

# Higher education and the allocation of graduate jobs

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## Abstract

In an economy where graduate jobs are allocated by tournament, and some of the potential participants cannot borrow against their expected future earnings, the government can increase ex ante equity and efficiency by redistributing wealth or, if that is too costly, borrowing wholesale and lending to potential participants. Both policies replace some of the less able rich with some of the more able poor.

*Key words:* higher education, matching, tournaments, credit.

*JEL:* C72, C78, D82, H42, I22, J24

## 1 Introduction

The present paper examines the effects of policy intervention, in particular in the form of student loans, in a situation where graduate jobs are allocated by a matching tournament.<sup>1</sup> The properties of this kind of contest are studied by Hoppe et al. (2009) in a context where the number of jobs and the number of workers are finite, and by Hopkins (2012) in a context where there is a continuum of both. Jobs and workers are differentiated by a quality parameter. Additionally, workers may differentiate themselves by investing in higher education. This investment may or may not enhance the graduate's working capacity. The quality of each job and the educational level of each worker are common knowledge, but the quality of each worker is private information. Workers are ranked according to their educational level, and matched with graduate jobs in such a way that the candidate with the highest educational level gets the highest quality job, the candidate with the second-highest educational level gets the second-highest quality job, and so on ("assortative matching"). In a separating equilibrium, strategies and payoffs depend on all the characteristics of both workers and jobs. If wages are flexible, all workers overinvest in education. If

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<sup>1</sup>For evidence of that see, among others, Bratti et al. (2004) and Castagnetti and Rosti (2009).

they are sticky, high-ability workers overinvest, but low-ability ones underinvest. Fernandez and Gali (1999) show that, in the presence of credit rationing, tournaments dominate conventional markets in terms of matching efficiency.

We consider a richer environment where workers are differentiated not only by quality, but also by wealth, and distinguish between jobs that can only be carried out by university graduates ("graduate jobs") from jobs for which such an education is not required ("non-graduate jobs"). The former are assigned by a matching tournament. The latter are assigned by a conventional market. In the absence of policy, if credit is rationed, workers with insufficient wealth endowments will be excluded from graduate jobs. That is undesirable not only on equity, but also on efficiency grounds. Assuming that (inherited) wealth is uncorrelated with talent, some graduate jobs will in fact be occupied by rich but low-quality workers, while some non-graduate jobs will be carried out by poor but high-quality workers. We show that the government can increase ex ante equity and efficiency by redistributing wealth. Alternatively, if detecting and redistributing wealth is too costly, the government can still increase ex ante equity and efficiency by borrowing wholesale and offering student loans. Both policies have the effect of replacing some of the less able rich with some of the more able poor in the performance of graduate jobs. In the student loan case, the matching equilibrium has the interesting feature that, contrary to what we are used to see in tournament models, graduate jobs of the same quality are assigned to graduates with the same educational level, but different quality.

## 2 Framework

The agents are school leavers. There is a continuum of agents differentiated by wealth,  $y$ , and learning ability,  $z$ . Wealth takes only two values,  $y \in \{0, \bar{y}\}$  with  $\bar{y} > 0$ . Learning ability is distributed over "poor" ( $y = 0$ ) and "rich" ( $y = \bar{y}$ ) agents with the same distribution function  $G(z)$  and density function  $g(z)$ , such that  $g(z|0) = g(z|\bar{y}) \forall z \in [0, \bar{z}]$ . The Lebesgue measure of the rich is a proper fraction  $\alpha$  of that of the total agent population.<sup>2</sup> An agent can go into the labour market straight after leaving school, or after a period in higher education. There is also a continuum of graduate jobs differentiated by quality,  $s \in [0, \bar{s}]$ , with distribution function  $H(s)$ . We can think of  $s$  as an index of technological sophistication or entrepreneurial ability. The Lebesgue measure of graduate jobs is a fraction  $\beta \leq \alpha$  of that of rich agents. Therefore, not all agents (possibly not even all the rich ones) can get a graduate job. Those who do not will take a non-graduate job, and earn a fixed wage  $w_0$ .<sup>3</sup> As our focus is on the allocation of graduate jobs, we assume that there are enough graduate and non-graduate jobs to occupy all school leavers, but nothing of substance

<sup>2</sup>If the number of agents were finite, we would be saying that the number of rich agents may be different from the number of poor agents.

<sup>3</sup>It would be more realistic to assume that the non-graduate wage increases with  $z$  or with some other index of individual ability, but this would make no difference of substance to the results.

changes if we allow for unemployment.

Let  $x$  denote the educational level achieved by an agent who attended university. We can think of this as either a degree level (e.g., BA, MA, Ph.D.) or a degree mark. The output produced by a graduate with learning ability  $z$  and education  $x$ , employed in a job of quality  $s$ , is  $\pi(s, x, z)$ , with  $\pi_s > 0$ ,  $\pi_x > 0$ ,  $\pi_z > 0$ ,  $\pi_{xx} < 0$ ,  $\pi_{sx} = 0$ ,  $\pi_{zs} > 0$  and  $\pi_{zx} \geq 0$ . The fifth of these assumptions is required for integrability, the sixth and seventh for stability of the matching equilibrium. Assuming that at least a certain amount of  $x$ ,  $x_0 > 0$ , is required to carry out a graduate job, the function  $\pi(\cdot)$  is defined only for  $x \geq x_0$ . The cost of acquiring  $x$  units of education for an agent of ability  $z$  is  $c(x, z)$ , with  $c(0, \cdot) = 0$ ,  $c_x > 0$ ,  $c_z \leq 0$  and  $c_{xz} < 0$ . Therefore,  $z$  has a multiplicity of roles. First, it reduces the cost of  $x$ . Second, it increases the productivity of  $x$ . Third, it directly increases output. One way to justify the last two roles is to say that  $x$  correctly measures the work capacity of the newly appointed worker, but that the worker's future ability to learn from work experience and adapt to changing circumstances depends positively on  $z$ . If we use this justification, we must interpret  $\pi$  as the present value of an output stream.

The pay-off of buying  $x$  units of university education for an agent of ability  $z$  who will be employed in a job of quality  $s$  is

$$u(s, x, z) = y + w(s, x, z) - c(x, z), \quad (1)$$

where  $w(s, x, z)$  is the wage of a worker with educational level  $x$  and learning ability  $z$ , employed in a graduate job of quality  $s$ . The pay-off of not buying a university education is simply  $w_0$ .

### 3 First best

In first best,  $s$ ,  $x$  and  $z$  are common knowledge. The policy maker prescribes educational investments to agents, and assigns graduate jobs to graduates, so as to maximize the aggregate surplus

$$\int_z \int_s [\pi(s, x, z) - c(x, z)] ds dz.$$

Koopmans and Beckmann (1957) demonstrate that this maximization implies assortative matching in  $(s, z)$ , and that there will be a value of  $x$  for each value of  $z$  in the maximizing allocation. There is then be a threshold value of  $z$ ,  $\tilde{z}$ , defined by

$$G(\tilde{z}) = 1 - \beta, \quad (2)$$

such that all agents with  $z \geq \tilde{z}$  will attend university independently of their  $y$ . This subpopulation of agents is distributed with distribution function  $\frac{G(z) - (1 - \beta)}{\beta}$ , and density function  $\frac{g(z)}{\beta}$ . The first-best level of university education for a school leaver of ability  $z \geq \tilde{z}$  matched with a job of quality  $s$  is  $x_{FB}(s, z) = \arg \max \pi(s, x, z)$ —

$c(x, z)$ , and will thus satisfy

$$\pi_x(s, x, z) - c_x(x, z) = 0. \tag{3}$$

Clearly,  $x_{FB}(s, z) \geq x_0$ .

## 4 Laissez faire

In laissez faire, individual educational investments and the quality of the graduate jobs to be assigned are common knowledge, but the ability of each agent is private information. Following Hopkins (2012), we represent the equilibrium process as a two-stage game. At the first (non-cooperative) stage, the agents choose their own educational investments. Assuming, like Fernandez and Gali (1999), that these agents cannot borrow against their expected future wages,<sup>4</sup> the choice of  $x$  of an agent of ability  $z$  and wealth  $y$  maximizes  $u(s, x, z)$  subject to the liquidity constraint

$$y - c(x, z) \geq 0. \tag{4}$$

At the second (cooperative) stage, graduate jobs are allocated by a matching tournament, and the products of the resulting matches shared between the parties in such a way that the matching scheme will be stable. As  $x_0$  is positive, the poor cannot invest in higher education, and are thus excluded from the tournament. Without loss of generality, we assume that there are as many graduate jobs as there are rich agents ( $\alpha = \beta$ ). We further assume that  $x_0$  is not so high that some of the rich (those with a very low  $z$ ) will not participate in the tournament and thus leave some of the graduate jobs vacant. As all the rich take graduate jobs, the support of the ability distribution of graduate workers is wider than in first best where only agents with  $z \geq \tilde{z}$  do. As the poor are excluded from the matching tournament, the properties of the laissez-faire equilibrium are essentially the same as in Hopkins (2012). In this section, we will then limit ourselves to summarizing the results of that article, and simply point out the differences arising from the existence of a non-graduate wage. In subsequent sections, we will see how the set-up and the properties of the associated equilibrium are modified by government intervention.

In a separating laissez-faire equilibrium, all rich agents adopt a symmetric, differentiable and strictly increasing investment strategy  $x_{LF}(z)$ . Let  $F(x)$  be the distribution function of  $x$  induced by the distribution of  $z_i$ ,  $G(z_i)$ . The position  $F(x(z_i))$  of an agent of ability  $z_i \in [0, \bar{z}]$  buying  $x_{LF}(z_i)$  will then be equal to this agent's rank  $G(z_i)$  in the ability distribution. The only stable matching is the positive assortative one, where a worker of ability  $z_i$  is matched with a job of quality  $s_i \in [0, \bar{s}]$ , such that

$$G(z_i) = F(x_i) = \phi(G(z_i)) = H(s_i), \tag{5}$$

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<sup>4</sup>Stiglitz and Weiss (1981) show that, if the characteristics of would-be borrowers (in our case, the ability of would-be graduates) are private information, credit may be rationed in equilibrium.

where  $\phi: [0, 1] \rightarrow [0, 1]$  is the matching function. This defines the function

$$s(z) = H^{-1}(G(z)),$$

which associates a job of quality  $s$  with an agent of ability  $z$ . The first derivative of this function is

$$s'(z) = \frac{g(z)}{h(s(z))}.$$

The equilibrium pay-off of an agent of ability  $z$  is

$$u(s(z), x(z), z) = y + w(s(z), x(z), z) - c(x(z), z). \quad (6)$$

Graduate wages are bargained between employers and employees.<sup>5</sup> For the equilibrium to be stable, the product of the match between a job and a graduate of the same rank,  $\pi(s(z), x(z), z)$ , must be divided between employer and employee in such a way (i.e., the wage schedule must be such) that no match with partners of different rank would make either of them better-off. Having assumed  $\pi_{sx}(s, x, z) = 0$ , the wage schedule implied by this condition does not depend on the functional form of  $x_{LF}(\cdot)$ , and is thus the same as with complete information. The stability condition may then be written as

$$w(z + \varepsilon, s(z + \varepsilon), x) + \pi(s(z), x, z) - w(z, s(z), x) \geq \pi(s(z), x, z + \varepsilon). \quad (7)$$

Let  $\underline{w}$  denote the lowest graduate wage. Above  $\underline{w}$ , the wage schedule will be

$$w(z, s(z), x) = \int_0^z \pi_z(t, s(z), 0) dt + \int_0^x \pi_x(z, s(z), t) dt + \underline{w}, \quad (8)$$

where  $t$  is a running variable. At  $w = \underline{w}$ ,  $w_x = \pi_x$ . In contrast with Hopkins (2012), where all jobs are assigned by tournament,  $\underline{w}$  cannot be set arbitrarily, but must satisfy  $\underline{w} \geq w_0 + c(x_0, 0)$ . Competition among graduates will ensure that this constraint is satisfied as an equation.

In a separating equilibrium, it is unprofitable for an agent of ability  $z$  to choose the level of  $x$  appropriate for an agent of ability  $z' \neq z$ . Exploiting this (no-mimicking) condition and (8), Hopkins demonstrates that, for all participating agents other than those with  $z = 0$ ,

$$x'_{LF}(z) = \frac{\pi_z(z, s(z), x)}{c_x(x, z) - \pi_x(z, s(z), x)}. \quad (9)$$

This tells us that  $c_x(x, z)$  is greater than  $\pi_x(z, s(z), x)$  for all  $z > 0$ , and thus that all agents with ability higher than the minimum will invest more than would be efficient given the job allocation. The agents with  $z = 0$ , will choose  $x$  so that  $c_x(x, 0) = \pi_x(0, s(0), x)$ . Integrating (9) from the value of  $x$  chosen by a rich agent of ability  $z = 0$  gives us the laissez-faire equilibrium

<sup>5</sup>This is the transferable utility case. Hopkins considers also the non-transferable utility case where wages are sticky.

investment  $x_{LF}(z)$ . The equilibrium is inefficient for two reasons. First, because all graduate jobs other than the one of quality  $s(\bar{z})$  are occupied by graduates of lower ability than in first best. Second, because  $x$  is inefficiently high for all  $z > 0$ . The former derives from the fact that the agents excluded from the tournament are the poor rather than the less able. The latter derives from the fact that, as graduate workers are ranked according to their educational level, all rich agents other than the marginal ones (those who are indifferent between taking a graduate or a non-graduate job) try to improve their match by investing more.

## 5 Wealth redistribution and price subsidies

If  $y$  is observable, one way to improve the allocation is to redistribute wealth. If redistribution is carried far enough to leave every agent with the same wealth  $y_{WR}$ , and assuming that  $y_{WR}$  is large enough to make the liquidity constraint (4) slack for everyone, all agents with  $z \geq \tilde{z}$  will then buy enough  $x$  to participate in the matching tournament. The resulting equilibrium (WR) will be different from the laissez faire in that the participant with the lowest ability will now have  $z = \tilde{z} > 0$ . The support of the ability distribution of participating agents is then  $[\tilde{z}, \bar{z}]$  as in first best, and thus narrower than in laissez faire. Again as in first best, the distribution function is now  $\frac{G(z) - (1 - \beta)}{\beta}$ . The investment profile may be different, however, because  $x$  is not prescribed by the social planner. In the WR equilibrium, the job assigned to the agent of ability  $z$  is now

$$s_{WR}(z) = H^{-1} \left( \frac{G(z) - (1 - \beta)}{\beta} \right),$$

and the educational investment carried out by this agent satisfies

$$x'_{WR}(z) = \frac{\pi_z(z, s_{WR}(z), x)}{c_x(x, z) - \pi_x(z, s_{IR}(z), x)}. \quad (10)$$

Integrating (10) from the value of  $x$  chosen by the agent of ability  $z = \tilde{z}$ , we obtain the WR equilibrium investment function  $x_{WR}(z)$ . The boundary condition is thus given by the value of  $x$  that maximizes  $\pi(\tilde{z}, x, 0) - c(x, \tilde{z})$ .

Condition (10) implies that, for  $z \geq \tilde{z}$ , the WR equilibrium value of  $x$ ,  $x_{WR}(z)$ , is higher than the first-best value  $x_{FB}(s, z)$  and lower than the laissez-faire value  $x_{LF}(z)$ . In comparison with the first best, there is then overinvestment. The comparison with the laissez faire is not straightforward. On the one hand, the less able are excluded from the tournament. Some of these (those with  $y = 0$  before redistribution) would have been excluded also in laissez faire. The rest (those with  $y = \bar{y}$  before redistribution) would have participated in the tournament in laissez faire, but are excluded from it due to the redistribution. Therefore, the agents at the lower end of the ability distribution will invest the same or less than in laissez faire. On the other hand, however, those at the upper end of the ability distribution will invest less than in laissez faire

if the redistribution made them poorer, more if it made them richer. Therefore, aggregate educational investment may rise or fall. Given, however, that any overinvestment will occur at the upper end of the ability distribution, and given also that the agents excluded are those who would have been excluded also in first best, wealth redistribution raises efficiency. There is an equity gain too, because every agent of ability  $z$  will have the same pay-off,

$$u_{WR}(s_{IR}(z), x_{IR}(z), z) = y_{WR} + u_{WR}(s_{IR}(z), x_{WR}(z), z) - c(x_{WR}(z), z),$$

irrespective of  $y$ .

A variant of this policy is to reduce the cost of higher education through a price subsidy financed by a wealth tax. As this would not affect  $\tilde{z}$ , the matching scheme would be the same as under straight redistribution, but all participants would now invest more. Therefore, the efficiency gain would be lower than under straight redistribution.

## 6 Student loans

If  $y$  is imperfectly observable, redistribution may be impossible or very costly. Provided, however, that it can borrow wholesale against its future tax revenue, the government can still raise equity and efficiency by lending to individual agents at stage 1 of the game, and recovering the credit at stage 2. If the amount that an agent can borrow from the government is large enough to make (4) slack for all agents with  $z \geq \tilde{z}$ , all these agents will participate in the tournament by investing  $x_{WR}(z)$ , where  $x_{WR}(\cdot)$ . Where efficiency is concerned, the effect will be the same as that of perfect wealth redistribution. Not so where equity is concerned, however, because the rich will continue to have a higher pay-off than the poor for all  $z$ .

Alternatively, suppose that a student can borrow from the government only up to a certain amount  $b$  (e.g., because the government itself is rationed in the wholesale money market), and that this amount is lower than would be required to reproduce the allocation associated with perfect wealth redistribution. That would change the structure of the equilibrium because some of the rich and some of the poor would participate in the tournament, but the amount of  $x$  bought by a poor agent of ability  $z$  would not necessarily be the same as that bought by a rich agent of the same ability.

The liquidity constraint is now

$$y + b - c(x, z) \geq 0. \tag{11}$$

We show in Appendix A that there is only one equilibrium for every  $b \geq 0$ . Such an equilibrium will either allow at least the most able agent to participate in the tournament or coincide with the laissez faire. In the first case, (11) will not be binding for  $z = \bar{z}$ .<sup>6</sup> A poor of ability  $z = \bar{z}$  will then buy the same amount of  $x$  as a rich of the same ability, but a poor of ability  $z < \bar{z}$  will buy less  $x$  than a

<sup>6</sup>There cannot exist an equilibrium where some rich agents of ability  $z \leq \bar{z}$  buy more  $x$

rich of the same ability. Let  $\bar{x}$  denote the amount of education bought by a rich or poor agent of ability  $\bar{z}$ . For each  $x < \bar{x}$ , there will be two levels of  $z$ , a lower one if the agent is rich and a higher one if the agent is poor. As agents and jobs are matched on the basis of their observable characteristics ( $x$  for the former,  $s$  for the latter), jobs with the same  $s$  are assigned at random to agents with the same  $x$ . Once employed, however, the agent's ability will be deduced from the output of the match and the agent's educational investment. Therefore, agents with different  $z$  will receive different wages even though they have the same  $x$ , and are thus matched with jobs of the same quality  $s$ .

Graduate wages must satisfy two restrictions. The first is that the associated equilibrium must be stable. The second is that an agent of ability  $z$  and wealth  $y$  must have no interest in mimicking an agent of different ability  $z'$  and same wealth  $y$ . The stability conditions are analogous to (7), and the resulting wage schedules analogous to (8). As the same  $x$  will be chosen by rich agents of ability  $z$  and poor agents of ability  $z' > z$ , however, there will a wage schedule for the rich

$$w_R(z, s_R(z), x) = \int_0^z \pi_z(t, s_R(z), 0) dt + \int_0^x \pi_x(z, s_R(z), t) dt + \underline{w}$$

and a wage schedule for the poor,

$$w_P(z, s_P(z), x) = \int_0^z \pi_z(t, s_P(z), 0) dt + \int_0^x \pi_x(z, s_P(z), t) dt + \underline{w}.$$

The functions that allocate jobs to agents,  $s_R(\cdot)$  for the rich and  $s_P(\cdot)$  for the poor, will be derived from the equilibrium strategies of the two categories,  $x_R(z)$  and  $x_P(z)$ , and from the matching condition,

$$F(x) = \alpha F_R(x) + (1 - \alpha) F_P(x) = H(s), \quad (12)$$

where  $F_R(x)$  is the distribution of  $x$  induced by  $x_R(z)$ , and  $F_P(x)$  that induced by  $x_P(z)$ . There will then be a threshold level of  $x$ ,  $\hat{x}$ , such that<sup>7</sup>

$$F(\hat{x}) = \alpha F_R(\hat{x}) + (1 - \alpha) F_P(\hat{x}) = 1 - \beta. \quad (13)$$

We now bring in the no-mimicking conditions. As in *laissez faire*, it must be unprofitable for a rich agent of ability  $z$  to choose the level of  $x$  appropriate for a rich agent of ability  $z' \neq z$ . For all the participating rich other than the least able ones, the investment strategy  $x_R(z)$  must then satisfy

$$x'_R(z) = \frac{\pi_z(z, s_R(z), x)}{c_x(x, z) - \pi_x(z, s_R(z), x)}. \quad (14)$$

than the poor of ability  $z = \bar{z}$ , because there would then be a level of  $x$ ,  $x^m$ , such that the rich for whom it is optimal to buy at least  $x^m$  separate themselves from the rest. That, however, cannot be an equilibrium because the employer hiring a graduate with education level  $x^m$  would be better-off hiring a worker with  $x < x^m$  instead.

<sup>7</sup>If the number of agents were finite, we would be saying that the total number of participants (some rich, some poor) equals the number of graduate jobs.



Given  $\hat{x}$ , the ability  $\hat{z}_R$  of the least able rich participating in the tournament will satisfy<sup>8</sup>

$$c_x(x, z_R) = \pi_x(z_R, s(z_R), \hat{x}). \quad (15)$$

In the equilibrium with student loans, the amount invested by a rich of ability  $z > \hat{z}_R$  is then

$$\int_{\hat{z}_R}^{\bar{z}} x'_R(z) dz + \hat{x}. \quad (16)$$

As the poor choose  $x$  so that  $b = c(x, z)$ , there is no question of them choosing the  $x$  appropriate for a different  $z$ . Given  $\hat{x}$ , the ability  $\hat{z}_P$  of the least able poor participating in the tournament is determined by

$$c(\hat{x}, z) = b.$$

The investment strategy,  $x_P(z)$ , of the poor with ability higher than that minimum will satisfy

$$x'_P(z) = -\frac{c_z}{c_x}. \quad (17)$$

In the equilibrium with student loans, the amount invested by a poor of ability  $z > \hat{z}_P$  is then

$$x_P(z) = \int_{\hat{z}_P}^{\bar{z}} x'_P(z) dz + \hat{x}. \quad (18)$$

The equilibrium values of  $s_P(z)$  for each  $z > \hat{z}_P$  and  $s_R(z)$  for each  $z > \hat{z}_R$  are derived together with  $\hat{z}_R$ ,  $\hat{z}_P$  and  $\hat{x}$  by solving (12), (13), (15) and (16) – (18). Recalling that a poor of ability  $z < \bar{z}$  will buy less  $x$  than a rich of the same ability, we will have that  $\hat{z}_P > \hat{z}_R$ . Therefore, more rich than poor agents will attend university and enter a graduate occupation.

The equilibrium is illustrated in Figure 1. The steeper of the two continuous curves is the graph of the investment strategy of the poor, and the flatter one that of the rich, for a given value of  $b$ ,  $b^0$ . The investment strategies associated with a larger value of  $b$ ,  $b'$ , are represented by the dashed curves. As  $b$  rises from  $b^0$  to  $b'$ ,  $\bar{x}$  rises from  $\bar{x}^0$  to  $\bar{x}'$ ,  $\hat{x}$  rises from  $\hat{x}^0$  to  $\hat{x}'$ , and both investment curves become steeper. As  $\hat{z}_R$  will then increase from  $\hat{z}_R^0$  to  $\hat{z}_R'$ , and  $\hat{z}_P$  fall from  $\hat{z}_P^0$  to  $\hat{z}_P'$ , some of the less able rich who would have participated in the tournament for  $b = b^0$  will not do so for  $b = b'$ . Conversely, some of the poor who would not have participated in the tournament at the lower value of  $b$  will do so at the higher one. The new investment curve of the rich will cut the old one because the new marginal rich (the one of ability  $\hat{z}_R'$ ), who would have overinvested at the lower  $b$ , will now invest at the efficient level. Furthermore, the two investment curves will be closer to each other at the higher than at a lower value of  $b$  because, on the one hand, the credit rations of the poor are relaxed and, on the other, the rich face more competition. Therefore,  $\hat{z}_P - \hat{z}_R$

<sup>8</sup>In laissez faire,  $\hat{z}_R = 0$ .

will be smaller, and the more able agents will invest more, at the higher than at a lower value of  $b$ . The difference between the equilibrium pay-off of the rich and the equilibrium pay-off of the poor also will smaller for every  $z$ . For  $b$  sufficiently high, the two investment curves will coincide, and  $\hat{z}_P = \hat{z}_R = \tilde{z}$ . As already pointed out, the equilibrium allocation would then be the same as with perfect wealth redistribution, but the utility distribution would not be the same because the rich would remain better-off than the poor. On aggregate, we cannot then say whether overinvestment increases or decreases as  $b$  gets larger. As the poor will improve their matches while the rich will do the opposite, and given that some of the additional investment will come from poor agents who would otherwise have been excluded from the tournament to the advantage of less able rich, the overall allocation is likely to become more efficient as  $b$  gets larger.

## 7 Conclusion

We have shown that, in an economy where graduate jobs are allocated by a matching tournament, and some of the potential participants cannot acquire the required education not because they are not sufficiently able, but because they are poor and cannot borrow against their expected future earnings, the government can increase ex ante equity and efficiency by redistributing wealth. Alternatively, the government can increase equity and efficiency by borrowing wholesale and lending to potential participants. Both policies have the effect of replacing some of the less able rich with some of the more able poor. In the student loan case, the equilibrium has an interesting feature. Contrary to what we are used to see in tournament models, graduate jobs of the same quality are assigned to graduates with the same educational level, but different abilities. As poor agents cannot invest in their education more than the government is prepared to lend them, these agents will in fact enter the tournament with less education than rich participants of the same ability. Both policies increase equity, but perfect equity can be achieved only by perfect wealth redistribution. We have also shown that an education price subsidy financed by a wealth tax would raise efficiency less than a purely redistributive tax because it would encourage overinvestment. If it can, the government should then go for straight wealth redistribution.<sup>9</sup> If it cannot, but can borrow wholesale, it should go for student loans. Cigno and Luporini (2009) argue that, in the presence of uncertainty and moral hazard, student loans are dominated by a scholarship scheme financed by a graduate tax. In that article, graduate jobs are allocated by a conventional labour market, but the result would hold even if they were allocated by a matching tournament as in the present one.

Overinvestment occurs in our as in all models of the same type because the agent's native ability is not directly observable by the employer ex ante, and education is a signal of ability. In the presence of wealth inequalities and imperfect credit markets, however, education is a distorted signal. As emphasized in Hoff

<sup>9</sup>A similar conclusion is reached by Hoff and Lyon (1995) in a non-tournament setting.

and Lyon (1995), lenders have no way and no reason to distinguish between a borrower who is willing to offer collateral because his educational project has a high probability of success, and one who does so because the collateral staked is only a small fraction of his wealth. All means of directly ascertaining the native ability of a worker, from cognitive tests to the gathering of "soft information" as advocated by Gary-Bobo (2008), are thus beneficial not only because they reduce unproductive signalling, but also because they tend to redress the distortions induced by such signalling, and thus to raise allocative efficiency. As we have shown, however, the same effect can be achieved also by relaxing the borrowing constraints of poor high-ability students.

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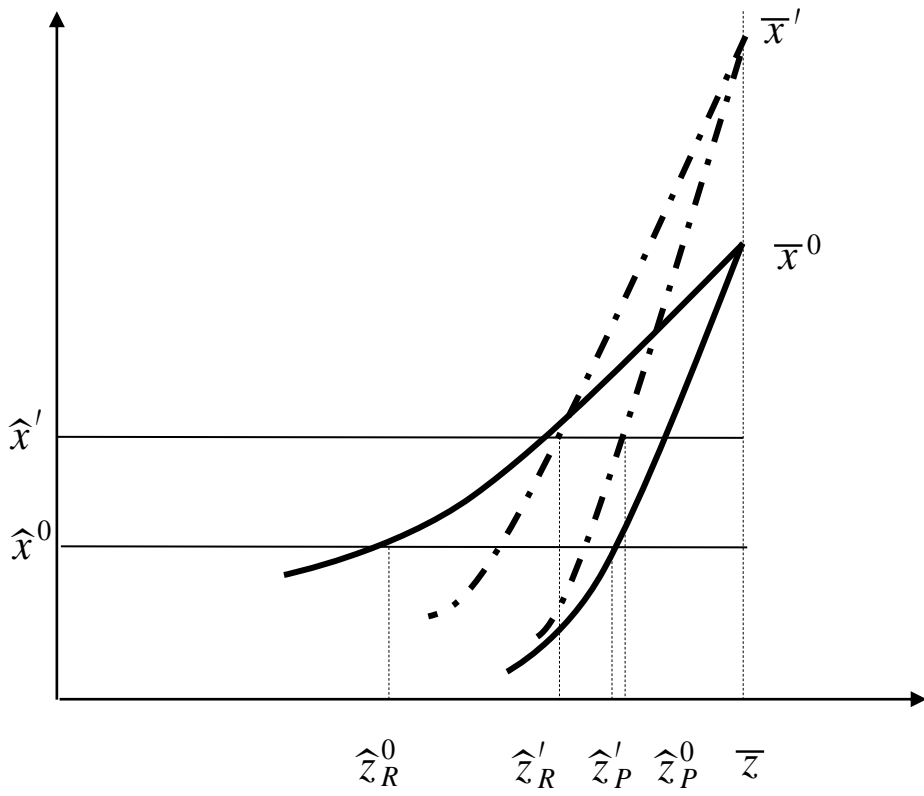


Figure 1