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The social rate of discount, climate change and real options

by

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1.Introduction

The effect of dynamic uncertainty on the determination of a social rate of discount (the SRD) is one of the critical points of cost benefit analysis. From the theoretical point of view, an abundant literature exists on the interpretation to give to the SRD, on its meaning, on the estimation techniques (for a valid taxonomy see Munroe, 1981 and for an updated review, Frederick, Lowenstein and O' Donoghe, 2002). Different authors have conceived the SRD as the pure rate of time preference of society, as the rate of discount based on the elasticity of the marginal utility of consumption with respect to the changes of per-capita consumption, as the rate that measures the decline in time of the value of an additional unit of public income, as the opportunity cost of capital for society or, more simply, as a tool to choose projects in the context of a limited capital account budget. The recent literature (Dixit and Pindyck, 1994), which extends the real option methodology to project evaluation, in particular, seems to suggest that the discount rate should be increased to take into account of project specific sources of dynamic uncertainty.

These considerations assume peculiar significance in the light of the growing importance of climate change and the need to evaluate both mitigation and adaptation programs. The amount of resources that society should be willing to commit to these programs, in fact, critically depends on the value assigned to expected benefits over a long time horizon. As the controversy arisen by the Stern review (2006) has demonstrated[1], the choice of different discount rates causes profound differences in the value assigned to expected damages from climate change, both because of the very long period of gestation that these damages entail and because of the deep uncertainty characterizing the size and the timing of the damages.

[1] See, for example, Nordhaus (2007) and Weitzman (2007).

While climate change is often characterized only as an increase in average temperature, a second, relevant feature is its volatility. Volatility, measured as the annualized standard deviation (ASD) of the change of a random variable over time (called a stochastic process), is a measure of the uncertainty ensuing from time varying (technically "non stationary") probability distributions. As a key indicator of this change, volatility measures the variability of a stochastic process per unit of time (usually a year) and can be reported both in absolute value, as ASD or in percentage of the mean value of the change per unit of time. When the process on hand is the percentage change of a random variable, volatility is directly measured in percentages.

Time changing variability constitutes a characterizing feature of climate change. In fact, weather changes are perturbed by "nuisances" that appear to be persistent, increasing and irregular, so that measuring variances (e.g., swings in weather to greater extremes) has been central to assessing the biological and economic consequences of climate change (Albritton et al. 2001). The significance of volatility is twofold. First, the uncertainty on the future value of climate change as a stochastic process combining temperature , rainfall and other critical variables, depends critically on how much the spread of the underlying distribution is changing over time. This effect, which has been called the "funnel of uncertainty", is the consequence of the fact that, because of the central limit theorem, when uncertainty derives from the sum of sufficiently many random phenomena, the variance of the process tends to increase linearly with time. Thus, the larger the volatility (the variability per unit of time), the proportionally larger the variability over time. Second, the greater the volatility, the greater the value of the any "real option" that can be exercised by implementing an investment that yields benefits as a function of the process considered. This second effect depends on the fact that a real option consists of a faculty that can be exercised profitably under some, but not all, states of nature. The greater the uncertainty on whether the favorable states will occur, the greater the incentive to hold, rather than to exercise, the option. This implies that holding the option will have higher value the higher the volatility of the underlying stochastic process.

If one uses the usual Ramsey formula, the impact of climate change uncertainty on the discount rate appears in principle to be positive, since the same parameter of the utility function, the elasticity of marginal utility of consumption, governs both risk aversion and the rate of time preference. The impact of uncertainty on optimal growth, however, is clearly called into question and, in addition to risk aversion, "prudence" should also be considered . Leland (1968) first demonstrated that non zero precautionary savings imply convex marginal utility in addition to risk aversion, but only in 1990 Kimball (1990) proposed $\gamma(w) = -U'''(w)/U''(w)$ as a measure of absolute prudence, and $\rho(w) = -wU'''(w)/U''(w)$ as a measure of relative prudence. Kimball showed that, while risk aversion can be interpreted as a measure of the intensity of the desire for insurance, prudence can be similarly interpreted as a measure of the intensity of the desire to defer consumption (and accumulate savings) for precautionary reasons. Thus, climate change uncertainty should affect the willingness of economic agents to defer consumption. At the same time, it is likely to impact on the rate of return to capital, by reducing its marginal productivity , depressing investment, and increasing depreciation of all forms of capital, as well as depletion of natural resources.

These considerations take further meaning if we consider the two related issues of food security and human capital. Climate change impacts food security in a major way, since it threatens to make obsolete the traditional strategies of food production based on increasing average productivities through specialized cultivars and selected seeds. This implies a shift to policies that reduce exposure and sensitivity to climate change uncertainties, by diversifying production and adopting long term strategies based on tree crops, forestry and conservation of natural capital. Building up human capital to create higher adaptability and resilience is also a more likely response than traditional investment in infrastructure and irrigation. The economic attractiveness of both these strategies, however, depend on the level of the social rate of discount, i.e. on the willingness of society to defer consumption in the short run for longer term projects as well as for precautionary investment .

From a formal point of view, the social rate of discount is linked to optimal stochastic growth as a dual solution both of a consumption and a production problem. From the consumption side , the social rate of discount rules the utility function and thus determines the conditions at which intertemporal allocation should favor consumption versus savings. From the production side, the social rate determines production allocation over time and thus capital accumulation. With complete markets the two problems can be solved independently of each other by virtue of Fisher's separation theorem. However, elementary equilibrium considerations require that in order to achieve optimal growth with no distortions the two rates are the same. Furthermore, dynamic uncertainty and investment irreversibility imply that production can be used to create real options , which in turn create opportunity costs for capital accumulation in addition to those generated by foregone consumption.

2. Optimal stochastic growth

A recent literature has evolved on the costs of climate change (CC). This literature (see, for example, World Bank, 2010), deals with CC essentially as an adaptation problem by applying the methodology of cost effectiveness evaluation. Thus, a baseline is defined, representing the situation without climate change and costs of adaptation are obtained by estimating the additional costs that climate change and a certain level of adaptation would entail with respect to the baseline. An "adaptation deficit" is also defined and estimated as a difference between the desirable and achievable level of adaptation[2], the difference being due to a lack of "capacity" (World Bank, 2010, p.18).

[2] According to the World Bank study cited (World Bank, 2010, p.17-18), *adaptation deficit* has two meanings in the literature on climate change and development. One captures the notion that countries are underprepared for current climate conditions, much less for future climate change. Presumably, these shortfalls occur because people are under-

Although this way of proceeding may provide an effective first cut to estimate the additional resources needed to face climate change, it misses one important dimension: the nature of adaptability as a form of contingent wealth in the hands of individuals and communities. In addition to the costs to perform specific measures of adaptation to face some broadly predictable effects of climate change, in fact, economic rationality requires resources to be devoted to preventive measures that increase the preparedness and the flexibility of the communities threatened and enhance their capacity to deal with the unpredictable side of the future. Even in a situation without expected climate change, the capability to endure unpredictable environmental changes is valuable.

This preventive enhancement of the capability to face future threats (and perhaps opportunities) is what we call adaptability. Since adaptability is, at the same time, a capability that can be exploited and can be further developed, it is important to distinguish the adaptation actions, i.e. the actions that can be implemented to exercise existing options, from the actions that increase the number, the variety and the values of the options to adapt in the face of future and uncertain change Scandizzo, 2010). The exercise of adaptation options in the attempt to cope with changes in the environment that have already occurred or are expected to occur may return a flow of expected benefits (or higher or lower levels of adaptation at a cost) , but will also have an effect, in general, on further ability to adapt. The effects on adaptability of adaptation are not necessarily positive and occur mainly through the creation and the destruction of other options, and these are particularly relevant because of the uncertainty and the irreversibility characterizing climate change.

A long term strategy of adaptation thus requires an investment in adaptability, that may go beyond and may partly contradict the logic of "coping" with climate change through short term adaptations. In particular, in order to counter the destruction of options threatened by the joint action of current economic activities (some of which are of adaptive nature) and climate change, a new set of capabilities is needed to improve the adaptability of the local economic system. These capabilities should in part address the need to slow down or, if possible, to suppress the negative interactions between the local economy and the ecosystem. In part, they should also allow the local stakeholders to deal in new and creative ways with the challenges of climate and the governance of the commons under deep uncertainty.

In line with this approach, in this paper I assume that climate change has two distinct effects on the economy, one that de-stabilizes established consumption patterns and one that affects capabilities in production and capital accumulation. I also assume that these two effects occur as a consequence of two parallel stochastic processes of the geometric Brownian motion (GBM) variety. On the consumption side, de-stabilization is mostly a threat to food security, while on the production side, it is human and natural capital that are mostly involved in the new, critical choices.

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informed about climate uncertainty and therefore do not rationally allocate resources to adapt to current climate events…. The second, perhaps more common, use of the term captures the notion that poor countries have less capacity to adapt to change, whether induced by climate change or other factors, because of their lower stage of development."

In the case of consumption, a primary GBM process threatens the established level of consumption and growth, by modifying both the trend and the volatility of consumption changes over time:

(1) $dC(t) = g(t)C(t)dt + \sigma C(t)d\xi$

In (1) $C(t)$ is consumption at time t, $g(t)$ growth at time t and $d\xi$ is the increment of a Weiner process, i.e. a random variable with mean zero and variance equal to dt.

For production, on the other hand, the CC effects can be described as the impact on production capacity over time, given a production function of the type:

(2) $Q(t) = A(t)F(K(t))$

where $Q(t)$ is production $K(t)$ denotes capital, and $A(t)$ is the Solow residual . This residual, which is generally identified with neutral technological progress, is here taken to represent the net effect of climate change as an exogenous shock affecting both production capabilities and investment behavior . This shock is also assumed to evolve according to a Brownian motion with drift:

(3) $dA(t) = -\alpha(t)A(t)dt + \sigma_p A(t)d\zeta$,

where $d\zeta$ is the increment of a Weiner process, -$\alpha(t)$ is the expected effect on production of the exogenous shock at time t and σ_p is the standard deviation of the process.

Consider the stochastic optimal control problem, where C and A are ruled by the processes in (1) and (2). Assuming a well behave welfare function concave in consumption , optimal stochastic growth can be defined as the solution of the problem :

(4) $$\max_C E\int_0^T e^{-\phi t} U(C(t))dt$$

$$\dot{K} = AF(K) - \frac{AF(K)}{\beta_1} - (\delta + \alpha)K - EC$$

$\lim_{c \to 0} EU(C) = \propto$ Inada condition

$\lim_{T \to \propto} e^{-\phi T} EU_c(C(T))K(T) = 0$ Transversality condition

In (4) and all the subsequent formulas, E indicates the forward expectation operator, i.e. the expectation over the future states of the world, given the information available at the time when the operator is applied, ϕ is a rate of discount , $U(C(t)$ is a well behaved utility function and the objective function is an Ito integral. The investment condition states that capital formation equals the amount produced through a convex production function, adjusted for uncertainty (the option value of investment) minus net capital depreciation adjusted for the climate change trend $\delta + \alpha$, minus expected consumption.

The differential equation for capital formation in (4) can be derived using well known results from real options theory (Dixit and Pyndyck, 1994), which implies that aggregate benefits for irreversible investment can be expressed as the following extended net present value (ENPV):

(5) $ENPV = AF(K) - \dot{K} - bA^{\beta_1}$

where β_1 is the positive root of the characteristic equation: $\rho + \beta_1\alpha - \frac{\beta_1(\beta_1 - 1)}{2}\sigma_p^2 = 0$. The term bA^{β_1} represents the value of the option to wait before investing due to the dynamic uncertainty affecting technological progress.

Optimality requires:

(6) $AF_k - \dot{K} - A^{\beta_1} - = 0$ Value matching

(7) $\frac{1}{\beta_1} AF_k = bA^{\beta_1}$ Smooth pasting

This last condition identifies the value of the option to wait as equal , at the time of investment adoption , to a fraction $\frac{1}{\beta_1}$ of production, which is larger the larger is uncertainty, and for the limiting value of $\frac{1}{\beta_1}$=1 (infinite uncertainty) equals the entire production. The existence of this option depends on the expandability nature of production and is thus a direct function of adaptability, in the sense that production can be used to create consumption goods, investment goods for immediate deployment and investment goods for future deployment (waiting or expansion options). In the face of climate change, I also assume that aggregate investment is irreversible, in the sense that it is implemented through a combination of investment goods and foregone opportunities that, once realized, cannot be fully recovered .

Dropping for simplicity the time argument from the variables , the Hamiltonian from (4) is:

(8) $$H = e^{-\phi t} EU(C) + \mu E[\frac{\beta_1 - 1}{\beta_1} AF(K) - (\delta + \alpha)K - C]$$

FOC for the control variable are:

(9) $$\frac{\partial H}{\partial C} = EU_c - \mu = 0$$

where U_c is the first derivative of the utility function.

Differentiating the LHS of (6A) w.r.t. time and applying Ito's lemma, we obtain the rate at which the value of the numeraire falls over time.

(10a) $$-\frac{\dot{\mu}}{\mu} = \frac{d(e^{-\phi t} E(U_c(C))/dt}{e^{-\phi t} E(U_c(C))} = \phi + \eta g - \eta\gamma\sigma^2$$

where $\eta = -\frac{U_{cc}}{U_c} C$ is the coefficient of relative risk aversion and $\gamma = -\frac{1}{2}\frac{EU_{ccc}}{EU_{cc}} C$ is the index of relative prudence defined by Kimball[3] (1990). For an

[3] Kimball defines prudence as " the propensity to prepare and forearm oneself in the face of uncertainty". He shows that, given a consumption function $C_0(w)$ as a function of consumer's wealth, in conditions of zero uncertainty, for a small uncertainty σ^2, the consumption function satisfies : $C_0(w + \pi(\sigma_y^2, s)) = C_0(w)$, where $s = w - C_0(w)$ is the level of savings in the absence of uncertainty and $\pi(\sigma_y^2, s) = -\frac{1}{2}\frac{U'''(s)}{U''}\sigma_y^2 + o(\sigma_y^2).$

iso-elastic utility function (constant relative risk aversion or CRRA) $U = \frac{C^{1-\eta}}{1-\eta}$, in particular, equation (10a) yields:

$$\text{(10b)} \qquad -\frac{\dot{\mu}}{\mu} = \phi + \eta(g - \frac{1}{2}(1+\eta)\sigma^2)$$

As expressions (10a) and (10b) show, if we take consumption as the numeraire, its value falls over time with the rate of growth (future consumption is less valuable today the higher the expectations to be richer in the future) and raises with volatility and prudence (the higher uncertainty, the more valuable future consumption for a prudent individual) .

Along the optimum path, μ, the opportunity cost of capital equals the expected value of the marginal utility of consumption. The social discount rate from the production side can be obtained from the FOC for the state variable as the rate at which capital opportunity cost decreases with time:

$$\text{(11)} \quad \dot{\mu} = -\frac{\partial H}{\partial K} \rightarrow \qquad -\frac{\dot{\mu}}{\mu} = [\frac{\beta_1 - 1}{\beta_1} AF_k - \delta - \alpha]$$

Note that the term $\frac{\beta_1 - 1}{\beta_1}$, which multiplies the marginal productivity of capital , equals the inverse of the so called "hurdle rate", i.e. the amount by which investment returns should be greater than expected returns to adopt irreversible investments under dynamic uncertainty (Dixit and Pyndick, 1994). From the production side, the social discount rate under uncertainty is thus the net marginal productivity of capital, reduced of the so called "hurdle rate", minus depreciation plus technical progress. In other words, the "supply" discount rate , i.e. the rate at which the value of the numeraire (private consumption) falls over time as a consequence of greater accumulation of capital, is lowered by the uncertainty and the negative expected effects of climate change ,while it is increased by efficiency (a higher marginal productivity of capital).

Substituting (10) into (11):

$$\text{(12)} \quad \phi + \eta g - \eta\gamma\sigma^2 = \frac{\beta_1 - 1}{\beta_1} AF_k - (\delta + \alpha)$$

The LFS of expression (12) is the Consumption Rate of Interest or CRI, while the RHS is the marginal social rate of return to capital or MSSRI.

Solving for g:

(13a) $g = \frac{1}{\eta}(\frac{\beta - 1}{\beta} AF_k - \phi - (\delta + \alpha) + \gamma\sigma^2$

and, in the case of a CRRA function:

(14) $g = \frac{1}{\eta}[(\frac{\beta_1 - 1}{\beta_1})AF_k - \phi - (\delta + \alpha)] + \frac{1}{2}(1+\eta)\sigma^2$

Optimal stochastic growth is achieved when the Consumption Rate of Interest (CRI) equals the Marginal Social rate of Return to Capital (MSSRI) and can be divided into two components: the first, analogous to the Ramsey-Solow optimal growth, is proportional to the difference between the marginal productivity of capital corrected to account for the option value (e.g. the investment opportunities) created by the uncertainty of climate change, the rate of time preference in the objective function and the expected negative effect of climate change. The second, proportional to the volatility of consumption, is the result of the accumulation of prudential savings. Because marginal productivity will go to zero for a sufficiently large accumulation of capital, the solution in (13) and (14) implies that , in the absence of technical progress, long run growth may still go on, but it is entirely contingent on the availability of prudential savings(i.e. savings accumulated because of prudence and volatility of consumption).

2. The dual solution: the social rate of discount

We consider now the problem of optimal growth from the dual point of view, namely the determination of an appropriate system of shadow prices and their variation over time to value investment. This will give us the opportunity to achieve at the same time higher simplicity and generality, while solving the same optimal stochastic growth problem.

Our first parameter is the so called social rate of discount (SRD). As remarked before, this measures the rate at which the social numeraire loses its value with time . The SRD should take into account the fact that the opportunity cost of public investment is produced: a) from foregoing alternative investment opportunities; b) from the effects of investment on private consumption; c) from the effects of investment on public income. Accordingly, the SRD can be expressed as follows:

$$SRD = CRI + \Delta v$$

where CRI represents the consumption rate of interest, i.e. the private rate of time preference for present goods in comparison to future goods (reflected in the rates of interest on the capital market) and Δv represents the decrement (or, in some cases, the increase) of the weight of the public income in comparison to private consumption per unit of time. The magnitude of Δv depends on subjective-political judgment, so that, without loss of generality , I will assume that it is zero.

The basis for computing the *SRD* is thus the Consumption Rate of Interest (*CRI*), which we have already encountered in the solution of the primal problem, but can be now directly defined as the rate at which the present value of utility falls over time. This means that, given a utility function $U(C)$, the *CRI* can be determined as:

$$(14)\quad CRI = -\frac{\partial(e^{-\phi t}U_c(C(t))/\partial t}{e^{-\phi t}U_c(C(t))} = \phi + \eta g$$

where t stands for time, ϕ is the rate of time preference, $\eta = -\frac{U_{cc}}{U_c}C$ is the elasticity of the marginal utility of consumption (primes denote derivatives), and $g = \frac{dC/dt}{C(t)}$ is the rate of growth of consumption.

Assume again that, as a consequence of climate change, consumption is affected by random disturbances that make it follow a stochastic process of the Brownian motion variety:

$$(15)\quad dC = gCdt + \sigma Cd\xi$$

where $d\xi$ is a random variable with mean zero and variance equal to dt. Applying Ito's lemma, we find:

$$(16)\quad CRI = -\frac{d(e^{-\phi t}E(U_c(C))/dt}{e^{-\phi t}E(U_c(C))} = \phi + \eta g - \eta\gamma\sigma^2 \ ,$$

where $\gamma = -\frac{1}{2}\frac{EU_{ccc}}{EU_{cc}}C$ is the index of relative prudence defined by Kimball[4] (1990). For an iso-elastic utility function (constant relative risk aversion or CRRA) of the type considered in section 1, $U = \frac{C^{1-\eta}}{1-\eta}$, in particular, equation (3) yields:

[4] Kimball defines prudence as " the propensity to prepare and forearm oneself in the face of uncertainty". He shows that, given a consumption function $C_0(w)$ as a function of consumer's wealth, in conditions of zero uncertainty, for a

(17) $CRI = \phi + \eta g - \frac{1}{2}\eta(1+\eta)\sigma^2$

In the *CRRA* case, where absolute risk aversion increases with consumption, the stochastic nature of the underlying process determines a reduction of the social rate. For example, assume that the pure rate of time preference is 2%, the elasticity of marginal utility of consumption is 1 and the growth rate of consumption is 5%. If we do no take account uncertainty, the *CRI* would be equal to 7%. If we consider the "prudence" term of expression (17), however, even for a modest value of the volatility of 5%, the *CRI* would be reduced to 2%. For values of the volatility higher than 7%, the *CRI* would become negative.

What is the reason of this result, which, of course, is reversed in the case of absolute risk aversion decreasing with consumption? Basically, the idea is that when risk aversion (in the absolute sense) increases with the level of consumption, prudent behavior should induce people to reduce present consumption (and increase precautionary savings) as a precaution for future states of natures that not only may be unfavorable, but, because of the increasing risk aversion, are weighed more heavily than earlier unfavorable ones. Thus, if present consumption with no uncertainty is C_0, for a given level of wealth, a level of wealth higher of an amount equal to the third term on the RHS of (17) will be needed for an uncertainty level measured by the parameter σ^2.

3. The option value of capital and the equilibrium rate of growth of consumption

According to welfare economics, the consumption rate of interest is only one of the two measures necessary to quantify the social rate of discount. The CRI, in fact, is the *marginal social rate of time preference*, which reflects society's rational bias in favor of consumption sooner rather than later (or vice versa), as a consequence of the preferences of the consumer. In addition to this demand based measure, however, a second measure reflects producers' behavior and is therefore supply based. This is the risk adjusted *marginal social rate of return* (MSRR) from investment, which reflects the returns that the private sector sacrifices when resources are diverted to public projects. Thus, while the CRI reflects society's preference for a dollar's worth of consumption today rather than tomorrow; the MSRR reflects the

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small uncertainty σ^2, the consumption function satisfies : $C_0(w + \pi(\sigma_y^2, s)) = C_0(w)$, where $s = w - C_0(w)$ is the level of savings in the absence of uncertainty and

$$\pi(\sigma_y^2, s) = -\frac{1}{2}\frac{U'''(s)}{U''}\sigma_y^2 + o(\sigma_y^2).$$

opportunity cost of what that dollar could have returned, if it had not been consumed, but productively employed between today and tomorrow. The CRI and the MSRR will be identical in a situation of social optimum, where consumers and producers are both satisfied and capital demand is equal to supply. In general, however, the two rates need not be the same and the social rate of discount will have to be an appropriate weighted average of both.

Consider the problem of measuring the MSRR. If there are no distortions in the capital markets, the opportunity cost of capital should be equal to the value of its marginal productivity minus depreciation :

(18) $MSRR = AF_k - \delta$

where *K* denotes capital, A is the Solow residual, which generally is taken to measure factor neutral technological progress, but that can be more generally characterized as the sum of exogenous shocks that may affect positively or negatively factor productivity. *AF(K)* is the production function where variable inputs are optimized for the given level of *K* and prices. F_k its marginal productivity. With output stochastic, however, output price will also be stochastic. Assume that the exogenous shock is dominated by climate change and that it evolves stochastically over time according to a Brownian motion with drift:

(19) $dA = -\alpha A dt + \sigma_p A dz$,

where dz is the increment of a Weiner process, α (> 0) is a drift parameter and σ_p is the standard deviation of the process.

The solution to the problem of maximizing the expected value under uncertainty and irreversibility (Dixit and Pyndick, 1994) is to invest when marginal productivity significantly exceeds marginal cost:

(20) $AF_k = \dfrac{\beta_1}{\beta_1 - 1}\rho$

where $\rho = MSSR + \delta + \alpha$ is the marginal social cost of capital, $MSSR$ is the marginal social rate of return, $(\delta + \alpha)$ is the net rate of capital depreciation and β_1 is the positive root of the characteristic equation:

(21) $MSSR + \beta_1 \alpha - \dfrac{\beta_1(\beta_1 - 1)}{2}\sigma_p^2 = 0$

i.e.:

(22) $$\beta_1 = (-\frac{\alpha}{\sigma_p^2} - \frac{1}{2}) + [(-\frac{\alpha}{\sigma_p^2} - \frac{1}{2})^2 + \frac{2MSSR}{\sigma_p^2}]^{1/2}$$

As can be ascertained from the explicit solution in (21), β_1 increases with $MSSR$ and declines with price volatility σ_p.
From (20), solving for *MSRR*, we obtain:

(23) $$MSRR = (\frac{\beta_1 - 1}{\beta_1})AF_k - (\delta + \alpha)$$

The *MSRR* under dynamic uncertainty is significantly below the value of marginal productivity of capital per unit of capital cost and the more so the higher uncertainty.

With no distortions in the economy, the social rate of return can be either calculated as a *CRI* or as an *MSRR* . If distortions exist, however, the supply of savings will not equal the demand for new investment, and the two expressions should be conveniently combined to reflect this difference. A simple linear approximation of the *SRD* in this case will be:

(24) $$SRD = wCRI + (1 - w)MSRR$$

where w is the relative amount of savings shortfall and $1 - w$ is the correspondent, relative amount of excess investment.

By equating the *CRI* and the *MSRR*, we may solve for the expected rate of growth at which the demand price equals the supply price (the opportunity cost) of capital. This implies:

(25) $$g = \frac{1}{\eta}(\frac{\beta - 1}{\beta} AF'(K) - \phi - (\delta + \alpha)) + \gamma\sigma^2$$

which the case of the iso-elastic function becomes:

(26) $$g = \frac{1}{\eta}(\frac{\beta - 1}{\beta} AF'(K) - \phi - (\delta + \alpha)) + \frac{1}{2}(1 + \eta)\sigma^2$$

Thus, the expected equilibrium growth rate[5], at which the consumption and the production rates of interest are equal, is larger, the larger is uncertainty, the smaller investment cost, and the larger the marginal productivity of capital. In other words, the higher the level of uncertainty determined by climate change, as measured by the volatility of consumption and the relative efficiency of capital, the higher the levels of growth required to reconcile demand and supply (i.e. savings and investment). Without uncertainty, no depreciation and no trend in climate change, expressions (25) and (26) converge to optimal neoclassical growth:

(27) $g^* = \frac{1}{\eta}(AF'(K) - \phi)$.

Given this result, equation (26) can be written as:

(28) $g = g^* - \frac{\alpha}{\eta} + \gamma\sigma^2 - \frac{AF'(K)}{\beta_1}$

As capital accumulates, the marginal productivity of capital will fall and eventually go to zero, so that, if the exogenous shock maintains a negative tendency in the long run:

(29) $\lim_{k \to \infty} g = \frac{1}{\eta}(-\alpha - \phi - \delta) + \gamma\sigma^2$

The first term on the LHS is the component of growth that is due to the trend in climate change, net of a positive shock which could be due to exogenous technological progress. In the long run, even without significant technical progress, optimal growth would be positive if the combination of prudence and volatility were to outweigh the sum of the pure rate of time preference , depreciation and climate change. In practice, this means that optimal growth under uncertainty will exceed deterministic neoclassical growth , since prudence will require an extra-accumulation of capital that will fuel growth over and beyond the excess of its return over the discount rate. With no uncertainty, in other words, postponing consumption to achieve a higher consumption in the future (which is what we call growth) will be convenient only to the extent that the capital returns exceed the rate of time preferences. Under dynamic uncertainty, however, it will pay to postpone consumption at a higher rate to exploit the information acquired by the passage of

[5] Note that g is the drift term of the stochastic process affecting consumption and equals to expected growth. In fact, by virtue of equation (2), we have:

$$E\frac{d\log C}{dt} = g + \sigma E d\xi = g \quad \text{since} \quad Ed\xi = 0$$

time . This rate should be higher, the higher the prudence of the decision maker and the higher the volatility of consumption.

Note also how the results obtained relate to the questions of sustainable growth. In a broad sense, growth under the uncertainty created by climate change would not be sustainable if it were limited to the optimal neoclassical growth (Ramsey-Solow) paradigm, for two distinct reasons. First, neoclassical growth (NG) would come to halt if the rate of technical progress were insufficient to counteract the negative effects of climate change and the decline in the marginal productivity of capital due to its expansion under decreasing returns. Second, NG would over-concentrate consumption in the short run, since it would not take into account of its increasing volatility over time. Thus , long run NG would not be sustainable because it would be jeopardized by the joint effect of declining capital returns and insufficient accumulation. Sustainable growth, on the other hand, would require a larger accumulation of capital of NG to counteract the threats and take advantage of the opportunities (the option values) that uncertainty would generate in the course of time.

4. The development country perspective

The results obtained are specially relevant for the economic evaluation of public investment projects from the perspective of individual countries . Here, we consider in particular developing countries as the subjects most interested in selecting development projects financed by their governments and by the international institutions. Moreover, while dynamic uncertainty has been a traditional element of their decision problem, climate change has recently added a more powerful set of concerns as to the threats and the opportunities that the future may bring about.

A simple classification of developing countries that reflects their differences in facing public investment choices is the following:

1. Poor countries with downward trends in income : as a consequence of climate change, these countries will typically tend to face also high volatilities in consumption and , as a consequence, a major threat to food security. Low risk aversion, and negative growth prospects will all affect negatively consumption discount rates, while lower prudence parameters may somewhat attenuate this tendency. Although it may seem paradoxical, because of their low levels of

consumption, these countries should display the highest propensity to defer gratification and invest in long term projects. This propensity would be attenuated if absolute risk aversion were decreasing with income, but even in this somewhat unlikely case, it remains a solid argument to concentrate development projects in the poorest areas of the countries. The welfare created by the same project, for example, would be higher for poor farmers facing increasing poverty, than for any other group, since the lower discount rate implies that for the same yearly returns, the expected net present value of the project would be higher.

2. Poor countries, with upward trend in incomes : these countries will have a higher risk aversion, comparably higher prudence, but somewhat higher discount rates because of a lower threat to food security and because their expectations that the future will bring about more prosperity.
3. Non poor countries with downward trends in incomes : these countries will tend to have high risk aversion and high prudence parameters , but the prospect of lower growth will determine coeteris paribus lower discount rates .
4. Non poor countries with upward trends in incomes : these countries will tend to have lower risk aversion, lower prudence and higher consumption discount rates.

Table 1 shows conjectural estimates of discount rates for different country types. For poor countries, plausible hypotheses on the different parameters involved show consumption rates of discount ranging from 0% to 7.6%. Both because of risk aversion and prudence, poor countries with declining and variable incomes should thus prefer longer term investment, such as tree crops and forestry, while poor countries with good growth prospects and low volatility of consumption could concentrate on shorter term projects. This difference appears even more important for non poor countries.

Table 1 : Plausible consumption rates of interest (CRI) for different types of countries

Present condition	**Pure rate of time preference**	**Risk aversion parameter**	**Expected Growth (%)**	**Volatility of consumption**	**Prudence parameter**	**Consumption Rate of Discount (%)**
Poor	**0,020**	**1,0**	**2,0**	**0,2**	**1,0**	**-1,0**
Poor	**0,020**	**1,5**	**5,0**	**0,1**	**1,87**	**7,66**
Non Poor	**0,015**	**2,0**	**5,0**	**0,15**	**3,0**	**4,08**

Non Poor	**0,015**	**2,0**	**1,0**	**0,10**	**3,0**	**1,85**

In sum, consumption rates of interest under uncertainty are likely to vary in such a way that long term development projects would tend to be more attractive for poorer countries facing the prospect of increasing poverty and higher volatility of consumption, and progressively less attractive for poor countries whose conditions are improving, richer countries whose conditions are worsening and richer countries benefitting from economic growth. Paradoxically, lower discount rates should also favor longer term environmental projects for poorer countries, with incentives decreasing along the ranking indicated above.

Consumption rates of discount, however, are equal to the marginal social rate of return to capital (MSRR) only along the optimal path, while in practice various distortions may prevent the economy from reaching this ideal solution. We must also take into account, therefore, the production side, where the social rate of discount should essentially reflect the fact that public investment faces the opportunity cost of displacing private investment. As shown by equation (23), uncertainty has the effect of depressing the MSRR below the marginal productivity of capital of an amount that is equivalent to the inverse of the "hurdle rate" that private investors require to adopt risky investments. This effect will depend on the size of uncertainty, with beta tending to 1 for infinite uncertainty and to infinity for complete certainty. Other complicating factors are capital depreciation and technical progress which will move in the opposite direction to render less or more costly to adopt a public investment project.

Resorting again to a simple classification of developing countries that reflects their plausible differences in facing public investment choices, we hypothesize the following country typologies:

1. Poor countries with high rates of return to capital (specially human capital), mild negative effects, but high volatility of climate change. High volatility and negative growth prospects will all affect negatively MSSRs, just as in the case of consumption rates of interest, while higher rates of return to capital and low depreciation rates may work in the opposite direction. However, depletion of natural resources, for example by oil economies, would further increase (natural) capital depreciation and thus face lower MSSRs. Again, somewhat paradoxically, these countries should display the highest propensity to defer gratification and invest in long term projects. The argument in this case is that uncertainty and natural resource depletion make less costly any displacement of private investment and more likely the hypothesis that public investment may serve a complementary role, by concentrating on innovation and other public goods, as well as by ensuring a sufficient level of reinvestment .

2. Poor countries, with lower rates of return to capital , and moderately negative, but still variable, prospects of climate change impact on the economy. These countries will presumably have higher rates of depreciation and somewhat higher MSRRs.
3. Non poor countries with lower rates of return to capital and slight and less variable impact from climate change : these countries will also tend to have higher depreciation rates, especially if natural resource depletion is involved, and the uncertainty and the comparatively low rates of return to capital will conjure up lower SSRMs, of an intermediate same order of magnitude with respect to the two previous categories of poor countries.
4. Non poor countries with lower rates of return to capital, high depreciation rates and low climate change impact. With reasonably high uncertainty, these countries will also tend to have relatively low MSRRs, similar to the first category of poor countries.

   Table 2 shows plausible orders of magnitudes for MSSRs for these country typologies.

Table 2 : Plausible marginal Social Rates of Return to Investment (MSRR) for different types of countries

Present condition	Capital Marginal Productivity	Depreciation Rate	Net Effect of Climate change	Volatility	SSRM (%)
Poor	**0,25**	**0,04**	**-0,005**	**0,5**	**3,83**
Poor	**0,20**	**0,02**	**-0,01**	**0,4**	**5,23**
Non Poor	**0,18**	**0,03**	**-0,015**	**0,3**	**4,5**
Non Poor	**0,15**	**0,04**	**-0,0015**	**0,3**	**3,35**

Table 3 presents plausible ranges of discount rates based on the combination of the values presented in Tables 1 and 2. In the main, even though they are significantly lower, the rates highlighted in this table do not differ dramatically from the ones that are typically recommended for developing countries. However, they consistently suggest that poor countries with low growth prospects should give higher priority to investment focusing on building long term capabilities, rather than on immediate returns.

Table 3: Plausible ranges of social discount rates (%)

	CRI	MSRR	0.5CRI+ 0.5MSRR	0.8CRI+ 0.2MSRR	0.2CRI+ 0.8MSRR
Poor	**-1,000**	**3,83**	**1,415**	**-0,034**	**2,864**
Poor	**7,600**	**5,23**	**6,415**	**7,126**	**5,704**
Non Poor	**4,800**	**4,5**	**4,65**	**4,74**	**4,56**
Non Poor	**1,850**	**3,35**	**2,6**	**2,15**	**3,05**

How do these values compare with values that can be estimated for specific countries? Table 4 shows the results of an estimation exercise on the basis of the country CRI estimates from a World Bank document (Lopez, 2008) for Latin America. Using estimates of growth based on a combination of historical records and (optimistic) forecasts, CRI estimates would range between a minimum of 2,4% for "poor" Honduras and a maximum of 6,4 for "non poor" Brazil. After consideration of CC impact, however, this range would become 0,62 to 3,5 for moderate volatility and 0,38 to 2,98 for high volatility. As highlighted in Tables 1-3 , by using CC adjusted discount rates while poorer/low growth countries would have the highest incentive to concentrate on long term projects, adjusted discount rates would be significantly lower (of factor greater than 2) than unadjusted ones for all countries.

	Table 4: Estimates of CRI for Latin American Countries						
	Projections	Projections	Risk	CRI	Prudence	CRI	CRI

	of Growth*	of Growth with CC	Aversion-Income Distribution Parameter*	with no CC impact	Parameter	with CC: Hp 1**	with CC: Hp 2***
Argentina	2,7	1,2	1,3	3,61	1,49	1,51	1,19
Bolivia	3,1	1,6	1,5	4,75	1,88	2,33	1,94
Brazil	3,5	2	1,8	6,4	2,52	3,50	2,98
Chile	3,5	2	1,3	4,65	1,49	2,55	2,24
Columbia	2,1	0,6	1,8	3,88	2,52	0,98	0,46
Honduras	2,1	0,6	1,1	2,41	1,16	0,62	0,38
Mexico	2,7	1,2	1,3	3,61	1,15	1,51	1,20
Nicaragua	2,6	1,1	1,4	3,74	1,68	1,48	1,13
Peru	2,1	0,6	1,9	4,09	2,26	1,03	0,46
Average	2,8	1,3	1,5	4,3	1,88	1,88	1,49

** Lopez (2008), ** Hp 1: Volatility =20%; *** Hp2 : Volatility=30%
Source: our estimates, based on Lopez (2008)

Conclusions

In this paper I have looked at the controversial problem of the choice of the social discount rate in development projects, by focusing on the investment required to adapt to climate change, considering the threats to food security and the needs for human and natural capital, especially for developing countries. Because climate change introduces negative trends and time increasing volatilities both in production and in consumption, social rates of discount can only be estimated within a framework of dynamic uncertainty. For this purpose, climate change can be modeled as a twin stochastic process of the geometric Brownian motion variety, affecting both consumption and productive capacity.

Under these hypotheses, the determination of the social rate of discount can be seen as a dual problem: (i) the estimate of the discount rate at which the economy attains the optimal rate of growth and , (ii) the estimate of a discount rate that should be used in the evaluation of public investment in order to achieve an optimal rate of growth for the economy. The solution to this dual problem is unique under the hypothesis of perfect and complete capital markets and, as in the case of neoclassical growth, is attained at the point where the discount rate measured as the rate of fall of the value of the numeraire over time equals the marginal social cost of capital given by the value of private investment displaced by the public project. In the case where markets are not complete and/or perfect, the social rate of discount is an appropriate average of these two rates.

Unlike the case of deterministic neoclassical growth, however, and contrary to the usual estimates for project evaluation, the stochastic nature of climate changes makes the social discount rate (SDR) depend on volatility in two distinct and important ways. On the side of consumption and growth, the SDR is reduced by the likely negative effects of climate change (CC) on growth and food security. It also becomes dependent on the fact that the volatility of growth favors the accumulation of precautionary savings and thus reduces the rate of fall of the value of consumption over time.

On the side of production capacity, the SDR is also reduced by the negative effect of CC on the productivity of capital and by the fact that the opportunity cost of the displacement of private investment under dynamic uncertainty is lowered by the value of the options to invest when more information will be available.

When all these considerations are taken into account to determine plausible values to use in appraising public projects in developing countries, they indicate much lower values of the SDR than those generally used in project evaluation by local governments and international agencies. These values are especially low (around 1 %) for poor countries with low food security and growth prospects and high volatilities and suggest that these countries should mainly focus their public investment programs on long run increases in human and non human capital (capabilities) rather than in fast paced , cash intensive projects. A specific application to Latin American countries confirms this finding and further suggests that even middle income countries, when confronted with the threats of climate change, should adopt sharply lower discount rates to evaluate their investment programs.

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