

Inequality as a multiple equilibria signaling game

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Abstract

We develop a signaling model of wage determination where the signal consists in the number of hours worked. By endogenizing the cost of providing the signal, we obtain multiple separating equilibria, which induce a spurious positive correlation between income and inequality. We present some preliminary evidence that the signal is indeed positively correlated with measures of inequality.

1 Introduction

For decades economists have wondered whether inequality is bad or good for aggregate wealth accumulation and ultimately long-term growth (Bénabou, 1996). On the one hand, entrenched inequality threatens to create an underclass whose members' inadequate education and low skills leave them with poor prospects for full participation in the economy as earners or consumers. It can cause political instability and thus poses risks to investment and growth. On the other hand, some argue that because inequality puts more resources into the hands of capitalists (as opposed to workers), in a world of imperfect capital markets it promotes savings and investment and catalyzes growth.

In this paper, we enrich this trade off by considering that the link between inequality and growth might be spurious, and in particular follow complex patterns intertwined with the functioning of the labor market. We develop a dynamic signaling model where workers compete for good jobs by providing a signal to firms, for instance in terms of their attitude to work long hours. The signal—in addition to possibly revealing some information about the true ability of a worker—has a value in itself, as an input of the production function (Spence, 2002). Because the outcome (high or low wages) feeds back into the cost of providing the signal, the model produces multiple equilibria: for every wage level associated to bad jobs, there are two equilibrium levels for good jobs wages, which differ for the amount of the signal (hours worked) the rich workers are willing to provide. Hence, the same economic structure is able to support two different levels of inequality and wealth accumulation. This provides a spurious correlation between inequality and aggregate income.

Testing the implications of our model in the data proves difficult, as evidence of multiple equilibria is hard to collect: cross-country variation is affected by many confounders, while within-country variation over time (from one to the other equilibria) is unlikely due to a strong persistence of work attitudes, which conceivably change only in response to some major shocks—these shocks (say the German reunification, for instance) typically affecting the structure of economic incentives at large.

These difficulties notwithstanding, we present one empirical exercises that suggest that the outcome is indeed positively related to the signal. Using the dataset assembled in Roine et al. (2009) (RVW hereafter), we test whether—at an aggregate level and over a long time

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span— wage inequality is correlated with the average number of hours worked.¹ We show that the average number of hours worked is an important determinant of wage inequality. Incidentally, we also show that RVW’s results are very sensitive to the introduction of this additional variable, with many controls losing their significance.

We describe the related literature in section 2, our simple theoretical model in section 3, and the empirical analysis in section 4; section 5 concludes.

2 The literature

Our work is related to two different strands of the literature. A first strand looks directly into the relationship between inequality and growth. Many models of income distribution are characterized by multiple steady states, typically arising from capital markets imperfections which create a negative feedback from inequality to social mobility.² For instance, investment may involve a minimum project size or sunk cost: agents with a level of wealth below a threshold therefore do not invest. This requires more wealth concentration in the first stages of development, in order to escape a poverty trap. As the economy grows richer, the same level of inequality reduces growth because decreasing returns to investment. Such mechanisms highlight the relevance of intervening variables (such as the initial distribution of wealth) on the relationship between the treatment (redistribution) on the outcome (growth). This path dependency then implies that small differences in the treatment can give rise over time to big differences in outcome. This is not, however, a scholarly case of multiple equilibria, which would require that two or more level of outcomes are consistent with the same level of the treatment and of all intervening variables. Our simple model provides such a case.

The second strand investigates the role of (unpaid) overtime work as a signal in the labor market. In such models, workers signal their value to the employer by supplying a number of hours in excess to their contract (Anger, 2008). This helps explaining the observed correlation between the number of hours worked, and in particular unpaid overtime, and unit (hourly) wages (Bell and Freeman, 2001; Booth and Frank, 2003; Pannenberg, 2005; Aaronson and French, 2009).³ From a theoretical perspective, many mechanisms can account for a positive correlation between hours worked and unit wages, including labor market segmentation between part-timers and full-timers (Struyk, 1973), quasi-fixed training or vacancy costs (Rosen, 1976), and interdependencies between hours worked and productivity (Barzel, 1973). Our innovation with respect to the signaling models in the literature is to consider an endogenous variable (wealth), rather than exogenous characteristics (ability) as a determinant of the cost of providing the signal. This endogeneity leads to a multiplicity of separating equilibria.

3 The model

We consider an economy populated by two agents. The two agents have different wealth endowments, $W_H > W_L$. There are two jobs in this economy, paying different remunerations, $Y_H > Y_L$. The two agents compete to obtain the best job through overtime. More precisely, given two levels of hours worked $h_H > h_L$, the best job is assigned to the agent who works more. In case of a break even, job assignment is random.⁴

Providing overtime is costly, as described by $c(h, W)$, with $c_h > 0$, $c_{hh} < 0$. The cost comprises not only a non-monetary component (disutility from work and foregone leisure),

¹We thanks Jesper Roine, Jonas Vlachos and Daniel Waldenström for having made their data public, and in particular Waldenström for having provided the final dataset and the code in order to exactly replicate their results.

²See Bénabou (1996)

³From an empirical perspective, being hours and wages jointly determined, examination of the issue requires the use of instrumental variables (typically family structure, social Security rules, non-labor income and disability status —see (Aaronson and French, 2009).

⁴Any input that enters the production function, is a choice variable for the individual at every period and is not explicitly required by the work contract is a valid candidate to act as a signal. Examples are portfolios of potential clients and the acquisition of new skills.

but also a monetary component (the need to buy extra family care).

If we assume that $c_{hW} < 0$, then the model has the structure of a signaling game, since the single crossing condition is satisfied. This condition requires the cost of providing overtimes to increase at a slower rate for the rich individual than for the poor individual. This is reasonable as poor individuals are likely to be liquidity constrained in their ability to buy out family duties.

Given that the condition above holds, the best separating equilibrium —known as Riley equilibrium— is such that the poor agent provides no overtime ($h_L = 0$), while the rich agent provides enough overtime to make the poor indifferent between low income with no overtime and high income with overtime:

$$Y_L = Y_H - c(h^*, W_L). \quad (1)$$

Equation 1 allows us to determine h^* , given endowments — W_L and W_H — and rewards — Y_L and Y_H .

One reasonable upper bound for $h^* = h_H$ is given by the condition $W_L \geq c(h^*, W_L)$, identifying the maximum disutility from overtime that the low-wealth agent can afford, which we denote by \bar{h}^* . If the low-wealth individual cannot afford more than \bar{h}^* , then the high-wealth individual has no convenience in providing more overtime than \bar{h}^* .

Notice that for given W_H and W_L , there exist many triples (Y_H, Y_L, h^*) that satisfy equation (1). We assume that total output in this economy depends on the level of h and is given by $F(h)$, where F is a production function such that $F' > 0$ and $F'' < 0$. We also assume that all output is distributed as income, i.e., $F(h^*) = Y_H + Y_L$, which closes a degree of freedom giving a unique triple (Y_H, Y_L, h^*) for every pair (W_H, W_L) .

In a dynamic setup, we can think that W_H is the wealth of the long working hours type and depends on the income Y_H he earned in the previous period, and similarly for the short working hours type. Let us assume, for simplicity, that next period W_H is equal to this period Y_H , and similarly for type L .

We look for the steady state equilibria where $W_H = Y_H$ and $W_L = Y_L$. Differentiating (1) and $F(h^*) = Y_H + Y_L$ with respect to Y_H , Y_L , and h^* , we obtain:

$$\frac{dY_H}{dY_L} = \frac{F'(h^*)(1 + c_W(h^*, W_L)) + c_h(h^*, W_L)}{F'(h^*) - c_h(h^*, W_L)}. \quad (2)$$

The steady state equilibria look like in figure 1.

The red curve projects the equilibria on the plane (Y_L, Y_H) , while the black curve projects the equilibria on the plane (Y_L, h) . Looking at the curves, we can understand how the three variables Y_L , Y_H , and h change along the continuum of equilibria. For each level of $Y_L > 0$ but the highest, we have two different levels of Y_H and h that form an equilibrium triple. One triple has higher Y_H , higher h and, hence, both higher inequality and higher output.

4 Empirical analysis

4.1 Empirical strategy

As already discussed, looking for an empirical analog of figure 1 is hazardous. Though equilibrium selection is likely to be correlated among individuals and firms, we can expect aggregation to wipe out differences in outcomes to a great extent. However, both the low and the high equilibrium trajectories share a common feature: they display by and large a positive relationship between the strength of the signal and (i) the level of inequality, (ii) the average wage. As the literature has already established a positive effect of hours on unit wage (see section 2 above), we now investigate the relationship between hours worked and inequality.

4.2 Descriptive evidence

Figure 2 shows the aggregate relationship between the average annual number of hours worked in 2010 and the Gini coefficient (computed on household disposable income) in the late 2000s, for the Oecd countries. The relationship is positive; moreover, there is a lot of

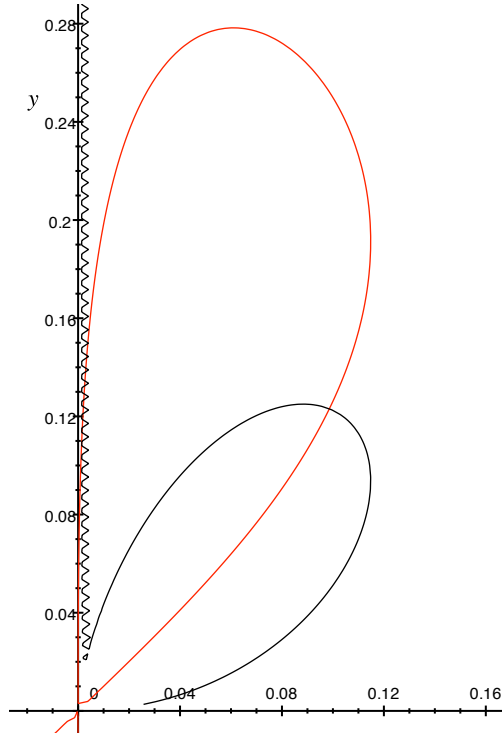


Figure 1: Steady state equilibria. The x -axis measures Y_L while the y -axis measures alternatively Y_H (red curve) and h^* (black curve). Curves are depicted for the following specific functional forms: $F(h) = h^{\frac{1}{2}}$ and $c(h, W) = h^2/W$.

variability meaning that countries with similar work attitudes show big differences in the level of inequality.



Figure 2: Average annual number of hours worked for dependent employees and income inequality, Oecd countries. Hours worked are measured in 2010. The Gini coefficient is computed on household disposable income and refers to the late 2000s. Source: Oecd.

A similar pattern is found when focusing (on a restricted number of countries due to data availability) only on full time dependent employees and on wage inequality. A positive relationship also occurs between work attitude as measured in the World Value Survey (as the fraction of respondents who strongly agree to the statement “Work should come first

Δ_{top10}	(1)	(2)	(3)	(4)	(5)	(6)
Dgdppc	-5.563** (2.727)	0.000 (0.000)	0.000 (0.000)	1.301 (3.532)	0.000 (0.000)	0.000 (0.000)
Dpop	-9.833 (11.543)	0.000 (0.000)	0.000 (0.000)	-24.089** (12.025)	0.000 (0.000)	0.000 (0.000)
Dcgov	-22.391*** (8.526)	0.000 (0.000)	0.000 (0.000)	-23.962*** (8.886)	0.000 (0.000)	0.000 (0.000)
Dfindev	0.530 (0.621)	0.000 (0.000)	0.000 (0.000)	1.890*** (0.664)	0.000 (0.000)	0.000 (0.000)
Dopen	-3.291 (4.400)	0.000 (0.000)	0.000 (0.000)	-0.145 (5.049)	0.000 (0.000)	0.000 (0.000)
Dhours		0.012*** (0.000)			0.012*** (0.000)	
Dmargtax1				-10.219*** (2.210)	0.000 (0.000)	0.000 (0.000)
Observations	99	52	52	77	46	46
Number of countries	12	9	9	10	8	8

FDGLS estimations allowing for country specific AR(1) processes and heteroskedasticity in the error terms. Robust standard errors in parentheses. Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table 1: The determinants of inequality: top 10% income share.

even if it means less spare time”) and the Gini index provided by the World Bank.⁵

While definitely not conclusive and admittedly only little informative, this raw evidence is at least not in stark contrast with the implications of our model.

4.3 The effects of hours worked on inequality

To test the effects of hours worked on inequality, we introduce this additional variable in the RVW analysis. As we use Oecd data for the number of hours worked, we are forced to restrict the original RVW sample from 25 to 14 countries⁶ and the period of observation from 1900-2000 to 1950-2000. We implement the same first differenced GLS (FDGLS) estimation procedure as in the original RVW paper, which further reduces the sample to 9 countries.⁷

Rather than providing separate results for the top 1%, the top 10%-top 1%, and the bottom 90% as in RVW, we only report results for the top 10%. The explanatory variables are, as in RVW, per capita GDP (*gdppc*), population (*pop*), central government expenditure (*cgov*, as a share of GDP), an indicator of financial development (*findev*: total capitalization, measured as the sum of bank deposits and market capitalization, as a share of GDP), an indicator of trade openness (*open*: the sum of imports and exports, as a share of GDP) and the top marginal tax rate (*margtax1*).

Table 1 reports the results. Following RVW, columns (1)-(3) exclude the top marginal tax rate from the controls, while columns (4)-(6) include it (this causes the Netherlands to drop off the sample).

The specification in columns (1) and (4) use the same RVW controls on the original RVW sample: should we have used the same outcome variables, we would have obtained exactly the same results as in Table 5 of their paper. Columns (2) and (5) include the number of hours worked. Columns (3) and (6) apply the same RVW controls, but on the restricted sample on which specifications (2) and (4) are estimated.

⁵Figures available upon request.

⁶Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Japan, Netherlands, New Zealand, Portugal, Spain, United Kingdom, United States.

⁷Canada, Finland, France, Germany, Netherlands, New Zealand, Spain, United Kingdom, United States.

$d9/d1$	(1)	(2)	(3)	(4)	(5)	(6)
Dgdppc	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Dpop	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Dcgov	-1.185 (0.752)	0.000 (0.000)	0.000 (0.000)	-0.823 (0.899)	0.000 (0.000)	0.000 (0.000)
Dfindev	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Dopen	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Dhours		0.004*** (0.000)			0.004*** (0.000)	
Dmargtax1				0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Observations	57	35	35	43	24	24
Number of countries	12	8	8	9	6	6

FDGLS estimations allowing for country specific AR(1) processes and heteroskedasticity in the error terms. Robust standard errors in parentheses. Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table 2: The determinants of inequality: $d9/d1$ interdecile ratio.

Two things are to be noted. First, RVW results loose significance once the sample is restricted along the ways described above. Second, the coefficient of the number of hours worked is always positive and significant (indeed, it is the *only* significant coefficient).

As a robustness test, we replicate the exercise using the ratio of the 9th to 1st decile in gross earnings of full-time dependent employees as an outcome variable (table 2). Again, the number of hours worked is the only significant variable.

Now, of course the number of hours worked might be partly endogenous: in more unequal societies, one might argue, people develop a more intensive work attitude in order to climb the social ladder, or avoid sliding down (Bell and Freeman, 2001). In other words, what we might be catching up in our analysis is not only a movement along one of the equilibrium trajectories (high or low), but also a movement between equilibrium trajectories, from low to high or viceversa: in more unequal societies wealthy people can afford buying more signal (and have to do so in order to prevent poor individuals from overtaking them), and this is consistent with that higher level of inequality. Unfortunately we cannot discriminate between the two stories, but they are both consistent with the implications of our model.

5 Conclusions

In this paper we have developed a signaling model of wage determination where workers compete for good jobs by working longer hours. Our crucial assumption is that the cost of deferred leisure is lower for richer individuals. If this holds true, the model predicts a multiplicity of separating equilibria. In a low inequality, low income equilibrium, rich people work only slightly more than poor individuals, and the little additional income they receive is sufficient to deter poor individuals from competing with them. In a high inequality, high income equilibrium rich people work more and earn more, but again the extra effort they put in is barely sufficient to make good jobs unattractive to poor individuals.

As this mechanism is based on relative wages, it could be at work at all stages of development, thus shedding new light on the (absence of) a Kuznet curve in the aggregate inequality-income relationship (Deininger and Squire, 1998).

Our work could be extended on a number of directions. From an empirical perspective,

smarter tests should be devised in order to test the implications of the mechanism we have drawn attention to. From a theoretical viewpoint, by making the supply of good and bad jobs endogenous one might offer an explanation different from the standard agglomeration externality story ([Moretti, 2012](#)), to why some areas thrive, with high wages, high growth and high inequality, which in the end benefits everybody, while others languish or decline.

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