Production theory: accounting for firm heterogeneity and technical change*

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Abstract

The paper presents a new framework to assess firm level heterogeneity and to study the rate and direction of technical change. Building on the analysis of revealed short-run production function by Hildenbrand (1981), we propose the (normalized) volume of the zonotope composed by vectors-firms in a narrowly defined industry as an indicator of inter-firm heterogeneity. Moreover, the angles that the main diagonal of the zonotope form with the axes provides a measure of the rates and directions of technical change over time. The proposed framework can easily account for n-inputs and m-outputs and, crucially, the measures of heterogeneity and technical change do not require many of the standard assumptions from production theory.

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1 Introduction

In recent years an extremely robust evidence regarding firm– and plant– level longitudinal microdata has highlighted striking and persistent heterogeneity across firms operating in the same industry. A large body of research from different sectors in different countries (cf. Baily et al.; 1992; Baldwin and Rafiquzzaman; 1995; Bartelsman and Doms; 2000; Disney et al.; 2003; Dosi; 2007; Syverson; 2011, among many others) documents the emergence of the following "stylized facts": first, wide asymmetries in productivity across firms; second, significant heterogeneity in relative input intensities even in presence of the same relative input prices; third, high intertemporal persistence in the above properties; fourth, such heterogeneity is maintained also when increasing the level of disaggregation.

The latter property is particularly puzzling and has been vividly summarized by Griliches and Mairesse (1999): "*We [...] thought that one could reduce heterogeneity by going down from general mixtures as "total manufacturing" to something more coherent, such as "petroleum refining" or "the manufacture of cement." But something like Mandelbrot's fractal phenomenon seems to be at work here also: the observed variability-heterogeneity does not really decline as we cut our data finer and finer. There is a sense in which different bakeries are just as much different from each others as the steel industry is from the machinery industry.*"

The bottom line is that firms operating in the same industry display a large and persistent degree of technological heterogeneity, while there does not seem to be any clear sign that either the diffusion of information on different technologies, or the working of the competitive mechanism bring about any substantial reduction of such an heterogeneity, even when involving massive differences in efficiencies, as theory would predict.

This evidence poses serious challenges not only to theory of competition and market selection, but also to any theoretical or empirical analysis which relies upon some notion of industry or sector defined as a set of production units producing under rather similar input prices with equally similar technologies, and the related notion of "the technology" of an industry represented by means of a sectoral production function.

Note that these problems do not only concern the neoclassical production function, whose well known properties may either not fit empirical data or fit only spuriously,[1] but also non neoclassical representations of production at the industry level. If input-output coefficients *à la* Leontief (1986) are averages over a distribution with high standard deviation and high skewness, average input coefficients may not provide a meaningful representation of the technology of that industry. Moreover, one cannot take for granted that changes of such coefficients can be interpreted as indicators of technical change as they may be just caused by some changes in the distribution of production among heterogeneous units, characterized by unchanged technologies.

How does one then account for the actual technology - or, better, *technologies* - in such industry? Hildenbrand (1981) suggests a direct and agnostic approach which instead of estimating some aggregate production function, offers a representation of the empirical production possibility set of an industry in the short run based on actual microdata. Each production unit is represented as a point in the input-output space whose coordinates are input requirements and output levels at full capacity. Under the assumptions of divisibility and additivity of production processes,[2] the production possibility set is represented geometrically by the space formed by the finite sum of all the line segments linking the origin and the points representing each production unit, called a *zonotope* (see also below). Hildenbrand then derives the actual "production function" (one should more accurately say "feasible" functions) and shows that "*short-run efficient production functions do not enjoy the well-known properties which are frequently assumed in production theory. For example, constant returns to scale never prevail, the*

[1] Shaikh (1974), for instance, shows that Cobb-Douglas production functions with constant returns to scale, neutral technological change and marginal products equal to factor rewards in presence of constant distributional shares of labour and capital (wages and profits) tend to yield a good fit to the data for purely algebraic reasons.

[2] Already not entirely innocent assumptions: for a discussion cf. Dosi and Grazzi (2006).

production functions are never homothetic, and the elasticities of substitution are never constant. On the other hand, the competitive factor demand and product supply functions [...] will always have definite comparative static properties which cannot be derived from the standard theory of production" (Hildenbrand; 1981, p. 1095).

In this paper we move a step forward and show that by further exploiting the properties of zonotopes it is possible to obtain rigorous measures of heterogeneity and technical change without imposing on data a model like that implied by standard production functions. In particular, we develop measures of technical change that take into consideration the entire *observed* production possibility set derived from *observed* heterogeneous production units, instead of considering only an *efficient frontier*. The promise of the methodology is checked in this work against the evidence on micro data of Italian industries and the dynamics of their distributions.

The rest of the work is organized as follows. Section 2 builds on the contribution of Hildenbrand (1981) and introduces the (normalized) volume of the zonotope as a measure of industry heterogeneity. We then proposes a measure of technical change based on the zonotope's main diagonal and we assess the role of firm entry and exit on industry level productivity growth. Section 3 presents an empirical application on manufacturing firms in narrowly defined industries. Section 4 discuss the implications of this work and further applications of the proposed methodology.

2 Accounting for heterogeneous micro-techniques

Without loss of generality it is possible to represent the *ex post* technology of a production unit by means of a *production activity* represented by a vector (Koopmans; 1977; Hildenbrand; 1981)

$$a = (\alpha_1, \ldots, \alpha_l, \alpha_{l+1}) \in \mathbb{R}_+^{l+1}.$$

A production unit, which is described by the vector a, produces during the current period α_{l+1} units of output by means of $(\alpha_1, \ldots, \alpha_l)$ units of input.[3] Also notice that in this framework it is possible to refer to the *size* of the firm as to the lenght of vector a, which can be regarded as a multi-dimensional extension of the usual measure of firm size, often proxied either by the number of employees, sales or value added. In fact, this measure allows to employ both measures of input and output in the definition of firm size.

In this framework, as noted by Hildenbrand (1981), the assumption of constant returns to scale (with respect to variable inputs) for individual production units is not necessary: indeed it is redundant if there are "many" firms in the industry. Anyhow, the short run production possibilities of an industry with N units at a given time are described by a finite family of vectors $\{a_n\}_{1 \le n \le N}$ of production activities. In order to analyze such a structure Hildenbrand introduces a novel *short-run feasible industry production function* defined by means of a *Zonotope* generated by the family $\{a_n\}_{1 \le n \le N}$ of production activities. More precisely let $\{a_n\}_{1 \le n \le N}$ be a collection of vectors in $\mathbb{R}^{l+1}$, $N \ge l + 1$. To any vector a_n we may associate a line segment

$$[0, a_n] = \{x_n a_n \mid x_n \in \mathbb{R}, 0 \le x_n \le 1\}.$$

Hildenbrand defines the *short run total production set* associated to the family $\{a_n\}_{1 \le n \le N}$ as the Minkowski sum

$$Y = \sum_{n=1}^{N} [0, a_n]$$

of line segments generated by production activities $\{a_n\}_{1 \le n \le N}$. More explicitly, it is the Zonotope

[3] Our considerations hold also for the multi-output case.

$$Y = \{y \in \mathbb{R}^{l+1}_+ \mid y = \sum_{n=1}^{N} \phi_n a_n, 0 \leq \phi_n \leq 1\}.$$

Remark 2.1 *Geometrically a Zonotope is the generalization to any dimension of a Zonohedron that is a convex polyhedron where every face is a polygon with point symmetry or, equivalently, symmetry under rotations through* 180°. *Any Zonohedron may equivalently be described as the Minkowski sum of a set of line segments in three-dimensional space, or as the three-dimensional projection of an hypercube. Hence a Zonotope is either the Minkowski sum of line segments in an l-dimensional space or the projection of an $(l+1)$-dimensional hypercube. The vectors from which the Zonotope is formed are called its generators.*[4]

Analogously to parallelotopes and hypercubes, Zonotopes admit diagonals. We define the main diagonal of a Zonotope Y as the diagonal joining the origin $O = (0, \dots, 0) \in Y \subset \mathbb{R}^{l+1}$ with its opposite vertex in Y. Algebraically it is simply the sum $\sum_{n=1}^{N} a_n$ of all generators, that is, in our framework, the sum of all production activities in the industry. In the following, we will denote by d_Y such diagonal and we will call it production activity of the industry.

Denote by D the projection of Y on the firsts l coordinates, i.e.

$$D = \{v \in R^l_+ \mid \exists x \in \mathbb{R}_+ \text{ s.t. } (v, x) \in Y\}$$

and the production function $F : D \longrightarrow \mathbb{R}_+$ associated with Y as

$$F(v) = \max\{x \in \mathbb{R}_+ \mid (v, x) \in Y\}.$$

In the definition above the aggregation of the various production units is achieved by associating to the level v_1, ..., v_l of inputs for the industry the maximum total output which is obtainable by allocating, without restrictions, the amounts v_1, ... v_l of inputs in a most efficient way over the individual production units. As argued by Hildenbrand (1981)it might well be that the technology and/or the market structure and organization of the industry is such that the efficient production function is neither from a positive nor from a normative point of view a relevant concept. In this respect our work distinguishes from the contribution in the Data Envelopment Analysis (DEA) tradition, see Farrell (1957); Charnes et al. (1978) for the original contributions and Murillo-Zamorano (2004) for a review. In the DEA approach one is interested in providing a measure of firm's efficiency and that is provided by the distance between any single firm and the efficient frontier. Hence each firm that is not on the efficiency frontier is compared to a similar firm on the frontier or with a convex combination of similar firms on the frontier. In our approach, the way in which a firm contributes to industry heterogeneity depends on how such firms *combines* with *all* other firms. A similar argument, see below, applies to how technical change is measured.

The representation of any industry at any one time by means of the Zonotope provides a way to assess and measure the degree of heterogeneity. As we shall show below, it allows also to account for its variation of production techniques adopted by firms in any industry and, at least as important, it allows to ascertain the rate and direction of technical change.

2.1 Volume of Zonotopes and heterogeneity

Let us remark that if all firms in an industry with N enterprises were to use the same technique in a given year, all the vectors of the associated family $\{a_n\}_{1 \leq n \leq N}$ of production activities would be multiples of the same vector. Hence they would lie on the same line and the generated Zonotope would coincide with the diagonal $\sum_{n=1}^{N} a_n$, that is a degenerate Zonotope of null

[4]The interested reader can refer to Ziegler (1995) for a survey on Zonotopes.

volume. This is the case of one technology only and perfect homogeneity among firms. At the opposite extreme one has the case of maximal heterogeneity. In such a case in the industry there are firms that can produce a large quantity of output with a quantity of inputs nearly close to zero, and at the other extreme, much inefficient companies that produce few output with a large quantity of inputs. This case of maximal heterogeneity is geometrically described by vectors that generate a Zonotope which is almost a rectangular cuboid.

Remark 2.2 *The representation of production in the industry by means of zonotope implies perfect divisibility of the production process. Production can always be scaled up/down by any factor. A plane that would cut the zonotope would always provide a combination of K and L that attains a level of Y, even if such combination is not feasible or it is not observed in the data.*

In the following we provide the formula to compute the volume of the Zonotope.

Let $A_{i_1,\dots,i_{l+1}}$ be the matrix whose rows are vectors $\{a_{i_1},\dots,a_{i_{l+1}}\}$ and $\Delta_{i_1,\dots,i_{l+1}}$ its determinant. In our framework, the first l entries of each vector provide the amount of the inputs used in the production process by each firm, whether the last entry of the vector is output. It is well known that the volume of the zonotope Y in $\mathbb{R}^{l+1}$ is given by:

$$Vol(Y) = \sum_{1\leq i_1<\dots<i_{l+1}\leq N} \mid \Delta_{i_1,\dots,i_{l+1}} \mid$$

where $\mid \Delta_{i_1,\dots,i_{l+1}} \mid$ is the module of the determinant $\Delta_{i_1,\dots,i_{l+1}}$.

Our main interest lies in getting a pure measure of the heterogeneity in techniques employed by firms within any given industry that allows for comparability across firms and time; that is, a measure which is independent both from the unit in which inputs and output are measured and from the number of firms making up the sector. The volume of the Zonotope itself depends both from the units of measure involved and from the number of firms. In order to solve these issues we introduce a way to normalize the zonotope's volume and we get a new index which is dimensionless and independent from the number of firms.

The normalization we introduce is a generalization of the well known Gini index, which we call *Gini volume* of the Zonotope. Analogously to the original index, we will consider the *ratio* of the volume of the Zonotope Y generated by the production activities $\{a_n\}_{1\leq n\leq N}$ over a *total* volume of an industry with production activity $d_Y = \sum_{n=1}^N a_n$. It is an easy remark that the Parallelotope is the Zonotope with largest volume if the main diagonal is fixed. If P_Y is the parallelotope of diagonal d_Y, its volume $Vol(P_Y)$, i.e. the product of the entries of d_Y, is obviously the maximal volume that can be obtained once we fix the industry production activity $\sum_{n=1}^N a_n$, that is the *total* volume of an industry with production activity $d_Y = \sum_{n=1}^N a_n$.

Note that alike the complete inequality case in the Gini index, i.e. the case in which the index is 1, also in our framework the complete heterogeneity case is not feasible, since in addition to firms with large values of inputs and zero output it would imply the existence of firms with zero inputs and non zero output. It has to be regarded as a *limit* similarly to the 0 volume in which all techniques are equal, i.e. the vectors $\{a_n\}_{1\leq n\leq N}$ are proportional and hence lie on the same line.

In what follows we consider the Gini volume defined above for the short run total production set Y:

$$G(Y) = \frac{Vol(Y)}{Vol(P_Y)} \quad . \tag{1}$$

2.2 Unitary production activities

An interesting information is provided by comparison of the Gini volume $G(Y)$ of the short run total production set Y and the same index computed for the Zonotope $\overline{Y}$ generated by

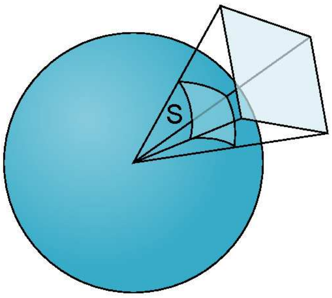

Figure 1: The solid angle of a pyramid generated by 4 vectors.

the normalized vectors $\{\frac{a_n}{\|a_n\|}\}_{1\le n\le N}$, i.e. the unitary production activities. The Gini volume $G(\overline{Y})$ evaluates the heterogeneity of the industry in a setting in which all firms have the same size (norm is equal to one). Hence the only source of heterogeneity is the difference in adopted techniques, since differences in firm size do not contribute to the volume. This allows to insulate against possible contribution to heterogeneity that comes through size differences among firms.

Comparing the Gini volume of the Zonotope Y with that of the unitary Zonotope $\overline{Y}$ will be informative about how much large or small firms contribute to the heterogeneity in techniques within the given industry. Indeed, intuitively, if the Gini volume $G(Y)$ of Y will be bigger than $G(\overline{Y})$ then it means that the big firms contribute to the heterogeneity more than the small ones, while if, viceversa, the volume $G(Y)$ is smaller than $G(\overline{Y})$ then small firms contribute to heterogeneity more than bigger ones.

2.3 Solid Angle and external production activities

Let us move further and introduce the *external* Zonotope Y_e, which although different from Y and $\overline{Y}$, is related to them. In order to define it we need to introduce the notion of solid angle. Let us start with the solid angle in a 3-dimensional space, the notion can be easily generalized to an n-dimensional one.

In geometry, a solid angle (symbol: Ω) is the two-dimensional angle in three-dimensional space that an object subtends at a point. It is a measure of *how large* the object appears to an observer looking from that point. In the International System of Units, a solid angle is a dimensionless unit of measurement called a steradian (symbol: sr). The measure of a solid angle Ω varies between 0 and 4π steradian.

More precisely, an object's solid angle is equal to the area of the segment of a unit sphere, centered at the angle's vertex, that the object covers, as shown in figure 1.

In our framework the production activities are represented by a family $\{a_n\}_{1\le n\le N}$ of vectors. Their normalization $\{\frac{a_n}{\|a_n\|}\}_{1\le n\le N}$ will generate an arbitrary pyramid with apex in the origin. Note that in general, not all vectors a_i, $i = 1, \ldots, N$ will be edges of this pyramid. Indeed it can happen that one vector is *inside* the pyramid generated by others. We will call *external* vectors those vectors $\{e_i\}_{1\le i\le R}$ of the family $\{a_n\}_{1\le n\le N}$ such that their normalizations $\{\frac{e_i}{\|e_i\|}\}_{1\le i\le R}$ are edges of the pyramid generated by the vectors $\{\frac{a_n}{\|a_n\|}\}_{1\le n\le N}$. All the others will be called *internal*.

This pyramid will subtend a solid angle Ω, smaller or equal than $\frac{\pi}{2}$ as the entries of our vectors are positive. We will say that the *external* vectors of the family $\{a_n\}_{1\le n\le N}$ subtend the solid angle Ω if it is the angle subtended by the generated pyramid.

Define the external Zonotope Y_e as the one generated by vectors $\{e_i\}_{1\le i\le R}$. A pairwise comparison between $G(Y_e)$ and $G(Y)$ shows the relative importance of the *density* of internal activities in affecting our proposed measure of heterogeneity.

Solid angle of an arbitrary pyramid. In $\mathbb{R}^3$ the solid angle of an arbitrary pyramid defined by the sequence of unit vectors representing edges $\{s_1, s_2, \ldots, s_n\}$ can be efficiently computed by

$$\Omega = 2\pi - \arg \prod_{j=1}^{n} (< s_j, s_{j-1} >< s_j, s_{j+1} > - < s_{j-1}, s_{j+1} > +i \mid s_{j-1} s_j s_{j+1} \mid) \tag{2}$$

where parentheses $< s_j, s_{j-1} >$ are scalar products, brackets $\mid s_{j-1} s_j s_{j+1} \mid$ are scalar triple products, i.e. determinants of the 3×3 matrices whose rows are vectors s_{j-1}, s_j, s_{j+1}, and i is the imaginary unit. Indices are cycled: $s_0 = s_n$ and $s_{n+1} = s_1$ and arg is simply the argument of a complex number.

The generalization of the definition of solid angle to higher dimensions simply needs to account for the n-sphere in an $n+1$-dimensional space.

2.4 Technical Change

Let us consider a non zero vector $v = (x_1, x_2, \ldots, x_{l+1}) \in \mathbb{R}^{l+1}$. Using the trigonometric formulation of the Pythagoras' theorem we get that if θ_i is the angle that v forms with the x_i axis and $\|v\|$ is the norm of v, i.e. the length of the vector v, then

$$\cos \theta_i = \frac{x_i}{\|v\|}.$$

In our framework we are primarily interested in the angle θ_{l+1} that the diagonal of the zonotope, i.e. the vector d_Y, forms with the output axis x_{l+1}. This can easily be generalized to the case of multiple outputs, so that if we have m different outputs we will consider the angles θ_i for $l < i \leq l + m$.

In order to assess if and to what extent productivity is growing in a given industry, it is possible to analyze how the angle θ_{l+1} varies over the years. For example if the angle θ_{l+1} decreases then productivity increases. This is indeed equivalent to state that the industry is able to produce more output, given the quantity of inputs, than it was able to. On the contrary, an increase in θ_{l+1} stands for a productivity reduction.

Also notice that it is possible to study how the relative inputs use changes over the years. In this case it is enough to consider the angles that the input vector, i.e. the vector with entries given by only the inputs of d_Y, forms with different input axis. More precisely, if there are l inputs and m outputs and the vectors of production activities are ordered such that the first l entries are inputs, then we can consider the projection function:

$$\begin{aligned} pr : \mathbb{R}^{l+m} &\longrightarrow \mathbb{R}^l \\ (x_1, \ldots, x_{l+m}) &\mapsto (x_1, \ldots, x_l) \quad . \end{aligned}$$

The change over time of the angle φ_i between the projection vector $pr(d_Y)$ and the x_i axis, $1 \leq i \leq l$, captures the changes in the relative intensity of input i over time with respect to all the other inputs.

It is also relevant to measure the changes in the *normalized* angles $\overline{\theta}_i$. Indeed, as we have done for volumes, we can consider the normalized production activities $\{\frac{a_n}{\|a_n\|}\}_{1 \leq n \leq N}$. Call $d_{\overline{Y}}$ the resulting industry production activity, of course, one can study how it varies over time and this is equivalent to study how the productivity of an industry changes independently from the size of the firms. In particular the comparison of the changes of two different angles, θ_i and $\overline{\theta}_i$, is informative on the relative contribution of bigger and smaller firms to productivity changes and hence, on the possible existence of economies/diseconomis of scale.

For the sake of simplicity we study the variation of the cosine of angles instead of angles themselves. Remark that if an angle increases then its cosine decreases and viceversa.

2.5 Entry and exit

Under what circumstances does the entry of a new firm increase or decrease the heterogeneity of a given industry? In order to compute how entries and exits impact on industry heterogeneity it is enough to remark that, by definition of volume, given a zonotope Z in the space $\mathbb{R}^{l+1}$ generated by vectors $\{a_n\}_{1\leq n\leq N}$ and a vector $b=(x_1,\ldots,x_{l+1})\in\mathbb{R}^{l+1}$, the volume of the new zonotope X generated by $\{a_n\}_{1\leq n\leq N}\cup\{b\}$ can be computed as follow:

$$Vol(X)=Vol(Z)+V(x_1,\ldots,x_{l+1})$$

where $V(x_1,\ldots,x_{l+1})$ is a real continuous function on $\mathbb{R}^{l+1}$ defined as:

$$V(x_1,\ldots,x_{l+1})=\sum_{1\leq i_1<\ldots<i_l\leq N}\mid\Lambda_{i_1,\ldots,i_l}\mid,$$

$\Lambda_{i_1,\ldots,i_l}$ being the determinant of the matrix $B_{i_1,\ldots,i_l}$ whose rows are vectors $\{b,a_{i_1},\ldots,a_{i_l}\}$.

If $d_Z=(d_1,\ldots,d_{l+1})$ is the diagonal of the Zonotope Z, then the diagonal of X will be $d_X=d_Z+b=(d_1+x_1,\ldots,d_{l+1}+x_{l+1})$. The heterogeneity for the *new* industry will be the continuous real function

$$G(X)=\frac{Vol(Z)+V(x_1,\ldots,x_{l+1})}{Vol(P_X)}=\frac{Vol(Z)+V(x_1,\ldots,x_{l+1})}{\Pi_{i=1}^{l+1}(d_i+x_i)}$$

and the cosine of the angle with the output axis will be the continuous real function

$$cos\theta_{l+1}(x_1,\ldots,x_{l+1})=\frac{d_{l+1}+x_{l+1}}{\|d_X\|}$$

Studying the variation (i.e. gradient, hessian etc...) of these real continuous functions is equivalent to analyze the impact of a new firm on the industry. So, for example, when these functions increase then the new firm positively contributes both to industry heterogeneity and productivity. We consider as an example the entry of a firm in the 3-dimensional case. If Z is the Zonotope generated by vectors $\{a_n\}_{1\leq n\leq N}$ in $\mathbb{R}^3$ with entries $a_n=(a_n^1,a_n^2,a_n^3)$, the function $V(x_1,x_2,x_3)$ for a generic vector $b=(x_1,x_2,x_3)$ is

$$V(x_1,x_2,x_3)=\sum_{1\leq i<j\leq N}\mid x_1(a_i^2a_j^3-a_i^3a_j^2)-x_2(a_i^1a_j^3-a_i^3a_j^1)+x_3(a_i^1a_j^2-a_i^2a_j^1)\mid.$$

The diagonal of the new Zonotope X is

$$d_X=(\sum_{i=1}^N a_i^1+x_1,\sum_{i=1}^N a_i^2+x_2,\sum_{i=1}^N a_i^3+x_3).$$

We get the Gini volume for X as:

$$G(X)=\frac{Vol(Z)+\sum_{1\leq i<j\leq N}\mid x_1(a_i^2a_j^3-a_i^3a_j^2)-x_2(a_i^1a_j^3-a_i^3a_j^1)+x_3(a_i^1a_j^2-a_i^2a_j^1)\mid}{\sum_{i,j,k=1}^N(a_i^1+x_1)(a_j^2+x_2)(a_k^3+x_3)}\quad,\quad(3)$$

where $Vol(Z)$ and $\{a_n^1,a_n^2,a_n^3\}_{1\leq n\leq N}$ are constants and the cosine of the angle with the output axis as:

$$cos\theta_3(x_1,x_2,x_3)=\frac{\sum_{i=1}^N a_i^3+x_3}{\sqrt{(\sum_{i=1}^N a_i^1+x_1)^2+(\sum_{i=1}^N a_i^2+x_2)^2+(\sum_{i=1}^N a_i^3+x_3)^2}}\quad.$$

#	Year 1 L	Year 1 K	Year 1 VA	Year 2 L	Year 2 K	Year 2 VA	Year 3 L	Year 3 K	Year 3 VA	Year 4 L	Year 4 K	Year 4 VA
1	7.0	4.0	9.0	7.0	4.0	9.0	**7.0**	**4.0**	**9.0**	**7.0**	**4.0**	**9.0**
2	1.0	4.0	5.0	1.0	4.0	5.0	**1.0**	**4.0**	**5.0**	**1.0**	**4.0**	**5.0**
3	**6.0**	**2.0**	**9.0**	**6.0**	**2.0**	**9.0**	**6.0**	**2.0**	**9.0**	**6.0**	**2.0**	**9.0**
4	**1.5**	**8.0**	**10.0**	**1.5**	**8.0**	**10.0**	**1.5**	**8.0**	**10.0**	**1.5**	**8.0**	**10.0**
5	5.0	2.0	8.0	5.0	2.0	8.0	5.0	2.0	8.0	5.0	2.0	8.0
6	**1.0**	**3.0**	**8.0**	**1.0**	**3.0**	**8.0**	**1.0**	**3.0**	**8.0**	**1.0**	**3.0**	**8.0**
7	**2.0**	**2.0**	**7.0**	**2.0**	**2.0**	**7.0**	**2.0**	**2.0**	**7.0**	**2.0**	**2.0**	**7.0**
8	3.0	5.0	7.0	3.0	5.0	7.0	3.0	5.0	7.0			
9	**2.5**	**2**	**2**	**2.5**	**2**	**2**						
10	**5.0**	**6.0**	**4.0**	4.0	4.0	6.0	**4.0**	**4.0**	**6.0**	**4.0**	**4.0**	**6.0**

Table 1: Production schedules in year 1 to 4, Number of employees, Capital and Output. External production activities in bold.

If we fix the output setting x_3 constant or we fix the norm of b, i.e. the size of the firm, setting $x_3 = \sqrt{\|b\| - x_1^2 - x_2^2}$ then $G(X)$ and $cos\theta_3(x_1, x_2, x_3)$ become two variables functions, $G(X) = G(X)(x_1, x_2)$ and $cos\,\theta_3(x_1, x_2)$, which can be easily studied from a differential point of view.

It is important to notice that all the foregoing measures not only can be easily applied to any n-dimensional case with multi-dimensional outputs (i.e., for example, l inputs and m outputs in the space $\mathbb{R}^{l+m}$), but also to the more general case of a vector space V over a field $\mathbb{K}$. Indeed all the tools we introduced hold for any finite dimensional vector space. In that respect recall that the set $\mathcal{H}om(V, W)$ of all linear maps between two vector spaces V and W over the same field $\mathbb{K}$ is a vector space itself. Hence we can consider the vector space $\mathcal{H}om(\mathbb{R}^l, \mathbb{R}^m)$ in which a vector is a linear function from $\mathbb{R}^l$ to $\mathbb{R}^m$. More in general, our model applies to all finite dimensional topological vector spaces such as, for example, the space of degree n polynomials over a field $\mathbb{K}$, the finite dimensional subspaces of smooth functions on $\mathbb{R}$ and so on.

2.6 A toy illustration

Consider the production schedules of 10 hypothetical firms composing an industry as reported in Table 1, with two inputs, labor, on the x axis, and capital, on the y axis, and one output, on the z axis, measured in terms of value added; "external" production activities are in bold. Figure 2 reports the solid angles in year 1 and 2, respectively.[5]

In order to better evaluate the proposed measure of heterogeneity and technical change, and, even more relevant, their evolution over time, we allow for a change in only one of the firm (vector) making up our hypothetical industry in going from one period to the other, as reported in Table 1. In particular, from period 1 to 2 only the production schedule of firm 10 changes with unequivocal productivity increases as both inputs decrease while output increases. Then, from period 2 to period 3 the nineth firm exits the industry. The property of the vector representing the nineth firm is that it is an "external" vector, hence removing it, affects significantly the shape of the zonotope. Finally, from period 3 to 4 firm 8 leaves the industry. However this time it is a firm represented by an "internal" vector. How do these changes, i.e. a firm increasing productivity and two different firm exiting, affect industry heterogeneity and the extent and direction of technical change?

Let us introduce a few notations in order to study this easy example. Denote by $a_j^t \in \mathbb{R}^3$ the 3-dimensional vector representing the production activity of the firm j in the year t, $1 \leq j \leq 10$

[5] Numerical calculations for this toy illustration as well as for the empirical analysis that follows have been performed using the software zonohedron, written by Federico Ponchio. The code and instructions are available at: `http://vcg.isti.cnr.it/~ponchio/zonohedron.php`.

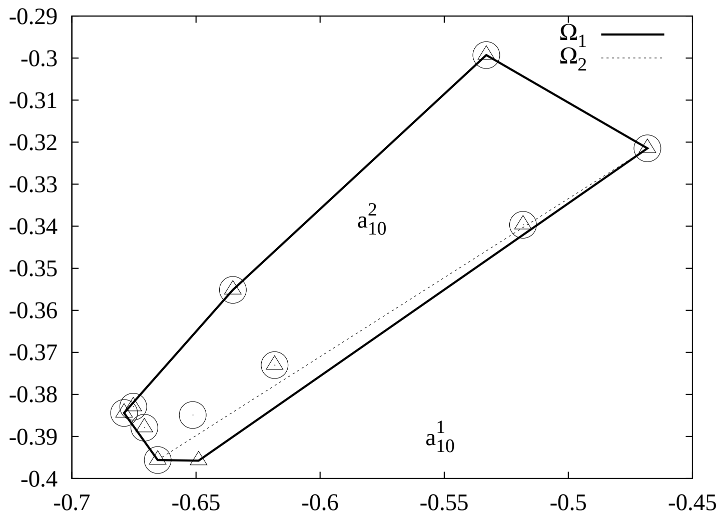

Figure 2: Solid angle in year 1 and 2.

and $1 \leq t \leq 4$ (e.g. $a_1^1 = (7.0, 4.0, 9.0)$ and $a_2^2 = (1.0, 4.0, 5.0)$). The zonotope at year t will be denoted by Y^t and the industry production activity will be $d_{Y^t} = \sum_{j=1}^{10} a_j^t$, $1 \leq t \leq 4$.

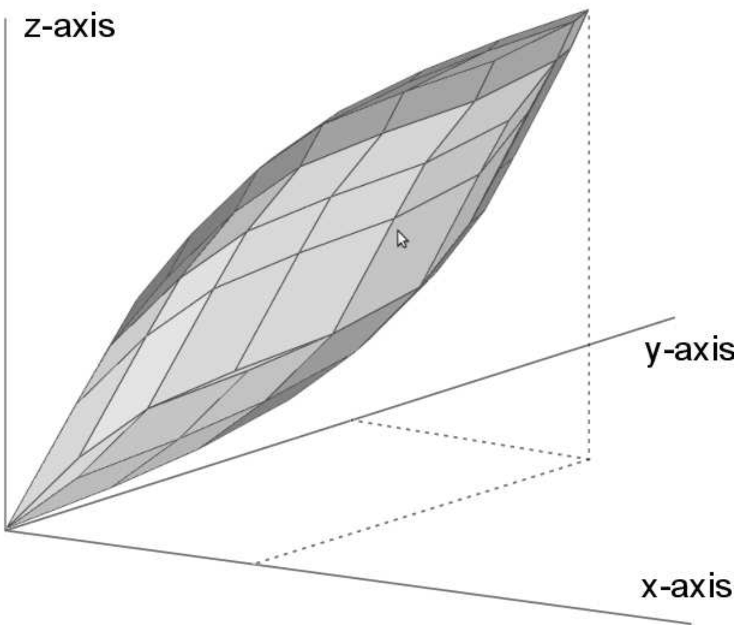

Figure 3: 3D representation of the zonotope of the toy illustration.

Then the matrices described in section 2.1 will be 3×3 matrices $A_{i,j,k}^t$ with vectors a_i^t, a_j^t, a_k^t as columns and determinants $\Delta_{i,j,k}^t$.

Under the foregoing notations, the volumes of zonotopes Y^t are given by

$$Vol(Y^t) = \sum_{1 \leq i < j < k \leq 10} \mid \Delta_{i,j,k}^t \mid \quad , \qquad 1 \leq t \leq 4$$

and yielding the following values:

	Year 1	Year 2	Year 3	Year 4
$G(Y^t)$	0.09271	0.07196	0.06518	0.06880
$G(\overline{Y}^t)$	0.09742	0.07905	0.06795	0.07244
$G(Y_e^t)$	0.12089	0.09627	0.07297	0.07297
Solid Angle	0.28195	0.22539	0.15471	0.15471
$G(Y^t)$ / $G(Y_e^t)$	0.70593	0.74748	0.89324	0.94285
$\cos\theta_3^t$	0.80423	0.82391	0.83385	0.84050
$\cos\varphi_1^t$	0.84089	0.82885	0.83960	0.81194

Table 2: Measures of Volume, normalizations and solid angles in the four years of the toy illustration.

$$Vol(Y^1) = 8265 \quad Vol(Y^2) = 6070 \quad Vol(Y^3) = 4664.5 \quad Vol(Y^4) = 3402.$$

The norm of the 3-dimensional diagonal vector $d_{Y^t} = (d_1^t, d_2^t, d_3^t)$, $1 \le t \le 4$, is $\|d_{Y^t}\| = \sqrt{(d_1^t)^2 + (d_2^t)^2 + (d_3^t)^2}$ and we get for its cube the following numerical values:

$$\|d_{Y^1}\|^3 = 631547 \quad \|d_{Y^2}\|^3 = 639929 \quad \|d_{Y^3}\|^3 = 566597 \quad \|d_{Y^4}\|^3 = 401372.$$

The Gini volume will be:

$$G(Y^t) = \frac{Vol(Y^t)}{d_1^t d_2^t d_3^t}$$

and the numerical results for $1 \le t \le 4$ are shown in Table 2.

As illustrated in Section 2.4 the variation over time of the angle θ_3 that the diagonal of the zonotope Y^t forms with the output axis z can be used to assess if and to what extent productivity is growing in a given industry; similarly if φ_1^t is the angle that the diagonal of Y^t forms with the x axis, then $\cos\varphi_1^t$ allows to study how the relative inputs use changes over the years. Using the notation introduced above, they are given, respectively, by

$$\cos\theta_3^t = \frac{d_3^t}{\|d_{Y^t}\|} \quad \text{and} \quad \cos\varphi_1^t = \frac{d_1^t}{\|(d_1^t, d_2^t, 0)\|},$$

where the first one is the index of the technical evolution of the output and the second one is the index for the first input (the second one can be obtained as $\cos\varphi_2^t = 1 - \cos\varphi_1^t$). Table 2 displays the values of Gini volume for the zonotopes Y^t, the zonotopes $\overline{Y}^t$ generated by the normalized production activities $\{\frac{a_j^t}{\|a_j^t\|}\}_{1 \le j \le 10}$ and the zonotopes Y_e^t generated by the external production activities which are in bold in Table 1. Moreover it also reports the solid angle, the ratio of the Gini volumes of Y^t over the Gini volumes of Y_e^t and the angles that account for the rate and direction of technical change.

In going from year 1 to 2 firm 10 reports an unequivocal increase in productivity. As shown in Figure 2 the normalized vector accounting for the production activity of firm 10 rotates inward: in period 1 $\overline{a}_{10}^1$ is a boundary (normalized) vector, whether in period 2, $\overline{a}_{10}^2$ is an "internal" vector. Since a boundary vector (firm) shifts inward, production techniques are more similar in period 2, hence heterogeneity within the industry reduces. This is captured by our proposed measures which all vary in the expected direction. The Gini index, $G(Y)$, the Gini index on normalized, $G(\overline{Y})$ and "external" vectors reduce from year 1 to year 2. As apparent from Figure 2 also the solid angle reduces. The ratio $G(Y^t)$ / $G(Y_e^t)$ increases suggesting that internal vectors now contribute more to the volume as compared to external vectors. The variation of the angle θ_3 that the diagonal of the zonotope forms with the output axis is our measure of technical change. From year 1 to 2, firm 10, the least efficient, becomes more productive, and this *within* effect positively contributes to productivity growth at the industry level as captured by the increase in the cosine of the angle θ_3. The last indicator of Table 2 is informative of the direction of technical change. The decrease in $\cos\varphi_1^t$ suggest that technical change was capital saving.

From year $t = 2$ to year $t = 3$ firm 9, an external vector, leaves the industry.[6] The outcomes are smaller Gini volumes for all our Zonotopes. The solid angle reduces, too, whether the cosine of the angle θ_3 increases, suggesting the exit of firm 9 resulted in a a further efficiency gain for the industry. Technical change is labor saving as $\cos\varphi_1^t$ increases.

From period $t = 3$ to $t = 4$ an "internal" vector, firm 8, drops the industry. In this case all our measures of Gini volumes point to an increase in heterogeneity, except, obviously, $G(Y_e^t)$ since the boundary vectors do not change. Again the exit of firm 8 positively contributes to productivity growth in the industry, as shown by the increase in $\cos\theta_3^t$. Technical change is now capital saving, $\cos\varphi_1^t$ reduces.

More in general, the graph in figure 4 shows how the heterogeneity changes when a generic firm of value added equals to 5.0 enters the industry in year 1. The function plotted in Fig. 4 is the function G(X) in equation (3) with $Z = Y^1$, $N = 10$ and vectors $a_n = a_n^1$.

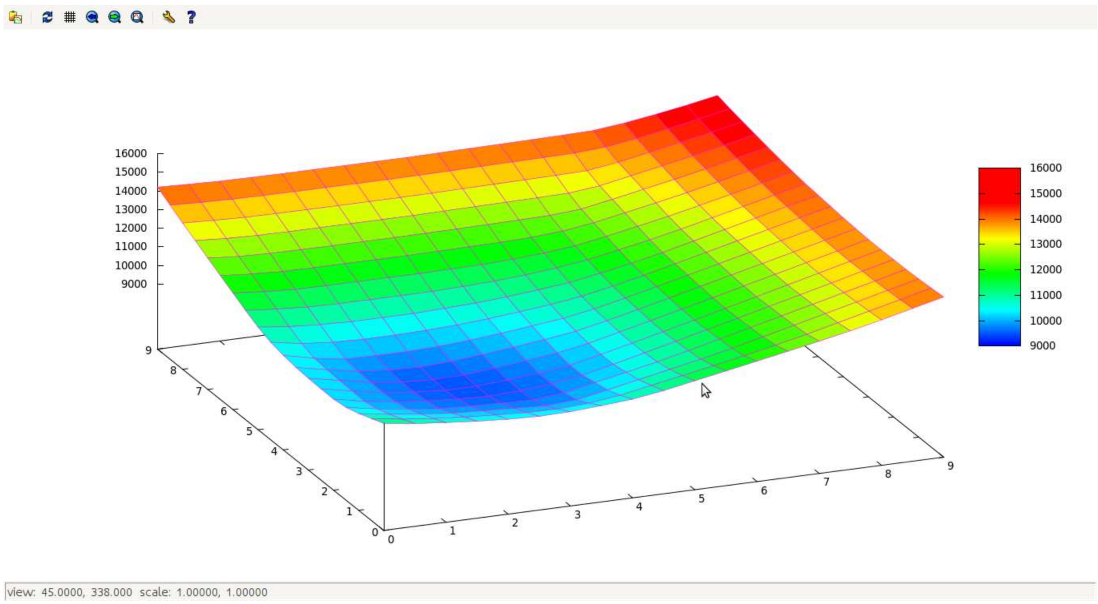

Figure 4: Variation of heterogeneity (on the z axis) when a firm of labor x, capital y and fixed value added enters the industry.

3 An empirical application

In the following we put the model at work on longitudinal firm-level data of an ensemble of Italian 4-digit industries (chosen on the grounds of the numerosity of observations) over the period 1998-2006. Values have been deflated with the industry-specific production price index. Output is measured as valued added (thousands of euro), capital is tangible assets (thousands of euro) and labour is the number of employees (full time equivalent). More details on the databank are in Appendix A at the end of the paper. The list of sectors and the number of observations is reported in Table 3 together with the number of external vectors, in brackets.

[6] Note that, intuitively, external vectors are the analogous to the support of an empirical distribution.

Nace Code	Description			
1513	Meat and poultrymeat products	162 (7)	162 (10)	190 (9)
1721	Cotton-type weaving	139 (9)	119 (11)	113 (7)
1772	Knitted & crocheted pullovers, cardigans	137 (8)	117 (10)	100 (7)
1930	Footwear	616 (9)	498 (6)	474 (9)
2121	Corrugated paper and paperboard	186 (7)	176 (9)	199 (11)
2222	Printing n.e.c.	297 (11)	285 (10)	368 (8)
2522	Plastic packing goods	204 (7)	217 (10)	253 (11)
2524	Other plastic products	596 (9)	558 (9)	638 (10)
2661	Concrete products for construction	208 (8)	231 (11)	272 (7)
2663	Ready-mixed concrete	103 (8)	114 (8)	147 (10)
2751	Casting of iron	94 (7)	77 (9)	88 (9)
2811	Metal structures and parts of structures	402 (9)	378 (8)	565 (10)
2852	General mechanical engineering	473 (11)	511 (8)	825 (11)
2953	Machinery for food & beverage processing	131 (6)	134 (7)	159 (6)
2954	Machinery for textile, apparel & leather	191 (10)	170 (10)	154 (12)
3611	Chairs and seats	205 (8)	201 (10)	229 (7)

Table 3: Nace sectors for the empirical analysis. Number of observations in 1998, 2002 and 2002. In brackets the number of external vectors in each year.

Figure 5 is the real world analog of Figure 2 and it shows the coordinates of the normalized vectors on the unit sphere for firms making up the industry in 2002 and 2006. Both plots show that the solid angle provides a snapshot of the extreme techniques at use in a given industry. For the same reason, this measure can change a lot following a variation in the adopted technique by one firm only. Hence we will not refer to the solid angle as our measure of heterogeneity, but we'd rather focus on some normalized measures of the zonotope's volume.

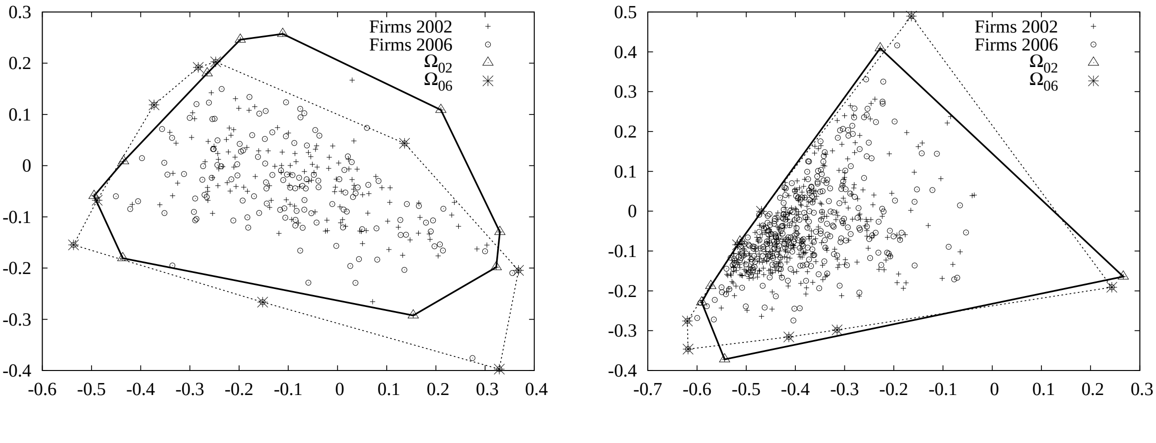

Figure 5: Production set and solid angle, sector 1513 (left) and 1930 (right) in 2002 and 2006, balanced sample.

3.1 Within Industry Heterogeneity and its dynamics

Table 4 reports the normalized volumes for the sectors under analysis. Notice that figures reported in Table 4 and following are measures and not estimates. The first set of columns report for 1998, 2002 and 2006 $G(Y)$ which is the ratio between the zonotope's volume and the volume of the parallelotope build on the zonotope's main diagonal. Notice that the volume of the cuboid (denominator) is much bigger than that of the zonotope (nominator) because, intuitively, the

Nace Code	I $G(Y)$ '98	'02	'06	II $G(\overline{Y})$ '98	'02	'06	III $G(Y_e)$ '98	'02	'06	IV Solid Angle '98	'02	'06
1513	0.059	0.051	0.062	0.082	0.062	0.096	0.391	0.201	0.301	0.071	0.056	0.094
1721	0.075	0.068	0.103	0.075	0.078	0.124	0.135	0.120	0.133	0.048	0.037	0.127
1772	0.160	0.122	0.136	0.154	0.126	0.130	0.142	0.273	0.172	0.175	0.121	0.122
1930	0.108	0.139	0.150	0.110	0.115	0.123	0.361	0.562	0.249	0.264	0.375	0.261
2121	0.108	0.043	0.062	0.081	0.064	0.081	0.257	0.105	0.178	0.078	0.041	0.045
2222	0.062	0.077	0.087	0.077	0.086	0.115	0.239	0.328	0.356	0.064	0.069	0.425
2522	0.065	0.061	0.070	0.071	0.064	0.074	0.197	0.261	0.266	0.135	0.040	0.061
2524	0.089	0.083	0.094	0.097	0.088	0.096	0.458	0.269	0.307	0.141	0.126	0.171
2661	0.079	0.088	0.099	0.100	0.094	0.110	0.376	0.234	0.352	0.127	0.046	0.165
2663	0.066	0.067	0.088	0.111	0.106	0.111	0.306	0.192	0.277	0.112	0.068	0.072
2751	0.035	0.037	0.070	0.064	0.055	0.073	0.174	0.107	0.184	0.028	0.028	0.040
2811	0.105	0.109	0.109	0.117	0.113	0.122	0.327	0.480	0.416	0.215	0.215	0.365
2852	0.088	0.102	0.110	0.100	0.103	0.111	0.227	0.395	0.391	0.154	0.092	0.124
2953	0.072	0.095	0.096	0.098	0.104	0.111	0.233	0.155	0.248	0.028	0.031	0.030
2954	0.078	0.074	0.093	0.086	0.130	0.113	0.170	0.141	0.352	0.016	0.117	0.071
3611	0.078	0.099	0.118	0.107	0.096	0.121	0.288	0.233	0.281	0.148	0.055	0.150

Table 4: Normalized volumes in 1998, 2002 and 2006 for selected 4 digit sectors.

parellelotope is formed by production activities that produce no output with positive amounts of inputs, and conversely, produce a high quantity of output with no input. That is why the ratio, $G(Y)$, although small in absolute value, points to relevant differences in the production techniques employed by firms in the industry. The trend over time of the ratio within any one industry allows to investigate how heterogeneity in the adopted techniques evolves over time. $G(Y)$ display an increase over time for most sectors,suggesting that heterogeneity is not shrinking, if anything it has rather increased.[7] Since $G(Y)$ is a ratio, we can also compare this measure of heterogeneity across industry and rank sectors according to the diversity of techniques that are employed. As it might be expected, there exist relevant differences in the degree of heterogeneity displayed by industries, $G(Y)$ varies in the range .03-.16. Nace sectors 1772, Knitted & crocheted pullovers, cardigans and 1930, Footwear, both display a normalized volume much bigger than most other sectors. Even more interestingly, also sectors that are supposed to produce rather homogeneous output, such as 2661, Concrete, and 2663, Ready-mixed concrete, display a degree of heterogeneity comparable, if not higher, to that of many other sectors.

The second set of columns reports the value of $G(\overline{Y})$ that is the Gini volume of the *unitary* zonotope. As recalled in Section 2, in this case the zonotope is formed by vectors having the same (unitary) length; hence in measuring industry heterogeneity all firms get assigned the same weight, and size plays no role. For most of sectors, $G(\overline{Y})$ is bigger than $G(Y)$ suggesting that, within any industry, smaller firms contribute relatively more to heterogeneity than bigger ones. In particular, in some industries, such as 2663, industry heterogeneity almost doubles when all firms are rescaled to have the same size. Finally, also $G(\overline{Y})$ display an increasing trend over time, from 2002 to 2006, pointing to growing differences in the techniques adopted by firms.

$G(Y_e)$ (column III) reports the Gini volume for the zonotope built on the external vectors only. As it could be expected, for all sectors $G(Y_e)$ is bigger than $G(Y)$. The subset of external vectors contribute to industry heterogeneity relatively more than *all* vectors, as in $G(Y)$.

[7] This result is coherent with the evidence shown in Dosi et al. (2012) on Italian firms, although employing a different methodology to explore heterogeneity.

3.2 Assessing industry level technical change

In this section we take to the data the investigation of technical change by means of the angle that the main diagonal of the zonotope forms with the output with the input plane. Notice that the nominal trend has been removed with the industry-specific production price index.

As shown in the toy illustration, Section 2.6 an increase in the cosine of the angle with output is evidence of an increase of efficiency of the industry. The first three columns of Table 5 reports the value of $\cos\theta_3$ respectively, in 1998, 2002 and 2006.[8]

Some sectors, such as Nace 1513, Meat and poultrymeat products, 2121 Corrugated paper and paperboard or 2522, Plastic packing goods do not display any increase in efficiency over the sample period, on the contrary the cosine with output is smaller for more recent years. Overall, not many sectors display a constant increase of the angle in all periods, as that was a decade of almost no growth for Italian manufacturing firms, see Dosi et al. (2012) and OECD (2008). Notice that, similarly, also the values of the *unitary* zonotope point to the same trend.

Nace	(a) $\cos\theta_3$			(b) $\cos\overline{\theta_3}$		
Code	1998	2002	2006	1998	2002	2006
1513	0.9379	0.9221	0.9036	0.9212	0.9209	0.9161
1721	0.8897	0.9071	0.9138	0.9346	0.9307	0.9454
1772	0.9848	0.9840	0.9908	0.9811	0.9754	0.9826
1930	0.9897	0.9903	0.9938	0.9869	0.9852	0.9879
2121	0.8474	0.8298	0.8048	0.8804	0.8743	0.8699
2222	0.9347	0.8947	0.8997	0.9518	0.9207	0.9305
2522	0.8777	0.8323	0.7806	0.8802	0.8644	0.8519
2524	0.9393	0.9183	0.9189	0.9487	0.9362	0.9369
2661	0.9551	0.9603	0.9418	0.9621	0.9670	0.9537
2663	0.9158	0.9440	0.9170	0.9574	0.9586	0.9525
2751	0.9051	0.8161	0.8474	0.9199	0.8358	0.8876
2811	0.9756	0.9783	0.9746	0.9796	0.9806	0.9824
2852	0.9702	0.9619	0.9634	0.9756	0.9682	0.9709
2953	0.9943	0.9960	0.9956	0.9954	0.9953	0.9956
2954	0.9948	0.9947	0.9916	0.9952	0.9952	0.9929
3611	0.9908	0.9865	0.9865	0.9890	0.9870	0.9854

Table 5: Angles of the zonotope's main diagonal. (a) Cosine with output, original zonotope; (b) *unitary* zonotope.

The change over time of the angle φ_i between the projection vector $pr(d_Y)$ and the x_i axis captures the changes of the quantity of input i over time with respect to all the other inputs. Results are reported in Table 6. For most of sectors the value of $\cos\varphi_i$ decreases over time, suggesting that industries have moved towards more labor intensive techniques. Notice that assigning the same same weight to all firms, irrespectively of their size, sometime results in a different pattern over time.

[8] Changing the unit of measurement, i.e. considering value added in milions (rather than thousands) of euro of course changes the value of the angle, but the variation over time - our proxy of technical change - is not affected by the unit of measure.

Nace	(a) $\cos\varphi_1$			(b) $\cos\overline{\varphi_1}$		
Code	1998	2002	2006	1998	2002	2006
1513	0.0434	0.0391	0.0370	0.0458	0.0441	0.0497
1721	0.0476	0.0467	0.0540	0.0657	0.0622	0.0774
1772	0.1449	0.1475	0.1761	0.1951	0.1585	0.1751
1930	0.2133	0.1850	0.1941	0.2527	0.2133	0.2184
2121	0.0256	0.0254	0.0250	0.0417	0.0374	0.0402
2222	0.0523	0.0382	0.0410	0.0752	0.0580	0.0679
2522	0.0330	0.0268	0.0228	0.0417	0.0358	0.0368
2524	0.0557	0.0465	0.0499	0.0740	0.0635	0.0679
2661	0.0704	0.0603	0.0557	0.0944	0.0804	0.0824
2751	0.0458	0.0272	0.0313	0.0512	0.0330	0.0404
2811	0.1092	0.1017	0.1013	0.1432	0.1261	0.1489
2852	0.0987	0.0755	0.0752	0.1254	0.0955	0.0987
2953	0.1811	0.1836	0.1668	0.2368	0.1938	0.2074
2954	0.1945	0.1905	0.1674	0.2410	0.2557	0.2122
3611	0.1723	0.1561	0.1587	0.2165	0.1848	0.1946

Table 6: Angles of the zonotope's main diagonal. (a) Angles on production inputs plane, original zonotope; (b) *unitary* zonotope.

4 Conclusions

Robust evidence at the level of the firm has shown that firms choose different relative input intensities and that also their ex-post performance is much heterogeneous, even within the same industry. The present paper presents a new framework to assess firm level heterogeneity and to study the rate and direction of technical change.

The present paper ....

These findings raise questions for future research. [...]

How industries evolve following a perturbation such as the introduction of an innovation. If the adoption of the new technology is taking place rapidly, then one would observe a relative fast convergence towards the mix of inputs required with the new technology. This trend would be captured by a decrease in the (normalized) volume of the zonotope. On the contrary, the co-existence of the "old" and the "new" technology is suggested by an in increase in the (normalized) volume.

Appendix

The database employed for the analyses, Micro.3, has been built through to the collaboration between the Italian statistical office, ISTAT, and a group of LEM researchers from the Scuola Superiore Sant'Anna, Pisa.[9]

Micro.3 is largely based on the census of Italian firms yearly conducted by ISTAT and contains information on firms above 20 employees in all sectors[10] of the economy for the period 1989-2006. Starting in 1998 the census of the whole population of firms only concerns companies with more than 100 employees, while in the range of employment 20-99, ISTAT directly monitors only a "rotating sample" which varies every five years. In order to complete the coverage of firms in the range of employment 20-99 Micro.3 resorts, from 1998 onward, to data from the

[9]The database has been made available for work after careful censorship of individual information. More detailed information concerning the development of the database Micro.3 are in Grazzi et al. (2009).

[10]In the paper we refer to the Statistical Classification of Economic Activities known as NACE, Revision 1.1 (final draft 2002).

financial statement that limited liability firms have to disclose, in accordance to Italian law.[11]

In order to undertake intertemporal comparison, we deflate our data on current value variables making use of the sectoral production price index provided by ISTAT and taking 2000 as the reference year.[12] The deflators are available from 1991 onward.

Because of the purpose of the present work we investigate the dynamics within narrowly defined industry, to get a broader picture of the Italian manufacturing industry in the period of interest we refer to Dosi et al. (2012) and (OECD; 2008).

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[11]Limited liability companies (*società di capitali*) have to hand in a copy of their financial statement to the Register of Firms at the local Chamber of Commerce

[12]Istat provides the time series for the Italian economy at: *http://con.istat.it/default.asp*

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