Generational conflict and labor market turnover: a tale of employment protection and retirement age

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Abstract

This paper considers the political economy of employment protection and retirement age and analyzes interaction between labor market and the pension system in a setting where young outsiders are the median group in the electorate. These voters choose a bundle of policies which defines an optimal labor market turnover and realises their preferred allocation of income and unemployment risk across time and states of the world. Shocks on search costs or on the productivity gap between different types of workers cause variations in opposite directions in the degree of employment protection and in the mandatory retirement age.

In equilibrium this defines an inverse relationship between the degree of employment protection and the mandatory retirement age, and provides an interpretation, alternative to the "lump of labor fallacy", for the absence of empirical relationship between early retirement and youth unemployment, in the presence of imperfect labor markets.

JEL Classification: D72, H55, J63, E24.

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1 Introduction

Pension system reform is a widely debated issue in all industrialized countries and especially in Europe. A major factor in the debate is early retirement, which in the last thirty years has caused a steep increase in the dependency ratio and a dramatic reduction in labor force participation of elderly workers, particularly among the less educated (Gruber and Wise (1999); Blöndal and Scarpetta (1997)). A significant deterioration in the viability of unfunded pension systems has been worsened by an increase in expected life-span (Disney (2000)) and the recent slowdown in the world economy.

Encouraging continued work has been seen as the main way of alleviating the current financial distress of pension systems. This solution is proposed in the literature by Cremer and Pestieau (2003). It crucially relies on the assumption that competitive labor markets can fully adjust variations in the labor supply.

But an increase in retirement age raises two sets of problems.

The first has to do with the economic rationale for public pension programs. Many authors[1] consider pensions as a means to induce retirement, which allow younger, unemployed workers to take over jobs from older employed persons. Hence any reform which reduces the scope for this "forced" turnover is likely to meet strong opposition.

A second set of problems emerges if the assumption of competitive markets is dropped and equilibrium unemployment is included in the analysis. The benefits of late retirement do not stand out so clearly in this alternative scenario, because the pension system provides insurance against unemployment risk, as noted by Casamatta and De Paoli (2007), and because an increase in retirement age also leads to an increase in labor supply.

This second rationale[2] was used in some countries during the 1970s and 1980s to justify the relaxation of pension eligibility criteria in exchange for the employment of young workers[3]. A major problem with this argument is that there is currently no clear empirical evidence on the link between unemployment rate and retirement age. Some authors find no relationship at all (Diamond (2005)) while others even find a positive impact on employment (Keuschnigg and Keuschnigg (2004)). This has led to these policies being widely discarded as the result of the "lump of labor fallacy"[4].

The fact that real economies display involuntary unemployment conflicts with the evidence that labor supply variations following a change in retirement age do not affect unemployment. It is therefore necessary to supplement existing analyses to find an explanation for this puzzle which also has sizeable implications for the current debate on pension reform.

The present work integrates the study of the labor market and of

[1] See for instance Bhattacharya et al. (2001), Mulligan (2000a), Mulligan (2000b), Sala-i-Martin (1996)

[2] As reported by Gruber and Wise (1999) and Gruber, Milligan, and Wise (2009).

[3] Conde-Ruiz and Galasso (2003) provide a classification of early retirement institutions which underlines this exchange.

[4] See Mulligan and Sala-i-Martin (1999) for a description of the "lump of labor fallacy".

the pension system. It focuses on the relationship between the degree of employment protection and the retirement age which jointly define the level of turnover between young outsiders and old insiders on the labor market. In the spirit of Hoj et al. (2006) the analysis seeks to extend the basis for understanding the political economy influence prompting or hindering consensus around structural reforms in the Social Security System.

The main contribution of the paper is to provide an explanation, alternative to the "lump of labor", for the absence of an empirical relationship between retirement age and youth employment. It further sheds some light on the determinants which in a political equilibrium define the degree of employment protection and the retirement age.

The analysis considers a framework where young outsiders are median voters[5] in the choice over a bi-dimensional policy platform including the degree of employment protection and the legal retirement age. This is of course a limitation of the study, as in the real world effective retirement depends on a complex bunch of provisions. But as noted by the OECD (2011), p.20, "The retirement age is the most visible parameter of the pension system. As such it sends a clear signal for people in choosing when to cease to work". Moreover the observed and well-established trend in most countries toward a tightening of the qualifying conditions for early retirement[6] means a closer correspondence between legal and effective retirement age can be predicted for the future. In this sense the legal retirement age is interpreted in the present setting as a rough and ready indicator of the attitude of the Social Security System toward the timing of retirement.

In this setting there is no lump of labor. Indeed not only do employment protection and retirement age drive the substitution of old insiders with young workers, but they also affect overall efficiency and labor demand. The drivers of the equilibrium in fact concern efficiency. They are labor market frictions, namely the search cost paid by firms hiring new employees, and the productivity gap between high and low productivity workers. A shock in any of these structural parameters changes the median voter preferences and produces variations in opposite directions, in the degree of employment protection and in the retirement age.

As a consequence a pension system reform which reduces retirement age in response, for instance, to a change in the production technology

[5] Note that young outsiders are not only unemployed youngsters. The median voter is a new entrant worker who has as yet no well defined career path. This definition also includes those who are employed but have a short CV and little experience or no seniority.

[6] See OECD (2011).

which narrows the productivity gap among different types of workers (or to an increase in search costs), needs to be implemented jointly with an increase in the degree of employment protection. This was seen for instance in France where measures encouraging early retirement were implemented in 1971 and 1977 and administrative approval for layoffs was made compulsory in 1974. In a dual labor market where youngsters are the outsiders, the net effect on youth unemployment is necessarily ambiguous. On one hand the turnover between insiders and outsiders increases due to earlier retirement, on the other hand it is reduced by stronger protection of the insiders.

A second finding concerns pension system reforms. A negative relationship is found between the degree of employment protection and the retirement age in political equilibrium. This suggests that in the presence of an external constraint which make it necessary to postpone retirement (for instance to maintain the balance of the pension system), a political constituency will form if a reduction in the degree of employment protection is also implemented. The recent increase in retirement age in many developed countries thus makes greater labor market flexibility necessary. This is well illustrated by the case of Italy, which in 1995 simultaneously passed major reforms of the pension system and the labor market, implementing an increase in the legal retirement age and a reduction in employment protection.

The model is confirmed by data from a sample of 27 OECD countries. The graph below plots the five-year average of the standard deviations from the country mean over the period 1985-2009[7], of the Early Retirement Index (ERI) calculated by DICE at CESifo[8] and of the index of overall strictness of employment protection (EPI) calculated by the OECD and lagged one period[9].Using a robust regression technique to

[7] The countries included in the sample are: Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Japan, Korea, Mexico, Netherlands, Norway, Poland, Portugal, Slovak Republic, Spain, Sweden, Turkey, United Kingdom, United States. For the period 2005-2009, most of the data for 2009 is missing so that in fact the variable is a 4-year average. The same procedure, i.e. a reduction of the period considered, has been followed where other data was missing.

[8] The ERI is defined as 100% minus the participation rate for men aged 55-64. Its main drawback is that it describes the behavior of a paradigmatic agent and does not directly account for changes in the legislation. So the adoption of the ERI relies on the assumption that countries which provide more and better options for early retirement are also those where people in fact retire earlier.

[9] See OECD (2004) for a description of how the index is compiled. The reasons for this different timing have to do with the different nature of the indexes. The EPI directly accounts for variations in the legislation, but the ERI records changes in the behavior of economic agents following a variation in the legislation. In particular, a

control for outliers, the estimated coefficient associated with the ERI is positive and statistically significant for a 1% confidence interval.

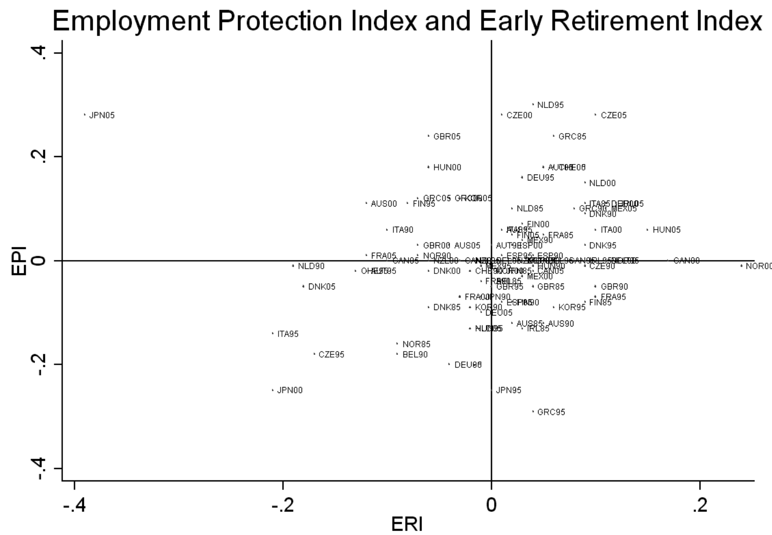

Figure 1

It appears that employment protection and retirement age move together and tend to be negatively correlated. Regimes with high employment protection display also low retirement age and viceversa.

Labor market dynamics are pivotal for this result. By means of modifications in the parameters of the Social Security System, young agents optimally allocate risk and income across time and states of the world in order to smooth consumption. An increase in retirement age, for a given degree of employment protection, transfers income from the present to the future, due to reduced turnover between insiders and outsiders. An increase in the degree of employment protection, for a given retirement age, produces analogous effects by transferring unemployment risk from the future to the present. The need for employment protection decreases when the retirement age is high; in a sense they are "substitute goods" for the young whose (implicit) prices in terms of efficiency are jointly determined by the productivity gap between different workers and by the size of the search costs on the labor market. This defines the optimal bundle of policies chosen by the young.

As noted by Saint-Paul (2000) economic institutions are often complementary, in the sense that the existence of one increases the political support for another. So there is also a form of substitutability of the

change in the legislation affecting retirement is likely to be recorded by the ERI with a time lag.

type already recognized for employment protection and unemployment benefit schemes (Buti and Sapir, 1998; Boeri et al., 2003; Sapir, 2005). Both employment protection and early retirement in fact serve as devices to smooth consumption and it is crucial to take account of this in designing intervention, in appropriately combining and sequencing labor market and pension system reforms.

1.1 Related literature

Analysis of the political preferences on the Social Security System in an environment where the pension system and an imperfect labor market interact is a fairly new field of research. It is largely a hybrid of two strands of research focusing on the effect of retirement on unemployment on one hand, and political preferences on specific aspects of the Social Security System on the other.

The paper by Casamatta and De Paoli (2007) is the closest to the present approach. The authors focus on retirement age and study an optimal pension system maximizing life-cycle utility when labor market imperfections define different levels of (exogenous) unemployment risk. They find that if unemployment benefits are low, when unemployment increases, reducing the optimal retirement age is optimal. In the voting equilibrium a lower than the optimal retirement age and a higher payroll tax rate are chosen. The median voter, closer to retirement than the newborn individual, in fact has a shorter horizon.

Conde-Ruiz and Galasso (2003) also study political preferences on retirement age. In the equilibrium of the voting game, an extensive pension system with generous early retirement provisions is supported by elderly with an incomplete working history and low-ability young who expect to retire early. The choice between work and leisure drives the results and the risk of unemployment is ignored.

Other authors, starting from the seminal contribution by Saint-Paul (2002), study the political economy of employment protection. In an insider-outsiders model, Boeri et al. (2003) find a trade-off between unemployment benefits and employment protection which are used alternatively as insurance devices against unemployment risk. Two main outcomes emerge in the voting game and depend on the composition of the electorate in terms of workers with different ability endowments. When low-ability insiders are in the majority, the equilibrium includes low unemployment benefits and high employment protection. If the number of high-skilled workers (either insiders or outsiders) and low-skilled unemployed is large enough, high unemployment benefits and low employment protection are chosen.

The present analysis shares with this literature the same focus on the interplay between different Social Security System institutions and contributes to the debate in two ways. First, a specific relationship, between employment protection and retirement age, which other authors do not consider, is highlighted. Moreover, an attempt is made to provide a micro foundation to labor market dynamics. This is lacking in the previous models where unemployment risk level and distribution are determined by exogenous parameters.

Labor market dynamics are studied by another strand of the literature which aims to assess the impact of pension reforms on efficiency and unemployment. In a union wage-setting model, Corneo and Marquardt (2000) find that contributions to the retirement system are harmful for growth. Moreover a shift from a PAYG to a funded system entails a Pareto improvement[10]. Keuschnigg and Keuschnigg (2004) consider an OLG setting with involuntary unemployment. Calibrating the model to Austrian data, they show that a rise in retirement age produces both an increase in growth and a reduction in unemployment.

These works all use a reverse angle perspective. Instead of studying the effect of labor market dynamics on voter preferences to assess the feasibility of a pension system reform, they show its potential benefits and take for granted that a political constituency forms. The present analysis is thus a useful complement to this literature.

1.2 Outline of the paper

The paper has the following structure: Section 2 presents the model describing the labor market and the pension system. Section 3 studies firm behavior, agent preferences and election outcomes and characterizes the equilibrium. Section 4 concludes.

2 The Economy

Consider an economy where each agent lives for two periods, and is young in the first and old in the second. The population is stationary and the size of each generation is normalized to one. Worker's utility is a function, $U(c_t, c_{t+1})$, of present consumption, c_t, and future consumption, c_{t+1} and, as in Casamatta and De Paoli (2007), does not include an inter temporal discount factor:

$$U(c_t, c_{t+1}) = u(c_t) + u(c_{t+1}).$$

Utility within a single period, $u(c_t)$ is a concave function of present consumption which exhibits DARA and CRRA:

$$u(c_t) = \nu \cdot (c_t)^{1-\mu}.$$

[10] A similar result is found also by Demmel and Keuschnigg (2000)

As a consequence the inter temporal elasticity of substitution is also constant. Agents have no disutility from working and labor supply is perfectly inelastic for any positive wage. Output is not storable and there are no savings. This assumption greatly simplifies the analysis and also appears in Boeri et al. (2003) and in Casamatta and De Paoli (2007). There are no unemployment benefits and workers without a job earn no income. The effects of these simplifying assumptions are discussed in detail in Section 3.4.

The Social Security System includes a pay-as-you-go pension system. Pensions are financed by worker contributions only. Each retiree receives a fraction of the net wage rate $(1-\tau_t)\,W_t$ which is equal to the inverse of the dependency ratio. Under the hypothesis that wages are set by a monopoly union whose preferences are biased towards insiders, the replacement rate of the pension system equals the employment rate $n_t \leq 1$, as it will be made clear in Section 3.1. Hence pension payments, P_t, which do not depend on the contributions to the system[11] are defined by:

$$P_t = (1-\tau_t)\, n_t \cdot W_t$$

where τ_t is the contribution rate which guarantees balanced budget condition to hold.

Retirement is mandatory at date $\gamma + \theta\,(1-\gamma)$ with $1 > \gamma > 0$[12] and $\theta \in [0,1]$. The elderly supply labor for a fraction $\gamma + \theta\,(1-\gamma)$ of the period, and retire in the last fraction $(1-\theta)\,(1-\gamma)$ of it. Youngsters remain on the market (either employed or actively looking for a job) for the whole period.

Firms produce a unique good which is sold on a competitive market and is also the numeraire. Each firm is randomly matched with one worker. Every time a vacancy is filled through random matching, the firm bears a search cost which depends on labor market tightness. Following Mortensen and Pissarides (1998), the search process can be described as follows. Each unmatched firm has a list of all unemployed workers and makes phone calls to contact them; each unemployed worker has a list of all potential employers and makes phone calls to contact

[11] The same assumption is made by Casamatta and De Paoli (2007). Assuming that workers who did not contribute to the pension system receive only a fraction of their last net wage does not change the results.

[12] γ is the earliest date for retirement which is an arbitrary point in time, different from the beginning of the period, when the selection process takes place. This is required for a meaningful labor market dynamics and for model tractability. The latest date for retirement is the end of the period; in this case there is no retirement at all.

them. Analytically this is defined by the function:

$$s(\phi, k, V, U, \beta) = \phi(k + \beta \cdot V - U)$$

where $k \geq 1$ is a fixed search cost[13], V and U denote respectively the number of vacancies and the number of unemployed workers at a given point in time, $\beta \geq 0$ is a technological parameter which summarizes both the search intensity of firms and workers, and $1 \geq \phi > 0$ is a scale factor. Search costs increase with V which defines the probability that a firm contacts a worker who has already been hired by another firm. The probability of being contacted by an unemployed worker decreases as V gets larger. But as U gets larger, the probability of a firm contacting an already hired worker decreases and its probability of being contacted by an unemployed worker increases.

Every period a fraction, $\eta \in [0, 1]$, of all the firms in the market is allowed to fire current employees with low labor productivity. Among the elderly who are employed there are high and low productivity workers in the same proportion; young agents on the contrary are all alike. A firm output is 1 if it is matched with a low productivity worker and $1 + \alpha$ if it is matched with a high productivity type, where $\alpha > 0$ is the productivity gap between them. If a firm hires a young agent the output produced is $1 + \frac{1}{2} \cdot \alpha$, i.e. the average productivity in the economy. Unemployed old agents are all low productivity workers.

The dynamics of labor productivity describe a process of human accumulation through learning by doing. People who are hired might accumulate human capital in excess of its depreciation, or they might not, depending on personal characteristics. High types increase human capital and display above average productivity. Low types and unemployed old workers are only affected by human capital depreciation and have below average productivity.

Whether an old agent is high or low type is revealed at different points in time to the worker him/herself and to the employer. At the beginning of period t, the elderly find out their productivity and elections take place. Hence voters differ by age, productivity and working status. Those in the old generation who are employed can be either high productivity (HP) or low productivity agents (LP). Those who do not have a job are low type unemployed (U). Finally there are the young (Y) whose present productivity is known and equals $1 + \frac{\alpha}{2}$ but whose future type is uncertain.

The parameters defining the mandatory retirement age, θ, and the degree of employment protection $1 - \eta$ are chosen through majority vot-

[13] It is required for $k \geq 1$ to have $s(\phi, k, V, U, \beta) \geq 0$ for any value of V and U.

ing. A problem in this setting arises because the young generation represents half of the electorate. If the whole old generation chooses the same pair (η, θ) which is different from that chosen by the young, a tie emerges in the voting process. When this happens it is assumed that the choice of the old prevails; this is coherent with the findings of a number of papers where the pressure exerted by the elderly drives the outcomes of the political process[14].

After the elections a monopoly union defines the wage rate W_t and the employment level n_t, in order to maximize the function:

$$\Omega\left(W_t, n_t\right) = \left(W_t\right)^{\sigma}\left(n_t\right)^{1-\sigma}$$

where σ is a parameter which captures the relative intensity of the two targets. Firms are wage-takers. Once W_t is observed, existing firms decide whether they are going to stay on the market while new firms decide whether to enter. Hence n_t is the total number of active firms in period t[15]. Only when the production starts is the type of an old worker revealed to the employer. But in spite of this, both existing firms and new entrants produce until the end of the period even if losses are generated[16].

The timing of the game is the following:

1. Old agent type is revealed and the political process defines θ and η.
2. The monopoly union sets W_t and n_t.
3. Firms decide whether to remain on/enter the market or not.
4. Firms find out old worker productivity level and the selection process takes place.
5. At time $\gamma + \theta\left(1 - \gamma\right)$ old workers retire.

3 Equilibrium Analysis

3.1 Labor Market Dynamics

Consider a steady state equilibrium where all the relevant variables are constant.

[14]See Galasso and Profeta (2002).

[15]In a sense they retain the "right to manage" the employment level.

[16]For instance there might be fees and penalties due to customers and suppliers with contracts in progress, or fixed costs to be covered by proceeds of sales.

Firm i decides to remain on or enter the market if expected profits are non negative, i.e. if:

$$E\left[\pi^{i}\left(\alpha;\theta;\eta;W;n;k;\phi\right)\right]=E\left[y^{i}\right]-W-E[s\left(\phi,k,V,U,\beta\right)]\geq 0$$

where y^i defines firm i expected production and sale proceeds. The aggregate labor demand of the economy is given by the total number of firms on the market, n.

Consider now the maximization problem of the monopoly union. When setting the wage rate, it must take company decisions to remain on or leave the market as given. Since $\Omega'_W \geq 0$ and $\Omega'_n \geq 0$ hold, it is optimal to choose for each level of the aggregate demand, n, the wage rate which realizes the zero-profit condition for individual firms.

Tractability of the model requires the assumption that union preferences are biased toward the older generation. The choice of W is restricted to all cases where $n \leq 1$ and the wage rate exceeds the level which allows the young to enter immediately the employment pool. So when the elderly retire, youngsters are hired and at the beginning of the next period, only old workers are in employment. No new firm enters the market.

This means at date θ for every retiree the number of employed workers is n, which is also the replacement rate for the pension system. A balanced budget condition for the pension system thus is defined by the equality:

$$n\cdot W\left(1-\tau\right)\left(1-\gamma\right)\left(1-\theta\right)=n\cdot W\cdot\tau.$$

and the equilibrium contribution rate is:

$$\tau^{*}=\frac{\left(1-\gamma\right)\left(1-\theta\right)}{1+\left(1-\gamma\right)\left(1-\theta\right)}.$$

In a steady state, at the beginning of period t, firm i can face three different circumstances:

1. Initially matched with an HP old worker; this occurs with probability $\frac{1}{2}$.

2. Initially matched with an LP old worker and permitted to fire the employee; this occurs with probability $\frac{\eta}{2}$.

3. Initially matched with an LP old worker and not permitted to fire the employee; this occurs with probability $\frac{1-\eta}{2}$.

In the first case, there are no incentives to fire current employees whose productivity exceeds that of a new match with a young agent.

As a consequence, firm i produces the output $1+\alpha$ up to date $\gamma + \theta(1-\gamma)$, and after retirement, is matched with a young worker with average productivity $1+\frac{\alpha}{2}$ until the end of the period. Note that firms never hire low productivity unemployed old workers whose productivity is lower than that of a young agent. This is the case because $n \leq 1$.

Firms which are initially matched with an LP old worker increase profits by firing current employees to hire an unemployed youngster. So with probability η, the selection process matches these firms with a young worker until the end of the period. With complementary probability they produce output 1 until retirement and are then matched with a young worker.

Consider now search costs and the size and composition of the flows in and out the unemployment pool. After the selection process, $\frac{n \cdot \eta}{2}$ firms fire their low productivity employees who are replaced through a random draw from the young generation. Total unemployment at this date is $U = 1$ and old unemployed who are never hired, are classified as discouraged workers. Each unemployed youngster thus has a probability $\frac{n \cdot \eta}{2}$ of being hired at the beginning of each period. After retirement, old workers leave their job which means that $n\left(1-\frac{\eta}{2}\right)$ new vacancies are created. This is also the (*ex ante*) probability which a young worker is hired at that moment in time. Total unemployment at this date is $U = 1 - \frac{n \cdot \eta}{2}$. The expected value of the search cost borne by each individual firm is thus given by:

$$
\begin{aligned}
& E\left[s\left(\phi, k, V, U, \beta\right)\right] \\
= & \phi\left\{k - 1 + \beta \cdot n\left[\left(\frac{\eta}{2}\right)^2 + \left(1-\frac{\eta}{2}\right)^2\right] + n\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}.
\end{aligned}
$$

As a consequence the zero-profit condition for firm i can be rewritten as:

$$
\begin{aligned}
& E\left[\pi^i\left(\alpha; \theta; \tau; \eta; W; n; k; \phi\right)\right] \\
= & 1 + \frac{\alpha}{2}\left\{1 + \frac{\eta}{2}\left[\gamma + \theta\left(1-\gamma\right)\right]\right\} - W + \\
& -\phi\left\{k - 1 + \beta \cdot n\left[\left(\frac{\eta}{2}\right)^2 + \left(1-\frac{\eta}{2}\right)^2\right] + n\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\} = 0.
\end{aligned}
$$

The inverse aggregate labor demand function is implicitly defined by:

$$
\begin{aligned}
W = & 1 + \frac{\alpha}{2}\left\{1 + \frac{\eta}{2}\left[\gamma + \theta\left(1-\gamma\right)\right]\right\} + \\
& -\phi\left\{k - 1 + \beta \cdot n\left[\left(\frac{\eta}{2}\right)^2 + \left(1-\frac{\eta}{2}\right)^2\right] + n\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}
\end{aligned}
$$

Assume that $2 > k \geq 1$ and $\phi \leq 1$ so that $W \geq 0$ always holds for $n \leq 1$.

The maximization problem for the union is the following:

$$
\begin{aligned}
&Max_{W,n}\left(W\right)^{\sigma}\left(n\right)^{1-\sigma}\\
&s.t.\\
W \leq\; & 1+\frac{\alpha}{2}\left\{1+\frac{\eta}{2}\left[\gamma+\theta\left(1-\gamma\right)\right]\right\}+\\
&-\phi\left\{k-1+\beta\cdot n\left[\left(\frac{\eta}{2}\right)^{2}+\left(1-\frac{\eta}{2}\right)^{2}\right]+n\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}.
\end{aligned}
$$

The Kuhn-Tuker conditions are:

$$\frac{\delta\Lambda}{\delta W}=\sigma\left(\frac{n}{W}\right)^{1-\sigma}-\lambda=0$$

$$\frac{\delta\Lambda}{\delta n}=\left(1-\sigma\right)\left(\frac{W}{n}\right)^{\sigma}-\lambda\cdot\phi\left\{\beta\left[\left(\frac{\eta}{2}\right)^{2}+\left(1-\frac{\eta}{2}\right)^{2}\right]+\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}=0$$

and

$$
\begin{aligned}
\frac{\delta\Lambda}{\delta\lambda}=\;&1+\frac{\alpha}{2}\left\{1+\frac{\eta}{2}\left[\gamma+\theta\left(1-\gamma\right)\right]\right\}-W+\\
&-\phi\left\{k-1+\beta\cdot n\left[\left(\frac{\eta}{2}\right)^{2}+\left(1-\frac{\eta}{2}\right)^{2}\right]+n\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}=0.
\end{aligned}
$$

Substituting $\lambda=\sigma\left(\frac{n}{W}\right)^{1-\sigma}$ in the second equation and solving W gives:

$$W=n\cdot\frac{\sigma}{1-\sigma}\cdot\phi\left\{\beta\left[\left(\frac{\eta}{2}\right)^{2}+\left(1-\frac{\eta}{2}\right)^{2}\right]+\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}.$$

Substitute now the last condition in the equation above and solve n to obtain:

$$n^{*}=\left(1-\sigma\right)\frac{1+\frac{\alpha}{2}\left\{1+\frac{\eta}{2}\left[\gamma+\theta\left(1-\gamma\right)\right]\right\}-\phi\left(k-1\right)}{\phi\left\{\beta\left[\left(\frac{\eta}{2}\right)^{2}+\left(1-\frac{\eta}{2}\right)^{2}\right]+\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}}.$$

Note that for high enough σ it is always the case that total employment includes less than one whole generation.

Lemma 1 *For every $\theta\in[0,1]$ and $\eta\in[0,1]$ there exists a cutoff $0\leq\hat{\sigma}<1$ such that for every $\sigma\geq\hat{\sigma}$ the inequality $n^{*}\leq1$ holds.*

Proof. In order to have $n^{*}\leq1$, it must be the case that:

$$1\geq\left(1-\sigma\right)\frac{1+\frac{\alpha}{2}\left\{1+\frac{\eta}{2}\left[\gamma+\theta\left(1-\gamma\right)\right]\right\}-\phi\left(k-1\right)}{\phi\left\{\beta\left[\left(\frac{\eta}{2}\right)^{2}+\left(1-\frac{\eta}{2}\right)^{2}\right]+\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}}.$$

Therefore it is always possible to find a σ large enough for the previous inequality to hold, since its right hand side approaches 0 as σ approaches 1. ■

Note now that since $\frac{\delta n^*}{\delta \alpha} \geq 0$ and $\frac{\delta n^*}{\delta \phi} \leq 0$, assuming that an upper and a lower bound exists respectively for the productivity gap, $\bar{\alpha} > 0$, and for the scale factor, $\underline{\phi} > 0$, it is possible to find a threshold for the parameter σ such that $n^* \leq 1$ holds for all the possible values of α and ϕ.

Definition 2 *Define $\hat{\sigma}$ as the level for σ such that if $\alpha = \bar{\alpha}$ and $\phi = \underline{\phi}$, for $\sigma \geq \hat{\sigma}$, $n^* \leq 1$ holds for every $\theta \in [0, 1]$ and $\eta \in [0, 1]$.*

Assume then that $\sigma \geq \hat{\sigma}$. It is possible now to solve W^*:

$$W^* = \sigma \left(1 + \frac{\alpha}{2} \left\{1 + \frac{\eta}{2} [\gamma + \theta (1 - \gamma)]\right\} - \phi (k - 1)\right)$$

Before proceeding further, it is useful to analyze some properties of the equilibrium values of the wage rate and of total employment. Note initially that increasing retirement age causes an increase in both the wage rate and in total employment since:

$$\frac{\delta W^*}{\delta \theta} = \sigma \cdot \frac{\alpha \cdot \eta}{4} (1 - \gamma) \geq 0$$

and

$$\frac{\delta n^*}{\delta \theta} = (1 - \sigma) \frac{\frac{\alpha \cdot \eta}{4} (1 - \gamma)}{\phi \left\{\beta \left[\left(\frac{\eta}{2}\right)^2 + \left(1 - \frac{\eta}{2}\right)^2\right] + \left(1 - \frac{\eta}{2}\right) \frac{\eta}{2}\right\}} \geq 0$$

hold. This is due to the fact that the time span where workforce composition is at its best gets longer, hence increasing firm profits.

A similar result is obtained with regard to employment protection. Consider what happens to the equilibrium wage rate if employment protection is reduced:

$$\frac{\delta W^*}{\delta \eta} = \sigma \cdot \frac{\alpha}{4} [\gamma + \theta (1 - \gamma)] \geq 0.$$

W^* increases because a larger fraction of firms is allowed to substitute low productivity workers. An analogous effect is also at work when total employment is considered. But a further element contributes to the definition of the net effect of η on n^*, namely the variation in search costs. Note that lower employment protection on one hand increases vacancies and search costs, and on the other hand reduces them by increasing the probability that a firm is involved in the selection process.

In fact the probability of hiring a new worker is shifted from a date where unemployment is low (when retirement happens) to a date where it is high (at the beginning of the period when the selection process takes place). The net effect of a change in employment protection is ambiguous and defined by:

$$\frac{\delta n^*}{\delta \eta} = (1-\sigma)\frac{\frac{\alpha}{4}\left[\gamma+\theta\left(1-\gamma\right)\right]}{\phi\left\{\beta\left[\left(\frac{\eta}{2}\right)^2+\left(1-\frac{\eta}{2}\right)^2\right]+\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}}+$$
$$+(1-\sigma)(1-\eta)\left(\beta-\frac{1}{2}\right)\frac{1+\frac{\alpha}{2}\left\{1+\frac{\eta}{2}\left[\gamma+\theta\left(1-\gamma\right)\right]\right\}-\phi\left(k-1\right)}{\phi\left\{\beta\left[\left(\frac{\eta}{2}\right)^2+\left(1-\frac{\eta}{2}\right)^2\right]+\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}^2}.$$

Since the available evidence predicts a positive effect on the employment rate of a reduction in employment protection (of an increase in η)[17], it is here assumed that $\beta \geq \frac{1}{2}$ so that $\frac{\delta n^*}{\delta \eta} \geq 0$ holds for every $\theta \in [0,1]$ and $\eta \in [0,1]$. It is worth noting that both variations in η and θ affect the equilibrium of the labor market in a way which contrasts with the hypothesis of a "lump of labor".

3.2 Agent Preferences

Election outcomes crucially depend on agent preferences. The present analysis characterizes the maximization problem of the different workers.

3.2.1 HP Workers

The horizon of an old worker is limited to one period and he/she maximizes current utility. HP agents face no uncertainty and simply maximize the sum of labor income earned until date θ, and pension payments received until the end of the period. They solve the problem:

$$Max_{\eta,\theta} u^{HP}(c)$$
$$= \nu\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\left[\gamma+\theta\left(1-\gamma\right)+n^*\left(1-\theta\right)\left(1-\gamma\right)\right]^{1-\mu}$$

which defines the following preferences:

[17] The empirical evidence on the effect of a decrease in the degree of employment protection is mixed (see Bassanini and Duval (2006) for a detailed review). But greater consensus exists in the literature on the fact that more rigid employment laws are associated with high unemployment for the young (Botero et al., (2004)). Moreover as noted by Bassanini and Duval (2006), the absence of clear-cut evidence on the impact of employment protection on aggregate unemployment may depend upon the fact that EPL on regular contracts exerts upward pressure on unemployment and EPL on temporary contracts pushes in the opposite direction. Since the present setting mainly concerns labor market dynamics regarding young workers and insiders of the labor market (with regular contracts), it is not too far-fetched to assume a positive effect of a decrease of employment protection on young outsider employment.

Proposition 3 *High productivity worker optimal choice entails no employment protection and no retirement, i.e.* $\eta^{HP} = 1$ *and* $\theta^{HP} = 1$.

Proof. See the Appendix. ■

A reduction in employment protection increases the average productivity of the economy. This results in higher expected firm profits and allows the monopoly union to raise both wage and employment rates. Since the selection process does not affect HP old workers who are always employed until retirement, their only concern is efficiency and their preferred choice is no employment protection.

An increase in the retirement age enhances the effects of the selection process. This again results in an increase in the net wage rate due both to a rise in W^* and a reduction in τ^*. Hence, HP old workers who are better off working rather than retiring, because $P \leq (1-\tau^*) W^*$ holds, choose no retirement.

3.2.2 Unemployed Old Workers

In equilibrium, there are $1 - n^*$ unemployed old workers who are never hired during the whole period. Hence they maximize overall payments obtained by the pension system and solve the problem:

$$Max_{\eta,\theta} u^U(c) = \nu \left[\frac{W^*}{1 + (1-\gamma)(1-\theta)}\right]^{1-\mu} \left[n^*(1-\theta)(1-\gamma)\right]^{1-\mu}$$

which defines the following preferences:

Proposition 4 *Unemployed old worker optimal choice entails no employment protection and early retirement, i.e.* $\eta^U = 1$ *and* $\theta^U = 0$.

Proof. See the Appendix. ■

Improving the efficiency of the selection process allows firms to pay higher wages. In turn this also raises pension benefits. Unemployed old workers thus choose no employment protection. A decrease in the mandatory retirement age increases pension payments to these agents. Hence they choose to retire as soon as possible.

3.2.3 LP Workers

LP old workers amount to $\frac{n^*}{2}$ and maximize expected current utility in two possible states of the world:

- They are fired during the selection process; this happens with probability η and implies that the worker is unemployed until retirement.

- They are not fired; this happens with probability $(1-\eta)$ and implies that the worker is employed until retirement.

The maximization problem for these agents is thus:

$$Max_{\theta,\eta}u^{LP}(c)=\eta\cdot u^{U}(c)+(1-\eta)\,u^{HP}(c)$$

LP workers are the only agents who lose their job during the selection process, so their optimal choice η^{LP} crucially depends on the trade-off between efficiency and insurance which is defined by the joint action of two effects. One is the standard gain in efficiency which allows the firms to pay higher wages and also increases pension payments. The second effect is negative and depends on the increase in unemployment risk due to a reduction in employment protection. The choice of θ^{LP} also results from a trade-off between efficiency and insurance. A reduction in retirement age shortens the unemployment time span but at the cost of a loss in efficiency and of a decrease in the equilibrium wage.

3.2.4 Young Workers

Young outsiders maximize expected utility over two periods. In the first period they face three possible situations:

- They are hired after the selection process and work until the end of the period. This happens with probability $\frac{\eta\cdot n^{*}}{2}$.
- They are hired after retirement and work for a fraction $(1-\gamma)(1-\theta)$ of the period. This happens with probability $n^{*}\left(1-\frac{\eta}{2}\right)$.
- They are not hired in the first period. This happens with probability $1-n^{*}$.

In the second period young agents have the same probability, $\frac{n^{*}}{2}$, of obtaining the expected utility of an LP or of an HP old worker. With probability $1-n^{*}$ they are U workers. Hence their maximization problem is the following:

$$\begin{aligned}&Max_{\eta,\theta}u^{Y}(c)\\=&\frac{\eta\cdot n^{*}}{2}\cdot\nu\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}+n^{*}\left(1-\frac{\eta}{2}\right)\cdot\nu\left[\frac{W^{*}(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}+\\&+\frac{n^{*}}{2}\cdot u^{HP}(c)+\frac{n^{*}}{2}\cdot u^{LP}(c)+(1-n^{*})\cdot u^{U}(c).\end{aligned}$$

The effects described above for the maximization problems of the elderly characterize the utility of the young in the second period. They are weighted according to the probability that each state of the world occurs. Hence the choice of young outsiders is also influenced by the fact that a variation in η has a direct effect on the probability of being fired for an LP worker. By changing the equilibrium level of employment, n^*, moreover, both η and θ indirectly affects the probability attached to each state of the world of the second period.

Focus now on the first period. The impact of lowering employment protection is only positive. Making the selection process more effective by widening the set of firms which fire current employees and by increasing overall employment increases the probability of an early entry into the employment pool. The standard positive effect on the equilibrium wage also remains.

Consider now the mandatory retirement age. The impact of an increase in the retirement age on the expected utility of the first period is ambiguous. The positive effects on W^* and n^* are balanced by an increase in the expected length of unemployment resulting from a delayed generational turnover.

Note finally that since young worker future productivity is uncertain, their expected income (net of the effects on overall efficiency) is invariant to both inter and intra-generational redistribution. As a consequence, the main trade-off is between total expected income and its allocation across time and states of the world.

3.3 Elections

The present section considers the equilibrium for the elections; as in Casamatta and De Paoli (2007) and Boeri et al. (2003), the policy space is bi-dimensional, (θ, η), and a Condorcet winner which defines a Nash equilibrium for the game does not exists. The analysis thus focuses, following Shepsle (1979), and more recently Persson and Tabellini (2000), on the class of structure-induced equilibria. Restrictions are introduced in the model in order to partition the voting process in two unidimensional voting stages and define a simultaneous issue-by-issue voting game[18].

Proposition 5 *The pair $\left(\eta^Y, \theta^Y\right)$ which maximizes young agent utility is a structure-induced equilibrium of the game.*

Proof. In order to prove that the pair $\left(\eta^Y, \theta^Y\right)$ is a structure induced equilibrium for the game, it must be shown that young agents are median voters in both the voting on η and on θ.

[18] See Conde-Ruiz and Galasso (1999 and 2003) for a discussion of the required assumptions and Boeri et al. (2003) for a description of the concept of structure induced equilibrium.

Consider then the choice over the mandatory retirement age. As proven in Propositions 3 and 4, the preferences of HP and U workers are single-peaked and monotone since $\frac{\delta u^{HP}(c)}{\delta\theta} \geq 0$ and $\frac{\delta u^{U}(c)}{\delta\theta} \leq 0$. Hence for any value of $\eta \in [0,1]$ there is always a majority sustaining the young worker preferred solution, $\theta^{Y}(\eta)$, against any other alternative.
Focus now on the voting over the degree of employment protection. In this case too the preferences of HP and U agents are single-peaked and monotone since $\frac{\delta u^{HP}(c)}{\delta\eta} \geq 0$ and $\frac{\delta u^{U}(c)}{\delta\eta} \geq 0$[19]. Consider now the first derivative of $u^{Y}(c)$ with respect to η: using simple algebra it is possible to obtain:

$$\begin{aligned}
\frac{\delta u^{Y}(c)}{\delta\eta} = & \frac{n^*}{2}\cdot\nu\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\left[1-(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}\right]+ \\
& +\frac{1}{2}\cdot\frac{\delta n^*}{\delta\eta}\cdot\eta\cdot\nu\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}+ \\
& +\frac{\eta\cdot n^*}{2}\cdot\frac{\nu(1-\mu)\frac{\delta W^*}{\delta\eta}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}+ \\
& +\frac{1}{2}\cdot\frac{\delta n^*}{\delta\eta}(1-\eta)\nu\left[\frac{W^*(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}+ \\
& +\frac{1}{2}\cdot\frac{\delta n^*}{\delta\eta}\left\{\nu\left[\frac{W^*(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}-u^{U}(c)\right\}+ \\
& +n^*\left(1-\frac{\eta}{2}\right)\nu(1-\mu)\frac{\frac{\delta W^*}{\delta\eta}(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}+ \\
& +\frac{\delta n^*}{\delta\eta}\cdot\frac{1}{2}\left[u^{HP}(c)-u^{U}(c)\right]+\frac{n^*}{2}\cdot\frac{\delta u^{HP}(c)}{\delta\eta}+\frac{\delta n^*}{\delta\eta}\cdot\frac{1}{2}\cdot u^{LP}(c)+ \\
& +(1-n^*)\frac{\delta u^{U}(c)}{\delta\eta}+\frac{n^*}{2}\cdot\frac{\delta u^{LP}(c)}{\delta\eta}.
\end{aligned}$$

Note that for every $\theta \in [0,1]$ all the terms in the previous equality are positive except for $\frac{\delta u^{LP}(c)}{\delta\eta}$. Hence when $\frac{\delta u^{Y}(c)}{\delta\eta} = 0$ holds, it must be the case that $\frac{\delta u^{LP}(c)}{\delta\eta} < 0$. This has two main implications. The first is that $\eta^{Y}(\theta) > \eta^{LP}(\theta)$ holds and $\eta < \eta^{Y}(\theta)$ is never an equilibrium outcome because $\eta^{Y}(\theta)$ is supported by a majority including the young, HP and U agents. The second implication is that $\frac{\delta u^{Y}(c)}{\delta\eta} \geq \frac{\delta u^{LP}(c)}{\delta\eta}$ holds, implying that any alternative $\eta > \eta^{Y}(\theta)$ is never an equilibrium either. $\eta^{Y}(\theta)$ in fact is supported by a majority including the young and LP workers.

[19] This again is proven in Propositions 3 and 4.

Note indeed that for $\eta > \eta^Y(\theta)$ it is:

$$u^Y(\eta) - u^Y\left(\eta^Y(\theta)\right) = \int_{\eta^Y}^{\eta} \frac{\delta u^Y(t)}{\delta \eta} dt < 0$$

since $\eta^Y(\theta)$ is a point of absolute maximum; moreover $\frac{\delta u^Y(c)}{\delta \eta} \geq \frac{\delta u^{LP}(c)}{\delta \eta}$ holds, implying that:

$$0 > \int_{\eta^Y(\theta)}^{\eta} \frac{\delta u^Y(t)}{\delta \eta} dt > \int_{\eta^Y(\theta)}^{\eta} \frac{\delta u^{LP}(t)}{\delta \eta} dt$$

and

$$u^{LP}(\eta) - u^{LP}\left(\eta^Y(\theta)\right) = \int_{\eta^Y(\theta)}^{\eta} \frac{\delta u^{LP}(t)}{\delta \eta} dt < 0.$$

holds. As a consequence $\eta^Y(\theta)$ is preferred by LP workers to any $\eta > \eta^Y$.
■

Note that the steady state dynamics of the labor market implies that agent preferences too are constant across times and re-voting produces the same outcome in every period.

In considering the equilibrium of the voting stage an important issue is whether the maximization problem of young agents involves corner solutions. In fact, when this is the case, one or both the policies considered are invariant to exogenous shocks and a direct relationship with the parameters of the economy is hardly identified.

Proposition 6 *There exists cutoffs $1 > \hat{\mu} > 0$ and $\hat{\alpha} > 0$, such that for $\alpha \leq \hat{\alpha}$ and $\mu \geq \hat{\mu}$, the young agent maximization problem has an interior solution $\left(\eta^Y, \theta^Y\right)$.*

Proof. See the Appendix. ■

Definition 7 *Define μ^1 and $\hat{\alpha}$ respectively as the level for μ and α such that for $\mu \geq \mu^1$ and $\alpha \leq \hat{\alpha}$, a pair $\left(\eta^Y, \theta^Y\right) \in (0,1) \times (0,1)$ exists which satisfies the conditions $\frac{\delta u^Y(c)}{\delta \eta}|_{\eta=\eta^Y} = 0$, $\frac{\delta u^Y(c)}{\delta \theta}|_{\theta=\theta^Y} = 0$, $\frac{\delta'' u^Y(c)}{\delta'' \eta}|_{\eta=\eta^Y} \leq 0$ and $\frac{\delta'' u^Y(c)}{\delta'' \theta}|_{\theta=\theta^Y} \leq 0$.*

In order to have an interior solution which satisfies the first and the second order conditions for young agent maximization problem, the efficiency gain deriving from worker selection and the inter temporal elasticity of substitution cannot be too large. The size of the productivity gap in fact plays the same role as the interest rate in a standard Ramsey growth model, and drives the inter-temporal allocation of consumption and income. When $\frac{1}{\mu}$ and α are high, the young are willing to adjust consumption growth to increase expected income. So a corner solution emerges which involves either no employment protection, or no retirement, or neither of these, because these choices enhance efficiency and income. But if $\frac{1}{\mu}$ and α are small, the consumption smoothing motive is very strong and the previous solutions are discarded.

3.3.1 Determinants of the Political Equilibrium

The present sections analyzes the determinants of the political equilibrium: young agent preferences, namely the degree of risk aversion and the intertemporal elasticity of substitution, and the efficiency of the labor market. We now focus on labor market efficiency which is a field where economic policies can effectively intervene, and study how the equilibrium combination of employment protection and retirement age is affected by a change in the productivity gap between high and low productivity workers and by the size of the search cost. A preliminary result is worth noting.

Lemma 8 *For every $\theta \in (0,1)$ and $\eta \in (0,1)$, there exists a cutoff $1 > \hat{\mu} > 0$, such that for $\mu \geq \hat{\mu}$, $\frac{\delta'' u^Y(c)}{\delta\eta\delta\theta} \geq 0$ holds.*

Proof. See the Appendix. ■

Definition 9 *Define μ^2 as the level for μ such that for $\mu \geq \mu^2$, $\frac{\delta'' u^Y(c)}{\delta\eta\delta\theta}|_{\eta=\eta^Y,\theta=\theta^Y} \geq 0$ holds.*

Consider now the effects on θ^Y and η^Y of shock on α. A further preliminary result must be mentioned.

Lemma 10 *For every $\eta \in (0,1)$ and $\theta \in (0,1)$ there exist a cutoff $1 > \hat{\mu} > 0$ such that for $\mu \geq \hat{\mu}$, $\frac{\delta'' u^Y(c)}{\delta\eta\delta\alpha} \geq 0$ and $\frac{\delta'' u^Y(c)}{\delta\theta\delta\alpha} \geq 0$ hold.*

Proof. See the Appendix. ■

Definition 11 *Define μ^3 as the level for μ such that for $\mu \geq \mu^3$, $\frac{\delta'' u^Y(c)}{\delta\eta\delta\alpha}|_{\eta=\eta^Y,\theta=\theta^Y} \geq 0$ and $\frac{\delta'' u^Y(c)}{\delta\theta\delta\alpha}|_{\eta=\eta^Y,\theta=\theta^Y} \geq 0$ hold.*

Proposition 12 *If $\mu \geq \max\{\mu^1, \mu^2, \mu^3\}$ and $\alpha \leq \hat{\alpha}$, $\frac{\delta\theta^Y}{\delta\alpha} \geq 0$ and $\frac{\delta\eta^Y}{\delta\alpha} \geq 0$ hold.*

Proof. By the implicit function theorem in equilibrium the following equality holds:

$$\begin{aligned}\frac{\delta\theta^Y}{\delta\alpha} =& -\frac{\frac{\delta'' u^Y(c)}{\delta\theta\delta\alpha}|_{\eta=\eta^Y,\theta=\theta^Y} \cdot \frac{\delta'' u^Y(c)}{\delta''\eta}|_{\eta=\eta^Y,\theta=\theta^Y}}{\frac{\delta'' u^Y(c)}{\delta''\eta}|_{\eta=\eta^Y,\theta=\theta^Y} \cdot \frac{\delta'' u^Y(c)}{\delta''\theta}|_{\eta=\eta^Y,\theta=\theta^Y} - \left(\frac{\delta" u^Y(c)}{\delta\eta\cdot\delta\theta}|_{\eta=\eta^Y,\theta=\theta^Y}\right)^2} + \\ &+ \frac{\frac{\delta" u^Y(c)}{\delta\eta\cdot\delta\theta}|_{\eta=\eta^Y,\theta=\theta^Y} \cdot \frac{\delta'' u^Y(c)}{\delta\eta\delta\alpha}|_{\eta=\eta^Y,\theta=\theta^Y}}{\frac{\delta'' u^Y(c)}{\delta''\eta}|_{\eta=\eta^Y,\theta=\theta^Y} \cdot \frac{\delta'' u^Y(c)}{\delta''\theta}|_{\eta=\eta^Y,\theta=\theta^Y} - \left(\frac{\delta" u^Y(c)}{\delta\eta\cdot\delta\theta}|_{\eta=\eta^Y,\theta=\theta^Y}\right)^2} \\ &\geq 0.\end{aligned}$$

By the result of Proposition 6 it must be the case that the denominator of the fraction is positive. Consider now the numerator. By Lemma 10 $\frac{\delta'' u^Y(c)}{\delta\eta\delta\alpha}|_{\eta=\eta^Y,\theta=\theta^Y} \geq 0$ and $\frac{\delta'' u^Y(c)}{\delta\theta\delta\alpha}|_{\eta=\eta^Y,\theta=\theta^Y} \geq 0$ hold. Hence it also is $\frac{\delta\theta^Y}{\delta\alpha} \geq 0$ because the inequalities $\frac{\delta" u^Y(c)}{\delta\eta\cdot\delta\theta}|_{\eta=\eta^Y,\theta=\theta^Y} \geq 0$ and $\frac{\delta" u^Y(c)}{\delta"\eta}|_{\eta=\eta^Y,\theta=\theta^Y} \leq 0$ are verified.

A symmetrical argument proves $\frac{\delta\eta^Y}{\delta\alpha} \geq 0$. ■

An increase in the productivity gap causes both an increase in the mandatory retirement age and a reduction in the degree of employment protection, implying that a direct relationship exists between η^Y and θ^Y. This is the case because if α increases, the equilibrium level of employment also increases and reduces overall uncertainty from the perspective of young agents. The probability of being hired during the selection process increases and the probability of being unemployed in the second period decreases. A lower risk of unemployment means there is less income variability across states of the world and more room to exploit the efficiency gains triggered by the increase in the productivity gap. This occurs because of a reduction in employment protection which increases unemployment risk in the old age (partially offset by a reduction in the young age) and of an increase in the mandatory retirement age which causes an increase in unemployment risk in every state of the world except for HP agents. On the other hand, more employment protection and more early retirement are present when a reduction in the productivity gap occurs.

Analyze now the effect of labor market frictions and study how η^Y and θ^Y are affected by a change in ϕ.

Lemma 13 *For every $\eta \in (0,1)$ and $\theta \in (0,1)$ there exist a cutoff $1 > \hat{\mu} > 0$ such that for $\mu \geq \hat{\mu}$, $\frac{\delta'' u^Y(c)}{\delta\eta\delta\phi} \geq 0$ and $\frac{\delta'' u^Y(c)}{\delta\theta\delta\phi} \geq 0$ hold.*

Proof. See the Appendix. ■

Definition 14 *Define* μ^4 *as the level for* μ *such that for* $\mu \geq \mu^4$, $\frac{\delta'' u^Y(c)}{\delta\phi\delta\eta}|_{\eta=\eta^Y,\theta=\theta^Y} \geq 0$ *and* $\frac{\delta'' u^Y(c)}{\delta\phi\delta\theta}|_{\eta=\eta^Y,\theta=\theta^Y} \geq 0$ *hold.*

Proposition 15 *If* $\mu \geq \max\{\mu^1, \mu^2, \mu^4\}$ *and* $\alpha \leq \hat{\alpha}$, $\frac{\delta\theta^Y}{\delta\phi} \leq 0$ *and* $\frac{\delta\eta^Y}{\delta\phi} \leq 0$ *hold.*

Proof. Analogous to the proof of Proposition 12. ■

An increase in the search cost triggers both a decrease in the mandatory retirement age and an increase in the degree of employment protection, implying that, in this case too, a direct relationship exists between η^Y and θ^Y. More labor market frictions cause the equilibrium level of employment to decrease. From the perspective of young agents, an increase in income uncertainty follows, for opposite reasons to those previously mentioned. Hence more insurance is introduced in the economy by means of a decrease in η^Y and in θ^Y. A lower income variability is obtained at the cost of a loss of productivity and of an increase of the search costs. An increase in employment protection in fact concentrates most of the turnover on the date of retirement. Search costs thus increase because most of the vacancies are filled at the moment where unemployment is at its lowest. On the other hand, less employment protection and later retirement are the consequence of lower search costs.

3.3.2 The Case of an Exogenous Binding Constraint over the Political Equilibrium

Consider now what happens when a parameter of the equilibrium is exogenously set, and study the effects on η^Y of a change in θ^Y or vice versa. This may occur if for instance, an increase in the expected lifespan requires an increase in the retirement age to guarantee the economic viability of the pension system. Alternatively, consider the circumstance where an economy is increasingly exposed to international competition and needs to increase its overall efficiency through a reduction in the degree of employment protection.

Proposition 16 *If* $\mu \geq \max\{\mu^1, \mu^2\}$ *and* $\alpha \leq \hat{\alpha}$, $\frac{\delta\theta^Y}{\delta\eta^Y} \geq 0$ *holds and a direct relationship exists between the equilibrium values of* η^Y *and* θ^Y.

Proof. By the implicit function theorem, in equilibrium the following equality holds:

$$\frac{\delta\theta^Y}{\delta\eta^Y} \cdot \frac{\delta'' u^Y(c)}{\delta'' \theta}|_{\eta=\eta^Y,\theta=\theta^Y} \cdot \Delta\eta + \frac{\delta'' u^Y(c)}{\delta\eta \cdot \delta\theta}|_{\eta=\eta^Y,\theta=\theta^Y} \cdot \Delta\eta = 0$$

implying also that:

$$\frac{\delta\theta^Y}{\delta\eta^Y} = -\frac{\frac{\delta" u^Y(c)}{\delta\eta\cdot\delta\theta}|_{\eta=\eta^Y,\theta=\theta^Y}}{\frac{\delta" u^Y(c)}{\delta"\theta}|_{\eta=\eta^Y,\theta=\theta^Y}} \geq 0.$$

Note indeed that Lemma 8 proves $\frac{\delta" u^Y(c)}{\delta\eta\cdot\delta\theta}|_{\eta=\eta^Y,\theta=\theta^Y} \geq 0$ while from Proposition 6 it follows that $\frac{\delta" u^Y(c)}{\delta"\eta}|_{\eta=\eta^Y,\theta=\theta^Y} \leq 0$ and $\frac{\delta" u^Y(c)}{\delta"\theta}|_{\eta=\eta^Y,\theta=\theta^Y} \leq 0$ hold. In equilibrium, if the mandatory retirement age exogenously increases, employment protection decreases and a positive correlation emerges between early retirement and employment protection. The same argument proves $\frac{\delta\eta^Y}{\delta\theta^Y} \geq 0$. ■

If a binding constraint causes an exogenous variation in one of the parameters defining the political equilibrium, for instance in the legal retirement age, a reform of the labor market is also required to meet the preferences of the median voter and to increase the constituency supporting these changes.

A pivotal role in this context is played by insider/outsider turnover on the labor market which defines the size of income transfers from the future to the present. Consider what happens if retirement age decreases. The elderly are forced out of employment sooner, and income in the first period increases. In order to smooth consumption, an increase in future expected income is required. This is obtained by increasing employment protection and reducing the probability of the state of the world where a LP worker is fired. The reverse happens when θ increases and hence reduces the turnover generated by non-selective exits from employment by the elderly. An increase in the transfers from the future to the present is required and is realized through a reduction in employment protection. Retirement age and employment protection move in opposite directions if an exogenous shock forces one of them to vary.

3.4 Discussion of the Results

The current analysis relies on a set of assumptions which need to be discussed in detail.

A first issue regards the lack of an inter temporal discount factor. Extending the model to include this element does not significantly alter the results of the model as long as the young are not too myopic. Even though some authors identify agent myopia as a rationale for the introduction of a pension system[20], there is no unanimous consensus in the literature. The present analysis thus provides a useful insight for the benchmark framework where young agents are far-sighted. In the case

[20] See Cremer and Roeder (2012).

where these young agents, which remain the median group, have a high discount factor, the utility of the second period plays a smaller role in their maximization problem and the intertemporal allocation of income is not relevant for them. Hence they choose a corner solution involving no employment protection, because it maximizes expected income in the first period.

Focus now on the absence of capital markets. Note that the presence of a storage device which permits savings, only affects the maximization problem of young and low productivity workers, since unemployed old workers are likely to be constrained on their possibility to save and high productivity worker incentives do not change if savings are introduced. As a consequence the impact of savings on the political equilibrium is limited and the median voter does not change

In the maximization problem of young agents, a pivotal role is played by the need to transfer income across time and states of the world through an optimal allocation of the unemployment risk. As a consequence, if the capital market allows young outsiders to obtain the preferred resource allocation by self-insuring against labor market risk, they will act roughly as risk-neutral agents. In this case overall efficiency is maximized and the political equilibrium entails no employment protection and no retirement. Hence capital market imperfections affect agent preferences toward risk and consumption smoothing, and favor the adoption of low employment protection and late retirement. This result supports the predictions of the model because it squares with the observed differences in the Social Security Systems between Anglo-Saxon countries, and other (specially European) countries. The former economies in fact display low levels of both EPL and the ERI indexes compared to those countries where capital markets are less developed[21]. Lastly, unemployment benefits are not considered here; the effects of this assumption on the result are not straightforwardly identified in the present framework. But if a Bismarckian, corporative, unemployment insurance scheme of the type prevailing in European countries is considered, it is plausible to expect a limited impact on the results. If unemployment benefits are awarded only to people who have previously contributed to the insurance scheme, the scheme is irrelevant for both high productivity workers and for old unemployed workers who are not entitled to receive them. Furthermore the maximization problem of young agents does not change significantly because they are not entitled to receive the benefit in the first period either. Only people who are fired

[21] In our sample, the country average of the EPL and ERI indexes calculated over the period 1985-2008 for US, UK, Canada, Ireland and Australia are below the sample average.

after the selection process are covered by the insurance scheme. So it is not too far fetched to suppose that the present results are robust with regard to this extension of the model.

4 Final Remarks

The present analysis studies the interaction between the labor market and the pension system. In particular it focuses on how the degree of employment protection and the mandatory retirement age affect labor market dynamics and insider/outsider turnover. Firm behavior and bargaining over the wage and employment rates, as well as the distribution of unemployment risk across time and states of the world are described in detail. This provides a useful insight on worker preferences on the setup of the Social Security System and highlights the complementarities existing between different institutions.

The main result is the characterization of the determinants of the political equilibrium defining the setup of the Social Security System. It emerges that shocks on search costs or on the productivity gap between different types of workers cause variations in opposite directions of the degree of employment protection and of the mandatory retirement age. This is the case because from the perspective of young workers, median voters in the electorate, they represent "substitute goods". By influencing both selective and non-selective exits from the labor market, these parameters in fact define the size and the direction of income transfers from the present to the future or *vice versa*. Hence they serve as devices to smooth consumption and have different costs in terms of the efficiency of the economy.

In equilibrium this defines an inverse relationship between the degree of employment protection and the mandatory retirement age and provides an interpretation, alternative to the "lump of labor fallacy", for the lack of empirical relationship between early retirement and youth unemployment, in the presence of imperfect labor markets. It further warns against the potential lack of political constituency for pension system reforms which are not appropriately combined and sequenced with a labor market reform. In fact, the recent round of pension system reforms has shown that even in the presence of an exogenous constraint, the median voter requires pension reform to be accompanied by a lower level of employment protection.Bhattacharya, J., Mulligan, C. B., and R. R. Reed (2001), Labor Market Search and Optimal Retirement Policy. NBER Working Paper No. 8591.

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5 Appendix: Main Proofs

5.1 Proof of Proposition 3

The maximization problem of HP agents yields the following first order condition with respect to η:

$$\frac{\delta u^{HP}(c)}{\delta \eta}$$
$$= \frac{\nu(1-\mu)}{1+(1-\gamma)(1-\theta)} \left\{ \frac{W^*[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)]}{1+(1-\gamma)(1-\theta)} \right\}^{-\mu} \cdot$$
$$\cdot \left\{ \frac{\delta W^*}{\delta \eta}[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)] + \frac{\delta n^*}{\delta \eta}(1-\gamma)(1-\theta)W^* \right\} \geq 0$$

since $\frac{\delta W^*}{\delta \eta} \geq 0$ and $\frac{\delta n^*}{\delta \eta} \geq 0$. The first order condition with respect to θ is:

$$\frac{\delta u^{HP}(c)}{\delta \theta}$$
$$= \frac{\nu(1-\mu)}{1+(1-\gamma)(1-\theta)} \left[\frac{W^*}{1+(1-\gamma)(1-\theta)} \right]^{-\mu} \cdot \left[\frac{\delta W^*}{\delta \theta} + \frac{W^*(1-\gamma)}{1+(1-\gamma)(1-\theta)} \right] \cdot$$
$$\cdot [\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)]^{1-\mu} +$$
$$+\nu(1-\mu) \left[\frac{W^*}{1+(1-\gamma)(1-\theta)} \right]^{1-\mu} \left[1-n^*+\frac{\delta n^*}{\delta \theta}(1-\theta) \right] \cdot$$
$$\cdot (1-\gamma)[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)]^{-\mu} \geq 0$$

since $\frac{\delta W^*}{\delta \theta} \geq 0$, $\frac{\delta n^*}{\delta \theta} \geq 0$ and $n^* \leq 1$.

5.2 Proof of Proposition 4

The maximization problem of U agents yields the following first order condition with respect to η:

$$\frac{\delta u^{U}(c)}{\delta \eta}$$
$$= \frac{\nu(1-\mu)(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)} \left[\frac{W^* \cdot n^*(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)} \right]^{-\mu} \left(\frac{\delta W^*}{\delta \eta} \cdot n^* + \frac{\delta n^*}{\delta \eta} \cdot W^* \right) \geq 0.$$

The first order condition with respect to θ is:

$$\frac{\delta u^{U}(c)}{\delta \theta}$$
$$= \frac{\nu(1-\mu)(1-\gamma)}{1+(1-\gamma)(1-\theta)} \left[\frac{W^* \cdot n^*(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)} \right]^{-\mu} \cdot$$
$$\cdot \left\{ n^*(1-\theta) \left[\frac{\delta W^*}{\delta \theta} + \frac{W^*(1-\gamma)}{1+(1-\gamma)(1-\theta)} \right] - W^* \left[n^* - \frac{\delta n^*}{\delta \theta}(1-\theta) \right] \right\} \leq 0$$

since

$$n^* (1-\theta) \left[\frac{\delta W^*}{\delta\theta} + \frac{W^* (1-\gamma)}{1+(1-\gamma)(1-\theta)} \right] \leq W^* \left[n^* - \frac{\delta n^*}{\delta\theta} (1-\theta) \right].$$

In fact, substituting n^* and $\frac{\delta n^*}{\delta\theta}$ and using simple algebra gives:

$$2(1-\theta) \frac{\delta W^*}{\delta\theta} \leq \frac{W^*}{1+(1-\gamma)(1-\theta)}.$$

Substitute further W^* and $\frac{\delta W^*}{\delta\theta}$, simplify and reorder the terms to obtain:

$$-\frac{\alpha}{2} \left[1 + \frac{\eta}{2} (2 \cdot \gamma + 2 \cdot \theta - 2 \cdot \gamma \cdot \theta - 1) \right] \leq 1 - \phi (k-1).$$

Evaluate the left hand side of the previous inequality at $\eta = 1$ where an absolute maximum is reached and obtain the inequality:

$$-\frac{\alpha}{2} \left[\frac{1}{2} + \gamma (1-\theta) + \theta \right] \leq 1 - \phi (k-1)$$

which is always verified.

5.3 Proof of Proposition 6

The proof follows straightforwardly from the following lemmas.

Lemma 17 *For every $\eta \in [0,1]$ there is a cutoff $1 > \hat{\mu} > 0$ such that for $\mu > \hat{\mu}$, an interior value $\theta^Y \in (0,1)$ exists which satisfies the conditions $\frac{\delta u^Y(c)}{\delta\theta}|_{\theta=\theta^Y} = 0$ and $\frac{\delta'' u^Y(c)}{\delta''\theta}|_{\theta=\theta^Y} \leq 0$.*

Proof. The maximization problem of young agents yields the following first order condition with respect to θ:

$$\begin{aligned}
&\frac{\delta u^Y(c)}{\delta\theta} \\
&= \frac{\eta}{2} \cdot \frac{\delta n^*}{\delta\theta} \cdot \nu (2-\mu) \left[\frac{W^*}{1+(1-\gamma)(1-\theta)} \right]^{1-\mu} \left[1 - (1-\gamma)^{1-\mu} (1-\theta)^{1-\mu} \right] + \\
&\quad + \frac{\eta}{2} \cdot \frac{n^* \cdot \nu (1-\mu)(1-\gamma)}{1+(1-\gamma)(1-\theta)} \left[\frac{W^*}{1+(1-\gamma)(1-\theta)} \right]^{1-\mu} \left[1 + (1-\gamma)^{-\mu} (1-\theta)^{-\mu} \right] + \\
&\quad + \frac{\delta n^*}{\delta\theta} \cdot \nu (2-\mu) \left[\frac{W^* (1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)} \right]^{1-\mu} + \\
&\quad - \frac{n^* \cdot \nu (1-\mu)}{1+(1-\gamma)(1-\theta)} \left[\frac{W^* (1-\gamma)}{1+(1-\gamma)(1-\theta)} \right]^{1-\mu} (1-\theta)^{-\mu} + \frac{\delta u^U(c)}{\delta\theta} + \\
&\quad + \left(1 - \frac{\eta}{2} \right) \frac{\delta n^*}{\delta\theta} \left[u^{HP}(c) - u^U(c) \right] + n^* \left(1 - \frac{\eta}{2} \right) \left[\frac{\delta u^{HP}(c)}{\delta\theta} - \frac{\delta u^U(c)}{\delta\theta} \right].
\end{aligned}$$

Evaluate it at $\theta = 1$ and note that $u^U(c)|_{\theta=1} = 0$, $u^{HP}(c)|_{\theta=1} = \nu (W^*|_{\theta=1})^{1-\mu}$and

$$\frac{\delta u^{HP}(c)}{\delta\theta}|_{\theta=1} = \nu(1-\mu)(W^*|_{\theta=1})^{-\mu}\left[\frac{\delta W^*}{\delta\theta}|_{\theta=1} + W^*|_{\theta=1}(1-\gamma)(2-n^*|_{\theta=1})\right]$$

hold. Since $\frac{\delta u^U(c)}{\delta\theta}|_{\theta=1} = -\infty$, also $\frac{\delta u^Y(c)}{\delta\theta}$ approaches $-\infty$ as θ approaches 1 and $\theta = 1$ is never a solution for the maximization problem of young agents.

Evaluate now $\frac{\delta u^Y(c)}{\delta\theta}$ at $\theta = 0$ to obtain:

$$\begin{aligned}
&\frac{\delta u^Y(c)}{\delta\theta}|_{\theta=0} \\
=&\frac{\eta}{2}\cdot\frac{\delta n^*}{\delta\theta}|_{\theta=0}\cdot\nu(2-\mu)\left(\frac{W^*|_{\theta=0}}{2-\gamma}\right)^{1-\mu}\left[1-(1-\gamma)^{1-\mu}\right]+ \\
&+\frac{\eta}{2}\cdot\frac{n^*|_{\theta=0}\cdot\nu(1-\mu)(1-\gamma)}{2-\gamma}\left(\frac{W^*|_{\theta=0}}{2-\gamma}\right)^{1-\mu}\left[1+(1-\gamma)^{-\mu}\right]+ \\
&+\frac{\delta n^*}{\delta\theta}|_{\theta=0}\cdot\nu(2-\mu)\left[\frac{(1-\gamma)W^*|_{\theta=0}}{2-\gamma}\right]^{1-\mu}-\frac{n^*|_{\theta=0}\cdot\nu(1-\mu)}{2-\gamma}\left[\frac{(1-\gamma)W^*|_{\theta=0}}{2-\gamma}\right]^{1-\mu}+ \\
&+\frac{\delta u^U(c)}{\delta\theta}|_{\theta=0}+\left(1-\frac{\eta}{2}\right)\frac{\delta n^*}{\delta\theta}|_{\theta=0}\cdot\left[u^{HP}(c)|_{\theta=0}-u^U(c)|_{\theta=0}\right]+ \\
&+n^*|_{\theta=0}\cdot\left(1-\frac{\eta}{2}\right)\left[\frac{\delta u^{HP}(c)}{\delta\theta}|_{\theta=0}-\frac{\delta u^U(c)}{\delta\theta}|_{\theta=0}\right].
\end{aligned}$$

Consider what happens if μ approaches 1. First note that:

$$u^{HP}(c)|_{\theta=0} = \nu\left(\frac{W^*|_{\theta=0}}{2-\gamma}\right)^{1-\mu}\left[\gamma+(1-\gamma)n|_{\theta=0}\right]^{1-\mu}$$

and

$$u^U(c)|_{\theta=0} = \nu\left(\frac{W^*|_{\theta=0}}{2-\gamma}\right)^{1-\mu}\left[(1-\gamma)n^*|_{\theta=0}\right]^{1-\mu}$$

hold implying:

$$\lim_{\mu\to 1}\left[u^{HP}(c)|_{\theta=0}-u^U(c)|_{\theta=0}\right]=0.$$

Consider further that:

$$\begin{aligned}
&\frac{\delta u^{HP}(c)}{\delta\theta}|_{\theta=0} \\
=&\frac{\nu(1-\mu)}{2-\gamma}\left(\frac{W^*|_{\theta=0}}{2-\gamma}\right)^{-\mu}\left[\gamma+(1-\gamma)n^*|_{\theta=0}\right]^{1-\mu}\left[\frac{\delta W^*}{\delta\theta}|_{\theta=0}+\frac{(1-\gamma)W^*|_{\theta=0}}{2-\gamma}\right]+ \\
&+\nu(1-\mu)\left(\frac{W^*|_{\theta=0}}{2-\gamma}\right)^{1-\mu}\left[\gamma+(1-\gamma)n^*|_{\theta=0}\right]^{-\mu}(1-\gamma)\left(1-n^*|_{\theta=0}+\frac{\delta n^*}{\delta\theta}|_{\theta=0}\right)
\end{aligned}$$

and

$$\frac{\delta u^{U}(c)}{\delta\theta}|_{\theta=0}$$

$$=\frac{\nu(1-\mu)(1-\gamma)}{1+(1-\gamma)}\left[\frac{W^{*}|_{\theta=0}\cdot n^{*}|_{\theta=0}(1-\gamma)}{1+(1-\gamma)}\right]^{-\mu}\cdot$$

$$\cdot\left\{n^{*}|_{\theta=0}\left[\frac{\delta W^{*}}{\delta\theta}|_{\theta=0}+\frac{W^{*}|_{\theta=0}(1-\gamma)}{1+(1-\gamma)}\right]-W^{*}|_{\theta=0}\left(n^{*}|_{\theta=0}-\frac{\delta n^{*}}{\delta\theta}|_{\theta=0}\right)\right\}$$

hold. Since it is $\lim_{\mu\to1}\frac{\delta u^{HP}(c)}{\delta\theta}|_{\theta=0}=\lim_{\mu\to1}\frac{\delta u^{U}(c)}{\delta\theta}|_{\theta=0}=0$,

$$\lim_{\mu\to1}\frac{\delta u^{Y}(c)}{\delta\theta}|_{\theta=0}=\frac{\delta n^{*}}{\delta\theta}|_{\theta=0}\cdot\nu=\frac{\nu(1-\sigma)\frac{\alpha\cdot\eta}{4}(1-\gamma)}{\phi\left\{\beta\left[\left(\frac{\eta}{2}\right)^{2}+\left(1-\frac{\eta}{2}\right)^{2}\right]+\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}}\geq0$$

holds.

For every $\eta\in[0,1]$ thus there exist a $0<\hat{\mu}<1$ large enough that if $\mu\geq\hat{\mu}$, $\frac{\delta u^{Y}(c)}{\delta\theta}|_{\theta=0}\geq0$ holds. Moreover by the continuity of $\frac{\delta u^{Y}(c)}{\delta\theta}$ and since it is $\frac{\delta u^{Y}(c)}{\delta\theta}|_{\theta=1}\leq0$, if $\mu\geq\hat{\mu}$, there exists a θ^{Y} such that $\frac{\delta u^{Y}(c)}{\delta\theta}|_{\theta=\theta^{Y}}=0$ and $\frac{\delta'' u^{Y}(c)}{\delta''\theta}|_{\theta=\theta^{Y}}\leq0$. Note now that at the extremes of the dominion there are two points of local minimum. Hence an interior maximum must exist where $\frac{\delta'' u^{Y}(c)}{\delta''\theta}|_{\theta=\theta^{Y}}\leq0$ holds. ■

Lemma 18 *For every $\theta\in[0,1]$ there are cutoffs $\hat{\alpha}>0$ and $1>\hat{\mu}>0$ such that for $\alpha\leq\hat{\alpha}$ and $\mu\geq\hat{\mu}$, an interior value $\eta^{Y}\in(0,1)$ exists which satisfies the conditions $\frac{\delta u^{Y}(c)}{\delta\eta}|_{\eta=\eta^{Y}}=0$ and $\frac{\delta'' u^{Y}(c)}{\delta''\eta}|_{\eta=\eta^{Y}}\leq0$.*

Proof. The first order condition with respect to η is:

$$\frac{\delta u^{Y}(c)}{\delta\eta}$$

$$=\frac{n^{*}\cdot\nu}{2}\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\left[1-(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}\right]+$$

$$+\frac{\eta\cdot n^{*}}{2}\cdot\frac{\nu(1-\mu)\frac{\delta W^{*}}{\delta\eta}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}+$$

$$+\frac{\delta n^{*}}{\delta\eta}\left(1-\frac{\eta}{2}\right)\nu\left[\frac{W^{*}(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}+$$

$$+n^{*}\left(1-\frac{\eta}{2}\right)\frac{\nu(1-\mu)\frac{\delta W^{*}}{\delta\eta}(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^{*}(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}+\frac{\delta u^{U}(c)}{\delta\eta}+$$

$$+\left[\frac{\delta n^{*}}{\delta\eta}\left(1-\frac{\eta}{2}\right)-\frac{n^{*}}{2}\right]\left[u^{HP}(c)-u^{U}(c)\right]+n^{*}\left(1-\frac{\eta}{2}\right)\left[\frac{\delta u^{HP}(c)}{\delta\eta}-\frac{\delta u^{U}(c)}{\delta\eta}\right].$$

Evaluate it at $\eta = 1$ to get:

$$\frac{\delta u^{Y}(c)}{\delta \eta}|_{\eta=1}$$
$$= \frac{n^*|_{\eta=1} \cdot \nu}{2} \left[\frac{W^*|_{\eta=1}}{1+(1-\gamma)(1-\theta)} \right]^{1-\mu} \left[1 - (1-\gamma)^{1-\mu} (1-\theta)^{1-\mu} \right] +$$
$$+ \frac{n^*|_{\eta=1}}{2} \cdot \frac{\nu (1-\mu) \frac{\delta W^*}{\delta \eta}|_{\eta=1}}{1+(1-\gamma)(1-\theta)} \left[\frac{W^*|_{\eta=1}}{1+(1-\gamma)(1-\theta)} \right]^{-\mu} +$$
$$+ \frac{\delta n^*}{\delta \eta}|_{\eta=1} \cdot \frac{\nu}{2} \left[\frac{(1-\gamma)(1-\theta) W^*|_{\eta=1}}{1+(1-\gamma)(1-\theta)} \right]^{1-\mu} +$$
$$+ \frac{n^*|_{\eta=1} \cdot \nu}{2} \cdot \frac{(1-\mu)(1-\gamma)(1-\theta) \frac{\delta W^*}{\delta \eta}|_{\eta=1}}{1+(1-\gamma)(1-\theta)} \left[\frac{(1-\gamma)(1-\theta) W^*|_{\eta=1}}{1+(1-\gamma)(1-\theta)} \right]^{-\mu} +$$
$$+ \frac{\delta u^{U}(c)}{\delta \eta}|_{\eta=1} + \frac{1}{2} \left(\frac{\delta n^*}{\delta \eta}|_{\eta=1} - n^*|_{\eta=1} \right) \left[u^{HP}(c)|_{\eta=1} - u^{U}(c)|_{\eta=1} \right] +$$
$$+ \frac{n^*|_{\eta=1}}{2} \left[\frac{\delta u^{HP}(c)}{\delta \eta}|_{\eta=1} - \frac{\delta u^{U}(c)}{\delta \eta}|_{\eta=1} \right].$$

Consider what happens if a approaches 0 and note that:

$$\lim_{\alpha \to 0} \frac{\delta W^*}{\delta \eta}|_{\eta=1} = \lim_{\alpha \to 0} \frac{\delta n^*}{\delta \eta}|_{\eta=1} = 0.$$

This implies:

$$\lim_{\alpha \to 0} \frac{\delta u^{Y}(c)}{\delta \eta}|_{\eta=1}$$
$$= \lim_{\alpha \to 0} \left\{ \frac{n^*|_{\eta=1} \cdot \nu}{2} \left[\frac{W^*|_{\eta=1}}{1+(1-\gamma)(1-\theta)} \right]^{1-\mu} \left[1 - (1-\gamma)^{1-\mu} (1-\theta)^{1-\mu} \right] \right\} +$$
$$+ \lim_{\alpha \to 0} \frac{\delta u^{U}(c)}{\delta \eta}|_{\eta=1} + \lim_{\alpha \to 0} \left\{ \frac{1}{2} \left(\frac{\delta n^*}{\delta \eta}|_{\eta=1} - n^*|_{\eta=1} \right) \left[u^{HP}(c)|_{\eta=1} - u^{U}(c)|_{\eta=1} \right] \right\} +$$
$$+ \frac{n^*|_{\eta=1}}{2} \left[\frac{\delta u^{HP}(c)}{\delta \eta}|_{\eta=1} - \frac{\delta u^{U}(c)}{\delta \eta}|_{\eta=1} \right].$$

Note now that:

$$u^{HP}(c)|_{\eta=1} = \nu \left[\frac{W^*|_{\eta=1}}{1+(1-\gamma)(1-\theta)} \right]^{1-\mu} \left[\gamma + \theta(1-\gamma) + (1-\theta)(1-\gamma) n^*|_{\eta=1} \right]^{1-\mu}$$

and

$$u^{U}(c)|_{\eta=1} = \nu \left[\frac{W^*|_{\eta=1} \cdot n^*|_{\eta=1} (1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)} \right]^{1-\mu}$$

Hence it is:

$$\begin{aligned}
&\lim_{\alpha\to 0}\left[u^{HP}(c)|_{\eta=1}-u^{U}(c)|_{\eta=1}\right]\\
=&\nu\left[\frac{W^{*}|_{\eta=1}}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\cdot\\
&\cdot\left\{\left[\gamma+\theta(1-\gamma)+(1-\theta)(1-\gamma)n^{*}|_{\eta=1}\right]^{1-\mu}-\left[n^{*}|_{\eta=1}(1-\theta)(1-\gamma)\right]^{1-\mu}\right\}.
\end{aligned}$$

Consider then:

$$\begin{aligned}
&\frac{\delta u^{HP}(c)}{\delta\eta}|_{\eta=1}\\
=&\frac{\nu(1-\mu)}{1+(1-\gamma)(1-\theta)}\left\{\frac{W^{*}|_{\eta=1}\cdot\left[\gamma+\theta(1-\gamma)+n^{*}|_{\eta=1}\cdot(1-\theta)(1-\gamma)\right]}{1+(1-\gamma)(1-\theta)}\right\}^{-\mu}\cdot\\
&\cdot\frac{\delta W^{*}}{\delta\eta}|_{\eta=1}\cdot\left[\gamma+\theta(1-\gamma)+(1-\theta)(1-\gamma)n^{*}|_{\eta=1}\right]+\\
&+\frac{\nu(1-\mu)}{1+(1-\gamma)(1-\theta)}\left\{\frac{W^{*}|_{\eta=1}\cdot\left[\gamma+\theta(1-\gamma)+n^{*}|_{\eta=1}\cdot(1-\theta)(1-\gamma)\right]}{1+(1-\gamma)(1-\theta)}\right\}^{-\mu}\cdot\\
&\cdot\frac{\delta n^{*}}{\delta\eta}|_{\eta=1}\cdot(1-\gamma)(1-\theta)W^{*}|_{\eta=1}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\delta u^{U}(c)}{\delta\eta}|_{\eta=1}=&\frac{\nu(1-\mu)(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^{*}|_{\eta=1}\cdot n^{*}|_{\eta=1}\cdot(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\cdot\\
&\cdot\left(\frac{\delta W^{*}}{\delta\eta}|_{\eta=1}\cdot n^{*}|_{\eta=1}+\frac{\delta n^{*}}{\delta\eta}|_{\eta=1}\cdot W^{*}|_{\eta=1}\right).
\end{aligned}$$

Since $\lim_{\alpha\to 0}\frac{\delta u^{HP}(c)}{\delta\theta}|_{\eta=1}=\lim_{\mu\to 1}\frac{\delta u^{U}(c)}{\delta\theta}|_{\eta=1}=0$ holds it also is:

$$\begin{aligned}
&\lim_{\alpha\to 0}\frac{\delta u^{Y}(c)}{\delta\eta}|_{\eta=1}\\
=&\lim_{\alpha\to 0}\left\{\frac{n^{*}|_{\eta=1}}{2}\cdot\nu\left[\frac{W^{*}|_{\eta=1}}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\left[1-(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}\right]\right\}+\\
&-\lim_{\alpha\to 0}\left\{\frac{n^{*}|_{\eta=1}}{2}\cdot\nu\left[\frac{W^{*}|_{\eta=1}}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\right\}\cdot\\
&\cdot\lim_{\alpha\to 0}\left\{\left[\gamma+\theta(1-\gamma)+(1-\theta)(1-\gamma)n^{*}|_{\eta=1}\right]^{1-\mu}-\left[(1-\theta)(1-\gamma)n^{*}|_{\eta=1}\right]^{1-\mu}\right\}\\
\leq&0
\end{aligned}$$

by the concavity of the utility function. Hence for every $\theta\in[0,1]$ there exists an $\hat{\alpha}>0$ small enough for $\frac{\delta u^{Y}(c)}{\delta\eta}|_{\eta=1}\leq 0$ to hold.

Evaluate now $\frac{\delta u^{Y}(c)}{\delta \eta}$ at $\eta = 0$ to obtain:

$$\begin{aligned}
&\frac{\delta u^{Y}\left(c\right)}{\delta\eta}|_{\eta=0} \\
&= \frac{n^{*}|_{\eta=0}\cdot\nu}{2}\left[\frac{W^{*}|_{\eta=0}}{1+\left(1-\gamma\right)\left(1-\theta\right)}\right]^{1-\mu}\left[1-\left(1-\gamma\right)^{1-\mu}\left(1-\theta\right)^{1-\mu}\right]+ \\
&+\frac{\delta n^{*}}{\delta\eta}|_{\eta=0}\cdot\nu\left[\frac{\left(1-\gamma\right)\left(1-\theta\right)W^{*}|_{\eta=0}}{1+\left(1-\gamma\right)\left(1-\theta\right)}\right]^{1-\mu}+ \\
&+n^{*}|_{\eta=0}\cdot\frac{\nu\left(1-\mu\right)\left(1-\gamma\right)\left(1-\theta\right)\frac{\delta W^{*}}{\delta\eta}|_{\eta=0}}{1+\left(1-\gamma\right)\left(1-\theta\right)}\left[\frac{\left(1-\gamma\right)\left(1-\theta\right)W^{*}|_{\eta=0}}{1+\left(1-\gamma\right)\left(1-\theta\right)}\right]^{-\mu}+ \\
&+\frac{\delta u^{U}\left(c\right)}{\delta\eta}|_{\eta=0}+\left(\frac{\delta n^{*}}{\delta\eta}|_{\eta=0}-\frac{n^{*}}{2}|_{\eta=0}\right)\left[u^{HP}\left(c\right)|_{\eta=0}-u^{U}\left(c\right)|_{\eta=0}\right]+ \\
&+n^{*}|_{\eta=0}\cdot\left[\frac{\delta u^{HP}\left(c\right)}{\delta\eta}|_{\eta=0}-\frac{\delta u^{U}\left(c\right)}{\delta\eta}|_{\eta=0}\right].
\end{aligned}$$

Consider what happens if μ approaches 1 and note that:

$$u^{HP}\left(c\right)|_{\eta=0}=\nu\left[\frac{W^{*}|_{\eta=0}}{1+\left(1-\gamma\right)\left(1-\theta\right)}\right]^{1-\mu}\left[\gamma+\theta\left(1-\gamma\right)+\left(1-\theta\right)\left(1-\gamma\right)n^{*}|_{\eta=0}\right]^{1-\mu}$$

and

$$u^{U}\left(c\right)|_{\eta=0}=\nu\left[\frac{W^{*}|_{\eta=0}\cdot n^{*}|_{\eta=0}\left(1-\theta\right)\left(1-\gamma\right)}{1+\left(1-\gamma\right)\left(1-\theta\right)}\right]^{1-\mu}$$

hold implying that:

$$\lim_{\mu\to 1}\left[u^{HP}\left(c\right)|_{\eta=0}-u^{U}\left(c\right)|_{\eta=0}\right]=0.$$

Consider then:

$$\begin{aligned}
&\frac{\delta u^{HP}\left(c\right)}{\delta\eta}|_{\eta=0} \\
&=\frac{\nu\left(1-\mu\right)\frac{\delta W^{*}}{\delta\eta}|_{\eta=0}}{1+\left(1-\gamma\right)\left(1-\theta\right)}\left[\frac{W^{*}|_{\eta=0}}{1+\left(1-\gamma\right)\left(1-\theta\right)}\right]^{-\mu}\cdot \\
&\cdot\left[\gamma+\theta\left(1-\gamma\right)+\left(1-\theta\right)\left(1-\gamma\right)n^{*}|_{\eta=0}\right]^{1-\mu}+ \\
&+\nu\left(1-\mu\right)\frac{\delta n^{*}}{\delta\eta}|_{\eta=0}\left(1-\gamma\right)\left(1-\theta\right)\left[\frac{W^{*}|_{\eta=0}}{1+\left(1-\gamma\right)\left(1-\theta\right)}\right]^{1-\mu}\cdot \\
&\cdot\left[\gamma+\theta\left(1-\gamma\right)+\left(1-\theta\right)\left(1-\gamma\right)n^{*}|_{\eta=0}\right]^{-\mu}
\end{aligned}$$

and

$$\frac{\delta u^{U}(c)}{\delta \eta}|_{\eta=0}$$
$$=\frac{\nu(1-\mu)(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^{*}|_{\eta=0}\cdot n^{*}|_{\eta=0}\cdot(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\cdot$$
$$\cdot\left(\frac{\delta W^{*}}{\delta \eta}|_{\eta=0}\cdot n^{*}|_{\eta=0}+\frac{\delta n^{*}}{\delta \eta}|_{\eta=0}\cdot W^{*}|_{\eta=0}\right).$$

Since $\lim_{\mu\to 1}\frac{\delta u^{HP}(c)}{\delta \eta}|_{\eta=0}=\lim_{\mu\to 1}\frac{\delta u^{U}(c)}{\delta \eta}|_{\eta=0}=0$ it is also:

$$\lim_{\mu\to 1}\frac{\delta u^{Y}(c)}{\delta \eta}|_{\eta=0}$$
$$=\frac{\delta n^{*}}{\delta \eta}|_{\eta=0}\cdot\nu=\nu(1-\sigma)\frac{\frac{\alpha}{4}\left[\gamma+\theta(1-\gamma)\right]}{\phi\cdot\beta}+(1-\sigma)\frac{\left(\frac{1}{2}-\beta\right)}{\phi\cdot\beta^{2}}\left[1+\frac{\alpha}{2}-\phi(k-1)\right]\geq 0.$$

and for every $\theta\in[0,1]$ there exists a $0<\hat{\mu}<1$ large enough for $\frac{\delta u^{Y}(c)}{\delta \eta}|_{\eta=0}\geq 0$ to hold. If $\mu\geq\hat{\mu}$, it is thus possible to exclude that young agent preferred choice is the corner solution $\eta=0$. If $\alpha\leq\hat{\alpha}$ also holds, $\frac{\delta u^{Y}(c)}{\delta \eta}|_{\eta=1}\leq 0$ and $\frac{\delta u^{Y}(c)}{\delta \eta}|_{\eta=0}\geq 0$ hold too so that by the continuity of $\frac{\delta u^{Y}(c)}{\delta \eta}$ there exists a η^{Y} such that $\frac{\delta u^{Y}(c)}{\delta \eta}|_{\eta=\eta^{Y}}=0$ holds. Furthermore since at the extremes of the dominion there are two points of local minimum, an interior maximum must exist where the inequality $\frac{\delta'' u^{Y}(c)}{\delta'' \eta}|_{\eta=\eta^{Y}}\leq 0$ is verified. ■

5.4 Proof of Lemma 8

Consider $\frac{\delta u^Y(c)}{\delta\theta}$ and derive with respect to η to obtain:

$$
\begin{aligned}
&\frac{\delta'' u^Y(c)}{\delta\eta\delta\theta} \\
&= \frac{\delta n^*}{\delta\theta}\cdot\frac{\nu}{2}(2-\mu)\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\left[1-(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}\right]+ \\
&+\frac{\eta}{2}\cdot\frac{\delta'' n^*}{\delta\eta\delta\theta}\cdot\nu(2-\mu)\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\left[1-(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}\right]+ \\
&+\frac{\eta\cdot\frac{\delta n^*}{\delta\theta}}{2}\cdot\frac{\nu(2-\mu)(1-\mu)\frac{\delta W^*}{\delta\eta}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\cdot \\
&\cdot\left[1-(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}\right]+ \\
&+\frac{n^*}{2}\cdot\frac{\nu(1-\mu)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\left[1+(1-\gamma)^{-\mu}(1-\theta)^{-\mu}\right]+ \\
&+\frac{\eta}{2}\cdot\frac{\nu(1-\mu)(1-\gamma)\frac{\delta n^*}{\delta\eta}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\left[1+(1-\gamma)^{-\mu}(1-\theta)^{-\mu}\right]+ \\
&+\frac{\eta\cdot n^*}{2}\cdot\frac{\nu(1-\mu)^2(1-\gamma)\frac{\delta W^*}{\delta\eta}}{\left[1+(1-\gamma)(1-\theta)\right]^2}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\left[1+(1-\gamma)^{-\mu}(1-\theta)^{-\mu}\right]+ \\
&+\frac{\delta'' n^*}{\delta\eta\delta\theta}\cdot\nu(2-\mu)\left[\frac{W^*(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}+ \\
&+\frac{\delta n^*}{\delta\theta}\cdot\frac{\nu(2-\mu)(1-\mu)\frac{\delta W^*}{\delta\eta}(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}+ \\
&-\frac{\nu(1-\mu)\frac{\delta n^*}{\delta\theta}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}(1-\theta)^{-\mu}+ \\
&-\frac{n^*\cdot\nu(1-\mu)^2\frac{\delta W^*}{\delta\eta}(1-\gamma)}{\left[1+(1-\gamma)(1-\theta)\right]^2}\left[\frac{W^*(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}+\frac{\delta'' u^U(c)}{\delta\eta\delta\theta} \\
&-\frac{1}{2}\cdot\frac{\delta n^*}{\delta\theta}\left[u^{HP}(c)-u^U(c)\right]+\left(1-\frac{\eta}{2}\right)\frac{\delta'' n^*}{\delta\eta\delta\theta}\left[u^{HP}(c)-u^U(c)\right]+ \\
&+\left(1-\frac{\eta}{2}\right)\frac{\delta n^*}{\delta\theta}\left[\frac{\delta u^{HP}(c)}{\delta\eta}-\frac{\delta u^U(c)}{\delta\eta}\right]+\frac{\delta n^*}{\delta\eta}\cdot\left(1-\frac{\eta}{2}\right)\left[\frac{\delta u^{HP}(c)}{\delta\theta}-\frac{\delta u^U(c)}{\delta\theta}\right]+ \\
&-\frac{n^*}{2}\left[\frac{\delta u^{HP}(c)}{\delta\theta}-\frac{\delta u^U(c)}{\delta\theta}\right]+n^*\left(1-\frac{\eta}{2}\right)\left[\frac{\delta'' u^{HP}(c)}{\delta\eta\delta\theta}-\frac{\delta'' u^U(c)}{\delta\eta\delta\theta}\right].
\end{aligned}
$$

Take the limit for μ approaching 1 to get:

$$\lim_{\mu\to 1}\frac{\delta'' u^{Y}(c)}{\delta\eta\delta\theta}$$
$$=\frac{\delta'' n^{*}}{\delta\eta\delta\theta}\cdot\nu+\frac{\delta'' n^{*}}{\delta\eta\delta\theta}\left(1-\frac{\eta}{2}\right)\lim_{\mu\to 1}\left[u^{HP}(c)-u^{U}(c)\right]+$$
$$-\frac{1}{2}\cdot\frac{\delta n^{*}}{\delta\theta}\cdot\lim_{\mu\to 1}\left[u^{HP}(c)-u^{U}(c)\right]+\frac{\delta n^{*}}{\delta\theta}\left(1-\frac{\eta}{2}\right)\lim_{\mu\to 1}\left[\frac{\delta u^{HP}(c)}{\delta\eta}-\frac{\delta u^{U}(c)}{\delta\eta}\right]+$$
$$+\frac{\delta n^{*}}{\delta\eta}\left(1-\frac{\eta}{2}\right)\lim_{\mu\to 1}\left[\frac{\delta u^{HP}(c)}{\delta\theta}-\frac{\delta u^{U}(c)}{\delta\theta}\right]-\frac{n^{*}}{2}\cdot\lim_{\mu\to 1}\left[\frac{\delta u^{HP}(c)}{\delta\theta}-\frac{\delta u^{U}(c)}{\delta\theta}\right]+$$
$$+n^{*}\left(1-\frac{\eta}{2}\right)\lim_{\mu\to 1}\left[\frac{\delta'' u^{HP}(c)}{\delta\eta\delta\theta}-\frac{\delta'' u^{U}(c)}{\delta\eta\delta\theta}\right]+\lim_{\mu\to 1}\frac{\delta'' u^{U}(c)}{\delta\eta\delta\theta}$$

and note that $\lim_{\mu\to 1}\frac{\delta u^{HP}(c)}{\delta\eta}=\lim_{\mu\to 1}\frac{\delta u^{HP}(c)}{\delta\theta}=\lim_{\mu\to 1}\frac{\delta u^{U}(c)}{\delta\eta}=\lim_{\mu\to 1}\frac{\delta u^{U}(c)}{\delta\theta}=0$ and $\lim_{\mu\to 1}u^{HP}(c)=\lim_{\mu\to 1}u^{U}(c)=\nu$ hold. Hence it is:

$$\lim_{\mu\to 1}\frac{\delta'' u^{Y}(c)}{\delta\eta\delta\theta}=\frac{\delta'' n^{*}}{\delta\eta\delta\theta}\cdot\nu+n^{*}\left(1-\frac{\eta}{2}\right)\cdot\lim_{\mu\to 1}\left[\frac{\delta'' u^{HP}(c)}{\delta\eta\delta\theta}-\frac{\delta'' u^{U}(c)}{\delta\eta\delta\theta}\right]+\lim_{\mu\to 1}\frac{\delta'' u^{U}(c)}{\delta\eta\delta\theta}.$$

Consider now $\frac{\delta u^{HP}(c)}{\delta\theta}$ and derive with respect to η to obtain:

$$\frac{\delta'' u^{HP}(c)}{\delta\eta\delta\theta}$$

$$= -\frac{\nu(1-\mu)\mu\cdot\frac{\delta W^*}{\delta\eta}}{[1+(1-\gamma)(1-\theta)]^2}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu-1}\left[\frac{\delta W^*}{\delta\theta}+\frac{W^*(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]\cdot$$
$$\cdot[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)]^{1-\mu}+$$
$$+\frac{\nu(1-\mu)^2\frac{\delta n^*}{\delta\eta}(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\cdot$$
$$\cdot\left[\frac{\delta W^*}{\delta\theta}+\frac{W^*(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right][\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)]^{-\mu}+$$
$$+\frac{\nu(1-\mu)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\left[\frac{\delta''W^*}{\delta\eta\delta\theta}+\frac{\frac{\delta W^*}{\delta\eta}\cdot(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]\cdot$$
$$\cdot[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)]^{1-\mu}+$$
$$+\frac{\nu(1-\mu)^2\frac{\delta W^*}{\delta\eta}(1-\gamma)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\left[1-n^*+\frac{\delta n^*}{\delta\theta}(1-\theta)\right]\cdot$$
$$\cdot[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)]^{-\mu}+$$
$$-\nu(1-\mu)\mu\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)]^{-\mu-1}\cdot$$
$$\cdot\frac{\delta n^*}{\delta\eta}(1-\theta)(1-\gamma)^2\left[1-n^*+\frac{\delta n^*}{\delta\theta}(1-\theta)\right]+$$
$$+\nu(1-\mu)(1-\gamma)\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)]^{-\mu}\cdot$$
$$\cdot\left[1-\frac{\delta n^*}{\delta\eta}+\frac{\delta''n^*}{\delta\eta\delta\theta}(1-\theta)\right].$$

Note that $\lim_{\mu\to1}\frac{\delta'' u^{HP}(c)}{\delta\eta\delta\theta}=0$.

Consider now $\frac{\delta u^U(c)}{\delta\theta}$ and derive with respect to η to obtain:

$$\frac{\delta'' u^U(c)}{\delta\eta\delta\theta}$$
$$= -\frac{\nu\cdot\frac{\delta W^*}{\delta\eta}(1-\mu)\mu}{[1+(1-\gamma)(1-\theta)]^2}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu-1}[n^*(1-\theta)(1-\gamma)]^{1-\mu}\cdot$$
$$\cdot\left[\frac{\delta W^*}{\delta\theta}+\frac{W^*(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]+$$
$$+\frac{\nu(1-\mu)^2\frac{\delta n^*}{\delta\eta}(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*\cdot n^*(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\cdot$$
$$\cdot\left[\frac{\delta W^*}{\delta\theta}+\frac{W^*(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]+$$
$$+\frac{\nu(1-\mu)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}[n^*(1-\theta)(1-\gamma)]^{1-\mu}\cdot$$
$$\cdot\left[\frac{\delta'' W^*}{\delta\eta\delta\theta}+\frac{\frac{\delta W^*}{\delta\eta}(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]+$$
$$+\frac{\nu\cdot\frac{\delta W^*}{\delta\eta}(1-\mu)^2(1-\gamma)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*\cdot n^*(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\left[\frac{\delta n^*}{\delta\theta}(1-\theta)-n^*\right]+$$
$$-\nu(1-\mu)\mu\left[\frac{W^*(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}[n^*(1-\theta)]^{-\mu-1}\frac{\delta n^*}{\delta\eta}(1-\theta)\cdot$$
$$\cdot\left[\frac{\delta n^*}{\delta\theta}(1-\theta)-n^*\right]+$$
$$+\nu(1-\mu)\left[\frac{W^*(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}[n^*(1-\theta)]^{-\mu}\cdot\left[\frac{\delta'' n^*}{\delta\eta\delta\theta}(1-\theta)-\frac{\delta n^*}{\delta\eta}\right].$$

Note that $\lim_{\mu\to 1}\frac{\delta'' u^U(c)}{\delta\eta\delta\theta}=0$ holds. Hence also holds that:

$$\lim_{\mu\to 1}\frac{\delta'' u^Y(c)}{\delta\eta\delta\theta}=\frac{\delta'' n^*}{\delta\eta\delta\theta}\cdot\nu=(1-\sigma)\frac{\frac{\alpha}{4}[\gamma+\theta(1-\gamma)]+\frac{\alpha\cdot\theta}{4}(1-\gamma)}{\phi\left\{\beta\left[\left(\frac{\eta}{2}\right)^2+\left(1-\frac{\eta}{2}\right)^2\right]+\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}}+$$
$$+(1-\sigma)\frac{(1-\eta)\left(\frac{1}{2}-\beta\right)\frac{\alpha\cdot\eta}{4}(1-\gamma)}{\phi\left\{\beta\left[\left(\frac{\eta}{2}\right)^2+\left(1-\frac{\eta}{2}\right)^2\right]+\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}^2}\geq 0$$

implying that for every pair $\theta\in(0,1)$ and $\eta\in(0,1)$ there exists a large enough $0<\hat{\mu}<1$ for $\frac{\delta'' u^Y(c)}{\delta\eta\delta\theta}\geq 0$ to hold.

5.5 Proof of Lemma 10

Consider $\frac{\delta u^{Y}(c)}{\delta\theta}$ and derive with respect to α to obtain:

$$\begin{aligned}
&\frac{\delta'' u^{Y}(c)}{\delta\alpha\delta\theta} \\
&= \frac{\eta\cdot\frac{\delta'' n^*}{\delta\alpha\delta\theta}}{2}\cdot\nu(2-\mu)\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\left[1-(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}\right]+ \\
&+\frac{\eta\cdot\frac{\delta n^*}{\delta\theta}}{2}\cdot\frac{\nu(2-\mu)(1-\mu)\frac{\delta W^*}{\delta\alpha}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\cdot \\
&\cdot\left[1-(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}\right]+ \\
&+\frac{\eta\cdot\frac{\delta n^*}{\delta\alpha}}{2}\cdot\frac{\nu(1-\mu)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\left[1+(1-\gamma)^{-\mu}(1-\theta)^{-\mu}\right]+ \\
&+\frac{\eta\cdot n^*}{2}\cdot\frac{\nu(1-\mu)^2\frac{\delta W^*}{\delta\alpha}(1-\gamma)}{\left[1+(1-\gamma)(1-\theta)\right]^2}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\left[1+(1-\gamma)^{-\mu}(1-\theta)^{-\mu}\right]+ \\
&+\frac{\delta'' n^*}{\delta\alpha\delta\theta}\cdot\nu(2-\mu)\left[\frac{W^*(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}+ \\
&+\frac{\delta n^*}{\delta\theta}\cdot\frac{\nu(2-\mu)(1-\mu)\frac{\delta W^*}{\delta\alpha}(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}+ \\
&-\frac{\frac{\delta n^*}{\delta\alpha}\cdot\nu(1-\mu)(1-\theta)^{-\mu}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}+ \\
&-\frac{n^*\cdot\nu(1-\mu)^2\frac{\delta W^*}{\delta\alpha}(1-\gamma)}{\left[1+(1-\gamma)(1-\theta)\right]^2}\left[\frac{W^*(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}+ \\
&+\left(1-\frac{\eta}{2}\right)\frac{\delta'' n^*}{\delta\alpha\delta\theta}\left[u^{HP}(c)-u^{U}(c)\right]+\left(1-\frac{\eta}{2}\right)\frac{\delta n^*}{\delta\theta}\left[\frac{\delta u^{HP}(c)}{\delta\alpha}-\frac{\delta u^{U}(c)}{\delta\alpha}\right]+ \\
&+\frac{\delta n^*}{\delta\alpha}\left(1-\frac{\eta}{2}\right)\left[\frac{\delta u^{HP}(c)}{\delta\theta}-\frac{\delta u^{U}(c)}{\delta\theta}\right]+ \\
&+n^*\left(1-\frac{\eta}{2}\right)\left[\frac{\delta'' u^{HP}(c)}{\delta\alpha\delta\theta}-\frac{\delta'' u^{U}(c)}{\delta\alpha\delta\theta}\right]+\frac{\delta'' u^{U}(c)}{\delta\alpha\delta\theta}.
\end{aligned}$$

Take the limit for μ approaching 1 to obtain:

$$\begin{aligned}
&\lim_{\mu\to 1}\frac{\delta'' u^{Y}(c)}{\delta\theta\delta\alpha} \\
&=\frac{\delta'' n^{*}}{\delta\alpha\delta\theta}\cdot\nu+\frac{\delta'' n^{*}}{\delta\alpha\delta\theta}\left(1-\frac{\eta}{2}\right)\lim_{\mu\to 1}\left[u^{HP}(c)-u^{U}(c)\right]+ \\
&+\frac{\delta n^{*}}{\delta\theta}\left(1-\frac{\eta}{2}\right)\lim_{\mu\to 1}\left[\frac{\delta u^{HP}(c)}{\delta\alpha}-\frac{\delta u^{U}(c)}{\delta\alpha}\right]+ \\
&+\frac{\delta n^{*}}{\delta\alpha}\left(1-\frac{\eta}{2}\right)\lim_{\mu\to 1}\frac{\delta u^{HP}(c)}{\delta\theta}+n^{*}\left(1-\frac{\eta}{2}\right)\lim_{\mu\to 1}\frac{\delta'' u^{HP}(c)}{\delta\alpha\delta\theta}+ \\
&-\frac{\delta n^{*}}{\delta\alpha}\left(1-\frac{\eta}{2}\right)\lim_{\mu\to 1}\frac{\delta u^{U}(c)}{\delta\theta}+\left[1-n^{*}\left(1-\frac{\eta}{2}\right)\right]\lim_{\mu\to 1}\frac{\delta'' u^{U}(c)}{\delta\alpha\delta\theta}
\end{aligned}$$

and note that $\lim_{\mu\to 1} u^{HP}(c)=\lim_{\mu\to 1} u^{U}(c)=\nu$ and $\lim_{\mu\to 1}\frac{\delta u^{HP}(c)}{\delta\theta}=\lim_{\mu\to 1}\frac{\delta u^{U}(c)}{\delta\theta}=0$ hold. This implies:

$$\begin{aligned}
&\lim_{\mu\to 1}\frac{\delta'' u^{Y}(c)}{\delta\theta\delta\alpha} \\
&=\frac{\delta'' n^{*}}{\delta\alpha\delta\theta}\cdot\nu+\frac{\delta n^{*}}{\delta\theta}\left(1-\frac{\eta}{2}\right)\lim_{\mu\to 1}\left[\frac{\delta u^{HP}(c)}{\delta\alpha}-\frac{\delta u^{U}(c)}{\delta\alpha}\right]+ \\
&+n^{*}\left(1-\frac{\eta}{2}\right)\lim_{\mu\to 1}\frac{\delta'' u^{HP}(c)}{\delta\alpha\delta\theta}+\left[1-n^{*}\left(1-\frac{\eta}{2}\right)\right]\lim_{\mu\to 1}\frac{\delta'' u^{U}(c)}{\delta\alpha\delta\theta}.
\end{aligned}$$

Focus now on $\frac{\delta u^{HP}(c)}{\delta\alpha}$, $\frac{\delta u^{U}(c)}{\delta\alpha}$, $\frac{\delta'' u^{HP}(c)}{\delta\alpha\delta\theta}$ and $\frac{\delta'' u^{U}(c)}{\delta\alpha\delta\theta}$. Consider initially:

$$\begin{aligned}
&\frac{\delta u^{HP}(c)}{\delta\alpha} \\
&=\frac{\nu(1-\mu)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\left[\gamma+\theta(1-\gamma)+n^{*}(1-\theta)(1-\gamma)\right]^{-\mu}\cdot \\
&\cdot\left\{\frac{\delta W^{*}}{\delta\alpha}\left[\gamma+\theta(1-\gamma)+n^{*}(1-\theta)(1-\gamma)\right]+W^{*}\cdot\frac{\delta n^{*}}{\delta\alpha}\cdot(1-\theta)(1-\gamma)\right\}
\end{aligned}$$

and note that $\lim_{\mu\to 1}\frac{\delta u^{HP}(c)}{\delta\alpha}=0$ holds. Derive with respect to θ to get:

$$\frac{\delta'' u^{HP}(c)}{\delta\theta\delta\alpha}$$
$$=-\frac{\nu(1-\mu)\mu\cdot\frac{\delta W^*}{\delta\alpha}}{\left[1+(1-\gamma)(1-\theta)\right]^2}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu-1}\cdot$$
$$\cdot\left[\frac{\delta W^*}{\delta\theta}+\frac{(1-\gamma)W^*}{1+(1-\gamma)(1-\theta)}\right]\left[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)\right]^{1-\mu}+$$
$$+\frac{\nu(1-\mu)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\left[\frac{\delta'' W^*}{\delta\theta\delta\alpha}+\frac{\frac{\delta W^*}{\delta\alpha}(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]\cdot$$
$$\cdot\left[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)\right]^{1-\mu}+$$
$$+\frac{\nu(1-\mu)^2\frac{\delta W^*}{\delta\alpha}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\cdot$$
$$\cdot\left[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)\right]^{-\mu}(1-\gamma)\left[1-n^*+\frac{\delta n^*}{\delta\theta}\cdot(1-\theta)\right]+$$
$$+\nu(1-\mu)^2\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\cdot\left[\frac{\delta W^*}{\delta\theta}+\frac{(1-\gamma)W^*}{1+(1-\gamma)(1-\theta)}\right]\cdot$$
$$\cdot\left[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)\right]^{-\mu}\cdot\frac{\delta n^*}{\delta\alpha}(1-\theta)(1-\gamma)+$$
$$-\nu(1-\mu)\mu\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\left[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)\right]^{-\mu-1}\cdot$$
$$\cdot\left[1-n^*+\frac{\delta n^*}{\delta\theta}(1-\theta)\right]\frac{\delta n^*}{\delta\alpha}(1-\theta)(1-\gamma)^2+$$
$$+\nu(1-\mu)\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\left[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)\right]^{-\mu}\cdot$$
$$(1-\gamma)\left[\frac{\delta'' n^*}{\delta\theta\delta\alpha}(1-\theta)-\frac{\delta n^*}{\delta\alpha}\right].$$

In this case too $\lim_{\mu\to 1}\frac{\delta'' u^{HP}(c)}{\delta\theta\delta\alpha}=0$ holds. Consider:

$$\frac{\delta u^{U}(c)}{\delta\alpha}$$
$$=\frac{\nu(1-\mu)(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*\cdot n^*(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\left(\frac{\delta W^*}{\delta\alpha}\cdot n^*+\frac{\delta n^*}{\delta\alpha}\cdot W^*\right)$$

and note that $\lim_{\mu\to1}\frac{\delta u^U(c)}{\delta\alpha}=0$ holds. Derive then with respect to θ to get:

$$\frac{\delta'' u^U(c)}{\delta\theta\delta\alpha}$$

$$\begin{aligned}
&= -\frac{\nu(1-\mu)\mu}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\left[\frac{\delta W^*}{\delta\theta}+\frac{W^*(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]\cdot\\
&\cdot\frac{\frac{\delta W^*}{\delta\alpha}}{1+(1-\gamma)(1-\theta)}\cdot\left[n^*(1-\gamma)(1-\theta)\right]^{1-\mu}+\\
&+\frac{\nu(1-\mu)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\left[\frac{\delta'' W^*}{\delta\theta\delta\alpha}+\frac{(1-\gamma)\frac{\delta W^*}{\delta\alpha}}{1+(1-\gamma)(1-\theta)}\right]\cdot\\
&\cdot\left[n^*(\eta,\theta)\cdot(1-\gamma)(1-\theta)\right]^{1-\mu}+\\
&+\frac{\nu(1-\mu)^2\frac{\delta W^*}{\delta\alpha}(1-\gamma)}{1+(1-\gamma)(1-\theta)}\cdot\left[\frac{W^*\cdot n^*(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\left[\frac{\delta n^*}{\delta\theta}(1-\theta)-n^*\right]+\\
&+\frac{\nu(1-\mu)^2\frac{\delta n^*}{\delta\alpha}(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*\cdot n^*(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\cdot\\
&\cdot\left[\frac{\delta W^*}{\delta\theta}+\frac{W^*(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]+\\
&-\nu(1-\mu)\mu\left[\frac{W^*(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}(n^*)^{-\mu-1}\left[\frac{\delta n^*}{\delta\theta}(1-\theta)-n^*\right]\frac{\delta n^*}{\delta\alpha}(1-\theta)^{-\mu}+\\
&+\nu(1-\mu)\left[\frac{W^*(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\left[n^*(1-\theta)\right]^{-\mu}\left[\frac{\delta'' n^*}{\delta\theta\delta\alpha}(1-\theta)-\frac{\delta n^*}{\delta\alpha}\right].
\end{aligned}$$

In this case too $\lim_{\mu\to1}\frac{\delta'' u^U(c)}{\delta\theta\delta\alpha}=0$ and

$$\lim_{\mu\to1}\frac{\delta'' u^Y(c)}{\delta\theta\delta\alpha}=\nu\cdot\frac{\delta'' n^*}{\delta\alpha\delta\theta}=\frac{\nu\cdot\frac{1-\sigma}{2}\left[1+\frac{\eta}{2}(1-\gamma)\right]}{\phi\left\{\beta\left[\left(\frac{\eta}{2}\right)^2+\left(1-\frac{\eta}{2}\right)^2\right]+\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}}\geq0$$

holds. For every pair $\theta\in(0,1)$ and $\eta\in(0,1)$ there thus exists a large enough $0<\hat{\mu}<1$ for $\frac{\delta'' u^Y(c)}{\delta\eta\delta\alpha}\geq0$ to hold.

Consider now $\frac{\delta u^Y(c)}{\delta \eta}$ and derive with respect to α to obtain:

$$\frac{\delta'' u^Y(c)}{\delta\alpha\delta\eta}$$

$$= \frac{\delta n^*}{\delta\alpha}\cdot\frac{\nu}{2}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\left[1-(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}\right]+$$

$$+\frac{n^*}{2}\cdot\frac{\nu(1-\mu)\frac{\delta W^*}{\delta\alpha}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\left[1-(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}\right]+$$

$$+\frac{\eta}{2}\cdot\frac{\delta n^*}{\delta\alpha}\cdot\frac{\nu(1-\mu)\frac{\delta W^*}{\delta\eta}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}+$$

$$-\frac{\eta\cdot n^*}{2}\cdot\frac{\nu(1-\mu)\mu\cdot\frac{\delta W^*}{\delta\alpha}\cdot\frac{\delta W^*}{\delta\eta}}{[1+(1-\gamma)(1-\theta)]^2}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu-1}+$$

$$+\frac{\eta\cdot n^*}{2}\cdot\frac{\nu(1-\mu)\frac{\delta'' W^*}{\delta\alpha\delta\eta}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}+$$

$$+\frac{\delta'' n^*}{\delta\alpha\delta\eta}\left(1-\frac{\eta}{2}\right)\nu\left[\frac{W^*(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}+$$

$$+\frac{\delta n^*}{\delta\eta}\left(1-\frac{\eta}{2}\right)\frac{\nu(1-\mu)\frac{\delta W^*}{\delta\alpha}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}+$$

$$+\frac{\delta n^*}{\delta\alpha}\left(1-\frac{\eta}{2}\right)\frac{\nu(1-\mu)\frac{\delta W^*}{\delta\eta}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*(1-\gamma)(1-\theta)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}+$$

$$-n^*\left(1-\frac{\eta}{2}\right)\frac{\nu(1-\mu)\mu\cdot\frac{\delta W^*}{\delta\alpha}}{[1+(1-\gamma)(1-\theta)]^2}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu-1}\cdot$$

$$\cdot\frac{\delta W^*}{\delta\eta}(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}+$$

$$+n^*\left(1-\frac{\eta}{2}\right)\frac{\nu(1-\mu)\frac{\delta'' W^*}{\delta\alpha\delta\eta}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}+$$

$$+\left[\frac{\delta'' n^*}{\delta\alpha\delta\eta}\left(1-\frac{\eta}{2}\right)-\frac{\delta n^*}{\delta\alpha}\cdot\frac{1}{2}\right]\left[u^{HP}(c)-u^U(c)\right]+$$

$$+\left[\frac{\delta n^*}{\delta\eta}\left(1-\frac{\eta}{2}\right)-\frac{n^*}{2}\right]\left[\frac{\delta u^{HP}(c)}{\delta\alpha}-\frac{\delta u^U(c)}{\delta\alpha}\right]+$$

$$+\frac{\delta n^*}{\delta\alpha}\left(1-\frac{\eta}{2}\right)\left[\frac{\delta u^{HP}(c)}{\delta\eta}-\frac{\delta u^U(c)}{\delta\eta}\right]+$$

$$+n^*\left(1-\frac{\eta}{2}\right)\left[\frac{\delta'' u^{HP}(c)}{\delta\alpha\delta\eta}-\frac{\delta'' u^U(c)}{\delta\alpha\delta\eta}\right]+\frac{\delta'' u^U(c)}{\delta\alpha\delta\eta}.$$

Take the limit for μ which approaches 1 and note that it is:

$$\lim_{\mu\to 1}\frac{\delta'' u^{Y}(c)}{\delta\alpha\delta\eta}$$

$$=\nu\cdot\frac{\delta'' n^{*}}{\delta\alpha\delta\eta}+\lim_{\mu\to 1}\frac{\delta'' u^{U}(c)}{\delta\alpha\delta\eta}\left[1-n^{*}\left(1-\frac{\eta}{2}\right)\right]+\lim_{\mu\to 1}\frac{\delta'' u^{HP}(c)}{\delta\alpha\delta\eta}\cdot n^{*}\left(1-\frac{\eta}{2}\right)$$

Focus on $\frac{\delta'' u^{HP}(c)}{\delta\alpha\delta\eta}$ and $\frac{\delta'' u^{U}(c)}{\delta\alpha\delta\eta}$ and consider initially:

$$\frac{\delta'' u^{HP}(c)}{\delta\alpha\delta\eta}$$

$$=-\frac{\nu(1-\mu)\mu\cdot\frac{\delta W^{*}}{\delta\alpha}\cdot\frac{\delta W^{*}}{\delta\eta}}{[1+(1-\gamma)(1-\theta)]^{2}}\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{-\mu-1}\cdot$$

$$\cdot[\gamma+\theta(1-\gamma)+n^{*}(1-\theta)(1-\gamma)]^{1-\mu}+$$

$$+\frac{\nu(1-\mu)^{2}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\cdot$$

$$\cdot[\gamma+\theta(1-\gamma)+n^{*}(1-\theta)(1-\gamma)]^{-\mu}\frac{\delta W^{*}}{\delta\eta}\cdot\frac{\delta n^{*}}{\delta\alpha}(1-\theta)(1-\gamma)+$$

$$+\frac{\nu(1-\mu)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\cdot$$

$$\cdot[\gamma+\theta(1-\gamma)+n^{*}(1-\theta)(1-\gamma)]^{1-\mu}\frac{\delta'' W^{*}}{\delta\alpha\delta\eta}+$$

$$+\frac{\nu(1-\mu)^{2}\frac{\delta W^{*}}{\delta\alpha}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\cdot$$

$$\cdot[\gamma+\theta(1-\gamma)+n^{*}(1-\theta)(1-\gamma)]^{-\mu}\frac{\delta n^{*}}{\delta\eta}(1-\gamma)(1-\theta)+$$

$$-\nu(1-\mu)\mu\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}[\gamma+\theta(1-\gamma)+n^{*}(1-\theta)(1-\gamma)]^{-\mu-1}\cdot$$

$$\cdot\frac{\delta n^{*}}{\delta\alpha}\cdot\frac{\delta n^{*}}{\delta\eta}(1-\gamma)^{2}(1-\theta)^{2}+$$

$$+\nu(1-\mu)\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}[\gamma+\theta(1-\gamma)+n^{*}(1-\theta)(1-\gamma)]^{-\mu}\cdot$$

$$\cdot\frac{\delta'' n^{*}}{\delta\alpha\delta\eta}(1-\gamma)(1-\theta).$$

is the case that $\lim_{\mu\to1}\frac{\delta'' u^{HP}(c)}{\delta\alpha\delta\eta}=0$ holds. Consider now:

$$
\begin{aligned}
&\frac{\delta'' u^{U}(c)}{\delta\alpha\delta\eta}\\
&= -\frac{\nu(1-\mu)\mu}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu-1}\frac{\frac{\delta W^*}{\delta\alpha}\left[n^*(1-\theta)(1-\gamma)\right]^{1-\mu}\frac{\delta W^*}{\delta\eta}}{1+(1-\gamma)(1-\theta)}+\\
&\quad+\frac{\nu(1-\mu)^2}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*\cdot n^*(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\frac{\delta n^*}{\delta\alpha}(1-\theta)(1-\gamma)\frac{\delta W^*}{\delta\eta}+\\
&\quad+\frac{\nu(1-\mu)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\left[n^*(1-\theta)(1-\gamma)\right]^{1-\mu}\frac{\delta'' W^*}{\delta\alpha\delta\eta}+\\
&\quad+\frac{\nu(1-\mu)^2\frac{\delta W^*}{\delta\alpha}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*\cdot n^*(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}(1-\theta)(1-\gamma)\frac{\delta n^*}{\delta\eta}+\\
&\quad-\nu(1-\mu)\mu\left[\frac{W^*(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}(n^*)^{-\mu-1}\frac{\delta n^*}{\delta\alpha}\cdot\frac{\delta n^*}{\delta\eta}+\\
&\quad+\nu(1-\mu)\left[\frac{W^*(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}(n^*)^{-\mu}\frac{\delta'' n^*}{\delta\alpha\delta\eta}.
\end{aligned}
$$

In this case too $\lim_{\mu\to1}\frac{\delta'' u^{U}(c)}{\delta\alpha\delta\eta}=0$ and

$$
\begin{aligned}
\lim_{\mu\to1}\frac{\delta'' u^{Y}(c)}{\delta\alpha\delta\eta}=\nu\cdot\frac{\delta'' n^*}{\delta\alpha\delta\eta}&=(1-\sigma)\frac{\frac{1}{4}\left[\gamma+\theta(1-\gamma)\right]}{\phi\left\{\beta\left[\left(\frac{\eta}{2}\right)^2+\left(1-\frac{\eta}{2}\right)^2\right]+\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}}+\\
&-(1-\sigma)\frac{(1-\eta)\left(\beta-\frac{1}{2}\right)\frac{1}{2}\left\{1+\frac{\eta}{2}\left[\gamma+\theta(1-\gamma)\right]\right\}}{\phi\left\{\beta\left[\left(\frac{\eta}{2}\right)^2+\left(1-\frac{\eta}{2}\right)^2\right]+\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}^2}\geq 0.
\end{aligned}
$$

holds. For every pair $\theta\in(0,1)$ and $\eta\in(01)$ there thus exists a large enough $0<\hat{\mu}<1$ for $\frac{\delta'' u^{Y}(c)}{\delta\eta\delta\alpha}\geq 0$ to hold.

5.6 Proof of Lemma 13

Derive $\frac{\delta u^{Y}(c)}{\delta\theta}$ with respect to ϕ to obtain:

$$
\begin{aligned}
&\frac{\delta'' u^{Y}(c)}{\delta\phi\delta\theta}\\
=&\frac{\eta}{2}\cdot\frac{\delta'' n^{*}}{\delta\phi\delta\theta}\cdot\nu(2-\mu)\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\left[1-(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}\right]+\\
&+\frac{\eta}{2}\cdot\frac{\delta n^{*}}{\delta\theta}\cdot\frac{\nu(2-\mu)(1-\mu)\frac{\delta W^{*}}{\delta\phi}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\cdot\\
&\cdot\left[1-(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}\right]+\\
&+\frac{\eta}{2}\cdot\frac{\delta n^{*}}{\delta\phi}\cdot\frac{\nu(1-\mu)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\left[1+(1-\gamma)^{-\mu}(1-\theta)^{-\mu}\right]+\\
&+\frac{\eta\cdot n^{*}}{2}\cdot\frac{\nu(1-\mu)^{2}\frac{\delta W^{*}}{\delta\phi}(1-\gamma)}{\left[1+(1-\gamma)(1-\theta)\right]^{2}}\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\left[1+(1-\gamma)^{-\mu}(1-\theta)^{-\mu}\right]+\\
&+\frac{\delta'' n^{*}}{\delta\phi\delta\theta}\cdot\nu(2-\mu)\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}+\\
&+\frac{\delta n^{*}}{\delta\theta}\cdot\frac{\nu(2-\mu)(1-\mu)\frac{\delta W^{*}}{\delta\phi}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}+\\
&-\frac{\delta n^{*}}{\delta\phi}\cdot\frac{\nu(1-\mu)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}(1-\gamma)^{1-\mu}(1-\theta)^{-\mu}+\\
&-n^{*}\cdot\frac{\nu(1-\mu)^{2}\frac{\delta W^{*}}{\delta\phi}}{\left[1+(1-\gamma)(1-\theta)\right]^{2}}\left[\frac{W^{*}}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}(1-\gamma)^{1-\mu}(1-\theta)^{-\mu}+\\
&+\left(1-\frac{\eta}{2}\right)\frac{\delta'' n^{*}}{\phi\delta\theta}\left[u^{HP}(c)-u^{U}(c)\right]+\left(1-\frac{\eta}{2}\right)\frac{\delta n^{*}}{\delta\theta}\left[\frac{\delta u^{HP}(c)}{\delta\phi}-\frac{\delta u^{U}(c)}{\delta\phi}\right]+\\
&+\frac{\delta n^{*}}{\delta\phi}\cdot\left(1-\frac{\eta}{2}\right)\left[\frac{\delta u^{HP}(c)}{\delta\theta}-\frac{\delta u^{U}(c)}{\delta\theta}\right]+\\
&+n^{*}\left(1-\frac{\eta}{2}\right)\left[\frac{\delta'' u^{HP}(c)}{\delta\phi\delta\theta}-\frac{\delta'' u^{U}(c)}{\delta\phi\delta\theta}\right]+\frac{\delta'' u^{U}(c)}{\delta\phi\delta\theta}.
\end{aligned}
$$

Take the limit for μ approaching 1 to obtain:

$$
\begin{aligned}
&\lim_{\mu\to 1}\frac{\delta'' u^{Y}(c)}{\delta\phi\delta\theta}\\
=&\frac{\delta'' n^{*}}{\delta\phi\delta\theta}\cdot\nu+\left(1-\frac{\eta}{2}\right)\frac{\delta n^{*}}{\delta\theta}\cdot\lim_{\mu\to 1}\left[\frac{\delta u^{HP}(c)}{\delta\phi}-\frac{\delta u^{U}(c)}{\delta\phi}\right]+\\
&+n^{*}\left(1-\frac{\eta}{2}\right)\lim_{\mu\to 1}\left[\frac{\delta'' u^{HP}(c)}{\delta\phi\delta\theta}-\frac{\delta'' u^{U}(c)}{\delta\phi\delta\theta}\right]+\lim_{\mu\to 1}\frac{\delta'' u^{U}(c)}{\delta\phi\delta\theta}
\end{aligned}
$$

Focus on $\frac{\delta u^{HP}(c)}{\delta\phi}$, $\frac{\delta u^{U}(c)}{\delta\phi}$, $\frac{\delta'' u^{HP}(c)}{\delta\phi\delta\theta}$ and $\frac{\delta'' u^{U}(c)}{\delta\phi\delta\theta}$ and consider initially:

$$\frac{\delta u^{HP}(c)}{\delta\phi}$$
$$= \frac{\nu(1-\mu)\frac{\delta W^*}{\delta\phi}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu} \cdot [\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)]^{1-\mu} +$$
$$+\nu(1-\mu)\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)]^{-\mu} \cdot$$
$$\cdot\frac{\delta n^*}{\delta\phi}(1-\theta)(1-\gamma).$$

Note that $\lim_{\mu\to 1}\frac{\delta u^{HP}(c)}{\delta\phi} = 0$ holds. Derive then $\frac{\delta u^{HP}(c)}{\delta\phi}$ with respect to θ to obtain:

$$\frac{\delta'' u^{HP}(c)}{\delta\phi\delta\theta}$$

$$= -\frac{\nu \cdot \frac{\delta W^*}{\delta\phi}(1-\mu)\mu}{[1+(1-\gamma)(1-\theta)]^2}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu-1} \cdot$$

$$\cdot [\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)]^{1-\mu} \cdot \left[\frac{\delta W^*}{\delta\theta}+\frac{W^*(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]+$$

$$+\frac{\nu \cdot (1-\mu)^2}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu} \cdot [\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)]^{-\mu} \cdot$$

$$\cdot \frac{\delta n^*}{\delta\phi}(1-\theta)(1-\gamma)\left[\frac{\delta W^*}{\delta\theta}+\frac{W^*(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]+$$

$$+\frac{\nu(1-\mu)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)]^{1-\mu} \cdot$$

$$\cdot \left[\frac{\delta'' W^*}{\delta\phi\delta\theta}+\frac{\frac{\delta W^*}{\delta\phi} \cdot (1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]+$$

$$+\frac{\nu \cdot \frac{\delta W^*}{\delta\phi}(1-\mu)^2}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu} \cdot [\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)]^{-\mu} \cdot$$

$$\cdot (1-\gamma) \cdot \left[1-n^*+\frac{\delta n^*}{\delta\theta}(1-\theta)\right]$$

$$-\nu(1-\mu)\mu\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)]^{-\mu-1} \cdot$$

$$\cdot \frac{\delta n^*}{\delta\phi}(1-\theta)(1-\gamma)^2\left[1-n^*+\frac{\delta n^*}{\delta\theta}(1-\theta)\right]+$$

$$+\nu(1-\mu)\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}[\gamma+\theta(1-\gamma)+n^*(1-\theta)(1-\gamma)]^{-\mu} \cdot$$

$$(1-\gamma) \cdot \left[1-\frac{\delta n^*}{\delta\phi}+\frac{\delta'' n^*}{\delta\phi\delta\theta}(1-\theta)\right].$$

In this case too $\lim_{\mu\to 1}\frac{\delta'' u^{HP}(c)}{\delta\phi\delta\theta}=0$ holds. Focus now on $\frac{\delta u^U(c)}{\delta\phi}$:

$$\frac{\delta u^U(c)}{\delta\phi}$$

$$=\frac{\nu(1-\mu)(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^* \cdot n^*(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\left(\frac{\delta W^*}{\delta\phi} \cdot n^*+\frac{\delta n^*}{\delta\phi} \cdot W^*\right)$$

and note that $\lim_{\mu\to1}\frac{\delta u^U(c)}{\delta\phi}=0$ holds. Derive $\frac{\delta u^U(c)}{\delta\phi}$then with respect to θ to obtain:

$$\begin{aligned}
&\frac{\delta'' u^U(c)}{\delta\phi\delta\theta}\\
=&-\frac{\nu\cdot\frac{\delta W^*}{\delta\phi}(1-\mu)\mu}{[1+(1-\gamma)(1-\theta)]^2}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu-1}[n^*(1-\theta)(1-\gamma)]^{1-\mu}\cdot\\
&+\frac{\nu(1-\mu)^2}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*\cdot n^*(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\cdot\\
&\cdot\frac{\delta n^*}{\delta\phi}(1-\theta)(1-\gamma)\left[\frac{\delta W^*}{\delta\theta}+\frac{W^*(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]+\\
&+\frac{\nu(1-\mu)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}[n^*(1-\theta)(1-\gamma)]^{1-\mu}\cdot\\
&\cdot\left[\frac{\delta'' W^*}{\delta\phi\delta\theta}+\frac{\frac{\delta W^*}{\delta\phi}(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]+\\
&+\frac{\nu\cdot\frac{\delta W^*}{\delta\phi}(1-\mu)^2(1-\gamma)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*\cdot n^*(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\left[\frac{\delta n^*}{\delta\theta}(1-\theta)-n^*\right]+\\
&-\nu(1-\mu)\mu\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}[n^*(1-\theta)(1-\gamma)]^{-\mu-1}(1-\gamma)^2\cdot\\
&\cdot\frac{\delta n^*}{\delta\theta}(1-\theta)\left[\frac{\delta n^*}{\delta\theta}(1-\theta)-n^*\right]+\\
&+\nu(1-\mu)\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}[n^*(1-\theta)(1-\gamma)]^{-\mu}(1-\gamma)\cdot\\
&\cdot\left[\frac{\delta'' n^*}{\delta\phi\delta\theta}(1-\theta)-\frac{\delta n^*}{\delta\theta}\right].
\end{aligned}$$

In this case too $\lim_{\mu\to1}\frac{\delta'' u^U(c)}{\delta\theta\delta\phi}=0$ and

$$\lim_{\mu\to1}\frac{\delta'' u^Y(c)}{\delta\phi\delta\theta}=\nu\cdot\frac{\delta'' n^*}{\delta\phi\delta\theta}=-\nu(1-\sigma)\frac{\frac{\alpha\cdot\eta}{4}(1-\gamma)}{\phi^2\left\{\beta\left[\left(\frac{\eta}{2}\right)^2+\left(1-\frac{\eta}{2}\right)^2\right]+\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}}\leq0$$

holds. For every pair $\theta\in(0,1)$ and $\eta\in(01)$ there thus exists a large enough $0<\hat{\mu}<1$ for $\frac{\delta'' u^Y(c)}{\delta\phi\delta\theta}\leq0$ to hold.

Consider $\frac{\delta u^Y(c)}{\delta \eta}$ and derive it with respect to ϕ to obtain:

$$\frac{\delta'' u^Y(c)}{\delta\phi\delta\eta}$$

$$= \frac{\delta n^*}{\delta\phi}\cdot\frac{\nu}{2}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}\left[1-(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}\right]+$$

$$+\frac{n^*}{2}\cdot\frac{\frac{\delta W^*}{\delta\phi}(1-\mu)}{1+(1-\gamma)(1-\theta)}\nu\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}\left[1-(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}\right]+$$

$$+\frac{\eta}{2}\cdot\frac{\delta n^*}{\delta\phi}\cdot\frac{\nu(1-\mu)\frac{\delta W^*}{\delta\eta}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}+$$

$$-\frac{\eta\cdot n^*}{2}\cdot\nu\frac{\frac{\delta W^*}{\delta\eta}\cdot\frac{\delta W^*}{\delta\phi}(1-\mu)\mu}{[1+(1-\gamma)(1-\theta)]^2}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu-1}+$$

$$+\frac{\eta\cdot n^*}{2}\cdot\frac{\nu(1-\mu)\frac{\delta'' W^*}{\delta\phi\delta\eta}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}+$$

$$+\frac{\delta'' n^*}{\delta\phi\delta\eta}\left(1-\frac{\eta}{2}\right)\nu\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}+$$

$$+\frac{\delta n^*}{\delta\eta}\left(1-\frac{\eta}{2}\right)\frac{\nu(1-\mu)\frac{\delta W^*}{\delta\phi}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}+$$

$$+\frac{\delta n^*}{\delta\phi}\left(1-\frac{\eta}{2}\right)\frac{\nu(1-\mu)\frac{\delta W^*}{\delta\eta}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}+$$

$$+n^*\left(1-\frac{\eta}{2}\right)\frac{\nu(1-\mu)^2\frac{\delta W^*}{\delta\phi}\cdot\frac{\delta W^*}{\delta\eta}}{[1+(1-\gamma)(1-\theta)]^2}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu-1}(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}+$$

$$+n^*\left(1-\frac{\eta}{2}\right)\frac{\nu(1-\mu)\frac{\delta'' W^*}{\delta\phi\delta\eta}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}(1-\gamma)^{1-\mu}(1-\theta)^{1-\mu}+$$

$$+\left[\frac{\delta'' n^*}{\delta\phi\delta\eta}\left(1-\frac{\eta}{2}\right)-\frac{\delta n^*}{\delta\phi}\cdot\frac{1}{2}\right]\left[u^{HP}(c)-u^U(c)\right]+$$

$$+\left[\frac{\delta n^*}{\delta\eta}\left(1-\frac{\eta}{2}\right)-\frac{n^*}{2}\right]\left[\frac{\delta u^{HP}(c)}{\delta\phi}-\frac{\delta u^U(c)}{\delta\phi}\right]+$$

$$+\frac{\delta n^*}{\delta\phi}\left(1-\frac{\eta}{2}\right)\left[\frac{\delta u^{HP}(c)}{\delta\eta}-\frac{\delta u^U(c)}{\delta\eta}\right]+$$

$$+n^*\left(1-\frac{\eta}{2}\right)\left[\frac{\delta'' u^{HP}(c)}{\delta\phi\delta\eta}-\frac{\delta'' u^U(c)}{\delta\phi\delta\eta}\right]+\frac{\delta'' u^U(c)}{\delta\phi\delta\eta}.$$

Take the limit for μ approaching 1 to obtain:

$$\lim_{\mu\to1}\frac{\delta''u^{Y}\left(c\right)}{\delta\phi\delta\eta}$$
$$=\frac{\delta''n^{*}}{\delta\phi\delta\eta}\left(1-\frac{\eta}{2}\right)\nu+n^{*}\left(1-\frac{\eta}{2}\right)\lim_{\mu\to1}\left[\frac{\delta''u^{HP}\left(c\right)}{\delta\phi\delta\eta}-\frac{\delta''u^{U}\left(c\right)}{\delta\phi\delta\eta}\right]+\lim_{\mu\to1}\frac{\delta''u^{U}\left(c\right)}{\delta\phi\delta\eta}.$$

Focus on $\frac{\delta''u^{HP}(c)}{\delta\phi\delta\eta}$ and $\frac{\delta''u^{U}(c)}{\delta\phi\delta\eta}$. Consider initially:

$$\frac{\delta''u^{HP}\left(c\right)}{\delta\phi\delta\eta}$$
$$=-\frac{\nu\cdot\frac{\delta W^{*}}{\delta\phi}\left(1-\mu\right)\mu}{\left[1+\left(1-\gamma\right)\left(1-\theta\right)\right]^{2}}\left[\frac{W^{*}}{1+\left(1-\gamma\right)\left(1-\theta\right)}\right]^{-\mu-1}\cdot$$
$$\cdot\left[\gamma+\theta\left(1-\gamma\right)+n^{*}\left(1-\theta\right)\left(1-\gamma\right)\right]^{1-\mu}\frac{\delta W^{*}}{\delta\eta}+$$
$$+\frac{\nu\left(1-\mu\right)^{2}}{1+\left(1-\gamma\right)\left(1-\theta\right)}\left[\frac{W^{*}}{1+\left(1-\gamma\right)\left(1-\theta\right)}\right]^{-\mu}\cdot$$
$$\cdot\left[\gamma+\theta\left(1-\gamma\right)+n^{*}\left(1-\theta\right)\left(1-\gamma\right)\right]^{-\mu}\frac{\delta n^{*}}{\delta\phi}\left(1-\theta\right)\left(1-\gamma\right)\frac{\delta W^{*}}{\delta\eta}+$$
$$+\frac{\nu\left(1-\mu\right)}{1+\left(1-\gamma\right)\left(1-\theta\right)}\left[\frac{W^{*}}{1+\left(1-\gamma\right)\left(1-\theta\right)}\right]^{-\mu}\cdot$$
$$\cdot\left[\gamma+\theta\left(1-\gamma\right)+n^{*}\left(1-\theta\right)\left(1-\gamma\right)\right]^{1-\mu}\frac{\delta''W^{*}}{\delta\phi\delta\eta}+$$
$$+\frac{\nu\cdot\frac{\delta W^{*}}{\delta\phi}\left(1-\mu\right)^{2}}{1+\left(1-\gamma\right)\left(1-\theta\right)}\left[\frac{W^{*}}{1+\left(1-\gamma\right)\left(1-\theta\right)}\right]^{-\mu}\left[\gamma+\theta\left(1-\gamma\right)+n^{*}\left(1-\theta\right)\left(1-\gamma\right)\right]^{-\mu}\cdot$$
$$\cdot\frac{\delta n^{*}}{\delta\eta}\left(1-\gamma\right)\left(1-\theta\right)+$$
$$-\nu\left(1-\mu\right)\mu\left[\frac{W^{*}}{1+\left(1-\gamma\right)\left(1-\theta\right)}\right]^{1-\mu}\left[\gamma+\theta\left(1-\gamma\right)+n^{*}\left(1-\theta\right)\left(1-\gamma\right)\right]^{-\mu-1}\cdot$$
$$\cdot\frac{\delta n^{*}}{\delta\phi}\cdot\frac{\delta n^{*}}{\delta\eta}\left(1-\theta\right)^{2}\left(1-\gamma\right)^{2}+$$
$$+\nu\left(1-\mu\right)\left[\frac{W^{*}}{1+\left(1-\gamma\right)\left(1-\theta\right)}\right]^{1-\mu}\left[\gamma+\theta\left(1-\gamma\right)+n^{*}\left(1-\theta\right)\left(1-\gamma\right)\right]^{-\mu}\cdot$$
$$\cdot\frac{\delta''n^{*}}{\delta\phi\delta\eta}\left(1-\gamma\right)\left(1-\theta\right).$$

It is the case that $\lim_{\mu\to 1}\frac{\delta'' u^{HP}(c)}{\delta\phi\delta\eta}=0$ holds. Consider now:

$$\begin{aligned}
&\frac{\delta'' u^{U}(c)}{\delta\phi\delta\eta}\\
=&-\frac{\nu\cdot\frac{\delta W^*}{\delta\phi}(1-\mu)\mu}{[1+(1-\gamma)(1-\theta)]^2}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu-1}[n^*(1-\theta)(1-\gamma)]^{1-\mu}\frac{\delta W^*}{\delta\eta}+\\
&+\frac{\nu(1-\mu)^2\frac{\delta n^*}{\delta\phi}(1-\theta)(1-\gamma)\frac{\delta W^*}{\delta\eta}}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*\cdot n^*(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}+\\
&+\frac{\nu(1-\mu)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}[n^*(1-\theta)(1-\gamma)]^{1-\mu}\frac{\delta'' W^*}{\delta\phi\delta\eta}+\\
&+\frac{\nu\cdot\frac{\delta W^*}{\delta\phi}(1-\mu)^2\frac{\delta n^*}{\delta\eta}(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\left[\frac{W^*\cdot n^*(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{-\mu}+\\
&-\nu(1-\mu)\mu\left[\frac{W^*(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}(n^*)^{-\mu-1}\frac{\delta n^*}{\delta\phi}\cdot\frac{\delta n^*}{\delta\eta}+\\
&+\nu(1-\mu)\left[\frac{W^*(1-\theta)(1-\gamma)}{1+(1-\gamma)(1-\theta)}\right]^{1-\mu}(n^*)^{-\mu}\frac{\delta'' n^*}{\delta\phi\delta\eta}.
\end{aligned}$$

and note that $\lim_{\mu\to 1}\frac{\delta'' u^{U}(c)}{\delta\phi\delta\eta}=0$. This implies:

$$\begin{aligned}
&\lim_{\mu\to 1}\frac{\delta'' u^{Y}(c)}{\delta\phi\delta\eta}=\nu\cdot\frac{\delta'' n^*}{\delta\phi\delta\eta}\left(1-\frac{\eta}{2}\right)\\
=&-(1-\sigma)\frac{\frac{\alpha\cdot\theta}{4}[\gamma+\theta(1-\gamma)]}{\phi^2\left\{\beta\left[\left(\frac{\eta}{2}\right)^2+\left(1-\frac{\eta}{2}\right)^2\right]+\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}}+\\
&+(1-\sigma)\frac{(1-\eta)\left(\beta-\frac{1}{2}\right)}{\phi^2\left\{\beta\left[\left(\frac{\eta}{2}\right)^2+\left(1-\frac{\eta}{2}\right)^2\right]+\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}^3}\cdot\\
&\cdot\left(1+\frac{\alpha}{2}\left\{1+\frac{\eta}{2}[\gamma+\theta(1-\gamma)]\right\}-\phi(k-1)\right)+\\
&+(1-\sigma)\frac{(1-\eta)\left(\beta-\frac{1}{2}\right)(k-1)}{\phi\left\{\beta\left[\left(\frac{\eta}{2}\right)^2+\left(1-\frac{\eta}{2}\right)^2\right]+\left(1-\frac{\eta}{2}\right)\frac{\eta}{2}\right\}^2}\leq 0.
\end{aligned}$$

Hence for every pair $\theta\in(0,1)$ and $\eta\in(01)$ there exists a large enough $0<\hat{\mu}<1$ for $\frac{\delta'' u^{Y}(c)}{\delta\phi\delta\eta}\leq 0$ to hold.Bassanini, A., and R. Duval (2006), Employment Patterns in OECD Countries: Reassessing the Role of Policies and Institutions, OECD Economics Department Working Papers No. 486.

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