The Role of Mortality Rate in the Transition from Stagnation to Growth

Davide Fiaschi* Tamara Fioroni**

October 12, 2013

Preliminary draft: please do not circulate
Any comments are welcome

Abstract

This paper studies how a model with (exogenous) mortality reductions jointly with an ongoing improvement in technological progress can capture the salient features of the transition from an agricultural regime to a pre-modern regime with accumulation of only physical capital, and finally, to a modern-growth regime, with also the accumulation of human capital. Theoretical framework explicitly refers to the Unified Growth Theory proposed by Galor (2005), where altruistic agents live two periods, childhood and adulthood, but adults are subject to a risk to die. Mortality reductions have a positive effect by increasing the incentive to accumulate human, but also a negative effect by decreasing savings of adults. In the agriculture regime human capital accumulation is absent and the second effect prevails, while the opposite holds in the modern-growth regime. Finally, transition from a pre-modern to a modern regime shows a typical pattern of take-off with period of accelerating growth rates.

Keywords: Unified Growth Theory, Human Capital, Mortality Rate, Nonlinear Dynamics
JEL code: O10, O40, I20,

*Dipartimento di Economia, Università di Pisa, Via Ridolfi 10, Pisa. *E-mail address:* dfiaschi@ec.unipi.it.
**Dipartmento di Scienze Economiche, Università di Verona, Vicolo Campofiore, 2. *E-mail address:* tamara.fioroni@univr.it.

Contents

1 Introduction **3**

2 The Model **6**

2.1 Production 6

2.2 Consumption and Total Transfers 8

2.3 Optimal choices 9

2.3.1 Consumption and Total Transfers 9

2.3.2 Physical and Human Capital 10

3 The Stages of Development **12**

3.1 The Role of Mortality Rate 14

3.2 Endogenous Fertility 16

4 Concluding Remarks **19**

A Optimal Choices **23**

A.1 Exogenous Fertility 23

A.2 Endogenous Fertility 24

A.3 Thresholds 25

B Proof of Proposition 1 . **25**

1 Introduction

In literature there is no agreement on which are the main determinants of the extraordinary development in the last five centuries of Western economies and of the related phenomenon denoted Great Divergence.

Many scholars (see, e.g., Glaeser et al., 2004) following a literature starting in 1960' (see, e.g., Cipolla (1962)) argue that the (differences in the) accumulation of human capital is the main source of long-run growth, and therefore the root(s) of this development (and divergence) should be searched in the factors affecting its accumulation. However, other scholars point to the quality of institutions as the main determinant of the long-run growth of a country. In one of the most important contributions Acemoglu et al. (2001) argument that a lower settler mortality, favoring a better quality of institutions, explains the differences in income between North American and Center and Southern American countries.

These two explanations can be also viewed as complementary, but the prevalence of one or the other has crucial policy implications. For example, if the quality of institutions is the key factor of development then the adoption of Western institutions (e.g. democracy) is the main policy recommendation to poor countries; differently, the attention should be on all the factors benefiting the accumulation of human capital (e.g., public expenditure in education).

The aim of this paper is to discuss how changes in mortality (jointly with a change in technological progress), affecting the accumulation of human and physical capital, can provide a plausible additional determinant of the long-run growth of countries.

In the literature, the role of life expectancy on economic growth is the subject of an active debate. Many contributions focus on the positive effects of gains in life expectancy in the process of development (see for example Cervellati and Sunde, 2005; Boucekkine et al., 2003; Soares, 2005; De la Croix and Licandro, 1999; Lagerlof, 2003; Weisdorf, 2004; Bar and Leukhina, 2010 among others). On the other hand, Acemoglu and Johnson (2007) argue that improvements in life expectancy, rising population growth, have a negative effect on income per capita. Cervellati and Sunde (2011) find a non linear effect of higher longevity on economic growth: a negative effect before the demographic transition and a strong and positive effect after the demographic transition. We reconcile these conflicting results by analyzing the effect of mortality reductions on economic growth in a theoretical framework which explicitly refers to the *Unified Growth Theory* proposed by Galor and Weil (2000) and Galor (2005).

We develop a unified growth model with three different regimes: (i) an *agricultural regime*, where output is produced only by an agricultural technology, whose factors are unskilled labour and land; (ii) a *pre-modern regime*, where an increasing share of aggregate output is produced using physical capital and unskilled labour in an industrial sector; and, finally, (iii) a *modern regime* where both physical and human capital are used in the industrial sector.

The transition from stagnation in the agricultural regime to long-run growth in the modern regime can be jointly driven by technological progress *and* and mortality reductions. However, at low level of income, mortality reductions can be harmful.

Following Preston (1975) and Easterlin (2004), changes in technological progress and mortality rate are assumed to be exogenous. Both authors convincingly argue that, at least for the period we consider here (i.e. the period from the 18th to the 20th century), health improvements are the result of the so-called "Mortality Revolution", which it is counterpart of Industrial Revolution for the improvements in technological progress. According to Easterlin (2004), both revolutions have the same source in the Scientific Revolution of the 17th century, with the mortality revolution displaying its main effects one century later with respect to Industrial Revolution. In Easterlin (2004)'s words: "the Industrial and Mortality Revolution are two of a kind. Both mark the onset of accelerated technological change in their respective fields. Both reflect the cumulation of empirically tested knowledge dating from the seventeenth century onward. ... In seeking an explanation of both the Industrial and Mortality Revolution, one must ask what is new on the scene. The answer suggested here is the emergence and growth of modern science ... "(see Easterlin, 2004, pp. 99-100). The idea that Industrial Revolution is mainly the result of a cultural revolution caused by the emergence of the new scientific method elaborated in the 17th century (which particularly permeated the English society in the 18th and 19 centuries) has strong advocates, among others see Mokyr (2002).[1]

We develop an overlapping generations model where agents potentially live two periods (childhood and adulthood), there exists a subsistence consumption and the saving rate is an increasing function of wealth. Childhood is a certainty and the risk of mortality occurs in adulthood[2]. In a first step of analysis every adult has an exogenous number of children, thus the size of working population directly depends on the number of children and inversely on the mortality rate. Agents devote the first period of their life to the acquisition of human capital (if any) and in the second period they allocate their income, given by the sum of their labor income and their (if any) bequest, between consumption and transfers to their offspring

[1]In (Mokyr, 2002) 's words: "... the interconnections between the Industrial Revolution and those parts of Enlightenment movement that sought to rationalize and spread knowledge may have played a more important role than recent writings have given them credit for ... This would explain the timing of Industrial Revolution following the Enlightenment and - equally important - why it did not fizzle out like similar bursts of macroinventiona in earlier times. It might also help explain why the Industrial Revolution takes place in western Europe ... "(see Mokyr, 2002, p. 29).

[2]The possibility to die also in the childhood is out of the scope of the paper.

(their transfers are positive only when their income is over a some threshold). The transfers (if positive) are invested in physical capital and in the children's education in order to maximize the future income of children (see Galor and Moav (2004)).

The optimal investment in human capital is always decreasing in the mortality rate, since a lower mortality rate increases the return on investing in education (the agent has more time to recover from her investment in education). However, a decrease in mortality rate has two opposite effects on transfers. On one hand, it raises the lifetime consumption of parent reducing, given their income, the amount of transfers to offspring; on the other hand, it increases their income (via an increase in their labour income) and hence their transfers. At low level of income (i.e. in the agricultural and pre-modern regimes) the first effect can dominate the second one, leading to a reduction in the bequest to the offspring and therefore in the amount of resources available in the economy, while at high level of income the first effect is dominated. Empirical evidence discussed in Cervellati and Sunde, 2011 (see, in particular, their Figure 5) on the U-shaped relationship between life expectancy and the growth rate of per capita income supports this finding.

In the agricultural regime, the very low level of income does not allow any saving, and therefore any bequest, given the presence of a subsistence consumption. A sufficiently increase in the productivity of agricultural sector can however grant the escape of such stagnant regime by devoting a part of income to saving, and therefore a switch to the pre-modern regime.

As discussed above in the pre-modern regime the impact of a fall in the mortality rate is ambiguous: this fall could indeed decrease the inherited wealth, harm the accumulation of physical capital, and therefore act as a counterbalancing force with respect to the increase of agents' income. In the worst scenario this could set back a country from a pre-modern regime to an agricultural regime. The opposite dynamics happens when the level of technological progress is sufficiently high, so that the decrease in mortality rate increases the size of bequest and the incentive to invest in human capital by raising the wage of skilled workers. In the best scenario this leads to a switch to the modern regime. A simultaneous increases in technological progress and reduction of mortality (as it happened for the Industrial and Mortality revolutions), tends to reinforce one another.

It is to remark that along the transition from a pre-modern to a modern regime we observe a period of increasing growth rates of per capita income followed by lower steady growth rates; this reproduces a take-off pattern of development typically observed in many western countries since the end of 19th century.

The introduction of endogenous fertility does not substantially affect the main results of the paper, but just adds a possible self-reinforcing mechanism to the stability and transition from a regime to the other. The agricultural regime assumes the typical characteristic of a Malthusian regime (as it is denoted in the Unified Growth Theory), where the increases in income are checked by the increased in the population. Once a country escape from the Malthusian regime the decreasing fertility further boost its growth rates by increasing the accumulation of physical and human capital (in per capita terms).

The paper proceeds as follows; Section 2 presents the model; Section 3 analyses the development process; in Section 4 we calibrate and simulate the model; section 5 concludes. .

2 The Model

The model is inspired by Galor and Moav (2004). We consider an economy populated by an overlapping generations of people who potentially live for two periods: childhood and adulthood. They live in childhood for sure but are subjected to a mortality risk during the adulthood.

More specifically, we denote the expected length of adulthood by $p \in (0, 1)$. For the sake of simplicity in the rest of the section we mantain p as constant. It is straightforward to show that relaxing this hypothesis has not any implication for the main results of analysis; a time-varying p is indeed considered in the numerical simulations in Section 4.

Denoted by L_t the total number of adults in the period t, because of a positive probability to die during the adulthood, the aggregate labor supply observed in the period is given by pL_t.

2.1 Production

In every period, the economy produces a single material good, the price of which is normalized to 1. Production may take place with two different methods:an agricultural technology that employs unskilled labour and land, and an industrial technology that employs physical capital and skilled labour. While the agricultural technology is always operating, the industrial technology, as we shall see below, will become available once technology has progressed enough (for the production structure we follow (Aghion and Howitt, 2009)).

The agricultural production function is given by:

$$Y_t^a = A^a(pL_t^a)^{1-\lambda}(T)^{\lambda}, \tag{1}$$

where A^a is a productivity parameter, pL_t is the supply of unskilled labour (L_t is the adult population size at the beginning of period t) and T is the quantity of land. The industrial production function is given by:

$$Y_t^m = A(ph_tL_t^m)^{1-\alpha}K_t^{\alpha} \tag{2}$$

where $\alpha \in (0,1)$ and $A > 0$ is a technological parameter.

When production is conducted using only the agricultural technology the wage rate is given by:

$$w_t^a = (1-\lambda)A^a p^{1-\lambda}(L^a)_t^{-\lambda}T^\lambda, \tag{3}$$

When industrial technology is operating the rate of return to capital r_t and the wage rate per efficiency unit of labor w_t^m are given by:

$$r_t = \alpha A p^{1-\alpha}\left(\frac{K_t}{h_t L_t^m}\right)^{\alpha-1}; \tag{4}$$

$$w_t^m = (1-\alpha)\, A p^{1-\alpha}\left(\frac{K_t}{h_t L_t^m}\right)^{\alpha}. \tag{5}$$

In the early stages of development production is conducted using the agricultural technology while the industrial technology is latent since is too inefficient to be used. The economy will start to employ the industrial technology alongside the agricultural technology when industrial technology has progressed enough. In particular, as will become clear below, the rise in the technological progress leads parents to leave a positive bequest to their children which activates the industrial technology.

When production is conducted using both technologies, total output is therefore:

$$Y_t = Y_t^a + Y_t^m. \tag{6}$$

Since individuals are perfectly mobile between the two sectors, wages are equalized across sectors, i.e. $w_t^a = w_t^m h_t$. This implies that employment in the agricultural sector is chosen in order to maximize profit (excluding the return to land), i.e. $L_t^a = \arg\max[A^a(pL_t^a)^{1-\lambda}(T)^\lambda - w_m h_t L_t^a]$ which solves for the following value:

$$L_t^a = \left[\frac{A^a p^{1-\lambda}(1-\lambda)}{w_t^m h_t}\right]^{1/\lambda} T. \tag{7}$$

The amount of labor employed in the industrial sector is therefore:

$$L_t^m = L_t - L_t^a, \tag{8}$$

where L_t is the size of working population. If we assume for simplicity that $\alpha = \lambda$ and that the productivity in the agricultural sector has the same trend that productivity in the industrial sector, that is $A^a = \phi A$ with $\phi < 1$, the aggregate production function is given [3]:

[3]Substituting Eq. (5) into Eq. (7) leads to:

$$L^a = \left[\frac{A^a p^{1-\lambda}(1-\lambda)(h_t L_t^m)^\alpha}{A p^{1-\alpha}(1-\alpha)h_t K_t^\alpha}\right]^{1/\lambda} T; \tag{9}$$

$$Y_t = Ap^{1-\alpha}L_t^{1-\alpha}(\phi^{1/\alpha}T + h_t^{\left(\frac{1-\alpha}{\alpha}\right)}K_t)^\alpha \tag{12}$$

When $K_t = 0$ the economy is entirely agrarian and the aggregate production is just the output produced in the agricultural.

The income per capita which agents earn during their life is therefore given:

$$y_t = Ap^{1-\alpha}\left(\phi^{1/\alpha}\frac{T}{L_t} + h_t^{\left(\frac{1-\alpha}{\alpha}\right)}k_t\right)^\alpha \tag{13}$$

where $y_t = Y_t/L_t$ and $k_t = K_t/L_t$.

2.2 Consumption and Total Transfers

Consider an overlapping generations economy where agents live two periods: childhood and as adults.

In the first period, individuals acquire education and make no decisions. In the second period, individuals work, have n_{t+1} children, invest in their children's human capital, and save for the future wealth of their offspring.

To analyze adults behavior, it is useful to conceptualize adulthood (of length p) as divided into time increments (for example years or months). At each time increments agents allocate their income between consumption c_{t+1} and a transfer to their offspring b_{t+1}:

$$y_{t+1} = pc_{t+1} + pb_{t+1}. \tag{14}$$

The transfer b_{t+1}, in turn, is allocated between the spending in children's education e_{t+1} and saving s_{t+1} for the future wealth of children:

$$pb_{t+1} = ps_{t+1} + pe_{t+1}. \tag{15}$$

The investment in education is devoted to increase children's human capital. In particular, children with parental investment in education pe_t acquire:

$$h_{t+1} = h(pe_t) = (1 + pe_t)^\gamma, \text{with } \gamma < 1, \tag{16}$$

with $\lambda = \alpha$, it yields:

$$L_t^a = \left(\frac{\phi}{h_t}\right)^{1/\alpha}\frac{h_t L_t^m T}{K_t}. \tag{10}$$

Thus from equation (8) it follows that the labour share in the agricultural sector is given by:

$$\frac{L_t^a}{L_t} = \frac{(\phi/h_t)^{1/\alpha}\frac{h_t T/L_t}{k_t}}{1 + (\phi/h_t)^{1/\alpha}\frac{h_t T/L_t}{k_t}}, \tag{11}$$

thus we it follows that $\partial(L_t^a/L_t)/\partial h_t < 0$, $\partial(L_t^a/L_t)/\partial k_t < 0$ and $\partial(L_t^a/L_t)/\partial T > 0$

units of human capital, where $h(0) = 1$, $h'(0) = \gamma$ and $\lim_{e_t \to \infty} h'(pe_t) = 0$ (Galor and Moav, 2004, 2006). Allowing for the case $\gamma \geq 1$ implies that human capital accumulation can sustain a positive long-run growth of per capita income.

Individual preferences are defined over a consumption above a *subsistence level* $\tilde{c} > 0$ and the transfer to the children b_{t+1}. The expected utility function of altruistic parents is therefore:[4]:

$$U = p[(1-\beta)\log(c_{t+1}) + \beta \log(b_{t+1} + \theta)], \tag{18}$$

where $\beta \in (0,1)$ is the discount factor and $\theta > 0$ implies that children receive a positive transfer only when parent's income is sufficiently high (see Eq. (22) below).

2.3 Optimal choices

2.3.1 Consumption and Total Transfers

Parents choose the level of consumption and the transfer to the offspring so as to maximize their expected utility, that is:

$$\left(c^*_{t+1}, b^*_{t+1}\right) = \underset{c_{t+1}, b_{t+1}}{\arg\max} \{p[(1-\beta)\log(c_{t+1}) + \beta \log(b_{t+1} + \theta)]\}, \tag{19}$$

subject to:

$$\begin{aligned} y_{t+1} &= pc_{t+1} + pb_{t+1}; \\ c_{t+1} &\geq \tilde{c}; \text{ and} \\ b_{t+1} &\geq 0. \end{aligned}$$

Given the following condition on parameters (which ensures that, for low levels of income, the optimal consumption is increasing with respect to income while optimal bequest is zero, i.e. $\tilde{y} < y^b$):

$$\tilde{c} < \frac{(1-\beta)\theta}{\beta}, \tag{20}$$

the optimal levels of consumption and bequest are given as follows:[5]

[4]Following Rosen (1988) we assume the expected utility in the second period is given by the utility of state "life "given by the utility from consumption and the bequest to the children and the utility of state "death "given by M which is assumed to be equal to zero for simplicity:

$$U = p[(1-\beta)\log(c_{t+1}) + \beta \log(b_{t+1} + \theta)] + (1-p)M, \tag{17}$$

[5]See Appendix A.

$$c_{t+1}^{*} = \begin{cases} \dfrac{y_{t+1}}{p} & \text{if } y_{t+1} \in (\tilde{y}, y^b] \\ \\ \dfrac{(1-\beta)(y_{t+1}+p\theta)}{p} & \text{if } y_{t+1} \in (y^b, \infty) \end{cases} \tag{21}$$

and:

$$b_{t+1}^{*} = \begin{cases} 0 & \text{if } y_{t+1} \in (\tilde{y}, y^b] \\ \\ \dfrac{\beta y_{t+1} - \theta(1-\beta)p}{p} & \text{if } y_{t+1} \in (y^b, \infty) \end{cases} \tag{22}$$

where $\tilde{y} = p\tilde{c}$ and $y^b = \theta(1-\beta)p/\beta$ are two thresholds of income.

Henceforth we denote with $^-$ all expected variables, which are also the levels observed in the period; thus $\bar{c}_{t+1} = pc_{t+1}$ is the consumption during the adulthood per individual, $\bar{b}_{t+1} = pb_{t+1}$ is the bequest which each parent give to their children during their life. From Eq. (21) notice that the rise in longevity, when income is above the subsistence level, increases the expected consumption , i.e. $\partial\bar{c}_{t+1}/\partial p = \partial y_{t+1}/\partial p + (1-\beta)\theta > 0$. while the expected bequest decreases, i.e. $\partial\bar{b}_{t+1}/\partial p = -(1-\beta)\theta$. Therefore the rise in longevity has two opposite effects on

When production is conducted using only the agricultural technology, we assume that technology is always sufficiently high to ensure a consumption at least equal to the subsistence level $\tilde{c}$, that is (for technical details see Appendix A):

$$A \geq A^{MIN}. \tag{23}$$

where:

$$A^{MIN} = \frac{p^{\alpha}\tilde{c}}{\phi}\left(\frac{L_t n_t}{T}\right)^{\alpha}. \tag{24}$$

and n_t denote the constant fertility rate (see below Eq. (26)).

2.3.2 Physical and Human Capital

The economy begins to accumulate physical capital only when parents leave a positive transfer to their children (which happens when $y_t > y^b$ according to Eq. (22)). Eq. (15) shows that bequest is allocated between saving, i.e. accumulation of physical capital, and education, i.e. accumulation of human capital.

The capital stock in period $t+1$ is therefore given by:

$$K_{t+1} = pL_t s_t = pL_t(b_t - e_t), \tag{25}$$

where $e_t \geq 0$.

Adult population at time $t+1$, L_{t+1}, is:

$$L_{t+1} = n_t L_t; \tag{26}$$

where L_t is adult population at time t and n_t is the fertility rate.

The capital/labour ratio is therefore equal to:

$$k_{t+1} = \frac{p(b_t - e_t)}{n_t} \tag{27}$$

Parents choose the amount to invest in children's education in order to maximize the future income of offspring i.e. y_{t+1}. In the early stages of development, when the productivity in the industrial sector is relatively low with respect to the productivity in the agricultural sector, agents do not have incentive to invest in human capital of their children. However, as the level of industrial technology improves, human capital will be demanded and parents will have an incentive to invest in the human capital of their children. Thus from equations (59) and (16) it follows that:

$$e_t^* = \underset{e_t \in [0, b_t]}{\arg\max} \left[A p^{1-\alpha} \left(\phi^{1/\alpha} \frac{T}{L_{t+1}} + (1 + p e_t)^{\frac{\gamma(1-\alpha)}{\alpha}} k_{t+1} \right)^{\alpha} \right], \tag{28}$$

where k_{t+1} is given by Eq. (27).

The optimal choice of education in per capita terms, i.e. $\bar{e}_t = e_t^*$, is therefore given by:

$$\bar{e}_t = \begin{cases} 0 & \text{if } \bar{b}_t \in [0, \tilde{b}]; \\ \\ \dfrac{\bar{b}_t - \tilde{b}}{1 + \tilde{b}} & \text{if } \bar{b}_t \in (\tilde{b}, \infty) \end{cases} \tag{29}$$

where:

$$\tilde{b} = \frac{\alpha}{(1-\alpha)\gamma}. \tag{30}$$

The optimal level of education is therefore positive only if the expected transfer in period t is sufficiently high, that is $b_t > \tilde{b}$. From Eqq. (29)and (59) we have that parents start investing in education only if their income is above y^E:

$$y_E = A p^{1-\alpha} \left(\phi^{1/\alpha} \frac{T}{L_t} + \tilde{b} \right)^{\alpha} \tag{31}$$

The (optimal) saving of parents during their life, i.e. $\bar{s}_t = p s_t$, is given by:

$$\bar{s}_t = \begin{cases} \bar{b}_t & \text{if } 0 < \bar{b}_t \le \tilde{b}; \\ \\ \dfrac{\tilde{b}\left(1 + \bar{b}_t\right)}{1 + \tilde{b}} & \text{if } \bar{b}_t > \tilde{b}. \end{cases} \tag{32}$$

Therefore, when bequest is $0 < b_t \leq \tilde{b}$ the optimal choice for education is zero and total transfer is entirely devoted to finance the accumulation of wealth of children. When $\bar{b}_t > \tilde{b}$, both $\bar{e}_t$ and $\bar{s}_t$ increase with respect to $\bar{b}_t$.

From Eqq. (27), (29) and (32) we obtain the capital-labor ratio in period $t+1$ as follows:

$$k_{t+1} = \begin{cases} \dfrac{\bar{b}_t}{n_t} & \text{if } 0 < \bar{b}_t \leq \tilde{b}; \\ \\ \dfrac{\tilde{b}}{n_t}\left(\dfrac{1+\bar{b}_t}{1+\tilde{b}}\right) & \text{if } \bar{b}_t > \tilde{b}. \end{cases} \tag{33}$$

3 The Stages of Development

From Eqq. (22), (29) and (33) we can now characterize the dynamic of aggregate transfers in period $t+1$, i.e. $\bar{b}_{t+1}$, as function of the aggregate intergenerational transfers in the preceding period $\bar{b}_t$:

$$\bar{b}_{t+1} = \begin{cases} \max\left[\dfrac{\beta A p^{1-\alpha}\phi T^{\alpha}}{n_t^{\alpha} L_t^{\alpha}} - \theta(1-\beta)p, 0\right] & \text{if } \bar{b}_t = 0; \\ \\ \dfrac{\beta A p^{1-\alpha}}{n_t^{\alpha}}\left(\phi^{1/\alpha}\dfrac{T}{L_t} + \bar{b}_t\right)^{\alpha} - \theta(1-\beta)p & \text{if } \bar{b}_t \in (0, \tilde{b}); \text{and} \\ \\ \dfrac{\beta A p^{1-\alpha}}{n_t^{\alpha}}\left[\phi^{1/\alpha}\dfrac{T}{L_t} + \tilde{b}\left(\dfrac{1+\bar{b}_t}{1+\tilde{b}}\right)^{\dfrac{\gamma(1-\alpha)+\alpha}{\alpha}}\right]^{\alpha} - \theta(1-\beta)p & \text{if } \bar{b}_t \in [\tilde{b}, +\infty). \end{cases} \tag{34}$$

The three ranges of $\bar{b}_t$ identify three distinct regimes: the *agricultural regime*, i.e. $\bar{b}_t = 0$, where production is conducted using agricultural technology; a *pre-modern-growth regime* $\bar{b}_t \in (0, \tilde{b})$, where output is the result of using physical capital and unskilled labour in an industrial sector; and a *modern-growth regime*, i.e. $\bar{b}_t > \tilde{b}$ where both physical and human capital are used in the industrial sector. Simple calculations show a smoothing transition from the pre-modern regime to the modern regime that is, $\lim_{\bar{b}_t \to \tilde{b}^-} \bar{b}_{t+1} = \lim_{\bar{b}_t \to \tilde{b}^+} \bar{b}_{t+1}$.

Proposition 1 states the conditions on under which we observe one or more than one equilibria in these three regimes.

Proposition 1 *Under some (not so restrictive) conditions on the model's parameters reported in Appendix B:*

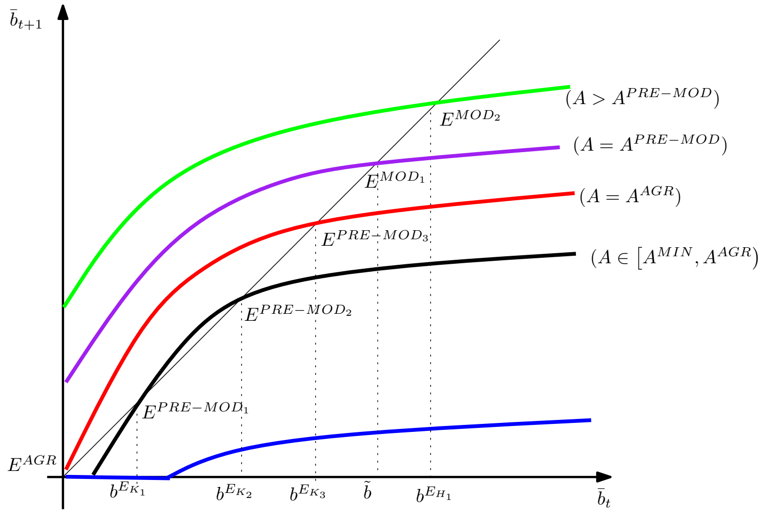

Figure 1: Stages of growth in terms of transfers to offspring

- *if* $A \in \left[A^{MIN}, A^{AGR}\right)$, *then there exists one stable equilibrium in the agriculture regime and possibly one unstable and one stable equilibrium in the pre-modern regime, where* A^{MIN} *is defined in Eq. (24) and*

$$A^{AGR} = \frac{\theta\,(1-\beta)\,L_t^{\alpha} n_t^{\alpha} p^{\alpha}}{\beta\phi T^{\alpha}}. \tag{35}$$

- *If* $A \in \left[A^{AGR}, A^{PRE-MOD}\right]$, *then there exists one stable equilibrium in the pre-modern regime:*

$$A^{PRE-MOD} = \frac{n_t^{\alpha}\left[\tilde{b}p^{\alpha-1} + \theta\,(1-\beta)\,p^{\alpha}\right]}{\beta\left(\phi^{1/\alpha}\frac{T}{L_t} + \tilde{b}\right)^{\alpha}}. \tag{36}$$

- *Finally, if* $A > A^{PRE-MOD}$ *there exists just one stable equilibrium in modern regime.*

The assumption of $\gamma < 1$ prevents the per capita income to growth in the long run without any increase in technological progress.

Proof. See Appendix B. ■

Figure 1 provides a graphical exposition of the results contained in Proposition 1.

At low level of A, i.e. above but around A^{MIN}, the only equilibrium of economy is in the agricultural regime, i.e. E^{AGR}. A^{MIN} depends on the level of mortality rate p, the level of population at period t L_t, and the fertility rate n_t. In this regard A^{MIN} could change over time; for example if L_t were increasing over time this would imply a continuous increase in A^{MIN}. A^{AGR} is identified as the level of A such that the intercept is equal to zero; it is straightforward indeed to check that in such case the equilibrium in agricultural regime disappears. For value of A below A^{AGR} it is possible to have also a stable equilibrium in the pre-modern regime (e.g. $E^{PRE-MOD_2}$). For A greater than $A^{PRE-MOD}$ just an equilibrium in the modern regime is possible (e.g. E^{MOD_2}; the level of $A^{PRE-MOD}$ corresponds to the level of A such that the equilibrium in the pre-modern regime is at the bound of the range of no accumulation of human capital (i.e. $\bar{b}_t = \tilde{b}$).

Taken as given the mortality rate and the population we have therefore that as A increases over time the economy will pass through all three regimes. The transition from agricultural to pre-modern regime is driven by the increase in the agricultural production, which allows to satisfy consumption and to make some transfers to offspring: this is the source of accumulation of capital. The transition from pre-modern regime to modern regime is instead driven by the higher accumulation of capital (transfers) generated by the increase of productivity in manufacturing sector (this happen also in agriculture but the share of the latter on total output is declining in income): such accumulation increases the return to the investment in human capital and therefore incentive such type of investment.

The model can produce under very mild conditions multiple equilibria. The bold black line case in Figure 1) corresponds to the case of existence of a poverty trap, with the economies with a level of transfers below $b^{E_{k_1}}$ converging to an equilibrium in the agricultural regime, and the economies with a level above $b^{E_{k_1}}$ converging to the equilibrium $E^{PRE-MOD_2}$ in pre-modern regime.

Taken as given the level of mortality rate it is easy to see that in presence of a positive and constant (and exogenous) fertility rate, i.e. $n_t > 0$, the threshold on A of Proposition 1 should be emended. If A were constant over time the only possibility will be the equilibrium in the agricultural regime. The same happen if A were increasing at a growth rate lower than $(n_t)^\alpha$ (in fact the ratio A/L_t^α would be decreasing over time; in the contrary if A were increasing at a growth rate higher than $(n_t)^\alpha$ the only possible equilibrium would be in the modern regime. For many Western countries the latter case has been an historical fact.

3.1 The Role of Mortality Rate in the Transition between Regimes

Below we analyze the effect of change in the adult mortality on the economic development. In the analysis we take as constant the population (i.e. $L_t = L$ and $n = 1$).

In Figure 2 we have identify in the space (A, p) the regions corresponding to the cases described in Proposition 1. A^{MIN} and A^{AGR} are both increasing and concave, while $A^{PRE-MOD}$ it is decreasing until a given p^T and then increasing;[6] for the sake of simplicity we consider the case where $p^T > 1$.

Consider first the case of an economy with a low level of technological progress and a high

[6]The level of p^T is equal to $(1-\alpha)\tilde{b}/\theta(1-\beta)\alpha$.

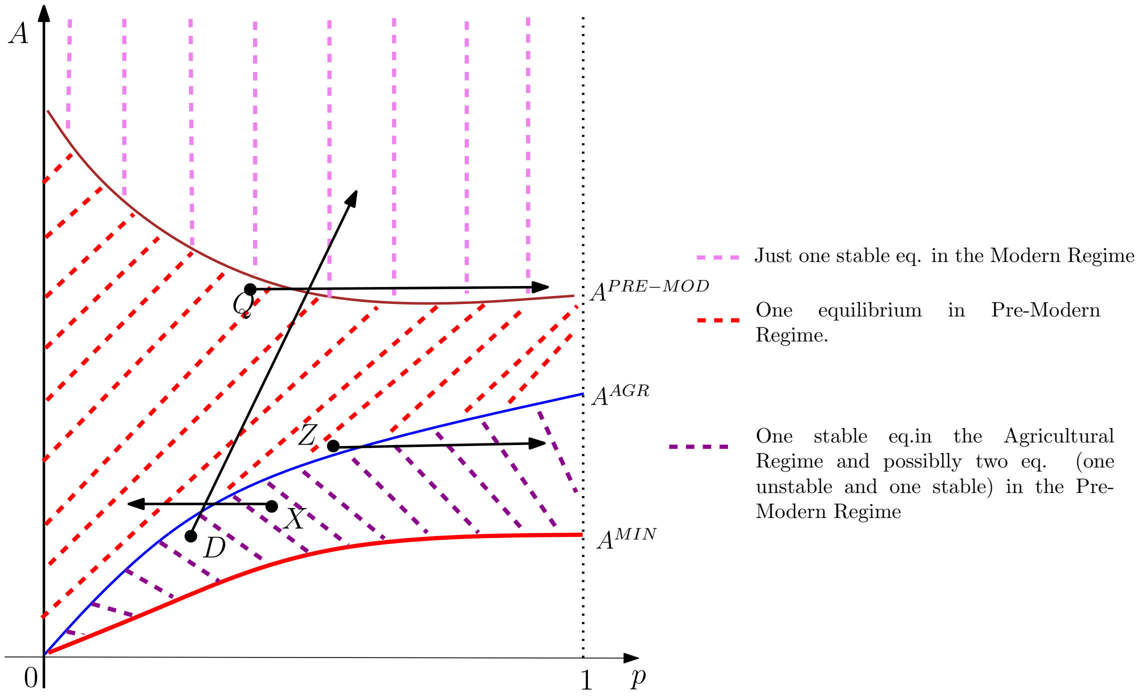

Figure 2: Dynamic between regimes

mortality rate (i.e. point D in Figure 2). The transition through the different regimes is driven by the contemporaneous rise of technological progress and longevity. This latter seems to be the growth path followed by the most of actual developed countries. When technological progress does not change and its actual level is very low, the rise in longevity alone cannot allow the transition to the (pre-)modern growth regime. Differently, if the level is high, an increase in p alone allows for a change in regime (see, e.g., trajectory starting from point Q).

The decline in adult mortality $(1-p)$ has however two opposing effects on intergenerational transfers. On the one hand, higher longevity increases consumption of parents (see equation 21), thus reducing transfer to their offspring ; on the other hand, parents who live longer, work for a longer period, thus increasing income and raising transfers to their children. When the initial level of income is sufficiently high, the second effect always prevails whereas at low levels of income the first effect could prevail. In Figure 2 the comparison of the two trajectories starting from Q and Z should clarify the point.

The basic motivation underlying this result are the diminishing returns of labor at low level of income. That is, at low level of income, the rise in population due to a decline in mortality has a less than proportional effect on output because the presence of the fixed factor land. This implies that when longevity rises above a certain threshold, at low levels of income, the rise in income is not sufficient to compensate the rise in consumption. On the other hand, at high levels of income the economy can accumulate human capital, and, therefore, the rise in longevity always allows a level of income sufficiently high to compensate for the rise in consumption, thus allowing the transition to the modern regime. Indeed, if income is sufficiently high, the rise in longevity increases the return on investment in education and therefore high income perpetuates. These results are in line with the empirical evidence discussed in Cervellati and Sunde, 2011 which show a non linear relationship between life expectancy and economic growth. In particular, they show that this relationship is negative before the onset of the demographic transition and strongly positive after its onset.

Finally, the path starting from the point X in Figure 2 shows a scenario in which the rise in mortality, as for example an epidemic such as the Black Death, can have a positive effect on economic growth. In this case, indeed, the population reduction, increasing income per capita, can push the economy from the agricultural regime to the pre-modern regime.

3.2 Endogenous Fertility

If we assume endogenous fertility, we have that agents preferences are defined over consumption, the total number of children n_{t+1} and the transfer to their children b_{t+1}. Thus the optimal problem of parents is given as follows:

$$\left(c^*_{t+1}, b^*_{t+1}, n^*_{t+1}\right) = \underset{c_{t+1}, b_{t+1}, n_{t+1}}{\arg\max} \; \{p[(1-\beta)\log(c_{t+1}) + \epsilon \log(n_{t+1}) + \beta \log(b_{t+1} + \theta)]\}, \tag{37}$$

subject to:

$$\begin{aligned} y_{t+1} &= pc_{t+1} + pb_{t+1} + \delta n_{t+1} y_{t+1}; \\ c_{t+1} &\geq \tilde{c}; \\ b_{t+1} &\geq 0. \end{aligned}$$

where δ is the opportunity cost of raising children, that is the fraction of parents time required in order to raise each child (Galor, 2005) [7].

Assuming that condition (20) holds[8], then from the first order conditions the optimal levels

[7] We can consider income in the previous section net of the cost of raising children. If you consider explicetely the cost of raising children in the budget constraint when fertility is exogenous results do not change

[8] this condition ensures that $\tilde{y} < y^c < y^{b'}$

of consumption, bequest and the optimal number of children are given as follows:[9].

$$c^*_{t+1} = \begin{cases} \dfrac{\tilde{c}}{p} & \text{if } y_{t+1} \in (\tilde{y}, y^c] \\[2ex] \dfrac{(1-\beta)y_{t+1}}{p(1-\beta+\epsilon)} & \text{if } y_{t+1} \in (y^c, y^{b'}] \\[2ex] \dfrac{(1-\beta)(y_{t+1}+p\theta)}{p(1+\epsilon)} & \text{if } y_{t+1} \in (y^{b'}, \infty) \end{cases} \tag{38}$$

and:

$$b^*_{t+1} = \begin{cases} 0 & \text{if } y_{t+1} \in (\tilde{y}, y^c] \\[2ex] 0 & \text{if } y_{t+1} \in (y^c, y^{b'}] \\[2ex] \dfrac{\beta y_{t+1} - \theta(1-\beta+\epsilon)p}{p(1+\epsilon)} & \text{if } y_{t+1} \in (y^{b'}, \infty) \end{cases} \tag{39}$$

and:

$$n^*_{t+1} = \begin{cases} \dfrac{y_{t+1} - p\tilde{c}}{\delta y_{t+1}} & \text{if } y_{t+1} \in (\tilde{y}, y^c] \\[2ex] \dfrac{\epsilon}{\delta(1-\beta+\epsilon)} & \text{if } y_{t+1} \in (y^c, y^{b'}] \\[2ex] \dfrac{\epsilon(y_{t+1}+\theta p)}{\delta y_{t+1}} & \text{if } y_{t+1} \in (y^{b'}, \infty) \end{cases} \tag{40}$$

where $\tilde{y} = p\tilde{c}$ and $y^c = \frac{p\tilde{c}(1-\beta+\epsilon)}{1-\beta}$ (this threshold implies that $c_{t+1} > \tilde{c}$) and $y^{b'} = \frac{\theta(1-\beta+\epsilon)p}{\beta}$ (this threshold ensures that $b_{t+1} > 0$).

Fig. 3 describes the evolution of the fertility rate, the bequest, the human capital investment for children and the per capita consumption. When income is sufficiently low, that is $y_{t+1} < y^c$ the per capita consumption is at subsistence level, the optimal choice for bequest is zero while the optimal number of children increases with respect to income. The economy is therefore in a Malthusian trap where production is conducted using only the agricultural technology and any increase in income is channeled towards larger population. Thus per capita income, due to the diminishing returns to labour, stagnates to a subsistence level. When income increases (due to the increases in technological progress or mortality reduction) above y^c, parents escape from the subsistence level, consumption starts to increases and the fertility rate becomes constant. However, the economy is still in the agricultural regime since parents do not have a sufficient

[9]See Appendix A

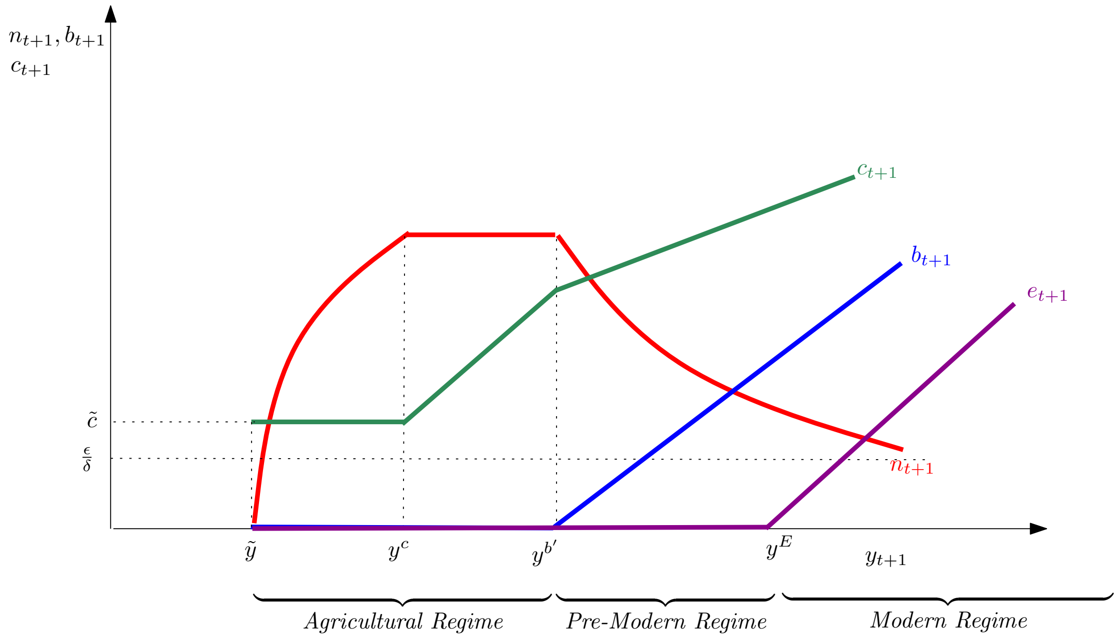

Figure 3: Optimal Choices

level of income to leave a positive transfer to their children. If income continues to increases, the constant fertility rate ensures that, at certain point, i.e. $y_{t+1} > y^{b'}$, the economy enters into pre-modern regime where parents stars to devote a fraction of their income to the transfer to their children and the relationship between income and population growth becomes negative. Finally, when income is sufficiently high to lead parents investing in education the economy enters into Modern Regime. The inverted U-shaped relationship between the fertility rate and income presented in Fig. 3 is consistent with the experience of most pre-industrial economies. For example Coale and Treadway (1986), Dyson and Murphy (1985) show that the fertility rate in most of Western Europe, increased over the 18th century and the beginning of the 19th century and then it declined.

From Eqq. (39) and (40) we can now characterize the dynamic of aggregate transfers in period $t+1$, as follows:

$$\bar{b}_{t+1} = \frac{1}{1+\epsilon}\begin{cases} \max\left[\frac{\beta A p^{1-\alpha}\phi T^{\alpha}}{n(y_t)L_t^{\alpha}}\left[\frac{\delta(1-\beta+\epsilon)}{\epsilon}\right]^{\alpha} - \theta(1-\beta+\epsilon)p, 0\right] & \text{if } \bar{b}_t = 0; \\ \frac{\beta A p^{1-\alpha}}{n(y_t)^{\alpha}}\left(\phi^{1/\alpha}\frac{T}{L_t} + \bar{b}_t\right)^{\alpha} - \theta(1-\beta+\epsilon)p & \text{if } \bar{b}_t \in (0, \tilde{b}); \\ \frac{\beta A p^{1-\alpha}}{n(y_t)^{\alpha}}\left[\phi^{1/\alpha}\frac{T}{L_t} + \tilde{b}\left(\frac{1+\bar{b}_t}{1+\tilde{b}}\right)^{\frac{\gamma(1-\alpha)+\alpha}{\alpha}}\right]^{\alpha} - \theta(1-\beta+\epsilon)p & \text{if } \bar{b}_t \in [\tilde{b}, +\infty). \end{cases} \tag{41}$$

where $n(y_t)$ is given by equation (40) and income at time t given as follows:

$$y_t = \begin{cases} A p^{1-\alpha}\phi\left(\frac{T}{n_{t-1}L_{t-1}}\right)^{\alpha} & \text{if } \bar{b}_{t-1} = 0; \\ \frac{A p^{1-\alpha}}{n_{t-1}^{\alpha}}\left(\phi^{1/\alpha}\frac{T}{L_{t-1}} + \bar{b}_{t-1}\right)^{\alpha} & \text{if } \bar{b}_{t-1} \in (0, \tilde{b}); \\ \frac{A p^{1-\alpha}}{n_{t-1}^{\alpha}}\left[\phi^{1/\alpha}\frac{T}{L_{t-1}} + \tilde{b}\left(\frac{1+\bar{b}_{t-1}}{1+\tilde{b}}\right)^{\frac{\gamma(1-\alpha)+\alpha}{\alpha}}\right]^{\alpha} & \text{if } \bar{b}_{t-1} \in [\tilde{b}, +\infty). \end{cases} \tag{42}$$

From eq. (41), it follows that the economy moves from the agricultural regime to the pre-modern regime only when technological progress is sufficiently high, that is: $A > A^{AGR'}$, where:

$$A^{AGR'} = \frac{\theta}{\beta}(1-\beta+\epsilon)^{1-\alpha}\left(\frac{\epsilon p L_t}{\delta T}\right)^{\alpha} \tag{43}$$

4 Concluding Remarks

This paper contributes to the literature on the role of mortality reductions on economic growth by accounting for the differential effects of life expectancy during the various stages of economic development.

We find that the rise in technological progress always allows the transition from agricultural (stagnant) regime to a modern regime. However, the rise in longevity can have important effects on the transition. It has a positive effect on intergenerational transfer at high levels of income and a non-linear effect at low levels of income: this effect is positive if longevity remains below a threshold but becomes negative if longevity exceeds such threshold. The basic motivation underlying this result is the presence of the fixed factor land which leads to diminishing returns of labor in agriculture. Thus, if longevity increases above a certain threshold, at low levels of income, the rise in income is insufficient to compensate the rise in consumption. Reduction in intergeneration transfer, in turn reduces physical capital accumulation, pushing the economy towards an agricultural regime. On the other hand, at high income levels, rising longevity doesn't has the same opposite effects. The rise in longevity, indeed, increasing the return on investment in education, stimulates investment in human capital and increases labour income. Thus the rise in income is sufficiently high to compensate for the increase in consumption, leading to higher intergenerational transfers.

The presence of endogenous fertility dampens the effects of shocks on the economy but does not change qualitatively our results.

Finally, the introduction of endogenous mortality should not affect the qualitative results of the paper but just adding a possible self-reinforcing mechanism to the transition from stagnation to growth.

References

Acemoglu, D. and S. Johnson (2007). "Disease and Development: the Effect of Life Expectancy on Economic Growth". *Journal of Political Economy 115*(61), 925985.

Acemoglu, D., S. Johnson, and J. A. Robinson (2001). "The Colonial Origins of Comparative Development: An Empirical Investigation". *American Economic Review 91*(5), 1369–1401.

Aghion, P. and P. Howitt (2009). *The Economicsof Growth* . Cambridge, Massachusetts: The MIT Press.

Bar, M. and O. Leukhina (2010). "The role of mortality in the transmission of knowledge". *Journal of Economic Growth 15*(4), 291321.

Boucekkine, R., D. D. la Croix D., and O. Licandro (2003). "Early Mortality Declines at the dawn of modern growth". *Scandinavian Journal of Economics 105*(3), 401–418.

Cervellati, M. and U. Sunde (2005). "Human Capital Formation, Life Expectancy and the Process of Development". *American Economic Review 95*, 1653–1672.

Cervellati, M. and U. Sunde (2011). "Life Expectancy and economic growth: the role of the demographic transition". *Journal of economic growth 16*, 99–133.

Cipolla, C. (1962). *The Economic History of World Population.* Harmondsworth, Penguin Books.

De la Croix, D. and O. Licandro (1999). "Life Expectancy and Endogenous Growth". *Economic Letters* (65), 225–263.

Dyson, T. and M. Murphy (1985). "The onset of of fertility transition'. *Population and Development Review 11*, 399–440.

Easterlin, R. A. (2004). *The Reluctant Economist Perspectives on Economics, Economic History, and Demography.* Cambridge University Press.

Galor, O. (2005). "From Stagnation to Growth: Unified Growth Theory ". *in The Handbook of Economic Growth (A 'Classical' Prespective) 1*(Edited by Philippe Aghion and Steven Durlauf), 171–293.

Galor, O. and O. Moav (2004). "From Physical to Human Capital Accumulation: Inequality and the Process of Development". *Review of Economic Studies 71*, 1001–1026.

Galor, O. and O. Moav (2006). "Das Human Kapital: A Theory of the Demise of Class Structure". *Review of Economic Studies 73*, 85–117.

Galor, O. and D. Weil (2000). "Population, Technology and Growth: From the Malthusian Regime to the Demographic Transition and Beyond". *American Economic Review) 90*, 806–828.

Glaeser, E. L., R. L. Porta, F. L. de Silanes, and A. Shleifer (2004). "Do Institutions Cause Growth? ". *Journal of Economic Growth 9*(3), 271–303.

Lagerlof, N. P. (2003). "From Malthus to Modern Growth: Can Epidemics Explain the three Regimes". *International Economic Review 44*, 755–777.

Mokyr, J. (2002). *The Gifts of Athena.* Priceton: Princeton University Press.

Preston, S. (1975). "The Changing Relation between Mortality and Level of Economic Development". *Population Studies 29*(2), 231–248.

Rosen, S. (1988). "The Value of Changes in Life Expectancy". *Journal of Risk and Uncertainty 1*(3), 285–304.

Soares, R. (2005). "Mortality Reductions, Educational Attainment and Fertility Choice". *American Economic Review 95*(3), 580–601.

Weisdorf, J. L. (2004). "From stagnation to Growth: Revisiting three historical regimes". *Journal of Population Economics 17*, 455–472.

A Optimal Choices

A.1 Exogenous Fertility

The agent's maximization problem is given:

$$\left(c_{t+1}^*, b_{t+1}^*\right) = \underset{c_{t+1},b_{t+1}}{\arg\max}\{p[(1-\beta)\log(c_{t+1}) + \beta\log(b_{t+1}+\theta)]\}, \tag{44}$$

subject to:

$$y_{t+1} = pc_{t+1} + pb_{t+1};$$

$$c_{t+1} \geq \tilde{c};$$

$$b_{t+1} \geq 0.$$

The Lagrangian for problem (44) is given by:

$$\mathscr{L} = p[(1-\beta)\log\left(\frac{y_{t+1} - pb_{t+1}}{p}\right) + \beta\log(b_{t+1}+\theta)] + \lambda b_{t+1} + \mu\left(\frac{y_{t+1} - pb_{t+1}}{p} - \tilde{c}\right) \tag{45}$$

and the first order conditions are:

$$\frac{\partial \mathscr{L}}{\partial b_{t+1}} = -\frac{(1-\beta)p}{y_{t+1} - pb_{t+1}} + \frac{\beta}{b_{t+1}+\theta} + \lambda - \mu = 0.$$

$$\lambda b_{t+1} = 0$$

$$\mu\left(\frac{y_{t+1} - pb_{t+1}}{p} - \tilde{c}\right) = 0$$

Thus we can have different cases:

1. $c_{t+1} = \tilde{c}$ and $b_{t+1} = 0$. Thus we have that $p\tilde{c} = y_{t+1}$.

2. $c_{t+1} > \tilde{c}$ and $b_{t+1} = 0$. Thus we have that $c_{t+1} = \frac{y_{t+1}}{p}$.

3. $c_{t+1} > \tilde{c}$ and $b_{t+1} > 0$. Thus solving the first order conditions we get:

$$c_{t+1}^* = \frac{(1-\beta)(y_{t+1} + p\theta)}{p} \tag{46}$$

$$b_{t+1}^* = \frac{\beta y_{t+1} - \theta(1-\beta)p}{p} \tag{47}$$

We don't consider the case $c_{t+1} = \tilde{c}$ and $b_{t+1} > 0$.

A.2 Endogenous Fertility

When fertility is endogenous the agent's maximization problem is given:

$$\left(c_{t+1}^{*}, b_{t+1}^{*}, n_{t+1}^{*}\right) = \underset{c_{t+1}, b_{t+1}, n_{t+1}}{\arg\max} \; \{p[(1-\beta)\log(c_{t+1}) + \epsilon \log(n_{t+1}) + \beta \log(b_{t+1}+\theta)]\}, \tag{48}$$

subject to:

$$y_{t+1} = \delta y_{t+1} + p c_{t+1} + p b_{t+1};$$

$$c_{t+1} \geq \tilde{c};$$

and

$$b_{t+1} \geq 0.$$

The Lagrangian for this optimization problem is given as follows:

$$\mathscr{L} = p\left[(1-\beta)\log c_{t+1} + \epsilon \log\left(\frac{y_{t+1} - p(c_{t+1}+b_{t+1})}{\delta y_{t+1}}\right) + \beta \log(b_{t+1}+\theta)\right] + \lambda b_{t+1} + \mu\left(c_{t+1} - \tilde{c}\right) \tag{49}$$

and the first order conditions are:

$$\frac{\partial \mathscr{L}}{\partial c_{t+1}} = \frac{(1-\beta)}{c_{t+1}} - \frac{\epsilon p}{y_{t+1} - p(c_{t+1}+b_{t+1})} + \mu = 0. \tag{50}$$

$$\frac{\partial \mathscr{L}}{\partial b_{t+1}} = -\frac{\epsilon p}{y_{t+1} - p(c_{t+1}+b_{t+1})} + \frac{\beta}{b_{t+1}+\theta} + \lambda = 0. \tag{51}$$

$$\lambda b_{t+1} = 0$$

$$\mu\left(c_{t+1} - \tilde{c}\right) = 0$$

Thus we can have different cases:

1. $c_{t+1} = \tilde{c}$ and $b_{t+1} = 0$. Thus given $c_{t+1} = \tilde{c}$, from the budget constraint we get:

$$n_{t+1}^{*} = \frac{y_{t+1} - \tilde{c}}{\delta y_{t+1}} \tag{52}$$

2. $c_{t+1} > \tilde{c}$ and $b_{t+1} = 0$. Given that $\mu = 0$, we solve the first order condition (50) with respect to c_{t+1}:

$$c_{t+1}^{*} = \frac{(1-\beta) y_{t+1}}{p(1-\beta+\epsilon)} \tag{53}$$

Substituting this solution into the budget constraint, the optimal number of children, is given by:

$$n_{t+1} = \frac{\epsilon}{\delta(1-\beta+\epsilon)} \tag{54}$$

3. $c_{t+1} > \tilde{c}$ and $b_{t+1} > 0$ Thus given $\mu = 0$ and $\lambda = 0$, from the first order conditions (50), (51) and the budget constraint we get:

$$c^*_{t+1} = \frac{(1-\beta)(y_{t+1} + p\theta)}{p(1+\epsilon)} \tag{55}$$

$$b^*_{t+1} = \frac{\beta y_{t+1} - \theta(1-\beta+\epsilon)p}{p(1+\epsilon)} \tag{56}$$

$$n^*_{t+1} = \frac{\epsilon(y_{t+1} + \theta p)}{\delta y_{t+1}} \tag{57}$$

We don't consider the case $c_{t+1} = \tilde{c}$ and $b_{t+1} > 0$.

A.3 Thresholds

From equations (59) when production is conducted using agricultural technology per-capita income is given by:

$$y_t = Ap^{1-\alpha}\phi\left(\frac{T}{L_t}\right)^{\alpha} \tag{58}$$

Thus, from equation (21), per capita income when production is conducted using agricultural technology ensures a consumption at least equal to the subsistence level $\tilde{c}$ if:

$$Ap^{-\alpha}\phi\left(\frac{T}{L_t n_t}\right)^{\alpha} \geq \tilde{c} \tag{59}$$

which implies that:

$$A \geq A^{MIN} = \frac{p^{\alpha}\tilde{c}}{\phi}\left(\frac{L_t n_t}{T}\right)^{\alpha}. \tag{60}$$

B Proof of Proposition 1 .

Given equation (42) the economy shows one stable equilibrium in the agricultural regime and possibly one unstable and one stable equilibrium in the pre-modern regime if the following conditions hold:

$$\lim_{\bar{b}_t \to 0} \bar{b}_{t+1} < 0,$$

$$\lim_{\bar{b}_t \to \tilde{b}^-} \bar{b}_{t+1} \leq \tilde{b},$$

$$\lim_{\bar{b}_t \to \tilde{b}^+} \bar{b}_{t+1} \leq \tilde{b},$$

- The first condition holds if $A < A^{AGR}$, where:

$$A^{AGR} = \frac{\theta(1-\beta)p^{\alpha}}{\beta\phi}\left(\frac{n_t L_t}{T}\right)^{\alpha} \tag{61}$$

  where $A^{MIN} < A^{AGR}$ if assumption 20 holds.

- The second and third conditions hold if:

$$A \leq A^{PRE-MOD} = \frac{n_t^{\alpha}[\tilde{b}p^{\alpha-1} + \theta(1-\beta)p^{\alpha}]}{\beta(\phi^{1/\alpha}\frac{T}{L_t} + \tilde{b})^{\alpha}} \tag{62}$$

  where $\lim_{p\to 0} A^{PRE-MOD} = \infty$ and $\partial A^{PRE-MOD}/\partial p < 0$ if:

$$p < p^T = \frac{(1-\alpha)\tilde{b}}{\theta(1-\beta)\alpha} \tag{63}$$

  where we assume for simplicity $p^T >$,1 that is $\theta < (1-\alpha)\tilde{b}/(1-\beta)\alpha$.

Some calculations show that, when $p = 1$, $A^{AGR} < A^{PRE-MOD}$ if:

$$\theta < \frac{\tilde{b}\left(\frac{T}{T\phi^{1/\alpha}+\tilde{b}}\right)^{\alpha}}{(1-\beta)[\frac{1}{\phi} - \left(\frac{T}{T\phi^{1/\alpha}+\tilde{b}}\right)^{\alpha}]} \tag{64}$$

An economy shows one stable equilibrium in the pre-modern regime if:

$$\lim_{\bar{b}_t \to 0} \bar{b}_{t+1} \geq 0,$$

$$\lim_{\bar{b}_t \to \tilde{b}^-} \bar{b}_{t+1} \leq \tilde{b},$$

$$\lim_{\bar{b}_t \to \tilde{b}^+} \bar{b}_{t+1} \leq \tilde{b},$$

- The first condition holds if $A \geq A^{AGR}$

- The second and third conditions hold if $A \leq A^{PRE-MOD}$

An economy shows one stable equilibrium in the modern regime if:

$$\lim_{\bar{b}_t \to 0} \bar{b}_{t+1} > 0,$$

$$\lim_{\bar{b}_t \to \tilde{b}^-} \bar{b}_{t+1} > \tilde{b},$$

$$\lim_{\bar{b}_t \to \tilde{b}^+} \bar{b}_{t+1} > \tilde{b},$$

- The first condition holds if $A > A^{AGR}$
- The second and third conditions hold if $A > A^{PRE-MOD}$