Persuasion with Reference Cues and Elaboration Costs

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Abstract

We develop a model of persuasion where, consistent with the psychological literature on dual process theory, the persuadee has to sustain a cognitive effort – the elaboration cost – in order to fully and precisely elaborate information. The persuader makes an offer to the persuadee and, aware that she is a dual process reasoner, also sends her a costly signal – the reference cue – which refers the offer to a category of offers whose average quality is known by the persuadee. Initially, the actual quality of the offer by the persuader is hidden to the persuadee, while the signal is visible. Then, the persuadee can either rely on cheap low elaboration and form expectations on the basis of the signal – thinking coarsely, i.e., by category – or engage in costly high elaboration to attain knowledge of the actual quality of the offer. This signaling setup allows us to keep the assumption that agents are both rational and Bayesian and, at the same time, to match many of the findings emphasized by well established psychological models of persuasion – such as the Elaboration Likelihood Model and the Heuristic-Systematic Model. In addition, the model provides novel theoretical results such as the possibility of separating equilibria that do not rely on the single-crossing property and a new rationale for the phenomenon of counter-signaling.

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Keywords: persuasion, coarse reasoning, peripheral and central route, heuristic and systematic reasoning, counter-signaling.

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1 Introduction

Activities aimed at persuading are ubiquitous in social life, especially in the economic and political spheres. The ability to influence and persuade others is a critical skill if you are in business. Indeed, firms need for their survival to convince consumers that their products are worth to be purchased. To do this, marketing communication generates an enormous flow of messages, that go through radio and television, occupy intervals of sport events, attract your attention with advertising posters in motorways and roads, and even use strategic placement on shelves in supermarkets to reach you. Investment financing provides a further motivation for persuasion activities: every day a large number of managers leading big or small companies try to convince bankers to lend them money in order to realize their investment plans. Political campaigns also rely on persuasion, as they are organized efforts aimed at convincing the majority of electors that a candidate is the best choice for some office. Here the variety is ample: we range from high-budget campaigns to national government – that use virtually all campaign media – to low-budget campaigns to local offices that are based on direct mail and direct voter contact. The relevance of persuasion is not limited to business and politics. For instance, fundraising activities carried on by non-profit institutions often rely on persuasive communication; the likelihood of success in asking for contribution heavily depends on fundraiser ability to arise emotions and participation in the potential donor.[1]

In this paper we want to contribute to a better understanding of the phenomenon of persuasion, by nesting on a rigorous economic model the insights from psychology about dual process reasoning and categorical thinking. More precisely, we follow most of the economic literature in framing persuasion activities within a sender-receiver model – the sender is the *persuader*, the receiver is the *persuadee* – assuming that agents are both rational and Bayesian (see Subsection 2.1 for the related economics literature). In addition, we introduce cognitive limitations along the lines suggested by social and cognitive psychology with regard to how the persuadee can elaborate information and how the persuader can take advantage of this (see Subsection 2.2 for the related psychological literature). In particular, we follow Dewatripont and Tirole (2005) by assuming that the persuadee has to pay a cognitive cost to process information fully and precisely.[2,3] However, we depart from Dewatripont and

[1] Della Vigna et al. (2012) found in a field experiment that a door-to-door fundraising activity is more effective for charitable purposes.

[2] Other recent contributions considering the costly acquisition of information are Dewatripont (2006), Caillaud and Tirole (2007), Tirole (2009) and Butler et al. (2013).

[3] Brocas and Carrillo (2008) and Brocas (2012) stress that the evidence provided by brain sciences on

Tirole (2005) in the modeling of cues – i.e., a simple signal sent by the persuader which does not require great cognitive effort to be processed – which they assume to contain information on the trustworthiness of the persuader; instead, we propose to model cues as signals that refer an object to a category of objects, and to follow Mullainathan (2002) in assuming that individuals may suffer from "coarse thinking", i.e., they might be unable to distinguish objects falling in the same category.[4,5] More precisely, we assume that the persuadee suffers from coarse thinking whenever she chooses not to bear the cognitive cost of elaborating carefully the message sent by the persuader. Thanks to this we can investigate the strategic use of reference cues by the persuader and we are able to match many findings of psychological theories of persuasion. In particular, this assumption allows us to model persuasion activities such as the one considered in the field experiment by Bertrand et al. (2010) – where prospective borrowers are offered loans by mailed advertisement cards with different cues – which can hardly fit the model proposed by Dewatripont and Tirole (2005) since the latter only considers cues related to the sender's expertise.[6]

We briefly sketch the basic working of our model. The persuader makes an offer of unknown quality to the receiver, and provides a reference cue – like the packaging of a product – that gives summary information about the expected quality of the offer; the persuadee can then decide to exert a low cognitive effort in processing the offer – *low* elaboration – and take a decision on the sole basis of the reference cue, or she can scrutinize the offer carefully – *high* elaboration – and obtain a precise knowledge of the offer received but also bear a

the multi-system nature of the human brain should be a fundamental source of inspiration for the economic modeling of decision-making (see Brocas and Carrillo, 2012a, for a focused survey).

[4]Other papers considering coarse thinking or thinking by categories are: Mullainathan et al. (2008) on persuasion; Mohlin (2009), Peski (2011) and Fryer and Jackson (2008) on optimal categorization, Ettinger and Jehiel (2010) on coarse understanding of opponents' play, and Mengel (2012) on the evolution of coarse categorization.

[5]Models of decision-making that consider cognitive limitations often focus on how the decision-maker treats, and possibly reduces, the information to be processed. A growing body of literature considers the case where the decision-maker's attention is drawn to those payoffs or characteritics which are most different or salient relative to the average or to a reference point (Bordalo et al., 2012; Kőszegi and Szeidl, 2013; Bordalo et al., 2013b). A different but related model is about local thinking (Gennaioli and Shleifer, 2010), where only the information relative to the most representative scenarios is retrieved from memory.

[6]The marketing literature focuses on the strategic use of cues to influence how consumers perceive quality, and points out that very effective cues might have little to do with actual quality of products (Teas and Agarwal, 2000). Experimental and survey evidence indicates that these cues can exert a sizeable impact on consumers' valuation (see, e.g., Sáenz-Navajas et al., 2013; Woodside, 2012, for wine and food products, respectively). Our model can be seen as a way to rationalize the use of such cues in the attempt to persuade consumers to buy (also, see Bagwell, 2007, for detailed references on persuasive advertisement).

cognitive cost due to information processing. Importantly, we do not treat low elaboration as a behavioral shortcut, but as ignorance that the persuadee is aware of and able to quantify, so that she can form expectations and choose the elaboration level to maximize expected utility. We stress that this is where coarse thinking crucially comes in: instead of investigating all benefits and costs of the current offer, the persuadee uses the reference cue to assign the offer to a category of similar offers – all associated with the same reference cue — and then evaluates the offer according to the average quality of its category, taking a decision on the sole basis of this information and her priors. The trade-off at play is the following: low elaboration leads to take a decision that is good on average, but that is not necessarily the best choice for the current offer (only high elaboration guarantees to always take the best choice available); however, low elaboration allows to save on cognitive costs, and hence the persuadee can well decide not to engage in high elaboration, especially when beliefs about the quality of the current offer are quite extreme (in which case the persuadee does not expect to learn much from high elaboration). Since the persuader anticipates all this, he can make a strategic use of the reference cue.

Despite its simplicity, this model and its extensions prove able to predict behavior that is in line with well established findings on persuasion in the psychological literature (Petty and Cacioppo, 1986a; Eagly and Chaiken, 1993). The persuasive message sent by the persuader is interpreted by the persuadee differently under high elaboration and low elaboration (Remark 1). A greater motivation or a better cognitive ability by the persuadee makes the recourse to high elaboration more likely (Remarks 2 and 3). The use of reference cues by the persuader affects the elaboration effort and the reaction to the offer by the persuadee (Remark 4). Persuasion obtained under high elaboration by the persuadee is stabler than if obtained under low elaboration (Remark 5). Antecedents of elaboration such as arousal or prior knowledge can influence the intensity of elaboration and induce biased elaboration (Remarks 6 and 7).

Our analysis also shows that explicitly considering elaboration costs in a signaling framework where signals are reference cues allows for the emergence of a variety of possible equilibrium behaviors, some of which cannot be easily accommodated in a standard signaling model. First, there are pooling equilibria where all types of persuaders employ the same reference cue, either high or low. Second, there are separating equilibria where different types send different reference cues. As one might expect, there is a separating equilibrium where high quality persuaders employ high reference cues while low quality persuaders employ low reference cues – this outcome resembling standard separating equilibria of signaling models. We stress, however, that separating equilibria can exist in our model even if different types incur the same costs for reference cues and obtain the same gains from having their offer accepted, i.e., the single-crossing property does not hold. Standard separation of types is still possible because if the persuadee decides to engage in high elaboration then she can condition acceptance on types, and in such a case different types face different expected gains. We also stress that separating equilibria can exist only for elaboration costs that are mild relatively to the stake associated with the offer. Indeed, if elaboration costs are sufficiently low, then the persuadee relies on high elaboration whatever cue she observes, and hence both persuader types find it convenient to save money and pool on the cheap reference cue; if, instead, elaboration costs are sufficiently high, then the persuadee never relies on high elaboration, and similarly there is no reason for either persuader type to choose the more costly cue, with the result that both types pool on the cheap reference cue also in this case.

Perhaps more interestingly, a different form of separation between types can emerge in this setup: separating equilibria where high quality persuaders go with low reference cues and low quality persuaders go with high reference cues. This is an instance of what is called counter-signaling. However, the reason why this outcome emerges in equilibrium differs from previous literature. In Feltovich et al. (2002), a sender has a quality that can be mistaken only for close qualities, and this allows a counter-signaling outcome where medium-quality senders choose high signals to separate from low-quality senders, while high-quality senders choose low signals to separate from medium-quality senders. Our counter-signaling result, instead, is triggered by a totally different mechanism. A low quality persuader relies on a high cue that refers to a category where average quality is high enough, so that the persuadee chooses to accept the offer without exerting high elaboration effort. A high quality persuader prefers to save on the cost of the signal, choosing a cheaper cue that refers to a category where average quality is intermediate, so that the persuadee exerts high elaboration effort and accepts the offer after discovering it is of high quality. The low quality persuader does not find it convenient to choose the cheaper cue, because his offer would be discovered to be of low quality and hence rejected. We note that this result cannot arise in the model by Dewatripont and Tirole (2005) as in their setup the sender is not allowed to manipulate the cue, but only to either send a truthful one or do not send it at all. We also stress that our result hinges on coarse thinking by the persuadee, which makes the manipulation of the cue potentially worthwhile (Mullainathan et al., 2008). In this sense, the counter-signaling arising in our model can be seen as a case of profitable deception similar to that studied in Heidhues et al. (2012), where naive consumers overestimate the net value of products that they buy; however, differently from Heidhues et al. (2012), in our model agents are not necessarily naive (or sophisticated), but can decide to be so in order to save on the cognitive effort, sustaining deception in equilibrium.

The paper is organized as follows. Section 2 surveys the relavant literature on persuasion, distinguishing between contributions from economics (Subsection 2.1) and those from psychology (2.2). Section 3 presents the model in three steps: the modeling of the elaboration of information (Subsection 3.1), the optimal behavior of the persuadee in terms of elaboration level and reaction to the offer (Subsection 3.2), and the optimal behavior of the persuader who anticipates the behavior of the persaudee and tries to take advantage of this (Subsection 3.3); furthermore, a discussion of the potential interpretations of the model is provided (Subsection 3.4). Section 4 presents all persuasion equilibria of our model, distinguishing between pooling equilibria (Subsection 4.1), standard separating equilibria (Subsection 4.2), and separating equilibria where counter-signaling emerges (Subsection 4.3); uniqueness and existence of equilibria are also discussed and related to the degree of coarse thinking (Subsection 4.4). Section 5 discusses how our model relates to psychological findings. Section 6 extends our stylized model in various directions, with the aim of checking the robustness of results: many offer qualities (Subsection 6.1), many offer categories and reference cues (Subsection 6.2), a continuum of elaboration intensities (Subsection 6.3), and cues with fully endogenous quality (Subsection 6.4). Finally, Section 7 concludes, summarizing the contribution and showing lines for future research.

2 Literature on Persuasion

In this section we review the main contributions on persuasion, distinguishing between economic and psychological literature. We focus mainly, but not only, on theory. For a recent survey of empirical evidence regarding persuasion activities, especially related to economics and politics, see Della Vigna and Gentzkow (2010).

2.1 Persuasion in Economics

In recent years several models have been proposed that study how a message sent by a persuader can affect a persuadee's behavior. We can distinguish among them on the basis of different criteria.

A first criterion is whether the act of persuasion is belief-based or non-belief-based – i.e., preference-based. Non-belief based persuasion affects behavior independently of beliefs. In such a case persuasion is obtained because the message itself impacts on utility and, hence, on behavior (Stigler and Becker, 1977; Becker and Murphy, 1993). This is also reminiscent of old models of persuasive advertising (Braithwaite, 1928). Instead, belief-based persuasion affects behavior by changing persuadees' beliefs. For instance, a persuadee can be persuaded by informative communication (Stigler, 1961; Telser, 1964).

Among the models where persuasion is understood as belief-based, a further distinction can be made between models where agents are perfect Bayesian updaters and models where they are not. When agents are Bayesian updaters, a persuader can persuade them by sending appropriate signals that can be costly (Nelson, 1970) or not (Gentzkow and Shapiro, 2006; Kamenica and Gentzkow, 2011) and that are correctly elaborated. Instead, non-fully Bayesian agents have limitations in the way they elaborate the signal: e.g., they are constrained by limited memory (Mullainathan, 2002; Shapiro, 2006), they double-count repeated information (De Marzo et al., 2003), or they neglect the incentives of the sender (Eyster and Rabin, 2010).

Finally, models can be distinguished on the basis of the nature of the information sent: hard versus soft. Hard information is actually verifiable, while soft information is not. Cheap talk models typically rely on soft information (Crawford and Sobel, 1982), while models that exploit the strategic use of verifiable information (Milgrom and Roberts, 1986) can also consider a verification cost (Caillaud and Tirole, 2007).

2.2 Persuasion in Psychology

In psychology the term persuasion refers to a broader and vaguer phenomenon than in economics: activities aimed at influencing others' behaviors. However, the most relevant difference is that psychologists assume that the persuadee can process the message sent by the persuader at different degrees of effectiveness, which are associated with different cognitive costs.

Psychologists often stress that persuasion activities exploit the fact that individuals have two distinct ways of processing information when they receive a message and have to take decisions based on it (Chaiken and Trope, 1999). Theories in cognitive and social psychology that refer to this idea are typically labelled as dual process theories (Evans, 2003), and the two ways of processing information are also called System 1 and System 2. Kahneman (2003) refers to System 1 and System 2 as, respectively, *intuition* and *reasoning*. Recent neurological research (Goel et al., 2000) suggests that different parts of our brain are actually activated when using System 1 and System 2, respectively. Dual process theories have been applied to explain human behaviors in different setups (Gawronski and Creighton, 2013): persuasion, attitude-behavior relations, prejudice and stereotyping, impression formation, dispositional attribution.

There are two workhorse models of persuasion in psychology. One is the Elaboration Likelihood Model (ELM) (Petty and Cacioppo, 1986a), where the persuadee can use the "central route" – characterized by a high cognitive effort – or the "peripheral route"– characterized by a low effort. These two routes can be understood as an approximation of a continuum of elaboration intensities which a subject can use when processing information: the higher an individual's cognitive effort, the more likely that she processes all relevant information. At the extremum characterized by highest level of elaboration individuals use all available information and integrate it with already stored information. On the contrary, at the extremum characterized by lowest level of elaboration individuals minimally scrutinize relevant information, extensively using short-cuts to process information.

The other model is the Heuristic-Systematic Model (HSM) (Chaiken et al., 1989), where the persuadee can use "systematic elaboration" – characterized by careful scrutiny – or "heuristic elaboration" – characterized by the use of simple heuristics, rule of thumbs and categorizations. The basic idea is very similar to that of the ELM. A fully systematic processing of information requires high cognitive effort and considers all relevant information. In contrast, a purely heuristic processing requires minimal cognitive effort and considers only a small amount of information.

One important feature of these models is that the persuader may have the chance to affect the choice of elaboration intensity. This naturally gives rise to a strategic interaction between the persuader and the persuadee.

3 The model

We introduce the model in three steps. Firstly, we describe the message received by the persuadee (to whom we refer as "she") and how she can elaborate the information it contains. Secondly, we study the her behavior with respect to both the choice of the elaboration level and the reaction to the offer. Finally, we introduce the persuader (to whom we refer as "he") and we analyze his strategic choice regarding the reference cue. At the end of the section a discussion of the potential interpretations of the model is provided.

3.1 Message processing: High and low elaboration

We follow the psychological literature in assuming that the decision-maker (DM) can process the message at two different levels of elaboration. More precisely, DM can decide to exert a high cognitive effort and scrutinize the message carefully. This can be interpreted as the use of the “central route” in the ELM or the use of the “systematic elaboration” in the HSM. Alternatively, DM can decide to exert a low cognitive effort and adopt simple heuristics, categorizations, and rules of thumb. This can be interpreted as the use of the “peripheral route” in the ELM or the use of the “heuristic elaboration” in the HSM. We stress that DM chooses the elaboration level and is fully aware of the elaboration cost.

More formally, message processing is modeled as follows. DM faces a two-part message $(q, r) \in \{G, B\} \times \{x, y\}$ associated with an offer which she has to decide upon. Part $r \in \{x, y\}$ of the message is a reference cue, i.e., a piece of information which allows DM to refer the offer to a specific category of offers and, through this, to infer the expected quality of the offer she is facing. Part $q \in \{G, B\}$ of the message contains the information regarding the actual quality of the offer: if $q = G$ then quality is good, while if $q = B$ then quality is bad.

Whenever DM is aware of a message (q, r), she chooses a level of cognitive effort $e \in \{H, L\}$ to elaborate it. In particular, DM can exert a low effort of elaboration L and only acquire knowledge of part r of the message, or exert a high effort of elaboration H and acquire also part q of the message. The elaboration level L is assumed to be costless and to occur as soon as the message is received, so that r is always observed. Instead, the elaboration level H can be activated after observing part r and requires to bear a cost of elaboration, denoted with $c_e > 0$.

We emphasize that part r of the message is informative in the sense that it refers to a category of objects of which DM knows the average quality. This is a simple way to model the fact that, when choosing $e = L$, DM is affected by “coarse thinking”, i.e., DM puts different offers with a common characteristic in the same mental category and treats them all in the same way (Fryer and Jackson, 2008; Mullainathan et al., 2008).

Figure 1 is a graphical representation of the different information on quality that can be drawn from the same message depending on the elaboration level chosen by DM.

3.2 Persuadee behavior: Elaboration level and reaction

We now study the costs and benefits for DM with regard to her choices about the elaboration level and the reaction to the offer. To help focus attention on the behavior of the persuadee, for now we abstract from the decision problem of the persuader – in a sense, we solve the model backwards.

DM initially processes the message received under $e = L$, so she observes the reference cue r while the actual quality q of the offer remains hidden. At this stage DM has a precise belief on the offer quality, that is conditional of the reference cue observed. Given such a belief, DM evaluates the convenience of increasing effort to $e = H$, paying c_e and acquiring

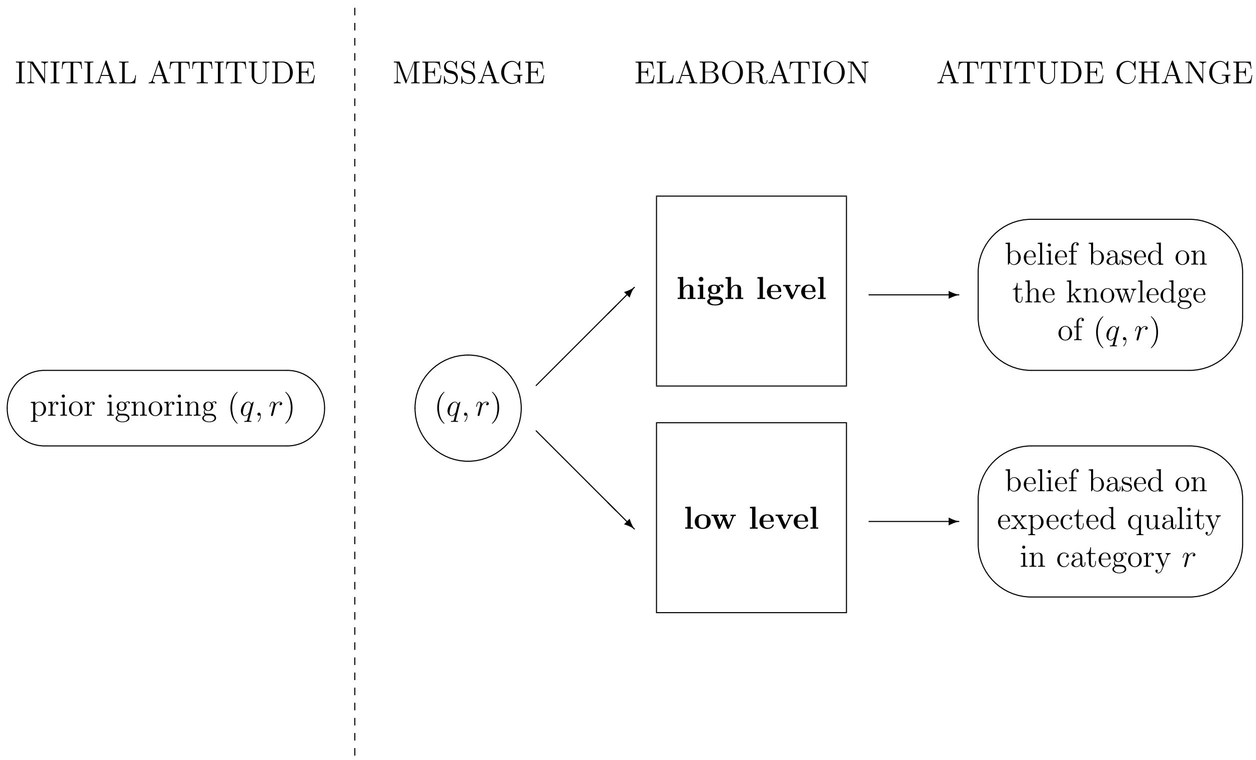


Figure 1: A graphical representation of message processing. Two elaboration levels are possible: high elaboration H which allows to observe (q, r) and low elaboration L which allows to observe only r.

the knowledge of q. Finally, DM has to decide whether to accept the offer, which we denote with Y, or reject it, which we denote with N.

The payoffs for DM are the following. If DM accepts the offer when $q = G$, then she obtains $U_G > 0$. If instead she accepts the offer when $q = B$, then she obtains $U_B < 0$. If DM rejects the offer, then she obtains a null payoff independently of q. In any case, if DM exerts $e = H$, then she also has to bear the elaboration cost c_e.

In most cases we will stick to the interpretation that Y means acceptance and N means rejection. However we stress that other interpretations are possible. For instance, if the offer is a consumer good to be bought, then Y could mean to buy a large quantity of the good and N to buy a small quantity; also, Y could mean to pay a high price per unit while N to pay a low price per unit. Both cases can easily be accommodated by the model since what really matters for the results is that Y is preferred when quality is G and N is preferred when quality is B. We observe that U_G and U_B can be interpreted as relative convenience to choose Y over N when, respectively, quality is G and quality is B.

Given the structure of DM's payoffs, we immediately recognize that, if DM exerts $e = H$, then she finds it convenient to choose Y in case $q = G$ and N in case $q = B$. We indicate such behavior with HYN. Other behaviors are possible, that make use of high elaboration but do not react optimally to the knowledge of actual quality. To simplify the analysis, we neglect those behaviors. We denote behaviors that make use of low elaboration with LY and LN, respectively leading to acceptance and rejection. Therefore, a strategy for DM can be described by a function $\delta : \{x, y\} \to \{LY, LN, HYN\}$.

We now proceed to compare the expected payoffs that DM obtains by choosing HYN, LY, and LN. We denote with μ the belief about the offer quality, conditional on the observed reference cue. More precisely, $\mu \in [0, 1]$ is the probability that the offer is of quality G, after a reference cue has been observed. At this stage of the analysis, we treat μ as a parameter. We observe that:

- the choice of LY leads to an expected utility of $\mu U_G - (1-\mu)|U_B|$;
- the choice of HYN leads to an expected utility of $\mu U_G - c_e$;
- the choice of LN leads to an expected utility of 0.

By solving, with respect to μ, for the highest expected payoff or utility across the above ones, we directly obtain the following proposition, which proof is hence omitted.

Proposition **1** (Persuadee's optimal behavior)**.**

- *HYN is optimal if and only if: $\mu \geq \frac{c_e}{U_G}$ and $\mu \leq 1 - \frac{c_e}{|U_B|}$;*
- *LY is optimal if and only if: $\mu \geq 1 - \frac{c_e}{|U_B|}$ and $\mu \geq \frac{|U_B|}{U_G + |U_B|}$;*
- *LN is optimal if and only if: $\mu \leq \frac{c_e}{U_G}$ and $\mu \leq \frac{|U_B|}{U_G + |U_B|}$.*

Note that HYN turns out to be optimal for some intermediate range of values of μ only if $c_e \leq \frac{U_G |U_B|}{U_G + |U_B|}$, although when equality holds there is just one value of μ for which HYN is optimal and for that value DM is indifferent between HYN, LN, and LY. The optimal behavior by DM as a function of μ is summarized in Figure 2.

Intuitively, DM is less likely to choose H if the elaboration cost c_e is higher, and never uses it if c_e is too high. Moreover, the expected benefits for DM to choose H over L decrease when beliefs on quality are more extreme – i.e., a high μ that gets closer to 1 or a low μ that gets closer to 0 – because less and less uncertainty remains while the same elaboration cost has to be borne.

(a) when $c_e < \frac{U_G|U_B|}{U_G+|U_B|}$.

(b) when $c_e > \frac{U_G|U_B|}{U_G+|U_B|}$.

Figure 2: DM's optimal behavior as a function of μ.

3.3 Persuader behavior: Strategic use of reference cues

We now introduce the persuader (P) and focus on the strategic use of reference cues. In particular, we study how P can induce in DM favorable beliefs about his offer through the sending of a reference cue r. This behavior on the part of P can be interpreted as an attempt to "frame" the offer (see, e.g., Mullainathan et al., 2008): the observation of r induces DM to associate P's offer to a specific mental category containing all offers sharing characteristic r, and consequently to evaluate P's offer by considering the average quality of offers in such a category.

Formally, in the first stage of the game Nature selects one among the following three possibilities: with probability $\alpha_x > 0$ the offer is not made by P and belongs to the category associated with reference cue x, with probability $\alpha_y > 0$ the offer is not made by P and belongs to the category associated with reference cue y, and with probability $\alpha_P = 1 - \alpha_x - \alpha_y > 0$ the offer is made by P. We stress that probabilities α_x, α_y, and α_P are intimately related to coarse thinking. Indeed, such probabilities can be interpreted as relative frequencies of occurrence and, hence, they are naturally thought of as dependent on the relative size of categories associated with x and y, which in turn is affected by the degree of coarse thinking. We elaborate more on this in Subsection 4.4.

If the offer does not come from P and belongs to category x, then it is of quality $q = G$ with probability β_x. Similarly, if the offer does not come from P and belongs to category y, then it is of quality $q = G$ with probability β_y. Hence, the parameters β_x and β_y represent the fraction of good quality offers in, respectively, category x and y, not taking into account the behavior of P. Without loss of generality we assume that $\beta_x > \beta_y$, i.e., on average x refers to a higher quality than y. An attempt to endogenize β_x and β_y is provided in Subsection 6.4.

Instead, if the offer comes from P, then the game unfolds as follows: the quality of the offer – i.e., P's type – is $q = G$ with probability α_G and $q = B$ with probability $\alpha_B = 1 - \alpha_G$, and then P chooses a reference cue $r \in \{x, y\}$ to be associated with the offer. Probabilities α_G and α_B can be interpreted as, respectively, the fraction of good quality offers and the fraction of bad quality offers when P is called into play. We remark that the quality of an offer should be interpreted in a broad sense, as something that correlates positively with DM's utility. The cost for P of choosing reference cue r is c_r. Since $\beta_x > \beta_y$, to rule out uninteresting cases we assume that $c_x > c_y = 0$, i.e., we posit that referring the offer to a category of higher average quality is more costly for P, and we normalize c_y to zero. Quality q is known to P, so a strategy for P is a function $\rho : \{G, B\} \longrightarrow \{x, y\}$ indicating which reference cue is chosen conditionally on the quality of the offer selected by Nature. The payoff for P is $V > c_x$ in case DM accepts the offer while it is 0 if DM rejects the offer. In any case, the cost c_r must be borne. We stress that the null payoff obtained by P when DM chooses N should be seen as a normalization, so to be consistent with different interpretations of Y and N; for instance, if N means to buy at a low price or a small quantity, then P would earn a low but possibly positive profit.

Under low elaboration DM suffers from coarse thinking, and hence she is unable to distinguish where the offer comes from and its quality, while only the reference cue is directly visible. After observing r, DM updates her belief μ (that $q = G$) and then decides whether to switch from $e = L$ to $e = H$ and acquire knowledge of q at the cost c_e. Finally, DM either accepts or rejects the offer, i.e., plays Y or N, potentally conditioning this choice upon the information acquired on q. A summary of the decision structure of this game is provided in Figure 3.

Given DM's behavior, the choices that maximize P's payoff are easily established as P obtains V only if DM reacts with Y, while P incurs the cost of reference c_r independently of DM's choices. Note that both types of P want to have the offer accepted by DM, but for type G it is enough that DM plays HYN, while type B needs that DM plays LY; moreover, everything else being equal, both types prefer to send y, since it costs less than x. The following proposition summarizes P's best replies to the optimal choices by DM, as analyzed in Proposition 1.

Proposition **2** (Persuader's optimal behavior)**.**

- $\rho(G) = \rho(B) = y$ *is optimal for P if DM's strategy is such that* $\delta(x) = \delta(y)$*;*
- $\rho(G) = \rho(B) = x$ *is optimal for P if DM's strategy is such that* $\delta(x) = LY$ *and* $\delta(y) = LN$*;*

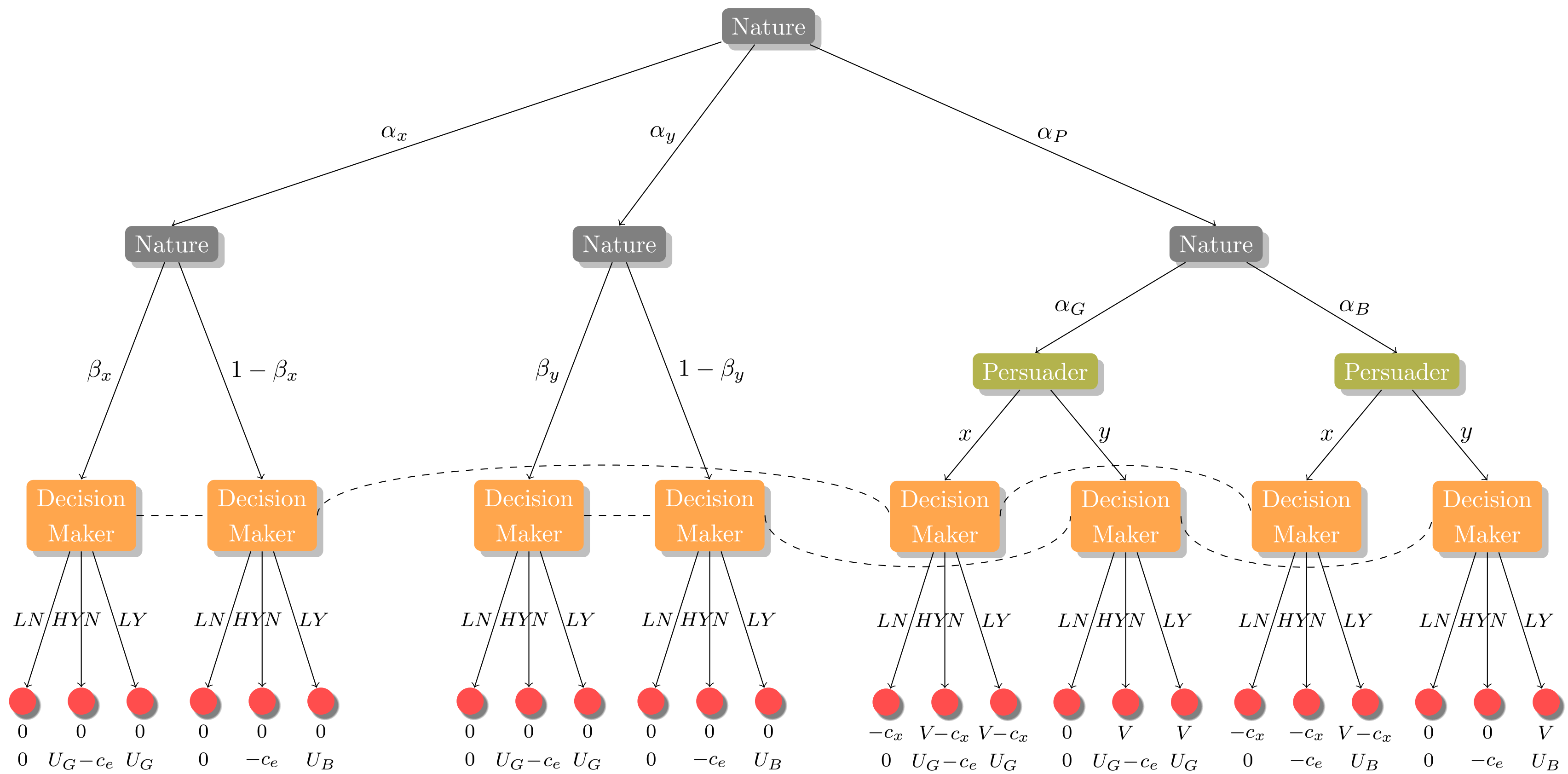


Figure 3: The game tree. In the left and the central branch DM faces an offer that does not come from P. In the left branch the offer is characterized by cue x and has probability β_x to be of quality G. In the central branch the offer is characterized by cue y and has probability β_y to be of quality G. Since in these parts of the game P is not involved, his payoff is set equal to 0. In the right branch DM faces an offer made by P, which has probability α_G to be of quality G. DM has two information sets each of which encompasses nodes from all three branches; one set is associated with reference cue x and the other set with reference cue y.

- *$\rho(G) = x$ and $\rho(B) = y$ is optimal for P if DM's strategy is such that $\delta(x) = HYN$ and $\delta(y) = LN$;*

- *$\rho(G) = y$ and $\rho(B) = x$ is optimal for P if DM's strategy is such that $\delta(x) = LY$ and $\delta(y) = HYN$.*

Proof. Suppose that DM chooses $\delta(x) = \delta(y)$. If $\delta(x) = \delta(y) = LY$ or $\delta(x) = \delta(y) = HYN$ and P is of type G, then P's payoff is $V - c_r$. If instead $\delta(x) = \delta(y) = LN$ or $\delta(x) = \delta(y) = HYN$ and P is of type B, then P's payoff is $-c_r$. Since $0 = c_y < c_x$, $r = y$ is optimal for P independently of his type.

Suppose that DM chooses $\delta(x) = LY$ and $\delta(y) = LN$. P's payoff is equal to $V - c_x > 0$, if $r = x$, and to 0, if $r = y$. Hence, $r = x$ is optimal for P independently of his type.

Suppose that DM chooses $\delta(x) = HYN$ and $\delta(y) = LN$. If P is of type G then his payoff is equal to $V - c_x > 0$, if $r = x$, and to 0, if $r = y$. Hence, $r = x$ is optimal for type G. If P is of type B then his payoff is equal to $-c_x$, if $r = x$, and to 0, if $r = y$. Since $0 < c_x$, $r = y$ is optimal for type B.

Suppose that DM chooses $\delta(x) = LY$ and $\delta(y) = HYN$. If P is of type G then his payoff is equal to $V - c_x$, if $r = x$, and to V, if $r = y$. Since $0 = c_y < c_x$, $r = y$ is optimal for type G. If P is of type B then his payoff is equal to $V - c_x > 0$, if $r = x$, and to 0, if $r = y$. Hence, $r = x$ is optimal for type B. □

Proposition 2 relates the optimal strategy of P to the behaviors that can be optimally adopted by DM, but it is silent on whether DM finds it actually optimal to adopt them. The analysis of mutually optimal decision between P and DM is carried out in the next section. Here we just want to introduce some further notation to take into account that average quality in a category depends on P's choice. We denote with $\hat{\beta}_r(\rho(G), \rho(B))$ the average quality of offers in the category associated with the reference cue r when P's choices, which are described by function ρ, are taken into account. The different values that $\hat{\beta}_r(\rho(G), \rho(B))$ can take as a function of ρ are summarized in the Appendix.

3.4 On some interpretations of the model

The model developed in this section is aimed at improving our undestanding of the phenomenon of persuasion. As such, and despite its simplicity, the model must prove to be flexible enough to fit the variety of persuading activities that, as we have argued in the Introduction, are many and widespread in our economic and social life. In the attempt to convince the reader that this is actually the case, we briefly discuss a wide range of situations that reasonably fit the model.

Milk is usually sold to consumers either in glass bottles or in plastic bottles. Glass is more expensive than plastic, but can be used by the seller to induce a higher expectation. Indeed, if consumers exert low elaboration effort and observe a glass bottle containing milk, then they generically think of products in glass containers, whose average quality can be higher than that of products in plastic containers. If consumers choose to rely on high elaboration, then they have to read and understand data on labels, recover information from memory, and put effort to fully assess the quality of the product.

A seller has to decide where to set up his shop, either in the costly location where most shops sell high quality products or in the cheap location where most shops sell low quality products. Consumers know the average quality of the products sold in each location, but in order to assess the quality of any single product they have to scrutinize it carefully, bearing an elaboration cost.

A loan agent working for a financial institution has to decide whether to lend money to an entrepreneur. The agent may put effort in studying the details of the investment plan for which the entrepreneur is asking money, but this can turn out to be a task requiring a substantial cognitive effort. Alternatively, the agent can look at how the entrepreneur presents himself considering, for instance, his dressing or his club memberships, and then can base her decision on the average quality of investment plans by similar entrepreneurs. Obviously, an elegant dress is more expensive than a casual one and a more exclusive club has higher fees than an unfashionable one.

A teacher, when marking a student's essay, may spend cognitive resources to scrutinize it carefully, or she can simply choose a mark on the basis of the length of the essay, considering that longer essays are usually better than shorter ones. Writing a longer essay is evidently more time demanding for a student.

A candidate who has applied for a job needs to convince the recruiter that she is qualified for that job. Before the interview the candidate can obtain a bunch of certified qualifications, which require time and effort to get. The recruiter can exert high effort and analyze each qualification in detail, trying to understand what is the real productivity of the candidate. Alternatively, the recruiter can rely on low elaboration and just consider that many qualifications typically go with a good candidate, while a bad candidate has only few qualifications.

A policy-maker who wants to carry out a project which is subject to approval by an assembly has to convince the members of the assembly that the benefits of the project are higher than its costs. The policy-maker can modify the project in such a way that it appears to belong to a category of projects that the assembly tends to consider favourably, e.g., improving environmental protection, defending human rights, or enforcing equality of oppotunities. By doing this, however, the policy-maker can incur in a cost due to a constrast with his own preferences or due to opportunity costs in the use of the resources available for the project. The voters in the assembly can exert low elaboration effort and evaluate the project on the basis of their preferences for the relevant category of projects, or otherwise they can exert high effort and analyze the details about costs and benefits.

A fundraiser working for a non-profit organization aims at collecting voluntary contributions from a potential donor to finance a charitable initiative. The fundraiser has a leaflet, full of detailed information about the destination of donations and the trustworthiness of the non-profit organization. He can decide to send the leaflet through the postal service for a very low fare, or to deliver it in person, which is more costly in terms of both time and money. The leaflet contains all the information needed by the donour to assess the merit of the initiative, but the scarce familiarity of the donor with the specific initiative makes it cognitively costly to extract all relevant information. Alternatively, the donor can take a decision on the basis of whether the leaflet was delivered in person or via mail. The average quality of initiatives for which fundraisers have personally talked to the potential donor may well be higher than the quality of initiatives where nobody has shown up.

Let us conclude with a remark on the interpretation of the elaboration cost. The model developed in this paper can be seen as a natural variant of Crawford and Sobel (1982) where talk is not cheap but costly in a twofold sense: the sender has to sustan a cost to send one signal, x, instead of another one, y, and the receiver has to incur a cost in order to learn what action is best for her, Y or N, without relying on the signal. Obviously, the cost that the receiver has to bear can have a non-psychological nature. It might be a search cost or just a time cost, or even a direct monetary cost to get verifiable information. Although the model fits well these cases too, we stress that we want to focus on situations where the cost that the receiver has to bear is psychological. We leave this alternative intepretation, and the related adjustments to the model and its extesions, for future research.

4 Persuasion equilibria

The arguments provided in the previous section almost led us to the identification of equilibria. In the present section we complete the analysis characterizing three types of equilibria and discussing both existence and uniqueness. We restrict our analysis to pure strategies.

Preliminarily, we note that as long as α_x, α_y, β_x, β_y are strictly comprised between 0 and 1, any combination of quality and reference cue occurs with positive probability. This implies that every information set of the game is reached with positive probability under any strategy profile. Therefore, every Nash equilibrium – to which we simply refer as equilibrium – is also sequential, and, hence, weak perfect Bayesian. We also note that the absence of out-of-equilibrium beliefs makes it impossible to refine the set of equilibria by applying criteria that rule out some scarcely plausible out-of-equilibrium beliefs (such as, e.g., the Intuitive Criterion or D1).

4.1 Pooling equilibria: High and low signals

A first type of equilibria is characterized by the persuader P sending a message with a reference cue that is independent of the actual quality of the offer, i.e., $\rho(G) = \rho(B) \in \{x, y\}$.

We say that an equilibrium is pooling with high signal if $\rho(G) = \rho(B) = x$. We note that the rejection of any offer associated with cue y, i.e., $\delta(y) = LN$, and the acceptance of any offer associated with cue x, i.e., $\delta(x) = LY$, is the only behavior by DM that can sustain a pooling equilibrium with high signal. Indeed, in such a case P can have his offer accepted only by using the reference cue x, and this independently of whether he is of type G or of type B. We also note that a pooling equilibrium with high signal can exist for low elaboration costs – so that HYN is convenient for intermediate values of expected quality – and for high elaboration costs – so high that HYN is never convenient. Figure 4 illustrates one such equilibrium when costs of elaboration are low. The following proposition provides the conditions for the existence of a pooling equilibrium with high signal.

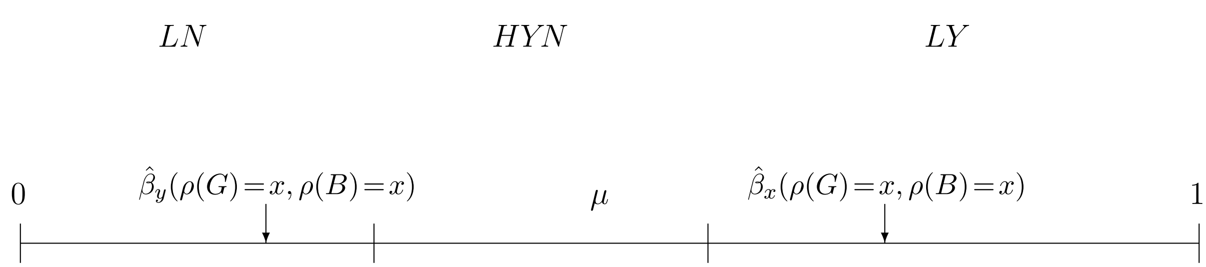


Figure 4: A pooling equilibrium where both G and B send cue x, and DM finds it optimal to reply with LY to cue x and with LN to cue y. Elaborations costs are low enough (i.e., $c_e < \frac{U_G|U_B|}{U_G+|U_B|}$) to make HYN a best reply for intermediate levels of expected quality.

PROPOSITION **3** (Pooling equilibrium with high signal)**.** *The profile* (ρ, δ) *such that* $\rho(G) = \rho(B) = x$*,* $\delta(x) = LY$*,* $\delta(y) = LN$ *is an equilibrium if and only if:*

(3.1) $\hat{\beta}_x(\rho(G), \rho(B)) \geq 1 - \frac{c_e}{|U_B|}$ *and* $\hat{\beta}_x(\rho(G), \rho(B)) \geq \frac{|U_B|}{U_G+|U_B|}$*;*

(3.2) $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{c_e}{U_G}$ *and* $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{|U_B|}{U_G+|U_B|}$*.*

There is no other equilibrium profile (ρ, δ) *such that* $\rho(G) = \rho(B) = x$*, whatever the values of* $\hat{\beta}_x(\rho(G), \rho(B))$ *and* $\hat{\beta}_y(\rho(G), \rho(B))$*.*

Proof. The last claim of the proposition follows directly from Proposition 2: for $r = x$ to be P's optimal choice independently of his type, DM's choice must be such that $\delta(x) = LY$ and $\delta(y) = LN$.

So, let (ρ, δ) be an equilibrium. Note that, along the equilibrium path, $\mu = \hat{\beta}_x(\rho(G), \rho(B))$ if DM sees $r = x$ and $\mu = \hat{\beta}_y(\rho(G), \rho(B))$ if DM sees $r = y$. Hence, from Proposition 1 follows that 3.1 must hold for DM to find $\delta(x) = LY$ optimal and that 3.2 must hold for DM to find $\delta(y) = LN$ optimal.

Suppose now that 3.1 and 3.2 hold. Then, from Proposition 1 follows that $\delta(x) = LY$ and $\delta(y) = LN$ is optimal for DM. Hence, by Proposition 2 we can conclude that the profile (ρ, δ) is an equilibrium. □

We say that an equilibrium is pooling with low signal if $\rho(G) = \rho(B) = y$. The existence of such an equilibrium depends on the convenience for DM to behave in the same way when observing x or y, i.e., $\delta(x) = \delta(y)$. Indeed, when this occurs, P clearly finds it convenient to choose y irrespectively of whether he is of type G or of type B, since DM's behavior is not affected by the choice of the reference cue, and y costs less than x. We note that there are variants of this type of equilibrium, depending on the behavior held by DM. If elaboration costs are so large that HYN is never optimal then there are two cases: either $\delta(x) = \delta(y) = LN$ or $\delta(x) = \delta(y) = LY$. Otherwise, if elaboration costs are not so large, then there is also a third possibility, namely that $\delta(x) = \delta(y) = HYN$. Figure 5 illustrates the case where DM always opts for HYN. The following proposition provides the conditions for the existence of a pooling equilibrium with low signal for each of the three variants.

PROPOSITION **4** (Pooling equilibrium with low signal)**.**

4.1 The profile (ρ, δ) *such that* $\rho(G) = \rho(B) = y$*,* $\delta(x) = \delta(y) = LN$ *is an equilibrium if and only if:*

4.1.1 $\hat{\beta}_x(\rho(G), \rho(B)) \leq \frac{c_e}{U_G}$ *and* $\hat{\beta}_x(\rho(G), \rho(B)) \leq \frac{|U_B|}{U_G+|U_B|}$*;*

4.1.2 $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{c_e}{U_G}$ *and* $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{|U_B|}{U_G+|U_B|}$.

4.2 The profile (ρ, δ) *such that* $\rho(G) = \rho(B) = y$, $\delta(x) = \delta(y) = HYN$ *is an equilibrium if and only if:*

4.2.1 $\hat{\beta}_x(\rho(G), \rho(B)) \geq \frac{c_e}{U_G}$ *and* $\hat{\beta}_x(\rho(G), \rho(B)) \leq 1 - \frac{c_e}{|U_B|}$;

4.2.2 $\hat{\beta}_y(\rho(G), \rho(B)) \geq \frac{c_e}{U_G}$ *and* $\hat{\beta}_y(\rho(G), \rho(B)) \leq 1 - \frac{c_e}{|U_B|}$.

4.3 The profile (ρ, δ) *such that* $\rho(G) = \rho(B) = y$, $\delta(x) = \delta(y) = LY$ *is an equilibrium if and only if:*

4.3.1 $\hat{\beta}_x(\rho(G), \rho(B)) \geq 1 - \frac{c_e}{|U_B|}$ *and* $\hat{\beta}_x(\rho(G), \rho(B)) \geq \frac{|U_B|}{U_G+|U_B|}$;

4.3.2 $\hat{\beta}_y(\rho(G), \rho(B)) \geq 1 - \frac{c_e}{|U_B|}$ *and* $\hat{\beta}_y(\rho(G), \rho(B)) \geq \frac{|U_B|}{U_G+|U_B|}$.

There is no other equilibrium profile (ρ, δ) *such that* $\rho(G) = \rho(B) = y$*, whatever the values of* $\hat{\beta}_x(\rho(G), \rho(B))$ *and* $\hat{\beta}_y(\rho(G), \rho(B))$.

Proof. The last claim of the proposition follows directly from Proposition 2: for $r = y$ to be P's optimal choice independently of his type, DM's choice must be such that $\delta(x) = \delta(y)$.

Suppose (ρ, δ) of 4.1 be an equilibrium. Note that, along the equilibrium path, $\mu = \hat{\beta}_x(\rho(G), \rho(B))$ if DM sees $r = x$ and $\mu = \hat{\beta}_y(\rho(G), \rho(B))$ if DM sees $r = y$. Hence, from Proposition 1 follows that 4.1.1 and 4.1.2 must hold for to find $\delta(x) = \delta(y) = LN$ optimal.

Suppose now that 4.1.1 and 4.1.2. Then, from Proposition 1 follows that $\delta(x) = \delta(y) = LN$ is optimal for DM. Hence, by Proposition 2 we can conclude that the profile (ρ, δ) is an equilibrium.

Claims 4.2 and 4.3 can be proved with analogous arguments. □

4.2 Separating equilibrium: High quality going with high signal

A second group of equilibria is characterized by the type of separation that is typical of signaling models where there are both high quality and low quality senders: persuaders whose offer is of quality G send the reference cue x associated with high quality, while persuaders whose offer is of quality B send the reference cue y associated with low quality. More precisely, we say that an equilibrium is separating with high quality going with high signal – or that it is a separating equilibrium with signaling – if $\rho(G) = x$ and $\rho(B) = y$. We note that this behavior by P can form an equilibrium only if DM chooses $\delta(x) = HYN$ and $\delta(y) = LN$. In such a case, the persuader of type G finds it convenient to incur the cost

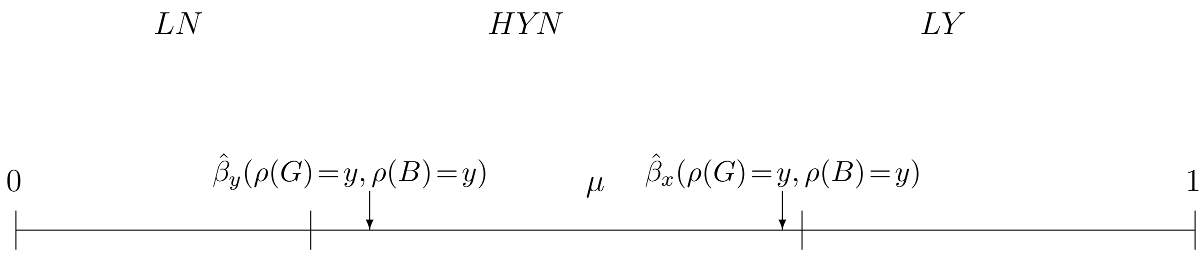


Figure 5: A pooling equilibrium where both G and B send cue y, and DM finds it optimal to reply with HYN to both cue x and y. Elaborations costs are low enough (i.e., $c_e < \frac{U_G|U_B|}{U_G+|U_B|}$) to make HYN a best reply for intermediate levels of expected quality.

of sending x, since this leads his offer to be accepted, while the persuader of type B prefers to save on costs and send reference cue y, since in no case his offer will be accepted. We also note that for this equilibrium to exist elaboration costs must be low enough so that DM actually best replies with HYN for intermediate values of μ. Figure 6 illustrates an example of such a separating equilibrium, while the following proposition provides the conditions for its existence.

PROPOSITION **5** (Separating equilibrium with signaling). *The profile (ρ, δ) such that $\rho(G) = x$, $\rho(B) = y$, $\delta(x) = HYN$, $\delta(y) = LN$ is an equilibrium if and only if:*

5.1 $\hat{\beta}_x(\rho(G), \rho(B)) \geq \frac{c_e}{U_G}$ *and* $\hat{\beta}_x(\rho(G), \rho(B)) \leq 1 - \frac{c_e}{|U_B|}$*;*

5.2 $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{c_e}{U_G}$ *and* $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{|U_B|}{U_G+|U_B|}$*.*

Proof. Let (ρ, δ) be an equilibrium. Note that, along the equilibrium path, $\mu = \hat{\beta}_x(\rho(G), \rho(B))$ if DM sees $r = x$ and $\mu = \hat{\beta}_y(\rho(G), \rho(B))$ if DM sees $r = y$. Hence, from Proposition 1 follows that 5.1 must hold for DM to find $\delta(x) = HYN$ optimal and that 5.2 must hold for DM to find $\delta(y) = LN$ optimal.

Suppose now that 5.1 and 5.2 hold. Then, from Proposition 1 follows that $\delta(x) = HYN$ and $\delta(y) = LN$ is an optimal choice for DM. Hence, by Proposition 2 we can conclude that the profile (ρ, δ) is an equilibrium. $\square$

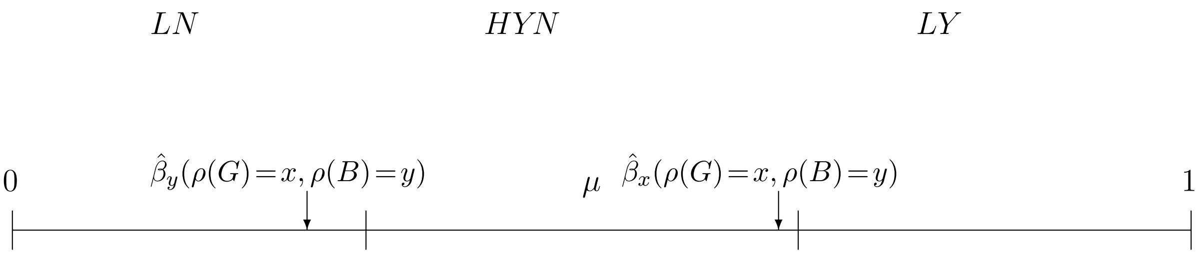


Figure 6: A separating equilibrium where G sends cue x and B sends cue y, and DM finds it optimal to reply with HYN to cue x and with LN to cue y. Elaboration costs are low enough (i.e., $c_e < \frac{U_G|U_B|}{U_G+|U_B|}$) to make HYN a best reply for intermediate levels of expected quality.

4.3 Separating equilibrium: High quality going with low signal

The last group of equilibria is perhaps the most interesting, since it is characterized by a sort of counter-signaling behavior: separation between high quality and low quality senders is attained because the low quality persuader B sends the reference cue x associated with high quality, while the high quality persuader G sends the reference cue y associated with low quality. More precisely, we say that an equilibrium is separating with high quality going with low signal – or that it is a separating equilibrium with counter-signaling – if $\rho(G)\!=\!y$ and $\rho(B)\!=\!x$. We note that this behavior by P can form an equilibrium only if DM chooses $\delta(y) = HYN$ and $\delta(x) = LY$. Indeed, in such a case the persuader of type G finds it convenient to save on costs and send y, since this leads nevertheless his offer to be accepted, while the persuader of type B finds it convenient to pay the cost of sending x, since this is the only way to have his offer accepted. We also note that, as for the other separating equilibrium, in order for this equilibrium to exist elaboration costs must be low enough so that DM actually best replies with HYN for intermediate values of μ. Figure 7 illustrates an example of a counter-signaling equilibrium, while the following proposition provides the conditions for its existence.

Proposition 6 (Separating equilibrium with counter-signaling)**.** *The profile (ρ, δ) such that $\rho(G) = y$, $\rho(B) = x$, $\delta(x) = LY$, $\delta(y) = HYN$ is an equilibrium if and only if:*

6.1 $\hat{\beta}_x(\rho(G), \rho(B)) \geq 1 - \frac{c_e}{|U_B|}$ *and* $\hat{\beta}_x(\rho(G), \rho(B)) \geq \frac{|U_B|}{U_G+|U_B|}$*;*

6.2 $\hat{\beta}_y(\rho(G), \rho(B)) \geq \frac{c_e}{U_G}$ *and* $\hat{\beta}_y(\rho(G), \rho(B)) \leq 1 - \frac{c_e}{|U_B|}$*.*

Proof. Let (ρ, δ) be an equilibrium. Note that, along the equilibrium path, $\mu = \hat{\beta}_x(\rho(G), \rho(B))$ if DM sees $r = x$ and $\mu = \hat{\beta}_y(\rho(G), \rho(B))$ if DM sees $r = y$. Hence, from Proposition 1 follows that 6.1 must hold for DM to find $\delta(x) = LY$ optimal and that 6.2 must hold for DM to find $\delta(y) = HYN$ optimal.

Suppose now that 6.1 and 6.2 hold. Then, from Proposition 1 follows that $\delta(x) = LY$ and $\delta(y) = HYN$ is an optimal choice for DM. Hence, by Proposition 2 we can conclude that the profile (ρ, δ) is an equilibrium. □

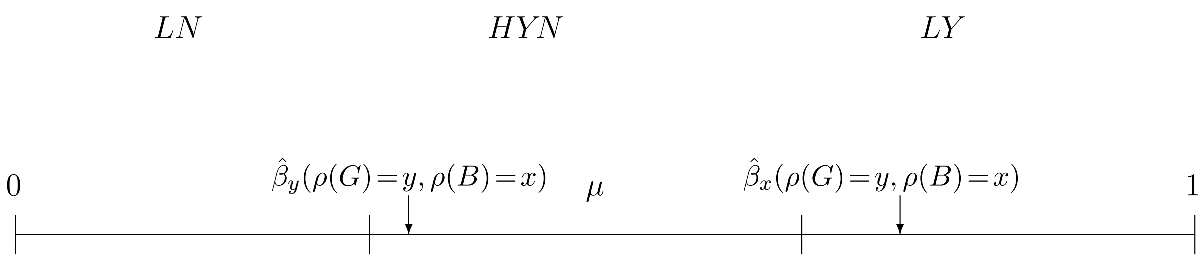


Figure 7: A separating equilibrium where G sends cue y and B sends cue x, and DM finds it optimal to reply with LY to cue x and with HYN to cue y. Elaboration costs are low enough (i.e., $c_e < \frac{U_G|U_B|}{U_G+|U_B|}$) to make HYN a best reply for intermediate levels of expected quality.

4.4 Uniqueness and existence of equilibria

We observe that some of the different types of equilibria described so far can actually coexist, at least for some set of parameter values. An example of such a multiplicity of equilibria is depicted in Figure 8, where the following two equilibria coexist: a pooling equilibrium with high signal and a counter-signaling equilibrium. We note, however, that not all types of equilibria can coexist. The following proposition summarizes:

Proposition **7** (Equilibrium multiplicity and coexistence)**.** *Multiple equilibria can exist, but only in the following pairs:*

- *a pooling where* $\rho(G) = \rho(B) = x$ *and a separating where* $\rho(G) = y$ *and* $\rho(B) = x$*;*
- *a pooling where* $\rho(G) = \rho(B) = y$ *and a separating where* $\rho(G) = x$ *and* $\rho(B) = y$*;*

- *a pooling where* $\rho(G) = \rho(B) = x$ *and a pooling where* $\rho(G) = \rho(B) = y$.

We provide a detailed proof of Proposition 7 in the Appendix, as it does not add much to intuition. Here we just want to spend a few words on why a counter-signaling equilibrium – where high quality goes with low signal – cannot coexist with a signaling equilibrium – where high quality goes with high signal. In a signaling equilibrium type G sends cue x and type B sends cue y, and for B to send cue y DM's belief conditional on cue x must be low enough not to induce DM to play LY – otherwise type B would find it convenient to deviate from sending cue y to sending cue x. On the contrary, in a counter-signaling equilibrium type G sends cue y and type B sends cue x, but for B to send cue x DM's belief conditional on cue x must be high enough to have DM play LY – so that B actually has the offer accepted if he sends cue x. This two conditions are incompatible because having G sending y and B sending x decreases the belief conditional on x with respect to having G sending x and B sending y, so that if beliefs are low enough to sustain a signaling equilibrium then they cannot be high enough to sustain a counter-signaling equilibrium.

Besides potential multiplicity we also observe that there are cases where no equilibrium actually exists. An example of equilibrium inexistence is depicted in Figure 9, where it can be easily checked that for any given behavior by P the best reply by DM is such that at least one type of persuader strictly gains by deviating.

Equilibrium inexistence and equilibrium multiplicity share a common root: the endogeneity of expected quality $\hat{\beta}_x$ and $\hat{\beta}_y$, and in particular their dependence on the behavior of P. This feature crucially hinges on the fact that DM is a fully Bayesian updater and hence takes into account P's behavior when forming her conditional expectations on quality along a path of play. We stress that the degree of endogeneity of $\hat{\beta}_x$ and $\hat{\beta}_y$ is mitigated by coarse thinking, namely by the fact that under low elaboration DM cannot distinguish an offer that comes from P from an offer that does not come from P. In particular, the dependency of DM's expectations on P's behavior decreases in the degree of coarse thinking that, in turn, positively affects the overall likelihood that an offer does not come from P.

To formalize this claim it is useful to introduce the parameter χ which measures the degree of coarse thinking, or, to say that in other words, how coarse is coarse thinking. We assume that χ ranges from 0 to ∞, where χ close to 0 means that low elaboration allows in any case a good level of elaboration, making DM uncertain between the current offer by P and only a few other offers, while χ very high means that thinking is very coarse when DM resorts to low elaboration, and hence the number of offers among which she is unable to distinguish is very large. More precisely, if we denote with N_x, N_y, and N_P the absolute number of offers that, respectively, fall into category x and are not made by P, fall into category y and are not made by P, and are made by P, we have that N_x is non-decreasing in χ and $N_x \to \infty$ when $\chi \to \infty$, and an analogous assumption is made for N_y, while we can reasonably assume that N_P does not depend on χ. We also define $\alpha_x = N_x/(N_x+N_y+N_P)$, $\alpha_y = N_y/(N_x + N_y + N_P)$ and $\alpha_P = N_P/(N_x + N_y + N_P)$. We then have that α_P is non-increasing in χ, $\alpha_P \to 0$ when $\chi \to \infty$, and both α_x and α_y are non-decreasing in χ and $\alpha_x + \alpha_y \to 1$ when $\chi \to \infty$.

Because more severe coarse thinking reduces the degree of endogeneity of $\hat{\beta}_x$ and $\hat{\beta}_y$, one can expect that for a large enough χ both equilibrium inexistence and equilibrium multiplicity are no longer an issue. The following proposition states that this is indeed the case:

PROPOSITION **8** (Equilibrium existence and uniqueness)**.** *If coarse thinking is strong enough, then almost always an equilibrium exists and is unique.*

Proof. We observe that $\hat{\beta}_x(\rho(G), \rho(B))$ converges to β_x and $\hat{\beta}_y(\rho(G), \rho(B))$ converges to β_y irrespective of ρ when the degree of coarse thinking grows larger and larger, i.e., χ tends to infinity. This is evident when looking at (1), (2), (3), (4), (5), (6), (7) and (8) in the Appendix.

We assume that β_x and β_y are interior points to the intervals of beliefs that determine DM's best choices according to Proposition 1. More precisely, β_x and β_y are both different from c_e/U_G and $1 - c_e/|U_B|$ if $c_e < U_G|U_B|/(U_G + |U_B|)$, and different from $|U_B|/(U_G + |U_B|)$ if $c_e \geq U_G|U_B|/(U_G + |U_B|)$. By so doing we are neglecting values that have measure zero in the parameters space.

We now build a profile that we then check to be an equilibrium. We set $\delta(x)$ and $\delta(y)$ equal to the best action by DM against a belief equal to β_x and β_y, respectively, as show by Proposition 1. We set $\rho(G)$ and $\rho(B)$ equal to the best action by P conditional on G and B, respectively, against $\delta(x)$ and $\delta(y)$, as shown by Proposition 2.

By construction, in the above profile P is best replying to δ, while DM is best replying given β_x and β_y, which are not equilibrium beliefs. However, we can choose χ high enough that, whatever ρ is chosen by P, $\hat{\beta}_x(\rho(G), \rho(B))$ and $\hat{\beta}_y(\rho(G), \rho(B))$ are very close to β_x and β_y, respectively. This means that DM is best replying even against $\hat{\beta}_x(\rho(G), \rho(B))$ and $\hat{\beta}_y(\rho(G), \rho(B))$, since β_x and β_y are interior points to the intervals of beliefs that determine DM's best choices.

We have just proved equilibrium existence. To understand that such equilibrium is unique, we simply observe that, when χ is large enough, the best reply by DM is uniquely determined whatever strategy is chosen by P, and P's best reply against such an optimal behavior by DM is uniquely determined as well. □

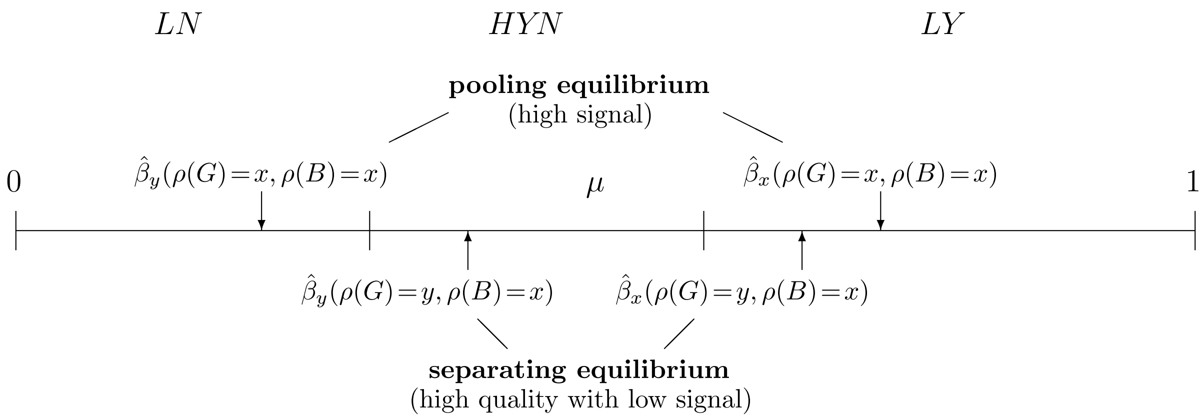


Figure 8: An example of the coexistence of two equilibria, for $c_e < \frac{U_G|U_B|}{U_G+|U_B|}$. One is a pooling equilibrium with high signal and the other is a separating equilibrium where high quality goes with low signal.

5 Consistency with psychological findings

The way in which we have modeled reference cues and costly elaboration turns out to be consistent with a number findings of dual process models of persuasion – and, in particular, of the ELM and the HSM. For clarity we organize them in remarks.

Remark **1.** *The persuadee interprets the persuasive message differently under high elaboration and under low elaboration (Petty and Cacioppo, 1986a; Eagly and Chaiken, 1993).*

This fundamental idea of dual process theories is directly embedded in the way we model how DM can extract information from the message (q, r): under L only r is observed, while under H also q is.

Remark **2.** *The recourse to high elaboration is more likely if persuadee's motivation is higher (Petty and Cacioppo, 1979, 1981; Cacioppo et al., 1983; Petty et al., 1981).*

To formally capture this fact, we may let U_G and U_B be multiplied by a parameter γ, with γ measuring motivation. We observe that, if $c_e > \gamma U_G U_B/(U_G + |U_B|)$, then an increase in γ may revert the inequality so that HYN passes from being never best reply to being so for some intermediate range of conditional beliefs on quality. Further, if $c_e < \gamma U_G|U_B|/(U_G + |U_B|)$, then an increase in γ enlarges the set of beliefs against which HYN is best choice.

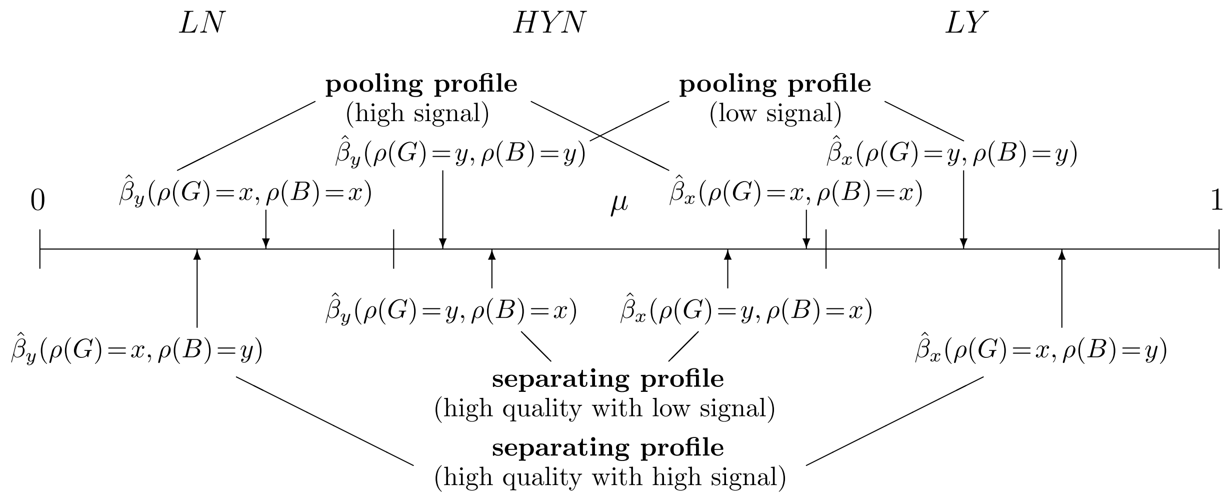


Figure 9: An example where no equilibrium exists, for $c_e < \frac{U_G|U_B|}{U_G+|U_B|}$. In both the separating profile where high quality goes with low signal and the pooling profile with high signal, the persuader of type B would prefer to send cue y and save on costs, as DM best replies with HYN to cue x. In both the pooling profile with low signal and the separating profile with high quality going with high signal, the persuader of type B has his offer rejected so that he would prefer to send cue x (so to have his offer accepted) since DM best replies with LY to cue x.

Remark **3.** *A greater ability to think and focus on the content of the message makes it more likely that the persuadee recurs to high elaboration (Petty et al., 1976; Petty and Cacioppo, 1986b).*

An increase in DM's cognitive skills can be translated into our model as a decrease of the elaboration cost required to find out the quality. We simply observe that a decrease of c_e has the same consequences as an increase of γ as discussed above.

Remark **4.** *The persuader can use cues to affect the persuadee's choice of the elaboration level and, through it, the persuadee's attitudes (Petty and Cacioppo, 1986b; Chaiken and Trope, 1999).*

To see why it is so in our model, note that if DM's optimal behavior given the belief $\hat{\beta}_x(\rho(G),\rho(B))$ differs from DM's optimal behavior given the belief $\hat{\beta}_y(\rho(G),\rho(B))$, then P can affect DM's decisions by choosing between x and y.

Remark **5.** *Persuasion under high elaboration is stabler than persuasion under low elaboration (Petty and Cacioppo, 1986b; Haugtvedt and Petty, 1992).*

To model this consider the following variant of our setup. Subsequently to the choice of the elaboration level, but prior to the choice to accept or reject the offer, DM receives a private signal with probability α_s that reveals q. Hence, now LY leads to accept only if no signal is received or the signal reveals G, LN leads to reject the offer only if no signal is received or the signal reveals B, while HYN leads to the same behaviors as before – i.e., it is independent of the signal. In particular, we can see that under HYN, the initial decision to accept or reject is maintained with probability 1. Instead, under LY, an initial decision to accept is maintained with probability $1 - \alpha_s + \alpha_s \beta < 1$, while under LN an initial decision to reject is maintained with probability $1 - \alpha_s + \alpha_s(1 - \beta) < 1$. We note that, incidentally, α_s acts against HYN, similarly to an increase of c_e.

Remark **6.** *Prejudice can affect the elaboration level and induce biased elaboration (Petty and Cacioppo, 1986b; Petty et al., 1999).*

Prejudice can be translated in our model by allowing DM to have biased priors about α_x, α_y, α_G, β_x and β_y, which lead to an expected quality $\tilde{\beta}_x$ and $\tilde{\beta}_y$ which are biased with respect $\hat{\beta}_x$ and $\hat{\beta}_y$. The consequence that the choice between H and L is affected by prejudice is immediate: both $\tilde{\beta}_x(\rho(G), \rho(B))$ and $\tilde{\beta}_y(\rho(G), \rho(B))$ directly depend on the (biased) values of α_x, α_y, α_G, β_x and β_y. For instance, if DM has a prejudice against x – i.e., $\tilde{\beta}_x(\rho(G), \rho(B))$ $< \hat{\beta}_x(\rho(G), \rho(B))$ – then she could choose HYN in place of LY or LN in place of HYN. Instead, to create the room for biased elaboration we have to modify the model further in order to have $\hat{\beta}_x$ and $\hat{\beta}_y$ playing a role also under high elaboration. This can be obtained by assuming that H does not allow to observe q directly, but to receive a signal on q which is truthful with probability greater than one-half and smaller than one. So $\tilde{\beta}_x$ and $\tilde{\beta}_y$, and hence prejudice, can affect the updated beliefs about q even under H.

Remark **7.** *The persuader can send arousing and other mood-affecting cues to induce low elaboration (Petty and Cacioppo, 1986b; Sanbonmatsu and Kardes, 1988).*

We can easily introduce in our model the possibility for P to induce arousal (or other mood-affecting factors) in DM, with the aim of increasing the likelihood that DM relies on low elaboration. Let us add, beside x and y, a further characteristic of the offer: a mood cue that we indicate with $m \in \{a, n\}$, where $m = a$ means that the cue induces arousal and $m = n$ that it does not. The categories of offers are hence four: (x, a), (y, a), (x, n), and (y, n). If the offer comes from Nature, then each category has a positive probability to contain the offer. If instead the offer comes from P, then he has to choose not only the reference cue between x and y, but also the mood cue between a and n, where choosing a costs $c_a > 0$ and

n costs nothing. Under L, DM observes both r and m, and if $m = a$ the cost of undertaking H increases from c_e to $c_e^a > c_e$. As a result we can have the persuader of type B that can use the mood cue together with a reference cue to induce in DM low elaboration and, thanks to this, acceptance of his offer. Figure 10 illustrates a case where the mood cue does not convey information on average quality – i.e., categories (x, a) and (x, n) have the same average quality, and similarly for categories (y, a) and (y, n) – but nevertheless the persuader of type B would like to send $m = a$. Indeed, B has his offer rejected if he pools with type G by sending (y, n) while, thanks to arousal, if B sends (x, a) then DM chooses LY and, hence, accepts B's offer. We note that persuader of type G has two reasons not to send $m = a$: first, it would cost c_a and, second, it would lead DM to choose LN and to reject G's offer.

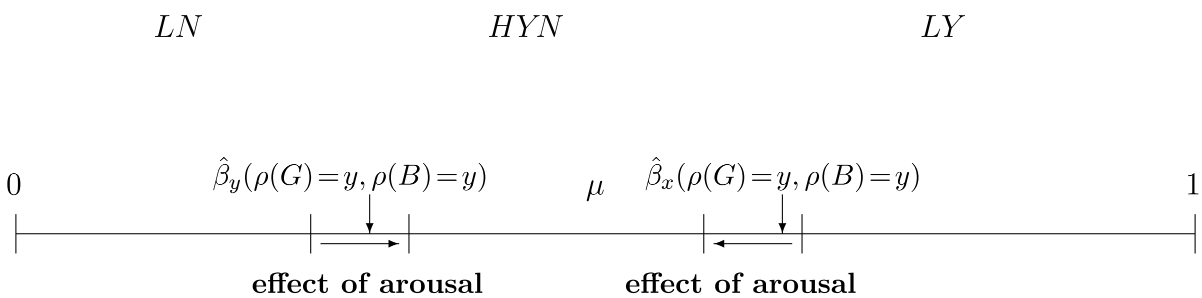


Figure 10: A pooling profile where both G and B send reference cue y and mood cue n, and where DM finds it optimal to reply with HYN to both cue x and y. It is assumed that the average quality of Nature's offers in category (y, a) is the same of category (y, n) – and similarly for categories (x, a) and (x, n) – and that coarse thinking is strong enough to have that P's behavior has a negligible impact on average qualities. So, expected quality $\hat{\beta}_x$ and $\hat{\beta}_y$ depend only on the reference cue, not on the mood cue. Without the possibility to send a mood cue, this profile would be a pooling equilibrium with low signal. However, if arousal can be induced then this is not an equilibrium anymore. Indeed, under mood cue a the range of beliefs for which HYN is a best reply shrinks, leading to a different outcome: if P sends x then DM reacts with LY, while if P sends y then DM reacts with LN. Hence, B would deviate sending (x, a).

6 Extensions

In this section we explore a number of extensions of our model with the aim of checking the robustness of our findings. Basically, we study if and how the characteristics of equilibria might change in a less stylized setup. We stress that the details of the model remain unchanged where not specified otherwise.

6.1 Many offer qualities

Consider the case where the quality of offers is not limited to two levels, G and B, but can take many possible values. Let us index qualities on the interval of the real line $[\underline{q}, \overline{q}]$, where $\underline{q} > 0$ is minimum quality and $\overline{q}$ is maximum quality. Nature determines the quality of the offer according to the cumulative distribution F^N in case N is chosen and according to the cumulative distribution F^P in case P is chosen. The values of βs are modified accordingly.

DM's utility is given by $U(q)$, which is strictly increasing in q and takes both positive and negative values over $[\underline{q}, \overline{q}]$. In particular, there exists $\tilde{q}$ such that $U(\tilde{q}) = 0$. DM would like to accept any offer of quality $q \geq \tilde{q}$ and reject any offer of quality $q \leq \tilde{q}$. Hence, optimal choice by DM is still described by Proposition 1, where U_G and U_B are replaced by, respectively, $\widetilde{U}_G = \int_{\tilde{q}}^{\overline{q}} U(q)\mathrm{d}F(q)$ and $\widetilde{U}_B = \int_{\underline{q}}^{\tilde{q}} U(q)\mathrm{d}F(q)$, with $F = \alpha_P F^P + (1 - \alpha_P)F^N$.

Further, persuaders of types $q \geq \tilde{q}$ find it optimal to behave like G in the model of Section 3, while persuaders of types $q \leq \tilde{q}$ find it optimal to behave like B. So, Proposition 2 still describes the optimal choice by P conditionally on the potentially optimal behavior by DM, where however $\rho(G)$ and $\rho(B)$ are interpreted as referring to, respectively, types in $[\tilde{q}, \overline{q}]$ and types in $[\underline{q}, \tilde{q}]$, and where again U_G and U_B are replaced by $\widetilde{U}_G$ and $\widetilde{U}_B$.

As a consequence, the substance of the findings reported in Proposition 3 through 8 remains true when we allow for many different qualities of the offer.

6.2 Many offer categories and reference cues

Consider the case where the categories of objects known by DM, and to which the offer might be referred to, are not just x and y, but possibly a large number. Let Z be the set of natural numbers $\{1, 2, \ldots, n\}$ indexing the different offer categories, with $n \geq 2$ and $z \in Z$ denoting the generic reference cue. Suppose also that both β_z, which denotes the average quality for category z, and c_z, which denotes the cost of sending a cue referring to category z, are strictly increasing in z. Finally, to rule out uninteresting cases let also $c_z < V$ for all $z \in Z$.

We note that the optimal choice by DM is still described by Proposition 1. However, to describe the optimal choice by P conditionally on the potentially optimal behavior by DM one needs to generalize Proposition 2 to the case of many offer categories. We do not enter the details of this, since the forces driving the choice by P remain the same. Indeed, P will choose a reference cue that leads his offer to be accepted, if such a cue exists. We note that, as in the basic setup, type G of P has more chances to attain this objective, since for him it is enough to choose a cue that induces HYN as best repy by DM, while type B of P needs a cue that leads to immediate acceptance – i.e., that leads DM to choose LY. Moreover, among cues that lead to the same best reply by DM, P will choose the one with the minimum index, since that cue is the least costly. The difference with respect to the setup of Section 3 is the larger number of choices that are available to P, and consequently the larger number of strategies for DM, since a strategy for her is now $\delta : Z \to \{LN, HYN, LY\}$. A proposition describing the best reply behavior by P should consider all possible behaviors by DM, and hence it would consist of many cases, whose explicit listing would add little to intuition.

From the detailed description of the potentially optimal choices by DM and P – i.e., the counterparts of Propositions 1 and 2 in this setup – one can obtain results that are in line with Proposition 3 through 6. To see that pooling and separating equilibria with both signaling and counter-signaling can emerge with many categories as well, it is enough to think of the equilibria described for the model of Section 3 and add some further categories whose average quality induces the same best reply by DM as done by x or y, but with the associated reference cues being more costly, so that the additional categories are never chosen by P. Moreover, it is easy to understand that as coarse thinking becomes stronger and stronger, existence and uniqueness of the equilibrium are almost always ensured, similarly to what happens in with Proposition 8.

Even if all types of equilibria remain possible in the presence of more than two categories, we remark that counter-signaling becomes the more likely outcome as the number of categories increases and average qualities are more spread all over $[0, 1]$. To show this formally, let us introduce a measure of how categories are densely distributed in terms of their average qualities. More precisely, given $0 \leq \beta_1 < \beta_2 < \ldots < \beta_n \leq 1$, we define ξ to be equal to the largest difference of two consecutive numbers in the above sequence. In other words, ξ is the minimum length that an interval of average qualities has to have to be sure that it contains at least the average quality of one category in Z. So, the lower ξ is, the more densely distributed average qualities are.

We are now ready to state the following result:

Proposition **9** (Counter-signaling with many offer categories)**.** *Suppose that $c_e < U_G|U_B|/(U_G + |U_B|)$ and $\xi < \min\{1 - c_e/|U_B| - c_e/U_G, c_e/|U_B|\}$. If coarse thinking is strong enough, then almost always there exists a profile (δ, ρ) that is the unique equilibrium and such that $\hat{\beta}_{\rho(G)} < \hat{\beta}_{\rho(B)}$, $\delta(\rho(G)) = HYN$, and $\delta(\rho(B)) = LY$.*

Proof. Since $c_e < U_G|U_B|/(U_G + |U_B|)$, then there exists an interval of beliefs against which

HYN is the best reply by DM. Moreover, since $\xi < \min\{1 - c_e/|U_B| - c_e/U_G, c_e/|U_B|\}$, we are sure that there exist at least one category whose average quality induces HYN as best reply, and at least one category whose average quality induces LY as best reply. We denote with z_{min}^{HYN} the category with the minimum index among those which are best replied with HYN, and we define z_{min}^{LY} analogously. We suppose that z_{min}^{HYN} and z_{min}^{HYN} are interior points in the intervals of beliefs that are best replied, respectively, with HYN and LY. By so doing we are neglecting values that have measure zero in the parameters space.

We now build a profile that we then check to be an equilibrium. For every $z \in Z$, we set $\rho(z)$ as the best action by DM against β_z, as shown by Proposition 1. Then we set $\delta(G) = z_{min}^{HYN}$ and $\delta(B) = z_{min}^{LY}$. We note that such δ selects for both G and B the least costly cue that allows them to have their offer accepted by DM. This shows the optimality of P's strategy against ρ. To understand that ρ is optimal against δ, it is enough to observe that, when coarse thinking is strong enough, $\hat{\beta}_{z_{min}^{HYN}}$ and $\hat{\beta}_{z_{min}^{LY}}$ are arbitrarily close to, respectively, $\beta_{z_{min}^{HYN}}$ and $\beta_{z_{min}^{LY}}$, and hence induce the same best action by DM, since $\beta_{z_{min}^{HYN}}$ and $\beta_{z_{min}^{LY}}$ are interior points in the intervals of best reply behavior by DM.

Finally, uniqueness follows exactly for the same argument shown in Proposition 8, which hence we do not repeat. $\square$

6.3 A continuum of elaboration intensities: Quality signal

Consider the case where DM has not to choose between high and low elaboration, but – as suggested by Petty and Cacioppo (1986b) themselves – has to select an elaboration intensity out of an elaboration continuum. The basic idea is that the likelihood of extracting q from the message sent by P increases in the elaboration intensity. To model this let us assume that DM never observes the part q of the message sent by P, but she can extract from it a signal σ which conveys information on q, and that the precision of σ depends on the elaboration intensity $e \geq 0$ that DM chooses. More precisely, $p(e)$ is the probability that the signal is correct – i.e., it says G when the quality is G and B when the quality is B – and $1 - p(e)$ is the probability that the signal is wrong – i.e., it says B when the quality is G and G when the quality is B. Further, we model the fact that the extraction of correct information about q is more likely under greater elaboration intensity by letting $p'(e) > 0$, and the fact that greater elaboration has increasing marginal costs by letting $p''(e) < 0$.

Clearly, choosing a positive level of elaboration intensity makes sense only if the acquired information is then used, i.e., only if also HYN is chosen. The expected utility of choosing HYN with elaboration intensity e^* is $\mu p(e^*)U_G - (1 - \mu)(1 - p(e^*))|U_B| - e^*$, while the expected utilities of choosing LY and LN with null elaboration intensity are as in the model of Section 3, i.e., $\mu U_G - (1-\mu)|U_B|$ and 0, respectively. Solving the inequalities among these expected utilities, we find that LN with $e^* = 0$ is best reply for DM when $\mu \leq (e^* + (1-p(e^*))|U_B|)/(p(e^*)U_G + (1-p(e^*))|U_B|)$ and $\mu U_G - (1-\mu)|U_B| \leq 0$, while LY with $e^* = 0$ is best reply when $\mu \geq (e^* + (1-p(e^*))|U_B|)/(p(e^*)U_G + (1-p(e^*))|U_B|)$ and $\mu U_G - (1-\mu)|U_B| \geq 0$.

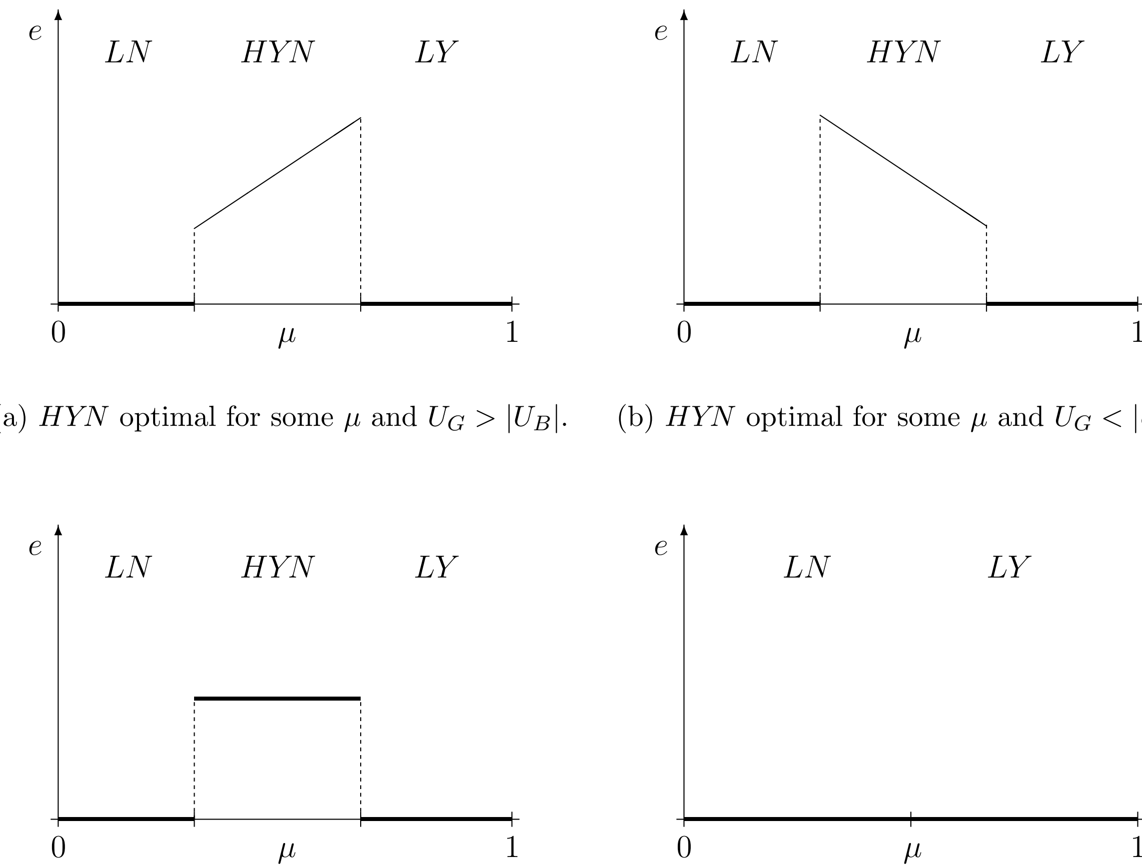
Figure 11 summarizes DM's optimal behavior as a function of μ, providing a counterpart of Proposition 1 for the current setup. We note that while the quality of DM's optimal behavior remains substantially unchanged with respect to the model of Section 3, the relaxation of the hypothesis of just two elaboration levels gives an extra role to U_G and U_B: under HYN the optimal elaboration intensity increases, decreases, or is constant in expected quality depending on whether $U_G > |U_B|$, $U_G < |U_B|$, or $U_G = |U_B|$. This is because the nature of the stake – i.e., whether it is most important to get the good offer or to avoid the bad one – determines whether direct information on quality and expected quality are complements or substitutes.

We also note that the optimal choice by P is still described by Proposition 2, meaning that the actual level of e^* under HYN is irrelevant to P. By the same token, Proposition 7 and Proposition 8 remain true. Further, the characterization of equilibria made by Proposition 3 through 6 still holds in qualitative terms, but the statements have to be adjusted by replacing conditions on c_e with the appropriate conditions from those described above.

Finally, we observe that the current extension could be slightly changed by assuming that with probability $p(e)$ the signal σ is correct, as before, but, differently from before, with probability $1 - p(e)$ the signal does not arrive at all. This represents the case where failure to extract from signal σ reliable information on q does not result in extracting wrong information, but just no information. While in this alternative setup the optimal choice of elaboration by DM would be partly different under HYN – potentially u-shaped – the substance of what discussed in this subsection remains true.

6.4 Cues with fully endogenous quality

Proposition 6 tells us that a counter-signaling equilibrium can arise in our model, and Proposition 9 states that this type of equilibrium is the only type that is possible when there are many categories and average qualities are sufficiently densely distributed. The combination of these results suggests that counter-signaling may be a quite relevant case. However, the assumed partial exogeneity of expected average quality that is associated with each reference cue can raise doubts about the relevance of counter-signaling. Indeed, in the model of Section 3 (and similarly in the model of Subsection 6.2) the average quality of offers not coming from



(a) HYN optimal for some μ and $U_G > |U_B|$. (b) HYN optimal for some μ and $U_G < |U_B|$.

(c) HYN optimal for some μ and $U_G = |U_B|$. (d) HYN never optimal.

Figure 11: In subfigures (a), (b) and (c) there exists an interval of beliefs against which HYN is best reply, which is the case for $p(e^*) > 1/2 + \frac{e^*(U_G+|U_B|)}{2U_G|U_B|}$. Moreover, LN is best reply for beliefs lower than $\frac{e^*+(1-p(e))|U_B|}{p(e)U_G+(1-p(e))|U_B|}$, while LY is best reply for beliefs higher than $\frac{p(e)|U_B|-e^*}{(1-p(e))U_G+p(e)|U_B|}$, and HYN is best reply for intermediate beliefs. In subfigure (d), instead, $p(e^*) < 1/2 + \frac{e^*(U_G+|U_B|)}{2U_G|U_B|}$ and HYN is never best reply, while LN and LY are best reply for beliefs, respectively, lower than and higher than $\frac{|U_B|}{U_G+|U_B|}$.

P, i.e., β_x and β_y, is exogenous and, in particular, one category of offers is of better average quality than the other by assumption, i.e., $\beta_x > \beta_y$. We observe that in a counter-signaling equilibrium the average quality remains higher in category x than in category y, despite the fact that when P is called to play he chooses cue y if his type is G and cue x if his type is

B. In order for this to occur, the decisions influencing the quality of offers in category x and y that are not made by P should be sustained by sound explanations. In the following we explore one particular explanation: for a subset of offers in each category, cues x and y denote characteristics which provide an intrinsic utility to DM, so that for such offers the choice between x and y directly affects the quality of the offer.

Consider the model presented in Section 3, but modified as depicted in Figure 12. Initially, either an offerer O or a persuader P are randomly selected with probability α_O and $\alpha_P = 1 - \alpha_O$, respectively. If P is chosen then the game unfolds as in the original model. Instead, if O is chosen then the game unfolds as follows. Firstly, a type for O is drawn, either the type A – who is endowed with advanced technology – or the type S – who is endowed with standard technology; types are selected with probability α_A and α_S, respectively. Secondly, O chooses between x and y. Lastly, DM observes the reference cue without knowing whether the offer comes from O or from P, and has to decide on both elaboration level and reaction.

The two types of player O are characterized by different technologies that induce different costs to choose x. In particular, the advanced technology allows type A to incur a lower cost than type S to employ x, i.e., $c_x^A < c_x^S$. We also assume that $V - c_x^A > 0$ and $V - c_x^S < 0$, meaning that type A finds it convenient to employ x, while type S prefers y.[7] Moreover, the offer made by O is such that the cue has an intrinsic value for DM, meaning that x qualifies the offer as good, while y qualifies the offer as bad. In other words, the offer is of good quality if x is chosen, while it is of bad quality if y is chosen. Finally, a strategy by O – which we denote by ω – is a choice between x and y as a function of the type, i.e., $\omega : \{A, S\} \to \{x, y\}$.

The following proposition shows that an equilibrium can arise where P counter-signals. We note that such an equilibrium is crucially sustained by the fact that average quality in category x is higher than average quality in category y, which in turn is possible thanks to the existence of a subset of offers whose quality is directly affected by x and y.

Proposition **10** (Counter-signaling with endogenous quality of cues)**.** *There exist values for c_e, α_0 and α_A such that the following profile (ω, ρ, δ) is equilibrium: $\omega(A) = x$, $\omega(S) = y$, $\rho(G) = y$, $\rho(B) = x$, $\delta(x) = LY$, $\delta(y) = HYN$.*

Proof. By Proposition 1 we know that if $c_e \leq \frac{U_G|U_B|}{U_G+|U_B|}$ then there exists some value for the

[7] We remark that the null payoff that type S obtains by choosing Y if DM replies with N should not be interpreted as "getting nothing", since payoff levels are normalized (see Subsection 3.4) and, hence, such an occurrence may represent the fact that DM buys at a lower price or a smaller quantity.

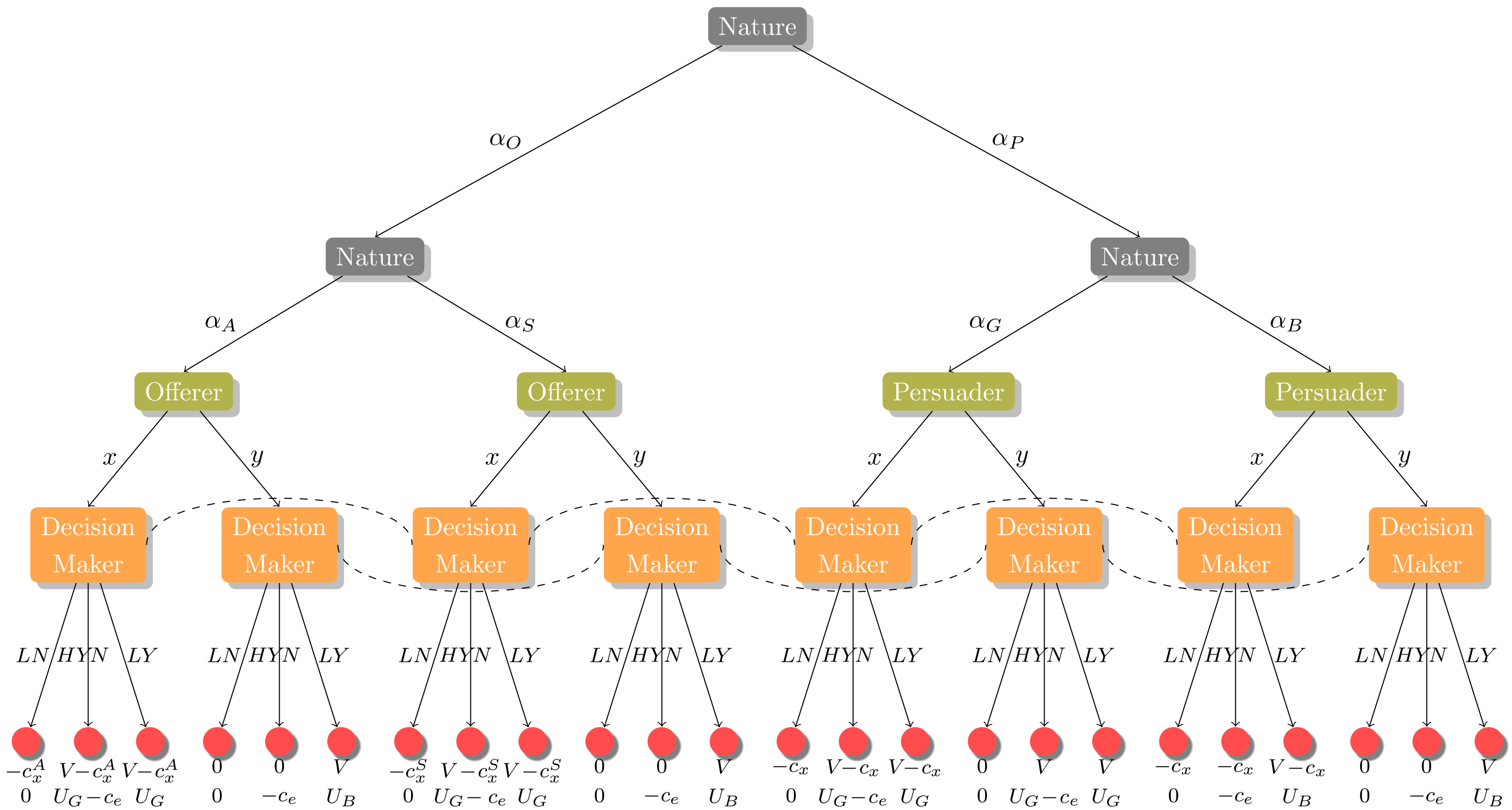


Figure 12: The game tree with endogenous quality of categories. In the left branch DM faces an offer that comes from O, which has probability α_A to be from type A (the offerer endowed with advanced technology). In the right branch DM faces an offer made by P, which has probability α_G to be of quality G. DM has two information sets each of which encompasses nodes from both branches; one set is associated with reference cue x and the other set with reference cue y.

belief on quality such that HYN is best reply for DM, and clearly for a belief on quality that is high enough LY is best reply.

We observe that $\hat{\beta}_x = \frac{\alpha_O \alpha_A}{\alpha_O \alpha_A + \alpha_P \alpha_B}$ and $\hat{\beta}_y = \frac{\alpha_P \alpha_G}{\alpha_P \alpha_G + \alpha_O \alpha_S}$. By inspection of these expressions – and remembering that $\alpha_O + \alpha_P = 1$ – we see that $\hat{\beta}_x$ approaches 1 if we choose α_O high enough. Given all other parameters, we pick a value of α_O such that LY is best reply against the corresponding value of $\hat{\beta}_x$. Now we turn our attention to $\hat{\beta}_y$. If $\hat{\beta}_y$ is such that the best reply by DM is LY, we can raise α_O until $\hat{\beta}_y$ decreases enough to take a value such that DM finds it optimal to reply with HYN. If, instead, $\hat{\beta}_y$ is such that the best reply by DM is LN – remembering that $\alpha_A + \alpha_S = 1$ – we can raise α_A until $\hat{\beta}_y$ increases enough to take a value such that DM finds it optimal to reply with HYN. In both cases, $\hat{\beta}_x$ gets higher, and hence LY remains best reply when x is observed.

Therefore, by choosing c_e low enough and by an appropriate choice of α_O and α_A we can obtain that DM has no incentive to deviate from the considered strategy profile. The optimality checks for O and P are straightforward, and hence omitted. □

7 Conclusions

In this paper we have proposed a model of persuasion that incorporates the insights of social and cognitive psychology on dual process reasoning. We have framed persuasion activities within a sender-receiver model where agents are both rational and Bayesian, but the persuadee has to pay a cognitive cost to extract all information from the message sent by the persuader. The main novelty of our approach is the combination of two distinct features borrowed from the psychological literature: the explicit consideration of elaboration costs to extract all information from a message – i.e., high elaboration is more informative, but also more costly in terms of cognitive resources, than low elaboration – and the modeling of low elaboration as coarse thinking – i.e., if the persuadee does not pay the cognitive cost to scrutinize carefully the offer then she is unable to distinguish among offers belonging to the same category. This setup allows us to endow the persuader with a novel strategic tool of persuasion – i.e., cues – that we model as references to categories of objects. We then proceed by studying the strategic use of reference cues by the persuader aimed at manipulating the beliefs of the persuadee.

The proposed model, despite its simplicity, has proved to be rich enough to provide predictions that are in line with well documented findings in the social psychology of persuasion. Moreover, it provides a novel reason for separating equilibria to arise even in the absence of the single-crossing property and an entirely new rationale for counter-signaling, both based on the persuader anticipating the costs and benefits of costly elaboration faced by the persuadee.

The next steps in this line of research would be, we think, to develop less stylized versions of the model to be applied to specific cases of interest as well as exploring dimensions overlooked in the present paper. Among the applications one might want to focus on there is, for instance, political campaigning (where politicians exploit slogans and ideological cues to gain the votes of electors), marketing through product packaging (where firms exploit packaging as reference cues to manipulate consumers' belief on quality), the engineering of financial products (where sellers exploit product complexity to induce low elaboration by potential purchasers), or fundraising activities (where fundraisers exploits mood cues to induce donations). One dimension left unexplored, which however is likely to play an important role in real persuasive activities, is competition among persuaders. This would go along the lines explored by Gentzkow and Kamenica (2011) for the case of purely Bayesian persuasion and by Bordalo et al. (2013a) for the case of consumers attention.

Let us conclude with a general remark. As a matter of fact, human cognitive resources are scarce and, as such, their allocation to alternative activities entails (at least opportunity) costs. So far and to our knowledge, only a few papers have explored the possibility of incorporating the cost of reasoning into models where agents have to take decisions based on the elaboration of information. As we noted in the Introduction, one important paper in this regard is Dewatripont and Tirole (2005). Another interesting attempt is the model of decision-making proposed by Dickhaut et al. (2009), where the informativeness of a signal (about the payoff associated with different options) increases in the effort that the decision-maker puts in the observation of the signal. Brocas and Carrillo (2012b) apply a similar approach to the modeling of how the human brain governs both memorization and retrieval of information from memory. The present paper adds to this list a further "piece of evidence" that introducing elaboration costs in strategic models of economic behavior is conducive of novel and interesting results. More specifically, it shows that a principal who is interested in motivating an agent to behave according to the principal's will can exploit tools that are not available in standard models with unbounded cognitive resources. We believe that our investigation on the relationship between cues and elaboration efforts, and how this can be exploited by a principal to affect the agent's behavior, can also be understood as a contribution to the development of a behavioral incentive theory, along the lines of Benabou and Tirole (2003) and Bénabou and Tirole (2006).

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A Appendix

A.1 Beliefs on quality conditional on cue and P's behavior

$$\begin{aligned}
\hat{\beta}_x(\rho(G)=y, \rho(B)=y) &= \beta_x && (1)\\
\hat{\beta}_x(\rho(G)=y, \rho(B)=x) &= \frac{\alpha_x \beta_x}{\alpha_x + \alpha_P \alpha_B} && (2)\\
\hat{\beta}_x(\rho(G)=x, \rho(B)=x) &= \frac{\alpha_x \beta_x + \alpha_P \alpha_G}{\alpha_x + \alpha_P} && (3)\\
\hat{\beta}_x(\rho(G)=x, \rho(B)=y) &= \frac{\alpha_x \beta_x + \alpha_P \alpha_G}{\alpha_x + \alpha_P \alpha_G} && (4)\\
\hat{\beta}_y(\rho(G)=y, \rho(B)=y) &= \frac{\alpha_y \beta_y + \alpha_P \alpha_G}{\alpha_y + \alpha_P} && (5)\\
\hat{\beta}_y(\rho(G)=y, \rho(B)=x) &= \frac{\alpha_y \beta_y + \alpha_P \alpha_G}{\alpha_y + \alpha_P \alpha_G} && (6)\\
\hat{\beta}_y(\rho(G)=x, \rho(B)=x) &= \beta_y && (7)\\
\hat{\beta}_y(\rho(G)=x, \rho(B)=y) &= \frac{\alpha_y \beta_y}{\alpha_y + \alpha_P \alpha_B} && (8)
\end{aligned}$$

A.2 Proof of Proposition 7

Preliminarily, we give the following straightforward results which will be used in the subsequent proof. For α_x, α_y, α_G, β_x, and β_y strictly comprised between 0 and 1, the following inequalities necessarily hold:

$$\hat{\beta}_x(\rho(G)=y, \rho(B)=x) < \hat{\beta}_x(\rho(G)=y, \rho(B)=y) \quad (9)$$
$$\hat{\beta}_x(\rho(G)=y, \rho(B)=x) < \hat{\beta}_x(\rho(G)=x, \rho(B)=x) \quad (10)$$
$$\hat{\beta}_x(\rho(G)=y, \rho(B)=y) < \hat{\beta}_x(\rho(G)=x, \rho(B)=y) \quad (11)$$
$$\hat{\beta}_x(\rho(G)=x, \rho(B)=x) < \hat{\beta}_x(\rho(G)=x, \rho(B)=y) \quad (12)$$
$$\hat{\beta}_y(\rho(G)=x, \rho(B)=y) < \hat{\beta}_y(\rho(G)=x, \rho(B)=x) \quad (13)$$
$$\hat{\beta}_y(\rho(G)=x, \rho(B)=y) < \hat{\beta}_y(\rho(G)=y, \rho(B)=y) \quad (14)$$
$$\hat{\beta}_y(\rho(G)=y, \rho(B)=y) < \hat{\beta}_y(\rho(G)=y, \rho(B)=x) \quad (15)$$
$$\hat{\beta}_y(\rho(G)=x, \rho(B)=x) < \hat{\beta}_y(\rho(G)=y, \rho(B)=x) \quad (16)$$

The following is a check that the above inequalities indeed hold:

$$\hat{\beta}_x(\rho(G)=y,\rho(B)=x)=\frac{\alpha_x\beta_x}{\alpha_x+\alpha_P\alpha_B}=\frac{\beta_x}{1+1-\frac{\alpha_P\alpha_B}{\alpha_x}}<\beta_x=\hat{\beta}_x(\rho(G)=y,\rho(B)=y) \tag{17}$$

$$\begin{aligned}\hat{\beta}_x(\rho(G)=y,\rho(B)=x)&=\frac{\alpha_x\beta_x}{\alpha_x+\alpha_P\alpha_B}<\frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P\alpha_B+\alpha_P\alpha_G}=\\&=\frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P}=\hat{\beta}_x(\rho(G)=x,\rho(B)=x)\end{aligned} \tag{18}$$

$$\begin{aligned}\hat{\beta}_x(\rho(G)=y,\rho(B)=y)&=\beta_x=\beta_x\left(\frac{\alpha_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P\alpha_G}\right)=\frac{\alpha_x\beta_x+\alpha_P\alpha_G\beta_x}{\alpha_x+\alpha_P\alpha_G}<\\&<\frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P\alpha_G}=\hat{\beta}_x(\rho(G)=x,\rho(B)=y)\end{aligned} \tag{19}$$

$$\begin{aligned}\hat{\beta}_x(\rho(G)=x,\rho(B)=x)&=\frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P}=\frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P\alpha_G+\alpha_P\alpha_B}<\\&<\frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P\alpha_G}=\hat{\beta}_x(\rho(G)=x,\rho(B)=y)\end{aligned} \tag{20}$$

$$\hat{\beta}_y(\rho(G)=x,\rho(B)=y)=\frac{\alpha_y\beta_y}{\alpha_y+\alpha_P\alpha_B}=\frac{\beta_y}{1+\frac{\alpha_P\alpha_B}{\alpha_y}}<\beta_y=\hat{\beta}_y(\rho(G)=x,\rho(B)=x) \tag{21}$$

$$\begin{aligned}\hat{\beta}_y(\rho(G)=x,\rho(B)=y)&=\frac{\alpha_y\beta_y}{\alpha_y+\alpha_P\alpha_B}<\frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P\alpha_B+\alpha_P\alpha_G}=\\&=\frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P}=\hat{\beta}_y(\rho(G)=y,\rho(B)=y)\end{aligned} \tag{22}$$

$$\begin{aligned}\hat{\beta}_y(\rho(G)=y,\rho(B)=y)&=\frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P}=\frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P\alpha_G+\alpha_P\alpha_B}\\&<\frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P\alpha_G}=\hat{\beta}_y(\rho(G)=y,\rho(B)=x)\end{aligned} \tag{23}$$

$$\begin{aligned}\hat{\beta}_y(\rho(G)=x,\rho(B)=x)&=\beta_y=\beta_y\left(\frac{\alpha_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P\alpha_G}\right)=\frac{\alpha_y\beta_y+\alpha_P\alpha_G\beta_y}{\alpha_y+\alpha_P\alpha_G}<\\&<\frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P\alpha_G}=\hat{\beta}_y(\rho(G)=y,\rho(B)=x)\end{aligned} \tag{24}$$

We note that inequality (17) follows from $(\alpha_P\alpha_B)/\alpha_x>0$, inequality (18) from $\alpha_P\alpha_G>0$ and (2) being strictly lower than 1, inequalities (19) and (24) from $\alpha_P\alpha_G\beta_x<\alpha_P\alpha_G$, inequalities (20) and (23) from $\alpha_P\alpha_B>0$, inequality (21) from $(\alpha_P\alpha_B)/\alpha_y>0$, and inequality (22) from $\alpha_P\alpha_G>0$ and (8) being strictly lower than 1.

Proof. From (9) follows that condition 6.1 is incompatible with conditions 4.1.1 and 4.2.1. Moreover, from (15) follows that condition 6.2 is incompatible with condition 4.3.2. This proves that a separating equilibrium where $\rho(G)=y$ and $\rho(B)=x$ and a pooling equilibrium where $\rho(G)=\rho(B)=y$ cannot coexist.

From (12) follows that condition 5.1 is incompatible with condition 3.1. This proves that a separating equilibrium where $\rho(G) = x$ and $\rho(B) = y$ and a pooling equilibrium where $\rho(G)=\rho(B)=x$ cannot coexist.

From (9) and (11) follows that $\hat{\beta}_x(\rho(G)=y, \rho(B)=x) < \hat{\beta}_x(\rho(G)=x, \rho(B)=y)$, which in turn implies that condition 6.1 is incompatible with condition 5.1. This proves that the two types of separating equilibria cannot coexist.

To see that the two types of pooling can coexists suppose that $|U_B|/(U_G+|U_B|) < 1$. If β_x is close enough to 1 so that 4.3.1 is satisfied and that β_y is close enough to 0 so that condition 3.2 is satisfied. For α_G large enough, also 3.1 is satisfied. To have also 4.3.2 satisfied it is enough to have α_P and α_G sufficiently large.

To see that a pooling equilibrium where $\rho(G) = \rho(B) = x$ and a separating equilibrium where $\rho(G)=y$ and $\rho(B)=x$ can coexist, note that from $\hat{\beta}_x(\rho(G)=y, \rho(B)=x) < \hat{\beta}_x(\rho(G)= x, \rho(B)=y)$ follows that condition 6.1 implies condition 3.1. Moreover, by (16) we can set c_e, U_G and U_B such that $U_G\hat{\beta}_y(\rho(G)=x, \rho(B)=x) \leq c_e \leq U_G\hat{\beta}_y(\rho(G)=y, \rho(B)=x)$, and $\hat{\beta}_y(\rho(G)=x, \rho(B)=x) \leq |U_B|/(U_G+|U_B|) \leq \hat{\beta}_y(\rho(G)=y, \rho(B)=x)$, so that both 6.2 and 3.2 are satisfied.

A similar argument can be applied to show that a pooling equilibrium where $\rho(G) = \rho(B)=y$ and a separating equilibrium where $\rho(G)=x$ and $\rho(B)=y$ can coexist. $\square$