Endogenous financial literacy, saving and stock market participation

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Abstract

Recent empirical literature provides evidence that financial literacy, human capital, savings and stock market participation are interconnected decisions. In this paper we contribute to this issue by building a framework that includes all these decisions in an encompassing model. In our model, earnings from work depend on optimally chosen human capital and financial literacy allows to access the stock markets and to invest in risky assets. Our results provide a theoretical foundation for the role and the determinants of financial literacy and explain several stylized facts on literacy and stock market participation.

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1. Introduction

Recent reforms of pension systems in several countries entail a higher degree of responsibility on workers' side. This, together with an increasing degree of complexity of individuals' financial decisions, has induced both scholars and policymakers to focus on the determinants and the role of financial literacy.

While quite a large number of empirical studies have shed light on the relevance of financial literacy as a key determinant of crucial life-time decisions such as retirement planning and stock market participation, (we review these contributions in section 2 and see also Mitchell and Lusardi 2011 for a full coverage on this subject), the theoretical literature on these issues is extremely thin.

Among the very few studies that cover the theoretical aspects, Jappelli and Padula (2011) develop a model where financial literacy affects the returns of non-stochastic saving: literacy is costly to acquire and there is thus a trade-off between the benefits and costs of investing in literacy; their findings show that, once financial literacy is endogenously determined, its optimal level is positively correlated with wealth and saving and that the introduction of social security systems reduces the incentives to acquire literacy.

Furthermore, Lusardi et al. (2011) develop a numerically-simulated life cycle model where endogenous financial literacy affects the stochastic returns from saving and where exogenous education determines income. Their results show that the acquisition of financial literacy follows a hump-shaped profile through the life-cycle and that different education groups, receiving different income, find convenient to acquire different amounts of literacy. Finally, Corsini and Spataro (2011) develop a theoretical model where individuals' decisions on pension plans are affected by complexity costs related to more sophisticated plans and investigate how the acquisition of financial literacy affect thus pension-saving decisions.

In this work we aim at making a step further in the theoretical literature on financial literacy by providing a unified framework that encompasses human capital formation, financial literacy achievement, savings and capital market participation. To the best of our knowledge, this has not been done so far.

In particular, differently from previous literature we focus *i)* on the fact that the acquisition of financial literacy allows individuals to access the stock market, to invest in risky assets and thus to obtain diversification of their savings and *ii)* on the interaction between financial literacy and education, with the latter being endogenously determined and affecting, at the same time, both the income and the cost to acquire financial literacy. As a consequence, within our model, savings, financial literacy, education and investment decisions strictly interact and are endogenously determined.

The results that we obtain from our model appear to explain well the observed evidence and also stir some further empirical questions that would be worth to address.

The work is organized as follows. In section 2 we sketch some stylized facts on financial literacy acquisition and stock market participation, stemming from the existing empirical literature. In section 3 we lay out the model concerning human capital investment, savings and financial literacy acquisition. In section 4 we derive the determinants of financial literacy investments and of stock market participation. In section 5 we describe the role of financial literacy in the accumulation of wealth. Concluding remarks end the work.

2. Some stylized facts on financial literacy and stock market participation

A number of empirical studies have analyzed the determinants of financial literacy and their consequences on investment behavior. Usually, literacy is assessed and measured through surveys that contain questions on the concepts of compound interest, inflation, purchasing power and risk diversification: on the basis of the correct answers to these questions a literacy score is computed for each individual. Some of these studies are single-country analyses[1] while others like Jappelli (2010) and Lusardi and Mitchell (2011a) contain cross-countries assessments. These studies highlight some common patterns in the determinants and the effects of financial literacy and they indicate that financial literacy:

- is lower among younger and older individuals.
- is lower among female individuals
- is usually higher for self-employed and individuals working in business related sectors
- is higher for higher income individuals
- is higher for individuals that are good in math and in numeracy
- increases with education (schooling) but the actual relationship can be complex
- affects positively the probability to plan for retirement
- affects positively, other things being equal, the accumulation of wealth
- is positively correlated with the degree of portfolio diversification
- is positively correlated with the participation to the stock market.

Some comments on the above list are worth doing.

First, education and financial literacy appear to be correlated but there is evidence that the field of study is also relevant in explaining how the level of schooling actually affects financial literacy (see Almenberg and Save-Soderbergh 2011). Moreover, when education and literacy are used contemporaneously as explicative variables in econometric regressions, they are usually both significant so that literacy appears to have an effect above and beyond education (see Lusardi and Mitchell 2011a). Second, literacy affects positively, other things being equal, wealth accumulation and in particular this causal effect persists even after controlling for factors like income, age and education.

The issue of participation to stock markets, apart for the above mentioned positive correlation with financial literacy, has also been the object of specific empirical analyses. Among the more recent studies, Bertaut and Haliassos (1995), Christiansen et al. (2007) and van Rooij et al. (2011) have highlighted some common features in participation. Summarizing, financial literacy has a positive effect on participation and, even after controlling for this aspect, participation to the stock market:

- increases in the level of income
- increases in the level of education
- decreases in the degree of risk aversion

[1] Analyses from Alessie et al. (2011), Almenberg and Säve-Söderbergh, (2011), Crossan et al. (2011), Bucher-Koenen and Lusardi (2011), Fornero and Monticone (2011), Klapper and Panos (2011), Lusardi and Mitchell (2011b) and Sekita (2011) cover, respectively, Netherlands, Sweden, New Zeland, Germany, Italy, Russia, United States and Japan. Evidence from United States on the effect of financial literacy on planning and wealth accumulation is also contained in Lusardi and Mitchell (2007). Finally, evidence from Italy on the effect of literacy on portfolio diversification is contained Guiso and Jappelli (2009) while evidence from Sweden on how less-sophisticated investors tend to achieve under-diversification is contained in Anderson (2013).

Moreover, the field of study is, even in this case, also relevant: in fact, Christiansen et al. (2007) find that individuals with higher education in economics-related disciplines are, all things considered, more likely to invest in the stock market.

These stylized facts highlight how financial literacy, education and participation to stock markets are strictly related, not only because the former appears to be a determinant of the latter, but also because several factors appear to concurrently have a role in the determination of all of them.

In the work that follows we provide a model that encompasses all these issues in an unified framework.

3. A Theoretical Model of Human Capital and Saving

We imagine an economy populated by individuals that live for two periods: in the first one they choose how much human capital to acquire and, depending on the latter, they receive an income that is partly consumed and partly saved; then, in the second period, they consume what they have saved.

Within this basic structure, we add three characterizing features to our model: *(i)* savings can be invested either in a safe or in a risky asset (or both); *(ii)* in order to invest in the risky asset, it is necessary to acquire financial literacy, which is costly and *(iii)* this literacy cost is an opportunity cost (given by the time lost to obtain literacy) but the share of time needed is decreasing in the level of human capital and education. Points *(ii)* and *(iii)* reflect the fact that investments in the risky assets require some knowledge of the mechanism behind stocks (something that is also suggested in Bertaut and Haliassos 1995) as well as some effort to track the performance of the assets. It follows that some time has to be lost to acquire financial literacy and our assumption is that the knowledge obtained from generic education effectively reduces the amount of time needed.

Education is thus particularly important within our model and, to all extent, time spent in education has three effects: 1) it increases human capital and thus the wage rate, 2) it decreases the amount of time spent working, 3) it decreases the amount of time needed to obtain financial literacy.

As for investment decisions, in our model individuals not only choose how much to save but they also decide whether to invest in financial literacy, thus differentiating their savings between two assets (i.e. participating in the stock market) or not to invest in financial literacy, thus saving only through the safe asset. We assume that risky assets provide higher returns although they entail higher volatility. To all extent the acquisition of financial literacy can be seen as an investment that allows diversification of the financial portfolio.

3.1 Basic Definitions

We now characterize the basic structure of the model and define its components.

3.1.1 Lifetime utility

Individuals live for two periods (representing working life and retirement) and their lifetime utility is given by

1) $U = -e^{-ac_1} - \beta e^{-ac_2}$

where c_1 and c_2 are consumption in period one and two respectively, a is the absolute risk aversion coefficient and β measures time preferences. The above utility function displays the CARA property and is

chosen to obtain closed form solutions: however, as we describe in details in sub-section 3.1.5, we also allow for an inverse relationship between the risk coefficient *a* and income, which is in line with observed evidence.

3.1.2 Human Capital

During their working life, individuals can invest part of their time in education to acquire human capital *h*. We assume that each individual is endowed with one unit of time and that human capital production function is given by

2) $h = \theta^{1/x}$

where θ represents the time (or the share of time) devoted to education and *x* measures how effective education is in producing human capital. We assume that *x>1* so that human capital production displays decreasing returns.

3.1.3 Income

The income per time-unit *w* depends on the amount of human capital acquired through education and on a scale factor *k>1*. The amount of income per unit of time is

3) $w = kh.$

3.1.4 Working life income

The working life income *W* is the product of income per time-unit *w* and time spent working (i.e. time not spent in education):

4) $W = kh(1 - \theta).$

Given eq. (2), the above can also be written as

5a) $W = kh(1 - h^x).$

Note that working life income *W* is an increasing function of *x* and *k* and has an inverted U-shape with respect to *h*.

3.1.5 Risk aversion

To make our model more realistic we also allow for an inverse relationship between risk aversion and income and therefore we assume that

5) $a = k^{-\alpha}$

where $\alpha > 0$ is the elasticity of risk aversion with respect to income.[2] Given the above function, we are assuming that only the exogenous component of income (that is, the parameter *k*) influences risk aversion. We make this latter assumption to guarantee that human capital and education do not influence directly

[2] This methodology to deal with income-decreasing risk aversion has been first proposed by Makarov and Schornick (2010) and can help explain the empirical finding that richer individuals have a preference for riskier assets (see Vissing-Jorgensen 2002).

risk aversion and to avoid that individuals, when determining their optimal human capital investment, are also choosing their own risk aversion.

Given eq. (5), an increase in α implies, other things being equal, a decrease in risk aversion. The parameter α also determines the behaviour of relative risk aversion with respect to income: relative risk aversion is increasing for $0 < \alpha < 1$, constant for $\alpha = 1$ and decreasing for $\alpha > 1$.

3.1.6 Savings

Individuals can save through two possible assets. A safe asset whose rate of return is *s* and a risky asset whose rate of return is normally distributed with mean *r* and variance σ^2. Given the complexity of financial markets, individuals that choose to invest in the risky asset have to spend some time to acquire financial literacy and to evaluate and track the performance of their asset. We indicate with *S* the amount invested in the safe asset and with *R* the amount invested in the risky asset.

3.1.6 Financial Literacy Costs

Financial literacy costs represent the amount of time needed to acquire the knowledge necessary to invest in the risky assets and to handle and track that investment. Consequentially, they are an opportunity cost as they represent the amount of time *c* lost to invest in financial literacy formation and to manage the risky assets. This amount of time depends inversely on the human capital (and thus on education) of the individual and, in particular:

$$6)\quad c(h) = 1 - h^{1-z}$$

where the parameter $0<z<1$ is a measure on how effective human capital (and education) is in facilitating the acquisition of financial literacy. Our idea is that different fields of education have different degree of complementarity with financial literacy skills and such a degree of closeness of human capital to financial education is captured by the parameter *z*. In the description of our model we assume that *z* is smaller than one: the case in which this parameter is equal to one is a special case which we will treat separately.

The specific form of equation (6) is chosen for analytical simplicity: however, it can be micro-funded using a more complex set-up where individuals invest time to acquire financial literacy which, in turn, abates the amount of time lost in understanding, evaluating and tracking the risky investments. In particular, eq. (6) can be directly derived assuming that individuals invest an optimal amount of time to acquire financial literacy and that literacy reduces the costs to access the financial markets: we show this in appendix C.

Finally, given that each individual has got one unit of time, $c(h)$ directly represents the share of time spent to acquire literacy and enter the financial markets the literacy and, from eq. (6), the actual cost paid is given by $w \cdot c(h)$.

3.2 The optimization problem

Each individual faces the decision on how much human capital to acquire and how much to save during working-life (which also determines consumption in period one and two). In addition, individuals have to choose whether to acquire financial literacy or not. Basically in the first case each individual has to choose the optimal amount of *h, S* and *R* whereas in the second case only *h* and *S* are chosen. These optimal amounts determine two different indirect expected utilities, one for each investment strategy. By

comparing these two indirect utilities, individuals will choose whether or not to invest in financial literacy and, accordingly, the optimal amount of education and savings. By convention, we assume that individuals do not invest in financial literacy when the two indirect utility are equal. We start describing the case where the individual chooses not to acquire financial literacy and we then move to the other case. At the end of this subsection we will be able to show the optimal choice of human capital, consumption and saving in the two cases and the resulting indirect expected utilities.

3.2.1 Safe assets only

If an individual does not acquire financial literacy then no literacy cost has to be paid but savings will be invested into the safe asset only. The individual budget constraint implies then

7) $c_1 = W - S = kh(1 - h^x) - S$

and

7a) $c_2 = S(1 + s)$.

Given the above equations for consumption, the individual maximizes the expected utility and the maximization problem is

8) $\max_{h,S} -e^{-a[kh(1-h^x)-S]} - \beta e^{-a[S(1+s)]}$.

The solution of the above allows us to obtain the indirect expected utility of an individual that chooses not to invest in financial literacy, which we will indicate with EU^*. In particular, the optimal solution implies

9) $h^* = \left(\frac{1}{1+x}\right)^{\frac{1}{x}}$

and

10) $S^* = \left[kh^*(1 - (h^*)^x) + \frac{\log \beta(1+s)}{a}\right]\frac{1}{2+s}$

where h^* and S^* are the optimal levels of human capital and saving: the derivation of eqs. (9) and (10) can be found in Appendix A.

The above equations also imply the following optimal lifetime income W^*

11) $W^* = kh^*(1 - h^*) = k\left(\frac{x}{1+x}\right)\left(\frac{1}{1+x}\right)^{\frac{1}{x}}$.

Finally, the indirect utility in this case is

12) $EU^* = -\left(\frac{2+s}{1+s}\right) e^{-a\left(\frac{1}{2+s}\right)\left((1+s)W^* - \frac{\log \beta(1+s)}{a}\right)}$.

3.2.2 Safe and Risky assets

An individual that has invested in financial literacy can invest savings in either the safe or risky asset, or both. However the individual needs to pay the cost to obtain literacy. Given the presence of this cost the budget constraint implies

13) $c_1 = W - c(h)w - S - R = kh(-h^x + h^{1-z}) - S - R$

and

13a) $c_2 \sim N(S(1+s) + R(1+r), R^2\sigma^2)$

where c_2 is now a stochastic variable (in fact, *r* is a stochastic variable).

Given the above equations for consumption, the maximization problem (for the expected utility EU_R) the individual is facing becomes[3]

14) $\max_{h,S,R} -e^{-a[kh(-h^x+h^{1-z})-S-R]} - \beta e^{-a\left[S(1+s)+R(1+r)-\frac{a\sigma^2R^2}{2}\right]}$.

The solution of the above allows us to obtain the indirect expected utility when an individual chooses to invest in financial literacy (and participate in the stock market), which we indicate with EU_F^*. In particular, the optimal solution implies

15) $h_F^* = \left(\frac{2-z}{1+x}\right)^{\frac{1}{x-1+z}}$

where h_F^* is the optimal amount of human capital when the individual decides to acquire the required financial literacy. Interestingly, if we compare eqs. (9) and (15) we have that $h_F^* > h^*$: in fact, given that human capital is also useful to abate financial literacy costs, individuals that choose to acquire financial literacy also find convenient to acquire more education. The above also yields the following lifetime net income (that is, life income net of the financial literacy costs).

16) $W_F^* = k\left(\frac{x-1+z}{2-z}\right)\left(\frac{2-z}{1+x}\right)^{\frac{1+x}{x-1+z}}$.

In addition, the optimal solution also implies:

17) $R_F^* = \frac{r-s}{a\sigma^2}$

and

18) $S_F^* = \left[W_F^* + \frac{\log\beta(1+s)}{a} - \left(2 + \frac{r+s}{2}\right)\frac{r-s}{a\sigma^2}\right]\left(\frac{1}{2+s}\right)$

where S_F^* and R_F^* are, respectively, the optimal amount of safe and risky investments. The derivation of eqs. (15), (17) and (18) can be found in Appendix A.

The indirect utility in this case is

19) $EU_F^* = -\frac{2+s}{1+s}e^{-a\left[(1+s)W_F^* - \frac{\log\beta(1+s)}{a} + \frac{(r-s)^2}{2a\sigma^2}\right]\left(\frac{1}{2+s}\right)}$.

[3] We exploit here a well-known property by which, for any given stochastic variable n_j distributed normally with mean *n* and variance σ_n^2, we have that $E(e^{-an_j}) = e^{-a(n-a\sigma_n^2/2)}$: see Varian (1993).

4 Acquisition of financial literacy and participation to the stock market

In the previous section we derived the optimal level of human capital, consumption and savings and the resulting indirect expected utilities. Within our set-up, individuals choose to acquire financial literacy and invest in the stock market when the resulting indirect utility is higher than the one obtaining in the alternative strategy. In terms of our model this happens when $EU_F^* > EU^*$, that is, given eqs. (11) and (18), for

$$20)\ -\left(\frac{2+s}{1+s}\right)e^{-a\left[(1+s)W_F^*-\frac{\log\beta(1+s)}{a}+\frac{(r-s)^2}{2a\sigma^2}\right]\left(\frac{1}{2+s}\right)} > -\left(\frac{2+s}{1+s}\right)e^{-a\left(\frac{1}{2+s}\right)\left[(1+s)W^*-\frac{\log\beta(1+s)}{a}\right]}$$

which implies

$$20a)\ W^* - W_F^* < \frac{1}{1+s}\frac{(r-s)^2}{2a\sigma^2}$$

and exploiting eq. (6) and inserting the value for the optimal lifetime incomes as given by eqs. (11) and (16) we obtain:

$$20b)\ \left(\frac{x}{1+x}\right)\left(\frac{1}{1+x}\right)^{\frac{1}{x}} - \left(\frac{x-1+z}{2-z}\right)\left(\frac{2-z}{1+x}\right)^{\frac{1+x}{x-1+z}} < \frac{1}{k^{1-\alpha}}\left[\frac{1}{1+s}\frac{(r-s)^2}{2\sigma^2}\right]$$

The above equation determines univocally whether individuals acquire financial literacy and, similarly, whether they invest in the stock market.

The left hand side of the eq. (20b) is decreasing in *x* and *z* for all the relevant values of the parameters (and for *z=1*, the LHS is always zero): therefore, individuals whose education is more effective in producing human capital or in abating financial literacy costs are more likely to acquire financial literacy and participate to the stock market. The analytical proof that the LHS (and $W^* - W_F^*$) are decreasing in *x* and *z* is contained in Appendix B.

The right hand side of the equation is increasing in the risk premium (*r-s*) and decreasing in the degree of risk aversion (in fact, the RHS is increasing in α), variance of the risky asset returns and returns from the safe asset, which implies that individuals are more likely to acquire financial literacy when the excess return of the risky asset is high, when its volatility is low and when their risk aversion is low. Basically these results tell us that when risky assets are more attractive individuals find more convenient investing in financial literacy.

The RHS also depends on income, as the latter influences both risk aversion and the opportunity costs to obtain literacy. However these effects are only due to the component *k* of income and the sign of the total effect depends on the value of α: for α<1 the component *k* has a negative effect on the RHS, for $\alpha > 1$ it has a positive effect and for $\alpha = 1$ it has no effect. Accordingly, for a given level of education, higher income individuals are more/less likely to acquire the literacy depending on the value of the elasticity of risk aversion with respect to income.

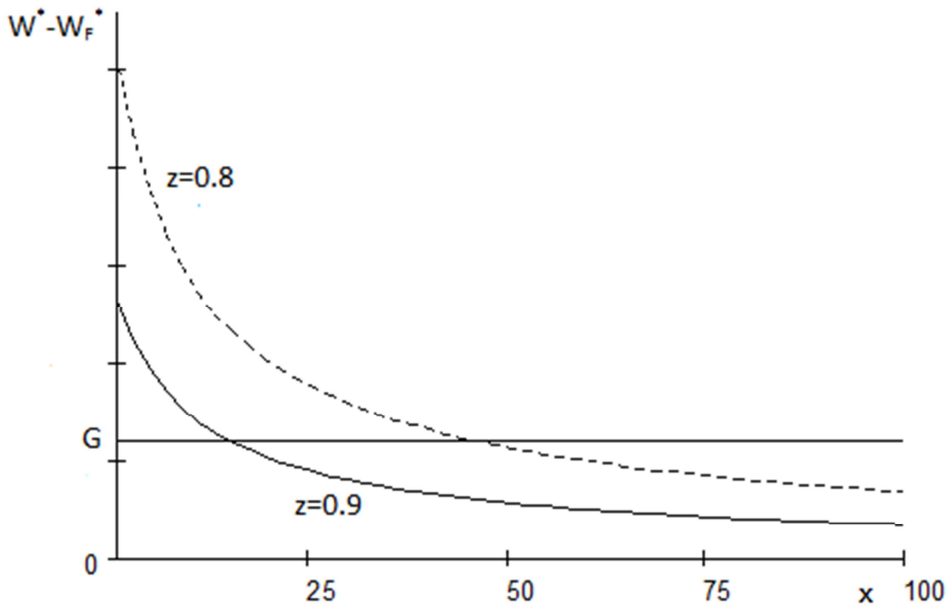

Figure 1

For illustrative purposes, we present in Figure 1 a graphic representation of inequality (20b), where $G \equiv \frac{1}{k^{1-\alpha}}\left[\frac{1}{1+s}\frac{(r-s)^2}{2\sigma^2}\right]$ is the RHS of the inequality and the two decreasing curves represent the LHS of inequality (20b), that is the difference between $W^* - W_F^*$, as a function of *x* for two distinct values of *z* (*z=0.8* and *z=0.9* for the higher and lower curve respectively).

For values of the curves below *G* individuals acquire financial literacy and enter the stock market. As it is clear from the decreasing pattern of the curves, only individuals with a value of *x* higher than a certain threshold actually acquire the financial literacy. Moreover, an increase in the value of *z* shifts the curve downward so that the threshold value of *x* becomes smaller.

4.1 The determinants of financial literacy and of stock market participation

In the light of the analysis carried out so far we can summarize our findings through the following propositions:

Proposition 1. *The share of individuals that acquire financial literacy and participate to the stock market is an increasing function of excess return of risky assets (r-s) and a decreasing function of the rate of return of the safe asset s and of the variance of the returns of the risky asset* (σ^2).

Proof. It follows naturally considering that the RHS of inequality (20a) is an increasing function of *(r-s)* and a decreasing function of *s* and σ^2. ■

Quite obviously, the share of individuals increases when the risky asset becomes more attractive in terms of mean returns and volatility. In addition, an increase of the return of the safe asset (keeping the differentials constant) reduces participation. This is due to the fact that when *s* increases the overall wealth increases and this magnifies the difference between W^* and W_F^* (as the former is strictly larger than the latter).

Proposition 2. *Individuals bestowed with larger values of x and z are more likely to acquire financial literacy and to participate the stock market.*

Proof. See Appendix B.

We can give a direct economic interpretation of these results. In fact, the variable *x* measures how effective individuals are in acquiring education and human capital (or the education system is in providing them): individuals with larger values of *x* acquire more human capital and thus, as a side effect of human capital and education, their cost to acquire financial literacy shrinks. Thus, they are more likely to acquire financial literacy and enter the stock market. The variable *z* measures how effective human capital and education are in abating the cost to acquire financial literacy: individuals with higher value of *z* have trained in education sectors closer to finance and their cost to acquire the financial literacy is smaller.

We now provide e corollary for a particular value of *z*, which helps to better interpret our results.

Corollary 1. *For $z = 1$ it descends: $c = 0$, $W^* - W_F^* = 0$ and $EU_F^* > EU^*$ so that r>s is a sufficient condition for individuals to obtain financial literacy and enter the stock market.*

Proof. Consider the case $z = 1$. From eq. (6) we have: $c(h) = 1 - h^{1-1} = 0$. From eqs. (11) and (16) we have $W^* - W_F^* = k\left(\frac{x}{1+x}\right)\left(\frac{1}{1+x}\right)^{\frac{1}{x}} - k\left(\frac{x-1+1}{2-1}\right)\left(\frac{2-1}{1+x}\right)^{\left(\frac{1+x}{x-1+1}\right)} = 0$. Finally, we know from (20a) that $EU_F^* > EU^*$ if and only if $W^* - W_F^* < \frac{1}{1+s}\frac{(r-s)^2}{2a\sigma^2}$ and given that, for $z = 1$, $W^* - W_F^* = 0$ then r>s is a sufficient condition to obtain $EU_F^* > EU^*$ and for individuals to obtain financial literacy and enter the stock market. ■

In words, when $z = 1$, we are depicting a case in which the main concepts of finance are taught and learnt within the standard education career of individuals and therefore the latter do not have to pay any extra costs to obtain financial literacy. In this case all individuals automatically obtain the financial literacy and find convenient to enter the stock market (though the amount of risky assets bought may be extremely low).

5 Savings and Wealth accumulation

In this section we explore the role that financial literacy has in determining wealth accumulation. Differently from what we have seen in previous section, where we analyzed the factors that determined the likelihood to acquire financial literacy for a given individual, we now assess whether individuals with financial literacy have a tendency, other things being equals, to accumulate more wealth than individuals without it. From this point of view, we need thus to compare individuals with and without the required literacy to invest in the risky assets.

For sake of simplicity, we define wealth as the total amount that has been accumulated during working-life, net of capitalization of interests: at the end of the section we discuss how our results are even more robust if we also include the capitalization of interests.

Preliminarily, we compute the total amount saved according to whether an individual acquires or not the literacy (and thus enters the stock market). In particular, if an individual does not acquire financial literacy the resulting total lifetime savings are equal to the amount invested in the safe asset:

1) $S^* = \left[W^* + \frac{\log\beta(1+s)}{a}\right]\frac{1}{2+s}$.

On the contrary, if an individual acquires financial literacy and enters the stock market, total life time savings are given by the sum of the amounts invested in the safe and in the risky asset, which is:

2) $S_F^* + R_F^* = \left[W_F^* + \frac{\log\beta(1+s)}{a}\right]\frac{1}{2+s} + \left(1 - \frac{r-s}{2}\frac{1}{2+s}\right)\frac{r-s}{2a\sigma^2}$.

Given the above results we can evaluate whether the acquisition of financial literacy implies higher wealth accumulation. Suppose then that we observe two different individuals: agent 1 has acquired financial literacy while agent 2 has not. The difference in their total saving is given by the following (the subscript identifies the agent):

3) $S_{F,1} - S_2 = \left[W_{F,1}^* - W_2^* + \frac{\log\beta_1(1+s)}{a_1} - \frac{\log\beta_2(1+s)}{a_2}\right]\frac{1}{2+s} + \left(1 - \frac{r-s}{2}\frac{1}{2+s}\right)\frac{r-s}{2a_1\sigma^2}$.

We can exploit the above to formulate the following proposition:

Proposition 3. *Suppose that two individuals are identical in terms of lifetime income, risk aversion and time preferences but that they differ in that one has acquired financial literacy and the other has not. Then, for* $r < 4 + 3s$*, the individual that has acquired financial literacy, and has entered the stock market, accumulates also more wealth.*

Proof. If individuals are identical in terms of their lifetime income, risk aversion and time preferences we have, by definition, $W_{F,1}^* - W_2^* = 0$, $a_1 = a_2$ and $\beta_1 = \beta_2$ and, consequently from eq. (23) we have $S_{F,1} - S_2 = \left(1 - \frac{r-s}{2}\frac{1}{2+s}\right)\frac{r-s}{2a_1\sigma^2}$. The latter is strictly positive for $r < 4 + 3s$. ■

The condition $r < 4 + 3s$ is satisfied for realistic values of the parameters so that our results imply that individuals that have acquired financial literacy accumulate, other things being equals, higher amount of wealth[4].

In our definition of wealth we did not include the capitalization of returns: if we had also included this factor, given that $r > s$, it is easy to see that the wealth of individuals with financial literacy would be, on average, larger than the wealth of those without literacy for an even less stringent condition than $r < 4 + 3s$. That is, literacy would imply, on average, an even larger accumulation of wealth. However, we focused our analysis on total savings because we want to stress that individual with financial literacy saves larger amounts and they accumulate larger wealth independently from the fact that risky assets have, on average, larger returns.

6 Main Findings and Reconciling Theory with Evidence

The model we have built delivers some clean-cut results on the determinants financial literacy acquisition and stock market participation. We now compare our theoretical findings with the stylized facts we described in section 2.

- **Education.** According to our results, there is a straightforward positive relationship between education and financial literacy. However, the degree of education and financial literacy are not

[4] Only for extremely high values of *r* the conclusion is reverted: this is probably due to income effect that, when *r* is extremely high, induces individuals that are investing in the stock market to consume more and save less.

perfectly correlated as the field of study (represented by *z* in our model) is also a key determinant. This is in line with the observed facts that schooling exerts a positive effect on financial knowledge although with a wide variability of financial literacy for a given level of schooling attainment and with literacy having an effect that goes beyond mere education level. Our model also indicates that education systems that are more effective in providing human capital (i.e. higher *x*) not only improve human capital accumulation, but also capital market participation and financial literacy acquirement.

- **Numeracy and math scores.** Within our model, individuals that have developed good abilities in these fields can be thought as having a higher *z* and thus, they should be more likely to acquire financial literacy, which is line with observed evidence.
- **Income.** Our results display a complex relationship between income and literacy, a relationship that can be broken down in three components. First, both income and literacy are positively influenced by education: this component implies a positive correlation but not a causal relationship between income and literacy. Beyond this correlation, however, we have a second component that is due to the fact risk aversion is decreasing income and thus, individuals with higher income would like to buy a higher amount of stocks and, consequentially, they find more convenient to acquire literacy in order to enter the stock market: therefore, this component implies then a causal positive relationship between income and literacy. Finally, higher income also implies higher opportunity costs and this induces a negative effect of income on the incentives to acquire financial literacy. Our model allows us to determine the overall effect of the latter two components and in particular we have that for $\alpha \geq 1$ (that is, when relative risk aversion is non-increasing in income) the overall effect is non-negative whereas it is negative otherwise. Consequentially, if we consider all the three components we can be certain that for $\alpha \geq 1$ income and literacy are positively correlated whereas for $\alpha < 1$ the sign of the relationship is undetermined. The assumption that relative risk aversion is non-increasing in income is usually the most realistic one (see MasColell et al. 2006, chapter 6) and thus, our model fully explains the observed positive correlation. Further empirical analysis aimed at disentangling the role of income from that of education would however be helpful to shed further lights on the ultimate nature of the relationship between these two factors.
- **Employment condition.** Individuals that are employed in sectors close to the business one are likely to have acquired forms of human capital which display higher complementarities with financial knowledge. In analytical terms, this implies higher *z* and thus, they should be more likely to acquire financial literacy. Similarly, self-employed should have higher *z* and, given the entrepreneurship nature of self-employment, they should probably have lower risk aversion (higher α in our model) further implying a higher probability of acquiring literacy. This mechanism is in line with the observed evidence.
- **Gender.** Our model does not include any gender differences and as such, it cannot account for gender gaps. However, our model shows how higher risk aversion reduces the incentive to acquire financial literacy. Given that there exists some evidence that female individuals have higher risk aversion (see Borghans et al. 2009) and are less likely to take risks in financial investments (see Powell and Ansic 1997), this could explain the gender differences in financial literacy. Further empirical analyses are in any case needed to discern whether risk aversion fully explains the gender gap or if a "pure" gender effect is present.
- **Wealth.** Our analysis shows that, for realistic values of the parameters, individuals that have acquired financial literacy accumulate, other things being equal, larger amounts of wealth. This result is due to the fact that these individuals save larger amounts and is therefore even more true if the access to stock market grants returns that are, on average, higher. These implications fit well

with observed evidence: in fact, Lusardi et al. 2007 shows that once we control for age, education and income, financial literacy affect positively wealth accumulation and several empirical analyses stress how individuals with literacy are more likely to plan to save more generously for their retirement.

- **Stock Market Participation.** Our model strongly relates the participation to stock markets to the acquisition of financial literacy. As such education and income, influencing literacy, also increase the likelihood to acquire stocks, something that has been observed in reality. Moreover, we highlight how schooling attainment alone is not enough to explain participation as the field of study is also a key determinant in participating in the stock market. Finally, and somehow obviously, risk aversion reduces the likelihood to acquire stocks. All these theoretical results find confirmation in the empirical evidence on stock market participation.
- **The attractiveness of risky assets.** Within our model, the more attractive (in terms of returns and volatility) the risky assets are, the larger the incentives to acquire financial literacy are. This aspect has not been explored empirically and it would be interesting to assess it, possibly exploiting differences in the attractiveness of financial markets of different countries.

6 Conclusions

In the present paper we have built a model that encompasses in a unified framework human capital investment, financial literacy acquirement, saving behavior and participation to the stock market. Our model seems to give precise account on the effects that education, field of study, income and employment conditions have on financial literacy and stock market participation. In addition, even if we do not include it directly in our model, the observed effect of gender can find a rationale within the framework we propose. Some of our theoretical results actually stimulate further empirical analysis: a first empirical implication is that individuals or countries with education systems that perform better in accumulating human capital should also have higher levels of financial literacy and higher participation rates to stock markets. A second empirical implication is that individuals from countries with education systems that allow for basic financial education should display higher participation rates to the stock market or higher financial literacy. Finally, it would be interesting to explore to what extent institutional factors such as gender wage-gap or more attractive capital markets can further explain differences in financial literacy attainment or stock market participation among individuals and countries. All these empirical implications are left for future empirical research.

APPENDIX A – Optimal levels of human capital and investment in safe and risky assets

We derive here the optimal levels h^* and S^* as given, respectively, by eqs. (9) and (10) and the optimal levels h_F^* , R_F^* and S_F^* as given, respectively, by eqs. (15), (17) and (18).

A.1 Derivation of h^* and S^*

Consider the optimization problem $\max_{h,S} -e^{-a[kh(1-h^x)-S]} - \beta e^{-a[S(1+s)]}$. The f.o.c imply

$$\text{A1)}\ \frac{\partial\left\{-e^{-a[kh(1-h^x)-S]} - \beta e^{-a[S(1+s)]}\right\}}{\partial h} = 0 \Rightarrow akh[1-(1+x)h^x]e^{-a[kh(1-h^x)-S]} = 0$$

and

$$\text{A2)}\ \frac{\partial\left\{-e^{-a[kh(1-h^x)-S]} - \beta e^{-a[S(1+s)]}\right\}}{\partial S} = 0 \Rightarrow ae^{-a[kh(1-h^x)-S]} - a\beta(1+s)e^{-a[S(1+s)]} = 0.$$

From (A1) we obtain h^*:

$$\text{A3)}\ h^* = \left(\frac{1}{1+x}\right)^{\frac{1}{x}}. \blacksquare$$

and from (A3), inserting h^*, we obtain S^*:

$$\text{A4)}\ S^* = \left[kh^*(1-h^{*x}) + \frac{\log\beta(1+s)}{a}\right]\frac{1}{2+s}. \blacksquare$$

A.2 Derivation of h_F^* , R_F^* and S_F^*

Consider the optimization problem $\max_{h,S,R} -e^{-a[kh(-h^x+h^{1-z})-S-R]} - \beta e^{-a\left[S(1+s)+R(1+r)-\frac{a\sigma^2R^2}{2}\right]}$. The f.o.c imply

$$\text{A5)}\ \frac{\partial\left\{-e^{-a[kh(h^{1-z}-h^x)-S-R]} - \beta e^{-a\left[S(1+s)+R(1+r)-\frac{a\sigma^2R^2}{2}\right]}\right\}}{\partial h} = 0 \Rightarrow$$

$$ak[(2-z)h^{1-z}-(1+x)h^x]e^{-a[kh(h^{1-z}-h^x)-S-R]} = 0;$$

$$\text{A6)}\ \frac{\partial\left\{-e^{-a[kh(-h^x+h^{1-z})-S-R]} - \beta e^{-a\left[S(1+s)+R(1+r)-\frac{a\sigma^2R^2}{2}\right]}\right\}}{\partial S} = 0 \Rightarrow$$

$$ae^{-a[kh(-h^x+h^{1-z})-S-R]} = a\beta(1+s)e^{-a\left[S(1+s)+R(1+r)-\frac{a\sigma^2R^2}{2}\right]};$$

and

$$\text{A7)}\ \frac{\partial\left\{-e^{-a[kh(-h^x+h^{1-z})-S-R]} - \beta e^{-a\left[S(1+s)+R(1+r)-\frac{a\sigma^2R^2}{2}\right]}\right\}}{\partial R} = 0 \Rightarrow$$

$$ae^{-a[kh(-h^x+h^{1-z})-S-R]} = a\beta[(1+r)-a\sigma^2R]e^{-a\left[S(1+s)+R(1+r)-\frac{a\sigma^2R^2}{2}\right]}.$$

From (A5) we obtain h_F^*:

A8) $h_F^* = \left(\frac{2-z}{1+x}\right)^{\frac{1}{x-1+z}}$. ■

Moreover, combining (A6) and (A7) we obtain R_F^*:

A9) $R_F^* = \frac{r-s}{a\sigma^2}$. ■

Finally, inserting (A9) in (A6) we obtain S_F^*:

A10) $S_F^* = \left[W_F^* + \frac{\log\beta(1+s)}{a} - \left(2 + \frac{r+s}{2}\right)\frac{r-s}{a\sigma^2}\right]\left(\frac{1}{2+s}\right)$. ■

APPENDIX B – Proof of Proposition 2

We prove here Proposition 2. From inequality (20a) we know that individuals acquire financial literacy and enter the stock market if $W^* - W_F^*$ is lower than $\frac{1}{1+s}\frac{(r-s)^2}{2a\sigma^2}$. Here we prove that $W^* - W_F^*$ is a decreasing function of *x* and *z* and, therefore, individuals with higher *x* or *z* are more likely to acquire financial literacy and enter the stock market.

Formally, we need to prove that

B1) $\frac{\partial(W^* - W_F^*)}{\partial x} < 0$

and

B2) $\frac{\partial(W^* - W_F^*)}{\partial z} < 0$.

Preliminarily, we define the following:

B3) $g \equiv \frac{1}{1+x}$

B4) $p \equiv \frac{2-z}{1+x}$

B5) $b(y) = \ln y + 1 - y$

B6) $f(y) = y^{\frac{y}{1-y}}$

B7) $h(y) = f(y)^{f(y)}$.

Note that by definition, we have that $0 < g < p < 1$ and that, for $y > 0$, $f(y) > 0$ and $h(y) > 0$.

Starting from eq. (11) and given definition (B3), we have

B8) $W^* = k\left(1 - \frac{1}{1+x}\right)\left(\frac{1}{1+x}\right)^{\frac{1}{x}} = k(1-g)(g)^{\frac{g}{1-g}}$.

Similarly, starting from eq. (16) and given definition (B3) and (B4) we have

B9) $W_F^* = k\left(1-\frac{2-z}{1+x}\right)\left(\frac{2-z}{1+x}\right)^{\frac{2-z}{x-1+z}} = k(1-p)(p)^{\frac{p}{1-p}}$

$$W_F^* = k\left(1-\frac{2-z}{1+x}\right)\left(\frac{2-z}{1+x}\right)^{\frac{2-z}{x-1+z}} = k(1-p)(ap)^{\frac{p}{1-p}}$$

We now present four lemmas and we use them to prove eqs. (B1) and (B2).

Lemma 1 *The following properties hold true:* $y > 0 \Rightarrow b(y) \leq 0$ *and* $y > 0 \cap y \neq 1 \Rightarrow b(y) < 0$.

Proof. Starting from definition (B5), we have that $\frac{\partial b(y)}{\partial y} = \frac{\partial(\ln y+1-y)}{\partial y} = \frac{1}{y} - 1$. Therefore $\frac{\partial[b(y)]}{\partial y}$ is positive for *y<1*, zero for *y=1* and negative for *y>1*. It follows that $b(y)$ has a global maximum for *y=1*. We have then that $\max_y b(y) = b(1) = 0$ which implies that $b(y) \leq 0$. Moreover, given that $b(y)$ is zero only for *y=1*, we also have that $b(y) < 0 \; \forall \; y > 0 \cap y \neq 1$. ■

Lemma 2. *The following property holds true:* $y > 0 \cap y \neq 1 \Rightarrow \frac{\partial f(y)}{\partial y} < 0$.

Proof. From definition (B5) and (B6) we compute $\frac{\partial f(y)}{\partial y} = \frac{y^{\frac{y}{1-y}}(\ln y+1-y)}{(y-1)^2} = \frac{y^{\frac{y}{1-y}}b(y)}{(y-1)^2}$. From Lemma 1 we know that $b(y) > 0 \; \forall \; y > 0 \cap y \neq 1$, thus: $\frac{\partial f(y)}{\partial y} < 0 \; \forall \; y > 0 \cap y \neq 1$. ■

Lemma 3. *The following property holds true:* $1/e < f(p) < f(g)$.

Proof. Given that $0 < g < p < 1$, from Lemma 2 we necessarily have $f(p) < f(g)$. Moreover, given Lemma 2, the infimum of $f(p)$ is obtained for the supremum of *p*, which, given the definition of *p,* is obtained for *z=0* and $x \to 1$. Therefore, $\inf f(p) = \lim_{x\to 1}\left(\frac{2}{1+x}\right)^{\frac{1+x}{x-1}}$. We prove now that $\lim_{x\to 1}\left(\frac{2}{1+x}\right)^{\frac{1+x}{x-1}} = 1/e$. We define $v \equiv \frac{1-x}{1+x}$ and rearranging $\lim_{x\to 1}\left(\frac{2}{1+x}\right)^{\frac{1+x}{x-1}}$ we obtain $\lim_{v\to 0}\left[\left(1+\frac{1}{v}\right)^{\frac{1}{v}}\right]^{-1}$ that, by definition, is equal to $1/e$. We then have $1/e < f(p) < f(g)$. ■

Lemma 4. *The following property holds true:* $0 < y \leq p \Rightarrow \frac{\partial h(y)}{\partial y} < 0$.

Proof. From definitions (B5), (B6) and (B7) we compute $\frac{\partial h(y)}{\partial y} = \frac{h(y)\frac{\partial f(y)}{\partial y}[1+\ln f(y)]}{y}$. For $0 < y \leq p$ we know: *i)* by definition, that *y>0* and *h(y)>0* are positive; *ii*) from Lemma 2, that $\frac{\partial f(y)}{\partial y} < 0$; and *iii)* from Lemma 3, that $[1+\ln f(y)] > 0$. Therefore we necessarily have that, for $0 < y \leq p$, $\frac{\partial h(y)}{\partial y} < 0$. ■

We use now the four lemmas to prove (B1) and (B2).

Proof of A1. We can rearrange $\frac{\partial(W^*-W_F^*)}{\partial x}$ as $\frac{\partial(W^*-W_F^*)}{\partial x} = \left(\frac{\partial W^*}{\partial g}\frac{\partial g}{\partial x} - \frac{\partial W_F^*}{\partial p}\frac{\partial p}{\partial x}\right)$. From the latter, using definition (B3), and (B4) and eqs. (B8) and (B9), we obtain $\frac{\partial(W^*-W_F^*)}{\partial x} = -kg\left(g^{\frac{g}{1-g}}\ln g^{\frac{g}{1-g}} - p^{\frac{p}{1-p}}\ln p^{\frac{p}{1-p}}\right)$ and, from definitions (B6) and (B7), we obtain $\frac{\partial(W^*-W_F^*)}{\partial x} = -kg\left[\ln f(g)^{f(g)} - \ln f(p)^{f(p)}\right] = -kg[\ln h(g) - \ln h(p)]$. Consider now $[\ln h(g) - \ln h(p)]$: we know from Lemma 4 that $0 < y \leq p \Rightarrow \frac{\partial h(y)}{\partial y} < 0$ and given that $g < p$ it follows that $h(g) > h(p)$ and $[\ln h(g) - \ln h(p)] > 0$. Then $\frac{\partial(W^*-W_F^*)}{\partial x} = -kg[\ln h(g) - \ln h(p)] < 0$. ■

Proof of A2. We can rearrange $\frac{\partial(W^*-W_F^*)}{\partial z}$ as $\frac{\partial(W^*-W_F^*)}{\partial z} = \left(\frac{\partial W^*}{\partial g}\frac{\partial g}{\partial z} - \frac{\partial W_F^*}{\partial p}\frac{\partial p}{\partial z}\right)$. From the latter, given that $\frac{\partial g}{\partial z} = 0$ and given eq. (B9), we obtain $\frac{\partial(W^*-W_F^*)}{\partial z} = -k\frac{p^{\frac{p}{1-p}}}{1-p}\frac{\partial p}{\partial z}\ln p$. From definition (B4) we have that $\frac{\partial p}{\partial z} = -g < 0$ and since *0<p<1* we have that $-k\frac{p^{\frac{p}{1-p}}}{1-p}\frac{\partial p}{\partial z}\ln p < 0$. ■

APPENDIX C – Financial literacy costs equation and the optimal financial literacy

In this appendix we provide a micro-foundation for eq. (6), which describes the opportunity costs that individuals face when entering financial market. The basic idea is that investments in risky assets are complex and they require time to evaluate, choose and keep track of the assets. Moreover, the time lost in this process depends, inversely, on the financial literacy of individuals. Therefore, individuals choosing to invest in risky assets may find convenient to invest part of their time also to acquire financial literacy to abate, through this, the time lost to participate to the financial market.

In particular, we show here that eq. (6) can be seen as the opportunity costs to enter the financial market once individuals have chosen, optimally, financial literacy.

Our assumption is that the time lost to participate to the financial markets is made of two components. The first one represents the time lost to evaluate, choose and track the risky assets and it thus depends inversely on the level of financial literacy *L*: in particular, we assume that the time lost for this reason, *q*, is given by the following function

C1) $q = 1 - 2L$

which is clearly decreasing in the level of financial literacy.

In addition, individuals can invest part of their time to acquire financial literacy and this makes up the second component related to the time lost when entering financial markets. In particular, we assume that individuals that invest *f* units of time to acquire literacy obtain the latter according to the following production function:

C2) $L = (fh^{1-z})^{1/2}$.

Clearly, the outcome in terms of literacy is increasing in time invested and, moreover, the marginal productivity of time depends on the education of individuals (*h*) and on how useful the specific education is in the acquisition of literacy (something that, as we have already seen, is measured by *z*). Basically, individuals with higher education or with a field of study closer to finance acquire more literacy per unit of time invested. Finally, we assume that the marginal productivity of time is decreasing.

Given these premises, the cost *c* to enter the financial markets is given by the sum of *q* and *f* which, from eq. (C1) and (C2) is thus given by:

C3) $c(L,h) = q + f = 1 - 2L + L^2 h^{z-1}$.

Eq. (C3) is the equivalent of eq. (6) once we explicitly consider the investment in financial literacy. Moreover, if we allow individuals to optimally choose financial literacy *L* we can reformulate the maximization problem of expected utility EU_F that individual face when investing in risky assets as follows:

C4) $\max_{h,L,S,R} -e^{-a[kh(1-2L+L^2h^{z-1})-S-R]} - \beta e^{-a\left[S(1+s)+R(1+r)-\frac{a\sigma^2R^2}{2}\right]}$.

The above maximization problem is equivalent to the one described by eq. (14) with the only difference that individuals now also choose their optimal level of financial literacy *L*.

From the above maximization problem of EU_R we can obtain the optimal value L^* of financial literacy, in particular:

C5) $\frac{\partial EU_R}{\partial L} = 0 \Rightarrow -akh(-2 + 2Lh^{z-1})e^{-a[kh(1-2L+L^2h^{z-1})-S-R]} = 0.$

From which we obtain:

C6) $L^* = h^{1-z}$

which, together with eq. (C3), implies:

C7) $c(L^*,h) = 1 - h^{1-z}$.

The above equation is identical to eq. (6) and it shows how the latter describe the costs to participate to financial markets once financial literacy is chosen optimally.

Moreover, eq. (C6) directly determines the optimal degree of financial literacy chosen by individual that invest in the financial markets, which is a positive function of education *h* (and thus of *x*) and of *z*.

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