Equilibrium earning premium and pension schemes: The long-run macroeconomic effects of the union

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Abstract

Using the theoretical framework based on the monopoly union model described in Kidd and Oswald (1987) and Jones (1987), this paper provides an explicit framework to assess the role of wage moderation in Italy in the last twenty years. There are two crucial ingredients to the model: the composition of union membership and the pension system. We show that the increase in pensioners' membership in the presence of a pay-as-you-go (PAYG) pension scheme has led unions to moderate wage claims. However, this result is reversed when we shift from a PAYG system to a fully-funded (FF) regime (recently adopted in Italy): in this case, the model predicts a rise in wages with respect to the standard model, regardless of the share of pensioners in the membership.

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1 Introduction

In offering an additional explanation for assessing wage moderation in Italy, this paper shows that wage moderation is, to a large extent, driven by the trade union pension policy in the light of membership composition. The composition of membership is an important determinant of union decisions and is often overlooked in the literature due to the central role attributed to the union earnings premium.[1] The literature on trade union recruitment has shown that gender, earnings level, industrial structure, socio-demographic features, education, qualifications and social factors may be determinants of union membership.[2] However, the propensity to unionize does not vary only with active worker characteristics and social context, but may involve those outside the labour market such as pensioners[3].

Using the theoretical framework based on the monopoly union model described in Kidd and Oswald (1987) and Jones (1987, henceforth K-O-J), this paper provides an explicit framework to assess the role of wage moderation in Italy in the last twenty years. The model has two crucial ingredients: the composition of union membership and the pension system. We show that in the last twenty years, the increase in the pensioners' membership in the presence of a pay-as-you-go (PAYG) pension scheme have brought unions to moderate wage demands. However, this result is reversed, when we shift from a PAYG system to a fully-funded (FF) regime (recently adopted in Italy): in this case, the model prospects a rise in wages with respect to the standard model, regardless of pensioners share in the membership.

Wage moderation is related here to a change in union preference for employment and not to an overall reduction in union bargaining power. We present the implications of a dynamic model of union behaviour based on the assumption that both present and future members enter into the intertemporal utility function, but where a share of these are pensioners.

In our model, pensioners are endogenous and determined by demographic factors. We show that if a steady state exists, it necessarily has a lower wage level than the corresponding conventional intertemporal model: since union membership is conditional upon both employment and pensioners, and because these

[1] See, amongst others in a vast literature, Oswald (1985), Creedy and McDonald (1991), Booth (1995), and among recent contributions, Checchi and Lucifera (2002), and Checchi and Visser (2005).

[2] In the 1940s Ross (1948) emphasized that unionism is a complex phenomenon not only in relation to economic factors. See, for instance, Booth (1983; 1984; 1985), Disney (1990), Wheeler and McClendon (1991), Mason and Bain (1993), Booth and Chatterji (1995), Naylor and Cripps (1993), Naylor and Raaum (1993), Riley (1997), Booth et al. (2000), to cite some of the literature.

[3] Askilden and Nilsen (2002) set out an empirical analysis of effects of union membership rates on wages, which include retired employees. Matsaganis (2007) analyzes union policy on pensions in the light of membership composition in Greece. A few works focus on the union preference formation and "seniority bias", stressing that membership composition leads union policies that benefit older and retired workers at the expense of younger workers. See for instance Boeri et al. (2001), Natali and Rhodes (2004). Yet, to the best of our knowledge, there is no theoretical framework including retired members in such models.

are linked in a trade-off between wages and pension benefits (via the social security tax rate) in a PAYG pension system, the union prefers steady-state higher employment. Intuitively, therefore, if there are more pensioners in the membership this tilts the union's preference towards a higher pension and hence a lower current wage for employed workers. However, the trade-off between wage and pension in union preferences is due to the pension scheme considered. When we include in the model a fully-funded pension scheme, there is no longer a trade-off between wages and pensions. In this situation unions bargain for a higher wage in order to ensure a higher pension in the future, regardless of pensioners' membership dynamics.

The model is a partial equilibrium model, where the only optimizing agent is the monopoly union.[4] Our aim is to reach a macroeconomic result in terms of union preferences, demographic factors and membership composition. We refer to future research the analysis in an OLG general equilibrium model to investigate the conflict between the young generation (active workers) and the old (pensioners) which endogenously emerges from the agents' utility maximization problem.

To analyze the changed preference of unions, we provide a description of membership trends of the leading Italian trade union confederations over the last 50 years, focusing on the composition of union membership, with a sharp drop among wage-earners and a considerable increase among pensioners. The very considerable increase in pensioner membership is a peculiar feature distinguishing Italian unions from those of other countries. We stress the implications of this change in the internal structure of union membership for the customary way to model union objectives as a function of wages and the level of employment.

The results of this paper raise several general questions about an ageing workforce and the rise of pensions in the European economy. The IMF predicts that the ratio of retirees to workers in Europe will double to 0.54 by 2050 (from four workers per retiree to two workers per retiree). Other studies predict a rise in the median age in Europe from 37.7 years old in 2003 to 52.3 by 2050, while the median age of Americans will rise to only 35.4 years old.[5] According to OECD estimates only 39% of Europeans between the ages of 55 to 65 work.

These changes in demographics will have a dramatic effect on the society in which we live and pose a major headache for future governments. The challenges of changing old age dependency ratios, and continuously increasing life expectancy at pensionable age raises two concerns, namely financial sustainability and adequacy of pensions. In France and Italy, trade unions have often opposed changes to pensions, but European governments will soon have to face the unpalatable problem of their current public pension schemes and their unions or pressure groups.

The paper is organized as follows. The following two sections highlight the role

[4] We are aware, at least from the contribution of Turnbull (1988), that the construction of an economic model of trade union behaviour requires interdisciplinary research. In this paper we use a well-known and simple framework to analyze the macro effects of the dynamic of union composition.

[5] See, amongst others, Carone and Costello (2006), OECD (2006, 2008).

of pensioners in the Italian leading trade unions and provide some data on wage moderation in Italy. Sections 4 and 5 illustrate the equilibrium implications for a monopoly union model of a high membership share of pensioners and compares the solution with the conventional one. Sections 6 and 7 report the contrasting results obtained under the two pension systems. Policy implications and results are discussed in the Concluding Remarks. The Appendix reports the dynamic analysis and discusses stability conditions of the models.

2 Wage moderation and pension benefits

It would appear odd that in the last 20 years, in a unionized labour market, there has been so much wage moderation. Figure 1 shows a much lower level of the Italian real wage in the last 20 years, compared to other EU members. Evidently, different patterns of productivity can account for differences in real wage growth. Although Italy has experienced slower productivity growth than other European countries, a growing wage gap has emerged at least in the last 25 years. Figure 2 depicts the wage-productivity gap (WG) measured as a ratio between labor productivity and real wage:

$$WG = \frac{VA/E}{Earnings/E} = \frac{\pi}{w}$$

where VA is value added, E is employment in terms of standar labor units (Eurostat definition), Earnings are the total amount of earnings.[6] The figure shows a persistent gap between productivity and wages, increasing in the 1980s and '90s and falling recently, particularly in the manufacturing sector, in the present recession[7].

Wage moderation has significantly slowed the rise in unit labour cost and has increased profits and employment. In the second half of the 1980s, unions and employers' organizations were oriented toward responsible strategies of collective action, pursuing public goals such as employment, growth and price stability. This corporatist governance led the union to agree to an income policy accord in 1993, setting up a national contract devoted to maintaining the purchasing power of wages. Since 1993 a union policy of wage moderation has been designed to underpin competitiveness (by curbing inflation) and employment.[8]

[6] It is worth recalling that this ratio, in aggregate terms, is equal to the labour share. Data are from ISTAT (National Institute of Statistics), National Accounts. Earnings and Value Added are deflated by the value added implicit deflator. Using a different price index for earnings does not significantly change our results.

[7] When the economy grows, the bargaining model is marked by a structural bias in favour of the capital share in income, that can be redressed only through a drop in labour productivity. The wage moderation of 1993-2000 was actually followed by a long period of productivity stagnation, which allowed the wage share to slowly recover.

[8] See, for instance, Fiorito (2003), Casadio (2003), Brandolini et al (2007), Visser (2007), Pastore (2009), and the papers in Acocella and Leoni (2007). Empirical analysis confirms that Italian income policy has altered the relationship between labour demand, productivity and wages. See, amongst others, Chiarini and Piselli (1997), De Stefanis et al (2005), Costantini and De Nardis (2007), Devicienti, Maida and Pacelli (2008).

To this end, the agreement provided that a first level of wage bargaining at the national level simply fixes basic wage growth at the officially forecast inflation rate, whereas a second level allows for further productivity-related increases at the sector and firm levels. This income policy has matched some of the conditions of the agreement proposal (inflation and employment) at the expense of others (wages). Although Italy experienced low productivity growth (in relation to other industrialized countries), the negative wage-productivity gap of recent decades, shown in Figure 2, has not allowed real wage gains.[9] The centralized wage fixing mechanism has reduced the sensitivity of real wages to productivity growth, curbing real wages.

In the absence of wage pressure, the unions have been involved in reforming the pension system. They have actively and crucially participated in the new pension reforms with negotiation power so as to ensure consistency between pay rises, employment, membership and fiscal consolidation targets.[10]

Prior to the 1990s reforms, the Italian pension system had been very generous and unsustainable. Pension benefits were based on the last five-year average earnings for private employees, while individuals with 40 years of contributions could obtain benefits amounting to around 80% of wages. Moreover, benefits were indexed to minimum contractual wages. For public employees, they were calculated using the last salary. A further anomalous feature of the Italian system was the right to obtain a seniority pension. This pushed many private sector employees and the self-employed to withdraw from the labour market and take retirement after 35 years of contributions, irrespective of the age at which they had started working. This anomaly was even more striking for public employees who were permitted to take retirement after 20 years of contributions for men and 15 for women. The official retirement for private employees was 60 for man and 55 for women, but individuals typically retired earlier, which has pushed up the overall costs of the pension system. Indeed, "Italian pensioners have benefited from one of the higher old-age pension gross replacement rates in the OECD area".[11] Combined with the demographic shocks, this system would have led to sharp increases in pension spending over the next few decades in the absence of reforms.[12] The 1995 Dini reforms changed the underlying structure

[9] On the factors which have affected the sharp deceleration of labour productivity see, amongst others, Hall, Lotti and Mairesse (2009), Brandolini et al (2007), Daveri and Jona-Lasinio (2005), European Commission (2006), Parisi, Schiantarelli, and Sembenelli (2006), Pianta and Vaona (2007).

[10] See Chiarini (1999), Baccaro (2002), Franco (2002) and Ferrera and Jessoula (2007).

[11] See OECD (2000). This economic survey of the OECD reports a detailed overview of the pension system prior to reforms and the pension reforms introduced in 1992 and 1995. See also Ferrera and Jessoula (2007) who report an historical overview of the funded schemes for employees in Italy. The 1992 Amato reform was the first of several significant reforms, but the crucial one was the Dini reform, adopted in 1995, which steered a pension system mostly based on a pay-as-you-go scheme toward a contribution-based system. Further adjustments to accelerate the phase-in of certain aspects of the previous changes were undertaken in 1997 (Prodi), 2004-5 (Maroni-Tremonti) and 2011 (Fornero-Monti). These adjustments led to a further tightening of the condition of seniority pensions and incentivize workers to participate in the complementary pillars.

[12] Several studies and Treasury forecasts indicated that the growth in pension spending in

of the system and further increased its long-term sustainability. The total of pension benefits received by an individual during retirement were to be directly linked to notional contributions over a person's working life.

Parallel to the increase in pensioners within union membership (as we shall see in the next section), in recent years Italy has witnessed a faster growth in pensions than in wages. Figure 3 plots the dynamics of nominal wages and average pensions in the period 1989-2008, taking 1989 as the base year (1989=100)[13]. In 20 years, wages have roughly doubled, while pensions have increased by 158 per cent.

3 Union membership trends

The membership trends of the Italian leading major confederations (CGIL, CISL and UIL) show a striking change in the structure of union membership, with a considerable increase in the number of pensioners (Santi 1988; Chiarini 1999). These trends are very important when related to labour market structure and provide significant implications for union bargaining. The following description of membership trends applies to the two largest trade unions (CGIL and CISL). Similar patterns hold for the other less important confederation (UIL).

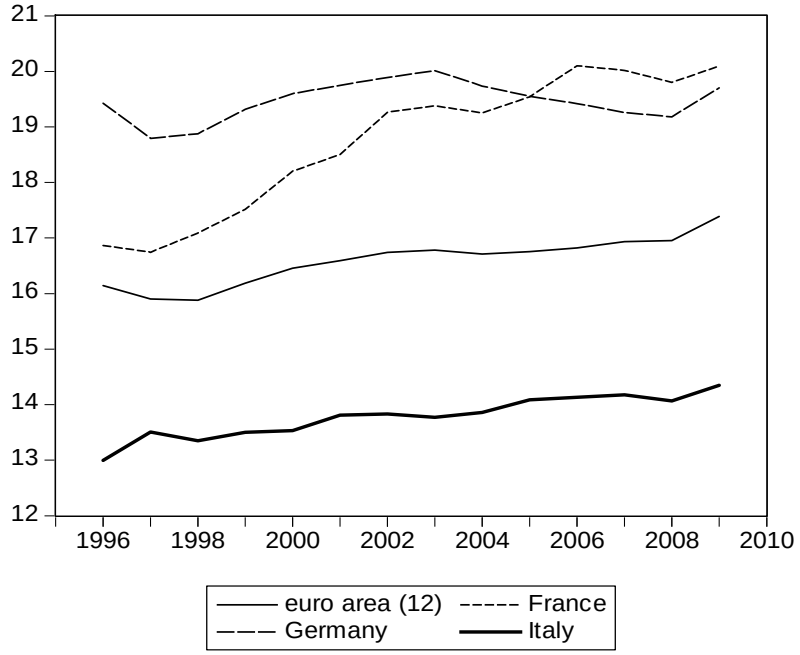
In the period 1968-1980 (Fig. 4), Italy experienced an extraordinary increase in unionization. This sharp rise in the unionization rate has been interpreted as a growth of members in already highly unionized sectors of industry and to a greater spread of union membership among white collar workers in industry and public administration.

During these years, a growing number of workers in the unionized sectors reached retirement age and a large share of them took an active part in union activities (Figure 5). The loyalty of many of these members was the strength behind the pensioners' federations. As a whole, the growing unionization of pensioners has considerably improved their position within the power distribution of union confederations. The question which naturally arises from this situation is whether the presence of retired workers in the membership confederations results in intergenerational solidarity or if it generates tension between the pensioners' federations and the active workers' leadership. In a PAYG financial system, the contribution of the working generations pay the pensions of other generations (already retired). The rates of return for successive generations in general differ due to changing demographic and economic factors, and these differences may lead to intergenerational conflict. Unions have traded off lower pay today for higher prospective pension benefits. This may constitute one of the main factors behind the decline of consensus among active workers' union membership and wage moderation depicted in Figures 1-2.

the absence of reforms would have reached at least 25 per cent of GDP by 2030.

[13] Sources: average pensions are from INPS (National Institute for Social Security); wages are proxied by earnings per employee from National Accounts.

Figure 1 Hourly real wage



Source: Elaborations on ECB and Eurostat data.
Annual hourly wages in euro are deflated by HICP price index (2005=100)

Fig. 3 Wage and pension dynamics
(1989=100)

Source: Elaborations on National Accounts data

Fig. 3 Wage and pension dynamics
(1989=100)

Source: Istat and INPS.

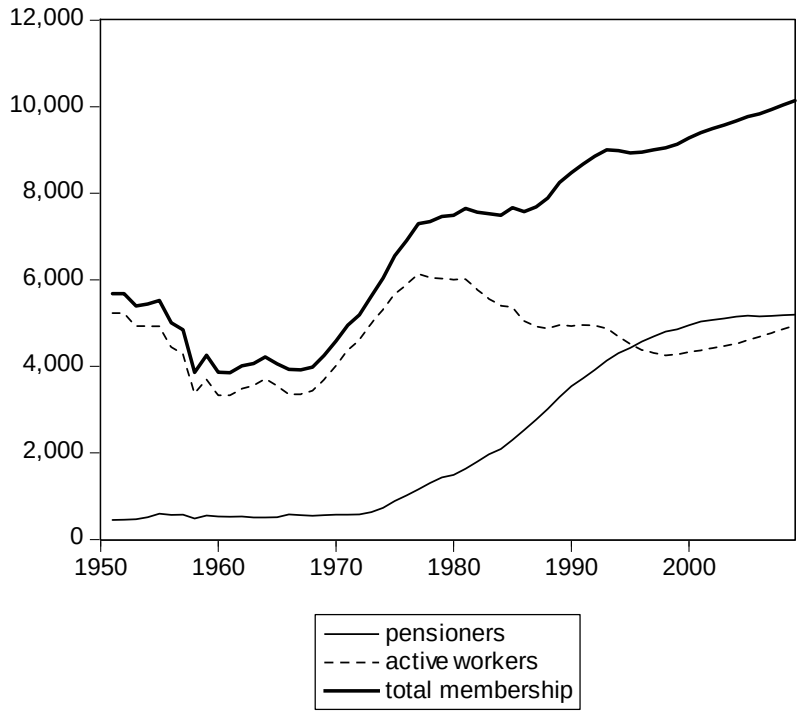
With regard to the two major confederations, Figure 5 shows that the number of pensioners as a percentage of total membership rose from 8 to 40 per cent during the period 1950-1990 and reached on average 50 per cent before 2000. On the other hand, the percentage of active workers in total membership shows a striking drop of about 42 % through the period under consideration, from 92 to less than 50 %.

The latest available data (2010) show that membership of the pensioners' federation amounted to 2.997 million for the CGIL out of a total membership of 5.643 million. The largest of the 16 affiliated federations numbered 409,389 workers. In the CISL confederation in 2009 there were 2,201,150 pensioners out of a total membership of 4.531 million. The largest of the 21 affiliated federations of active workers numbered 325,686 members.

Why Italian unions experienced from the mid-1970s a steep increase in pensioners membership is explained in Chiarini (1999) and Ferrera and Jessoula (2007) by a number of factors. These include the existence of an automatic check-off (*adesione-trattenuta per delega*); the professional union structures, organized through specific autonomous federations (within the confederation) and with a close rapport with their members; the ability to provide services and private benefits to union members (tax and legal assistance, information on

pensions, financial and insurance services and help to disentangle complex bureaucratic procedures etc.) who are elderly, which is the most vulnerable share of the population (elderly insurance motive); and the role of special organizations established at national, regional and local levels and controlled by unions, known as *patronati*, in providing many welfare services, etc. The growth of pensioners' membership is also linked to the Italian fiscal crisis associated with the predicted demographic trends, and the measures taken by the governments to resolve it. Since the mid-1990s, all the reforms have attempted to limit the imbalance in the social security system, involving retirement age, the minimum contribution period, the period of computing pensionable earnings and the different pension rules between private and public employees[14].

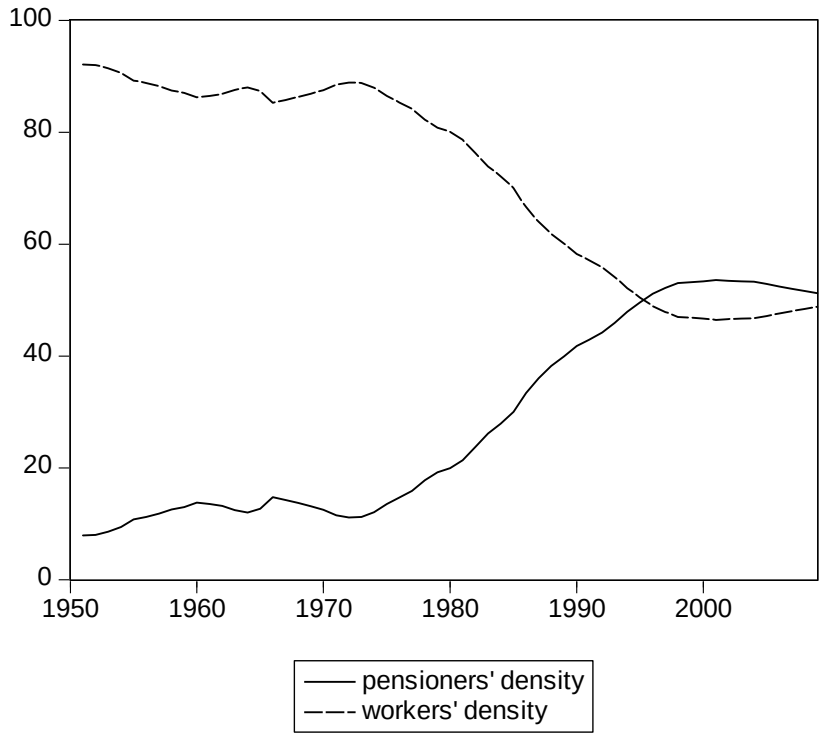
Figure 4. Union membership composition (thousands)



Source: CGIL, CISL.

[14] Trade unions differ in their willingness to adopt more or less broader perspectives. Their preferences are affected by the composition of their membership. On the different union role in Europe see, amongst others, Booth et al (2000); Boeri et al. (2001); Waddington (2000); Ebbinghaus (2004).

Figure 5. Workers and pensioners as percentage of total membership



Source: CISL,CGIL.

3.1 A simple long-run relationship

Is pensioners' unionization associated with evidence for a more moderate wage policy in Italy? Within a social security PAYG method of finance, for a given number of pensioners and a given amount of pension benefit, the social security financial balance requirement imposes a proportional tax on the average wage earned by active workers, such that benefits equal taxes. Obviously, in this simplified requirement scheme, the higher is the ratio of retirees to active workers the higher is the corresponding ratio of pension benefits to wages. Thus, this clear trade-off between wages and benefits (and between the number of aged retirees and workers) could not be ignored by a union subject to a composition of its members as that illustrated above. With regard to the pensioners-active workers ratio, it rises when population growth declines, but the substitution ratio between wages and pension benefits may be managed by the union, also influencing employment.[15] To provide further empirical support for this trade-

[15] For instance, the importance of trade union action in this respect is undeniable. The union confederations have made a considerable effort to reform their bargaining structure in

off between wages and pension benefits, with the available data, we estimate a long-run (cointegration) relationship among real wage w, labour productivity π and pensioner share in the membership $\frac{pensioners}{membership}$ (pd).[16] The equation (in logs) is the following:

$$w = \alpha_0 + \alpha_1 \pi + \alpha_2 pd$$

In Table 1 we report the results obtained with two different techniques: Phillips-Hansen (1990), for univariate relationships among *I(1)* variables, and the Johansen (1995) multivariate VAR approach.[17] In the latter case, we estimate a trivariate VAR of w, π and ps with two lags over the sample 1951-2009 (60 obs):

Table 1. Long-run relationships

(s.e. in parentheses)

	P-H	Johansen
α_0	-0.60	non-sign*
	(0.070)	
α_1	0.97	0.88
	(0.014)	*(0.027)*
α_2	-0.05	-0.14
	(0.009)	*(0.020)*

*The constant turns out to be non-significant when recstricted in the cointegration space.

There is clear evidence of a unit long-run elasticity of wages to productivity, but also of a significant and negative effect of pensioner share on wage level. Thus pensioners' membership has a stable relationship with real wages (hence employment). It is important to stress that restricting pensioners' membership component to zero in the cointegrating relations is rejected by the data.[18]

order to assist the process of adaptation to changes in the factors affecting the supply of and demand for labour. Moreover, unions, along with entrepreneurs and government have followed income policy arrangements to control wage pressure and increasing employment. In the past, unions facilitated the reduction of legal protection in organizing strikes and agreed to drop the automatic wage indexation mechanism, making the monetary stance of the central Bank more credible and hence the economy's adjustment to real shocks less difficult. Such union behaviour may appear less responsive to their active members. Actually, although bargaining over employment is relatively rare, the evidence quoted above suggests that Italian unions are concerned about employment.

[16] Variables are as defined in section 2.

[17] According to standard unit root tests (ADF with 1 lag), our variables prove to be I(1). Both techniques used above assume that variables are I(1). Tests are available upon request.

[18] These results are also consistent with those reported in Chiarini (1999). Tests and statistical models can be provided by the authors upon request.

In the rest of the paper, we seek to prove that this finding can be related to an intertemporal optimal union choice, which takes into account the share of pensioners in its membership when it bargains over wage and employment in a dynamic context.

4 The Kidd, Oswald and Jones model.

We illustrate the implications for a monopoly union model of a high pensioners' membership share, comparing the solution with the conventional K-O-J model.[19] The model specifies that the union cares about the sum of utilities of current and future members. If the union's objective function is specified as a utilitarian function (increases in membership raise utility), then the union prefers more employment in the steady state (employment and membership are positively correlated).
This model is characterized by a conventional demand curve for labour $w = f^{'}(n)$ and a single trade union, which faces a dynamic optimization problem considering the post-entry closed shop evolution of membership:

$$\begin{aligned} & \max_{n} \int U e^{-rt} dt \\ s.t. \quad w &= f'(n) \\ \dot{m} &= n - m \end{aligned} \tag{1}$$

where n is the level of employment, w is the wage, m stands for membership level, and $f(n)$ is a strictly concave production function. Union preferences are utilitarian, $U = nu(w) + (m - n)u(\xi)$, where $u(.)$ and $f(n)$ are non-negative and concave increasing functions and there exists an outside opportunity ξ for unemployed workers.

Having to apply to the Italian economy, the model deserves consideration on the dynamics of membership. Although crucial, the assumption of post-entry closed shop for Italy is only adopted to simplify the analysis.[20] In the period examined, union density for the whole trade union confederation ranged from about 50 to 30% of total active workers. If the union is assumed to be a closed shop, union density should be 100%. The closed shop assumption, however, does not be so restrictive if we consider the coverage. The national industry agreement establishes a minimum wage (that can be improved by local bargaining) and the coverage is in general quite high for sectoral contracts, although the latter only apply to the members of the signatory union. For

[19] Kidd and Oswald (1987) and Jones (1987) built a flexible framework for the analysis of union behaviour, in which a utilitarian union selects a time path for employment (and, implicitly, wage) by taking into account its membership dynamics. Lockwood and Manning (1989) set up a dynamic version of the right-to-manage and efficient bargaining model. Finally, Jones and McKenna (1994) proposed a more articulate dynamic union model.

[20] See, amongst others, Naylor and Cripps (1993), Booth (1995), Booth and Chatterji (1995), Chang et al. (1998).

instance, Visser (1996) estimates that in manufacturing coverage ranges between 70 and 90% of total employees, with metal manufacturing displaying the largest share, and textiles the lowest.

The well-known steady-state solution derived by the FOCs of this problem is:

$$\beta(n^*) = \frac{r}{1+r}u(\xi); \quad n^* = m^* \tag{2}$$

where $\beta(n) = u(f'(n)) + nu'(f'(n))f''(n)$.

As emphasized by K-O-J, $\beta^{'}(n) < 0$ if $f(n)$ is a conventional constant elasticity production function.[21]

5 The pensioners problem in a naive model.

Investigating the implications for employment (and wages) of a high membership share of pensioners, Chiarini (1997) includes the stock of pensioners in the membership, simply augmenting the membership state equation. The labour market is still characterized by a conventional demand curve for labour, $w = f^{'}(n)$, while the post-entry closed shop evolution of membership (both for workers and pensioners) becomes:

$$\dot{m} = n + P - m \tag{3}$$

where $\dot{m}$ is the derivative of union membership with respect to time and P represents the (given) level of pensioners.

Chiarini also adds an intertemporal constraint to the standard model, given by the social security PAYG system in force at that time. For a given number of pensioners P, pension benefit b and social security tax rate θ, the financial balance requirement of social security imposes that

$$\int (bP - \theta wn)e^{-rt}dt = 0 \tag{4}$$

Firms are assumed to retain the right to determine employment unilaterally, while the government is assumed to retain the right to determine the social

[21] Thus we use the well-known assumptions of this kind of model: we adopt a utilitarian function and the union comprises identical members (no distributional or normative judgments are made); the utilitarian function takes into account the fact that the union's wage claim may result in some union members being unemployed. Thus ξ may be interpreted both as alternative wages and as unemployment benefit; $u(\xi)$ is thus the utility of the unemployed workers. The utilitarian union function is non-decreasing in both membership and alternative wages; the model is a partial equilibrium model where ξ is exogenous (and affects only union preferences) and there are no feedbacks from the unemployed workers. See, amongst others, Booth (1995) and the works quoted therein, for a survey of these assumptions.

security tax rate θ.[22]

The union has to select a time path for wages (and implicitly, employment) and pension benefits b, considering outside opportunities. Now, the union's utility function is $U = n(u(w(1-\theta)-u(\xi))+P(u(b)-u(\xi))+mu(\xi)$, a utilitarian function which comprises the utilities of active workers and pensioners. It is worth noting that workers are now paid the net wage $w(1-\theta)$, while $w\theta$ is intended to finance the pension system. Membership dynamics still depends on employment only, but the level of pensions turns to be relevant. From the FOC, it is easy to obtain the following steady state:

$$\beta(n^*) + u'(b)\theta\Gamma(n^*) = \frac{r}{1+r}u(\xi) \quad (5)$$

$$m^* = n^* + P \quad (6)$$

$$b = \frac{wn\theta}{P} = \frac{f'(n)n\theta}{P} \quad (7)$$

where $\beta(n) = nu'(f'(n)(1-\theta))f''(n)+u(f'(n)(1-\theta))$ and $\Gamma(n) = f'(n)(1-\theta)+f''(n)n$. Moreover, $\Gamma(n) > 0$ and $\Gamma(n)$ is decreasing in n if $f(n)$ is a conventional constant elasticity production function. The crucial point that emerges from this simple model is that the term $u^{'}(b)\theta\Gamma(n^*)$ is positive and that taking into account pensioners, unions set in equilibrium a lower wage (higher employment) than the benchmark K-O-J model.

In accordance with the standard models, the equation predicts that a higher wage premium will reduce employment. Analogously, an increase in pension benefit lowers employment. In this simple model, when pensioners' level is given, there is an inverse relationship between employment and pension benefits. Higher pension benefits increase union's utility, which offsets the disutility of a lower stock of employees (members). As we show in the next section, this relationship is reversed, when pensioners change over time.

6 The endogenous pensioners

The way of modelling pension membership in the model above is quite unrealistic and provides a poor representation of the problem. The result, in fact, is totally driven by the specification of the union utility function. The above model neglects some important demographic factors which impinge on the dynamics of pensioners. To cope with this drawback, we assume that the following function drives pensioners:

$$\dot{P} = -\delta P + \lambda n \quad (8)$$

[22] Alternatively, we may consider the case where the constrained pay-as-you-go pension scheme is set to be equal to a given constant, k, or, yet, enters the problem as inequality, $\int (bP - \theta wn)e^{-rt}dt \succeq k$.

where δ is the elderly death rate, and λ represents the share of active workers which, in each point in time, changes status, becoming pensioners. The union optimization problem now becomes the following:

$$\max \int U e^{-rt} dt \tag{9}$$

$$\begin{aligned} s.t.\ w &= f'(n) && (10)\\ \dot{m} &= n + P - m && (11)\\ \dot{P} &= -\delta P + \lambda n && (12)\\ \int (bP - \theta w n) e^{-rt} dt &= 0 && (13)\\ m(0) &> 0\\ P(0) &> 0 \end{aligned}$$

From the FOCs, we obtain the following steady state[23]:

$$\begin{aligned} \beta(n^*) + u'(b)\theta\Gamma(n^*) &= \frac{r}{1+r}u(\xi) + \frac{\lambda}{\delta+r}[\frac{r}{1+r}u(\xi) - u(b) + u'(b)b] && (14)\\ \frac{P^*}{n^*} &= \frac{\lambda}{\delta} && (15)\\ m^* &= n^* + P^* && (16)\\ b &= \frac{\lambda f'(n)}{\delta} && (17) \end{aligned}$$

Equations (14)-(17) deserve some comments. First, equation (14) is a more general version of (5). To compare the two equations, we can rename the last term in the (14) as $A = \frac{\lambda}{\delta+r}[\frac{r}{1+r}u(\xi) - u(b) + u'(b)b]$. This term shows up when pension dynamics is modelled and depends on demographic parameters (λ, δ) and on the level of pension benefits (b). Moreover, if $A > 0$, wage in steady state is higher than (5) whereas if $A < 0$, wage in steady state is lower than (5). Since $(r/1+r) << 1$, and $u(b) > u^{'}(b)b$ for the standard concave utility function, it is easy to verify that $A < 0$. Thus, wage in steady state is lower and employment is higher when demographic dynamics is taken into account. Second, demographic factors pin down the steady-state value of the pensioners/workers ratio (15). This ratio represents the long-term (equilibrium) value of pensioners' and workers' dynamics within the unions. Looking at Italian data on membership, this ratio is roughly equal to 1 (see Figure 5), whereas it was a much lower value in the 1960s and '70s, when the share of pensioners was modest. Third, membership in steady state is equal to the sum of active workers and pensioners (16). This result is due to the hypothesis of closed-shop bargaining: all pensioners and workers join the union. Finally, it is worth recalling what we stated at the end of the previous section on the trade-off

[23] Appendix A.1 derives equilibrium and its stability.

between wages and pension benefits. Since A is an inverse function of b, in this case the relationship between pension benefits and employment is positive. As a result, now there is a trade-off between wages and pensions in the union's preferences: when pensions go up, wage is to decline in equilibrium. In a pay-as-you-go pension scheme, a higher level of pension benefits requires a higher stock of employment, that is a higher "wage bill", in order to finance the higher stock of pensions. In fact, for a decreasing marginal productivity function ($f''(n) < 0$), when n increases, w decreases but proportionally less, so that the wage bill wn increases.

6.1 Policy implications

The result that a higher utility related to pension benefits curbs wages and increases employment is in line with some studies of Italian industrial relations. Baccaro (2002), for instance, provides a comprehensive account of how this latter point is made in Italy, describing how pension reforms took a neo-corporatist form.[24] That is, they were based on centralized agreement between government and the labour unions. This allows room for policies to affect wages extending the retirement age and hence boost the labour supply of elderly workers. The considerable share of pensioners among union members, described above, may produce a sort of "seniority bias", pushing unions to favour elderly workers and pensioners, confirming that the composition of trade unions may exert a considerable influence on the pension-wage trade off. In this context, the union should adopt a broader perspective rather than the narrow interests of the protected (unionized) employees.
As regards demographic factors, equation (14) shows that the share of new pensioners has a negative effect on the wage level: a higher inflow of pensioners out of workers (λ) entails a higher stock of future pensions and requires a lower wage in order to increase labour demand. Conversely, as the death rate increases, pensioners (and pensions) diminish, and lower labour demand and a higher steady-state wage are predicted. Thus, the rise in life expectancy generates pressure on the demand for labour by requiring lower wages. Overall, demographic parameters are crucial to determine wages and employment, radically changing the role of the trade unions in the labour market.

7 Union model with fully-funded pension scheme

In the model above, we provide a theoretical reason for wage moderation in a unionised wage-setting, when pensioners in the union membership are explicitly modelled. In addition, we prove that, contrary to the naive case, in the model where pensioners dynamics is endogenised, a clear trade-off emerges between

[24] For a reconstruction of the process that led to pension reform in Italy, see also Baccaro and Locke (1996), OECD (2000), Franco (2002) and Ferrera and Jessoula (2007). See also Natali and Rhodes (2004). Matsaganis (2007) analyzes the relationship between Greek pensions and union policy, stressing that unions in Greece are incapable of pursuing all-encompassing interests.

wages and pensions in a PAYG pension scheme. In this section, we explore whether our result depends on the pension scheme considered. In the light of the progressive shift of the Italian pension system from a PAYG to a fully-funded financial system (summarized in section 2), it is worth trying to investigate the impact of a different intertemporal financial constraint on the union model.[25] Our model is the same as (9)-(13) except for the financial constraint. Here we introduce the fully-funded pension scheme via a simple dynamic equation for pension, which depends for each t both on the return of the pension fund and the share of wage accumulated in the fund for the future pension.

$$\dot{b} = \alpha w + r_b b \tag{18}$$

According to (18) each worker invests a share α of his/her salary in his/her own pension fund, which yields the rate r_b. Currently in Italy each worker leaves a share α to the National Social Insurance Institute, which is capitalised at the rate of change of nominal GDP.[26] Note that in this context, the pension is no longer a choice variable but a state variable, evolving according to a specific accumulation rule.
The union has to select a time path for wage (and implicitly, employment and pension benefits), considering outside opportunities and α as given. Formally:

$$\max_{n,b} \int U e^{-rt} dt \tag{19}$$

$$s.t.\ w = f'(n) \tag{20}$$

$$\dot{m} = n + P - m \tag{21}$$

$$\dot{P} = \delta P + \lambda n \tag{22}$$

$$\dot{b} = \alpha w + r_b b \tag{23}$$

$$m(0) > 0$$

$$P(0) > 0$$

The problem is solved by defining the current value Hamiltonian

$$n(u(w(1-\alpha))-u(\xi))+P(u(b)-u(\xi))+mu(\xi)+\mu(n+P-m)+\pi(-\delta P+\lambda n)+\Omega(\alpha w+r_b b) \tag{24}$$

[25] The 1995 Dini reform shifted the Italian pension system gradually toward a contribution-based scheme. It applies in full to anyone who entered the labour market after 1995. The current reform system mimics a funded system: the pension level of each retired employee will be based on the amount of contributions he/she paid into the public pension scheme during his/her working life.

[26] To be precise, the social security benefit is the annuity equivalent to the present value at retirement of past payroll taxes, capitalized at a five-year moving average of the nominal GDP growth rate. The new reforms are also envisaged for the supplementary funded schemes. The supplementary funded pillar is considered a necessary component of the reforms, but its development has been extremely slow. See amongst others, Franco (2002), Ferrera and Jessoula (2007).

FOC for n provides (see the Appendix A.2):

$$\beta(n) + \frac{\lambda}{\delta}\frac{nu'(b)}{r_b - r}\alpha f''(n) = \frac{r}{1+r}u(\xi) + \frac{\lambda}{\delta + r}[\frac{r}{1+r}u(\xi) - u(b)] \qquad (25)$$

Condition (25) implies that for standard values of parameters, n in steady state is smaller than in the standard case, as $\beta(n)$ is decreasing in n[27]. This solution may be explicitly rewritten for employment:

$$n = \frac{-\left[u(w(1-\alpha)) - (\frac{r}{1+r}(1+\frac{\lambda}{\delta+r})u(\xi)\right] - \frac{\lambda}{\delta+r}u(b)}{f''(n)\left[u'(w(1-\alpha)) + \frac{\frac{\lambda}{\delta}}{r_b - r}u'(b)\alpha\right]} \qquad (26)$$

The solution (26) shows that the union decision crucially depends on demographic factors and the gap between the return rate r_b and the intertemporal preference rate r. Unions would be willing to sacrifice some wages today, only if they are compensated by a rate of return r_b that should be sufficiently above r.[28].

Wage in equilibrium is going to be higher than that in the standard K-O-J model and, as a result, higher than that under the PAYG pension system. In this case including pensioners in union preferences does not result in a wage moderation effect, quite the reverse: a higher wage implies a higher pension for each member, regardless of pensioners' dynamics[29]. In other words, there is no longer a trade-off between wages and pensions, because unions bargain for a higher wage in order to ensure its members have a higher pension in the future.

7.1 Policy implications

Several policy implications can be drawn from our analysis. First of all, we emphasize that in the last two decades the policy of the union was to "exchange" wages with pension benefits, favoring the latter and increasing employment, whereas with the new pension scheme, we expect a recovery in wage claims and a reduction in employment. The issue which deserves particular concerns is the desirability of the pension system. The PAYG system does not generate macroeconomic effects only in terms of pension expenditure and sustainability of the system, as the debate often focuses, but it also has significant effects on the labour market. To be precise, in a unionized labour market, with a sizeable pensioners' membership, the pension system has significant equilibrium effects on wages and employment for a given level of productivity.[30] In a fully-funded context, unions prefer to support more active workers with a higher wage

[27] Solution is stable, but indeterminate (see Appendix A.2).

[28] This is the result of a concave (risk adversion) utility function. The concavity of $u(\cdot)$ determines the amount by wich r_b must excced r. See e.g. Barro and Sala-i-Martin (2004) for a general discussion.

[29] It is easy to prove that that this result still holds even when P is given in the membership function.

[30] It is worth recalling that in our model (labour) productivity is given. For the relationship between the pension schemes and the effects of productivity changes over time, see, for instance, Bilancini and D'Antoni (2009).

demand.
Interestingly, for a given contribution rate α, union demands may be affected by the "rate of return gap" ($r_b - r$): as the denominator of (26) clearly shows, the lower is the rate of return r_b, the higher are union wage demands (employment is lower). That is, for a given contribution rate α, high yields for accumulated pensions tend to reduce the union pressure on wage demand.

A further implication which can be drawn from the "rate of return gap", concerns the union wage claims over the cycle: since the social security benefit is the annuity equivalent to the present value at retirement of past payroll taxes, capitalised at a five-year moving average of the nominal GDP growth rate, then the rate of return r_b exerts a countercyclical effect on wage demands, with unions pushing for higher wage claims in periods of economic downturn.

A striking consideration, drawn from equation (25), is that now, and contrary to the previous solution achieved with the PAYG scheme, a higher inflow of pensioners out of workers (λ) entails a higher wage in order to increase pension benefits: higher λ brings P up, pushing for a larger weight on b, which can rise only if wages increase. A necessary corollary to this behaviour recalls the role of productivity: in countries where the old-age dependency ratio (those aged 65 or more as a percent of the working-age population) is increasing very sharply (in Italy according to the OECD, from the 1995 level of around 26 % to around 61 % by 2050), such union behaviour is sustainable only within the perspective of increasing productivity[31].

8 Concluding remarks

This paper offers an additional explanation for assessing wage moderation in Italy. We presented a dynamic model of union behaviour based on the assumption that both active workers and pensioners enter into the intertemporal objective function, endogenising pensioners' dynamics and exploring the effects of different pension regimes. The model is able to provide several significant results. First, we find that, when a PAYG pension system is in force, the steady state of the model provides a lower wage level than the corresponding conventional union monopoly model. Since union membership is conditional upon both active workers and pensioners, and because these are linked in a trade-off between wages and pension benefits (via the social security tax rate), the union necessarily prefers higher employment. In the model pensioners are defined as an endogenous variable and they depend on the death rate and the share of active workers who change status, becoming pensioners. In this framework, a higher share of pensioners brings down wages, allowing for policies which extend the retirement age. In Italy, the latest pension reforms, geared to budgetary savings, have extended the effective retirement age although unions have always opposed this extension. With the Dini pension reform in 1995, the Italian sys-

[31] Interestingly, in a recent paper, Bilancini and D'Antoni (2009) prove the desirability of an FF pension scheme as opposed to a PAYG, when productivity growth increases relative consumption of prospective pensioners.

tem gradually turned into a funded system. The model we set out to consider this shift in the pension scheme predicts a wage increase with respect both to the standard K-O-J model and the PAYG pension system union model.
A general result which can be drawn from this extension of the basic monopoly union model is that the composition of union membership is crucial to defining union decisions. Since both active workers and pensioners join the union as members, the union faces an intergenerational conflict, tied to wage-pension and employment-pensioners trade-offs. We show that these trade-offs are, in turn, related to the pension system. In a context with a sizeable pensioners' membership and a PAYG system, the union earnings premium proves less important with respect to employment, while for a context that approximates a fully funded system, the model predicts that union wage claims should increase once again.

Certainly, the models we present neglect several aspects of the product-market structure and endow the firm with a simple period horizon. Empirical and theoretical works may suggest several modifications to the models presented in the paper: one may assume the firm maximizes its discounted stream of profits, subject to the capital accumulation equation and a differentiable production function. A further extension should involve an OLG general equilibrium model with conflicting generations (active workers and pensioners). Finally, the analysis is limited to the steady state and the whole transition path between the two states of nature (active and retired) is neglected. These and other aspects will be considered for future research on the subject. However, we believe that this analysis of equilibria is essential for understanding the long-term trend in wages and pensions and, ultimately, how wages and pensions may be determined by changes in behaviour of unions caused by changing the pension system.

9 Appendices

A.1

The problem (9)-(13) is solved by defining the current value Hamiltonian

$$n(u(w(1-\theta))-u(\xi))+P(u(b)-u(\xi))+mu(\xi)+\mu(n+P-m)+\pi(-\delta P+\lambda n)+\Omega(bP-\theta wn) \tag{A.1}$$

FOCs are

$$\beta(n) - u(\xi) + \mu + \lambda\pi - \Omega\theta\Gamma(n) = 0 \tag{A.2}$$

$$Pu'(b) + \Omega P = 0 \tag{A.3}$$

$$-\dot{\mu} + r\mu = u(\xi) - \mu \tag{A.4}$$

$$-\dot{\pi} + r\pi = u(b) - u(\xi) + \mu - \delta\pi + \Omega b \tag{A.5}$$

$$bP - \theta f'(n)n = 0 \tag{A.6}$$

along with the two transversality conditions, one for each state variable[32]

$$\lim_{t\to\infty} e^{-rt}\mu(t)m(t) = 0 \tag{A.7}$$

$$\lim_{t\to\infty} e^{-rt}\pi(t)P(t) = 0 \tag{A.8}$$

Setting derivatives to zero,we obtain the steady state [33]
From (A.4)

$$\mu = \frac{u(\xi)}{1+r} \tag{A.9}$$

Using (A.9) in (A.5)

$$\pi = \frac{1}{\delta + r}[u(b) - \frac{r}{1+r}u(\xi) + \Omega b] \tag{A.10}$$

Using (A.9)-(A.10) in (A.2)

$$\beta(n) - u(\xi) + \frac{u(\xi)}{1+r} + \frac{\lambda}{\delta+r}[u(b) - \frac{r}{1+r}u(\xi) + \Omega b] - \Omega\theta\Gamma(n) = 0 \tag{A.11}$$

[32] The transversality conditions are standard in two-state variable models, but in general they are stronger than necessary (see Acemoglu 2009, p. 368, and Benhabib and Perli 1994).

[33] If the Hamiltonian is concave and both the transversality conditions are satisfied at the steady state, the solution is a maximum (sufficient condition). At this stage, we assume that this is the case.

and, after collecting some terms accordingly, we determine equation (14) in the text:

$$\beta(n^*) + u'(b)\theta\Gamma(n^*) = \frac{r}{1+r}u(\xi) + \frac{\lambda}{\delta+r}[\frac{r}{1+r}u(\xi) - u(b) + u'(b)b] \quad \text{(A.12)}$$

In order to investigate steady-state properties, we derive the system of differential equations around the steady state. First, as in Kidd and Oswald (1987), we eliminate costates from (A.4).

From (A.2) we get

$$\mu = -\beta(n) + u(\xi) - \lambda\pi + \Omega\theta\Gamma(n) \quad \text{(A.13)}$$

$$\dot{\mu} = -(\beta'(n) - \Omega\theta\Gamma'(n))\dot{n} - \lambda\dot{\pi} \quad \text{(A.14)}$$

From (A.5) we get

$$\dot{\dot{\pi}} = u(\xi) - u(b) - \mu + (\delta + r)\pi - \Omega b \quad \text{(A.15)}$$

which we substitute in (A.14).

So, (A.4) becomes

$$(\beta'(n) - \Omega\theta\Gamma'(n))\dot{n} = u(\xi) - (1+r)\mu - \lambda[u(\xi) - u(b) - \mu + (\delta + r)\pi - \Omega b]$$

or otherwise, by collecting some terms and by substituting for μ from (A.13)

$$\begin{aligned}(\beta'(n) - \Omega\theta\Gamma'(n))\dot{n} &= (1-\lambda)u(\xi) - (1+r-\lambda)[u(\xi) - \beta(n) - \lambda\pi + \Omega\theta\Gamma(n)] \\ &\quad -\lambda(-u(b) + (r+\delta)\pi - \Omega b)\end{aligned}$$

Setting $G(n) = \beta(n) - \Omega\theta\Gamma(n)$ and $G'(n) = \beta'(n) - \Omega\theta\Gamma'(n)$

$$\dot{\dot{\pi}} = (1 + r - \lambda)\frac{G(n)}{G'(n)} + \frac{-ru(\xi) + \lambda(u(b) + \Omega b) + \lambda(1 - \delta - \lambda)\pi}{G'(n)} \quad \text{(A.16)}$$

Using (A.13) in (A.15)

$$\dot{\dot{\pi}} = (\lambda + \delta + r)\pi + G(n) - u(b) - \Omega b \quad \text{(A.17)}$$

Now, we have 4 dynamic equations, $\dot{m}$, $\dot{P}$ from (3) and (4), (A.16) and (A.17) for $\dot{n}$ and $\dot{\pi}$. However, the system is highly nonlinear and non-autonomous due to equation (A.17). To make the system tractable and analyze steady-state properties, we restrict our analysis to local stability of the system, linearized around the steady state. In addition, we assume that $G^{'}(n)$ in (A.16) is roughly constant at the steady-state value, thus making the system autonomous. $G(n)$

is then approximated by $G(n) = g(n - n^{ss})$ where $g = G^{'}(n^{ss})$ is the first derivative of $G(n)$ evaluated at the steady state. The complete $4x4$ dynamic system linearized around SS is then

$$\begin{pmatrix} \dot{n} \\ \dot{m} \\ \dot{P} \\ \dot{\pi} \end{pmatrix} = \begin{pmatrix} (1+r-\lambda) & 0 & 0 & \gamma \\ 1 & -1 & 1 & 0 \\ \lambda & 0 & -\delta & 0 \\ g & 0 & 0 & \lambda+\delta+r \end{pmatrix} \begin{pmatrix} n-n^{ss} \\ m-m^{ss} \\ P-P^{ss} \\ \pi-\pi^{ss} \end{pmatrix} \tag{A.18}$$

where $\gamma = \frac{\lambda(1-\delta-\lambda)}{g}$.Eigenvalues are
-1,
$-\delta$,
$r + \frac{1}{2}\delta + \frac{1}{2}\sqrt{4g\gamma - 2\delta - 4\lambda + 4\lambda\delta + 4\lambda^2 + \delta^2 + 1} + \frac{1}{2}$,
$r + \frac{1}{2}\delta - \frac{1}{2}\sqrt{4g\gamma - 2\delta - 4\lambda + 4\lambda\delta + 4\lambda^2 + \delta^2 + 1} + \frac{1}{2}$.

Using γ's definition, the term under root reduces to $\delta^2 - 2\delta + 1$, which is between 0 and 1 for $0 < \delta < 1$, so that the third and fourth eigenvalues are both real and positive. As the number of unstable roots (2) equals the number of jump variables, the steady-state equilibrium is (locally) unique and stable.[34]

[34] This is the concept of saddle-path stability (see Acemoglu, 2009, p.271). The standard reference is Blanchard and Kahn (1980). The analogous characterization of solutions in continuous time is Buiter (1984). In our model there are two pre-determined variables m and P,which adjust slowly, while shadow price π and employment (the choice variable) n are the two forward-looking variables, which adjust (" jump") instantly to the equilibrium path. For the system to be determined, we need four boundary conditions: $m(0)$ and $P(0)$ for the pre-determined variables, while forward-looking variables are pinned down by the two transversality conditions (whereas $n(0)$ and $\pi(0)$ are free).

A.2

The Hamiltonian for model (19)-(23) is

$$n(u(w(1-\alpha))-u(\xi))+P(u(b)-u(\xi))+mu(\xi)+\mu(n+P-m)+\pi(-\delta P+\lambda n)+\Omega(\alpha w+r_b b) \tag{A.19}$$

and FOCs (necessary) for an interior solution are

$$\beta(n) - u(\xi) + \mu + \lambda\pi + \Omega\alpha f''(n) = 0 \tag{A.20}$$

$$\begin{aligned} -\dot{\mu} + r\mu &= u(\xi) - \mu && \text{(A.21)} \\ -\dot{\Omega} + r\Omega &= Pu^{'}(b) + \Omega r_b && \text{(A.22)} \\ -\dot{\pi} + r\pi &= u(b) - u(\xi) + \mu - \delta\pi && \text{(A.23)} \end{aligned}$$

along with the transversality condition

$$\lim_{t\to\infty} e^{-rt}\mu(t) = 0$$

$$\lim_{t\to\infty} e^{-rt}\Omega(t) = 0$$

$$\lim_{t\to\infty} e^{-rt}\pi(t) = 0$$

Pinnig down to 0 $\dot{\mu}, \Omega$, and $\dot{\pi}$, in order to obtain a value for costate variables in SS,

$\mu = \frac{u(\xi)}{1+r}$; $\Omega = \frac{Pu'(b)}{r_b - r}$; $\pi = \frac{1}{\delta+r}[u(b) - \frac{r}{1+r}u(\xi)]$

then FOC for n (A.20) is going to be

$$\beta(n) - u(\xi) + \frac{u(\xi)}{1+r} + \frac{\lambda}{\delta+r}[u(b) - \frac{r}{1+r}u(\xi)] + \frac{Pu'(b)}{r_b - r}\alpha f''(n) = 0 \tag{A.24}$$

and, after collecting some terms accordingly, we determine equation (25) in the text.

Following the same steps described in Appendix A.1, we derive the system of differential equation around the steady state.

From (A.20) we get

$$\mu = -\beta(n) + u(\xi) - \lambda\pi - \Omega\alpha f''(n) \tag{A.25}$$

$$\dot{\mu} = -(\beta'(n) + \Omega\alpha f'''(n))\dot{n} - \lambda\dot{\pi} - \dot{\Omega}\alpha f''(n) \tag{A.26}$$

So, using (A.25) in (A.21)

$$-(\beta'(n) + \Omega\alpha f'''(n))\dot{n} - \lambda\dot{\pi} - \dot{\Omega}\alpha f''(n) = (1+r)\mu - u(\xi) \tag{A.27}$$

From (A.23) we get

$$\dot{\pi} = u(\xi) - u(b) - \mu + (\delta + r)\pi \tag{A.28}$$

and from (A.22) we get

$$\dot{\Omega} = \Omega(r - r_b) - Pu^{'}(b) \tag{A.29}$$

which we substitute in (A.27).

$$\begin{aligned} & -(\beta'(n) + \Omega\alpha f'''(n))\dot{n} - \lambda[u(\xi) - u(b) - \mu + (\delta + r)\pi] - [\Omega(r - r_b) - Pu^{'}(b)]\alpha f''(n) \\ = \quad & (1 + r)\mu - u(\xi) \end{aligned}$$

Substituting (A.25) for μ and collecting terms

$$\begin{aligned} (\beta'(n) + \Omega\alpha f'''(n))\dot{n} \quad = \quad & (1 - \lambda)u(\xi) - (1 + r - \lambda)[u(\xi) - \beta(n) - \lambda\pi - \Omega\alpha f''(n)] \\ & -\lambda[-u(b) + (r + \delta)\pi] - [\Omega(r - r_b) - Pu^{'}(b)]\alpha f''(n) \end{aligned}$$

Setting $G(n) = \beta(n) + \Omega\alpha f''(n)$, $G'_n(n) = \beta'(n) + \Omega\alpha f'''(n)$ and $H(\Omega, P, n) = [\Omega(r - r_b) - Pu^{'}(b)]\alpha f''(n)$

$$\dot{n} = (1 + r - \lambda)\frac{G(n)}{G'_n(n)} + -\frac{H(\Omega, P, n)}{G'_n(n)}\frac{-ru(\xi) + \lambda u(b) + \lambda(1 - \delta - \lambda)\pi}{G'_n(n)} \tag{A.30}$$

Using (A.25) in (A.28)

$$\dot{\pi} = (\lambda + \delta + r)\pi + G(n) - u(b) \tag{A.31}$$

Now, we have 5 dynamic equations, $\dot{m}, \dot{P}$ from (A.21) and (A.22), (A.30), (A.31) and (A.29) for $\dot{n}$, $\dot{\pi}$ and $\dot{\Omega}$. However, the system is highly nonlinear and non-autonomous due to equation (A.30). To make the system tractable and analyze steady-state properties, we restrict our analysis to local stability of the system, linearized around the steady state. First order approximations are

$$G(n) = G^{'}_n(n - n^{ss}) + \alpha f''(n)(\Omega - \Omega^{ss}) \tag{A.32}$$

$$\begin{aligned} H(\Omega, P, n) \quad = \quad & (r - r_b)\alpha f''(n)(\Omega - \Omega^{ss}) + u^{'}(b)\alpha f''(n)(P - P^{ss}) \\ & +[\Omega(r - r_b) - Pu^{'}(b)]\alpha f'''(n)(n - n^{ss}) \end{aligned} \tag{A.33}$$

We set $g_1 = G^{'}_n$; $g_2 = \alpha f''(n)$. Moreover, we assume that $f''(n)$ is roughly constant around the steady state so that $f'''(n) = 0$ as well as the third term in (A.33).

The complete $5x5$ dynamic system linearized around SS is then

$$\begin{pmatrix} \dot{n} \\ \dot{m} \\ \dot{P} \\ \dot{\pi} \\ \dot{\Omega} \end{pmatrix} = \begin{pmatrix} (1+r-\lambda) & 0 & u_b g_2 & \frac{\lambda(1-\delta-\lambda)}{g_1} & [(1+r-\lambda)+(r-r_b)]\frac{g_2}{g_1} \\ 1 & -1 & 1 & 0 & 0 \\ \lambda & 0 & -\delta & 0 & 0 \\ g_1 & 0 & 0 & (\lambda+\delta+r) & g_2 \\ 0 & 0 & -u_b & 0 & (r-r_b) \end{pmatrix} \begin{pmatrix} n-n^{ss} \\ m-m^{ss} \\ P-P^{ss} \\ \pi-\pi^{ss} \\ \Omega-\Omega^{ss} \end{pmatrix} \tag{A.34}$$

Analytical solutions for a 5th-order characteristic polynomial are not available and the polynomial is too complex to be factorise. Moreover, as we are interested in local properties of the system, we work out a set of numerical solutions for a large span of possible values of parameters. Hence, we explicitely define some of the expressions in (A.34) in order to evaluate the matrix of coefficents numerically (table 2).

Table 2. Parameter values

Function / Parameter	Value/ Expression
$f(n)$	n^a
$u(.)$	$\sqrt{()}$
r_b	0.05
r	0.025
λ, δ	0.001
α	0.33
a	0.6
n, b	> 0

Using table 2 and the approximation $\beta'(n) \approx 2u'(f'(n))f''(n)+nu''(f'(n))[f''(n)]^2$, where in the last expression we used the assumption $f'''(n) = 0$,we get

$$\beta(n) = (an^{(a-1)})^{0.5} + 0.5n(an^{(a-1)})^{(0.5-1)}(a(a-1)n^{(a-2)})$$

$$\begin{aligned} \beta'(n) &= 2(0.5)[(an^{(a-1)})^{(0.5-1)}]a(a-1)n^{a-2} + \\ &\quad +0.5(0.5-1)(an^{(a-1)})^{(0.5-2)}(a(a-1)n^{(a-2)})^2 \end{aligned}$$

Then,

$$\begin{aligned} g_1 &= 2(0.5)[an\hat{}(a-1))\hat{}(0.5-1)]a(a-1)n^{a-2} + \\ &\quad 0.5(0.5-1)(an^{(a-1)})^{(0.5-2)}(a(a-1)n^{(a-2)}) \end{aligned}$$

and

$$g_2 = \alpha a(a-1)n^{a-2}$$

Substituting parameters and functions above in (A.34), we work out eigenvalues for a large set of values for n and b. In all cases, we get 3 negative roots and 2 positve ones. As there are more negative eigenvalues than state variables (2), the system is (locally) stable, but indeterminate[35].

[35] To pinpoint a unique equilibrium, additional conditions should be imposed on initial conditions; see Gandolfo (2010) and Buiter (1984). Changing parameters or functional form for $f(n)$ and $u()$ does not change our results.

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