GDP ratio (ca/y) were first seasonally adjusted. After logging y and c, and computing the interest rate spread (r) as the difference between each country and the Eurozone's (changing composition) euro spot interest rate on 1-year benchmark government bonds, all series were HP filtered with a smoothing parameter of 1600. Levene's test (H_0: no difference in standard deviations between samples) refers to the corresponding p-values for sample standard deviations (σ); Pearson's p-values are reported in parentheses for sample correlations (ρ). Full sample refers to the period 2002.1-2012.1; pre-crisis sample refers to 2002.1-2008.2; crisis sample refers to 2008.3-2012.1. The two quarters 2011.4-2012.1 for government bond yields have been excluded from computations in order to net out the effects of outliers in the historical series.
Sources: ECB, Datastream, author's calculations.

test, the volatility of both the current account and the interest rate spread experienced a significant shift in the crisis period; whereas the standard deviation of the current account decreased from 1.5 to 0.8 percent, the same statistic for the spread dramatically increased from 0.1 to 15.9 percent. Interestingly, Levene's test reveals that the volatility of fundamentals (i.e. real GDP and consumption) did not significantly change in the wake of the financial crisis. This result supports the view according to which the shift in market agents' pricing paradigm was not driven by changes in country-specific risk characteristics, but rather by exogenous events (e.g. the Lehman collapse and/or the upward revision by the Greek government of its expected fiscal deficit).

5.2 Calibration and Functional Forms

In this section, the benchmark calibration and results from the numerical solution and simulation of the model are presented. The functional form for the utility function of

Table 2: Specification of Model Parameters

parameter	value	target
θ	0.125	$\simeq$ 2 years of exclusion
σ_i	2	
β_i	0.99	4% annual rate
q^*	0.9943	Euro-area 1-year gov.t bond yield (2002.1-2011.4)
b^*	0.001	Average investor's exposure to gov.t bonds
ω_i	0.048	Greece's output costs of default (2011.4-2012.1)
σ_y^i	0.0135	Greece's log output (2002.1-2013.4)
ρ_y^i	0.8549	Greece's log output (2002.1-2013.4)
η	80, 150, 2000	model gov.t bond yield standard deviation

domestic households is:

$$u_i(c_t^i) = \frac{{c_t^i}^{1-\sigma_i}}{1-\sigma_i}.$$

The domestic endowment is assumed to follow a first-order Markov process with transition probability $p(y_t^i|y_{t-1}^i)$. The stochastic process for income then approximates the continuous AR(1) process around a long run mean μ_y^i below:[21]

$$y_t^i = \mu_y^i(1-\rho_y) + \rho_y^i y_{t-1}^i + \varepsilon_t^i, \tag{14}$$

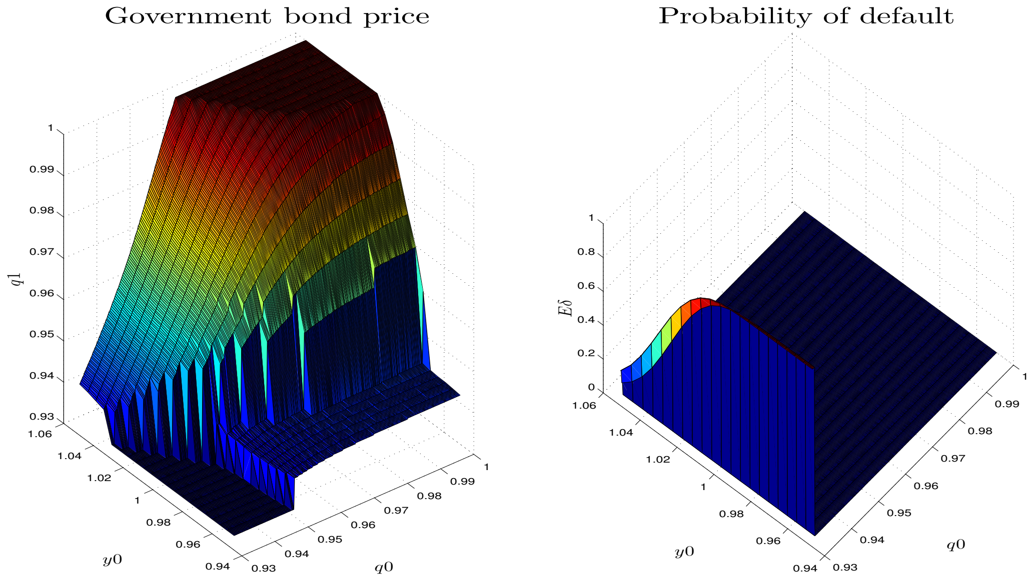
where $0 < \rho_y^i < 1$, $\varepsilon_t^i \sim N(0, {\sigma_y^i}^2)$. In order to solve the model, the state space is discretized. The support for the domestic endowment spans 21 equally spaced grids of the original processes steady state distribution. The state space for the government bond price includes 300 grid points.[22]

Table 2 reports the values of the calibrated parameters. The AR(1) process for income is fitted to the Greek detrended real GDP between the first quarter of 2002 and the first quarter of 2012. Autoregressive coefficient $\rho_y^i = 0.8549$, standard deviation for output shocks $\sigma_y^i = 0.0135$, whereas long run mean μ_y^i is standardized to one. The output cost of default $\omega_i = 0.048$ is obtained as the (percentage) deviation of output from its trend in the first quarter of 2012. International investors' sovereign debt portfolio index b^* is calibrated

[21]The values of the Markov transition matrix are then computed by integrating the underlying normal density over each interval (see Aguiar and Gopinath, 2006).

[22]An additional grid point for the price equal to infinity is included in order to compute the value function of the government with access to capital markets in the period after default.

Figure 5: Model Government Bond Price Menu and Probability of Default



to match the average exposure of euro-area monetary and financial institutions to euro-area non-domestic sovereign debt.[23] The aggregate bond price index q^* matches the long run average of the euro-area spot interest rate on 1-year benchmark government bonds (rescaled to a quarterly frequency). Parameter θ is set to 0.125, which corresponds to an expected duration of financial autarky of two years. This value is in line with the Greek experience, since the government could not tap international capital markets from the default event in March 2012 to April 2014.[24] Different specifications for the intratemporal risk aversion of the domestic investor are compared: notably, the model is solved for η equal to 80, 150, and 2000, corresponding to the low, baseline, and high elasticity scenarios, respectively. The choice for the remaining parameters is in line with the standard real business cycle literature: the coefficient of intertemporal risk aversion σ_i is set to 2 and the discount factor β_i corresponds to an annualized rate of 4 percent. However, each specification of η is associated with different maximum and minimum levels of debt-to-mean income b_{Gy}; hence, in order to preserve comparability between different specifications, I define the state space for the bond price q_t (which is linked to b_{Gy} through the aggregate CES demand for

[23] The data is collected from the ECB database; exposures refer to securities other than shares held by monetary and financial institutions other than the ECB and the European System of Central Banks.

[24] *The Financial Times*, April 9th 2014, "Greece: Gre-entry".

Table 3: Model Simulations

	Baseline	High elasticity	Low elasticity
	[1]	[2]	[3]
$\sigma(y)$	1.595	1.583	1.657
$\sigma(c)/\sigma(y)$	1.674	1.635	5.461
$\sigma(ca/y)$	1.601	1.103	6.762
$\sigma(r)$	4.056	0.888	6.315
$\rho(c, y)$	0.548	0.767	-0.066
	(0.000)	(0.000)	(0.008)
$\rho(ca/y, y)$	0.434	0.304	0.347
	(0.000)	(0.001)	(0.000)
$\rho(r, y)$	-0.391	0.113	-0.452
	(0.000)	(0.14)	(0.000)
$\rho(ca/y, r)$	-0.340	-0.295	-0.422
	(0.000)	(0.001)	(0.000)
mean debt-to-GDP	1.118	4.775	2.371
mean default	11.067	12.230	8.373

domestic sovereign debt) so that in every specification the set of debt-to-mean income between $100\omega_i$ and 50 percent is analyzed.

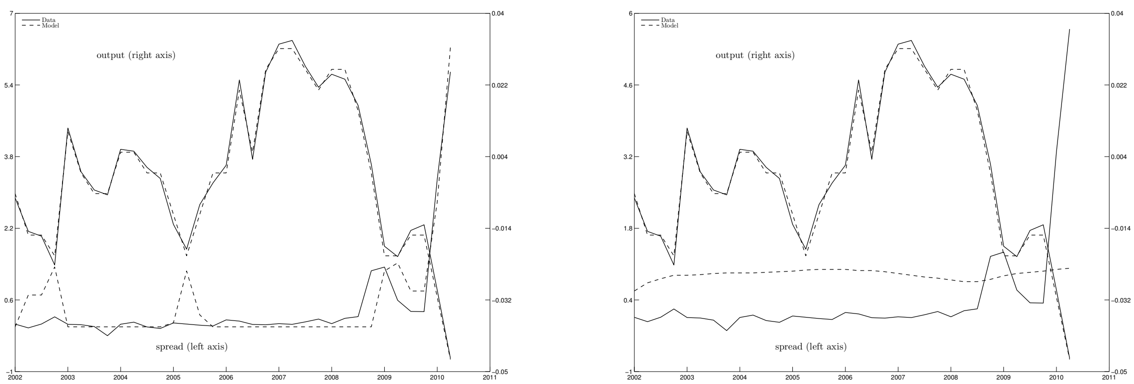
5.3 Simulations

The results of the numerical evaluation of the model with the baseline specification are showed in Figure 5. The optimal choice for the government bond price and the default probability are positively and negatively related to the state of the country's endowment today and the current government bond price, respectively.

The model is then simulated for 10,000 periods. From these 10,000 periods, subsamples that have the economy staying in the credit market for 40 periods before going into a default are taken to compute the economy's business cycle statistics. Hence, these statistics are comparable with actual sample statistics on 10 years of Greek data. This process is repeated 500 times, and the cycle statistics are the average of the statistics derived from each of these repetitions.

Table 3 reports results from the simulation of the model. As expected, the standard deviation of the simulated series for the government bond price decreases with the elasticity of substitution. More specifically, the standard deviation of the government bond spread when $\eta = 2000$ is just 0.888 percent and it sharply increases to 6.762 percent

Figure 6: Greece and Model Time series



Notes:.

when $\eta = 80$.[25] Hence, the model correctly predicts that in times of financial distress, when investors are relatively more risk averse and perceive bonds as imperfect substitutes, sovereign debt yields become more sensitive to macroeconomic fundamentals, so that their volatility increases. The model also delivers a higher volatility of consumption relative to output and countercyclical interest rates. Interestingly, as government bonds become more perfect substitutes (i.e. as η increases) and the perceived segmentation of the financial market shrinks, the level of countercyclicality of interest rates decreases; a number of studies (e.g. Neumeyer and Perri, 2005) reports that acyclical or mildly procyclical interest rates characterize business cycles in developed and financially integrated economies. However, the model cannot capture the countercyclicality of the current account. This result stems from the specification of the demand function for government bonds: since $\eta > 1$, an increase in the price translates into a decrease of a larger magnitude in the demand for bonds. As the government chooses high values in the bond price menu during good states (due to the persistence of the autoregressive process for output), the aggregate effect is a low borrowing level, so that output, which is higher than consumption, is used to repay outstanding debt and the country experiences net capital outflows, i.e. a current account surplus.

Figure 6 compares time series for output and government bond spread under the baseline (left-hand chart) and the high elasticity (right-hand chart) specification. I feed the model with the series for Greek (log) detrended output from the first quarter of 2012

[25] The spread is calculated as $1/q_t^i - 1/q^*$.

up to the second quarter of 2010, when the first bailout package was extended to Greece. In the high elasticity scenario, the model dynamics of the spread resemble the data in the pre-crisis period, characterized by smooth changes and a very low sensitivity to movements in fundamentals. Conversely, in the baseline scenario, the model can successfully replicate data during the crisis, when the decline in output generated a sharp increase in the spread. However, the high degree of investors' risk aversion induces considerable movements in the spread before the crisis as well. This result is consistent with empirical evidence on the misalignment between country-specific risk characteristics and investors' pricing paradigm.[26]

6 Conclusions

This paper models endogenous default risk in a stochastic dynamic framework of a large economic region composed of an infinite number of small open economies that feature incomplete financial markets and risk-averse international investors populating each country. International investors are risk averse along the intratemporal dimension: as bonds issued by different governments are perceived as imperfect substitutes, international lenders seek to optimally diversify their sovereign debt portfolios. In the paper, I argue that it is essential to consider such a specification for investors' preference in order to replicate salient empirical regularities observed in the euro-area sovereign debt market: the shift in investors' sentiments produced a reversal in the process of financial integration and determined the sharp increase in yield differentials in the wake of the recent crisis. Also, the modeling choice on the particular form of intratemporal risk aversion (i.e. a constant elasticity of substitution between different government bonds) reflects some key stylized facts. Further, different specifications of the elasticity of substitution are analyzed: as expected, a higher degree of intratemporal risk aversion (i.e. lower perceived financial integration) determines a higher sensitivities in movements of country-specific fundamentals, thus inducing a larger volatility of government bond spread compared a scenario with a low degree of risk aversion. Finally, the results show that the model can replicate important features of Greek government bond spreads and current account dynamics in the ten years preceding the sovereign default.

[26]In the literature on the determinants of euro-area government bond yields, the non-fundamental component of sovereign bond spreads, i.e. the part of bond spreads not explained by differences in fiscal and macroeconomic fundamentals have been often designated with the term mispricing (see, for instance, Grauwe and Ji, 2012, Cesare et al., 2012, Dewachter et al., 2014).

References

Abad, Pilar, Helena Chuliá, and Marta Gómez-Puig. 2010. "EMU and European government bond market integration." *Journal of Banking & Finance*, 34(12): 2851–2860.

Adam, Klaus, Tullio Jappelli, Anna Maria C. Menichini, Mario Padula, and Marco Pagano. 2002. "Analyse, Compare, and Apply Alternative Indicators and Monitoring Methodologies to Measure the Evolution of Capital Market Integration in the European Union." European Commission Report to the European Commission.

Aguiar, Mark, and Gita Gopinath. 2006. "Defaultable Debt, Interest Rates and the Current Account." *Journal of International Economics*, 69(1): 64–83.

Arellano, Cristina. 2008. "Default Risk and Income Fluctuations in Emerging Economies." *American Economic Review*, 98(3): 690–712.

Arellano, Cristina, and Yan Bai. 2013. "Linkages across Sovereign Debt Markets." Society for Economic Dynamics 2013 Meeting Papers 414.

Baele, Lieven, and Koen Inghelbrecht. 2008. "Time-varying integration, the euro and international diversification strategy." Directorate General Economic and Monetary Affairs (DG ECFIN), European Commission European Economy - Economic Papers 333.

Baele, Lieven, Annalisa Ferrando, Peter Hördahl, Elizaveta Krylova, and Cyril Monnet. 2004. "Measuring financial integration in the euro area." European Central Bank Occasional Paper Series 14.

Battistini, Niccolò, Marco Pagano, and Saverio Simonelli. 2014. "Systemic risk, sovereign yields and bank exposures in the euro crisis." *Economic Policy*, 29(78): 203–251.

Beirne, John, and Marcel Fratzscher. 2013. "The pricing of sovereign risk and contagion during the European sovereign debt crisis." *Journal of International Money and Finance*, 34(C): 60–82.

Bernoth, Kerstin, and Burcu Erdogan. 2012. "Sovereign bond yield spreads: A time-varying coefficient approach." *Journal of International Money and Finance*, 31(3): 639–656.

Bernoth, Kerstin, Jürgen von Hagen, and Ludger Schuknecht. 2012. "Sovereign risk premiums in the European government bond market." *Journal of International Money and Finance*, 31(5): 975–995.

Bolton, Patrick, and Olivier Jeanne. 2011. "Sovereign Default Risk and Bank Fragility in Financially Integrated Economies." *IMF Economic Review*, 59(2): 162–194.

Borensztein, Eduardo, and Ugo Panizza. 2009. "The Costs of Sovereign Default." International Monetary Fund IMF Staff Papers 4.

Bulow, Jeremy, and Kenneth Rogoff. 1989*a*. "A Constant Recontracting Model of Sovereign Debt." *Journal of Political Economy*, 97(1): 155–78.

Bulow, Jeremy, and Kenneth Rogoff. 1989*b*. "Sovereign Debt: Is to Forgive to Forget?" *American Economic Review*, 79(1): 43–50.

Cesare, Antonio Di, Giuseppe Grande, Michele Manna, and Marco Taboga. 2012. "Recent estimates of sovereign risk premia for euro-area countries." Bank of Italy, Economic Research and International Relations Area Questioni di Economia e Finanza (Occasional Papers) 128.

Chatterjee, Satyajit, Dean Corbae, Makoto Nakajima, and Jos?-V?ctor R?os-Rull. 2007. "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default." *Econometrica*, 75(6): 1525–1589.

Christiano, Lawrence J. 2006. "Imperfect Asset Substitutability and Other Things." *mimeo.*

Christiansen, Charlotte. 2014. "Integration of European bond markets." *Journal of Banking & Finance*, 42(C): 191–198.

Dewachter, Hans, Leonardo Iania, Marco Lyrio, and Maite de Sola Perea. 2014. "A macro-financial analysis of the euro area sovereign bond market." *Journal of Banking & Finance.*

Eaton, Jonathan, and Mark Gersovitz. 1981. "Debt with Potential Repudiation: Theoretical and Empirical Analysis." *Review of Economic Studies*, 48(2): 289–309.

ECB. 2013. "Financial Integration in Europe." European Central Bank ECB Annual Reports.

Engler, Philipp, and Christoph Große Steffen. 2013. "A theory of sovereign risk premia and financial intermediation in the euro area." *Working paper*.

Favero, Carlo. 2012. "Modelling and Forecasting Yield Differentials in the euro area. A non-linear Global VAR model." IGIER (Innocenzo Gasparini Institute for Economic Research), Bocconi University Working Papers 431.

Favero, Carlo, and Alessandro Missale. 2012. "Sovereign spreads in the eurozone: which prospects for a Eurobond?" *Economic Policy*, 27(70): 231–273.

Favero, Carlo, Marco Pagano, and Ernst-Ludwig von Thadden. 2010. "How Does Liquidity Affect Government Bond Yields?" *Journal of Financial and Quantitative Analysis*, 45(01): 107–134.

Grauwe, Paul De, and Yuemei Ji. 2012. "Mispricing of Sovereign Risk and Multiple Equilibria in the Eurozone." LICOS - Centre for Institutions and Economic Performance, KU Leuven LICOS Discussion Papers, LICOS - Centre for Institutions and Economic Performance, KU Leuven 30412.

Guerrieri, Luca, Matteo Iacoviello, and Raoul Minetti. 2012. "Banks, Sovereign Debt, and the International Transmission of Business Cycles." *NBER International Seminar on Macroeconomics*, 9(1): 181 – 213.

Hartmann, Philipp, Angela Maddaloni, and Simone Manganelli. 2003. "The Euro-area Financial System: Structure, Integration, and Policy Initiatives." *Oxford Review of Economic Policy*, 19(1): 180–213.

Lizarazo, Sandra Valentina. 2009. "Contagion of Financial Crises in Sovereign Debt Markets." University Library of Munich, Germany MPRA Paper 20795.

Lizarazo, Sandra Valentina. 2013. "Default risk and risk averse international investors." *Journal of International Economics*, 89(2): 317–330.

Manganelli, Simone, and Guido Wolswijk. 2009. "What drives spreads in the euro area government bond market?" *Economic Policy*, 24: 191–240.

Mendoza, Enrique G., and Vivian Z. Yue. 2012. "A General Equilibrium Model of Sovereign Default and Business Cycles." *The Quarterly Journal of Economics*, 127(2): 889–946.

Neumeyer, Pablo A., and Fabrizio Perri. 2005. "Business cycles in emerging economies: the role of interest rates." *Journal of Monetary Economics*, 52(2): 345–380.

Oliveira, Luís, José Dias Curto, and João Pedro Nunes. 2012. "The determinants of sovereign credit spread changes in the Euro-zone." *Journal of International Financial Markets, Institutions & Money*, 22: 278–304.

Pagano, Marco, and Ernst-Ludwig von Thadden. 2004. "The European Bond Markets under EMU." *Oxford Review of Economic Policy*, 20(4): 531–554.

Sandleris, Guido. 2008. "Sovereign defaults: Information, investment and credit." *Journal of International Economics*, 76(2): 267–275.

Schumacher, Julian, Christoph Trebesch, and Henrik Enderlein. 2013. "Sovereign Defaults in Court: The Rise of Creditor Litigation." *Working paper.*

Sgherri, Silvia, and Edda Zoli. 2009. "Euro Area Sovereign Risk During the Crisis." International Monetary Fund IMF Working Papers 09/222.

Sosa-Padilla, Cesar. 2013. "Sovereign Defaults and Banking Crises." *Working paper.*

Wagenvoort, Rien J.L.M., André Ebner, and Magdalena Morgese Borys. 2011. "A factor analysis approach to measuring European loan and bond market integration." *Journal of Banking & Finance*, 35(4): 1011–1025.

Yeyati, Eduardo Levy, and Ugo Panizza. 2011. "The elusive costs of sovereign defaults." *Journal of Development Economics*, 94(1): 95–105.

Yue, Vivian Z. 2010. "Sovereign default and debt renegotiation." *Journal of International Economics*, 80(2): 176–187.

Zettelmeyer, Jeromin, Christoph Trebesch, and Mitu Gulati. 2013. "The Greek debt restructuring: an autopsy." *Economic Policy, CEPR;CES;MSH*, 28(75): 513–563.

Appendix A Proofs of Theorems

In the following proofs, I assume that $h_t^i = 1$, for all i, i.e. every sovereign repaid its debt at $t-1$ and is in good standing, and can choose whether to repay or default on its outstanding debt at t. Also, country i's state at time t is given by $s_t^i = \{y_t^i, q_{t-1}^i, s_t^*\}$. As a reminder, domestic households' budget constraint under repayment and default is given by the conditions below, respectively,

$$c_t^i = y_t^i + q_t^i \left(\frac{q_t^i}{q_t^*}\right)^{-\eta} \frac{b_{t+1}^*}{\rho_t^*} - \left(\frac{q_{t-1}^i}{q_{t-1}^*}\right)^{-\eta} \frac{b_t^*}{\rho_{t-1}^*} \quad \text{and} \quad c_{Dt}^i = (1-\omega_i)y_t^i.$$

Proposition 1. *Let $\hat{q}_{t-1}^i > \bar{q}_{t-1}^i$, $\hat{s}_t^i = (y_t^i, \hat{q}_{t-1}^i, s_t^*)$, and $\bar{s}_t = (y_t^i, \bar{q}_{t-1}^i, s_t^*)$. If default is optimal in state $\hat{s}_t^i$, then it will be optimal in state $\bar{s}_t^i$, i.e. $\mathcal{D}_i(\hat{q}_{t-1}^i, s_t^*) \subseteq \mathcal{D}_i(\bar{q}_{t-1}^i, s_t^*)$.*

Proof. Suppose that $\hat{q}_{t-1}^i > \bar{q}_{t-1}^i$, so that the corresponding states of the union economy are $\hat{s}_t^i = (y_t^i, \hat{q}_{t-1}^i, s_t^*)$, and $\bar{s}_t^i = (y_t^i, \bar{q}_{t-1}^i, s_t^*)$, respectively. Notice that

$$\frac{\partial c_t^i}{\partial q_{t-1}^i} = \eta(q_{t-1}^i)^{-\eta-1}(q_{t-1}^*)^{\eta} b_t^* > 0 \quad \text{and} \quad \frac{\partial c_{Dt}^i}{\partial q_{t-1}^i} = 0,$$

since $0 < q_{t-1}^i \leq 1$, for all i, so that $\hat{c}_t^i > \bar{c}_t^i$ and c_{Dt}^i is independent of q_{t-1}^i; thus,

$$\begin{aligned} U_i^R(\hat{s}_t^i) &= u(\hat{c}_t^i) + \beta_i E_t\left[U_i^R(s_{t+1}^i)\right] \\ &> u(\bar{c}_t^i) + \beta_i E_t\left[U_i^R(s_{t+1}^i)\right] \\ &= U_i^R(\bar{s}_t^i), \end{aligned}$$

since $u_i'(\cdot) > 0$, which implies that the value of the debt contract under repayment is increasing in previous period's domestic bond price q_{t-1}^i. Now, suppose that $y_t^i \in \mathcal{D}_i(b_t^*, \hat{q}_t^i)$; then, by definition, $U_i^D(s_{Dt}^i) > U_i^R(\hat{s}_t^i)$. Therefore, $U_i^D(s_{Dt}^i) > U_i^R(\bar{s}_t^i)$ and $y_t^i \in \mathcal{D}_i(b_t^*, \bar{q}_t^i)$. □

Proposition 2. *Let the exogenous output costs of default be smaller than the debt-to-GDP ratio, i.e. $\omega_i < b_{t+1}^i/y_{t+1}^i$, and the debt level be strcitly positive, i.e. $b_{t+1}^i > 0$. Then, default is inversely related to the marginal utility of consumption next period, i.e. $Cov_t[(1-d_{t+1}^i), \lambda_{t+1}^i] \geq 0$.*

Proof. Assume that the sovereign does not default next period in any state of the domestic economy (i.e. $\mathcal{D}(q^i_{t-1}, s^*_t) = \emptyset$) given some bond price q^i_t; then $d^i_{t+1} = 0$, so that $Cov_t[(1 - d^i_{t+1}), \lambda^i_{t+1}] = 0$.

Similarly, assume that the sovereign defaults next period in every state of the domestic economy (i.e. $\mathcal{D}(q^i_{t-1}, s^*_t) = Y^i$, with Y^i denoting the whole set of realizations y^i_t) given some bond price q^i_t, then $d^i_{t+1} = 1$, so that $Cov_t[(1 - d^i_{t+1}), \lambda^i_{t+1}] = 0$.

In contrast, if the sovereign defaults next period only in some states of the domestic economy (i.e. $\mathcal{D}(q^i_{t-1}, s^*_t) \subset Y$), then either

(i) the sovereign repays, i.e. $d^i_{t+1} = 0$, and its wealth is $w^i_{Rt+1} = y^i_{t+1} - b^i_{t+1}$, or

(ii) the sovereign defaults, i.e. $d^i_{t+1} = 1$, and its wealth is $w^i_{Dt+1} = (1 - \omega_i) y^i_{t+1}$.

Now, for some choice of the bond price next period q^i_{t+1} and realizations of the aggregate variables s^*_{t+1}, the government's wealth is larger under default than under repayment, i.e. $w^i_{Rt+1} < w^i_{Dt+1}$, as long as the output costs of default are lower than the debt-to-GDP ratio, i.e. $\omega_i < b^i_{t+1}/y^i_{t+1}$, and $b^i_{t+1} > 0$. Then, due to concavity of the households' utility function $u_i(\cdot)$, the marginal utility of consumption is larger under repayment than under default, i.e. $\lambda_{Rt+1} > \lambda^i_{Dt+1}$. Therefore, a higher probability of default is associated with a lower marginal utility of consumption, so that $Cov_t[(1 - d^i_{t+1}), \lambda^i_{t+1}] \geq 0$. □

Appendix B Computational Algorithm

In order to reproduce the analytical solution of the model, the numerical solution of the model is implemented as follows. After calibrating the model parameters (including aggregate variables):

1. Define states $s = \{y, q\}$, as well as probability distribution $p(y'|y)$.
2. Guess initial values for value functions U^R and U^D.
3. Value function iteration:
   (a) Find optimal policy for q'.
   (b) Find optimal policy for d.
   (c) Use the value functions associated with the optimal policies as new guesses for U^R and U^D.
4. Update.

Finally, once convergence is achieved, it is possible to simulate the model and compute the relevant statistics predicted by the model for a given calibration. Notice that the algorithm above includes a unique iteration for the policy function q_I, as the level of debt is simultaneously identified via the endogenous demand for government bonds. This method innovates on the algorithms used in most of the sovereign default models; also, it is essential in order to guarantee an efficient computation of the model.