Evaluating Quantitative Easing: A DSGE Approach

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Abstract

When an economy is stuck in a liquidity trap or suffers from a liquidity shortage, the zero-lower bound of interest rates may challenge the conventional ways of conducting monetary policy. Therefore, Quantitative Easing (QE) becomes one of the main tools at the disposal of central banks in order to spur economic recovery. This paper develops a simple Dynamic Stochastic General Equilibrium (DSGE) model capable of evaluating the effect of large purchases of treasuries by central banks. The model exhibits imperfect asset substitutability between government bonds of different maturities and a feedback from the term structure to the macroeconomy. Both are generated through the introduction of portfolio adjustment frictions. As a result, the model is able to isolate a *portfolio rebalancing* channel of QE. This theoretical framework is employed to evaluate the impact on yields and the macroeconomy of large purchases of medium- and long-term treasuries recently carried out in the US and UK. In particular, the paper focuses on the QE2 phase in the US (from November 2010 to June 2011 - around $800 billion of purchases), and the first phase of the APF operations in the UK (from March 2009 to January 2010 - around £200 billion of purchases). The results, from both the calibrated and the estimated version of the model, are realistic and consistent with those obtained in the literature using different techniques. They suggest that large asset purchases of government assets have had beneficial effects in terms of lower long-term yields, and higher output and inflation. The size of the effects is nevertheless sensitive to the speed of the exit strategy chosen by monetary authorities.

KEYWORDS: unconventional monetary policy, quantitative easing, DSGE models, asset prices
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1 Introduction

When an economy is stuck in a liquidity trap or experiences a liquidity shortage, the zero-lower bound (ZLB) of interest rates may challenge the conventional ways of conducting monetary policy.[1] Hence, Quantitative Easing (QE) becomes one of the main tools at the disposal of central banks in order to spur economic recovery. QE can be defined as all policies carried out by central banks involving changes in the composition and/or size of the balance sheet aimed at, in a situation close to the ZLB, easing liquidity and credit conditions with the final goal of stimulating the economic system. There exist therefore a variety of different unconventional measures that fall under the label of QE, such as purchases of treasuries, purchases of private securities, and direct loans to banks, companies and households. Unconventional monetary policies are discussed in several studies (Krugman, 1998; Svensson, 2003; Bernanke and Reinhart, 2004; Orphanides, 2004). Figure 1 sketches strategies and policy options available for central banks facing ZLB problems, as well as the channels through which they may affect aggregate demand.

As the recent global downturn unfolded, many advanced economies experienced a serious liquidity shortage combined with an interest rate close to the ZLB, and their monetary authorities thus began to pursue QE measures. In particular, in the aftermath of the financial crisis of 2007, interbank money markets froze up due to some important bankruptcies (and, more generally, solvency concerns), a consequent widespread lack of confidence, and coordination failures among market participants. As a result, financial markets also broke down with dramatic consequences for the whole economic system. In an effort to spur economic activity and restore financial market functioning, several central banks intervened by reducing the short-term interest rate. The ZLB quickly became a serious concern for monetary institutions since, in such situations, the availability of credit tends to become irresponsive to quantity of liquidity present in the economic system.

In the US, when Lehman Brothers collapsed, the Fed engaged in dramatic cuts of the policy rate, and the ZLB was virtually reached in December 2008. As Figure 2 shows, this measure was accompanied by a huge expansion of the Fed's portfolio assets, which jumped by over $1,000 billion in a few weeks. Besides rescuing troubled companies, such as Bear Stearns and AIG, the Fed started a much more comprehensive program to provide liquidity and reduce risk premia along the term structure and across a variety of different assets.[2] Given improved conditions in financial markets, many of the programs introduced at the onset of the crisis were suppressed by the end of 2009 or throughout 2010. A second stage of QE, called by practitioners QE2 (in contrast with the first phase QE1), took place from October 2010 until June 2011, mainly consisting of purchases of medium- and long-term treasury securities.[3]

In September 2012, Bernanke announced that the Fed will purchase additional agency mortgage-backed securities at a pace of $40 billion per month, and will extend the average maturity of its holdings of securities. These actions are expected to increase the Fed's holdings of longer-term securities by about $85 billion each month until the end of the year. The declared objective of QE3 is to "put downward pressure on longer-term interest rates, support mortgage markets, and help to make broader financial

[1]The existence of *liquidity traps* was first hypothesized by Keynes (1936), during the years following the onset of the Great Depression, when, in a deflationary situation, short-term interest rates remained for a long time very close to zero.

[2]New specific programs include the Mortgage-Backed Securities (MBS) purchase program, which was intended to help mortgage and housing markets, the Term Asset-Backed Securities Loan Facility, aimed at providing credit to households and small companies, the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility, which provided funding to banks for their purchase of asset-backed securities, and the Term Auction Facility, which provided term funds to depository institutions.

[3]"QE1 directly supported struggling banks by buying their problematic assets. QE2 supports the government" (Bagus, 2010).

conditions more accommodative." (Board of Governors of the Federal Reserve System, 2012).

The QE approach of the Bank of England (BoE) has been quite different to that implemented by the Fed. As shown in Figure 3, a huge expansion of the balance sheet occurred just after the insurgence of the crisis. During this first stage, the central bank implemented some liquidity support measures, such as extensions to its lending operations, by allowing banks to borrow from a wider-than-normal range of collateral. The second stage of unconventional measures in the UK began with the establishment of the Asset Purchase Facility (APF) fund in March 2009, a separate subsidiary company of the BoE.[4] The goal of the APF was to improve market functioning by injecting money into the economy in the form of purchases of high-quality public and private assets. However, APF's operations were overwhelmingly oriented towards purchases of medium- and long-term governments bonds (Figure 4). Private securities accounted for a tiny proportion of the APF's purchases. Because of further recessionary pressures during the end of 2011, the Bank of England extended the program in October 2011, injecting additional liquidity into the economy, mainly in the form of medium- and long-term gilt purchases. Two more waves of purchases took place in February 2012 and July 2012. So far, the total amount of assets purchased by the BoE has reached a remarkable value of £375 billion. At the time of writing this paper, a date for a definitive exit strategy is still uncertain.

Recent events have inspired a growing body of empirical literature trying to assess whether unconventional monetary policies have been successful. However, gauging the effects of unconventional monetary policies remains a hard task. The reasons can be found both in the uncertain time lags between actions and effects, and in the difficulties related to disentangling other important factors, especially government policies and international developments. Another empirical concern is the identification of the channels through which QE may affect yields, premia, and other variables of interest. A substantial number of empirical contributions rely therefore on event studies, i.e. they focus on the patterns of specific variables, such as yields, within a narrow time interval between the announcement or the implementation of a policy. Evidence provided by event studies has been generally supportive of the effectiveness of QE policies, both in the US (Klyuev et al., 2009; Blinder, 2010; Gagnon et al., 2011; Glick and Leduc, 2011; Krishnamurthy and Vissing-Jorgensen, 2011; Swanson, 2011) and in the UK (Klyuev et al., 2009; Meier, 2009; Glick and Leduc, 2011; Joyce et al., 2011b).

Another strand of the empirical literature employs econometric techniques, such as simple regressions (Glick and Leduc, 2011; Gagnon et al., 2011), dynamic regressions (Gagnon et al., 2011), IV regressions (Stroebel and Taylor, 2009), panel data methods (D'Amico and King, 2010), and VARs and SVARs (Baumeister and Benati, 2010; Bridges and Thomas, 2012; Kapetanios et al., 2012), affine term structure models (Hamilton and Wu, 2012), and other finance models (Doh, 2010). These works generally find that unconventional monetary measures recently taken in the US and in the UK have been effective.

Lastly, more or less fully-fledged structural models have been used to assess the impact of unconventional monetary policies.[5] In standard Dynamic Stochastic General Equilibrium (DSGE) models, QE may only work through a *signaling channel*,[6] since the representation of the term structure and the financial sector is very stylized. In order to capture the effects of QE policies via other channels, it is

[4]The accounts of the APF are not consolidated with those of the central bank. Therefore, all the operations of the APF fund fall inside the category "other assets" in Figure 3.

[5]For a comparison of the different DSGE approaches to QE, see Cadlar et al. (2011). A large scale non DSGE model is used by Chung et al. (2012).

[6]See, for example, Eggertsson and Woodford (2003).

necessary to depart from the conventional DSGE framework by introducing specific financial frictions and structures.

A first attempt has been made by modeling financial intermediaries and banking frictions, in order to focus on the role of unconventional monetary policies in facilitating lending. These models are able to capture the *credit channel* of QE. Contributions in this area have been made by Cúrdia and Woodford (2010), Gertler and Kiyotaki (2010), Del Negro et al. (2011), Gertler and Karadi (2011), Brendon et al. (2011), and Chadha et al. (2012).

A different type of DSGE models features imperfect asset substitutability in order to isolate the *portfolio rebalancing channel* of QE. Within these frameworks, QE measures may affect asset prices and returns by changing the relative supplies of different assets. There has recently been a growing attention towards the contributions by Tobin (1969, 1982) about imperfect asset substitutability, whose portfolio approach has been employed in dynamic optimizing models by Andrés et al. (2004), Marzo et al. (2008), and, more recently, by Zagaglia (2011) and Falagiarda and Marzo (2012). Chen et al. (2011) and Harrison (2011, 2012), adopt this framework to study unconventional monetary policies. In models with imperfect asset substitutability, investors tend to rebalance their asset portfolios whenever the supply of different types of assets changes. Large asset purchases by the central bank vary the relative supply of assets of different maturities, inducing movements in their prices. As a result, aggregate demand may also be influenced.

By embracing this last approach, the present paper develops a DSGE model able to capture the effect of large asset purchases of treasuries by central banks. Partially drawing on Chen et al. (2011) and Harrison (2011, 2012), the model is characterized by imperfect asset substitutability and a feedback effect from the term structure to the macroeconomy, both generated through the introduction of portfolio adjustment frictions. In other words, agents pay a cost whenever they change the composition of their portfolio by shifting from short- to long-term assets, and vice versa. The model is therefore capable of isolating a *portfolio rebalancing* channel of QE, according to which large asset purchases of long-term treasuries by the central bank affect asset prices, inducing investors to rebalance their portfolios. By purchasing a particular asset, the monetary authority reduces the amount of that asset held by private agents usually in exchange of risk-free reserves. As a result, the price of that asset increases and the interest rate falls, creating favorable conditions for economic recovery through the traditional monetary transmission mechanisms.

Differently from Chen et al. (2011) and Harrison (2011, 2012), who employ perpetuities as long-term bonds, the model presented in this paper features a secondary market for bond trading, as proposed by Ljungqvist and Sargent (2004), allowing a straightforward modelization of zero-coupon government bonds of different maturities. Moreover, unlike Chen et al. (2011) the present model relies on a representative agent setting, avoiding the differentiation between restricted and unrestricted agents. Thanks to the general equilibrium nature of the model, it is possible to assess the effect of this type of QE policies on the macroeconomy, as well as on yields. To the best of my knowledge, this model represents the first attempt to evaluate the effects of large asset purchases within a relatively simple DSGE framework characterized by: a) representative agents; b) a stylized central bank's balance sheet; c) an endogenous term structure featuring imperfect asset substitutability between zero-coupon government bonds of different maturities.

The theoretical framework is then employed to simulate the impact of large purchases of medium- and long-term treasuries in the US during QE2 (from November 2010 to June 2011 - around $800 billion of purchases - Figure 2), and in the UK during the first phase of the APF program (from March 2009 to January 2010 - around £200 billion of purchases - Figure 3). The results both from the calibrated and the estimated version of the model are realistic and generally consistent with those obtained in the literature using different techniques. However, the estimated effects in the US have found to be slightly larger than in previous studies, while in the UK a bit smaller. More specifically, they suggest that large asset purchases of government assets have had beneficial effects both in terms of lower long-term yields and higher output and inflation. These effects seem to be generally larger for the UK than the US. This is not surprising, given that the purchases characterizing the phases of QE under consideration have been relatively larger in the UK. Still, the difference in the effects between the two countries is not as large as previously found in the literature. My preferred model specifications indicate that large asset purchases of QE2 in the US may have had a peak effect on long-term rates in annualized percentage rates between -51 and -64 basis points, on the level of real GDP between 0.95 and 1.07%, and on inflation between 0.11 and 0.39 percentage points. In the UK, the preferred model specifications suggest that the first phase of the APF program has had a peak effect on long-term rates between -34 and -69 basis points, on the level of real GDP between 1.15 and 1.33%, and on inflation between 0.55 and 0.96 percentage points. However, the size of the effects crucially depends on the speed of the exit strategy chosen by monetary authorities, and on the degree of substitutability among assets of different maturities.

All in all, the contribution of this paper is twofold. First of all, it provides a new and relatively simple setting through which the effects of large purchases of treasuries by central banks can be evaluated within a microfounded macro framework with optimizing agents. Second, it offers fresh empirical evidence on the effectiveness of the recent large asset purchase programs conducted in the US and in the UK.

The remainder of the paper is organized as follows. Section 2 elaborates the model and introduces its key features. Section 3 presents the results from the calibrated model, while Section 4 discusses the estimation results. Section 5 concludes.

2 The Model

A representative agent populates the economy and supply labor inputs. Monopolistically competitive firms hire labor and capital to produce differentiated goods. The government conducts fiscal and monetary policy. Since the deviations from a canonical DSGE setting concern the households and the government sectors, I start here with their discussion.

2.1 Households

There is a continuum of identical and infinitely-lived households. Households' preferences are defined over consumption C_t, real money balances $\frac{M_t}{P_t}$ and labor effort L_t, and are described by the infinite stream of utility:

$$U_t = \sum_{t=0}^{\infty} \beta^t v_t^{PR} u\left(C_t, \frac{M_t}{P_t}, L_t\right) \tag{1}$$

where β is the intertemporal discount factor, and v_t^{PR} is a preference shock that follows an AR(1) process:

$$\log v_t^{PR} = \phi_{PR} \log v_{t-1}^{PR} + \varepsilon_t^{PR} \tag{2}$$

where ε_t^{PR} is an i.i.d. shock with zero mean and standard deviation σ_{PR}.
The instantaneous utility function $u\left(C_t, \frac{M_t}{P_t}, L_t\right)$ is given by:

$$U\left(C_t, \frac{M_t}{P_t}, L_t\right) = \frac{(C_t - \gamma C_{t-1})^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + \frac{1}{1-\chi}\left(\frac{M_t}{P_t}\right)^{1-\chi} - \frac{\Psi}{1+1/\psi} L_t^{1+1/\psi} \tag{3}$$

where γ measures the importance of consumption habits, σ is the elasticity of intertemporal substitution, χ is the elasticity of money demand, and ψ is the Frisch elasticity of labor supply.

In this economy, each agent i can choose the composition of a basket of differentiated final goods. Preferences across varieties of goods have the standard constant elasticity of substitution (CES) form á la Dixit and Stiglitz (1977):

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\theta_t - 1}{\theta_t}} dj\right]^{\frac{\theta_t}{\theta_t - 1}} \tag{4}$$

where C_t is the aggregate consumption index of all the differentiated final goods produced in the economy under monopolistic competition. There are j-th varieties of final goods ($j \in [0, 1]$), and θ_t is a time-varying elasticity of substitution between different final goods varieties ($\theta_t > 1$).

Each agent is subject to the following budget constraint, which incorporates the secondary market for bond trading as in Ljungqvist and Sargent (2004):

$$\frac{B_t}{P_t R_t} + \frac{B_{L,t}^H}{P_t R_{L,t}}(1 + AC_t^L) + \frac{M_t}{P_t} + I_t(1 + AC_t^I) = \frac{B_{t-1}}{P_t} + \frac{B_{L,t-1}^H}{P_t R_t} + \frac{M_{t-1}}{P_t} + \frac{W_t}{P_t} L_t + q_t K_t - C_t - T_t + \Omega_t \tag{5}$$

Thus, agents allocate their wealth among money holding, accumulation of capital, which is rented to firms at the rental rate q_t, and holding of two types of zero-coupon bonds (B_t and $B_{L,t}^H$), which are purchased by households at their nominal price. They receive rental income $q_t K_t$, where K_t is capital, wage income $w_t L_t$, where w_t is the real wage, and a share of firms' profits Ω_t. They also pay a real lump-sum tax T_t. I_t is investment, and P_t is the aggregate price level.

Firms face quadratic adjustment costs of investment as in Kim (2000):

$$AC_t^I = \frac{\phi_K}{2}\left(\frac{I_t}{K_t}\right)^2 \tag{6}$$

The law of motion of capital stock is expressed in the following standard way:

$$K_{t+1} = I_t + (1 - \delta) K_t \tag{7}$$

where δ represents the depreciation rate of the capital stock.

The different zero-coupon government bonds are defined as money-market bonds B_t and long-term bonds $B^H_{L,t}$, whose yields are given, respectively, by R_t and $R_{L,t}$. Money-market bonds are considered as a proxy for 1-month-maturity bonds, and the long-term bonds for 10-year-maturity bonds.[7] The budget constraint incorporates the secondary market for bond trading as proposed by Ljungqvist and Sargent (2004). The strength of this approach is that it allows an explicit and straightforward modelization of assets of different maturities. The left-hand side of the budget constraint follows the usual formulation with bonds priced with their interest rates, since at time t, returns R and R_L are known with certainty and are risk-free from the viewpoint of agents. However, the right-hand side of (5) reveals the presence of the secondary market for bond trading as proposed by Ljungqvist and Sargent (2004), according to which long-term bonds are priced with the money-market rate. Even though these bonds represent sure claims for future consumption, they are subject to price risk prior to maturity. At time $t-1$, an agent who buys longer-maturity bonds and plans to sell them next period would be uncertain about the gains, since R_t is not known at time $t-1$. As stressed by Ljungqvist and Sargent (2004), the price R_t follows from a simple arbitrage argument, since, in period t, these bonds represent identical sure claims to consumption goods at the time of the end of the maturity as newly issued one-period bonds in period t.

As already mentioned, segmentation in financial markets is obtained by introducing portfolio adjustment frictions, which represent impediments to arbitrage behavior that would equalize asset returns. In particular, it is assumed, as in Falagiarda and Marzo (2012) and Harrison (2012), that the intratemporal trading between bonds of different maturities is costly to each agent. These bond transaction costs are given by:

$$AC^L_t = \left[\frac{\phi_L}{2}\left(\kappa_L \frac{B_t}{B^H_{L,t}} - 1\right)^2\right] Y_t \tag{8}$$

where κ_L is the steady state ratio of long-term bond holdings relative to short-term bond holdings. Thus, agents pay a cost whenever they shift the portfolio allocation between short and long maturity bonds. Transaction costs are paid in term of output, and are zero in the steady-state.

The rationale for including portfolio frictions is threefold. First of all, these costs can be viewed as a proxy for the behavior of agents towards risk (i.e. they rationalize a liquidity premium). The longer the maturity of a bond, the less liquid is considered the asset, and vice versa. Since long-term bonds are perceived as less liquid, there are liquidity costs associated with holding them. In other words, agents perceive longer-maturity assets as riskier, and hence associated with a loss of liquidity compared to the same investment in shorter-term bonds. It follows that, as they purchase longer-term bonds, they hold additional short-term bonds to compensate themselves for the loss of liquidity. Thus, agents self-impose a sort of ”reserve requirement” on their longer-term investments (Andrés et al., 2004). Another justification for including such portfolio frictions rests on the theory of preferred habitat, according to which agents have preferences over bond maturities (Vayanos and Vila, 2009). Third, these costs can be also considered as proxies for the shares of resources devoted to covering information costs, or the costs of managing bond portfolios.

[7]However, when calibrating the model, money-market bonds are assumed to include all government debt instruments with maturity up to one year, whereas long-term bonds government debt instruments with maturity longer than one year (see Paragraph 3.1).

2.1.1 Optimality Conditions

Households maximize their lifetime utility (1) subject to the budget constraint (5), and the capital accumulation equation (7). The first order conditions with respect to consumption, labor, money, money-market bonds, long-term bonds, capital and investment, are respectively given by:

$$\nu_t^{PR}(C_t - \gamma C_{t-1})^{-1/\sigma} - \nu_{t+1}^{PR}\beta\gamma E_t(C_{t+1} - \gamma C_t)^{-1/\sigma} = \lambda_t \tag{9}$$

$$\nu_t^{PR}\Psi L_t^{1/\psi} = \lambda_t \frac{W_t}{P_t} \tag{10}$$

$$\nu_t^{PR}\left(\frac{M_t}{P_t}\right)^{-\chi} + \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} = \lambda_t \tag{11}$$

$$\beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} = \frac{\lambda_t}{R_t} + \frac{\kappa_L \phi_L \lambda_t Y_t \left(\kappa_L \frac{b_t}{b_{L,t}^H} - 1\right)}{R_{L,t}} \tag{12}$$

$$\beta E_t \frac{\lambda_{t+1}}{\pi_{t+1} R_{t+1}} = \frac{\lambda_t}{R_{L,t}} + \frac{\phi_L \lambda_t Y_t \left(\kappa_L \frac{b_t}{b_{L,t}^H} - 1\right)^2}{2R_{L,t}} - \frac{\kappa_L \phi_L \lambda_t Y_t b_t \left(\kappa_L \frac{b_t}{b_{L,t}^H} - 1\right)}{b_{L,t}^H R_{L,t}} \tag{13}$$

$$\beta(1-\delta)E_t\mu_{t+1} = \mu_t - \lambda_t\left(q_t + \phi_K\left(\frac{I_t}{K_t}\right)^3\right) \tag{14}$$

$$\beta E_t \mu_{t+1} = \lambda_t\left(1 + \frac{3}{2}\phi_K\left(\frac{I_t}{K_t}\right)^2\right) \tag{15}$$

where λ_t and μ_t are the two Lagrange multipliers.

2.2 The Government

The consolidated government-central bank budget constraint is given by:

$$\frac{B_t}{P_t R_t} + \frac{B_{L,t}}{P_t R_{L,t}} + \frac{\Delta_t}{P_t} = \frac{B_{t-1}}{P_t} + \frac{B_{L,t-1}}{P_t R_t} + G_t - T_t \tag{16}$$

where G_t is government spending. As stressed in the previous subsection, money-market bonds are considered as a proxy for 1-month-maturity government debt assets, and the long-term bonds for 10-year-maturity government debt assets.

Drawing on Harrison (2012), Δ_t is defined as the change in the central bank balance sheet, equal to money creation and net asset purchases:

$$\frac{\Delta_t}{P_t} = \frac{M_t - M_{t-1}}{P_t} - \left[\frac{B^{CB}_{L,t}}{P_t R_{L,t}} - \frac{B^{CB}_{L,t-1}}{P_t R_t} \right] \tag{17}$$

where B^{CB} is the central bank's holdings of long-term government debt. Thus, the stylized central bank's balance sheet of this model includes long term treasuries on the asset side and money on the liability side. Central bank's holdings of long-term government bonds are a fraction x of the total amount of long-term bonds present in the economy:

$$B^{CB}_{L,t} = x_t B_{L,t} \tag{18}$$

The remaining proportion of long-term bonds is available to households and is given by:

$$B^{H}_{L,t} = (1 - x_t) B_{L,t} \tag{19}$$

Thus, asset purchases by the central bank are performed by varying the fraction x_t, which is modeled as a time-varying variable. In particular, x_t is assumed to follow an autoregressive process of order one:

$$\log\left(\frac{x_t}{X}\right) = \phi_x \log\left(\frac{x_{t-1}}{X}\right) + \varepsilon^x_t \tag{20}$$

where X is the steady-state value of the fraction of long-term bonds held by the central bank $\left(\frac{B^{CB}_{L,t}}{B_{L,t}}\right)$, and ε^x_t represents an i.i.d. shock to asset purchases with zero mean and standard deviation σ_X. This means that the central bank holds in the steady-state a quantity of long-term bonds X, and temporary fluctuations around this level are determined by (20). One limitation of this formulation is that it is assumed that the central bank gradually starts decumulating long-term asset holdings from the period just after the shock. The persistence of the shock is nevertheless carefully calibrated to mimic different plausible exit strategies conducted by the monetary authority.

The government spending, net of interest expenses, G_t follows an AR(1) process:

$$\log\left(\frac{G_t}{G}\right) = \phi_G \log\left(\frac{G_{t-1}}{G}\right) + \varepsilon^G_t \tag{21}$$

where ε^G_t is an i.i.d. shock with zero mean and standard deviation σ_G.

I introduce the following fiscal policy rule, according to which the total amount of tax collection T_t is a function of the total government's liabilities:[8]

$$T_t = \psi_0 + \psi_1 \left[\frac{b_{t-1}}{\pi_t} - \frac{b}{\pi} \right] + \psi_2 \left[\frac{b_{L,t-1}}{R_t \pi_t} - \frac{b_L}{R\pi} \right] \tag{22}$$

where ψ_0 is the steady-state level of T_t, and ψ_1 has been set to be equal for all bonds. Equation (22) tells us that the level of taxes reacts to deviations of the outstanding level of public debt from its steady-state level. In other words, taxes are not allowed to act independently from the stock of government liabilities

[8] In such a way, it is possible to prevent the emergence of inflation as a fiscal phenomenon (Leeper, 1991).

outstanding in the economy.[9]

The central bank is the institution devoted to set the money-market rate R_t, according to the following Taylor (1993) rule:

$$\log\left(\frac{R_t}{R}\right) = \alpha_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\alpha_R)\left\{\log\left(\frac{\pi_t^*}{\pi}\right) + \alpha_\pi\left[\log\left(\frac{\pi_t}{\pi}\right) - \log\left(\frac{\pi_t^*}{\pi}\right)\right] + \alpha_Y \log\left(\frac{Y_t}{Y}\right)\right\} + \varepsilon_t^R \quad (23)$$

where α_R, α_π, α_Y indicate the response of R_t with respect to lagged R_t, inflation and output. The policy rate is determined by the deviation of inflation and output from the steady-state with an interest rate smoothing component. The monetary policy shock ε_t^R is an i.i.d. with zero mean and standard deviation σ_R. π_t^* is a time-varying inflation target, as in Smets and Wouters (2003):

$$\log\left(\frac{\pi_t^*}{\pi}\right) = \phi_\pi \log\left(\frac{\pi_{t-1}^*}{\pi}\right) + \varepsilon_t^\pi \quad (24)$$

where ε_t^π is an exogenous shock with zero mean and standard deviation σ_π.

Finally, the supply of long-term bonds is assumed to follow a simple exogenous AR process, as in Marzo et al. (2008):

$$\log\left(\frac{b_{L,t}}{b_L}\right) = \phi_{BL} \log\left(\frac{b_{L,t-1}}{b_L}\right) + \varepsilon_t^{BL} \quad (25)$$

where ε_t^{BL} is a disturbance term with zero mean and standard deviation σ_{BL}. Thus, asset purchase shocks are assumed to affect only the composition of outstanding government liabilities.

2.3 Firms

The final step is to model the firms' sector, which follows a quite standard representation. Each j-th firm produces and sells differentiated final goods in a monopolistically competitive market. The production function is a standard Cobb-Douglas with labor and capital:

$$Y_t = A_t K_t^\alpha {L_t}^{1-\alpha} - \Phi \quad (26)$$

where α is the share of capital used in production, and Φ is a fixed cost to ensure that profits are zero in the steady-state. A_t is technology and follows an AR(1) process:

$$\log\left(\frac{A_t}{A}\right) = \phi_A \log\left(\frac{A_{t-1}}{A}\right) + \varepsilon_t^A \quad (27)$$

where ε_t^A is an i.i.d. shock with zero mean and standard deviation σ_A.

Firms' optimizing process is constrained by nominal rigidities á la Rotemberg (1982), i.e. firms face quadratic price adjustment costs:

$$AC_t^P = \frac{\phi_P}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - \pi\right)^2 Y_t \quad (28)$$

[9] A similar formulation has been employed, for instance, by Schmitt-Grohé and Uribe (2007).

Given the standard CES setting of equation (4), the demand function faced by each single firm j is:

$$Y_t(j) = \left[\frac{p_t(j)}{P_t}\right]^{-\theta_t} Y_t \Longrightarrow P_t(j) = \left[\frac{Y_t(j)}{Y_t}\right]^{-\frac{1}{\theta_t}} P_t \tag{29}$$

Thus, the demand function for each single good j is proportionally related to the general output level of the economy, and negatively to the price of good j. The elasticity of substitution of demand θ_t is time-varying around a mean θ:

$$\theta_t = \theta + \nu_t^{MU} \tag{30}$$

where ν_t^{MU} is a shock to price mark-up that follows an autoregressive process:

$$\log \nu_t^{MU} = \phi_{MU} \log \nu_{t-1}^{MU} + \varepsilon_t^{MU} \tag{31}$$

where ε_t^{MU} is an exogenous shock with zero mean and standard deviation σ_{MU}.

Following Kim (2000), the profit function for each firm j is:

$$P_t\Pi_t(j) = P_t P_t(j) Y_t(j) - P_t W_t L_t(j) - P_t q_t K_t(j) - P_t AC_t^P \tag{32}$$

After employing (28) and (29) into (32), the maximization problem of each firm becomes fully dynamic: each firm maximizes the expectation of the discounted sum of profit flows, given the information at time 0:

$$\Pi_0(j) = E_0\left[\sum_{t=0}^{\infty} \rho_t P_t \Pi_t(j)\right] \tag{33}$$

where ρ is the discount factor of firms.

Assuming that each agent in the economy has access to a complete market for contingent claims, I assume that the discount factors of households and firms are equal:

$$E_t\frac{\rho_{t+1}}{\rho_t} = E_t\beta\frac{\lambda_{t+1}}{\lambda_t} \tag{34}$$

Therefore, the necessary first order conditions of the maximization problem with respect to labor and capital are given respectively by:

$$\frac{W_t}{P_t} = (1-\alpha)\left(\frac{Y_t + \Phi}{L_t}\right)\left(1 - \frac{1}{e_t^Y}\right) \tag{35}$$

$$q_t = \alpha\left(\frac{Y_t + \Phi}{K_t}\right)\left(1 - \frac{1}{e_t^Y}\right) \tag{36}$$

where e_t^Y is the output demand elasticity:

$$\frac{1}{e_t^Y} = \frac{1}{\theta_t}\left\{1 - \phi_P(\pi_t - \pi)\pi_t + \beta\phi_P E_t\left[\frac{\lambda_{t+1}}{\lambda_t}(\pi_{t+1} - \pi)\pi_{t+1}^2 \frac{Y_{t+1}}{Y_t}\right]\right\} \tag{37}$$

which measures the gross price markup over marginal cost. It is easy to check that manipulations of the log-linearized version of (37) lead to the standard New Keynesian Phillips curve.

2.4 The Resource Constraint

The model is completed by specifying the resource constraint of the economy:

$$Y_t = C_t + G_t + I_t(1 + AC_t^I) + AC_t^P + \frac{b_{L,t}^H}{R_{L,t}}(AC_t^L) \tag{38}$$

The total output of the economy is allocated to consumption, government spending, investment (comprehensive of capital adjustment costs), price adjustment costs, and a component related to bond adjustment frictions.

2.5 Asset Markets: No Arbitrage and the Feedback

In order to appreciate the main features of the model, a deeper analysis of the asset market's structure is required. Combining the log-linearized version of the two first-order conditions for bond holdings, i.e. equations (12) and (13), yields:

$$\tilde{R}_{L,t} = \tilde{R}_t + A_1 E_t \tilde{R}_{t+1} + A_2 E_t \tilde{\lambda}_{t+1} - A_3 E_t \tilde{\pi}_{t+1} - \phi_L A_4 [\tilde{b}_t - \tilde{b}_{L,t}^H] \tag{39}$$

where A_ι ($\iota = 1, 2, 3, 4$) are convolutions of the parameters. Equation (39) reveals that the long-term rate depends positively on long-term bond supply, as desired, and positively on short-term bond supply, because of imperfect asset substitutability between the two assets. Thus, asset purchases carried out by the monetary authority, by reducing the supply of long-term bonds at the disposal of households, would lead to a reduction in the long-term yield. This is the so-called *portfolio rebalancing* channel of QE.[10] Conversely, an increase in the relative supply of the more illiquid asset will bid up the spread between the more illiquid asset and the more liquid asset. Moreover, notice the role of the transaction costs parameter ϕ_L that, by generating impediments to the arbitrage behavior of agents that would equalize returns, determines the degree to which relative bonds holding movements affect the long-term rate. When financial frictions are equal to zero, equation (39) boils down to the more usual formulation:

$$\tilde{R}_{L,t} = \tilde{R}_t + A_1 E_t \tilde{R}_{t+1} + A_2 E_t \tilde{\lambda}_{t+1} - A_3 E_t \tilde{\pi}_{t+1} \tag{40}$$

in which a sort of expectations hypothesis holds, and the long-term rate is not affected by changes in the relative holdings of bonds of different maturities.

An additional crucial feature of the model is the presence of a feedback channel from the term structure to the macroeconomy. This can be observed by combining the log-linearized versions of the first order conditions for consumption (9) and short-term bonds (12), in order to obtain the Euler equation for consumption, and employing then the first order condition of long-term bonds (13):

$$\tilde{c}_t = A_5 E_t \tilde{c}_{t+1} + A_6 E_t \tilde{\pi}_{t+1} + \cdots - A_7 \tilde{R}_t - A_8 \tilde{R}_{L,t} \tag{41}$$

[10]The other transmission channels of QE highlighted in Figure 1 are not considered in this model.

where A_ι ($\iota = 5, 6, 7, 8$) are convolutions of the parameters. Aggregate demand and, through general equilibrium forces, all the macro variables are therefore affected by the entire simple term structure of interest rate present in this model, and not only by the short-term rate as in standard DSGE frameworks.

The whole story behind the model can be summarized as follows. Long-term bond purchases by the central bank alter the relative supplies of assets and hence returns (equation (39)), which, in turn, stimulate the economy through standard general equilibrium mechanisms (equation (41)).

3 The Results from the Calibrated Model

The model is employed to simulate the effects of specific QE programs in the US and in the UK. More specifically, I focus my attention on QE2 in the US (from November 2010 to June 2011 - around $800 billion of purchases), and the first phase of the APF operations in the UK (from March 2009 to January 2010 - around £200 billion of purchases). As already mentioned, both phases were characterized exclusively by purchases of medium- and long-term government securities (Figure 2 and Figure 3). Therefore, it is possible to assess their effects using the model proposed in this paper. In this section, I first simulate the impact of such programs using a calibrated version of the model. In Section 4, the model is brought to the data, and a simulation exercise is conducted on the estimated model.

Since the model cannot be solved analytically, I log-linearized it around the steady-state. Appendix A reports the deterministic steady-state, while Appendix B contains the equations of the log-linearized model. I solved the model using both the MATLAB routine *Gensys* written by Christopher Sims, and *Dynare* developed by Adjemian et al. (2011).[11] In what follows, calibration issues are first discussed. I then analyze the results of the baseline case. Lastly, a sensitivity analysis is performed, shifting the focus to the variation of the key parameters of the model.

3.1 Calibration

The benchmark model is calibrated to match quarterly data over the most recent period prior to the financial crisis of 2008. Table 1 and Table 2 report, respectively, some steady-state values and the chosen calibration values for the standard parameters. Some parameters are chosen following previous studies and their calibrated value is quite standard in the literature. Among them: the elasticity of substitution across goods θ, set equal to 6 (Schmitt-Grohé and Uribe, 2004); the habit formation parameter γ, set equal to 0.7 (Smets and Wouters, 2007); the elasticity of intertemporal substitution σ, set equal to 0.5, which implies a coefficient of relative risk aversion of 2; the depreciation rate of capital δ calibrated to 0.025 (Christiano et al., 2005; Altig et al., 2011), which implies an annual rate of depreciation on capital equal to 10 percent; the share of capital in the production function α, set to 0.36 (Christiano et al., 2005; Altig et al., 2011); the parameter of the price adjustment cost ϕ_P, calibrated to 100 (Ireland, 2004); the elasticity of real money balances χ, set equal to 7 (Marzo et al., 2008); the Frisch elasticity ψ, set equal to 1 (Marzo et al., 2008).

The parameters of the fiscal and monetary policy rules are calibrated in a standard way, with the exception of α_R, which is chosen very close to one, in order to make the response of the policy rate to inflation/output changes not substantial (reflecting a situation close to the ZLB), and, at the same time, to avoid indeterminacy.

[11] The codes are available upon request.

The AR coefficients and the standard deviations of the shocks are set to $\phi_A = 0.95$, $\phi_G = \phi_{MU} = \phi_{PR} = \phi_\pi = \phi_{BL} = 0.90$, $\sigma_A = \sigma_{PR} = \sigma_{MU} = \sigma_{BL} = 0.01$, $\sigma_\pi = \sigma_R = 0.005$, $\sigma_G = 0.012$ (see, for example, Christiano and Eichenbaum, 1992; Kim, 2000; Andrés et al., 2004; Marzo et al., 2008; Altig et al., 2011; Zagaglia, 2011).

Some of the steady-states are obtained from the data, or following previous studies. Output is normalized to 1. The consumption-output ratio has been set to 0.57, and the taxes-output ratio to 0.1972 (Marzo et al., 2008). The ratio of market to non-market activities is set equal to 0.3. The steady-state values of the yields have been chosen identical for both countries, given the very similar recent trends of rates in the US and the UK, obtained from the Federal Reserve Economic Data and the Bank of England Statistical Interactive Database.

In order to simulate accurately the unconventional programs under consideration, the parameters and steady-states related to the new mechanisms proposed in this paper should be carefully chosen. Their values, reported in Table 3, are country-specific and significantly influence the impact of asset purchase policies. The ratio of total debt to GDP, the ratio of debt at different maturities to total debt, and the proportion of long-term debt held by households and the central bank, are obtained by combining data from the OECD Statistical Database, the Federal Reserve Statistical Release, the Bank of England Statistical Interactive Database, and the Bank of England APF Gilt Operational Results Dataset, and taking their values as they were just before the asset purchase shock occurred. In particular, the total debt on GDP ($B + B_L$) is the ratio of the total amount of marketable government debt to GDP. Short-term debt (B) includes money-market instrument plus bonds with maturity up to one year. Long-term debt (B_L) is calculated by subtracting the amount of short-term debt from the total amount of debt.

Also, the standard deviation of the asset purchase shock and the approximated duration of the shock should be carefully set. The magnitude of the asset purchase shock has been chosen equal to 1 for the US (i.e. there has been an increase of 100% in the long-term bonds held by the Fed during QE2), and 12 for the UK (i.e. the BoE increased its holding of long-term treasuries by 1200% during the first stage of the APF operations).[12] The duration of the asset purchase shock is approximated to be three quarters in the US, and four quarters in the UK.

The two *free* parameters of the model, namely the persistence of the asset purchase shock ϕ_x and the parameter of bond adjustment frictions ϕ_L, are not easily quantified. They are set equal, respectively, to 0.83, reflecting a medium-term exit strategy from QE (approximately six years after the asset purchase shock), and 0.01, following the calibration proposed by Harrison (2012). This means that 1% of agents' income is devoted to paying portfolio transaction costs.[13] In the next paragraphs some sensitivity analysis on these parameters is conducted.

Finally, the values of the remaining parameters and steady-states are computed using the deterministic steady-state solutions, as shown in the Appendix A.

3.2 Simulation Results: The Impact of Asset Purchases

The model impulse responses to an asset purchase shock are shown in Figure 5 for the US, and in Figure 6 for the UK. The simulated asset purchase shock in the US lasts for three quarters and its magnitude is such that central bank's long-term bond holdings double (left upper panel in Figure 5). This reduces the amount of long-term bonds at the disposal of households by around 23 percent. The reduction in long-

[12] See Figure 2 and Figure 3.

[13] In Chen et al. (2011), an analogous parameter has been set equal to 1.5%.

term bond supply pushes down the long-term rate by 48 basis points. Through the feedback mechanisms from the term structure to the macroeconomy, output and inflation experience a substantial increase of 0.71 percent and 0.29 percent, respectively. Notice that the term premium decreases almost as much as the long-term rate, given that the short-term rate, being close to the ZLB, does not move substantially.[14]

Figure 6 shows that the asset purchase shock in the UK takes place over four quarters and leads to an increase of 1200 percent of long-term bonds held by the central bank. As a result, long-term government bonds held by households decrease by approximately 27 percent, leading to a reduction in the long-term rate of 69 basis points. The positive effect on the macroeconomic variables is 1.33 percent for output and 0.55 percent for inflation.

Table 4 and Table 5 summarize these findings in annualized percentage rates in the *Baseline* row of *My calibrated model*, reporting also analogous results obtained by other studies using different techniques. The results obtained from the calibrated version of the model proposed in this paper are quite consistent with what has been previously found in the literature. In particular, for the US the effect on long-term yield, output, and inflation seems to be slightly larger than that obtained in other studies, whereas for the UK slightly smaller. Not surprisingly, given the different amount of assets purchased, the overall effect of large asset purchases on the economy is found to be larger in the UK than in the US. However, this difference is not as large as previously found in the literature.

In order to gain intuition about some of the key mechanisms at work in the model, it is useful to carry out a sensitivity analysis exercise. In particular, in what follows I analyze what happens when changing, first, the persistence of the asset purchase shock ϕ_x, and then the parameter concerning the portfolio adjustment frictions ϕ_L.

3.2.1 Sensitivity Analysis: The Role of the Persistence of the Asset Purchase Shock

In the benchmark calibration, it has been arbitrarily assumed that central banks, after purchasing long-term assets, undertake a medium term exit strategy, i.e. they wind down the program over the following six years by selling the assets accumulated during the QE phases. To illustrate how results change when varying the assumed length of the exit strategy, Figure 7 and Figure 8 plot the impulse response functions considering three different values for ϕ_x: the benchmark value (red line), a higher ϕ_x (0.88), which reflects a longer exit strategy from QE of approximately eight years (green line), and a lower ϕ_x (0.76), which corresponds to a faster exit strategy of four years (blue line).

When the parameter relative to the persistence of the asset purchase shock ϕ_x increases, the persistence of the response of the long-term yield increases as well, both for the US and the UK, while the magnitude of the response does not change significantly. Importantly, as far as the macroeconomic variables are concerned, not only the persistence of their response goes up, but also their impact effect. By contrast, a faster exit strategy is associated with a lower effect on the macroeconomy. This is completely in line with what is actually expected, since a longer exit strategy is likely to exert larger inflationary pressures, and a too fast exit strategy to have instead marginal effects on the economy. Moreover, inflation responds more strongly than output to changes in the length of the policy, a fact consistent with the findings of Chen et al. (2011), and due to the presence of nominal rigidities such as price stickiness.

The quantitative effects of the simulated asset purchase shock in annualized percentage rates for the

[14]The term premium ξ_t is calculated as follows: $\xi_t = R_{L,t} - \frac{1}{N}\sum_{j=0}^{N-1} E_t R_{t+j}$ Thus, the term premium represents deviations of the long-term yield $R_{L,t}$ from the level consistent with the expectations hypothesis. It is assumed that the short-term rate R_t is a proxy for the 1-month yield and the long-term rate $R_{L,t}$ for the 10-year rate. This implies that $N = 120$.

different persistence values are reported in Table 4 and Table 5. For the US, the effect on output is in the range of 0.68%-1.29%, while the effect on inflation is found to be in the range 0.24%-0.61%. For the UK, the effect on output is found to lie between 1.01% and 1.72%, and that on inflation between 0.35% and 0.82%. While these findings confirm that the effectiveness of such unconventional monetary policies seems to have been more pronounced in the UK than in the US, they also highlight that their predictions are subject to the uncertainty associated with the timing of the exit strategy from QE chosen by the monetary authority.

3.2.2 Sensitivity Analysis: The Role of Financial Frictions

As already noted, the magnitude of ϕ_L measures the extent of the impediments to the arbitrage behavior of agents, and therefore the degree of imperfect asset substitutability between short- and long-term bonds. Figure 9 and Figure 10 report the impulse response functions for the baseline case (red line), and the cases with higher (0.02) and lower (0.005) portfolio adjustment costs (green and blue line, respectively).

As expected, higher frictions generate larger obstacles to the arbitrage behavior of investors, making the two assets less substitutable. As a result, changes in the relative quantities of bonds held by households lead to a higher responsiveness of long-term yield. The macroeconomic effects are also amplified when ϕ_L increases, and vice versa. UK's variables seem to be less sensitive to changes in the parameter ϕ_L in comparison with the US. The results in annualized percentage changes for the different calibrations are contained in Table 4 and Table 5.

Lastly, notice that, when there are no frictions at all (ϕ_L=0), the two assets are perfect substitutes and a reduction in the supply of long-term bonds does not generate any effect on yields and on the macroeconomy, as agents can simply increase their holdings of short-term bonds by the same amount. In such a case, the identification of the *portfolio rebalancing* channel of large asset purchases would not have been possible.

4 The Results from the Estimated Model

The model has proved to do a good job in its calibrated version, providing some indications on the effectiveness of the large asset purchases recently conducted by the monetary authorities in the US and the UK. As discussed in the previous paragraphs, the results are nonetheless sensitive to the chosen calibration values of the key parameters, especially the persistence of the asset purchase shock. In this section, the model will be estimated using actual data and the effects of the two asset purchase programs assessed. At a first sight, it may seem strange to place side by side a calibration and an estimation exercise within the same paper. However, I consider the calibrated model more suitable for the understanding of the dynamics of the model, and for the fine-tuning of the key parameters. On the other hand, by carrying out the estimation, I am able both to check whether actual data supports the theoretical framework, and to provide robustness to the results of the calibrated model. The estimation is conducted on US and UK data at a quarterly frequency over the period 1987:3-2010:3 for the US, and 1987:3-2008:4 for the UK, excluding, in such a way, the period following the beginning of the two asset purchase programs. Simulations of the effects of large asset purchases in the US and in the UK are then performed.

4.1 The Estimation Technique

Estimation is performed using Bayesian techniques, which have become very popular in DSGE literature within the last decade.[15] Once the model has been log-linearized, it is possible to write it in a state-space representation, where the transition equation is given by:

$$S_t = f(S_{t-1}, W_t; \Theta) \tag{42}$$

where S_t is the vector of states, W_t is the vector of innovations, and Θ is the vector of the structural parameters.

The measurement equation is instead expressed in the following way:

$$Y_t = g(Y_{t-1}, Z_t; \Theta) \tag{43}$$

where Y_t are the observables and Z_t the measurement errors added to the observables.[16]

Given the data $Y^T \equiv \{Y_t\}_{t=1}^T$, the general expression of the likelihood function of the model is given by:

$$\mathcal{L}\left(\Theta|Y^T\right) = \prod_{t=1}^{T} p\left(Y_t|Y^{T-1}, \Theta\right) \tag{44}$$

The likelihood function is evaluated through the Kalman filter.[17] The Bayesian approach consists of finding the posterior distribution, given the likelihood function and the prior distribution ($\pi(\Theta)$). Bayes' theorem states that the posterior distribution of the parameters is given by:

$$\pi\left(\Theta|Y^T\right) = \frac{p(Y^T|\Theta)\pi(\Theta)}{\int p(Y^T|\Theta)\pi(\Theta)d\Theta} \tag{45}$$

Using a Random Walk Metropolis-Hastings algorithm,[18] I am able to obtain an empirical approximation of the posterior density function of the model, and I can finally perform inference.

It is evident that the inclusion of term structure observables is crucial in this type of model. The chosen observables are output, inflation, consumption, the short term rate, and the long-term rate. Quarterly data on real GDP, GDP deflator, consumption expenditures, and yields (respectively, Federal funds rate and 10-year yields - constant-maturity interest rates, in percent per year, for the US; the Sterling interbank lending rate 1 month and the 10-year nominal zero coupon yield on British government securities, for the UK) covering the period 1987:3-2010:3 for the US and 1987:3-2008:4 for the UK are employed, i.e. using only data prior to the start of the two asset purchase programs. They are obtained from the Federal Reserve Economic Data database, and the UK Office of National Statistics. I calculated log-differences of each series, with the exception of interest rates. The dataset is then detrended using a linear trend.[19]

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[15] The reasons are very well explained in Fernández-Villaverde (2010).

[16] Given that the observables used in the estimation are less than the structural shocks, measurement errors are not introduced.

[17] For more details, see Canova (2007).

[18] See Chib and Greenberg (1995), An and Schorfheide (2007), and Fernández-Villaverde (2010).

[19] Linearly detrended data has been used in the estimation of DSGE models, for instance, by Kim (2000), Dib (2004), Bouakez et al. (2005), and Nolan and Thoenissen (2009). I also estimated the model with data detrended using the HP filter with smoothing parameter equal to 1600, obtaining very similar results in terms of the posterior distribution of parameters.

4.2 The Prior Distribution of the Parameters

In setting the prior distributions I proceed as follows. Some parameters are not estimated, since they are either obtained through the steady-state solution, or usually treated as fixed in the literature, such as the coefficient of intertemporal substitution β. The persistence and magnitude of the asset purchase shock (ϕ_x and σ_x) are considered as fixed, by setting their value at 0.83 (as in the benchmark calibration of Section 3) and 0.001, in order to make this structural shock almost irrelevant for the estimation process. When simulating the impact of asset purchase shock I conveniently rescale the magnitude of the shock in order to match the actual one.

The remaining parameters are estimated, and their prior distribution is shown in the third column of Table 6. They are mainly chosen both following previous contributions in the literature (Smets and Wouters, 2003; Del Negro et al., 2007; Smets and Wouters, 2007; Zagaglia, 2011; Falagiarda and Marzo, 2012) and taking into account the calibration proposed in Section 3. In particular, the parameters concerning preferences and technology are assumed to be either normal distributed or beta distributed (if the parameter is restricted to the 0-1 range). Moreover, the priors on the parameters in the monetary policy reaction function are quite standard, with the coefficient on inflation set equal to 1.5, that on output to 0.4, and the parameter relative to monetary policy inertia to 0.5. The variances of the shocks follow an inverse gamma distribution with two degrees of freedom, so that the estimates are constrained to be positive. The distribution of the autoregressive parameters of the shocks is assumed to follow a beta distribution with mean 0.85 and standard error 0.1. Finally, notice that for the parameter of bond adjustment costs (ϕ_L) I choose a quite loose prior, in order to really check whether its calibrated value is plausible once data are taken into account.

4.3 The Estimation Results and the Impact of Asset Purchases

The posterior mean, and the 5 and 95 percentiles of the posterior distribution of the parameters are reported in Table 6.[20] The estimated posterior mean of almost all the parameters generally diverges from the prior mean, suggesting that the data have some informative content. Signals of poor identification can be observed for the elasticity of labor supply, the autoregressive parameters of the government spending shock and the mark-up shock, and, only for the US, the coefficient of price adjustment costs ϕ_P. Most of the estimation results are nevertheless in line with previous studies in the DSGE literature.

By looking at the estimate of the *free parameter* of the model, i.e. that regarding bond adjustment costs ϕ_L, it turns out that the mean of the posterior distribution is close to the mean of the prior assumptions both for the US and for the UK. These results provide, on the one hand, a strong empirical support to the theoretical framework and the calibration of previous section, but suggest, on the other hand, a weak identification, at least for the UK. The existence of weak identification of the parameter of portfolio adjustment frictions can be roughly verified by trying larger and smaller values for the prior mean. After doing that, I observe that the posterior distribution continues to stay generally close the prior one, pointing out that the data are not very informative for ϕ_L. Problems of weak identification of this type of parameters are common in the literature (Chen et al., 2011), and probably emerge because of the absence of any observable for the quantity of short-term and long-term debt.

[20] Estimates are obtained using two blocks of 100,000 replications each, of which the first 45 percent have been discarded. The convergence diagnostic tests indicate that the Markov chains converge. Their graphs have not been reported here, but are available upon request.

In order to assess the effect of asset purchase shocks on the observables, I report in Figure 11 and Figure 12 the posterior impulse response functions, i.e. the mean of a series of impulse response functions obtained by drawing from the parameter posteriors. The responses have been rescaled in order to capture the exact magnitude of the asset purchase shocks characterizing the phases of QE under scrutiny. The estimated effect of the government asset purchases of QE2 in the US has been a reduction of 38bp in the long-term rate, and an increase in output of 0.8 percent and inflation of 0.08 percent. Transforming these values in annualized percentage rates (Table 4), it is possible to observe that, except for the effect on output, which is still larger, they resemble the findings coming from previous studies.

As far as the UK is concerned, the simulated large asset purchases of treasuries of the first phase of the APF has led to an estimated reduction in the long-term rate of 34bp. Turning to the macroeconomic variables, the estimated peak impact on output amounts to 1.15 percent, and that on inflation to a remarkable 0.96 percent (see Table 5). Thus, there seems to be in the UK a much higher effect on inflation and a slightly smaller effect on output, in comparison with the results obtained in Section 3. This is probably due to the low degree of the estimated price stickiness ($\phi_P = 85.221$) with respect to its calibrated counterpart. As argued by Chen et al. (2011), nominal rigidities play indeed an important quantitative role in the response of inflation to large asset purchases. When prices are more flexible, one would expect a higher response of inflation to asset purchase shocks. However, it is worth noting that the estimated effect on inflation is now more in line with previous studies in the literature, whereas that on output and the long-term rate is a bit smaller.[21] In the remainder of this section, a sensitivity analysis analogous to that in Section 3 is performed.

4.3.1 Sensitivity of the Results

As highlighted in Section 3, the persistence of the asset purchase shock is a key parameter for determining the size and the length of the effects of such policies on the variables of interest. In this paragraph, different values for this parameter are checked. Consistently with the sensitivity analysis conducted in Section 3, the chosen values are 0.76 and 0.88, reflecting respectively a slow and a fast exit strategy from large asset purchase policies.

Figure 13 and Figure 14 quantify this comparison by showing the posterior impulse response functions when varying the persistence of the asset purchase shock. As expected, the impact of the shock on the variables of interest increases as the persistence of the shock increases, a fact consistent with Chen et al. (2011) and the analysis carried out in Section 3. More specifically, the estimated annualized effect on output and inflation in the US has been found between 0.93% and 1.24% for output, and between 0.08% and 0.15% for inflation (Table 4). In the UK, the plausible range for output oscillates between 0.88% and 1.33%, and for inflation between 0.71% and 1.43% (Table 5).

To sum up, also the results obtained from the estimated version of the model suffer from the uncertainty related to the timing of the exit strategy undertaken by the monetary authorities. They nevertheless point to the conclusion that such unconventional policies have been successful in reducing long-term yields and in stimulating output and inflation, both in the US and in the UK.

[21] "... higher price flexibility shifts the adjustment in response to asset purchase programs from GDP growth to inflation, by making its process more front-loaded." (Chen et al., 2011). A sensitivity analysis specifically conducted on ϕ_P actually has proved that a lower price stickiness is associated with a higher response of inflation and a lower response of output. The graphs have not been reported for the sake of space, but are available from the author upon request.

5 Concluding Remarks

This paper has developed a DSGE model capable of evaluating some of the effects of large purchases of treasuries by central banks. The model exhibits imperfect asset substitutability and a feedback from the term structure to the macroeconomy, both generated through the introduction of portfolio adjustment frictions. As a result, the model is able to isolate a *portfolio rebalancing* channel of QE. This theoretical framework is employed to evaluate the effects of recent specific large asset purchase programs in the US and in the UK. More specifically, the focus has been on QE2 in the US (from November 2010 to June 2011 - around \$800 billion of purchases), and the first phase of the APF operations in the UK (from March 2009 to January 2010 - around £200 billion of purchases). Both phases have been characterized exclusively by purchases of medium- and long-term government securities.

The simulation results, both from the calibrated and the estimated version of the model, are realistic and generally consistent with those obtained in the literature using different techniques. However, the estimated macroeconomic effects in the US have been found to be slightly larger than in previous studies, while in the UK a bit smaller. Overall, the findings suggest that large asset purchases of government assets have had beneficial effects both in terms of lower long-term yields and higher output and inflation in both countries. These effects seem to be generally larger for the UK than for the US. This is not surprising, given that the purchases characterizing the phase of QE under consideration has been more substantial, in relative terms, in the UK rather than in the US. More specifically, my preferred model specifications indicate that large asset purchases of QE2 in the US may have had a peak effect on long-term rates in annualized terms between -51 and -64 basis points, on the level of real GDP between 0.95 and 1.07%, and on inflation between 0.11 and 0.39 percentage points. In the UK, the preferred model specifications suggest that the first phase of the APF program has had a peak effect on long-term rates between -34 and -69 basis points, on the level of real GDP between 1.15 and 1.33%, and on inflation between 0.55 and 0.96 percentage points. The empirical results are nonetheless subject to some uncertainty associated with the degree of substitutability among assets of different maturities, and, more importantly, with the speed of the exit strategy chosen by monetary authorities.

All in all, the most substantive contribution of this paper is to provide a new setting through which the effects of large purchases of treasuries by central banks can be evaluated within a microfounded macro framework with optimizing agents. The model proposed in this paper points to further avenues for future research. First of all, considering the quantities of government debt as observables in the estimation would probably allow to better identify the parameter of the portfolio adjustment frictions. Moreover, the model can be easily extended in several directions, e.g. including an explicit and more structured modelization of a central bank's balance sheet, a wider term structure representation, or different types of assets, such as corporate bonds. Lastly, it would be worth combining this framework with those proposed by Cúrdia and Woodford (2010), Gertler and Kiyotaki (2010), Del Negro et al. (2011), Gertler and Karadi (2011) and Brendon et al. (2011), which capture some *credit channel* dynamics of QE.

Appendix A. The Steady-State

$$\lambda = (1 - \beta\gamma)[C(1 - \gamma)]^{-\frac{1}{\sigma}} \tag{46}$$

$$\beta = \frac{\pi}{R} \tag{47}$$

$$m^{-\chi} = \lambda\left(1 - \frac{1}{R}\right) \tag{48}$$

$$K = \frac{I}{\delta} \tag{49}$$

$$\frac{G}{Y} = 1 - \frac{C}{Y} - I_t(1 + AC_t^I) \tag{50}$$

$$w = (1 - \alpha)\left(\frac{Y + \Phi}{L}\right)\left(1 - \frac{1}{e^Y}\right) \tag{51}$$

$$q = \alpha\left(\frac{Y + \Phi}{K}\right)\left(1 - \frac{1}{e^Y}\right) \tag{52}$$

$$\Phi = \left[1 - \alpha\left(1 - \frac{1}{\theta}\right)\right]AK^{\alpha}L^{1-\alpha} \Longrightarrow \frac{\Phi}{Y + \Phi} = \left[1 - \alpha\left(1 - \frac{1}{\theta}\right)\right] \tag{53}$$

$$e^Y = \theta \tag{54}$$

Combining equations (14) and (15), I get the following formula, from which I can derive ϕ_K:

$$\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left\{(1 - \delta)\left[1 + \frac{3}{2}\phi_K\left(\frac{I_{t+1}}{K_{t+1}}\right)^2\right] + Q_{t+1} + \phi_K\left(\frac{I_{t+1}}{K_{t+1}}\right)^3\right\} = 1 + \frac{3}{2}\phi_K\left(\frac{I_t}{K_t}\right)^2 \tag{55}$$

Appendix B. The Log-linearized Model

$$
\begin{aligned}
&-\frac{1}{\sigma}(C-\gamma C)^{-\frac{1}{\sigma}-1}C\tilde{c}_t+\frac{\gamma}{\sigma}(C-\gamma C)^{-\frac{1}{\sigma}-1}C\tilde{c}_{t-1}+\frac{\beta\gamma}{\sigma}(C-\gamma C)^{-\frac{1}{\sigma}-1}C\tilde{c}_{t+1}-\frac{\beta\gamma^2}{\sigma}(C-\gamma C)^{-\frac{1}{\sigma}-1}C\tilde{c}_t+\\
&+(C-\gamma C)^{-\frac{1}{\sigma}}\tilde{v}_t^{PR}-\beta\gamma(C-\gamma C)^{-\frac{1}{\sigma}}\tilde{v}_{t+1}^{PR}=(1-\beta\gamma)(C-\gamma C)^{-\frac{1}{\sigma}}\tilde{\lambda}_t
\end{aligned}
\tag{56}
$$

$$
\tilde{l}_t=\psi(\tilde{\lambda}_t+\tilde{w}_t-\tilde{v}_t^{PR})
\tag{57}
$$

$$
\tilde{m}_t=-\frac{1}{\chi}\left[-\frac{\beta}{\pi-\beta}(\tilde{\lambda}_{t+1}-\tilde{\pi}_{t+1})+\frac{\pi}{\pi-\beta}\tilde{\lambda}_t-\tilde{v}_t^{PR}\right]
\tag{58}
$$

$$
\frac{\beta}{\pi}\tilde{\lambda}_{t+1}-\frac{\beta}{\pi}\tilde{\pi}_{t+1}=\frac{1}{R}\tilde{\lambda}_t-\frac{1}{R}\tilde{R}_t+\frac{\kappa_L\phi_L Y}{R_L}[\tilde{b}_t-\tilde{b}_{L,t}^H]
\tag{59}
$$

$$
\frac{\beta}{R\pi}\tilde{\lambda}_{t+1}-\frac{\beta}{R\pi}\tilde{\pi}_{t+1}-\frac{\beta}{R\pi}\tilde{R}_{t+1}=\frac{1}{R_L}\tilde{\lambda}_t-\frac{1}{R_L}\tilde{R}_{L,t}-\frac{\phi_L Y}{R_L}[\tilde{b}_t-\tilde{b}_{L,t}^H]
\tag{60}
$$

$$
\beta(1-\delta)\mu\tilde{\mu}_{t+1}=\mu\tilde{\mu}_t-\lambda Q\tilde{\lambda}_t-\lambda Q\tilde{q}_t-\frac{\phi_K\lambda I^3}{K^3}\tilde{\lambda}_t-\frac{3\phi_K\lambda I^3}{K^3}\tilde{i}_t+\frac{3\phi_K\lambda I^3}{K^3}\tilde{k}_t
\tag{61}
$$

$$
\beta\mu\tilde{\mu}_{t+1}=\lambda\tilde{\lambda}_t+\frac{3\phi_K\lambda I^2}{2K^2}\tilde{\lambda}_t+\frac{3\phi_K\lambda I^2}{K^2}\tilde{i}_t-\frac{3\phi_K\lambda I^2}{K^2}\tilde{k}_t
\tag{62}
$$

$$
K\tilde{k}_{t+1}=I\tilde{i}_t+(1-\delta)K\tilde{k}_t
\tag{63}
$$

$$
\begin{aligned}
&\frac{b}{R}\tilde{b}_t-\frac{b}{R}\tilde{R}_t+\frac{b_L^H}{R_L}\tilde{b}_{L,t}^H-\frac{b_L^H}{R_L}\tilde{R}_{L,t}+m\tilde{m}_t+\left(I+\frac{3}{2}\phi_K\frac{I^3}{K^2}\right)\tilde{i}_t-\phi_K\frac{I^3}{K^2}\tilde{k}_t=\\
&=\frac{b}{\pi}\tilde{b}_{t-1}-\frac{b}{\pi}\tilde{\pi}_t+\frac{b_L^H}{\pi R}\tilde{b}_{L,t-1}^H-\frac{b_L^H}{\pi R}\tilde{\pi}_t-\frac{b_L^H}{\pi R}\tilde{R}_t+\frac{m}{\pi}\tilde{m}_{t-1}-\frac{m}{\pi}\tilde{\pi}_t+Y\tilde{y}_t-C\tilde{c}_t-T\tilde{t}_t
\end{aligned}
\tag{64}
$$

$$
b_L^H\tilde{b}_{L,t}^H=b_L\tilde{b}_{L,t}-xb_L\tilde{b}_{L,t}-xb_L\tilde{x}_t
\tag{65}
$$

$$\tilde{y}_t = \frac{Y+\Phi}{Y}\left[\tilde{a}_t + \alpha\tilde{k}_t + (1-\alpha)\tilde{l}_t\right] \tag{66}$$

$$\tilde{w}_t = \frac{Y}{Y+\Phi}\tilde{y}_t - \tilde{l}_t + \frac{1}{\theta-1}\tilde{e}_t^Y \tag{67}$$

$$\tilde{q}_t = \frac{Y}{Y+\Phi}\tilde{y}_t - \tilde{k}_t + \frac{1}{\theta-1}\tilde{e}_t^Y \tag{68}$$

$$\tilde{e}_t^Y = \tilde{\theta}_t + \phi_P\pi^2\tilde{\pi}_t - \beta\phi_P\pi^3\tilde{\pi}_{t+1} \tag{69}$$

$$\begin{aligned}\frac{b}{R}\tilde{b}_t - \frac{b}{R}\tilde{R}_t + \frac{b_L}{R_L}\tilde{b}_{L,t} - \frac{b_L}{R_L}\tilde{R}_{L,t} + m\tilde{m} - \frac{xb_L}{R_L}\tilde{x}_t - \frac{xb_L}{R_L}\tilde{b}_{L,t} + \frac{xb_L}{R_L}\tilde{R}_{L,t} = \frac{b}{\pi}\tilde{b}_{t-1} - \frac{b}{\pi}\tilde{\pi}_t + \frac{b_L}{R\pi}\tilde{b}_{L,t-1} - \\ - \frac{b_L}{R\pi}\tilde{\pi}_t - \frac{b_L}{R\pi}\tilde{R}_t + \frac{m}{\pi}\tilde{m}_{t-1} - \frac{m}{\pi}\tilde{\pi}_t - \frac{xb_L}{R\pi}\tilde{x}_{t-1} - \frac{xb_L}{R\pi}\tilde{b}_{L,t-1} + \frac{xb_L}{R\pi}\tilde{\pi}_t + \frac{xb_L}{R\pi}\tilde{R}_t + G\tilde{g}_t - T\tilde{t}_t\end{aligned} \tag{70}$$

$$T\tilde{t}_t = \psi_1\frac{b}{\pi}(\tilde{b}_{t-1} - \tilde{\pi}_t) + \psi_2\frac{b_L}{R\pi}[\tilde{b}_{L,t-1} - \tilde{R}_t - \tilde{\pi}_t] \tag{71}$$

$$\tilde{R}_t = \alpha_R\tilde{R}_{t-1} + (1-\alpha_R)[\tilde{\pi}_t^* + \alpha_\pi(\tilde{\pi}_t - \tilde{\pi}_t^*) + \alpha_Y\tilde{y}_t] + \varepsilon_t^R \tag{72}$$

$$\tilde{\pi}_t^* = \phi_\pi\tilde{\pi}_{t-1}^* + \varepsilon_t^\pi \tag{73}$$

$$\tilde{a}_t = \phi_A\tilde{a}_{t-1} + \varepsilon_t^A \tag{74}$$

$$\tilde{g}_t = \phi_G\tilde{g}_{t-1} + \varepsilon_t^G \tag{75}$$

$$\tilde{b}_{L,t} = \phi_{BL}\tilde{b}_{L,t-1} + \epsilon_t^{BL} \tag{76}$$

$$\tilde{x}_t = \phi_x \tilde{x}_{t-1} + \epsilon_t^x \tag{77}$$

$$\log \nu_t^{MU} = \phi_{MU} \log \nu_{t-1}^{MU} + \varepsilon_t^{MU} \tag{78}$$

$$\log \nu_t^{PR} = \phi_{PR} \log \nu_{t-1}^{PR} + \varepsilon_t^{PR} \tag{79}$$

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Tables and figures

Table 1: Steady-state values of some variables

Notation	Description	SS value
Y	Output	1 (norm.)
C	Consumption-output ratio	0.57
I	Investment-output ratio	0.23
T/Y	Taxes-output ratio	0.1972
$L/(1-L)$	Ratio of market to non-market activities	0.3
R	Gross money-market rate	1.010
R_L	Gross long-term rate	1.014

Table 2: Benchmark calibration of some parameters

Notation	Description	Benchmark value
	Preferences and technology	
α	Share of capital in the production function	0.36
β	Intertemporal discount factor	0.994
σ	Elasticity of intertemporal substitution	0.5
χ	Elasticity of money demand	7
ψ	Elasticity of labor supply	1
γ	Habit formation	0.7
θ	Elasticity of substitution between varieties of goods	6
ϕ_P	Price adjustment costs	100
	Fiscal and monetary policy	
ψ_0	Fiscal policy constant	0.1972
ψ_1	Fiscal policy response to b	0.3
ψ_2	Fiscal policy response to long-term debt	0.3
α_π	Monetary policy response to inflation	1.5
α_Y	Monetary policy response to output	0
α_R	Monetary policy inertia	0.995
	Autoregressive parameters	
ϕ_A	Technology shock	0.95
ϕ_G	Government spending shock	0.90
ϕ_{MU}	Mark-up shock	0.90
ϕ_π	Inflation targeting shock	0.90
ϕ_{PR}	Preferences shock	0.90
ϕ_{BL}	LT bonds shock	0.90
	Standard deviations	
σ_A	Technology shock	0.01
σ_G	Government spending shock	0.012
σ_R	Monetary policy shock	0.005
σ_{MU}	Mark-up shock	0.01
σ_π	Inflation targeting shock	0.005
σ_{PR}	Preferences shock	0.01
σ_{BL}	LT bonds shock	0.01

Table 3: Calibration values of the key parameters and steady-states

Notation	Description	US	UK
ϕ_L	Portfolio adjustment frictions	0.01	0.01
$B + B_L$	Total debt on GDP	0.496	0.542
B	Total ST debt on total debt	0.188	0.052
B_L	Total LT debt on total debt	0.308	0.490
B_L^H	LT debt held by households	0.250	0.479
B_L^{CB}	LT debt held by the CB	0.058	0.011
σ_x	Magnitude of the asset purchases	1	12
ϕ_x	Persistence of the asset purchases	0.83[1]	0.83[1]
	Approximated duration of the shock	3Q	4Q

Notes: [1]A persistence of 0.83 reflects an exit strategy of approximately 6 years.
Sources: The values are calculated by combining data from the OECD statistical database, the Federal Reserve Statistical Release, the Bank of England Statistical Interactive Database, and the Bank of England APF Gilt Operational Results Dataset. Notice that they represent only approximations, given the difficulties of combining data with different frequency.

Table 4: Estimated effect of the LSAP2 on the LT rate,[1] output and inflation (US - annualized)

Study	Method	Total impact on LT Rate	Peak impact on Output	Peak impact on Inflation
KVJ (2011)[2]	Event study/regression	-33 bp	-	-
D'Amico et al. (2011)	Regressions	-55 bp	-	-
Chen et al. (2011)	DSGE model	-30 bp[3]	0.4%	0.05%
Chung et al. (2012)	FRB/US model	-20 bp[3]	0.6%	0.1%
My calibrated model	Specification			
	Baseline ($\phi_x = 0.83$)	**-64 bp**	**0.95%**	**0.39%**
	High persistence ($\phi_x = 0.88$)	-61 bp	1.29%	0.61%
	Low persistence ($\phi_x = 0.76$)	-65 bp	0.68%	0.24%
	Higher frictions ($\phi_L = 0.02$)	-75 bp	1.61%	0.77%
	Lower frictions ($\phi_L = 0.005$)	-47 bp	0.51%	0.17%
My estimated model	Specification			
	Baseline ($\phi_x = 0.83$)	**-51 bp**	**1.07%**	**0.11%**
	High persistence ($\phi_x = 0.88$)	-43 bp	1.24%	0.15%
	Low persistence ($\phi_x = 0.76$)	-59 bp	0.93%	0.08%

Notes: [1]10-year Treasury yield. [2]Krishnamurthy and Vissing-Jorgensen (2011). [3]Effect on the risk premium.

Table 5: Estimated effect of the first phase of the APF on the LT rate,[1] output and inflation (UK - annualized)

Study	Method	Total impact on LT Rate	Peak impact on Output	Peak impact on Inflation
Glick and Leduc (2011)	Event study	-49 bp	-	-
Joyce et al. (2011a)	Event study	-125 bp	-	-
Joyce et al. (2011b)	SVAR	-	1.5%	0.75%
Joyce et al. (2011b)	Red. form mod.	-	1.5-2.5%	0.75-2.25%
Kapetanios et al. (2012)	Time-series mod.	-	1.5%	1.25%
Bridges and Thomas (2012)	Time-series mod.	-	2%	1%
My calibrated model	Specification			
	Baseline ($\phi_x = 0.83$)	**-69 bp**	**1.33%**	**0.55%**
	High persistence ($\phi_x = 0.88$)	-66 bp	1.72%	0.82%
	Low persistence ($\phi_x = 0.76$)	-72 bp	1.01%	0.35%
	Higher frictions ($\phi_L = 0.02$)	-70 bp	1.39%	0.59%
	Lower frictions ($\phi_L = 0.005$)	-69 bp	1.22%	0.47%
My estimated model	Specification			
	Baseline ($\phi_x = 0.83$)	**-34 bp**	**1.15%**	**0.96%**
	High persistence ($\phi_x = 0.88$)	-36 bp	1.33%	1.43%
	Low persistence ($\phi_x = 0.76$)	-39 bp	0.88%	0.71%

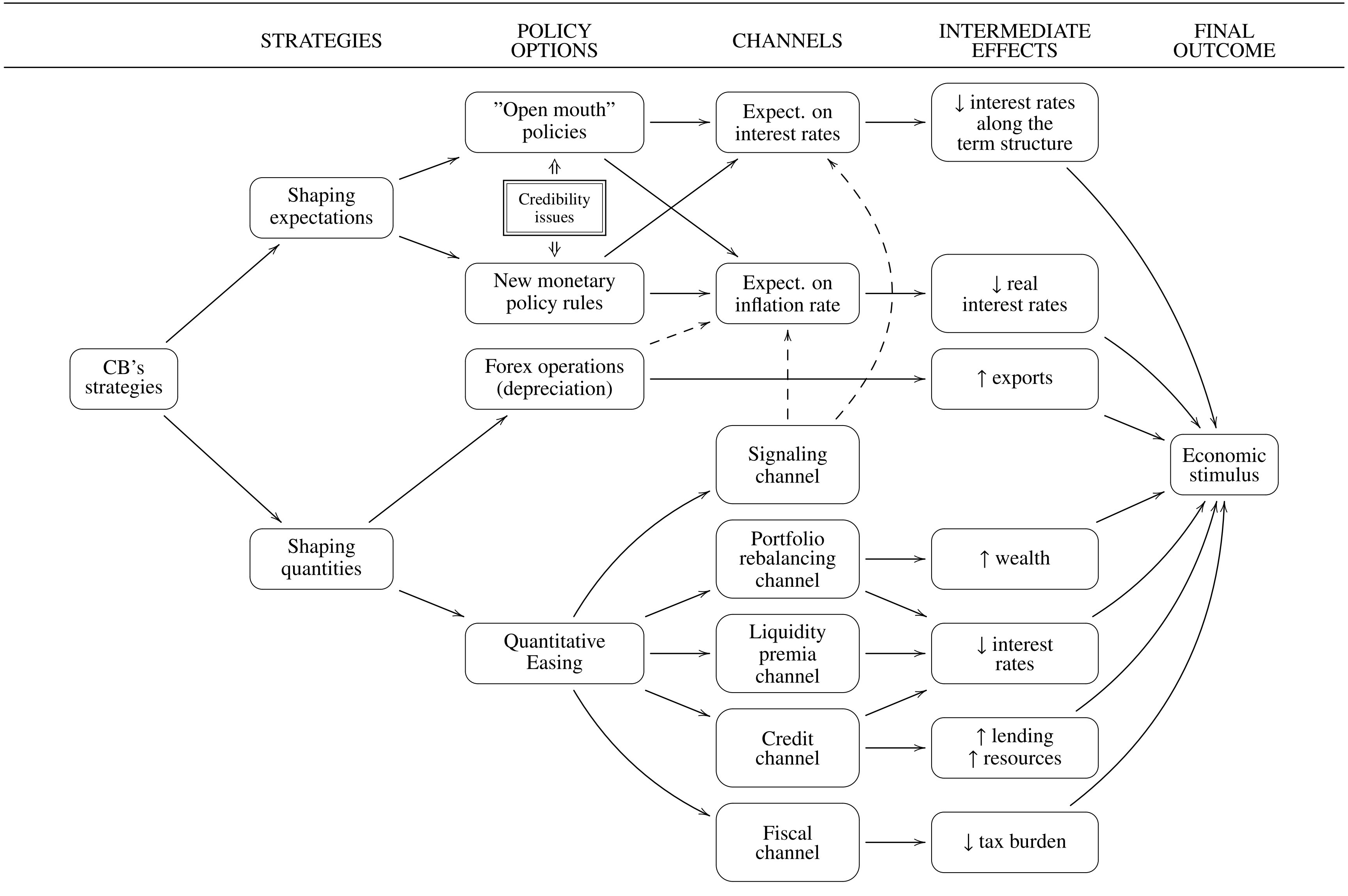
Notes: [1]10-year Treasury yield. [2]Benchmark calibration.

Table 6: Prior and posterior distribution (mean) of the parameters

Notation	Description	Prior distribution	Posterior distribution US			Posterior distribution UK		
			Mean	5%	95%	Mean	5%	95%
	Preferences and technology							
σ	Elasticity of intertemporal substitution	Normal (1,0.5)	0.543	0.711	1.160	0.538	0.300	0.792
ψ	Elasticity of labor supply	Normal (1,0.5)	1.015	0.853	1.176	0.922	0.746	1.094
γ	Habit formation	Beta (0.7,0.1)	0.284	0.166	0.416	0.382	0.250	0.514
θ	Elasticity of substitution between goods	Normal (6,3)	2.067	1.303	2.758	14.909	11.040	19.130
ϕ_P	Price adjustment costs	Normal (100,10)	101.782	85.039	118.787	84.759	66.958	103.142
δ	Capital depreciation	Beta (0.025,0.01)	0.011	0.005	0.018	0.016	0.008	0.024
	Monetary policy							
α_π	Monetary policy response to inflation	Normal (1.5,0.9)	3.252	2.071	4.101	1.135	1.039	1.311
α_Y	Monetary policy response to output	Normal (0.4,0.2)	0.938	0.711	1.160	0.465	0.248	0.693
α_R	Monetary policy inertia	Beta (0.5,0.2)	0.796	0.748	0.849	0.430	0.335	0.521
	Portfolio adjustment frictions							
ϕ_L	Bond adjustment costs	Normal (0.01,0.005)	0.020	0.015	0.026	0.012	0.006	0.018
	Autoregressive parameters							
ϕ_A	Technology shock	Beta (0.85,0.1)	0.731	0.329	0.999	0.668	0.452	0.975
ϕ_G	Government spending shock	Beta (0.85,0.1)	0.847	0.755	0.939	0.847	0.493	0.998
ϕ_{MU}	Mark-up shock	Beta (0.85,0.1)	0.872	0.691	0.999	0.857	0.712	0.999
ϕ_π	Inflation targeting shock	Beta (0.85,0.1)	0.765	0.681	0.856	0.905	0.834	0.987
ϕ_{PR}	Preferences shock	Beta (0.85,0.1)	0.788	0.633	0.918	0.203	0.108	0.293
ϕ_{BL}	Long-term bond supply shock	Beta (0.85,0.1)	0.678	0.583	0.776	0.356	0.234	0.448
	Standard deviations							
σ_A	Technology shock	Inv. Gamma (0.4,2[1])	0.646	0.089	1.601	0.660	0.418	0.888
σ_G	Government spending shock	Inv. Gamma (0.5,2[1])	1.857	1.519	2.190	0.321	0.112	0.753
σ_R	Monetary policy shock	Inv. Gamma (0.4,2[1])	0.079	0.063	0.093	0.372	0.302	0.439
σ_{MU}	Mark-up shock	Inv. Gamma (0.4,2[1])	1.549	0.167	2.738	0.619	0.098	1.435
σ_π	Inflation targeting shock	Inv. Gamma (0.1,2[1])	0.156	0.086	0.254	1.838	0.479	3.142
σ_{PR}	Preferences shock	Inv. Gamma (0.4,2[1])	0.911	0.584	1.231	1.150	0.817	1.548
σ_{BL}	Long-term bond supply shock	Inv. Gamma (0.4,2[1])	3.711	2.986	4.394	2.905	2.028	3.668

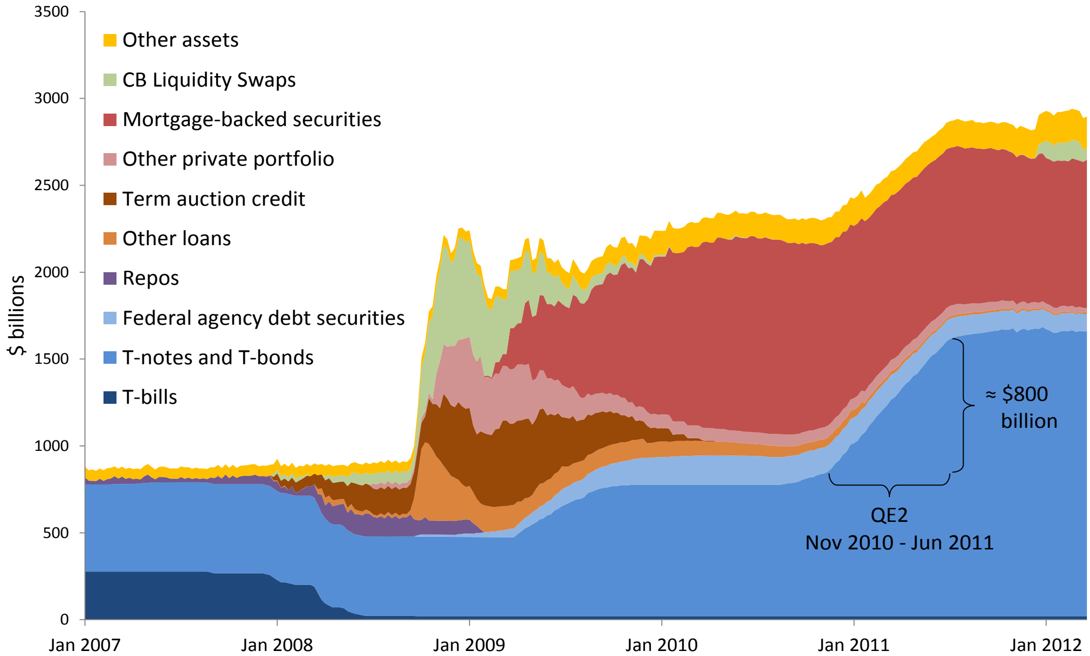
Notes: [1]For the inverted gamma distribution the degrees of freedom are indicated.

Figure 1: Facing the ZLB: Strategies, policy options and channels



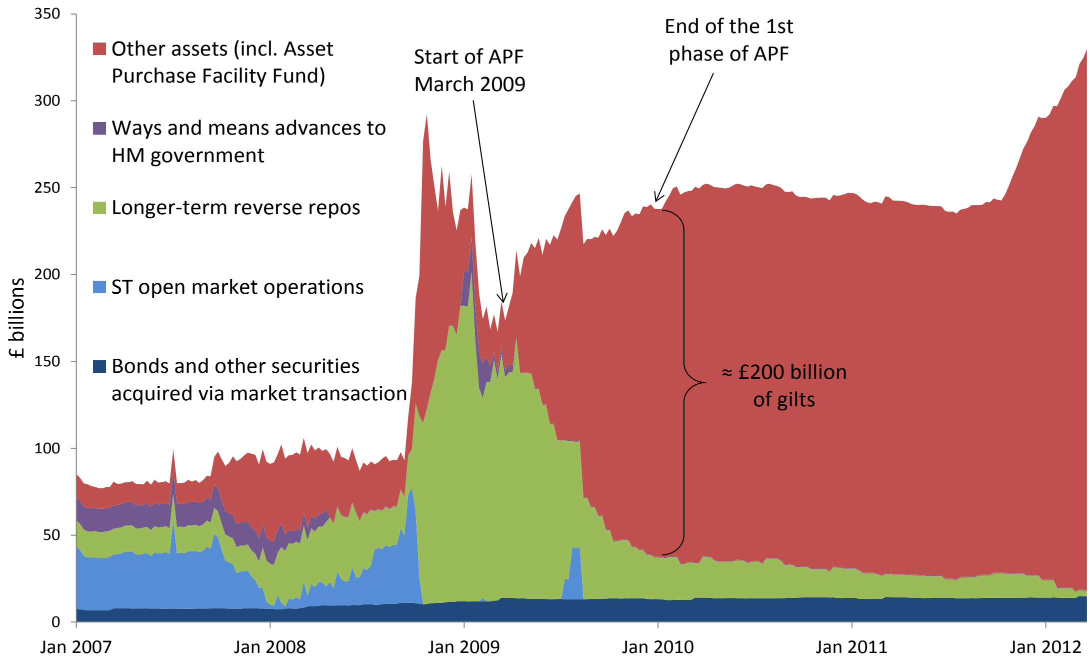
Source: Falagiarda (2012).

Figure 2: Evolution of Fed assets composition



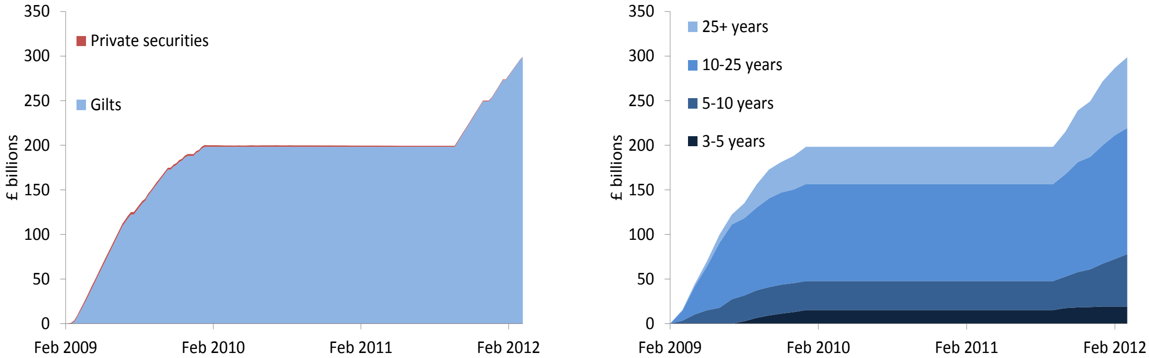
Source: Author's elaboration on data from the Federal Reserve Statistical Release.

Figure 3: Evolution of BoE assets composition



Source: Author's elaboration on data from the Bank of England Statistical Interactive Database.

Figure 4: Cumulative BoE asset purchases by type (a) and cumulative gilts purchases by maturity (b)



(a) (b)

Source: Author's elaboration on data from the Bank of England Statistical Interactive Database.

Figure 5: Impulse responses to the simulated Fed's asset purchase shock

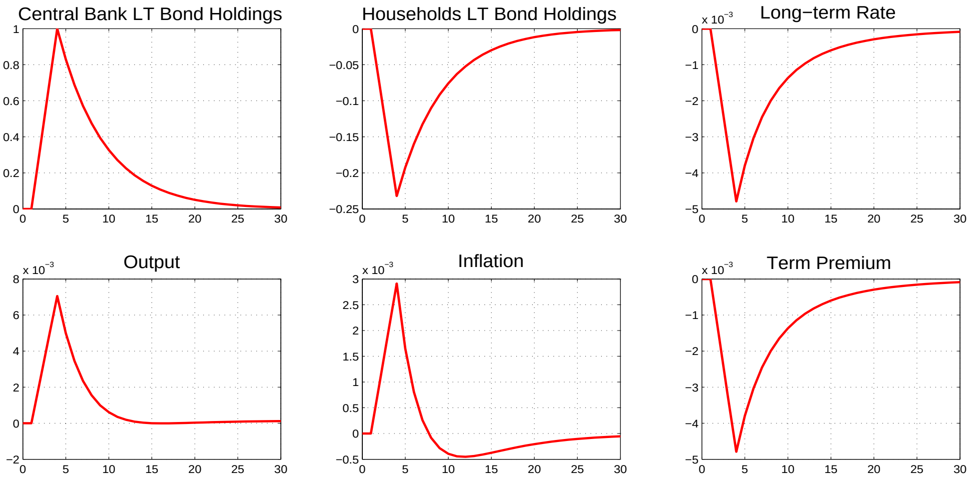


Figure 6: Impulse responses to the simulated BoE's asset purchase shock

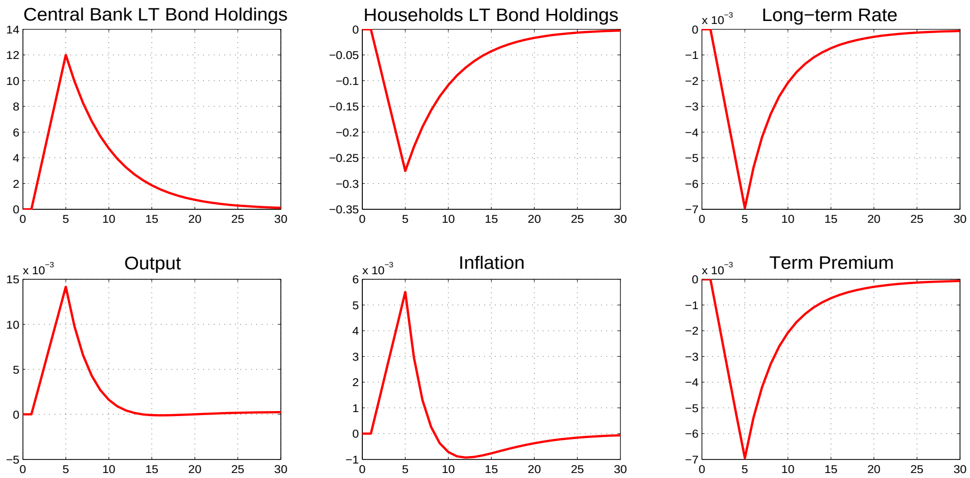


Figure 7: Impulse responses to the simulated Fed's asset purchase shock when varying the persistence of the shock

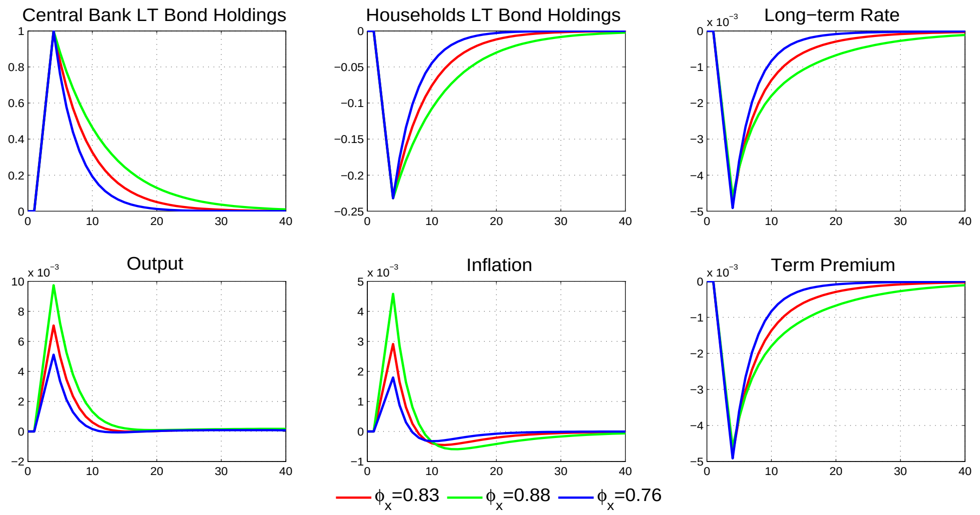


Figure 8: Impulse responses to the simulated BoE's asset purchase shock when varying the persistence of the shock

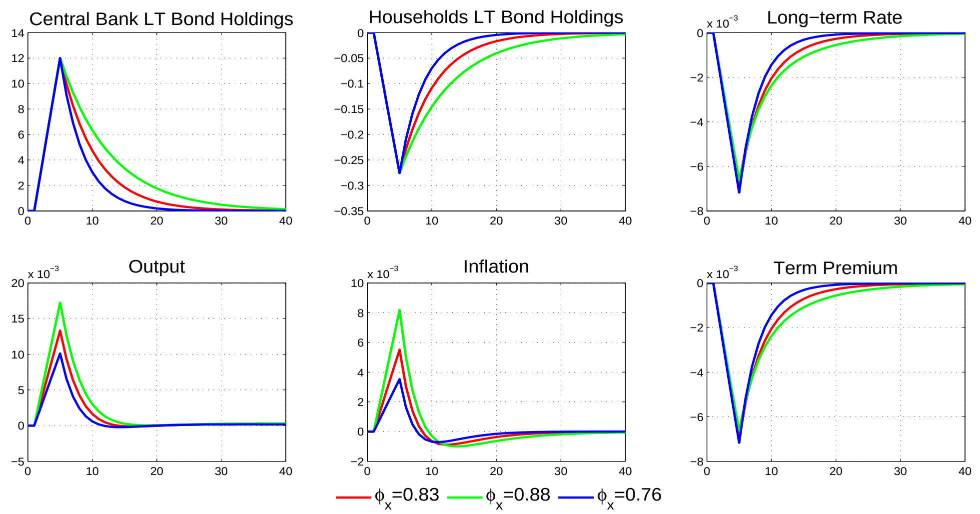


Figure 9: Impulse responses to the simulated Fed's asset purchase shock when varying bond transaction costs

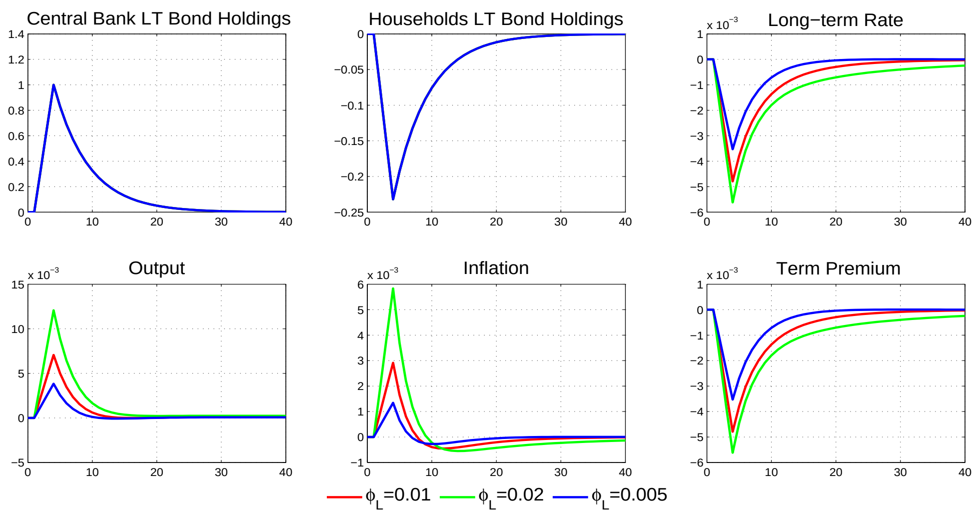


Figure 10: Impulse responses to the simulated BoE's asset purchase shock when varying bond transaction costs

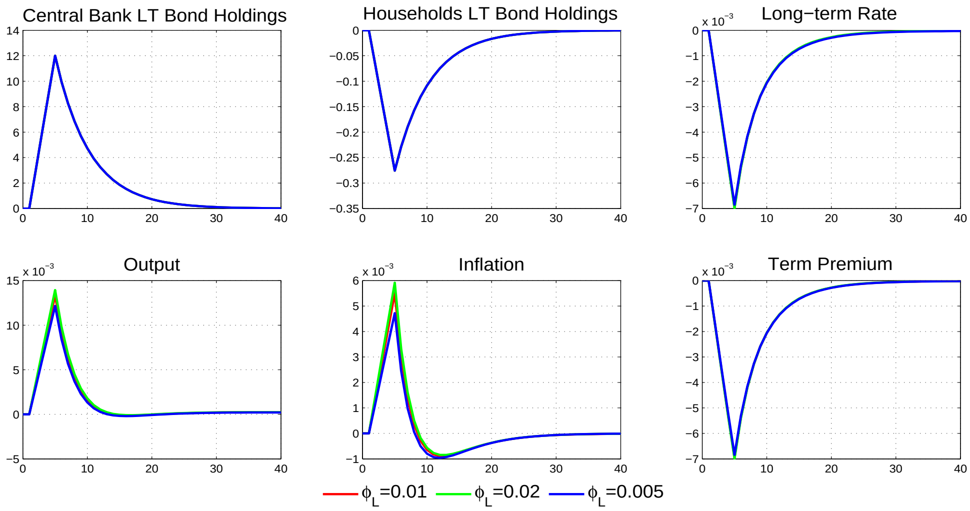
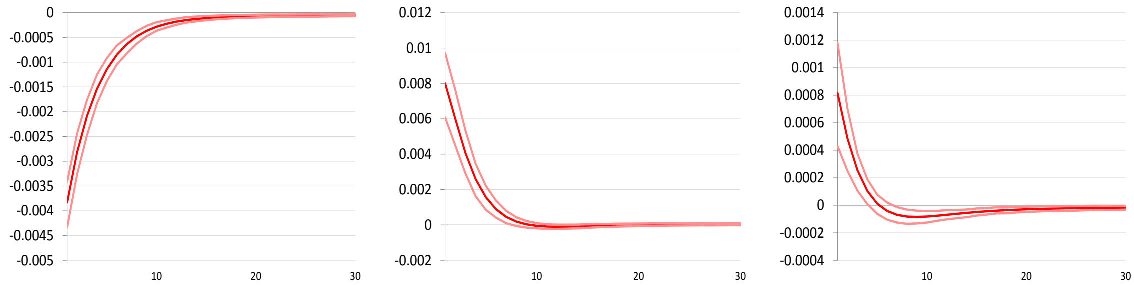
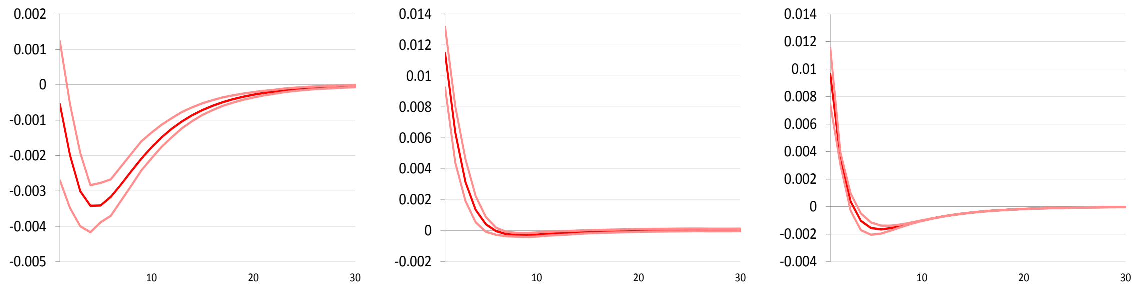


Figure 11: Posterior impulse responses to the simulated Fed's asset purchase shock (mean and 90% HPD interval)



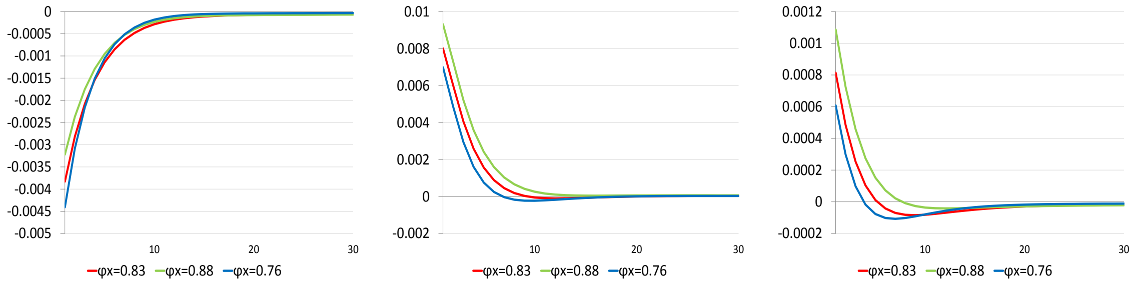
(a) Long-term rate (b) Output (c) Inflation

Figure 12: Posterior impulse responses to the simulated BoE's asset purchase shock (mean and 90% HPD interval)



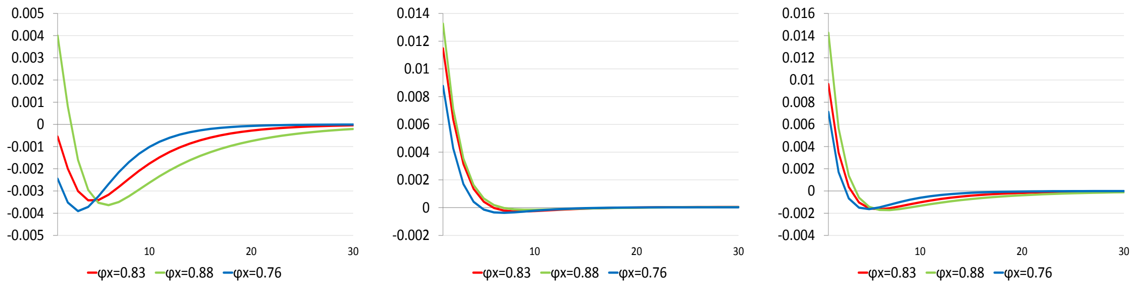
(a) Long-term rate (b) Output (c) Inflation

Figure 13: Posterior impulse responses to the simulated Fed's asset purchase shock for different values of ϕ_x (mean)



(a) Long-term rate (b) Output (c) Inflation

Figure 14: Posterior impulse responses to the simulated BoE's asset purchase shock for different values of ϕ_x (mean)



(a) Long-term rate (b) Output (c) Inflation