

Signalling, Social Status and Labor Income Taxes

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Abstract

We investigate the effects of introducing a linear labor income tax under the assumptions that individuals have concerns for social status, that they can signal their relative standing by spending on a conspicuous good, and that the tax revenue is redistributed by means of lump sum transfers. We show that the way social status is defined – i.e. how relative standing is computed and evaluated – crucially affects the desirability of the tax policy. More precisely, if status is ordinal then a labor income tax can decrease waste in conspicuous consumption only if the distribution of pre-tax incomes (or earning potentials) is not too unequal. The same applies for the tax to induce a Pareto improvement, but with the bound on pre-tax inequality being smaller. Instead, if status is cardinal then neither requirement applies: for any degree of pre-tax inequality we can find a cardinal notion of status such that the introduction of a labor income tax induces both a waste reduction and a strict Pareto improvement. However, under cardinal status a labor income tax is not necessarily more desirable than under ordinal status. Indeed, if status is cardinal in the sense that the status differential between being considered rich and being considered poor is strongly dependent on the income of the rich, then a labor income tax is more likely to increase social waste than under ordinal status.

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1 Introduction

When people care about their relative standing in society the labor market is likely to produce inefficient outcomes. Several contributions have investigated if and how taxing and redistributing labor income can alleviate such an inefficiency (see [Boskin and Sheshinski, 1978](#); [Layard, 1978](#); [Oswald, 1983](#); [Persson, 1995](#); [Ireland, 1994, 1998, 2001](#); [Corneo, 2002](#), and comments below). In this paper we show that, when people have concerns for their relative standing, the desirability of taxing and redistributing labor income depends on the shared notion of status, i.e. how people compute and evaluate their relative standing. More precisely, we analyze how the shared notion of status affects the desirability of a linear labor income tax when the revenue is redistributed by means of lump sum transfers.

In line with our objective, we assume that social status depends on relative labor income.¹ We also posit that agents can only observe the overall distribution of labor incomes and the amount of income spent on a conspicuous good. This naturally gives rise to a signalling game of conspicuous consumption where the amount of income earned plays the twofold role of generating social status and granting the purchasing power required for the signal. We stress that this feature of our model is an absolute novelty in the literature relating social status to signaling games where, typically, status is generated by an exogenously given resource (e.g. [Ireland, 1994, 1998](#); [Bilancini and Boncinelli, 2012](#)). In such a framework the choice of how much to work is evidently a strategic variable and individually optimal choices can possibly lead to inefficiencies. We show that the characteristics of such inefficiencies, and their possible cures, crucially depend on the shared notion of status.

This paper is related to a small but growing literature on optimal labor income tax under relative concerns. The first contribution to investigate the issue is the seminal book by [Duesenberry \(1949\)](#) where an entire chapter is devoted to proving that, if individuals care about the ratio between their consumption and a weighted average of others' consumption, then an income tax may be efficient. After a period of silence, [Boskin and Sheshinski \(1978\)](#) were the first to tackle the issue again. Assuming that people directly care about relative consumption, they find that welfare maximization requires higher linear taxes. This result has been later generalized by [Oswald \(1983\)](#) to non-linear tax rules.² Both studies rely on a welfare function to establish optimal tax schedules, hence taking into consideration

¹We abstract from other potentially relevant characteristics such as wealth or education, which would complicate the analysis while not being directly influential on the issue.

²Importantly, [Oswald \(1983\)](#) shows that the results of [Sadka \(1976\)](#) and [Seade \(1977\)](#) – that both the most and the least productive individual should not be taxed – are not robust to the introduction of relative concerns.

also equity issues. Such a welfarist approach has not been followed by [Persson \(1995\)](#) who has showed that, under assumptions similar to [Boskin and Sheshinski \(1978\)](#) and [Oswald \(1983\)](#), a linear income tax can induce a Pareto improvement. We too constrain the analysis to efficiency issues.

[Ireland \(1994, 1998\)](#) has been the first to follow the signalling approach which is similar to the one that we also pursue here. He has shown that, if people care about their rank in the distribution of income, an appropriate linear taxation policy can generate a Pareto improvement. In particular, if the range of pre-tax earning capabilities is not too large, then a Pareto improving income tax exists in which the poor gain from redistribution and the rich gain from a reduction in the expenditure required to signal their status.³ An important difference between our model and the one by [Ireland \(1998\)](#) is that in the latter status is assumed to depend on the gross earning potential (wages) while in our model status is assumed to depend on net earned income. If concerns for status are hardwired, then assuming social status to depend on the distribution of gross earning potentials does not seem unreasonable (see for instance [Rayo and Becker, 2007](#); [Samuelson, 2004](#), for a discussion on why Nature may want people to have status concerns). However, if one thinks of concerns for status as instrumental, i.e. arising because status provides the means for something else (e.g. [Cole et al., 1992, 1998](#)), then net earned income may seem more appropriate (see [Postlewaite, 1998](#), for a discussion of the advantages of the instrumentalist approach).⁴ Furthermore, our approach allows us to take into account the status-driven effects – possibly perverse – of redistributing the tax revenue through lump sum transfers. This could not be done properly in [Ireland \(1994, 1998\)](#) since, in equilibrium, social status is determined by exogenous individual characteristics.

A framework similar to [Ireland \(1998\)](#) is applied in [Ireland \(2001\)](#) to study the desirability of tax progressivity in the case of quasi-linear preferences. It turns out that status concerns do not qualitatively affect standard results at the finite or asymptotic endpoints at the top of the type distribution and that, although they imply steeper optimal tax schedules, they

³In [Ireland \(1994\)](#) it is also shown that universal benefits in cash or in kind can mitigate the waste due to signalling – although things are made more complex by means-testing because of its informational value.

⁴Consider, for instance, the case where status concerns are driven by concerns for the quality of social interactions (as in [Bagwell and Bernheim, 1996](#)). Owing to the instrumental approach it must be that the quality of interactions depends positively on status because people get more benefits by interacting with high status people. If we restrict to labor income as the source of such benefits then it seems reasonable to assume that benefits depend on consumption externalities. Hence, net earned income seems a better candidate than gross earning potential as the status-bearing asset – a person with a large potential that earns nothing cannot provide benefits to peers in terms of consumption.

do not imply a more convex tax function. This result is somewhat challenged, although not explicitly so, by [Corneo \(2002\)](#) who shows that, in a framework where preferences are not restricted to be quasi-linear, if income is observable then not only a progressive income tax can be Pareto improving but, in order to obtain undistorted choices of working hours, it is required a progressive tax whose degree of progressivity decreases in pre-tax income inequality. There is however an important difference between [Ireland \(2001\)](#) and [Corneo \(2002\)](#): while the former works with an unrestricted Mirrleesian income tax schedule, the latter imposes some restriction on the functional form of the income tax schedule.

Our contribution is twofold. In the first place, we analyze in detail the consequences of a labor income tax under ordinal status – i.e. when people care only about their rank in the distribution of labor incomes. Consistently with both [Ireland \(1998\)](#) and [Corneo \(2002\)](#), we show that much depends on the pre-tax wage distribution. In addition, we characterize the relevant threshold values of the pre-tax wage distribution. We find that while low income (low wage) people are always made better off by the introduction of a labor income tax, the implications for high income (high wage) people and social waste in conspicuous consumption depend on the degree of inequality in the wage distribution. If the wage distribution is highly unequal then waste is increased and high income people are made worse off. If the wage distribution is quite unequal then waste is decreased but high income people are still made worse off. Finally, if the wage distribution is only mildly unequal then waste in conspicuous consumption is decreased and high income people are made better off. These findings suggest that, under ordinal status, labor income taxes and wage inequality can be a kind of substitutes in mitigating the inefficiencies produced by status-seeking behavior.

In the second place, we analyze the consequences of a labor income tax when status is not ordinal but cardinal – i.e. when people also care about how far other people are in the distribution. We provide two novel findings in this regard. First, we show that under cardinal status a redistribution can be Pareto improving even if earning potentials are extremely unequal. In particular, we identify a necessary and sufficient condition for a marginal labor income tax to be both waste reducing and Pareto improving. Second, we prove that even in the presence of small differentials in pre-tax wage rates – a case which leads to a reduction in waste under ordinal status – the amount of waste in signalling may increase. Moreover, since a greater signalling induces high income individuals to earn more, this outcome can potentially make low income individuals worse off – as they may fall behind even further – notwithstanding the fact that they command a greater income. Our results suggest that, under cardinal status, labor income taxes and wage inequality need not be substitutes – actually, they might be complements – in mitigating the inefficiencies of

status-seeking behavior.

The intuition for the results under ordinal status is the following. A marginal increase in the labor income tax makes low income people better off because provides them with extra income that more than compensates the loss in terms of reduced net wage rate. This makes conspicuous consumption less costly for the poor because of decreasing marginal utility of both leisure and inconspicuous consumption, with the result of increasing social competition for status ([Hopkins and Kornienko, 2004, 2009a](#)). However, the labor tax also makes conspicuous consumption more costly in terms of the amount of work needed to afford it, contrasting the previous effect. Which of the two effects is greater determines whether waste increases or decreases. A greater inequality in the wage distribution strengthens the first effect – because of larger transfers – so making a waste reduction less likely. Furthermore, since a labor income tax necessarily decreases the earning potential of high income people, we have that a large enough waste reduction is required for high income people to be made better off.

Under cardinal status inequality has the additional effect of modifying the value of status itself, hence affecting the incentive to engage in wasteful social competition ([Bilancini and Boncinelli, 2012; Hopkins and Kornienko, 2009b](#)). In particular, if the status prize of the social competition diminishes then the incentive to compete decreases. When this latter effect dominates the sum of the effects described above, then a linear labor income tax is Pareto improving even if the pre-tax wage distribution (or earning potential) is extremely unequal. This explains our first result. The logic behind our second result is less evident. A labor income tax whose revenue is evenly redistributed has two direct effects: first, the income of the rich and the income of the poor become more similar and, second, it is easier for the poor to mimic the rich. As described above, under cardinal status the first effect is likely to reduce the status prize for the social competition and, therefore, to reduce the waste in signalling. The second effect, which is present under both cardinal and ordinal status, increases social competition and hence increases the waste in signalling. In addition to these, there is an indirect effect which arises because the object of signalling, i.e. own labor income, is a choice variable whose optimal value is positively affected by the signal. Indeed, a greater signalling by the rich does not only imply that they have to work more in order to buy more conspicuous goods, it also implies that their income is farther away from the income of the poor – with the result that the social prize of being considered rich is greater. If the status differential between the prize of being considered rich and the prize of being considered poor strongly depends on the income of the rich then total waste in signalling may increase even in the case where the ordinal effect alone – i.e. the first effect – would have made it decrease.

Interestingly enough, the indirect effect also generates the possibility – which is absent in the case of ordinal status – that a rise in the tax rate induces high income people to earn a greater net income despite their lower net wage. This means that, when status is cardinal in the sense described above, a greater tax rate may induce the top income earners to increase their work time substantially.

The paper is organized as follows. In the next section we describe the baseline model, providing the technical results which are required for our analysis. In section 3 we state our main results for the cases of both ordinal and cardinal status. Section 4 provides our conclusions and final remarks. All proofs are reported in the Appendix.

2 The Model

Our model is an extension of the one developed in [Bilancini and Boncinelli \(2012\)](#), that in turn resembles the model in [Bagwell and Bernheim \(1996\)](#). The novelty here is that the status-bearing asset is labor income and, therefore, it is endogenously determined. This turns out to be a non-trivial modification of the model and allows us to investigate how the notion of status affects the optimality of policies regarding the taxation and redistribution of labor income.

We consider a population of agents consisting of two types – one with high labor productivity, the other with low labor productivity – and whose income entirely depends on labor earnings, obtained in a competitive labor market. Hereafter, the subscript h will be used to refer to the highly productive type while the subscript l will be used to refer to the lowly productive type. No assumption is made on the size of population, that can be either finite or infinite, however agents are assumed not to take into account the effect of their labor supply on the government’s budget. A fraction $\beta \neq 0$ of population is of l -type agents and a fraction $(1 - \beta) \neq 0$ is of h -type agents. Types differ in their productivity which is, respectively, w_h and w_l , with $w_h > w_l > 0$. The time endowment is $Z > 0$ and is the same for everyone. Individuals are identical under any other respect.

Time can be allocated to either working or leisure while income can be allocated to the consumption of either a conspicuous or an inconspicuous good. The price of the inconspicuous good is normalized to 1; the price of the conspicuous good is p . Leisure is indicated with z , inconspicuous consumption with c and conspicuous consumption with x . Furthermore, we posit that one’s productivity, leisure and inconspicuous consumption are all unobservable to other individuals while conspicuous consumption is observable.

Utility is assumed to be additive in three components measuring the individual benefits

accruing from, respectively, inconspicuous consumption, leisure and status:

$$U(c, z, s) = \ln(c) + a \ln(z) + s, \quad (1)$$

where $a > 0$ represents the relative importance of leisure with respect to inconspicuous consumption and social status. We make a couple of remarks on the utility function. First, note that the conspicuous good does not generate utility directly. As explained afterwards, it serves only as a signal for labor income, and hence as a means to gain status.⁵ Second, the utility from inconspicuous consumption and leisure are assumed to be logarithmic. This is done because it allows us to keep the analysis tractable and more transparent. More precisely, when utility is logarithmic, and in the absence of status-seeking effects, an income tax leads to income and substitution effects on leisure which offset each other; this makes computations easier and allows us to isolate the impact of status-seeking behavior.

The component s is assumed to depend on how individual income compares to the overall income distribution. Let ϕ be an income (cumulative) distribution on the support $[0, Zw_h]$ – the range of feasible incomes – and let y be an income in $[0, Zw_h]$. We write $s(\phi, y)$ for the status of an individual who is believed to possess income y when the overall distribution of incomes in the population is ϕ . If individual incomes were public information, then there would have been no gain by conspicuous consuming. However, the income of every individual is private information. So, in order to attain status, individuals engage in a signalling activity by consuming the conspicuous good x . More precisely, let $\mu(x)$ be the belief function that associates the observation of the conspicuous consumption x with a distribution ϕ of incomes and a particular income y for the sender of signal x . Status is then given by $s(\mu(x))$.⁶

In the spirit of [Bilancini and Boncinelli \(2008, 2012\)](#), we are interested in understanding how the model predictions change when different notions of status are employed. In particular, we focus on two classes of status functions which have received attention from the literature, namely ordinal status and cardinal status. When status is ordinal people have concerns only for their rank in the distribution of incomes. Therefore, $s(\phi', y') = s(\phi, y)$ if $\phi(y) = \phi'(y')$ and $\phi^-(y) = \phi'^-(y')$.⁷ When status is cardinal, instead, people are possibly

⁵In [Bilancini and Boncinelli \(2012\)](#) we show that allowing x to be intrinsically beneficial does not change the quality of results, although it makes the analysis substantially more complex. In the present framework things would get even more complicated due to the endogeneity of the status-bearing asset.

⁶Alternatively, we might let status depend on the distribution of income net of the expenditure in signalling. Such a possibility has been explored, in a different setup, in [Bilancini and Boncinelli \(2012\)](#). We note that in the current model such an assumption does not represent a conceptual difficulty since, differently from [Bilancini and Boncinelli \(2012\)](#), the value of status is already endogenous.

⁷In order to distinguish between individuals with not greater income and with strictly less income, we

interested in features of the income distribution different from rank. For instance, under cardinal status it is likely to have $s(\phi', y) < s(\phi, y)$ when ϕ' first-order stochastically dominates ϕ , even if the rank of an individual with income y is the same in ϕ' and ϕ .

Finally, a linear tax τ is levied on income and its revenue is equally distributed to all individuals by means of a lump sum transfer T . Incomes of l -type agents and h -type agents are denoted, respectively, by $y_l = (1 - \tau)w_l(Z - z_l) + T$ and $y_h = (1 - \tau)w_h(Z - z_h) + T$. The hypothesis of balanced budget implies that $T = \frac{\tau}{1-\tau}(\beta y_l + (1 - \beta)y_h)$.

The decision problem of the generic individual of type i , with $i = h, l$, can be described as

$$\max_{c, z, x} [\ln(c) + a \ln(z) + s(\mu(x))], \quad \text{s.t. } c + px \leq y_i. \quad (2)$$

Since the budget constraint must hold with equality, (2) can be restated as

$$\max_{z, x} [\ln(w_i(Z - z)(1 - \tau) - px + T) + a \ln(z) + s(\mu(x))]. \quad (3)$$

We derive the optimal leisure for given s and x , and we obtain that⁸

$$z = \frac{a}{1 + a} \left(\frac{T - px}{w_i(1 - \tau)} + Z \right). \quad (4)$$

Next step is to choose an appropriate equilibrium concept for the model. We focus on symmetric Nash equilibria in pure strategies with consistent beliefs: a vector $(z_l^*, x_l^*, z_h^*, x_h^*, \mu^*)$ is an equilibrium if and only if:

1. (z_i^*, x_i^*) maximizes utility of type i given μ^* , $i = l, h$;
2. beliefs are consistent:

- (a) if $x_l^* \neq x_h^*$ then $\mu^*(x_l^*) = (y_l^*, \phi^*)$ and $\mu^*(x_h^*) = (y_h^*, \phi^*)$,
- (b) if $x_l^* = x_h^*$ then $\mu^*(x_l^*) = \mu^*(x_h^*) = (\beta y_l^* + (1 - \beta)y_h^*, \phi^*)$;

where $y_l^* = (1 + \tau)w_l(Z - z_l^*) + T$, $y_h^* = (1 - \tau)w_h(Z - z_h^*) + T$, and ϕ^* is the distribution where a fraction β of population earns y_l^* and a fraction $(1 - \beta)$ of population earns y_h^* . To

have used $\phi(y)$ and $\phi^-(y)$ respectively, with $\phi^-(y) = \lim_{\hat{y} \rightarrow y^-} \phi(\hat{y})$.

⁸The logarithmic shape of the utility function rules out corner solutions.

allow better readability of formulas, we set $L = s(y_l^*, \phi^*)$ and $H = s(y_h^*, \phi^*)$. Given ϕ^* , being considered to earn y_h^* is assumed to provide a higher status than being considered to earn y_l^* , namely $H > L$.

The above definition of equilibrium imposes only weak restrictions on out-of-equilibrium beliefs. In particular, beliefs are only required to be such that a deviation is not convenient for both l-type and h-type individuals. This great freedom in the choice of beliefs off the equilibrium path determines the existence of many pooling and separating equilibria, as in a standard signalling game. In order to get rid of this large multiplicity, and to have a unique prediction to use in comparative statics exercises, we adapt to the current setup the so-called Riley equilibrium, which is widely accepted as prominent equilibrium concept in signalling theory (see [Riley, 2001](#)). In particular, in the spirit of the Riley equilibrium, we look for a separating equilibrium where the lower income group spend nothing on signalling and the higher income group spend on signalling the minimum amount which makes a deviation not convenient for the lower income group. Unlike standard signalling models, income is not exogenously fixed, and both h -type individuals and l -type individuals can in principle end up with higher income. Proposition 1 shows that indeed a unique equilibrium exists, where the lower income group is composed of l -type individuals while the higher income group is composed of h -type individuals. Furthermore, equilibrium values of conspicuous consumption and leisure are derived for l -types and h -types, as well as the equilibrium lump sum transfer under balanced budget.

PROPOSITION 1. *There exists a unique equilibrium where l -types and h -types separate, the lower income group spend nothing on signalling, and the higher income group spend on signalling the minimum amount which makes a deviation not convenient for the lower income group. In such equilibrium:*

$$y_h^* > y_l^* , \tag{5}$$

$$x_l^* = 0 , \tag{6}$$

$$x_h^* = \left(1 - e^{\frac{L-H}{1+a}}\right) \frac{[w_l(1-\tau)Z + T]}{p} , \tag{7}$$

$$z_l^* = \frac{a}{1+a} \left(\frac{T}{w_l(1-\tau)} + Z \right) , \tag{8}$$

$$z_h^* = \frac{a}{1+a} \left[\frac{T e^{\frac{L-H}{1+a}} + \left(e^{\frac{L-H}{1+a}} - 1 \right) (1-\tau) Z w_l}{w_h (1-\tau)} + Z \right], \quad (9)$$

$$T = \frac{\tau(1-\tau)Z \left[\left((1-\beta)a \left(1 - e^{\frac{L-H}{1+a}} \right) + \beta \right) w_l + (1-\beta)w_h \right]}{(1+a)(1-\tau) + \tau a \left(\beta + (1-\beta)e^{\frac{L-H}{1+a}} \right)}. \quad (10)$$

For the sake of notation simplicity, from now on we will write x^* instead of x_h^* .

3 Income Taxation

3.1 Ordinal Status

We begin our analysis by considering the case where status is ordinal, that is, H and L are independent of the equilibrium income distribution. Differentiating (7) with respect to τ we get

$$\frac{dx^*}{d\tau} = \left(1 - e^{\frac{L-H}{1+a}} \right) \left(\frac{dT}{d\tau} - w_l Z \right) \frac{1}{p}. \quad (11)$$

From (10) and (11) we obtain the following preliminary results.

RESULT 1. A greater income tax reduces the waste in conspicuous consumption if and only if $dT/d\tau < w_l Z$.

RESULT 2. If $dT/d\tau < w_l Z$ at $\tau = 0$, then $dT/d\tau < w_l Z$ for all $\tau \in [0, 1]$.

From result 1 we see that a greater income tax decreases total wasting in conspicuous consumption if and only if the earning potential of l -types, $w_l Z$, is greater than the change in the transfer induced by the increase in τ . Intuitively, if l -types receive as a transfer more than what they can earn by having no leisure at all, then a greater income tax will necessarily make them richer with the consequence that h -types will have to spend more in conspicuous consumption in order to signal their status.

Moreover, from result 2 we see that if the introduction of a marginal labor income tax is waste reducing, then any further increase in the tax entails a further reduction in waste. The reason is that the marginal change in the amount of income transferred from h -types to l -types is bound to be smaller than its value at $\tau = 0$. This is because, under homothetic preferences, a flat labor income tax always decreases total income and, hence, a rising tax rate can only add a decreasing amount of income to the lump sum transfer.

For the rest of this section we will focus on the case $\tau = 0$. This will greatly simplify the analysis. Moreover, in the light of result 2, assuming $\tau = 0$ will allow us to take a conservative perspective on waste reduction. Under $\tau = 0$ the condition $dT/d\tau < w_l Z$ is satisfied if and only if

$$\frac{w_h}{w_l} < 1 + a \left(\frac{1}{1 - \beta} - \left(1 - e^{\frac{L-H}{1+a}} \right) \right) \equiv \sigma_x . \quad (12)$$

This shows that there is an upper bound to the degree of inequality in the distribution of wages, in line with the insights provided by Ireland (1998) and Corneo (2002). Note that for $a > 0$ the right hand side is larger than unity meaning that there always exists a range of w_h/w_l such that a waste reduction is possible.

Finally, we ask how the introduction of an income tax affects the equilibrium income of both l -types and h -types. The answer to this question is relevant in itself for obvious reasons. In addition, it will help to better understand the effects of the tax on individuals' utility.

RESULT 3. A greater income tax increases l -types' equilibrium income if and only if $dT/d\tau > w_l Z$. Moreover, a greater income tax always decreases h -types' equilibrium income.

From result 2 and 3 we see that an income tax decreases waste if and only if it decreases the equilibrium income of l -types. This is because a lower income makes l -types compete less fiercely for status – signalling becomes more costly for them – and, hence, it allows h -types to spend less in order to differentiate themselves from l -types. Then, from condition (12) we see that $w_h/w_l < \sigma_x$ implies that l -types' income decreases while $w_h/w_l > \sigma_x$ implies that l -types' income increases.

Result 3 also clarifies the impact of a greater tax rate on h -types' income. The intuition is the following. When l -types' income decreases, h -types find it convenient to decrease their income as well since they experience a lower net wage and they need less conspicuous consumption to differentiate themselves from l -types. When instead the income of l -types increases, then h -types spend more on conspicuous consumption but, because of the reduced net wage, they find it optimal to reduce their inconspicuous consumption even more. Consequently, a greater tax rate always makes the rich poorer. Interestingly enough, we will show in next section that this result only holds under ordinal status – i.e. under cardinal status h -types' income may increase as an effect of a rise in the tax rate.

We now turn our attention to individuals' utility. Differentiating utility functions at equilibrium with respect to τ we obtain

$$\left. \frac{dU_l}{d\tau} \right|_{\tau=0} = \frac{(1+a) \frac{dT}{d\tau}}{w_l Z} - 1, \quad (13)$$

$$\left. \frac{dU_h}{d\tau} \right|_{\tau=0} = \frac{e^{\frac{L-H}{1+a}} (1+a) \frac{dT}{d\tau} + \left(1 - e^{\frac{L-H}{1+a}}\right) w_l Z - w_h Z}{w_h Z - \left(1 - e^{\frac{L-H}{1+a}}\right) w_l Z}. \quad (14)$$

By imposing the positiveness of both (13) and (14) we get the following inequalities, respectively

$$\frac{w_h}{w_l} > 1 - a \left(1 - e^{\frac{L-H}{1+a}}\right) \equiv \sigma_l, \quad (15)$$

$$\frac{w_h}{w_l} < 1 + \frac{e^{\frac{L-H}{1+a}} (1-\beta) a \left(1 - e^{\frac{L-H}{1+a}}\right)}{1 - e^{\frac{L-H}{1+a}} (1-\beta)} \equiv \sigma_h. \quad (16)$$

Note that for $a > 0$ inequality (15) is always satisfied as the right hand side is strictly smaller than one. Moreover, for $a > 0$ the right hand side of (16) is strictly greater than one as the second term is positive. This implies that there is a range of wage distributions where a marginal increase of τ makes everyone strictly better off. By combining conditions (12), (15) and (16) we obtain the following:

PROPOSITION 2. *The introduction of a marginal income tax whose revenue is evenly distributed makes l -types better off. Moreover, it generates*

- i) a lower conspicuous consumption and a higher utility for h -types if $w_h/w_l < \sigma_h$;*
- ii) a lower conspicuous consumption and a lower utility for h -types if $\sigma_h < w_h/w_l < \sigma_x$;*
- iii) a greater conspicuous consumption and a lower utility for h -types if $w_h/w_l > \sigma_x$.*

The proof of the proposition can be found in the appendix – it substantially consists of demonstrating that $\sigma_h < \sigma_x$. Intuitively, if waste does not diminish then h -types cannot be better off as they suffer of both increased competition and lower potential income. Figure 1 shows the three relevant intervals of the wage distribution.

Further insights can be obtained by looking at how σ_x and σ_h vary in response to changes in the exogenous parameters of the model, i.e. a , β , $L - H$, p and Z . From (12), (15) and

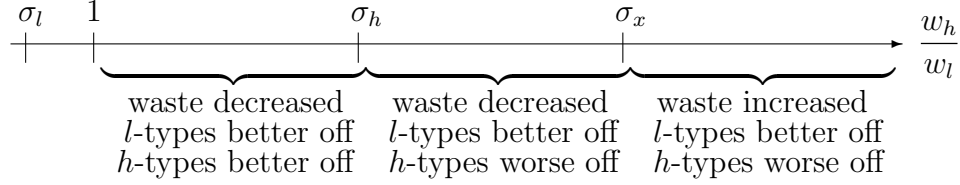


Figure 1: The effects of a marginal increase in τ as a function of w_h/w_l .

(16) we immediately see that p and Z play no role at all. The reasons are, respectively, that conspicuous consumption matters only for its market value – if p changes then x^* changes in such a way that the equilibrium values of px^* remains the same – and that types have identical endowments and homothetic preferences – changes in Z only have scale effects which leave σ_x and σ_h unaffected.

Instead, a smaller status differential $H - L$, i.e. a smaller net benefit of being considered rich instead of poor, induces a larger σ_x . This means that waste reduction is obtained for a larger range of wage distributions. The intuition here is that a smaller status differential implies that the rich have a smaller optimal expenditure in signaling which, in turn, implies that they have a smaller optimal labor income; hence, a marginal labor income tax transfers less money from the rich to the poor, making it less likely that the poor become rich enough to force the rich to spend more in signalling in order to differentiate from them.

Less obviously, the impact of a greater $H - L$ on σ_h is non-monotonic. More precisely, it is negative for $H - L < -\ln(1 - \sqrt{\beta})(1 + a)$ and positive for $H - L > -\ln(1 - \sqrt{\beta})(1 + a)$.⁹ This is because, besides the positive effect described for σ_x which is increasing in $H - L$, there is also a constant negative effect: a greater $H - L$ makes h -types work more and, hence, being taxed more. For small values of $H - L$ this latter effect dominates.

Finally, the impact of a greater preference for leisure a is positive on σ_x and ambiguous on σ_h . A greater a makes both l -types and h -types work less, and hence earn less. As a result a marginal tax transfers less money from the rich to the poor, making it less likely that the poor become rich enough to force the rich to spend more in signalling in order to differentiate from them. This explains why σ_x increases. A further effect of smaller earnings is that, depending on the relative change in incomes, the poor may find it relatively more or less attractive to engage in social competition for status through conspicuous consuming. If the poor find it more attractive then a marginal tax will make the rich save less on signaling

⁹The cutoff value can be obtained by differentiating σ_h with respect to $H - L$.

since the poor are now more costly to discourage. In this case, the direction of change in σ_h is ambiguous. If, instead, the poor find it less attractive to engage in social competition, then σ_h increases.

3.2 Cardinal Status

We now consider the case where status is cardinal, that is, both H and L depend on the equilibrium incomes y_l^* and y_h^* , which in turn implies that H and L depend on τ . Let L_{y_l} , L_{y_h} , H_{y_l} and H_{y_h} denote the derivatives of L and H with respect to y_l^* and y_h^* .¹⁰ Let us also assume, as it seems reasonable, that $L_{y_l} \geq 0$, $L_{y_h} \leq 0$, $H_{y_l} \leq 0$ and $H_{y_h} \geq 0$.

Our main point here is that, under cardinal status, the introduction of a labor income tax has an additional consequence which is otherwise absent under ordinal status: the prize of the social competition – i.e. the subjective value of status itself – may change. This can be seen by differentiating (7) with respect to τ and by opportunely rearranging terms (again, we conduct the analysis at $\tau = 0$):

$$\begin{aligned}
p \underbrace{\left(1 + e^{\frac{L-H}{1+a}} \frac{(L_{y_h} - H_{y_h})aw_l Z}{(1+a)^2} \right)}_{\text{Cardinal status indirect effect}} \frac{dx^*}{d\tau} &= \underbrace{\left(1 - e^{\frac{L-H}{1+a}} \right)}_{\text{Ordinal status effect, } <0 \Leftrightarrow \frac{w_h}{w_l} < \sigma_x} \left(\frac{dT}{d\tau} - w_l Z \right) + \\
- \underbrace{\frac{w_l Z e^{\frac{L-H}{1+a}}}{(1+a)^2} \left[(L_{y_l} - H_{y_l} + L_{y_h} - H_{y_h}) \frac{dT}{d\tau} - (L_{y_l} - H_{y_l})w_l Z - (L_{y_h} - H_{y_h})w_h Z \right]}_{\text{Cardinal status direct effect}} &. \tag{17}
\end{aligned}$$

By inspecting (17) we see that, besides the effect already seen in the case of ordinal status – represented by the first term of the right hand side – there are two additional effects. The first is direct and is represented by the second term of the right hand side. It accounts for the impact of τ on H and L through its net effect on the transfer and the net wage. Note that both sign and magnitude of the term in square brackets depend on $L_{y_l} - H_{y_l}$ and $L_{y_h} - H_{y_h}$, that is, on how the status differential is affected by the equilibrium incomes y_l^* and y_h^* . In principle, such a term could take any value because the definition of cardinal status is general enough to be consistent with a wide range of values for both $L_{y_l} - H_{y_l}$ and $L_{y_h} - H_{y_h}$. However, it seems reasonable to believe that in most cases the term will be positive – and, hence, that the cardinal direct effect will be negative. For instance, this is the case whenever we impose a bit of symmetry such as $L_{y_l} - H_{y_l} + L_{y_h} - H_{y_h} = 0$, i.e. an

¹⁰We implicitly assume that s is such that L and H are differentiable.

equal change in y_l^* and y_h^* leaves $H - L$ unaffected. Intuitively, while the transfer affects the status of l -types and h -types in the same way, the reduction in the income gap between h -types and l -types decreases $H - L$. Therefore, the status of being considered rich becomes relatively less desirable, thus decreasing the waste due to conspicuous consumption.

The second cardinal effect is represented by the coefficient of $dx^*/d\tau$ appearing on the left hand side of (17). It is indirect in the sense that it accounts for the change in H generated by the variation of y_h^* which, in turn, is generated by a change in x^* in the first place. The intuition is the following. Because of the increase in τ , the amount of conspicuous consumption which makes l -types indifferent between being considered rich and being considered poor also changes. This in turn affects the choice of h -types about how much to work and, hence, their income. Note that, however, it does not affect the equilibrium choice of l -types. Indeed, the coefficient contains the term $L_{y_h} - H_{y_h}$ but not the term $L_{y_l} - H_{y_l}$. As a consequence of the change in y_h^* , the status prizes H and L also change and this feedbacks on the amount of conspicuous consumption x^* which makes l -types indifferent between being considered rich and being considered poor.

We note that the coefficient of $pd x^*/d\tau$ could be interpreted as the reciprocal of a sort of waste multiplier. Since $L_{y_h} - H_{y_h} \leq 0$ such a coefficient is never greater than unity. Hence, if it is greater than zero, we have a reinforcing mechanism which magnifies the effects of an increase in τ , making labor taxation even more effective in reducing conspicuous consumption. In other words, a first reduction in x^* may trigger a mechanism which further reduces x^* . Thus, under cardinal status we can have two additional effects that make a labor income tax more likely to reduce the equilibrium waste.

However, the cardinal indirect effect may also work in the opposite direction if the difference $|L_{y_h} - H_{y_h}|$ is so large that the coefficient of $pd x^*/d\tau$ turns negative.¹¹ In such a case we have that the indirect effect triggered by the first change in x^* more than offsets the latter. For instance, we might have that an increase in τ has the direct effect of making the status prize less attractive and conspicuous consumption more costly but, because it makes the incomes of l -types and h -types more similar, it requires a greater conspicuous consumption for l -types to be indifferent between being considered rich and being considered poor; this, in turn, forces h -types to work more and hence increases both their income and the status prize of being considered rich; if the coefficient of $pd x^*/d\tau$ is negative then this latter effect

¹¹We abstract from the case where $(1 + a)^2 = -e^{\frac{L-H}{1+a}} (L_{y_h} - H_{y_h}) aw_l Z$ and therefore $dx^*/d\tau$ cannot be determined (the hypotheses of the Implicit Function Theorem are not met). Intuitively, a small variation of x^* is not sufficient to re-establish equilibrium conditions since it induces behaviors which in turn require a further and almost identical variation of x^* .

dominates resulting in an overall increase in x^* . This novel finding is a peculiar outcome of the interaction of two characteristics of our signalling model, namely the cardinality of status and the endogeneity of the status-bearing asset.^{12,13}

In conclusion, under cardinal status we have two additional effects of τ which can act to either increase or decrease the waste in conspicuous consumption. As a result, the range of wage distributions – i.e. values of w_h/w_l – for which waste decreases can differ both qualitatively and quantitatively from the range obtained under ordinal status. More precisely, there is a direct effect that, under reasonable assumptions, decreases waste and an indirect effect that operates in the same direction unless the status differential between being considered rich and being considered poor is too sensitive to the equilibrium income of h -types.

For the sake of concreteness, in next proposition we illustrate the link among the indirect cardinal effect, the change in the income gap, and the change in waste, for the special case where cardinal status depends on the income gap.¹⁴

PROPOSITION 3. *Let both L and H depend on the income gap $(y_h - y_l)$. Then, a marginal increase in τ generates a negative cardinal status indirect effect if and only if it increases the difference between equilibrium incomes of l -types and h -types, which in turn implies that the waste px^* increases, namely*

$$H_{y_h} - L_{y_h} > \frac{(1+a)^2}{aw_l Z e^{\frac{L-H}{1+a}}} \Leftrightarrow \frac{d(y_h - y_l)}{d\tau} > 0 \Rightarrow \frac{dx^*}{d\tau} > 0 \quad (18)$$

Proposition 3 tells us that two important things happen when status is cardinal in the sense that it depends on income differences. First, a greater sensitivity of the status prize to the income of the rich makes a greater tax rate more likely to increase waste. This means that, for a given value of w_h/w_l , we can have that a greater tax rate increases waste under cardinal status while it decreases waste under ordinal status and viceversa. Second, a greater tax rate can increase the equilibrium income gap only if waste increases. Therefore, a greater waste is a prerequisite for a greater tax rate to increase post-tax income inequality. In other words, in order for the income tax to be socially efficient it must not increase post-tax inequality

¹²In Bilancini and Boncinelli (2012) we do not observe such an effect because the status bearing-asset is exogenous.

¹³In the case of a negative cardinal status indirect effect, the feedback process might diverge. However, given that both labor supply and conspicuous consumption are bounded quantities and that both leisure and consumption are essential, divergence must be unless also utility from status diverges, which can be regarded as a rather exceptional case.

¹⁴This specification of concerns for status is rather common in the literature, see for instance Clark and Oswald (1998) and Cooper et al. (2001).

as measured by the income gap.¹⁵

One further aspect of cardinal status that is worth mentioning is that $dT/d\tau$ is not granted anymore to be decreasing in τ . In fact we have the following

RESULT 4. If $dT/d\tau < w_l Z$ at $\tau = 0$ and $d(H - L)/d\tau \leq 0$, then $dT/d\tau < w_l Z$ for all $\tau \in [0, 1]$.

The reason is that, if the impact of a greater τ on $H - L$ is positive, then people are induced to work more and, hence, the marginal transfer increases in τ .

In the following we will continue to focus on the case of $\tau = 0$.¹⁶ Besides providing a better analytical tractability, at $\tau = 0$ we can have a more neat comparison with the results obtained under ordinal status. However, from result 4, and more in general from the fact that cardinal effects may be large and of either sign, we see that assuming $\tau = 0$ is no longer a conservative perspective on waste reduction.

We now turn our attention to individuals' utility. Differentiating utility functions at equilibrium with respect to τ we get the counterparts of (13) and (14) under cardinal status

$$\left. \frac{dU_l}{d\tau} \right|_{\tau=0} = (1+a) \frac{dT}{d\tau} - w_l Z + \frac{dL}{d\tau}, \quad (19)$$

$$\left. \frac{dU_h}{d\tau} \right|_{\tau=0} = -1 + (1+a) \left[\frac{e^{\frac{L-H}{1+a}} \left(\frac{dT}{d\tau} - \frac{w_l Z}{(1+a)} \frac{d(H-L)}{d\tau} \right)}{w_h Z - \left(1 - e^{\frac{L-H}{1+a}} \right) w_l Z} \right] + \frac{dH}{d\tau}. \quad (20)$$

By manipulating (19) and (20) we get that utility increases when, respectively

$$\underbrace{w_h - w_l \left[1 - a \left(1 - e^{\frac{L-H}{1+a}} \right) \right]}_{\text{Ordinal status effect, } >0} + \underbrace{\left(L_{y_l} \frac{dy_l^*}{d\tau} + L_{y_h} \frac{dy_h^*}{d\tau} \right) \frac{1}{(1-\beta)Z}}_{\text{Cardinal status effect of } L} > 0, \quad (21)$$

¹⁵Proposition 3 holds for the case of ordinal status too. When status is ordinal we have that $H_{y_h} - L_{y_h} = 0$ implying that the income gap between h -types and l -types cannot increase. In the light of result 3 this means that under ordinal status the introduction of a marginal income tax decreases waste if and only if decreases the income of l -types not more than the income of h -types.

¹⁶Note that in $\tau = 0$ the marginal transfer $dT/d\tau$ is the same as under ordinal status.

$$\begin{aligned}
& \underbrace{w_l Z \left[\left((1 - \beta) a \left(1 - e^{\frac{L-H}{1+a}} \right) + \beta \right) + \left(1 - e^{\frac{L-H}{1+a}} \right) \right] - w_h Z \left(1 - e^{\frac{L-H}{1+a}} (1 - \beta) \right)}_{\text{Ordinal status effect, } >0 \Leftrightarrow \frac{w_h}{w_l} < \sigma_h} + \\
& + \underbrace{\left(L_{y_l} \frac{dy_l^*}{d\tau} + L_{y_h} \frac{dy_h^*}{d\tau} \right) w_l Z e^{\frac{L-H}{1+a}}}_{\text{Cardinal status effect of } L} + \underbrace{\left(H_{y_l} \frac{dy_l^*}{d\tau} + H_{y_h} \frac{dy_h^*}{d\tau} \right) (w_h - w_l) Z}_{\text{Cardinal status effect of } H} > 0. \quad (22)
\end{aligned}$$

From (21) we see that the impact of τ on y_l^* and y_h^* , and hence on L , can increase or decrease the threshold value of w_h/w_l for which l -types are made better off. In particular, differently from what seen for ordinal status, under cardinal status the introduction of a labor income tax may make l -types worse off: if L decreases enough to offset the positive ordinal status effect, then l -types' utility decreases. Notably, this may happen even if the equilibrium income of l -types increases. In fact, a higher tax rate may increase the expenditure in signalling by the h -types, and then induce them to work enough more to obtain a higher income, which reduces l -types' social status to an extent that may potentially lower their overall utility.

From (22) we see that also the threshold value of w_h/w_l for which h -types are made better off depends on how τ affects the equilibrium incomes and then the status prize. However, in this case both the variation in the status of poor and the variation in the status of rich matter, and the reason is that both H and L affect the equilibrium amount of conspicuous consumption x^* . In particular, we see that the new threshold is given by the sum of σ_h – which is got by imposing that the first term in (22) is greater than zero – and the net cardinal effects of L and H . The cardinal effect of L has the same sign of $L_{y_l}(dy_l^*/d\tau) + L_{y_h}(dy_h^*/d\tau)$ meaning that a rise in the status prize of being considered poor positively affects the utility of h -types. The reason is that a greater L makes l -types less inclined to compete for being considered rich and, therefore, it allows h -types to spend less on conspicuous consumption. On the contrary, a change in H has two effects which counteract each other. On the one side, an increase of H raises the equilibrium utility of H -types directly. On the other side, however, it increases the social prize of being considered rich and therefore, in equilibrium, it makes h -types spend more on wasteful conspicuous consumption in order to discourage l -types from emulation. As (22) reveals, the former effect always prevails, and the cardinal effect of H comes out to be of the same sign of $H_{y_l}(dy_l^*/d\tau) + H_{y_h}(dy_h^*/d\tau)$.

Together inequalities (21) and (22) give the necessary and sufficient conditions for the introduction of a marginal income tax to generate a Pareto improvement. The following proposition reports an important implication of such conditions.

PROPOSITION 4. *For any value of $w_h/w_l > 1$, there exist differentiable functions $H(y_h, y_l)$ and $L(y_h, y_l)$, with $L_{y_l} \geq 0$, $L_{y_h} \leq 0$, $H_{y_l} \leq 0$ and $H_{y_h} \geq 0$, such that the introduction of a marginal labor income tax whose revenue is evenly distributed induces both a reduction in waste and a strict Pareto improvement.*

The proof of proposition 4 is given in the Appendix. Here we just provide the intuition of the result. Fix w_h/w_l . If ordinal effects are already pushing towards a waste reduction and a Pareto improvement then it suffices to have the cardinal effect weak enough not to offset the ordinal effects. If, instead, ordinal effects push towards a waste increase and lower utility for h -types, then we can think of a cardinal definition of status such that the status of being considered rich, H , is not very much sensitive to the income of l -types and h -types while the status of being considered poor, L , is sensitive enough to induce a large change in L but not so much to have condition (18) satisfied. Under such a definition of status we have that taxing labor income and evenly redistributing the tax revenue makes l -types better off: l -types consume more inconspicuous goods, their status increases – as L increases – and they enjoy more leisure. Moreover, l -types find it less convenient to engage in social competition because the status prize, $H - L$, is now smaller. This decreases the amount of conspicuous consumption that h -types must use to separate themselves from l -types. Therefore, h -types can be made better off: h -types lose at most a little in terms of their status – because H does not change much – while they certainly increase both their inconspicuous consumption and their leisure due to the reduced competition for status – i.e. x^* decreases. This case is by no means exceptional. For instance, definitions of social status based on relative deprivation (Runciman, 1966) and upward-looking comparisons (Bowles and Park, 2005) do have similar characteristics.

4 Conclusions

In this paper we have investigated the impact of labor income taxes when agents can signal their relative standing by spending on a conspicuous good. We have assumed that tax revenue is redistributed by means of lump sum transfers and that status depends on the distribution of net incomes. Our main result is the characterization of how the desirability of a labor income tax depends on the definition of social status.

We contribute in two ways to the literature on optimal labor income taxation under status concerns. In the first place, consistently with the insights of Ireland (1998) and Corneo (2002), we have proved that under ordinal status the introduction of a labor income tax is desirable only if the pre-tax wage distribution is not too unequal. More precisely, we

have characterized the relevant threshold values of the pre-tax wage distribution showing that, while the low income people are always made better off, we have three different cases for what concerns waste in conspicuous consumption and satisfaction of high income people. If the pre-tax wage distribution is highly unequal then waste is increased and high income people are made worse off; if, instead, the pre-tax wage distribution is quite unequal then waste is decreased but high income people are still made worse off; finally, if the pre-tax wage distribution is only mildly unequal then waste is decreased and high income people are made better off.

In the second place, we have analyzed the effects of taxing and redistributing labor income under cardinal status. In particular, we have provided two novel findings. Firstly, we have shown that results obtained for ordinal status need not hold for cardinal status. Indeed, under cardinal status it is neither true that lowly productive individuals are always made better off by the introduction of a labor income tax, nor that a greater inequality in pre-tax wage rates makes waste reduction more likely. Most importantly, we have proved that under cardinal status a labor income tax can be Pareto improving even if pre-tax wage rates are extremely unequal. Secondly, we have shown that if status is cardinal in the sense that status depends on the income gap and the status differential between being considered rich and being considered poor is strongly dependent on the income of the rich, then a labor income tax is more likely to increase waste than it would be under ordinal status. This result is an outcome of a peculiar characteristic of our model: labor income plays the twofold role of generating social status and granting the purchasing power required for the signal. Thus, the introduction of a labor income tax might move the economy towards vicious equilibria sustained by the fact that a high conspicuous consumption requires a high income that in turn makes the status of being considered rich highly valuable (with respect to the status of being considered poor) and, hence, it makes conspicuous consumption worth its spending.

Our findings are relevant, we believe, for at least two reasons. The first is that they provide an argument in favor of the claim that, in models with status concerns, the applied definition of status should be carefully discussed and motivated ([Bilancini and Boncinelli, 2008](#)). In other words, the modeling of people's concerns for relative standing should be considered a major issue in the construction of status models. We emphasize this point because status models are quickly becoming a common fact in public economics, while we are still missing a serious investigation about what definitions of social status are more appropriate in the various cases of interest.

The second reason is more specific to the issue of the optimal tax policy. Especially in the light of our proposition 4, we can conclude that the degree of pre-tax wage inequality

does not imply much *per se* about the desirability of a labor income tax. In particular, under some definitions of status a greater wage inequality may ask for a greater taxation and redistribution whereas under some other definitions it may ask for exactly the opposite. Therefore, our contribution indicates that the question of what is the more desirable labor income tax under status concerns can only be answered by previously conducting an adequate (we think empirical) research on the way social status is computed and evaluated by people. From a logical standpoint, this is an obvious issue. However, despite its obviousness, it has not yet been given an appropriate assessment.

APPENDIX

A Proofs

A.1 Proof of Result 1

The result is immediately got from (11) by noticing that $e^{\frac{L-H}{1+a}} < 1$ for $L < H$.

A.2 Proof of Result 2

We take the first derivative of T with respect to τ and we obtain that

$$\frac{dT}{d\tau} = \frac{ZK [(1 - 2\tau)E - \tau(1 - \tau)E']}{E^2}, \quad (23)$$

where

$$K \equiv \left((1 - \beta)a \left(1 - e^{\frac{L-H}{1+a}} \right) + \beta \right) w_l + (1 - \beta)w_h, \quad (24)$$

$$E \equiv (1 + a)(1 - \tau) + \tau a \left(\beta + (1 - \beta)e^{\frac{L-H}{1+a}} \right), \quad (25)$$

$$E' \equiv \frac{dE}{d\tau}. \quad (26)$$

We take the second derivative of T with respect to τ and we obtain that

$$\frac{d^2T}{d\tau^2} = \frac{ZK}{E^3} [-1 + 4EE'\tau + \tau(1 - \tau)(E')^2]. \quad (27)$$

Note that a sufficient condition for (27) to be negative is $2\tau E \geq -\tau(1 - \tau)E'$, which is satisfied for every $\tau \in [0, 1]$.

A.3 Proof of Result 3

The equilibrium income of l -types is

$$y_l^* = w_l(Z - z_l^*)(1 - \tau) + T = \frac{Zw_l(1 - \tau) + T}{1 + a}. \quad (28)$$

Taking the derivative with respect to τ we get that $dy_l^*/d\tau > 0$ if and only if $dT/d\tau > Zw_l$. Moreover, the equilibrium income of h -types is

$$y_h^* = w_h(Z - z_h^*)(1 - \tau) + T = \frac{Zw_h(1 - \tau)}{1 + a} - \frac{a}{1 + a} \left[Te^{\frac{L-H}{1+a}} - \left(1 - e^{\frac{L-H}{1+a}}\right) (1 - \tau)Zw_l \right] + T. \quad (29)$$

Taking the derivative with respect to τ we get that $dy_h^*/d\tau > 0$ if and only if

$$w_hZ - \frac{dT}{d\tau} < \left(\frac{dT}{d\tau} - w_lZ \right) a \left(1 - e^{\frac{L-H}{1+a}} \right). \quad (30)$$

We note that for $\tau = 0$ the above inequality does not hold. Furthermore, since $d^2T/d\tau^2 < 0$ (as shown in proof of result 2), we conclude that inequality (30) never holds for $\tau \in [0, 1]$.

A.4 Proof of Result 4

We take the first derivative of T with respect to τ in the case of cardinal status, and we obtain that

$$\begin{aligned} \frac{dT}{d\tau} &= \frac{ZK [(1 - 2\tau)E - \tau(1 - \tau)E']}{E^2} + \\ &+ \frac{d(H - L)}{d\tau} \frac{(1 - \tau)\tau a Z e^{\frac{L-H}{1+a}}}{E^2} [(1 - \tau)w_l + \tau(1 - \beta)K]. \end{aligned} \quad (31)$$

From the proof of result 2 we know that the first term of the right hand side is decreasing in τ . Moreover, the second term of the right hand side is equal to 0 at $\tau = 0$. Therefore, if $d(H - L)/d\tau \leq 0$ and $dT/d\tau < w_lZ$ at $\tau = 0$, then $dT/d\tau < w_lZ$ for all $\tau \in [0, 1]$.

A.5 Proof of Proposition 1

Since types separate, $x_l \neq x_h$. We now prove that $y_h^* > y_l^*$. If $y_h^* = y_l^*$, then there would be no interest in signalling, and $x_l = 0 = x_h$, against the hypothesis of $x_l \neq x_h$. Suppose then that $y_l^* > y_h^*$. In equilibrium l -type individuals must find it not convenient to deviate from x_l to x_h , therefore

$$\begin{aligned} &\ln \left(\frac{w_l(1 - \tau)Z}{1 + a} - \frac{a}{1 + a} (T - px_l^*) - px_l^* + T \right) + a \ln \left(\frac{a}{1 + a} \left(\frac{T - px_l^*}{w_l(1 - \tau)} + Z \right) \right) + H \geq \\ &\geq \ln \left(\frac{w_l(1 - \tau)Z}{1 + a} - \frac{a}{1 + a} (T - px_h^*) - px_h^* + T \right) + a \ln \left(\frac{a}{1 + a} \left(\frac{T - px_h^*}{w_l(1 - \tau)} + Z \right) \right) + L. \end{aligned} \quad (32)$$

We will now prove that h -type individuals must strictly prefer choosing x_l^* than x_h^* , and so no equilibrium can exist for the case $y_l^* > y_h^*$. First note that if $x_l^* < x_h^*$ then it is immediate

to conclude that h -type individuals strictly prefer x_l^* to x_h^* . Hence, suppose $x_l^* > x_h^*$. We take the derivative with respect of w_l – evaluated at a generic w – of both the left hand side and right hand side of the above inequality, and we easily establish the following inequality:

$$\frac{(1-\tau)Z}{w(1-\tau)Z + T - apx_l} - \frac{a}{w + Zw(1-\tau)} > \frac{(1-\tau)Z}{w(1-\tau)Z + T - apx_h} - \frac{a}{w + Zw(1-\tau)},$$

which implies, together with (32), that h -type individuals strictly gain passing from x_h^* to x_l^* . Therefore, it must be that $y_h^* > y_l^*$.

By the above result and the assumption that the lower income group spend nothing on signalling, we get $x_l = 0$ and (6) is established. Consider the set of x_h^* such that there exist z_l^*, z_h^*, μ^* which, together with $x_l^* = 0$ and x_h^* , form a separating equilibrium. If the infimum of such a set did not belong to the set, then by a continuity argument we could not find any neighboring x_h^* belonging to the set, thus contradicting the definition of infimum. Then the infimum must belong to the set. Moreover, any infimum is unique. This completes the proof of the existence and uniqueness claim in the proposition.

We now illustrate how to derive the equilibrium values other than (6). At first we have to recognize that, in the unique equilibrium we are dealing with, l -type agents must be indifferent between their equilibrium choice and mimicking h -type agents by acquiring x_h . Clearly l -types cannot strictly prefer x_h , because in any Nash equilibrium they are optimizing at x_l . If they strictly preferred x_l , however, we could construct a belief function allowing h -types to save on signalling and still to separate from l -types. In such a case the requirement of minimum expenditure in signalling would be violated. Therefore, l -types must necessarily be indifferent between x_l and x_h . In the light of (4) and (6), the following condition must be satisfied:

$$\begin{aligned} & \ln\left(\frac{w_l(1-\tau)Z + T}{1+a}\right) + a \ln\left(\frac{a}{1+a}\left(\frac{T}{w_l(1-\tau)} + Z\right)\right) + L = \\ & = \ln\left(\frac{w_l(1-\tau)Z + T - px_h^*}{1+a}\right) + a \ln\left(\frac{a}{1+a}\left(\frac{T - px_h^*}{w_l(1-\tau)} + Z\right)\right) + H. \end{aligned} \tag{33}$$

Thanks to the log-specification, from (33) we can easily derive (7). Inserting (6) and (7) in (4) and exploiting equilibrium conditions, we obtain (8) and (9). Finally, we substitute (8) and (9) into the definition of balanced budget transfer T , and we obtain (10).

A.6 Proof of Proposition 2

We prove that $\sigma_x > \sigma_h$, and then the proposition follows from the inequalities (16), (15) and (12) established in the text. By using (16) and (12), the inequality $\sigma_x > \sigma_h$ can be written, after some simplifications, as

$$\left(\frac{1}{1-\beta} - \left(1 - e^{\frac{L-H}{1+a}} \right) \right) > \frac{e^{\frac{L-H}{1+a}} (1-\beta) \left(1 - e^{\frac{L-H}{1+a}} \right)}{1 - e^{\frac{L-H}{1+a}} (1-\beta)}.$$

Multiplying both sides by $1 - e^{\frac{L-H}{1+a}} (1-\beta)$, the above inequality becomes

$$\frac{1}{1-\beta} - e^{\frac{L-H}{1+a}} - \left(1 - e^{\frac{L-H}{1+a}} \right) + e^{\frac{L-H}{1+a}} (1-\beta) \left(1 - e^{\frac{L-H}{1+a}} \right) > e^{\frac{L-H}{1+a}} (1-\beta) \left(1 - e^{\frac{L-H}{1+a}} \right),$$

which simplifies to $1/(1-\beta) > 1$, that is always true for $\beta \in (0, 1)$.

A.7 Proof of Proposition 3

From (28) and (29) we get

$$y_h^* - y_l^* = \frac{Z(w_h - w_l)(1-\tau)}{1+a} + \frac{a}{1+a} \left[T \left(1 - e^{\frac{L-H}{1+a}} \right) - \left(1 - e^{\frac{L-H}{1+a}} \right) (1-\tau) Z w_l \right]. \quad (34)$$

Differentiating (34) with respect to τ at $\tau = 0$ we get

$$\frac{d(y_h^* - y_l^*)}{d\tau} = \frac{Z(w_h - w_l)}{1+a} + \frac{a}{1+a} \left[\frac{dT}{d\tau} \left(1 - e^{\frac{L-H}{1+a}} \right) - \left(1 - e^{\frac{L-H}{1+a}} - \frac{d \left(e^{\frac{L-H}{1+a}} \right)}{d\tau} \right) Z w_l \right]. \quad (35)$$

Since both L and H depend on $(y_h - y_l)$ we get that $L_{y_h} = -L_{y_l}$ and $H_{y_h} = -H_{y_l}$. This implies that

$$\frac{d \left(e^{\frac{L-H}{1+a}} \right)}{d\tau} = \frac{e^{\frac{L-H}{1+a}}}{(1+a)} (L_{y_h} - H_{y_l}) \frac{dy_h}{dy_l}. \quad (36)$$

Plugging (36) in (35) and assuming that the indirect cardinal effect is different from zero, we can solve for $d(y_h - y_l)/d\tau$ as follows:

$$\frac{d(y_h^* - y_l^*)}{d\tau} = \frac{\frac{Z(w_l - w_h)}{1+a} + \frac{a\left(1 - e^{\frac{L-H}{1+a}}\right)}{1+a} \left[\frac{dT}{d\tau} - Zw_l \right]}{(1+a)^2 - Zaw_l(L_{y_h} - H_{y_h})e^{\frac{L-H}{1+a}}}. \quad (37)$$

Considering the value of $dT/d\tau$ at $\tau = 0$ (see proof of result 2) it can be shown that the numerator of (37) is negative if and only if

$$w_h \left[\frac{a\left(1 - e^{\frac{L-H}{1+a}}\right)}{1+a} - 1 \right] + w_l \left[1 - a\left(1 - e^{\frac{L-H}{1+a}}\right) + a \frac{\left(1 - e^{\frac{L-H}{1+a}}\right)\left((1-\beta)a\left(1 - e^{\frac{L-H}{1+a}}\right) + \beta\right)}{1+a} \right] < 0. \quad (38)$$

The coefficient of w_h is negative while the coefficient of w_l might be either negative or positive. It follows that if (38) holds for $w_h = w_l$ then it holds for any $w_h > w_l$. Imposing $w_h = w_l$ we get that inequality (38) holds if and only if $(1-\beta)\left(1 - e^{\frac{L-H}{1+a}}\right) < 1$ which is always the case. Therefore, the numerator of (37) is negative. From this follows the equivalence result in (18).

The remaining part of the proposition can be proved by noting that

$$y_h^* - y_l^* = \frac{Zw_h(1-\tau) + T - apx^*}{1+a} - \frac{Zw_l(1-\tau) + T}{1+a}, \quad (39)$$

from which, differentiating with respect to τ at $\tau = 0$, we get

$$\frac{d(y_h^* - y_l^*)}{d\tau} = \frac{Z(w_l - w_h)}{1+a} - \frac{a}{1+a} p \frac{dx^*}{d\tau}. \quad (40)$$

Since the first term of (39) is negative, if expression (39) is positive then it must be that $dx^*/d\tau$ is positive.

A.8 Proof of Proposition 4

We want to show that, for any given value of $w_h/w_l > 1$, we can find an array of values for $H, L, H_{y_h}, H_{y_l}, L_{y_h}, L_{y_l}$ such that:

- (i) $H > L, H_{y_h} \geq 0, H_{y_l} \leq 0, L_{y_h} \leq 0, L_{y_l} \geq 0$;
- (ii) $\frac{dx^*}{d\tau} < 0$ at $\tau = 0$;
- (iii) $\frac{dU_l}{d\tau} > 0$ at $\tau = 0$;

$$(iv) \quad \frac{dU_h}{d\tau} > 0 \text{ at } \tau = 0.$$

Fix $w_h/w_l > 1$ and suppose that $H_{y_l} = L_{y_h} = 0$. Equation (17), (21) and (22) then become, respectively,

$$p \left(1 - e^{\frac{L-H}{1+a}} \frac{H_{y_h} a w_l Z}{(1+a)^2} \right) \frac{dx^*}{d\tau} = \left(1 - e^{\frac{L-H}{1+a}} \right) \left(\frac{dT}{d\tau} - w_l Z \right) - \frac{w_l Z e^{\frac{L-H}{1+a}}}{(1+a)^2} \left[L_{y_l} \left(\frac{dT}{d\tau} - w_l Z \right) - H_{y_h} \left(\frac{dT}{d\tau} - w_h Z \right) \right], \quad (41)$$

$$w_h - w_l \left[1 - a \left(1 - e^{\frac{L-H}{1+a}} \right) \right] + L_{y_l} \frac{dy_l^*}{d\tau} \frac{1}{(1-\beta)Z} > 0, \quad (42)$$

$$w_l Z \left[\left((1-\beta) a \left(1 - e^{\frac{L-H}{1+a}} \right) + \beta \right) + \left(1 - e^{\frac{L-H}{1+a}} \right) \right] - w_h Z \left(1 - e^{\frac{L-H}{1+a}} (1-\beta) \right) + L_{y_l} \frac{dy_l^*}{d\tau} w_l Z e^{\frac{L-H}{1+a}} + H_{y_h} \frac{dy_h^*}{d\tau} (w_h - w_l) Z > 0. \quad (43)$$

Consider then the case where $dT/d\tau < w_l Z$ – i.e. the ordinal effect on waste is negative – which implies that $dy_l/d\tau < 0$, that $dT/d\tau < w_h Z$, and that the first two terms of the right-hand side of (43) sum up to a positive amount. Then by setting $L_{y_l} = 0$ and $H_{y_h} < (1+a)^2 / \left(Z w_l e^{\frac{L-H}{1+a}} \right)$ we get that inequality (42) and (43) are satisfied and that $dx^*/d\tau < 0$. Note that this holds for any value of H and L such that $H > L$.

Consider now the case where $dT/d\tau > w_l Z$ – i.e. the ordinal effect on waste is positive – which implies that $dy_l/d\tau > 0$ and that the first two terms of the right-hand side of (43) sum up to a negative amount. Then by setting $H_{y_h} = 0$ we get that inequality (42) is satisfied while the negativity of $dx^*/d\tau$ and the positivity of the left-hand side of (43) are obtained, respectively, if and only if

$$L_{y_l} > \frac{\left(1 - e^{\frac{L-H}{1+a}} \right) (1+a)^2}{w_l Z e^{\frac{L-H}{1+a}}}, \quad (44)$$

$$L_{y_l} > \frac{(1+a) w_l \left[\left((1-\beta) a \left(1 - e^{\frac{L-H}{1+a}} \right) + \beta \right) + \left(1 - e^{\frac{L-H}{1+a}} \right) \right] - w_h \left(1 - e^{\frac{L-H}{1+a}} (1-\beta) \right)}{w_l Z e^{\frac{L-H}{1+a}} \left(w_h (1-\beta) - w_l \left(1 + a - (1-\beta) a \left(1 - e^{\frac{L-H}{1+a}} \right) - \beta \right) \right)}. \quad (45)$$

For given values of H and L the right-hand sides of (44) and (45) are finite numbers. Therefore, for such values, there exists $L_{y_l} > 0$ such that both (44) and (45) are satisfied.

The proof concludes by noting that for any given array of values for $H, L, H_{y_h}, H_{y_l}, L_{y_h}, L_{y_l}$ such that $H > L, H_{y_h} \geq 0, H_{y_l} \leq 0, L_{y_h} \leq 0, L_{y_l} \geq 0$, we can always find differentiable functions $H(y_h, y_l)$ and $L(y_h, y_l)$ that are consistent with such an array.

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