Economic (In)Stability under Monetary Targeting

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Abstract

Monetary growth targeting is often seen as an effective way of supporting macroeconomic stability. We scrutinize this property by checking whether multiplicity of equilibria, in the form of local indeterminacy (LI), can be both a possible and a plausible outcome of a basic model with an exogenous money growth policy rule. We address the question in different versions of the Sidrauski-Brock-Calvo framework, which isolates the contribution of monetary non-neutralities and monetary targeting. In line with previous literature, real effects of money are found to be a necessary condition for LI: we identify a single pattern for their magnitude if they are to be sufficient too. While the most elementary setups are unable to plausibly generate large enough real effects, LI becomes significantly more likely as one realistically considers additional channels of transmission of monetary expansions onto the real economy: in particular, we show that models in which holding money is valuable to both households and firms may yield a LI outcome for empirically relevant parameterizations, therefore casting doubt on the stabilizing properties of monetary monitoring.
JEL: E5, E58, E52, E41. *Keywords*: local indeterminacy, monetary targeting, real effects of money, money-in-the-utility-function, money-in-the-production-function.

1 Introduction

In the last decade – also following the adoption by the ECB of a monetary policy strategy assigning a prominent role to a monetary pillar – a number of studies have inquired into the role of monetary aggregates in the conduct of monetary policy. In reaction to new-keynesian and econometric criticisms (see, e.g., Dotsey and Hornstein, 2003, Ireland, 2004, Svensson and Woodford, 2003, Woodford, 2008), some authors have identified theoretical or empirical reasons supporting the relevance of money as an information variable for monetary policy decisions (see, e.g., Amisano and Fagan, 2010, Coenen *et al.*, 2005, Gerlach and Svensson, 2003, Nelson, 2003, or the overview in ECB, 2010). A more general consensus, also shared by the ECB monograph, appears to have emerged on the usefulness of money as a policy guide, acknowledging either the stability of the relations between money and other variables of interest in the conduct of monetary policy (e.g., Assenmacher Wesche and Gerlach, 2007, Dreger and Wolters, 2010, Hafer *et al.*, 2007), its robustness as a policy instrument in the face of model uncertainty (e.g., Gerdesmeier *et al.*, 2002, Kilponen and Leitemo, 2008), or its ability to prevent bad or multiple equilibria that could be caused by simple interest rate feedback rule strategies (see Christiano and Rostagno, 2001, Benhabib *et al.*, 2002, or more recently Christiano *et al.*, 2008, Atkeson *et al.*, 2010, and Minford and Srinivasan, 2009). Here we examine the property highlighted by this last set of papers, according to which targeting (also) a monetary aggregate would effectively support economic stability, by checking whether and how, under such a policy, multiple equilibrium paths, in the form of local indeterminacy (LI), can arise. To this extent, it is crucial to distinguish between the possibility and the plausibility of LI. One issue is whether there exist theoretical conditions for LI in the model which describes the economy, another is whether such conditions would find empirical support.

We address both issues in several versions of a framework as generic as the Sidrauski-Brock model with flexible prices, characterized by a money growth rule monetary policy.[1] In this model, money is held because it gives utility directly, proxying its role in providing services such as reducing transaction costs. We augment the framework *à la* Calvo (1979) by including money as a possible direct production input, representing working capital or the services that liquidity can provide to a firm, like for paying wages, purchasing inputs, marketing, and in general running plants: as firms too must execute transactions, the money-in-the-production-function (MIPF) approach is just as valid as money-in-the-utility-function (MIUF).[2] Both these ways of factoring money in are clearly shortcuts. Nevertheless, we find

[1] See Sidrauski (1967) and Brock (1974, 1975).

[2] Fischer (1974) offered microfoundations for MIPF. Finance theory offers many additional reasons why non-financial firms hold liquid assets - as part of the decision to hold a portfolio of assets, say, or as a buffer against cash-flow variability: a precautionary motive would induce firms to

the Sidrauski-Brock(-Calvo), or SBC, framework a very convenient tool for our exercise. It introduces in a simple way both a motive for holding money and a role for monetary non-neutralities and monetary targeting; it allows studying the stability properties of monetary targeting in isolation, as it abstracts from market imperfections, and therefore from renown routes to indeterminacy (IRS, externalities, and imperfect competition); and it ensures tractability and comparability with previous literature, while being general enough to encompass other approaches.[3]

In the SBC framework, the possibility of multiple equilibria hinges on the interplay between money and price expectations, which can be self-fulfilling. In steps, we develop the intuition of Benhabib and Farmer (1999) that in this kind of models LI can arise whenever around the steady state an excess supply of real balances turns into a net excess demand, since it is only then that self-fulfilling inflation expectations become compatible with stationary equilibria, giving rise to as many possible converging real allocations as there are price sequences of this sort. In line with those authors, we argue that a necessary condition for this reversal is that money be non-neutral, i.e. that it have real effects, impinging on marginal utilities and productivities, and ultimately on agents' propensity to demand money balances. If the growing nominal money supply is somehow able to stimulate the real economy enough to considerably alter agents' need for money for transaction or other purposes, alternative conjectures on sequences for prices, and hence real balances, and therefore for all real variables in the economy, could result in a converging equilibrium. Absent sufficiently large real effects, only explosive paths (if consistent with equilibrium) could constitute an alternative to the saddlepath equilibrium. We measure these real effects by the marginal proportional reaction of the propensity to hold real balances or consume goods to the monetary expansion. This enables us to define the necessary and sufficient conditions for LI in terms of a unique pattern for the (steady-state) elasticities of marginal utilities and productivities to monetary changes.

In the framework chosen, the real effects of money will be brought about either by preferences with a nonseparability in real balances or by MIPF. Benhabib and Farmer (1999, 2000) have shown that LI could be the outcome of a calibrated SBC economy with endogenous labor choice, but only if real balances can affect labor

demand money in order to avoid the opportunity costs related to missed investment opportunities and/or the costs of external finance to meet unanticipated cash needs. But the strong correlation between M3 holdings of non-financial corporations and gross value added in the sector suggests that the transaction motive is a major determinant of firms' money demand. For empirical evidence, among others, Mulligan (1997), Lotti and Marcucci (2007) and Bover and Watson (2005).

[3] The cash-in-advance, shopping time and transaction costs models all imply reduced forms that are special cases of the SBC framework: LI has been shown to be a possible outcome of these models under an exogenous money growth policy rule by, respectively, Wilson (1979), Woodford (1994), and Gray (1983).

supply or demand decisions strongly enough to induce a non-standard slope in one of the labor market schedules. Dissatisfaction with this assumption provides additional motivation for our work and justifies one of the restrictions we impose that, in order to identify possible sources of LI independent of a labor channel of this sort, the labor supply is assumed inelastical. In the most basic models that we examine, only unreasonable assumptions about the impact of money on the real economy could yield LI. However, as we include additional channels through which money can affect the real economy in general equilibrium, the chances of an indeterminate outcome increase substantially. In fact, in this paper they will be greater in our models featuring money both in the utility and in the production functions, creating conditions for LI that calibrated exercises will show to be compatible with the data.

We also use versions of the SBC framework used in other papers (e.g., Gray, 1983, Matsuyama, 1990, Obstfeld, 1984, or the survey by Benhabib and Farmer, 1999). In this respect, what distinguishes our work is the aim of checking whether LI could be both a possible and a plausible outcome of simple money growth rule economies, by examining new and old models in perspective and looking for a single pattern inducing LI in the framework, focusing only on local analysis (informative enough for our purposes), and imposing fewer a-priori restrictions on preferences and technology than in previous work.[4]

Sections 2 and 3 study how LI could arise, first in the basic SBC models where the mechanism at work is neater, and then applying that same mechanism in MIUF-MIPF models which include more channels for real effects of money. Section 4 concludes.

2 Local indeterminacy lessons from basic models

In this section, first we sketch out the general framework in which we conduct our analysis. Then, we review in a somewhat new and unified perspective its simplest versions, where driving forces and intuitions for indeterminacy are more readily grasped. The aim is to clarify the basic mechanisms inducing LI in the framework for immediate application in the slightly richer but analogous setups of Section 3.

The Sidrauski-Brock framework assumes perfect competition, no externalities, and flexible prices. Given the equivalence between indeterminacy in the neighborhood of a deterministic steady state and the existence of stationary sunspot equilibria (see Woodford, 1986), it is consistent with our aims to assume perfect foresight too. There is a unit-measure continuum of a single type of agent: infinitely-lived

[4] Our exercise recalls that of Benhabib *et al.* (2001), who looked at LI not only in flexible but also in sticky prices economies, under interest rate rules and alternative fiscal policies.

households that both consume and produce. In continuous time, they maximize

$$\int_0^\infty e^{-\beta t} U(c, m)\, dt\,,$$

where β is the rate of time preference, c is consumption, m is real balances, and utility U is assumed to satisfy the Inada conditions and to be strictly increasing, strictly concave and twice continuously differentiable, in each of its arguments. The flow budget constraint for each agent is of the form

$$\dot{m} = y - c + T - \pi m$$

where a dot indicates a time derivative, y is income, which may be exogenous or endogenous, T is fiscal transfers, and π is the actual and (given perfect foresight) also the expected rate of change in the price level P. Notice that wealth is diminished by the inflation tax if it is kept in the form of money. A no-Ponzi-game constraint is also imposed: with real total household wealth given by m, it simply amounts to $m_t \geq 0 \;\forall\, t$.

Money, M, is injected into the economy by the monetary authority according to an exogenous constant-growth rule

$$\frac{\dot{M}}{M} = \phi \tag{1}$$

i.e., in real terms, $\dot{m} = m(\phi - \pi)$: the steady state rate of inflation is constant and equal to the rate of monetary growth ϕ, assumed to be non-negative. Fiscal policy simply grants a lump-sum transfer T to each agent, financed by seignorage. The government budget constraint is then

$$PT = \dot{M}\,. \tag{2}$$

First, in order to derive some basic intuitions, we abstract from production and assume a fixed endowment. First-order conditions lead to the monetary Euler equation $-\dot{U}_c/U_c + \beta = U_m/U_c - \pi$ (with $\dot{U}_c = U_{cc}\dot{c} + U_{cm}\dot{m}$, and with a subscript indicating a derivative with respect to the corresponding argument). Equilibrium requires consumption to be always equal to endowment ($\dot{c} = 0$). Consider the separable utility case ($U_{cm} = 0$), so that a monetary expansion has no effect on the real part of the economy.[5] Taking policies (1) and (2) into account, the equilibrium solution is governed by a single differential equation

$$\frac{\dot{m}}{m} = \beta + \phi - \frac{U_m}{U_c}\;. \tag{3}$$

[5] This is one of the models originally studied by Brock (1974, 1975), who considered only stationary, hyperinflationary, and hyperdeflationary equilibria, ignoring non-stationary locally converging equilibria.

together with a transversality condition here amounting to $\lim_{t\to\infty} e^{-\beta t} m_t = 0$.

In order to explain how could LI be generated here, we resort to a thought experiment outlined by Benhabib and Farmer (1999). Starting from a monetary steady state m^* (itself a trivial stationary sequence), identified by setting (3) equal to zero, consider an unexpected, permanent, sufficiently small increase in M alone, without any accompanying change, and in particular holding prices and price expectations constant. An excess supply of real balances will result. If m were to increase from m^*, given fixed income the only change in (3) would be a decrease in U_m. Hence $\frac{\dot{m}}{m}$ would become positive in equilibrium. In other words, $\dot{P} < 0$: the excess supply could be driven down only by self-fulfilling expectations of lower prices, which would increase the return on holding money (equal to $-\pi$), and thus the demand for m. But on this pattern, immediately thereafter real balances will be even higher, so clearing the money market would require even greater price deflation to push demand further up. With this feedback process, steadily decreasing prices would cause real balances to explode: the differential equation is unstable, as depicted in Figure 1. Such a sequence would leave the neighborhood of the steady state; and it can be ruled out as an equilibrium sequence by the transversality condition.[6] In this case, the only equilibrium path is the stationary sequence that coincides with the steady state, achieved by means of an immediate offsetting jump in P.

The question is: can the excess supply in this thought experiment be absorbed without an immediate offsetting jump in prices? That is, under what conditions will it be possible for the economy to follow an equilibrium path towards m^* different from permanent steady state? Such multiplicity could be induced if the monetary expansion, by stimulating the economy, determined a suitable modification in the propensity to consume or to hold real balances, enough to transform the excess supply of real balances into excess demand. If expanding money stimulates real activity so that agents modify their demand for money for transactions or asset holding, then the extra demand for m that derives from this non-neutrality or *real effect of money* could make alternative conjectures for price sequences compatible with a stationary equilibrium. In fact, if the effects on the real side of the economy of a rise of m from m^* are such that a net excess demand for money is generated, then self-fulfilling expectations of future inflation can decrease the demand by depressing the rate of return of money as an asset (equal to the negative of the inflation rate). And steadily decreasing inflation will make it possible, at every following instant, to maintain equilibrium while real balances shrink to their original stationary level, and then hold constant again. This would hold for every sufficiently small jump in M: therefore, in the neighborhood of the steady state there will exist a continuum of converging perfect foresight equilibrium paths, each associated with a different

[6] The transversality condition requires m to grow asymptotically less rapidly than rate β. With Inada conditions in place and $\phi \geq 0$, violation obtains since $\lim_{m\to\infty} \frac{\dot{m}}{m} = \beta + \phi \geq \beta$.

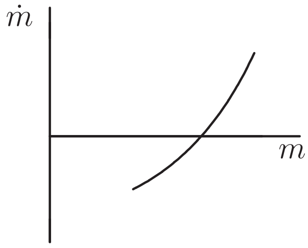


Figure 1:
Instability $\equiv$ Local Determinacy

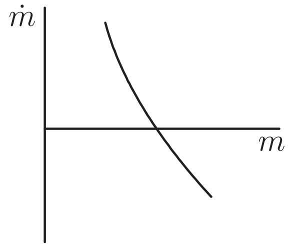


Figure 2:
Stability $\equiv$ Local Indeterminacy

initial condition for m and a different path for the price level. We emphasize that this intuition extends to the entire framework that we analyze. In the simple model above, let us suppose counterfactually that real effects of money exist. By acting either on output or directly on marginal utilities in (3) they could drive U_c down more than U_m after the monetary expansion: a marginal rate of substitution U_m/U_c *increasing* in m at m^* would invert the excess supply of m, turn the slope of the phase line negative and make the differential equation stable, thus yielding a multiplicity of possible equilibrium paths (as in Figure 2).

Proposition 1 *In the Sidrauski-Brock class of models, real effects of money are a necessary condition for local indeterminacy.*

Consistent with the foregoing, we measure the real effects of money as the marginal proportional reaction that the monetary expansion induces on the propensity to consume and to hold money balances and use this metric to identify conditions for LI. As an instrument, it is useful to define, for any two generic variables z and w, the elasticity of z with respect to w evaluated at the steady state as $\varepsilon_{z|w} \equiv -w^* z^*_w / z^*$.

The channel whereby the real effects of money can be added most readily to this basic setup is nonseparability between consumption and real balances ($U_{cm} \neq 0$), which yields a first set of necessary and sufficient conditions for LI (all proofs are in the Appendix):

Proposition 2 *In a Sidrauski-Brock economy with fixed endowment, equilibrium paths are locally indeterminate if and only if*

$$\textit{either} \qquad \varepsilon_{U_m|m} > \varepsilon_{U_c|m} > 1 \tag{4}$$

$$\textit{or} \qquad \varepsilon_{U_m|m} < \varepsilon_{U_c|m} < 1\,. \tag{5}$$

Proposition 2 spells out the pattern that in this simple model the impact of money on real activity should follow in order to create conditions for self-fulfilling inflation expectations compatible with stationarity of equilibrium. We will see how this same pattern will be confirmed, though in an enlarged format, in more general versions of the model. In any case, a closer look at this first, basic set of conditions reveals that condition (5) is associated with a rather pathological behavior, i.e. non-normality of m[7] and that a single explanatory feature underlies both conditions:

Proposition 3 *In a Sidrauski-Brock economy with fixed endowment, $U_{cm} < 0$ is a necessary condition for local indeterminacy.*

A statement analogous to Proposition 3 is already present in Matsuyama (1990), but on closer inspection in that model the condition is actually necessary to ensure just the positive autocorrelation of stationary sunspot equilibria, and not, as claimed, their existence. In either case the economic intuition underlying the proposition is as follows. A negative U_{cm} is a necessary condition for LI because if real balances are high today and expected to be lower tomorrow, then a rising inflation tax increases the opportunity cost of holding money today; however, increasing inflation under $U_{cm} < 0$ raises the expected U_c, creating an incentive to defer consumption, hence a greater propensity to hold money now. Forcing the explanation in terms of that outlined for the case of separability, a negative cross-derivative in the case of nonseparability is necessary to create conditions for the MRS to be increasing in m at m^*, but it might not be sufficient, as the magnitude of the overall variation in U_c induced by a change in m might not be large enough – in fact, it would be large enough only under non-normalities.[8]

As for plausibility, though, on the theoretical side $U_{cm} < 0$ is not the sort of restriction that one could derive starting from more detailed ways of introducing money.[9] On the empirical side, early work by Koenig (1990) is also against the hypothesis of $U_{cm} < 0$, while the ML estimations of DSGE models for the U.S. and the euro area by, respectively, Ireland (2004) and Andrés *et al.* (2006) both favor separability.

We then consider a second immediate, simple way to introduce real effects of money into the Sidrauski-Brock framework, namely including real balances directly as an input (the only variable input) in a strictly concave production function,

[7] Real balances are in fact normal if $U_{mm} < U_{cm}U_m/U_c$ (which amounts to $\varepsilon_{U_m|m} > \varepsilon_{U_c|m}$), as is assumed in the early works on this nonseparable case by Gray (1983) and Obstfeld (1984), who were therefore ruling out condition (5). As for condition (4), Obstfeld had something like $\varepsilon_{U_c|m} > 1$.

[8] $U_{cm} < 0$ would also be the key to LI in an MIUF model with capital k and a strictly concave technology $y = f(k)$ as in Calvo (1979) and Fischer (1979), who nevertheless assumed, respectively, $U_{cm} > 0$ and the weaker $U_{mm} < U_{cm}U_m/U_c$.

[9] Feenstra (1986) shows that the CIA model can be approximated by an MIUF model with $U_{cm} > 0$, and the same applies to a cash-good/credit-good model, while Croushore (1993) shows that the shopping-time model can be approximated by an MIUF model with $U_{cm} \geq 0$. Woodford (1990) and Mulligan and Sala-i-Martín (1997) argue that a possible justification of MIUF is that money is complementary with consumption, i.e. it helps to enjoy real consumption goods.

as in the analysis of Calvo (1979) and Benhabib and Farmer (1996, 1999). This simple MIPF economy naturally fulfils a necessary condition for LI analogous to the foregoing for the case of nonseparable utility: by increasing output and therefore equilibrium consumption, an increase in real balances would indirectly decrease U_c.[10] As for sufficiency, however, as above, after an increase in m from m^* the excess supply could actually be reversed, and LI occur, only if U_c were to fall sharply, i.e. if: i) m were so productive that y and hence, in equilibrium, c increase discretely, or else ii) U_c were very elastic to changes in consumption. More generally, LI would arise if and only if $\varepsilon_{U_c|c}\varepsilon_{y|m} > 1$, which, given the equilibrium equality between output and consumption, could be interpreted – if synthetically read as $\varepsilon_{U_c|m} > 1$ – as a subset of the conditions (4) derived for the case of nonseparable utility.[11]

How plausible is LI in this simple MIPF model? Specifying $y = m^\omega$ and a CRRA utility $U = \frac{c^{1-\alpha}}{1-\alpha}$, the necessary and sufficient condition for it tested by Benhabib and Farmer (1996) is $\alpha\omega > 1$. By a property of perfect competition, Benhabib and Farmer compute ω as the share of money in GDP, finding a value of about 0.01 for the U.S., which would imply unrealistically high values of α to induce LI.[12]

The conclusion of this section is therefore that, despite including the fundamental mechanisms that might in principle be conducive to LI, basic exogenous-money-growth-rule economies seem immune to the *effective* risk of local instability.

3 Local indeterminacy in MIUF-MIPF economies

In this section we outline the simplest versions of the SBC framework which turn out to be compatible with possible and plausible LI. Given the driving forces and mechanisms for LI indicated by the most basic models, all relying on real effects of money large enough to alter substantially the relative propensity to consume and to hold money, LI should be more likely in economies that have more channels through which a monetary expansion can affect real allocations. In the framework chosen, a natural first approximation to such a richer setup is given by a joint consideration of households' and firms' motives to hold money, i.e., a joint MIUF-MIPF economy.

[10] With $U = U(c)$ and $y = h(m)$ and strict concavity, at the margin the effect of an increase in m is $U_{cc}h_m < 0$.

[11] The elasticity of output with respect to real balances is defined in positive terms, as $\varepsilon_{y|m} = h_m m/y$.

[12] By endogenizing the choice of leisure and making it dependent on m, Benhabib and Farmer (1996, 2000) strengthen the output response of money through an indirect labor channel, which induces LI for a "wrong" labor supply slope. (They then exploit the indeterminacy to explain the output effect of money in the data – but, as they began by fulfilling the necessary condition for LI, i.e. that money has real effects, the exercise appears circular.)

3.1 MIUF-MIPF economies

Let us consider a version of the SBC framework with $U = U(c, m)$ and $y = h(m)$, $h(\cdot)$ strictly concave and twice continuously differentiable. This is a model sketched out in Benhabib and Farmer (1999) with very incomplete conclusions about conditions for LI, which we instead identify and relate to the pattern found for the basic models analyzed above.

The single equilibrium law of motion obviously combines MIUF and MIPF features:

$$\frac{\dot{m}}{m} = \frac{\beta + \phi - h_m - \frac{U_m}{U_c}}{1 + m h_m \frac{U_{cc}}{U_c} + m \frac{U_{cm}}{U_c}} \,. \tag{6}$$

Linearizing around a monetary steady state, one gets the following proposition:

Proposition 4 *In a Sidrauski-Brock-Calvo economy with nonseparable preferences $U = U(c, m)$ and strictly concave technology $y = h(m)$, equilibrium paths are locally indeterminate if and only if*

$$\textit{either} \quad \varepsilon_{U_m|m} + \varepsilon_{U_m|c}\varepsilon_{y|m}[1 + \frac{\varepsilon_{h_m|m}}{\varepsilon_{U_c|m}}] > \varepsilon_{U_c|m} + \varepsilon_{U_c|c}\varepsilon_{y|m} > 1 \tag{7}$$

$$\textit{or} \quad \varepsilon_{U_m|m} + \varepsilon_{U_m|c}\varepsilon_{y|m}[1 + \frac{\varepsilon_{h_m|m}}{\varepsilon_{U_c|m}}] < \varepsilon_{U_c|m} + \varepsilon_{U_c|c}\varepsilon_{y|m} < 1 \,. \tag{8}$$

These conditions are a natural extension of conditions (4)-(5) of the case of nonseparable-utility MIUF: the additional presence of indirect effects on the demand for money and for consumption goods deriving from MIPF (more precisely, from money that affects output and thus, in equilibrium, marginal utilities) reveals the full pattern for the real effects of money that could induce self-fulfilling inflation expectations compatible with stationary equilibria. This general pattern strictly includes the conditions from all the SBC models presented so far, and can be specified as

either (direct + indirect proportional change of U_m after a change in m) >
> (direct + indirect proportional change of U_c after a change in m) > 1

or (direct + indirect proportional change of U_m after a change in m) <
< (direct + indirect proportional change of U_c after a change in m) < 1 .

Conditions (7)-(8) show where and how the direct and indirect effects of money would play a role in making LI possible in this simple MIUF-MIPF economy: in this setup, only slightly more general than the basic ones, the number of monetary transmission channels in place is already so large that this possibility exists even without giving up normality of c or m, $U_{cm} \geq 0$, low elasticity of marginal utility or mild direct output effects of money. Evidently, conditions (7)-(8) for LI are more general than the previous ones and thus have a better chance of being realized in the empirical data.

In fact, unlike the simple MIUF model, even with the restrictions of normality of goods and no direct effect in utility, by setting $U_{cm} = 0$, there is a simple set of conditions under which LI can emerge:

$$\text{either} \quad \varepsilon_{U_m|m} > \varepsilon_{U_c|c}\varepsilon_{y|m} > 1$$

$$\text{or} \quad \varepsilon_{U_m|m} < \varepsilon_{U_c|c}\varepsilon_{y|m} < 1 \; .$$

We focus on this more realistic case of separability to evaluate the meaning, extent and plausibility of conditions for LI in a MIUF-MIPF economy. We take a specific example of this economy, namely a model characterized by a Cobb-Douglas technology with real balances as the only variable input

$$y = m^{\omega}$$

with $\omega \in (0, 1)$, and by separable CRRA preferences in consumption and real balances

$$U = \frac{c^{1-\alpha}}{1-\alpha} + Q\frac{m^{1-\gamma}}{1-\gamma}$$

Q being an arbitrary positive constant. The parameters α and γ are required to be positive in order to ensure concavity of the objective functional; as we know, they are equal to the elasticities of, respectively, U_c with respect to c and U_m with respect to m, and also to the reciprocal of the intertemporal elasticities of substitution of, respectively, c and m, labelled IES_c and IES_m: the smaller α (respectively, γ) is, the more slowly marginal utility falls as consumption (real money) rises, and so the more willing the household is to allow its consumption (or its real money balances) to vary over time. Under this specification, the single equilibrium law of motion is

$$\frac{\dot{m}}{m} = \frac{1}{1-\alpha\omega}\left[\beta + \phi - \omega m^{\omega-1} - Q m^{\alpha\omega-\gamma}\right]$$

and the necessary and sufficient conditions for LI are

$$\text{either} \quad \gamma > \alpha\omega > 1 \tag{9}$$

$$\text{or} \quad \gamma < \alpha\omega < 1 \; . \tag{10}$$

In terms of the thought experiment outlined in Section 2, both conditions refer to cases in which the demand for real balances might rise following a sudden expansion of the nominal money supply, sustaining self-fulfilling expectations on prices compatible with a multiplicity of real allocations. Under inequality (9), this could happen because of a strong motive for firms to want to hold money, when it plays a significant role in production ($\alpha\omega > 1$). Under inequality (10), instead, LI would mainly be the outcome of households' money preference. In fact, in this case LI is possible when both $IES_m > 1$ and $IES_m > IES_c$. This means that in order for multiple equilibrium paths to be possible, the representative household must be indifferent enough about when to hold real money balances. When households are sufficiently free in tailoring their real balances intertemporal profiles, an equilibrium sequence for m (and consequently for all variables in the economy) can be made out of a certain self-fulfilling expectation on prices. How much is "sufficiently"? Here, enough for the household to be quite willing to substitute real balances intertemporally, and more willing to do so for real balances than for consumption. Otherwise, according to the thought experiment, when there is an increase in the money supply the conditions for a sufficient increase in the demand could not exist. Or, in case of a change in marginal utilities, for whatever reason, the household would adjust its intertemporal choice profile mainly on the consumption side, damping the contribution to multiplicity that may come from the interaction between money balances and expectations.

The magnitude of ω, which measures the direct effect of real balances on output, does affect these inequalities. To assign a data-based range of values to ω, we use the strategy of Benhabib and Farmer (1996). Positing perfect competition, we compute ω as the share of real balances in production (im/y, where i is the nominal interest rate), both for the euro area and for the U.S. For the euro area (source: Statical Data Warehouse), averaging for the period 2003-2009, we compute a weighted average of gross interest rates on M2 deposits of non-financial corporations scaled it by gross value added in that sector. We find a value of 0.025. For the U.S., we use both a micro and a macro approach. At micro level, using data from Compustat for more than 15,000 non-farm non-financial corporations for the period 1982-2000 (to limit the effects of the swings induced by the use of "sweep" deposit accounts), we compute the panel ratio of cash plus short-term investments to sales, and multiply it by the M2 own rate provided by the FRED database, finding a value of 0.085. At the macro level, for the same years we use the Federal Reserve's flow-of-funds data for non-financial corporations[13] to compute the ratio of total deposits to sectoral gross value added, and multiply it by the 3-month T-Bill rate, finding a value of 0.01, i.e. the same value as in Benhabib and Farmer (1996). Further considering that these values should be slightly increased to account for currency in circulation and for other monetary items whose allocation is not sectoralized,[14] we consider any

[13] In the flow-of-funds data, non-financial sector liabilities might be underestimated because, while the holdings of each sector are computed exactly, households are assigned any money whose ownership cannot be accounted for (i.e., households' liabilities are computed subtracting the holdings of the other sectors from the aggregate values for the whole economy and assigning any difference to the households sector).

[14] For the euro area, the items that are included in the definition of money but are not sectoralized (and are therefore excluded from our calculations, which are based on official sectoralized data) are

value of ω in the range $[0.01, 0.1]$ as plausible.

For four values of ω in this range, Figure 1.3 shows (in red-solid) the LI loci for the current MIUF-MIPF model. The other parameters are calibrated as: $Q = 1$, $\beta = 0.025$, and $\phi = 0.02$ (implying steady state inflation of 2%).[15] The LI area on the left of each panel in Figure 1.3 corresponds to the high IES_m case of condition (10); the upper-right area, visible only in the bottom panels, corresponds to the case of a significant supply-side role of money, as in condition (9). This latter condition is shown to lead to LI only for values of ω on the high side of the interval: with $\omega = 0.1$, by condition (9) there would be LI for $\alpha \geq 10$ and $\gamma > 1$. On the low side, for ω equal to 0.01 or 0.03, condition (9) would support LI only for unrealistically high values of α.[16] By condition (10), however, LI could emerge for any ω, for the full range of realistic values of α and for $\gamma < 1$. How to calibrate a realistic range of values for γ? The inverse of this parameter is equal to the IES_m: in a simple MIUF model with separable preferences, it would also be equal to the absolute value of the interest rate elasticity of the demand for money. In this case, taking, say, 0.39, the value estimated by Chari *et al.* (2000) on U.S. data, γ should be set equal to 2.56; or taking the value of 1.2 (estimated by Kremer *et al.* (2003) for pre-euro Germany, a monetary-targeting country), γ should be set equal to 0.83, which is a value compatible with condition (10). But in a MIUF-MIPF model, in which money is demanded not only by households but also by firms with different objective functions and constraints, the CRRA parameter γ would no longer be equal to the inverse of the interest rate elasticity of the demand for money. In fact, in the current model with a single state variable, it would be a nonlinear function of m and of its ratio with c, which prevents a mapping with any empirically evaluated relation. The MIUF-MIPF model set out in Section 3.2, which distinguishes the demand for money by sector, will allow closer comparison between available estimates and the set of conditions for LI. Those results will further strengthen the conclusions that we draw here: on the one hand, the possibility of LI does exist in this simple MIUF-MIPF model (and is greater, the greater the output effect of money in the chosen range); on the other hand, its plausibility cannot be ruled out by empirical

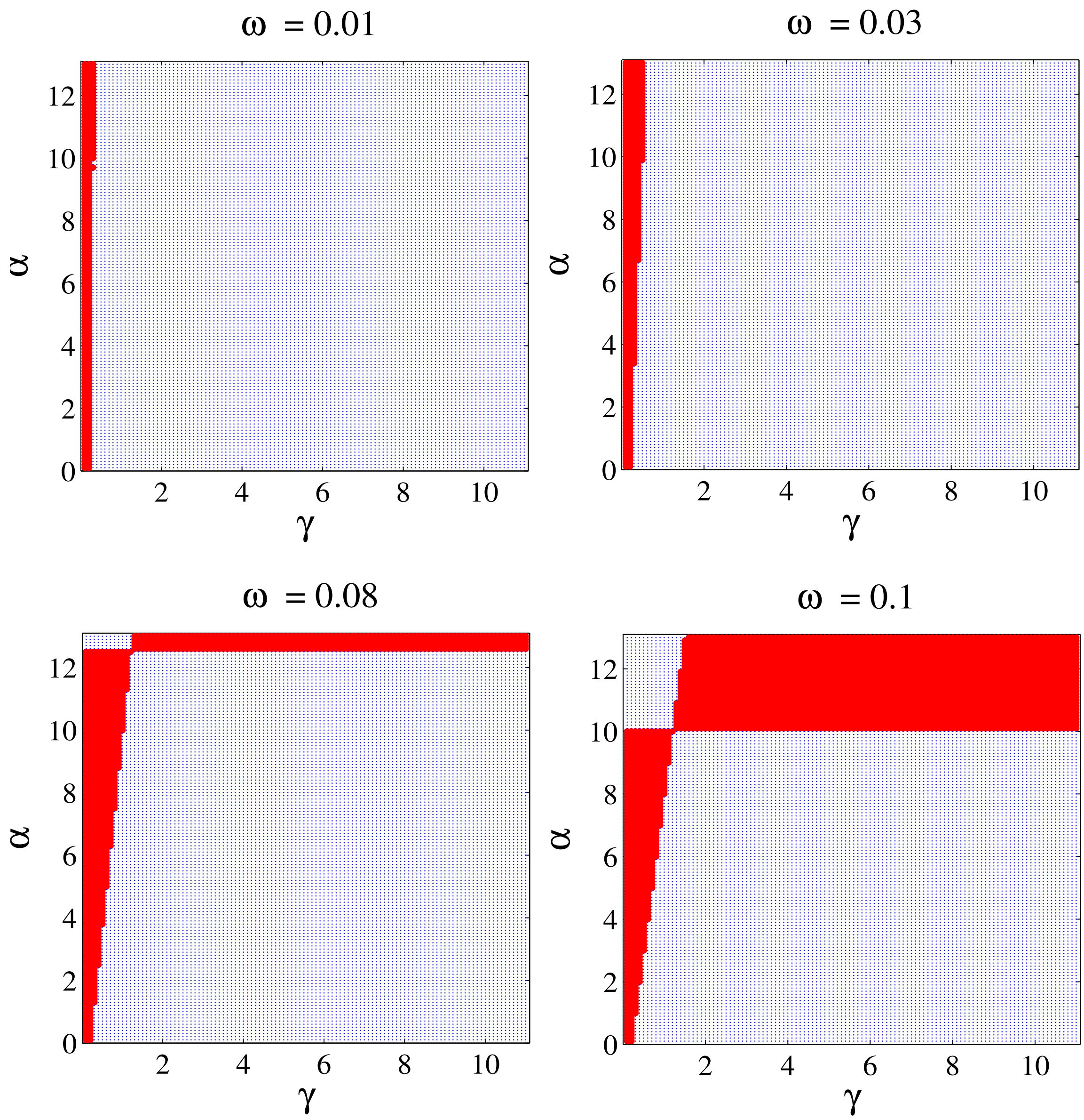
currency in circulation, money market fund shares/units and MFI debt securities with maturity up to two years. In the first quarter of 2006, these items accounted for 7.4%, 8.5% and 2.1% of the stock of M3 in the euro area, respectively, and, according to estimates by the ECB (2006), euro-area non-financial corporations were holding slightly less than 15% of the currency in circulation, more than one quarter of the money market fund shares/units, and 62.5% of the MFI debt securities with a maturity up to two years not held by non-monetary financial intermediaries.

[15] Robustness exercises with respect to these baseline parameters do not alter the basic picture depicted by Figure 1.3, although a lower value of Q, attenuating the impact of real balances on preferences, will obviously shrink in part the LI areas.

[16] This implausibility for low values of ω stems from the right hand side of condition (9), and corresponds to the implausibility found by Benhabib and Farmer (1996) in the simple MIPF model surveyed in our Section 2 (nested in the current model).

investigations to date.

Figure 1.3 - LI in an MIUF-MIPF economy
(LI in solid-red, saddlepath in dotted-blue)



3.2 MIUF-MIPF economies with distinct money

In the foregoing model, there is just one type of agent, a household-firm unit whose single holding of money facilitates at once both consumption and production. This specification thus does not allow extension to a decentralized economy, unless one distinguishes explicitly between households' money and productive money. In what follows we consider a model in which money used for utility purposes is distinguished from money used in production, as in the Taylor-rule model of Benhabib *et al.* (2001). Preferences are then defined over consumption and real balances for non-productive use, $U = U(c, m_{np})$; technology has instead real balances for productive use as the only variable input $y = h(m_p)$. With respect to the previous MIUF-MIPF model with no distinction, this setup delivers both a neater aggregate consistency condition on the money market (in nominal terms, it requires $M^s = M^d_{np} + M^d_p$ for equalization of supply and demand) and a modification of the LI sets that leaves the door open to an empirically effective possibility of multiple equilibria, hence of sunspot fluctuations.

In the equilibrium of this model with two distinct uses of money,[17] conditions $\frac{U_m}{U_c} = h_m$ and $h(m_p) = c$ lead to the following single law of motion in total real balances governing the economy

$$\frac{\dot{m}}{m} = \frac{\beta + \phi - h_m}{1 + \frac{\frac{[U_{cc}U_{mm} - (U_{cm})^2]}{U_c} m h_m + h_{mm} U_{cm} m}{\left(\frac{U_m}{U_c} U_{cc} - U_{mc}\right) h_m + \left(U_{mm} - \frac{U_m}{U_c} U_{cm}\right) + U_c h_{mm}}} \ . \tag{11}$$

Fixing one monetary steady state $m^* = m^*_{np} + m^*_p$, one has the following proposition.

Proposition 5 *In a Sidrauski-Brock economy with nonseparable preferences $U = U(c, m_{np})$ and strictly concave technology $y = h(m_p)$, equilibrium paths are locally indeterminate if and only if*

$$\frac{m^*}{U_c}\left(U_{cc}U_{mm} - U_{cm}^2 + h_{mm}\frac{U_{cm}}{U_m}U_c^2\right) > \left(U_{cm} - \frac{U_m}{U_c}U_{cc}\right) + \left(\frac{U_m}{U_c}U_{cm} - U_{mm}\right)\frac{U_c}{U_m} + \frac{U_c}{m_p^*}\varepsilon_{h_m|m_p} \tag{12}$$

As in the previous cases, condition (12) would be more likely to be fulfilled assuming $U_{cm} < 0$ and non-normality of both c and m_{np}. To avoid relying on these unrealistic features, we focus on conditions under which LI may emerge in the more restrictive case of $U_{cm} = 0$ (which implies normalities). This also ensures analytical convenience and comparability with our earlier results. In fact, with separable preferences the necessary and sufficient condition (12) translates into

$$\frac{\varepsilon_{U_c|c}\,\varepsilon_{y|m_p}\,\varepsilon_{U_m|m_{np}}}{m_p^*} + \frac{\varepsilon_{U_c|c}\,\varepsilon_{y|m_p}\,\varepsilon_{U_m|m^{np}}}{m_{np}^*} > \frac{\varepsilon_{U_c|c}\,\varepsilon_{y|m_p}}{m_p^*} + \frac{\varepsilon_{U_m|m_{np}}}{m_{np}^*} + \frac{\varepsilon_{h_m|m_p}}{m_p^*} \tag{13}$$

[17] When distinguishing, the subscript m refers to derivatives with respect to the correspondent type of real balances.

which we interpret as a condition on how important money has to be in the economy for LI to occur: it has to be that the weighted multiplicative impact of money on the economy be greater than the weighted additive impact. This is easier to see in an example economy: assuming the same functional forms as in the previous MIUF-MIPF model with no distinction between uses of money (i.e., assuming $U = \frac{c^{1-\alpha}}{1-\alpha} + Q\frac{m_{np}^{1-\gamma}}{1-\gamma}$ and $y = m_p^{\omega}$), condition (13) becomes

$$\frac{\alpha\omega\gamma}{m_p^*} + \frac{\alpha\omega\gamma}{m_{np}^*} > \frac{\alpha\omega}{m_p^*} + \frac{\gamma}{m_{np}^*} + \frac{1-\omega}{m_p^*} \tag{14}$$

The greater the impact of money on the economy, the more likely the interplay between money and price expectations that can result in multiple equilibrium real allocations.

We parameterize this economy just as in the previous Section 3.1, but with one additional refinement. While ω still ranges from 0.01 to 0.1, the calibration of γ can now be more precise. In fact, in the current model with distinct money the inverse of this parameter does correspond to (the absolute value of) the interest rate elasticity of the demand for money of the household sector. Most logarithmic estimates of demand for money performed on sectoral micro-data for both the euro area and the U.S. put this elasticity between 0.1 and 0.7: this is the interval covered by the estimates on U.S. data in Mulligan and Sala-i-Martín (2000), while point estimates for single European countries – e.g. by Attanasio *et al.* (2002) and Lippi and Secchi (2009) for Italy – range throughout that interval. Such values for the interest elasticity of household demand would correspond to values of γ between 1 and 11.[18] For this parameterization, Figure 1.4 shows that there is now a single boundary to the LI area, which has the shape of a translated equilateral hyperbola. In fact, condition (14) can be rewritten in homographic form as

$$\alpha\omega\gamma > \alpha\omega\frac{m_{np}^*}{m^*} + \gamma\frac{m_p^*}{m^*} + (1-\omega)\frac{m_{np}^*}{m^*}$$

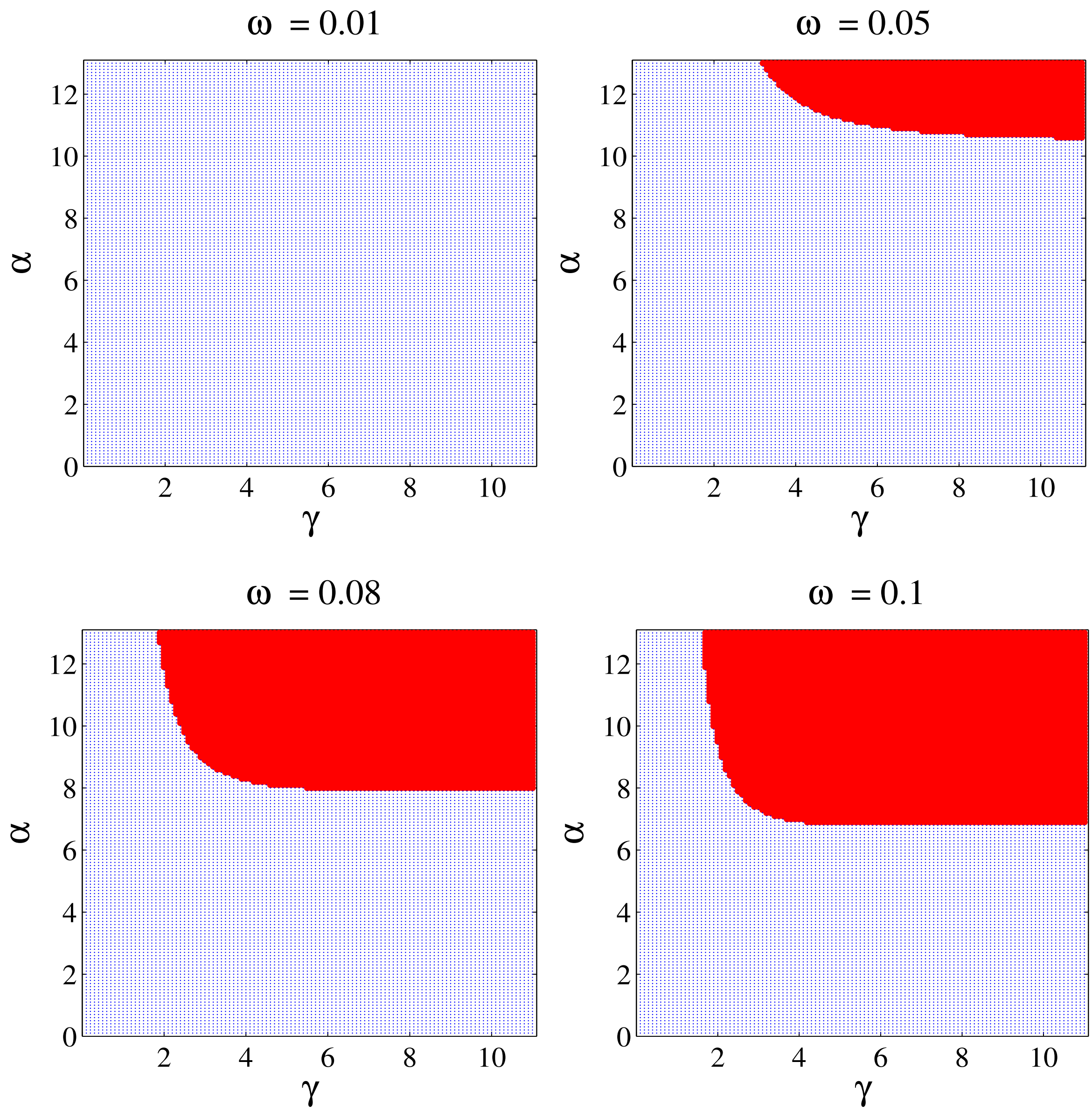
Above the boundary is the locus of combinations of α and γ that engender multiple equilibria: while for low ω no combination is plausible, as ω increases the area of plausibility expands. These are the cases in which around the steady state a sudden excess supply of real balances turns into an excess demand, which happens whenever the immediate consequence of an increase in money is a larger decrease in U_c than in U_m. How might this come about? Notice that a higher ω implies a lower own-elasticity of marginal productivity of real balances ($\varepsilon_{MP_m|m_p}$, here equal to

[18] The range should not to be seen as in contrast with the possibility of γ being below 1 in the MIUF-MIPF model with no distinction of money uses. In fact, the literature (ECB, 2006 and 2010, or Calza and Zaghini, 2010) reports that a greater weight of the portfolio motive for firms' demand for money induces much greater interest-rate sensitivity for that sector, and hence a greater interest-rate elasticity of the aggregate demand for money.

$1-\omega$). Suppose m rises from m^*, affecting m_{np} and m_p. If firms use more money, the decrease in h_m will be the smaller, the larger ω is. Equilibrium production, hence consumption, would increase, lowering U_c the more substantially, the higher the own-elasticity of marginal utility of consumption $\varepsilon_{U_c|c} = \alpha$. This would justify an increase in the demand for money holding on the part of households as well: in fact, by decreasing U_m, such an increase maintains the equilibrium condition $U_m/U_c = h_m$. At this point, larger aggregate money demand can trigger self-fulfilling inflation expectations compatible with equilibrium.

In this model with distinct money uses, the LI loci are different from our previous model. The area of possible multiplicity for low levels of γ disappears, while the upper-right area in the panels not only remains and is increasing in ω, but also intersects with a wider and more significant range of empirically relevant parameterizations. We take the existence of these areas in the current and previous MIUF-MIPF models as showing that LI is both a possible and a plausible outcome of SBC models with monetary targeting. Economic and policy analysis would do well to take this possibility into account when using models at least as complex as those posited here, like the typical DSGE models typically used as a basis for policy recommendations.

Figure 1.4 - LI in an MIUF-MIPF economy with distinct money
(LI in solid-red, saddlepath in dotted-blue)



4 Conclusions

The inclusion of monetary targeting among policy instruments is sometimes advocated as a tool for inoculating the economy from the instability risk associated with possible sunspot equilibria. This paper has checked the validity and scope of this property in the SBC framework, characterized by an exogenous money growth rule, a meaningful role for real balances, and perfect markets, thus isolating the role of money and monetary targeting. Benhabib and Farmer (1999, 2000) showed that local instability, in the form of multiple equilibria, might be induced by the interplay between the non-neutrality of real balances and price expectations. That is, a monetary expansion might stimulate the real economy to the point of generating excess net demand for real balances, which would make self-fulfilling inflation expectations compatible with stationary equilibria. In their analysis, however, this indeterminacy was deemed to be empirically relevant only when the effects of real balances spill over to labor supply and demand, heightening the impact on the real economy but inducing the considerable modeling cost of having non-standard slopes in labor market schedules.

This paper posits a basic economy with money and monetary targeting and produces two findings.

First, it identifies a general pattern for the real effects of money that can induce the shift in the propensity to demand real balances that is necessary for LI to occur. The pattern is identified by relating these real effects to the proportional reactions to a monetary change of marginal utilities and productivities. This pattern includes both direct and indirect effects; indirect effects via the labor market might play a role, but are not necessary – and are in fact excluded ex-ante from the analysis. Looking at the conditions that are nested in the pattern, we show that in more basic models LI can only arise under "wrong" assumptions on factors like cross-derivatives, normality of commodities, intertemporal elasticities of substitution, or direct output effects of money.

Based on the pattern found, we then pursued our principal purpose, which is to check whether LI could ever be both a possible and a plausible outcome in a monetary targeting economy with no non-standard features. We have shown that with some modest generalizations that enrich the ways in which a monetary expansion affects the real economy, such as jointly recognizing both to household and to firms a motive for holding money, LI can occur without unsatisfactory assumptions (on the labor market, say) and for empirically relevant parameter values. The conditions for LI that we derive in our simple MIUF-MIPF models are shown to have a reasonable chance of fitting the data.

One conclusion might be drawn in terms of model analysis and comparison to the real world. We have shown in a simple framework that with monetary targeting real effects of money are crucial for LI. But we have also shown that allowing multiple channels for these non-neutralities of money may make them "large enough" overall, and magnify their impact on the determinacy properties of the model. Thus our admittedly very simple model already serves to show the risks of instability that monetary targeting would entail in an economy in which money plays a significant role. But the point made by our simple model goes beyond its seemingly narrow scope, suggesting that in a model factoring in even a fraction of the larger set of sources of monetary non-neutrality that characterize actual economies there would be a much greater chance that LI will be both possible and plausible under monetary targeting. Along these lines, a model including features like nominal rigidities or financial frictions – which alter variables as relevant for economic choices as the real marginal cost of supplying a given output or the marginal productivity of non-monetary inputs – would be not only more realistic economically but also more likely to display multiple equilibria. The implication is that under monetary targeting a real-world economy would have an even greater risk of instability than in our simple examples.

A natural policy recommendation emerges as a bottom line. Our analysis warns of some limitations of policy rules based on monetary aggregates, and as such it could help define a correct monetary policy strategy. Making monetary targeting a part of the strategy may still be helpful for a long-run guide, but may neither guarantee that expectations remain anchored to fundamentals nor serve to help immunize the economy against the instability associated with possible multiple equilibria. In a world in which not only interest rate rules but also monetary targeting rules induce a risk of such instability, it would still appear to be prudent for monetary authorities to adopt a mixed strategy, with the composite use of various policy tools and attentive monitoring of a wide spectrum of monetary and real aggregates.

Appendix

Proof of Proposition 2: The equilibrium differential equation is

$$\frac{\dot{m}}{m} = \frac{\beta + \phi - \frac{U_m}{U_c}}{1 + m\frac{U_{cm}}{U_c}} \ .$$

Linearizing it in the neighborhood of the monetary steady state, so that derivatives are local, one has

$$\dot{m} = -\frac{\frac{m^*}{U_c}[U_{mm} - \frac{U_m}{U_c}U_{cm}]}{1 + m^*\frac{U_{cm}}{U_c}} (m - m^*) \ . \tag{15}$$

Given $U_c > 0$, stability of the linearized differential equation, and hence LI, will obtain if and only if the coefficient in (15) has a negative sign, i.e. if and only if

$$\text{sign}\left[m^*\left(U_{mm} - \frac{U_m}{U_c}U_{cm}\right)\right] = \text{sign}\left[1 + m^*\frac{U_{cm}}{U_c}\right].$$

The left-hand sign will be negative if $m^*\frac{U_{mm}}{U_m} < m^*\frac{U_{cm}}{U_c}$, which can be restated in the form $\varepsilon_{U_m|m} > \varepsilon_{U_c|m}$. The right-hand sign will be negative if $\varepsilon_{U_c|m} > 1$. Considering also the case of positive signs and combining, the Proposition holds. □

Proof of Proposition 3: In inequality (4), $\varepsilon_{U_c|m} > 1$ implies $U_{cm} < -\frac{U_c}{m^*}$, i.e. U_{cm} large and negative. Inequality (5) has $\varepsilon_{U_c|m} < 1$, so in principle U_{cm} either positive or negative but small; but it also has $\varepsilon_{U_m|m} < \varepsilon_{U_c|m}$, and hence $U_{cm} < \frac{U_c}{U_m}U_{mm}$. Given strict concavity, satisfying (5) therefore implies $U_{cm} < 0$. □

Proof of Proposition 4: Linearizing (6) around m^*, one has the following coefficient of the equilibrium equation in deviation form:

$$-\frac{m^*\{h_{mm}+\frac{1}{U_c}[(U_{mc}-\frac{U_m}{U_c}U_{cc})h_m+U_{mm}-\frac{U_m}{U_c}U_{cm}]\}}{1-\varepsilon_{U_c|c}\varepsilon_{y|m}-\varepsilon_{U_c|m}} \ .$$

Again, LI obtains whenever the sign of the numerator is the same as that of the denominator, which, recalling the proof of the analogous Proposition 2, is when one of the two inequalities (7) or (8) holds. □

Proof of Proposition 5: The steady state coefficient of the linearized monetary Euler equation (11) is

$$-\frac{h_{mm}m^*}{1 + \frac{\frac{h_m m^*}{U_c}\left(|H_U|+h_{mm}U_c^2\frac{U_{cm}}{U_m}\right)}{\left(\frac{U_m}{U_c}U_{cc}-U_{cm}\right)h_m+\left(U_{mm}-\frac{U_m}{U_c}U_{cm}\right)+U_c h_{mm}}} \ .$$

Using strict concavities and the equilibrium relation $U_m/U_c = h_m$, this coefficient will be negative if and only if the inequality in the Proposition is satisfied. □

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