

Organizational Structure and the Choice of Price vs. Quantity in a Mixed Duopoly

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Abstract We consider the choice of price/quantity by a public and a private firm in a mixed differentiated duopoly. First, we study the way in which the strategic choice of the market variable is affected by different given organizational structures (managerial or entrepreneurial) of the public and the private firm. Second, we investigate how the price/quantity choice interacts with the endogenous choice of the organizational structure, thus determining a subgame perfect equilibrium at which firms choose to behave as price-setters and to adopt a managerial structure.

JEL classification Numbers: D43, L22, L32.

Keywords mixed duopoly; strategic delegation; price competition; quantity competition

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1 Introduction

The present paper aims at investigating the relationship between firms' organizational structure and the optimal choice of market strategies, an issue which has been receiving an increasing attention among economics and business researchers. This objective is pursued by describing the driving forces behind the choice of the strategic variable played by firms on the market, price or quantity, when a management structure with centralized or decentralized market decision-making is either exogenously given or endogenously adopted.

Indeed, the Industrial Organization literature reflects a growing concern with the analysis of the factors affecting the endogenous choice of the market variable in multi-stage games; the latter is typically made at the first stage by firms competing at the market stage according to the selected strategy. In duopoly models with private firms, a number of articles have shown how the strategic decision between price and quantity is affected by the substitutability/complementarity relationship between goods (Singh and Vives, 1984), by demand uncertainty (Klemperer and Meyer, 1986; Reisinger and Ressler, 2009), and - in a framework with tacit collusion - by the size of the discount factor (Lambertini and Schultz, 2003). More recently, the analysis has been extended to duopolistic markets in which a public firm competes against a private firm: the price-quantity choice under mixed competition with differentiated products is studied by Matsumura and Ogawa (2012) in a standard duopoly and by Scrimatore (2013) in the presence of firm subsidization. While the former demonstrates the existence of incentives for both the types of firms to choose a price strategy at equilibrium, the latter shows that this result is conditioned to the presence of sufficiently low subsidies.¹

All the above works share the assumption of centralization of market decisions within owner-managed firms. Indeed, profit-maximizing firms in a private market, and a public welfare-maximizing competing against a private profit-maximizing firm in a mixed market, are assumed to directly choose the optimal price/output levels at the market stage. In the real world, however, firm decision-making involves a more complex process in which decisions can be decentralized. In this work we assume that decisions at the market stage can be taken by revenue-interested managers on the basis of delegation contracts offered by shareholders.² However, in our context the managerial contracts are not aimed, as prescribed by agency theory, to align the divergent preferences of owners and managers in order to achieve efficiency. In line with the strategic delegation theory (Vickers, 1985; Fershtman and Judd, 1987), we assume that contracts are designed with the aim to strategically exploit such a divergence of objectives and achieve a competitive advantage on the market. Indeed, through

¹The strategic choice of the strategy to play in a mixed duopoly has been also investigated by Choi (2012) in a model with wage-bargaining which shows the existence of a dominant strategy (price) only for the public firm.

²The separation of ownership and control observed in both private medium-sized or large firms leads to delegation of decision control from owners to professional managers (Fama and Jensen, 1983). The phenomenon also characterizes state-owned firms where 'an extreme form of separation of ownership and control' is argued to exist (Bolton, 1995, p. 2).

a delegation mechanism which shapes managers' incentives allowing them to pursue to some extent their own objectives, owners can credibly commit to a course of action granting an advantage to their own firms. By assuming that competition may involve firms with different organizational structures - with delegating managerial firms possibly coexisting with profit-maximizing entrepreneurial firms - the literature on strategic delegation has shown that the structure of the incentive contracts offered to managers and the advantages from delegation depend on the share of delegating firms - in duopoly on delegation being unilateral or bilateral - and on the nature of competition, i.e. quantity or price (Vickers, 1985; Fershtman and Judd, 1987; Scrimatore, 2012).

The above considerations open the question whether delegation is indeed optimal in different competitive environments. The strategic choice of hiring a manager to whom to delegate market decisions, namely the endogenous choice of the firm organization structure, has been investigated in different private duopoly frameworks. In particular, Basu (1995) and Mitrokostas and Petrakis (2013) identify the conditions under which symmetric and asymmetric delegation configurations arise at equilibrium, relating them to cost asymmetries; Lambertini (2000) finds that a managerial structure is symmetrically chosen by cost identical firms at the subgame perfect equilibrium of a game in which firms also choose to set quantities simultaneously.³ The optimality of delegation has been investigated also in the framework of mixed markets.⁴ Indeed, in a duopoly with quantity competition White (2001) finds an asymmetric equilibrium in which the public firm chooses to be entrepreneurial and the private firm to be managerial. This result contrasts with that obtained by Bárcena-Ruiz (2009) in a mixed duopoly with price competition, in which both the public firm and the private firm choose a managerial structure at equilibrium.

Summing up, at the state of the art the literature on mixed markets has treated the two issues of the mode of competition and the organizational structure of firms independently of each other. On the one side, the endogenization of the mode of competition has been studied under the standard assumption of a public welfare maximizing firm and a private profit-concerned firm. On the other side, the endogenization of the organizational structure has been investigated in given price-setting or quantity-setting setups. This paper aims at relating these two strategic dimensions of competition. In particular, it contributes to the existing literature by examining the price/quantity strategic decision in two scenarios of competition with product differentiation. In the first scenario the choice of quantity vs. price is made by assuming that the public firm and the private firm commit to a given organizational configuration. In a second scenario, the analysis of the strategic effects driving the choice between price and

³With respect to the mode of competition, Basu (1995) assumes that firms compete in quantities, Mitrokostas and Petrakis (2013) investigate the effects of both quantity and price competition, finally Lambertini (2000) assumes that firms endogenously choose to behave as price setters or quantity setters.

⁴The use of incentive contracts as strategic variable in mixed markets has been investigated, among others, by Barros (1995) in a quantity setting framework and by Nakamura and Inoue (2009) in a price setting framework.

quantity is enriched by considering as endogenous the same choice of the organizational structure. The analysis carried out under these two scenarios allows us to address two related questions. First, by assuming that firms commit to a given organizational type, we investigate the strategic effects of delegation on the outcome of market competition and on the choice of the market variable. Second, by endogenizing the choice of the organizational structure made by firms after deciding upon the market variable, we study how the decision upon firm organization can be strategically oriented according to the most advantageous mode of competition.

Our results show that when firms commit to an organizational structure, price is an equilibrium choice for both firms under unilateral delegation, with the private (public) firm being indifferent between price and quantity when acting as entrepreneurial. Moreover, under symmetric commitment to delegation, no equilibrium in pure strategies is shown to exist. Conversely, when the choice of the organizational structure is endogenous, a symmetric delegation equilibrium is found to be part of the sub-game perfect equilibrium at which firms choose to play according to price competition on the product market.

The rest of the paper is structured as follows. The basic setting presented in Section 2 is developed in Section 3 where a price/quantity game for any given configuration of firms' organizational structure is solved. In Section 4 both the optimal market variable and the optimal organizational structure are derived as solutions of the unique extended game. Section 4 also compares and discusses the results of the two frameworks, while Section 5 concludes.

2 The basic framework

We consider a mixed duopoly, in which the public firm (indexed with 1) and the private firm (indexed with 2) produce two imperfectly substitutable goods at a constant and identical average and marginal cost $c \in [0, 1)$. The public firm is owned by the government whose ultimate objective is social welfare, defined as the sum of the consumer surplus and the profits of the two firms; the owner of the private firm has an ultimate objective in terms of her own profits. We assume that consumers' preferences are described by the following utility function, defined over the two goods:

$$U(q_1, q_2) = q_1 + q_2 - \frac{(q_1^2 + q_2^2 + 2\gamma q_1 q_2)}{2}, \quad 0 < \gamma < 1$$

so that social welfare $W = CS + \pi_1 + \pi_2$, can be written as:

$$W = \frac{\left((1 - \gamma) \left((q_1)^2 + (q_2)^2 \right) + \gamma (q_1 + q_2)^2 \right)}{2} + \pi_1 + \pi_2 \quad (1)$$

with

$$\pi_i = (p_i - c) q_i \quad i = 1, 2 \quad (2)$$

and where q_i and p_i , $i = 1, 2$, denote respectively the quantity produced (consumed) of good i and its price.

As in Matsumura and Ogawa (2012) - henceforth MO - prior to any other decision firms choose whether to offer a price or a quantity contract to customers. Given the above hypothesis on preferences, according to the strategic variable chosen by firms we can formulate the demand functions faced by firms as follows.

If both firms choose a quantity contract, they face the inverse demand functions:

$$p_i = 1 - q_i - \gamma q_j \quad i, j = 1, 2, \quad i \neq j \quad (3)$$

If both firms choose a price contract, they face the direct demand functions:

$$q_i = \frac{(1 - \gamma) - p_i + \gamma p_j}{(1 - \gamma^2)} \quad i, j = 1, 2, \quad i \neq j \quad (4)$$

If firm i chooses a price contract, while firm j makes a quantity contract, their respective demand functions are:

$$q_i = 1 - p_i - \gamma q_j \quad (5a)$$

$$p_j = 1 - \gamma - q_j (1 - \gamma^2) + \gamma p_i \quad i, j = 1, 2, \quad i \neq j \quad (5b)$$

We extend the MO setting by assuming that each firm may be either entrepreneurial or managerial, according to White (2001) terminology. If a firm is entrepreneurial, it behaves at the market stage according to the ultimate objective function of its owner; if it is managerial, it delegates market decisions to a manager. In a strategic delegation vein (e.g. Fershtman and Judd, 1987), we assume that the objective function of the latter - which is mirrored in the incentive contract offered to the manager - is a linear combination of profits and revenues:

$$M_i = \theta_i \pi_i + (1 - \theta_i) p_i q_i \quad i = 1, 2 \quad (6)$$

where θ_i is optimally chosen by the owner, prior to market competition, in order to maximize its own objective function. Notice that consistently with the existing literature (Barros, 1995; White, 2001; Barcena-Ruiz, 2009), we are assuming that the structure of the manager's objective function is the same for the public and private firm; however, since the government is concerned with social welfare while the owner of the private firm is concerned with profits, the optimal contract offered by a managerial public firm and a by a managerial private firm, *ceteris paribus*, will be different. It must be stressed that the assumption that both firms, when delegating, adopt a managerial incentive contract as in (6) introduces an important asymmetry in the ability of the two firms to replicate at the market stage the behavior consistent with maximization of their ultimate objective function, through an appropriate setting of θ : clearly, while for the private firm $\theta = 1$ implies profit maximization, there is no value of θ through which the W function can be recovered from the M function.⁵

⁵We recall that $\theta < 1$ implies that managers are induced to under-estimate costs with respect to a standard profit maximizing criterion, and therefore to behave more aggressively as compared with the profit maximizing benchmark. On the contrary, $\theta > 1$ implies a less aggressive behavior of managers with respect to that benchmark.

Given the above setup, the interactions between the public and private firm will be described under two different hypotheses. In the next section we shall assume that firms are committed to a given organizational structure, i.e. their being entrepreneurial or managerial is given. This implies that we shall solve by backward induction the following three-stage game. At the first stage firms' owners choose whether to make a price or a quantity contract. At the second stage, the managerial firms choose the optimal incentive contract offered to their manager – in particular we shall investigate the three cases in which (a) only the public firm is managerial, (b) only the private firm is managerial, and (c) both firms are committed to a managerial contract.⁶ Finally, at the market stage, firms set the optimal value of the variable chosen at the first stage, according to the objective function inherited from the second stage.

We shall then turn to a game in which the decision whether to hire or not a manager is endogenized. Notice that when the managerial *vs* entrepreneurial structure of firms is endogenous, the owners' choice of the contract offered to customers (price *vs* quantity) is made by taking into account how this same choice affects the equilibrium organizational structure of the two firms, i.e. the objective function according to which they are going to behave at the market stage.

3 Price vs quantity choice in the presence of commitment to an organizational structure

In this section we analyze the price *vs* quantity choice of the public and private firm under different given configurations of their organizational structure. In particular, we investigate the two cases in which there is unilateral delegation to the manager by one or the other firm, and the case in which both firms are committed to be managerial.⁷

3.1 The case with managerial public firm and entrepreneurial private firm (ME)

If the private firm is entrepreneurial (*E*) and the public firm is managerial (*M*), their objective function at the market stage are respectively π_2 from (2) and M_1 from (6). In order to solve by backward induction the price-quantity game, we derive the solution of the subgames associated to the four alternative price-quantity pairs.

The ME qq-game. If both firms decide at the first stage to offer a quantity contract to customers, the demand functions they face are given by (3). The

⁶Clearly, the case in which both firms are committed to be entrepreneurial is covered by the MO model.

⁷All games discussed in this section have interior solutions, i.e. they imply positive prices and quantities set at equilibrium by both firms.

reaction functions of the M public firm and the E private firm are respectively:

$$q_1^M(q_2) = \frac{1}{2}(1 - \gamma q_2 - c\theta_1) \quad (7a)$$

$$q_2^E(q_1) = \frac{1}{2}(1 - \gamma q_1 - c) \quad (7b)$$

so that the solution of the market stage is:

$${}_{qq|}q_1^{ME} = \frac{2 - \gamma(1 - c) - 2c\theta_1}{4 - \gamma^2}$$

$${}_{qq|}q_2^{ME} = \frac{2(1 - c) - \gamma + \gamma c\theta_1}{4 - \gamma^2}$$

where the right superscript denotes the organizational structure of the public and private firm, and the left subscript denotes the subgame type. At the second stage the public managerial firm chooses its optimal delegation parameter. By substituting eqt (3), ${}_{qq|}q_1^{ME}$, and ${}_{qq|}q_2^{ME}$ into the welfare function (1), the welfare maximizing value of θ_1 turns out to be:⁸

$${}_{qq|}\theta_1^{ME} = \frac{8c - \gamma^2 - 4(1 - \gamma) - 4\gamma c - 2\gamma^2 c}{c(4 - 3\gamma^2)}$$

Therefore, if firms play a qq -game, the values of their objective functions are:

$${}_{qq|}W^{ME} = \frac{(1 - c)^2(7 - 6\gamma)}{2(4 - 3\gamma^2)} \quad (8a)$$

$${}_{qq|}\pi_2^{ME} = \frac{4(1 - c)^2(1 - \gamma)^2}{(4 - 3\gamma^2)^2} \quad (8b)$$

The ME pp-game. If both firms decide at the first stage to offer a price contract, the relevant demand functions are given by (4), so that the reaction functions of the market stage are:

$$p_1^M(p_2) = \frac{1}{2}(1 - \gamma + \gamma p_2 + \theta_1 c) \quad (9a)$$

$$p_2^E(p_1) = \frac{1}{2}(1 - \gamma + \gamma p_1 + c) \quad (9b)$$

the solution of which is:

$${}_{pp|}p_1^{ME} = \frac{2(1 + \theta_1 c) - \gamma(1 + \gamma) + \gamma c}{4 - \gamma^2}$$

$${}_{pp|}p_2^{ME} = \frac{2(1 + c) - \gamma(1 + \gamma) + \gamma\theta_1 c}{4 - \gamma^2}$$

⁸Notice that in this case, as in others, θ_1 turns out to be negative for some (or all) values of γ , if $c < 1/2$. This apparently paradoxical result can be explained by recalling that in this framework it is only through the optimal decision upon θ_1 that the public firm can show up in terms of market behavior the aggressiveness which is implicit in its ultimate objective function (social welfare). When costs are low, the under-estimation of costs optimally imposed upon managers ($\theta_1 < 1$), and consistent with the public firm's aggressiveness, might turn into managers being instructed to positively evaluate costs ($\theta_1 < 0$).

Along the same lines described above, we obtain the optimal value of the delegation parameter of the public firm:

$${}_{pp}|\theta_1^{ME} = \frac{(1-c)(4\gamma - 2\gamma^3 - \gamma^4) + 3\gamma^2(1-2c) + 8c - 4}{c(4-3\gamma^2)}$$

so that in a pp -game with managerial public firm, the two firms achieve:

$${}_{pp}W^{ME} = \frac{(1-c)^2(7 - \gamma^3 - 5\gamma^2 + \gamma)}{2(\gamma+1)(4-3\gamma^2)} \quad (10a)$$

$${}_{pp}\pi_2^{ME} = \frac{(1-c)^2(1-\gamma)(2-\gamma^2)^2}{(\gamma+1)(4-3\gamma^2)^2} \quad (10b)$$

The ME qp-game. If the public M firm chooses a quantity contract while the E private firm adopts a price contract, the relevant demand functions are given by (5a) and (5b). This implies the following reaction functions at the market stage:

$$q_1^M(p_2) = \frac{1 - \gamma - \theta_1 c + \gamma p_2}{2(1 - \gamma^2)} \quad (11a)$$

$$p_2^E(q_1) = \frac{1}{2}(1 + c - \gamma q_1) \quad (11b)$$

which yield the solution:

$${}_{qp}q_1^{ME} = \frac{2(1 - \theta_1 c) - \gamma(1 - c)}{4 - 3\gamma^2}$$

$${}_{qp}p_2^{ME} = \frac{2(1 + c) - (1 - \theta_1 c)\gamma - (1 + 2c)\gamma^2}{4 - 3\gamma^2}$$

Since the welfare maximizing value of the public firm delegation parameter is:

$${}_{qp}|\theta_1^{ME} = \frac{\gamma(1-c) + 2c - 1}{c}$$

the qp -game implies the following values of the objective functions:

$${}_{qp}W^{ME} = \frac{(1-c)^2(7-6\gamma)}{2(4-3\gamma^2)}, \quad (12a)$$

$${}_{qp}\pi_2^{ME} = \frac{4(1-c)^2(1-\gamma)^2}{(4-3\gamma^2)^2} \quad (12b)$$

The ME pq-game. Finally we consider the case in which the M public firm makes a price contract and its rival E private firm a quantity contract. Again the demand system is given by (5a)-(5b), the reaction functions being:

$$p_1^M(q_2) = \frac{1}{2}(1 + \theta_1 c - \gamma q_2) \quad (13a)$$

$$q_2^M(p_1) = \frac{1 - c - \gamma + \gamma p_1}{2(1 - \gamma^2)} \quad (13b)$$

At the market stage equilibrium,

$$\begin{aligned} {}_{pq|}p_1^{ME} &= \frac{2(1 + \theta_1 c) - \gamma(1 - c) - \gamma^2(2\theta_1 c + 1)}{4 - 3\gamma^2} \\ {}_{pq|}q_2^{ME} &= \frac{2(1 - c) - \gamma(1 - \theta_1 c)}{4 - 3\gamma^2} \end{aligned}$$

The delegation parameter set by the public firm is:

$${}_{pq|}\theta_1^{ME} = \frac{c(2 + \gamma) - 1}{c(1 + \gamma)}$$

while the values of the objective functions are:

$${}_{pq|}W^{ME} = \frac{(1 - c)^2(7 - \gamma^3 - 5\gamma^2 + \gamma)}{2(1 + \gamma)(4 - 3\gamma^2)} \quad (14a)$$

$${}_{pq|}\pi_2^{ME} = \frac{(1 - c)^2(1 - \gamma)(2 - \gamma^2)^2}{(4 - 3\gamma^2)^2(1 + \gamma)} \quad (14b)$$

A comparison of the payoffs earned by the two firms in the above subgames – eqts (8), (10), (12) and (14) – yields the following rankings of welfare and private profits:

$${}_{pp|}W^{ME} = {}_{pq|}W^{ME} > {}_{qq|}W^{ME} = {}_{qp|}W^{ME} \quad (15a)$$

$${}_{pp|}\pi^{ME} = {}_{pq|}\pi^{ME} > {}_{qq|}\pi^{ME} = {}_{qp|}\pi^{ME} \quad (15b)$$

The key feature of this price-quantity game with unilateral public delegation is that the firms' payoffs are univocally defined by the choice of the managerial public firm. Once the latter decides for a price or a quantity contract, the decision of the other firm is irrelevant in terms of prevailing prices, quantities and payoffs. This can be explained by referring to two properties of this game. The first is the equivalence between unilateral delegation – or any kind of optimal unilateral strategic manipulation of the objective function – and playing as leader at the market stage (Benassi *et al.*, 2013). The second is related to the nature of the reaction functions in the asymmetric price-quantity subgames: when one firm reacts by setting, say, its price to a given quantity set by its rival, it indeed sets that price at which it sells the quantity which would be the optimal response to the given quantity of the rival in a Cournot game (Singh and Vives, 1984, p.550). Indeed, the best replies of the asymmetric games, $p_i(q_j)$ in the (q_j, p_i) space or $q_i(p_j)$ in the (p_j, q_i) space, respectively translate, by means of the demand curves, into the best replies $q_i(q_j)$ in the (q_j, q_i) space or $p_i(p_j)$ in the (p_j, p_i) space. Unilateral delegation (or market leadership) allows the delegating (leader) firm to select a point on the reaction function of the rival,

thus defining the equilibrium value of market variables. Since in asymmetric games the selection by the delegating firm of a price (quantity) response of the rival to its own quantity (price) choice amounts to selecting the rival's quantity (price) response in the corresponding symmetric game, once the delegating firm has chosen a price (quantity) contract, the outcomes of the symmetric and asymmetric corresponding subgames are identical.

The solution of the three-stage game is now stated in Proposition 1.

Proposition 1 *In the presence of a public managerial firm and a private entrepreneurial firm, the subgame perfect equilibrium of the price-quantity game is characterized by the public firm offering a price contract, while the private firm is indifferent between a price or a quantity contract, the latter delivering the same overall outcome.*

Proof. It follows straightforwardly from the rankings of payoffs in (15a)-(15b). ■

Since the pp (or the equivalent pq) choice is a pair of dominant strategies (weakly dominant for the private), it also results as equilibrium of games in which the choice of the strategic variable is sequential, with public or private leadership.

3.2 The case with entrepreneurial public firm and managerial private firm (EM)

We consider now the case in which the public firm is entrepreneurial, while the private firm is managerial. Therefore, at the market stage the former maximizes welfare in (1), while the latter maximizes M_2 from (6). The solution of the four subgames associated to the possible price-quantity choice is obtained along the same lines as the previous case, with the private firm, rather than the public one, now choosing the optimal delegation parameter at the second stage. Their main features can be synthesized as follows.

The EM qq-game. Since the reaction functions are:

$$q_1^E(q_2) = 1 - c - \gamma q_2 \quad (16a)$$

$$q_2^M(q_1) = \frac{1}{2}(1 - c\theta_2 - \gamma q_1) \quad (16b)$$

the solution at the market stage is

$$\begin{aligned} {}_{qq}q_1^{EM} &= \frac{2(1-c) - \gamma(1-c\theta_2)}{2-\gamma^2} \\ {}_{qq}q_2^{EM} &= \frac{1-\gamma(1-c) - c\theta_2}{2-\gamma^2} \end{aligned}$$

At the second stage, the profit maximizing value of θ_2 is

$${}_{qq}\theta_2^{EM} = \frac{2c(1+\gamma) - \gamma^2(1-c)}{2c(1+\gamma)}$$

so that the payoffs of this subgame are:

$${}_{qq|}W^{EM} = \frac{(1-c)^2(7+\gamma)}{8(1+\gamma)} \quad (17a)$$

$${}_{qq|\pi_2^{EM}} = \frac{(1-c)^2(1-\gamma)}{4(1+\gamma)} \quad (17b)$$

The EM pp-game. In the price space, the optimal replies of firms are given by

$$p_1^E(p_2) = c(1-\gamma) + \gamma p_2 \quad (18a)$$

$$p_2^M(p_1) = \frac{1}{2}(1-\gamma + \theta_2 c + \gamma p_1) \quad (18b)$$

leading to

$${}_{pp|}p_1^{EM} = \frac{(2c+\gamma)(1-\gamma) + \gamma\theta_2 c}{2-\gamma^2}$$

$${}_{pp|}p_2^{EM} = \frac{(1+\gamma c)(1-\gamma) + \theta_2 c}{2-\gamma^2}$$

Since the private firm optimally sets

$${}_{pp|\theta_2^{EM}} = \frac{2c(1+\gamma) + \gamma^2(1-c)}{2c(1+\gamma)}$$

we get

$${}_{pp|}W^{EM} = \frac{(1-c)^2(7+8\gamma)}{8(1+\gamma)^2} \quad (19a)$$

$${}_{pp|\pi_2^{EM}} = \frac{(1-c)^2}{4(1+\gamma)^2} \quad (19b)$$

The EM qp-game. When the public entrepreneurial firm chooses a quantity contract, while the private managerial firm chooses a price contract, the reaction functions are shaped as follows:

$${}_{qp|}q_1^E = \frac{1-c}{1+\gamma} \quad (20a)$$

$$p_2^M(q_1) = \frac{1}{2}(1 + \theta_2 c - \gamma q_1) \quad (20b)$$

Since the public firm chooses its quantity independently of the price chosen by the private firm, the latter sets at equilibrium:

$${}_{qp|}p_2^{EM} = \frac{1 + \gamma c + \theta_2 c(1+\gamma)}{2(1+\gamma)}$$

Notice that private delegation cannot modify the optimal choice of the public firm; therefore the optimal delegation parameter for the private firm is ${}_{qp|\theta_2^{EM}} =$

1, so that this solution collapses to that of the qp -game with entrepreneurial firms described by MO:

$${}_{qp|}W^{EM} = \frac{(1-c)^2(7+8\gamma)}{8(1+\gamma)^2} \quad (21a)$$

$${}_{qp|\pi_2}{}^{EM} = \frac{(1-c)^2}{4(1+\gamma)^2} \quad (21b)$$

Moreover, the outcome of this game coincides with that of the EM pp -game: since we are assuming unilateral delegation, the price *vs* quantity choice of the non-delegating firm is irrelevant, for the same reasons discussed in the previous subsection.

The EM pq -game. Strategic independence at the market stage of the E public firm's decision from the choice of its M private rival also arises when the former chooses a price contract and the latter a quantity contract; the best reply functions are indeed:

$${}_{pq|}p_1^E = c \quad (22a)$$

$$q_2^M(p_1) = \frac{1-\gamma-\theta_2c+\gamma p_1}{2(1-\gamma^2)} \quad (22b)$$

so that

$${}_{pq|}q_2^{EM} = \frac{1-\gamma(1-c)-\theta_2c}{2(1-\gamma^2)}$$

and ${}_{pq|\theta_2}{}^{EM} = 1$. The irrelevance of the price-quantity choice of the non-delegating firm explains why the outcome of this subgame coincides with that of the EM qq -game:

$${}_{pq|}W^{EM} = \frac{(1-c)^2(7+\gamma)}{8(1+\gamma)} \quad (23a)$$

$${}_{pq|\pi_2}{}^{EM} = \frac{(1-c)^2(1-\gamma)}{4(1+\gamma)} \quad (23b)$$

The solutions of the four subgames – eqts. (17), (19), (21), and (23) – generate the following rankings of outcomes

$$\begin{aligned} {}_{qq|}W^{EM} &= {}_{pq|}W^{EM} > {}_{pp|}W^{EM} = {}_{qp|}W^{EM} \\ {}_{pp|\pi}{}^{EM} &= {}_{qp|\pi}{}^{EM} > {}_{qq|\pi}{}^{EM} = {}_{pq|\pi}{}^{EM} \end{aligned}$$

which allow us to establish Proposition 2.

Proposition 2 *In the presence of a public entrepreneurial firm and a private managerial firm, the subgame perfect equilibrium of the price-quantity game is*

characterized by the private firm making a price contract, while the public firm is indifferent between a price or a quantity contract, the latter delivering the same overall outcome.

Again the *pp* (or the *qp* equivalent) equilibrium is obtained as solution of the corresponding game in which the price-quantity choice is sequential, irrespective of the leader being the public or the private firm.

3.3 The case with managerial public firm and managerial private firm (MM)

Finally, we consider the case in which both firms are committed to be managerial. Both maximize at the market stage the objective functions (6) and both interact strategically in the setting of the delegation parameter at the second stage.

The MM qq-game. Since now both firms are managerial, in the *qq*-game the reaction functions of the public and the private firm are given respectively by (7a) and (16b). Solving these equations, we obtain:

$$\begin{aligned} {}_{qq|}q_1^{MM} &= \frac{2 - \gamma + c(\gamma\theta_2 - 2\theta_1)}{4 - \gamma^2} \\ {}_{qq|}q_2^{MM} &= \frac{2 - \gamma + c(\gamma\theta_1 - 2\theta_2)}{4 - \gamma^2} \end{aligned}$$

Given these solutions, at the second stage the welfare maximizing public firm and the profit maximizing private firm formulate their best reply functions in terms of the delegation parameter, which deliver the equilibrium values of θ_1 and θ_2 :

$$\begin{aligned} {}_{qq|}\theta_1^{MM} &= \frac{2c(4(2 - \gamma) - \gamma^2(4 - \gamma)) - 8(1 - \gamma) - \gamma^3(2 - \gamma)}{c(\gamma^4 - 8\gamma^2 + 8)} \\ {}_{qq|}\theta_2^{MM} &= \frac{2c(4 - 3\gamma^2) - \gamma^2(\gamma c(2 - \gamma) + 2(1 - \gamma))}{c(\gamma^4 - 8\gamma^2 + 8)} \end{aligned}$$

Therefore, the values of the objective functions in the *qq*-subgame are:

$$\begin{aligned} {}_{qq|}W^{MM} &= \tag{24a} \\ &= \frac{(1 - c)^2(2\gamma^7 - 5\gamma^6 - 24\gamma^5 + 104\gamma^3 + 40\gamma^4 - 96\gamma - 132\gamma^2 + 112)}{2(\gamma^4 - 8\gamma^2 + 8)^2} \end{aligned}$$

$${}_{qq|}\pi_2^{MM} = \frac{8(1 - c)^2(1 - \gamma)^2(2 - \gamma^2)}{(\gamma^4 - 8\gamma^2 + 8)^2} \tag{24b}$$

The other three subgames can be solved by following the same procedure. Since in the MM *pp*-game the reaction functions at the market stage are (9a)

and (18b), in the MM qp -game (11a) and (20b), and finally in the MM pq -game they are given by (13a) and (22b), it can be easily shown that the values of the objective functions at the three subgame solutions are the following.

The MM pp -game:

$$\begin{aligned} {}_{pp|}W^{MM} &= & (25a) \\ &= \frac{(1-c)^2 (\gamma^8 - 16\gamma^6 + 6\gamma^5 + 74\gamma^4 - 14\gamma^3 - 114\gamma^2 + 8\gamma + 56)}{(1+\gamma)(\gamma^4 - 8\gamma^2 + 8)^2} \end{aligned}$$

$${}_{pp|}\pi_2^{MM} = \frac{2(1-c)^2(1-\gamma)(2-\gamma^2)^3}{(1+\gamma)(\gamma^4 - 8\gamma^2 + 8)^2} \quad (25b)$$

The MM qp -game:

$${}_{qp|}W^{MM} = \frac{(1-c)^2 (8\gamma^4 - 3\gamma^3 - 29\gamma^2 + 4\gamma + 28)}{8(1+\gamma)(2-\gamma^2)^2} \quad (26a)$$

$${}_{qp|}\pi_2^{MM} = \frac{(1-c)^2(1-\gamma)}{2(1+\gamma)(2-\gamma^2)} \quad (26b)$$

The MM pq -game:

$${}_{pq|}W^{MM} = \frac{(1-c)^2 (3\gamma^2 + 32\gamma + 28)}{32(1+\gamma)^2} \quad (27a)$$

$${}_{pq|}\pi_2^{MM} = \frac{(1-c)^2 (2-\gamma^2)}{8(1+\gamma)^2} \quad (27b)$$

Therefore, the solution of the subgames yields the following rankings of firms' payoffs:

$$\begin{aligned} {}_{pp|}W^{MM} &> {}_{qp|}W^{MM} > {}_{qq|}W^{MM} > {}_{pq|}W^{MM} \\ {}_{pp|}\pi_2^{MM} &> {}_{qp|}\pi_2^{MM} > {}_{qp|}\pi_2^{MM} > {}_{qq|}\pi_2^{MM} \end{aligned}$$

which allow us to formulate Proposition 3.

Proposition 3 *If both the public and the private firm are committed to a managerial structure, there exists no equilibrium of the three-stage game in pure strategies.*

Proof. Since ${}_{pq}|\pi_2^{MM} > {}_{pp}|\pi_2^{MM}$, and ${}_{qp}|\pi_2^{MM} > {}_{qq}|\pi_2^{MM}$, the private firm deviates from any symmetric pair of price-quantity choice. Since ${}_{pp}|W^{MM} > {}_{qp}|W^{MM}$ and ${}_{qq}|W^{MM} > {}_{pq}|W^{MM}$, the public firm deviates from any asymmetric pair of price-quantity choice. Therefore, there is no deviation-proof pair of price-quantity choice. ■

Notice that should the public firm be leader in the choice of the strategic variable, it would adopt a quantity contract, with the private firm offering a price contract; if the private firm were leader in the price-quantity decision, both firms would offer a price contract.

When the strategic choice of the market variable in a mixed duopoly is analyzed under the assumption that one or both firms commit to a given organizational structure – with firms behaving according to (6) when managerial – the *pp* result by MO is only partially confirmed. The properties of models with unilateral delegation ensure that when the latter is observed, the (equilibrium) bilateral choice of price contracts is equivalent to an asymmetric choice, where the delegating firm offers a price contract, while the non-delegating firm is indifferent between a price or a quantity contract. Moreover, when both firms delegate, no equilibrium exists in pure strategies in a simultaneous game. Notice that the *pp* solution would be the equilibrium of the *MM* game in its sequential version when the private firm is the leader in the choice of the contract offered to customers, while an asymmetric *qp* solution arises when the public firm enjoys a time advantage in this choice.

4 Price vs quantity choice with endogenous organizational structure

In this section the game discussed in Section 3 is extended in order to endogenize the organizational structure of the public and private firm. The key difference with respect to the previous games is that competition in this game can involve up to four stages. Again, at the first stage the strategic market variable is chosen between price and quantity; at the second the government and the private firm's owner decide whether to hire a manager or keep an entrepreneurial structure; at the third stage, if a manager is hired, owners choose the optimal value of the parameter of the incentive contract. The last stage is the market competition stage. In the next subsections this multi-stage game is solved backwards for the subgame perfect Nash equilibrium. By comparing the equilibrium payoffs of both the public and the private firm inherited from Section 3 and from the MO model, we first define the optimal firms' choice of the organizational structure taken at the second stage for any given price-quantity pair. The analysis of the payoffs at these equilibrium choices will finally lead to the optimal choice between price and quantity as a solution of the first stage of the game.

4.1 The price-quantity subgames

We consider the following four subgames.

The qq-game. Assume both firms choose a quantity contract at the the first stage.

- If neither of them delegates to a manager, their payoffs are those of a standard mixed Cournot duopoly with imperfect substitutability (Fujiwara, 2007):

$$\begin{aligned} {}_{qq}|W^{EE} &= \frac{(1-c)^2(7-2\gamma^2-6\gamma+2\gamma^3)}{2(2-\gamma^2)^2} \\ {}_{qq}|\pi_2^{EE} &= \frac{(1-c)^2(1-\gamma)^2}{(2-\gamma^2)^2} \end{aligned}$$

- if only the public firm delegates, the analysis of Section 3 ensures that their payoffs are given by ${}_{qq}|W^{ME}$ and ${}_{qq}|\pi_2^{ME}$ from eqts. (8a) and (8b);
- if only the private firm delegates, the payoffs are ${}_{qq}|W^{EM}$ and ${}_{qq}|\pi_2^{EM}$ from eqts. (17a) and (17b);
- if both firms delegate, they earn ${}_{qq}|W^{MM}$ and ${}_{qq}|\pi_2^{MM}$ from eqts. (24a) and (24b).

These payoffs can be ranked as follows:

$$\begin{aligned} {}_{qq}|W^{EM} &> {}_{qq}|W^{MM} > {}_{qq}|W^{ME} > {}_{qq}|W^{EE} \\ {}_{qq}|\pi_2^{MM} &> {}_{qq}|\pi_2^{ME} > {}_{qq}|\pi_2^{EM} > {}_{qq}|\pi_2^{EE} \end{aligned}$$

which allow us to introduce Lemma 1.

Lemma 1. *If both firms offer a quantity contract to customers, the equilibrium organizational structure is characterized by a public entrepreneurial firm and a private managerial firm.*

Therefore, the equilibrium payoffs firms may achieve in the subgame with bilateral quantity contract are ${}_{qq}|W^{EM}$ and ${}_{qq}|\pi_2^{EM}$. Notice that Lemma 1 extends to a framework with differentiated product the asymmetric organization result by White (2001).

The pp-game. Assume both firms decide for a price contract at the the first stage. The optimal organizational structure in this case has been discussed by Barcena-Ruiz (2009).

Lemma 2. *Barcena-Ruiz (2009): when in a mixed duopoly the public and private firm compete with respect to prices, both choose to be managerial.*

This implies that the equilibrium payoffs in the subgame with bilateral price contract are ${}_{pp}W^{MM}$ from (25a) and ${}_{pp}\pi_2^{MM}$ from (25b).

The qp-game. Consider now the asymmetric case in which the public firm offers a quantity contract, while the private firm offers a price contract.

- If neither the public, nor the private firm delegate, their payoffs are:

$$\begin{aligned} {}_{qp}W^{EE} &= \frac{(1-c)^2(7+8\gamma)}{8(1+\gamma)^2} \\ {}_{qp}\pi_2^{EE} &= \frac{(1-c)^2}{4(1+\gamma)^2} \end{aligned}$$

which replicate, in our notation, the *qp*-game result in MO;

- if the public firm is managerial and the private firm is entrepreneurial, their payoffs are ${}_{qp}W^{ME}$ and ${}_{qp}\pi_2^{ME}$ from eqts. (12a) and (12b);
- if the private firm is managerial and the public firm is entrepreneurial, their payoffs are ${}_{qp}W^{EM}$ and ${}_{qp}\pi_2^{EM}$ from eqts. (21a) and (21b);
- if both firms are managerial, in the *qp*-game they earn ${}_{qp}W^{MM}$ and ${}_{qp}\pi_2^{MM}$ from eqts. (26a) and (26b).

Given that

$$\begin{aligned} {}_{qp}W^{MM} &> {}_{qp}W^{ME} > {}_{qp}W^{EM} = {}_{qp}W^{EE} \\ {}_{qp}\pi_2^{EM} &= {}_{qp}\pi_2^{EE} > {}_{qp}\pi_2^{MM} > {}_{qp}\pi_2^{ME} \end{aligned}$$

Lemma 3 can be established.

Lemma 3. *If the public firm offers a quantity contract and the private firm offers a price contract, the equilibrium organizational structure is characterized by both firms being managerial.*

Therefore the equilibrium payoffs of the *qp* subgame are ${}_{qp}W^{MM}$ and ${}_{qp}\pi_2^{MM}$.

The pq-game. Finally, we consider the asymmetric case in which the public firm chooses a price contract, while a quantity contract is adopted by the private firm.

- In the absence of delegation by both firms, their payoffs are

$$\begin{aligned} {}_{pq}W^{EE} &= \frac{(1-c)^2(7+\gamma)}{8(1+\gamma)} \\ {}_{pq}\pi_2^{EE} &= \frac{(1-c)^2(1-\gamma)}{4(1+\gamma)} \end{aligned}$$

which again replicate in our notation the corresponding MO result;

- if only the public firm delegates, firms earn ${}_{pq|}W^{ME}$ and ${}_{pq|\pi_2}{}^{ME}$ from eqts.(14a) and (14b);
- if only the private firm delegates, firms earn ${}_{pq|}W^{EM}$ and ${}_{pq|\pi_2}{}^{EM}$ from eqts.(23a) and (23b);
- if both delegate, their payoffs are ${}_{pq|}W^{MM}$ and ${}_{pq|\pi_2}{}^{MM}$ from from eqts.(27a) and (27b).

Since

$$\begin{aligned} {}_{pq|}W^{ME} &> {}_{pq|}W^{EM} = {}_{pq|}W^{EE} > {}_{pq|}W^{MM} \\ {}_{pq|\pi_2}{}^{MM} &> {}_{pq|\pi_2}{}^{ME} > {}_{pq|\pi_2}{}^{EM} = {}_{pq|\pi_2}{}^{EE} \end{aligned}$$

Lemma 4 holds.

Lemma 4. *If the public firm offers a price contract and private firm offers a quantity contract, the private firm chooses to be managerial, while the public firm decides to be entrepreneurial.*

Given the organizational structure of Lemma 4, the equilibrium payoffs of the pq subgame are ${}_{pq|}W^{EM}$ and ${}_{pq|\pi_2}{}^{EM}$.

4.2 The subgame perfect four-stage Nash equilibrium

We are now in the position to identify the subgame perfect equilibrium of our four-stage game. At the first stage, firms face the binary choice between a price contract or a quantity contract. Given the solutions of the subsequent stages, the payoff matrix at the first stage can be written as follows:

		Private firm	
		q	p
Public firm	q	$({}_{qq }W^{EM}, {}_{qq \pi_2}{}^{EM})$	$({}_{qp }W^{MM}, {}_{qp \pi_2}{}^{MM})$
	p	$({}_{pq }W^{EM}, {}_{pq \pi_2}{}^{EM})$	$({}_{pp }W^{MM}, {}_{pp \pi_2}{}^{MM})$

On the basis of this payoff matrix we can finally establish Proposition 4.

Proposition 4 *At the subgame perfect Nash equilibrium of the four-stage game, both firms choose a price contract, which implies that both decide to be managerial.*

Proof. Consider the pair of strategies qq . A straightforward comparison shows that ${}_{qp|\pi_2}{}^{MM} > {}_{qq|\pi_2}{}^{EM}$, so that the private firm would deviate and this pair cannot be an equilibrium. Consider now the pair of strategies pq . It can be checked that ${}_{pp|\pi_2}{}^{MM} > {}_{pq|\pi_2}{}^{EM}$; therefore, also in this case the private firm

would deviate and the pq pair cannot be an equilibrium. Should the pair of strategies be qp , then the inequality $_{pp|}W^{MM} > _{qp|}W^{MM}$ (see Subsection 3.3) implies that the public firm would now deviate. The pair qp is not an equilibrium. Finally, if the strategy pair chosen by firms is pp , the previous inequalities ensure that this pair is deviation-proof. ■

Proposition 4 shows that the endogenization of the organizational structure of firms has two important implications. On the one hand, it allows to solve the non-existence of equilibrium question arising in the simultaneous price-quantity game of subsection 3.3: if firms are committed to a managerial structure no equilibrium exists; if they are allowed to choose whether to hire or not a manager they do choose to be managerial, and both find it optimal to offer a price contract. When the managerial structure is endogenous, the private firm does not deviate from the symmetric pair pp since its deviation would be accompanied by a change in the organizational structure of the public firm from managerial to entrepreneurial, which makes the deviation unfavorable in terms of private profits. This lack of incentive to deviate does not occur when there is a irreversible commitment of the public firm to be managerial. On the other hand, we may interpret Proposition 4 as a proof of the robustness, in mixed duopolies, of the pp solution with respect to the perceived incentive of firms to strategically delegate decisions at the market stage.

5 Conclusions

In this paper we have investigated, in a mixed duopoly framework, the interrelations between two crucial strategic decisions of the competing firms: the choice of the mode of competition, price *vs* quantity, and the choice whether to delegate the market decision to a manager in order to better exploit the properties of the strategic interaction between firms. Our results contributes to the existing literature in several dimensions. We show that the established result that the public and private firms choose price at equilibrium is not independent of the their *assumed* organizational structure. There are organizational structures at which asymmetric modes of competition turn out to be equilibria equivalent to a (equilibrium) bilateral price choice, and there are structures at which no equilibrium exists of the price-quantity game. However, we show that offering bilaterally a price contract turns out to be the unique subgame perfect Nash equilibrium when the organizational structure is endogenous.

In a different perspective, our findings also contribute to univocally identify the optimal organizational structure of the competing firms. The previous literature has highlighted that commitments to different modes of competition imply different organizational structures, with price competition being associated with a bilateral managerial structure, and quantity competition leading to the private firm delegating to a manager and the public firm maximizing welfare at all stages. When the price-quantity choice is endogenized, with the public and private firm offering a price contract, the delegation to a manager stands out as the sequentially rational mode of firms' organization.

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