Child Labour and Inequality

Simone D'Alessandro*   Tamara Fioroni**

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Abstract

This paper focuses on the evolution of child labour, fertility and human capital in an economy characterized by two type of individuals, low and high skilled workers. This heterogeneity allows for an endogenous analysis of inequality generated by child labour. More specifically, according to empirical evidence, we offer an explanation for the emergence of a vicious cycle between child labour and inequality. The basic intuition behind this result arises from the interdependence between child labour and fertility decisions. The fertility differential between high and low skilled workers tends to increase the ratio between skilled and unskilled wage. This process during the transition leads to increasing inequality.

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*Dipartimento di Economia, Università di Pisa, Via Ridolfi 10, Pisa, Italy. Tel.: +39 050 2216333; Fax: +39 050 598040. *E-mail address:* s.dale@ec.unipi.it.

**Dipartmento di Scienze Economiche, Università di Verona, Vicolo Campofiore, 2. *E-mail address:* tamara.fioroni@univr.it.

1 Introduction

This paper presents a model where the interaction between child labour, fertility and human capital offers an explanation for the emergence of a vicious cycle between child labour and inequality. The basic intuition behind this result is the interdependence between child labour and fertility choices: unskilled parents tend to have a high number of children and send them to work, whereas skilled parents have low fertility rates and tend to invest more in education. In the long run, this fertility differential increases the fraction of unskilled workers in the labour market which in turns produces an increase in the wage differential between skilled and unskilled labour. The fact that children can offers only unskilled labour reinforces such process creating a vicious cycle between child labour and inequality.

As shown in Figure 1, we find empirical evidence of a positive relationship between inequality and child labour.[1] In this figure, we use the data on children not attending school (i.e.number of out-of-school children as a percentage of all primary school-age children) as a proxy of child labour given the shortage of data on child labour. Even if this measure presents the shortcoming that a child not attending school is not necessarily working, on the other hand it is more easier to monitor children not attending school with respect to children that are working. In addition, the rate of children out of school should give also a measure of children working within the household or engaged in unofficial which are not included in the number of children economically active (see Cigno and Rosati, 2002). Note also that while still positive, the relationship between child labour and inequality has begun to flatten in recent years. A possible explanation of this result could be the increasing attention by national and international organizations to child labour.

Our work is related to a large body of the literature which has devel-

[1] Variability bands in Figure 1 show the level of variability present in the estimate. In particular, their width is determined by an estimate of the standard error (Bowman and Azzalini, 1997).

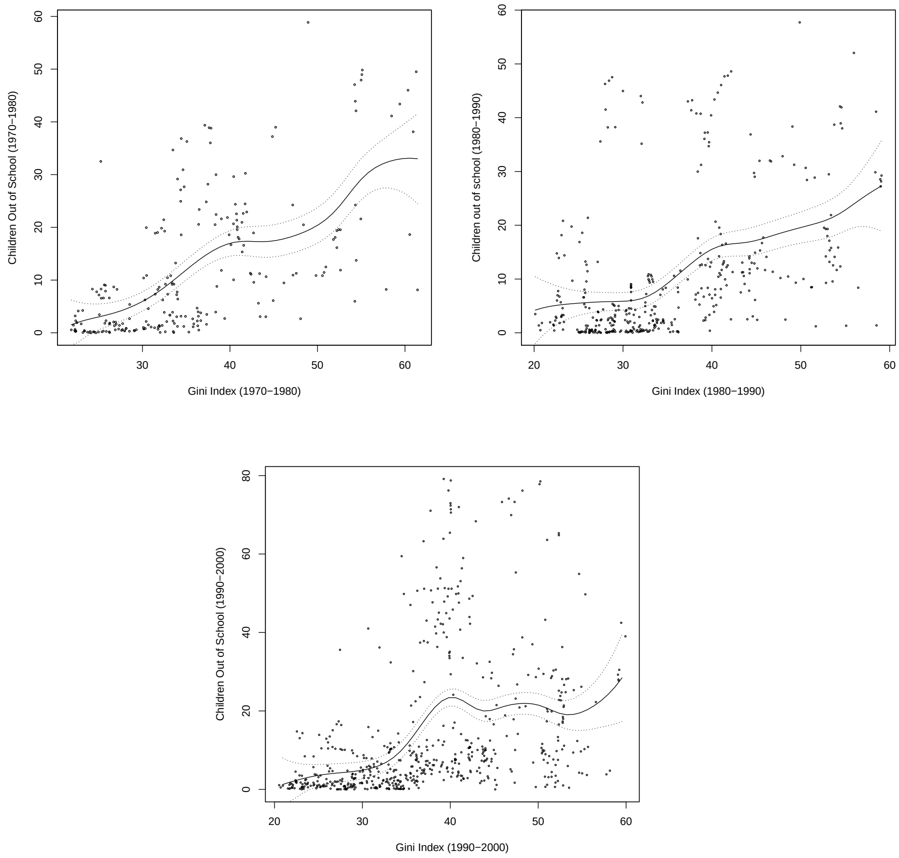

Figure 1: Children out of school and Gini Index (1970-1980, 1980-1990, 1990-2000). Nonparametric kernel smoother. Per capita GDP data are from Penn World Table 7.0. Gini Index data are from World Income Inequality Database. Children out of school data are from World Development Indicators (2010)

oped theoretical and empirical models to study the causes of child labour persistence (see for example Basu and Van, 1998; Basu, 1999, 2000; Baland and Robinson, 2000; Dessy, 2000; Dessy and Pallage, 2001, 2005; Ranjan, 1999, 2001). The benchmark framework is based on two main axioms: the luxury axiom and the substitution axiom (Basu and Van, 1998 and Basu, 1999). The luxury axiom implies that parents send children to work if their income is below a certain threshold. The substitution axiom implies that adult labour and child labour are substitutes. These axioms lead to multiple equilibria in the labour market, with one equilibrium where adult wage is low and children work and another where adult wage is high and children do not work.

This framework has been extended by Dessy (2000), Hazan and Berdugo (2002) and Doepke and Zilibotti (2005) which introduce endogenous fertility choices. They analyze the relationship between child labour, fertility and human capital showing the existence of multiple development paths. In early stages of development, the economy is in a development trap where child labour is abundant, fertility is high and output per capita is low. Technological progress, allows a take-off from this trap because it gradually increases the wage differential between parental and child labour and hence the return of investment in education. However, these contributions do not consider the presence of inequality, i.e. the economy can follow different paths of growth that are characterized – in equilibrium – by a unique level of human capital. We extend this framework taking into account two groups of individuals with two different levels of human capital. The presence of this heterogeneity can induce the high skilled dynasty to catch any increase in the return of human capital, by increasing the level of inequality during the transition.

In this respect, our framework is related to the literature on inequality, differential fertility, and economic growth. In particular, Moav (2005) develops a theory of fertility that offers an explanation for the persistence of poverty within and across countries. The basic idea is that the cost of child quantity increases with parent's human capital since the opportunity cost of time is high. High-income families choose low fertility rates and high investment in education. This implies that high income persists in the dynasty. On the other hand, poor households choose relatively high fertility rates with relatively low investment in their offspring's education. Therefore, their offspring are poor as well.[2] De la Croix and Doepke (2003, 2004) argue that a higher inequality, by increasing fertility differential between rich and poor families, lowers average education and, therefore, growth. The motivation for this result is that a large fertility differential implies more weight on children with little education. We show that the presence of child labour can reinforce this process.

The interaction between fertility differentials and inequality is also the focus of Dahan and Tsiddon (1998), Kremer and Chen (2002). Dahan and Tsiddon (1998) show that fertility and income distribution follow an inverted U-shaped dynamics during the transition to the steady state. In the first stage of development the high fertility among poor leads to a higher proportion of poor and thus to a greater inequality. When income inequality reaches a certain threshold, wage differential is high enough to lead some poor agents to invest in human capital. Thus the number of skilled agents rises, fertility declines and income becomes more equally distributed. Kremer and Chen (2002) develop a model that generates multiple steady states of inequality, when the initial proportion of skilled workers is high the economy converges to a steady state with low inequality, on the other hand when the initial proportion of skilled workers is too low the economy converges to a steady state with high inequality.

We show that, by incorporating child labour in this framework, a vicious cycle between inequality and child labour can persist for generations. In par-

[2] From a similar perspective, Galor and Zhang (1997) show that due to financial market imperfections countries with smaller average family size and with more equal distribution of income attain a high economic growth. Morand (1999) develops a model in which fertility decisions are motivated by old-age support. He shows that a rich economy with an equal distribution of income evolves on a growth path with increasing levels of human capital. By contrast, a poor economy can take off on the growth path only if there exists a class of relatively high skilled agents.

ticular we develop an overlapping generation model with two types of workers - low and high skilled. According to the existent literature, we assume that child labour is a perfect substitute for unskilled adult but children are relatively less productive. Adults allocate their time endowment between work and child raring. They choose the number of children and their time allocation between schooling and work. Hence, households can have two, possible, sources of income: income by parents and child income. As a result the model shows an intergenerational persistence in education levels. This effect together with the differential fertility between low and high skilled parents can produce a continuous increase in the inequality and child labour during the transition and an average impoverishment within the country in the long run. According to the model of Doepke (2004), our paper suggests that child labour regulation is essential in reducing permanently inequality.

Section 2 describes the basic structure of the model. In Section 3, we investigate the properties of the short run general equilibrium. Section 4 analyses the long run dynamics. Section 5 concludes.

2 The Model

We analyze an overlapping-generation economy which is populated by N_t individuals. Each of them is endowed with a level of human capital, h_t^i. This level is endogenously determined by parent's choice on the children's time allocation between labour and schooling. Adults can supply skilled or unskilled labour, while children can only supply unskilled labour. We depart from much of the literature on child labour by assuming that unskilled labour is not paid according to the level of human capital.[3]

2.1 Production

We assume that labour is the only production factor. Production occurs according to a constant-returns-to-scale technology using unskilled and skilled

[3]For instance, a similar way of modeling the labour market can be found in Galor et al. (2009).

labour as inputs. Thus the output produced at time t is

$$Y_t = \psi(H_t)^{\mu}(L_t)^{1-\mu} = \psi(s_t)^{\mu} L_t, \tag{1}$$

where $s_t \equiv H_t/L_t$ is the ratio of skilled H_t to unskilled labor L_t employed in production in period t, and $\psi > 0$ and $0 < \mu < 1$ are technological parameters. In each period t, firms choose the level of unskilled labour, L_t, and the efficiency units of labour, H_t, so as to maximize profits. Thus the wage of unskilled workers, i.e. w_t^u, and the wage rate per efficiency unit, w_t^s, are

$$w_t^u = \psi(1-\mu)\,(s_t)^{\mu}\,, \tag{2}$$

and

$$w_t^s = \psi\mu\,(s_t)^{\mu-1}\,. \tag{3}$$

2.2 Preferences

Members of generation t live for two periods: childhood and adulthood. In the childhood, individuals may either work, go to school or both. In the adulthood, agents supply unskilled or skilled labour. Individual's preferences are defined over consumption, i.e. c_t^i, the number of children n_t^i, and the human capital of children h_{t+1}^i.[4] The utility function of an agent i of generation t is given by

$$U_t^i = \alpha \ln c_t^i + (1-\alpha)\ln(n_t^i h_{t+1}^i), \tag{4}$$

where $\alpha \in (0,1)$ is the altruism factor.

We suppose that children born with some basic human capital a, which can be increased by attending school. In particular, human capital of children

[4] As it is clear from equation (4), we assume that parents are aware of the human capital of their children rather than their income. Although the results of the model are not crucially affected by this choice, we believe that this is a more realistic assumption, see for instance De la Croix and Doepke (2003) and Galor (2005) for discussion on this point.

in period $t+1$ is an increasing, strictly concave function of the time devoted to school (see e.g. Galor and Weil, 2000; Galor and Tsiddon, 1997; De la Croix and Doepke, 2004; Hazan and Berdugo, 2002), that is

$$h_{t+1}^i = a(b + e_t^i)^\beta, \tag{5}$$

where $a, b > 0$ and $\beta \in (0, 1)$.

Parents allocate their income between consumption and child rearing. In particular, raising each born child takes a fraction $z \in (0, 1)$ of an adult's income. In addition, parents allocates the time endowment of children between schooling, $e_t^i \in [0, 1]$, and labour force participation $(1 - e_t^i) \in [0, 1]$. We assume that, each child can offer only $\theta \in [0, z)$ units of unskilled labour, that is children are substitutes for unskilled adult workers but relatively less productive.[5] Therefore, each household have two potential sources of income: i) parent income, $I_t^i = \max\{w_t^s h_t^i, w_t^u\}$ and, ii) child income, $(1 - e_t^i)\theta w_t^u$. Indeed, while children can work only as unskilled workers, parents will choose to work in the sector that guarantees them the highest income. Hence, the budget constraint is

$$c_t^i \le (1 - z n_t^i) I_t^i + (1 - e_t^i)\theta w_t^u n_t^i. \tag{6}$$

2.3 Individual choices

Each household chooses c_t^i, n_t^i and e_t^i so as to maximize the utility function (4) subject to the budget constraint (6). Given the wage ratio, the optimal consumption, the optimal schooling and the optimal number of children chosen by member i of generation t are

$$c_t = \alpha I_t^i; \tag{7}$$

[5]The assumption $z > \theta$ means that the ratio between the income of child labour when the child just works and the cost of rising a child – i.e. the relative return of child rearing (θ/z) – is less than 1. This further implies that it is not possible to increase income by simply "producing" more children.

$$e_t^i = \begin{cases} 0 & \text{if } r_t^i \leq \frac{\theta(\beta+b)}{\beta z}, \\ \frac{r_t^i \beta z - \theta(\beta+b)}{\theta(1-\beta)} & \text{if } \frac{\theta(\beta+b)}{\beta z} \leq r_t^i \leq \frac{\theta(1+b)}{\beta z}, \\ 1 & \text{if } r_t^i \geq \frac{\theta(1+b)}{\beta z}; \end{cases} \tag{8}$$

and

$$n_t^i = \begin{cases} \frac{(1-\alpha) r_t^i}{z r_t^i - \theta} & \text{if } r_t^i \leq \frac{\theta(\beta+b)}{\beta z}, \\ \frac{(1-\alpha)(1-\beta) r_t^i}{z r_t^i - \theta(1+b)} & \text{if } \frac{\theta(\beta+b)}{\beta z} \leq r_t^i \leq \frac{\theta(1+b)}{\beta z}, \\ \frac{1-\alpha}{z} & \text{if } r_t^i \geq \frac{\theta(1+b)}{\beta z}; \end{cases} \tag{9}$$

where $r_t^i \equiv I_t^i / w_t^u$. In particular,

$$r_t^i = \begin{cases} 1 & \text{if } w_t^s h_t^i \leq w_t^u, \\ \frac{w_t^s h_t^i}{w_t^u} & \text{if } w_t^s h_t^i > w_t^u. \end{cases} \tag{10}$$

Since $h_t^i = a(b + e_{t-1}^i)^\beta$, the ratio r_t^i is a function of the level of education chose in period $t-1$.

Agent i, according to her level of human capital h_t^i, chooses to work as unskilled if, and only if, $w_t^s h_t^i < w_t^u$, while she works as skilled if, and only if, $w_t^s h_t^i > w_t^u$. Finally, if $w_t^s h_t^i = w_t^u$ agent i is indifferent to work as skilled or unskilled. Note that, from equation (10), if parents find convenient to work as unskilled, their choices on fertility and education do not depend on income since $r_t^i = 1$ – see equations (8) and (9). This feature results from the substitutability between child and unskilled labour.[6]

3 General Equilibrium

The last result highlights the emergence of a marked asymmetry between agents who offer skilled and unskilled work. For the sake of the argument, we assume that in the initial period, $t = 0$, population is divided in two

[6] According to the existent literature the model shows a trade-off between quantity and quality of children. See, for instance, Hazan and Berdugo (2002) for an analysis of a similar model under constant wages.

groups which are endowed with two different levels of human capital, a low level of human capital h_0^u and a high level h_0^s, with $h_0^u < h_0^s$. We show that such a difference may persist across generations. At any period t, if $h_t^u < h_t^s$, low skilled workers choose to work as unskilled if and only if $w_t^s h_t^u < w_t^u$, while if $w_t^s h_t^u > w_t^u$ they would prefer to work as skilled.[7] Since we assume perfect mobility of labour at equilibrium the ratio w_t^u / w_t^s must satisfy $w_t^s h_t^u \leq w_t^u$; otherwise all the labour force would offer skilled labour, which it is not possible given equation (2). A similar argument applies to high skilled workers; thus $w_t^s h_t^s \geq w_t^u$. Therefore, for any $h_t^u \leq h_t^s$, at equilibrium

$$h_t^u \leq \frac{w_t^u}{w_t^s} \leq h_t^s. \tag{11}$$

From equations (10) and (11), it holds

$$r_t^u = 1, \tag{12}$$

and

$$1 \leq r_t^s \leq \frac{h_t^s}{h_t^u}, \tag{13}$$

for all $t \in \mathbb{N}_0$.

Inequality (11) points out that three different regimes arise in this framework. Two regimes are corner solutions. If at equilibrium $w_t^s h_t^s = w_t^u$, high skilled workers are indifferent to work as skilled or unskilled. On the other hand, if at equilibrium $w_t^s h_t^u = w_t^u$, a fraction of low skilled workers work as skilled. In the other case, when $h_t^u < \frac{w_t^u}{w_t^s} < h_t^s$, low skilled only work as unskilled and high skilled only as skilled.

It is worth to point out that if in a certain period t, market equilibrium implies $w_t^s h_t^s = w_t^u$, in period $t+1$ there will be no difference between low and high skilled workers, since all the population gets the same adult income and makes the same schooling and fertility decisions. This argument does not apply when $w_t^s h_t^u = w_t^u$, since in that case high skilled workers get a higher income equal to $w_t^s h_t^s$ that is greater than w_t^u if $h_t^s > h_t^u$.

[7] The superscript u and s refers to low and high skilled workers respectively.

3.1 Internal equilibrium

Let us assume that in period t, $1 < r_t^s < \frac{h_t^s}{h_t^u}$. As we pointed out above, under such condition low skilled workers find convenient to work as unskilled and high skilled as skilled. Thus, at equilibrium – if it exists – the economy is characterized by two classes of income ($w_t^s h_t^s > w_t^u$), which make different fertility and schooling decisions – see equations (8) and (9).

Thus the aggregate demand is

$$D_t = c_t^u N_t^u + c_t^s N_t^s, \tag{14}$$

where N_t^u and N_t^s are, respectively, the number of low and high skilled agents, and from equations (2), (3) and (7),

$$c_t^u = \alpha\psi(1-\mu)s_t^{\mu}, \tag{15}$$

$$c_t^s = \alpha h_t^s \psi \mu s_t^{\mu-1}. \tag{16}$$

At time t, the supply of unskilled labour is given by the labour supplied by low skilled adults, i.e. $(1 - zn_t^u)N_t^u$, plus the labour supplied by the children of low and high skilled parents, i.e. $(1-e_t^u)n_t^u N_t^u$ and $(1-e_t^s)n_t^s N_t^s$. At equilibrium this supply must be equal to the total demand of unskilled labour, L_t, that is,

$$L_t = (1 - zn_t^u)N_t^u + \theta[(1-e_t^u)n_t^u N_t^u + (1-e_t^s)n_t^s N_t^s]. \tag{17}$$

Moreover, the supply of skilled labour must be equal to the demand of skilled labour H_t, that is

$$H_t = (1 - zn_t^s)h_t^s N_t^s. \tag{18}$$

From equations (1), (14), (15) and (16), the equilibrium in the goods market yields

$$L_t = \alpha\frac{(1-\mu)s_t N^u + \mu h^s N^s}{s_t}. \tag{19}$$

The ratio between equations (18) and (19) defines the equilibrium level of s_t

$$s_t^* = \frac{h_t^s}{1-\mu}\left[\frac{1-zn_t^s}{\alpha} - \mu\right]\frac{1}{x_t}, \tag{20}$$

where $x_t \equiv N_t^u/N_t^s$. Note that in period t, s_t^* depends only on the choice of n_t^s. The other variables N_t^s, N_t^u and h_t^s depend on choices made in period $t-1$. In order to understand the relation between s_t^* and n_t^s it is convenient to rewrite r_t^s. From equations (2), (3) and (20), we obtain,

$$r_t^s = \frac{w_t^s h_t^s}{w_t^u} = \frac{\mu\alpha x_t}{1 - zn_t^s - \mu\alpha}, \tag{21}$$

which depends only on n_t^s. This function takes different values according to the value of r_t^s.[8] Thus, at equilibrium

$$r_t^{s*} = \begin{cases} \frac{2\theta\alpha\mu x_t}{\theta(1-\alpha\mu)+z\alpha\mu x_t - \sqrt{\Delta_1(x_t)}} & \text{if} \quad x_t \le x_2 \\ \frac{2\theta(1+b)\alpha\mu x_t}{\theta(1+b)(1-\alpha\mu)+z\alpha\mu x_t - \sqrt{\Delta_2(x_t)}} & \text{if} \quad x_2 \le x_t \le x_3 \\ \frac{\mu}{1-\mu}x_t & \text{if} \quad x_3 \le x_t \end{cases} \tag{23}$$

and

$$n_t^{s*} = \begin{cases} \frac{\theta(1-\alpha\mu)-z\alpha\mu x_t + \sqrt{\Delta_1(x_t)}}{2\theta z} & \text{if} \quad x_t \le x_2 \\ \frac{\theta(1+b)(1-\alpha\mu)-z\alpha\mu x_t + \sqrt{\Delta_2(x_t)}}{2\theta z} & \text{if} \quad x_2 \le x_t \le x_3 \\ \frac{(1-\alpha)}{z} & \text{if} \quad x_3 \le x_t \end{cases} \tag{24}$$

where $\Delta_1(x_t) = [z\alpha\mu x_t - \theta(1-\alpha\mu)]^2 + 4\theta(1-\alpha)z\alpha\mu x_t$ and $\Delta_2(x_t) = [z\alpha\mu x_t - \theta(1+b)(1-\alpha\mu)]^2 + 4\theta(1+b)(1-\beta)(1-\alpha)z\alpha\mu x_t$; $x_2 = \frac{\theta(b+\beta)[b\alpha(1-\mu)-\beta(1-\alpha)]}{z\alpha\mu\beta b}$, $x_3 = \frac{\theta(1-\mu)(1+b)}{\mu\beta z}$.[9]

[8] From equation (9), the fertility choice of high skilled workers is given by

$$n_t^s = \begin{cases} \frac{(1-\alpha)r_t^s}{zr_t^s - \theta} & \text{if} \quad r_t^s \le \frac{\theta(\beta+b)}{\beta z}, \\ \frac{(1-\alpha)(1-\beta)r_t^s}{zr_t^s - \theta(1+b)} & \text{if} \quad \frac{\theta(\beta+b)}{\beta z} \le r_t^s \le \frac{\theta(1+b)}{\beta z}, \\ \frac{1-\alpha}{z} & \text{if} \quad r_t^s \ge \frac{\theta(1+b)}{\beta z}. \end{cases} \tag{22}$$

[9] The equilibrium values of r_t^{s*} and n_t^{s*} are obtained from equations (21) and (22).

Note that the equilibrium value of r_t^{s*} depends only on the ratio between the number of low and high skilled workers. Moreover, in an internal equilibrium it must hold that $1 < r_t^{s*} < \frac{h_t^s}{h_t^u}$. Thus it is possible that for some values of x_t does not exist an internal solution. Figure 2 clarifies this result. The function r_t^{s*} is a piecewise function defined in the interval $\underline{x} \leq x_t \leq \bar{x}$ – that implies $1 < r_t^{s*} < \frac{h_t^s}{h_t^u}$ – where an internal equilibrium always exists.[10] In the case presented in Figure 2, as long as x_t increases r_t^{s*} becomes equal to $\frac{h_t^s}{h_t^u}$ before reaching the level $\frac{\theta(1+b)}{\beta z}$, that is the level which ensures $e_t^s = 1$. In the Appendix we shows that the derivative of r_t^{s*} with respect of x_t is always positive.

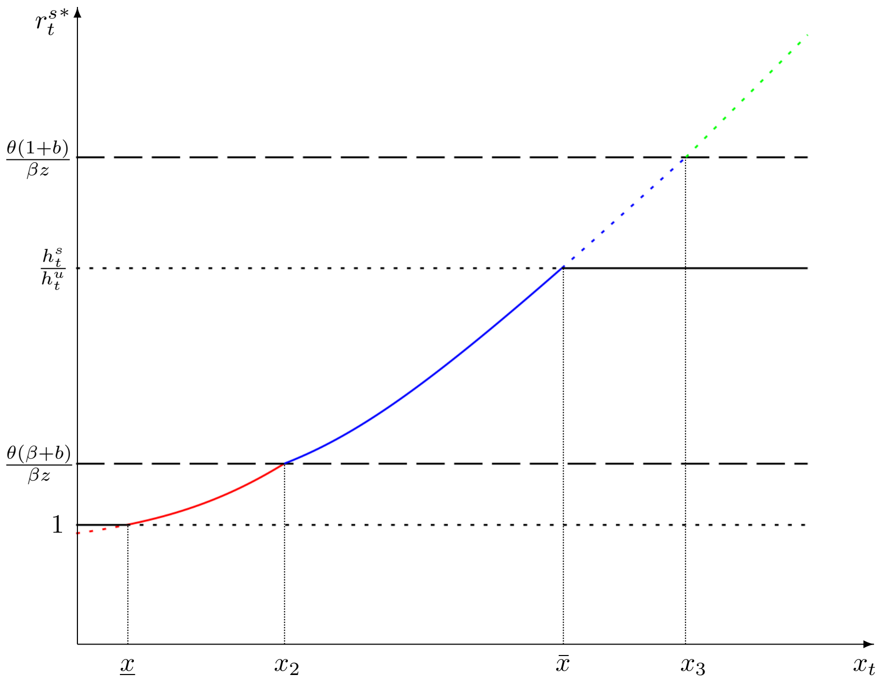

Figure 2: A numerical illustration of r_t^{s*} as a function of x_t. An internal equilibrium exists if and only if $\underline{x} \leq x_t \leq \bar{x}$. Value of parameters: $\alpha = 0.9$; $\mu = 0.3$; $z := 0.3$; $\theta = 0.25$; $\beta = 0.4$; $b = 0.2$; $a = 1$.

Given r_t^{s*}, it is easy to get the equilibrium values for all the other variables of the model. Note, that for $x_t \leq x_2$ high (and low) skilled workers do not

[10] The boundaries $\underline{x}$ and $\bar{x}$ can cross the function r_t^{s*} in each of the three intervals depending on the values of parameters. Thus many different cases may arise, but such an analysis does not give mach insight.

invest in education ($e_t^s = 0$), for $x_2 < x_t < x_3$ high skilled workers send children to work and to school ($0 < e_t^s < 1$), while for $x_3 \leq x_t$ they send children only to school ($e_t^s = 1$).

3.2 Corner solutions

The previous analysis shows that the equilibrium only depends on the level of x_t, that is the ratio between low and high skilled workers. If this ratio is smaller than $\underline{x}$, the number of high skilled is too high, given the technology available, and therefore the wage of the high skilled is equal to the wage of the unskilled ($w_t^s h_t^s = w_t^u$). This means that for any $0 \leq x_t \leq \underline{x}$, $r_t^u = r_t^{s*} = 1$ (see Figure 2).

On the other hand, if $x_t > \bar{x}$, high skilled workers are too small with respect to low skilled. Thus the wage for unit of human capital (w_t^s) is high enough to allow low skilled to work as skilled getting the same wage of unskilled workers ($w_t^s h_t^u = w_t^u$). This means that for any $\bar{x} \leq x_t$, $r_t^{s*} = \frac{h_t^s}{h_t^u}$ (see Figure 2).

4 Long Run

Fertility choices of the two groups affect the relative size of high and low skilled workers. In the long run, the fertility differential is crucial in determining the dynamics of the wage ratio, and hence the dynamics of human capital.

Since $N_{t+1}^u = n_t^u N_t^u$ and $N_{t+1}^s = n_t^s N_t^s$, the population dynamics is given by

$$N_{t+1} = n_t^u N_t^u + n_t^s N_t^s. \tag{25}$$

Thus the dynamics of $x_t \equiv N_t^u / N_t^s$ is given by

$$x_{t+1} = \frac{n_t^u}{n_t^{s*}} x_t, \tag{26}$$

where, from equations (9) and (12), the fertility of low skilled workers is

$$n_t^u = \begin{cases} \frac{(1-\alpha)}{z-\theta} & \text{if } 1 \leq \frac{\theta(\beta+b)}{\beta z}, \\ \frac{(1-\alpha)(1-\beta)}{z-\theta(1+b)} & \text{if } \frac{\theta(\beta+b)}{\beta z} \leq 1 \leq \frac{\theta(1+b)}{\beta z}, \\ \frac{1-\alpha}{z} & \text{if } 1 \geq \frac{\theta(1+b)}{\beta z}, \end{cases} \tag{27}$$

and n_t^{s*} is given by equation (24). Note that since both n_t^u and n_t^{s*} are decreasing function of income, $n_t^u \geq n_t^{s*}$.

We define a long run equilibrium a trajectory in which individual choices do not change over time. Since choices at any period t are affected by income, in a long run equilibrium wage ratio must be constant which means that there must be a constant proportion of skilled and unskilled labour. However, as long as there is inequality in the economy (the income of high skilled is higher than the income of low skilled), the fertility choices between the two groups are different and the ratio x_t changes over time. This implies that any short run internal equilibrium cannot be a long run equilibrium.

There are three limit cases, where inequality disappears after the first period.

First, if, $1 \geq \frac{\theta(1+b)}{\beta z}$, low skilled workers send their children only to school. The fertility choices of high and low skilled workers are the same and all the population will be characterized by the maximum level of human capital.

Second, if, $1 \leq \frac{\theta(\beta+b)}{\beta z}$ and in period $t = 0$, $x_0 \leq x_2$, high skilled workers send their children to work. Thus next generation will be characterized by the minimum level of human capital: the differences between the two classes disappear.

Third, if in period $t = 0$, $x_0 \leq \underline{x}$ thus high skilled workers get a level of wage equal to w_0^u. Also in this case, their choice of fertility and education are equal to the choice of low skilled workers.

Beside those cases, the economy follows a transition characterized by increasing inequality. For any $\max\{\underline{x}, x_2\} < x_t < \bar{x}$, the fertility of high skilled workers is permanently lower than the fertility of low skilled workers. Thus, from equation (26), during the transition x_t increases over time. This change directly affects the wage ratio since the supply of unskilled labour increases more than the supply of skilled one. However, the increase in x_t induces an increase in r_t^{s*} (see Figure 2), which in turns induces an increase in the human capital of children of high skilled parents, i.e. h_{t+1}^s. Both those consequences favor the group of high skilled workers. This process generates a continuous increase in the inequality and an increase in child labour, since generation by generation the number of high-skilled workers decreases and become richer whereas the number of low skilled workers increases and become poorer.

The descendent of the high skilled group cannot obtain an income lower than their parents, thus, the choice of education cannot decrease. In other words, the human capital of high skilled workers tends to increase over time. The increase in h_t^s leads also to an increase in $\bar{x}$, which may allow the dynamics of human capital to reach its maximum level. However, since the accumulation of human capital is bounded, the continuous increase in x_t implies that in a certain time period, for instance $t = \tilde{t}$, population ratio x_t reaches the threshold level $\bar{x}$ in which the wage ratio becomes constant, i.e. $w_t^u/w_t^s = h_t^u$ (see Figure 2). This implies that in the time interval $t > \tilde{t}$ low skilled workers are indifferent between working as skilled and unskilled and the proportion of skilled and unskilled workers will be constant to maintain a constant wage ratio.[11] Thus, for any $t > \tilde{t}$ the economy is in a long run equilibrium where wages do not change and individual choices are constant. However, the presence of differential fertility induces an increase of low skilled workers which send their children to work (at least partially). Thus the high-skilled fraction of population tend to zero, while an higher fraction of low skilled workers tend to work as skilled. In other words, the economy in the long run will be populated only by low skilled workers which continue to send children to work.

To summarize, during the transition inequality rises since the presence of fertility differential and child labour generate an increase in the return of human capital which is caught only by high skilled workers. However, the continuous rise of low skilled workers and the continuous decline of high

[11] This would imply that the share of low skilled workers who work as skilled increases over time.

skilled workers leads in the long run to a population totally composed only of low skilled workers and thus to the disappearance of inequality.

5 Final Remarks

This paper is built on the idea that the persistence of child labour is strictly linked to the presence of inequality within the country. For this reason we presented a model where the population is divided in two groups endowed with two different level of human capital. We study how this initial heterogeneity affect the distribution of income in the long run. The crucial result of this analysis is that the increase in the return of human capital is not sufficient to induce a transition to a high-skilled economy. The presence of two groups, with different levels of initial human capital, generates a continuous increase in the income of the high skilled workers with respect to those endowed with a low level of human capital. The presence of endogenous fertility induces low income group to make an higher number of children. Thus, child labour will increase. The substitutability between adult and child labour increases the resilience of this result: the economy is trapped in an equilibrium with a high fraction of the population with low income and low human capital. In other words, we find a vicious cycles between child labour and inequality.

This framework can be easily extended to evaluate the issues currently discussed in the literature. For instance, further research is needed to analyze the role of technical progress and international labour standards. With respect to the first issue, preliminary results seem to reject the hypothesis that technical progress can, by itself, induce the low-skilled group to invest in children's education. Another interesting application of the model is to evaluate public policies that through taxation on high-skilled individual may subsidies the low-skilled labour inducing them to invest in education. This policy may generate together a reduction of inequality and the disappearance of child labour.

A Appendix

In this section we show that r_t^s is always an increasing function of x_t. In order to simplify the notation we denote: $A = 2\theta\alpha\mu$, $B = \theta(1 - \alpha\mu)$, $C = z\alpha\mu$, $D = 4\theta(1 - \alpha)z\alpha\mu$. Given this simplification we can rewrite equation 28 as follows

$$r_t^{s*} = \begin{cases} \frac{Ax_t}{B+Cx_t-\sqrt{(Cx_t-B)^2+Dx_t}} & \text{if } x_1 \leq x_t \leq x_2 \\ \frac{A(1+b)x_t}{B(1+b)+Cx_t-\sqrt{[Cx_t-B(1+b)]^2+D(1-\beta)x_t}} & \text{if } x_2 \leq x_t \leq x_3 \\ \frac{\mu x_t}{1-\mu} & \text{if } x_3 \leq x_t \leq x_4 \end{cases} \tag{28}$$

Thus the derivative of the first line is positive if

$$2B\sqrt{(Cx_t - B)^2 + Dx_t} > 2B(Cx_t - B) - Dx_t, \tag{29}$$

where squaring both sides we get that it always holds

$$\alpha(1 - \mu) > 0 \tag{30}$$

The derivative of the second line is very similar. It is positve if

$$2B(1+b)\sqrt{(Cx_t - B(1 + b))^2 + D(1 - \beta)x_t} > 2B(1+b)(Cx_t-B(1+b))-D(1-\beta)x_t. \tag{31}$$

Squaring both sides we get that it always holds

$$b(1 - \alpha\mu) + \alpha(1 - \mu) + \beta(1 - \alpha) > 0 \tag{32}$$

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