Price Competition and Innovation When Substitutes Become Complements*

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Abstract

In this paper we analyze the effects of the introduction of a cocktail composed of a fixed proportion of two existing stand-alone products in a Bertrand duopoly with imperfectly substitutable goods, with a special focus on pharmaceutical markets. First, we find that a cocktail rises the Bertrand equilibrium prices as it introduces a certain degree of complementarity. Moreover, it creates an incentive to sell the separate components at a discount or at premium depending on the degree of substitutability. While discounts are always detrimental to consumers, premiums might increase consumer surplus. Finally, we study how such effects are altered in the presence of new substitutes that are not part of a cocktail. We show that a cocktail reduces the opportunity of a single-product entrant to earn positive profits, but that an incumbent can profitably introduce an innovation whenever goods are close substitutes or if they are highly differentiated. Interestingly, a higher quality incumbent's innovation can be at the same time profitable and welfare reducing, so that its introduction might harm consumers. On the other hand, it is never the case that an innovation that increases consumer surplus is not profitable.

Keywords: Complements, Price Discounts and Premia, Price Discrimination, Anticommons, Entry, Innovation.

JEL Codes: C7, D42, D43, K21, L11, L12, L13, L40, M21

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1 Introduction

Typically, goods are perceived by all consumers either as substitutes or complements. For instance, purchases in a supermarket and gasoline from the gas station near the shopping mall are (imperfect) substitutes. Hardware and software and (perfect) complements. Market interactions with either substitute or complement goods have been analyzed thoroughly: competition and pricing policy, quality choices by firms, investment in innovation, and bundling strategies have all been studied under countless different assumptions. Similarly, there is also a growing literature on markets in which both substitute and complement goods are produced at the same time. Hardware and software are produced in oligopolies, in which many firms produce substitute pieces of hardware and substitute software programs.[1].There are goods, however, that are perceived as substitutes by some consumers and as complements by others. About these goods, very little has been said, although they are very common and somehow crucial for consumers.

A very prominent example can be found in pharmaceutical markets, where patients very often take combinations of different drugs to improve the efficacy of a particular treatment or to weaken collateral effects. For instance, half of the new cholesterol reducing treatments entering phase 3 of clinical trials in 2007 were actually “cocktails” of drugs that had already been approved as single products to treat the same symptoms (Blume-Kohout and Sood, 2008). Similarly, in 2008, more than one-third of US colorectal cancer patients under chemiotherapy were under “cocktail” regime, and nowadays most HIV +/AIDS patients are still cured with a combination of two or more drugs (Lucarelli, Nicholson and Song, 2010). In other terms, it is sometimes the case that in a pharmaceutical market with a given number of stand-alone (imperfectly) substitutable drugs, a new “rival” product becomes available (after the approval by the Food and Drug Administration, that is after the “cocktail” proves to be either superior in efficacy or more convenient or with less side effects of its separate components), composed of a precise combination of existing products that act here as complementary components of a therapy. Similarly, tourist attractions (such as the museums of a city) can be substitutes for some consumers but complements for other. In fact, they are often explicitly offered in a "cocktail" through a "tourist card" and such card can be offered at a discount.

Clearly, cocktails will have an effect on price competition, but the direction of such influence is not obvious *a priori.* Do cocktail soften competition or increase it? At

[1] See Alvisi et al. (2011) and Alvisi and Carbonara (2010, 2011).

this purpose, in the first part of the paper we study the effects of the introduction of "cocktails" on equilibrium prices, profits and social welfare in markets where some (or all) competing products become complements for (at least) a portion of the demand. More specifically, we verify how the presence of a new "bundle" composed of a fixed proportion of two existing stand-alone products influences the competition in a Bertrand duopoly with imperfectly substitutable goods and how the new cocktail affects the incentives of selling products at a discount (or at a premium, if feasible). We find that the introduction of a cocktail rises the Bertrand equilibrium prices as it introduces a certain degree of complementarity between two goods that were previously substitutes. Interestingly, when the degree of substitutability is particularly low, the introduction of a cocktail decreases both duopolists' profits, so that it is not obvious that firms are in favor of new therapies composed of combinations of existing drugs being approved by the FDA. Moreover, while in a multiproduct monopoly cocktails do not induce any form of price discrimination, in a duopolistic setting it creates an incentive to sell the separate components of a cocktail at a discount when the degree of substitutability between stand-alone products is low and at premium when such degree is high. While discounts happen to be always detrimental to consumers, premiums might increase consumer surplus as they render the competition on single products fiercer, so that from an Antitrust perspective prohibiting price discrimination in drug markets might not be desirable in welfare terms.[2]

Such results also represent the benchmark for the second part of the paper, pointing directly to the question of entry in the market. In particular, would entry of a new product be facilitated by the presence of a cocktail? Our answer is that a cocktail reduces the opportunity of a single-product entrant to earn positive profits and at the same time enlarges the parameters' range such that the two incumbents find it profitable to offer a discount for the components of the cocktail. Conclusions are different, however, if the new product is supplied by an existing firm already participating to a cocktail. The new product will be a substitute of both the existing products and of the cocktail but will not participate to the cocktail, so that it can be interpreted as an "innovative",

[2] While two independent firms might not find feasible to directly coordinate on a premium when their goods are combined in a cocktail by consumers, they could do that indirectly adopting strategic packaging as a form of price discrimination. In fact, if good 1(2) is not sold in single units but in packages of $d_1(d_2)$ units only, and if the ratio d_1/d_2 is different from the required proportion of the two goods in the cocktail, then the effective price of one unit of the cocktail could be higher than the ones for one unit of the two separate products.

possibly more effective, product. In the early 2000s, for example, Abbott launched Kaletra, a drug for treating HIV/AIDS. At the time Abbott was already selling Norvir, which was used in a cocktail regimen to help boost the performance of its competitor's drug. We investigate in which cases an existing firm has the incentive to become a multiproduct duopolist and introduce the new product (which we call "innovation") and how is price competition affected by such move.We find that such duopolist will raise the price of its old product in order to divert part of the demand to the new product. Consistently with our findings, shortly after the launch of Kaletra, Abbott increased Norvir's price four-fold while pricing Kaletra more competitively, presumably to drive customers from the cocktail regimen to its new stand-alone regimen. Specifically, in opur model the new product will be profitable only if all products are close substitutes or if they are highly differentiated in their quality level. Interestingly, it exists a parameters' region such that the introduction of the new good is at the same time profitable and welfare reducing, so that the presence of new therapies in the market might also harm consumers. This may happen even if the new product improves on the existing ones, i.e., it is of higher quality. On the other hand, it is never the case that an innovation that increases consumer surplus is not profitable. In other words, all innovations that improve consumer welfare are always introduced. Finally, we find that the introduction of the new stand-alone product reduces the price of the cocktail because the increase in the price of the old component is accompanied by a more than proportional decrease in the price of the rival's product. Proofs of Propositions and Lemmas can be found in the Appendix.

1.1 Related literature

Firms often bundle or tie their own products for various reasons: to price discriminate (McAfee et al., 1989), to leverage monopoly power in one market by foreclosing sales and discouraging entry in another market (Whinston, 1990, Chen, 1997, Nalebuff, 2004), but little is known from a theoretical standpoint about price changes when a firm's product is bundled with those of its rival and about the welfare effects of such practice.

More is known about the incentives for rivals to provide a joint discount when consumers purchase from all firms participating to such agreement. Gans and King (2006) study the optimal discounting strategy of two rivals bundling two independent products as a way to relax price competition against a set of stand-alone competitors. Specifically, Gans and King (2006) prove that such firms can profit from offering a bundled discount to the detriment of other firms and of consumers, whose preferences are farther removed from the bundled brands. Indeed, when both pairs of firms negotiate bundling arrangements, there are no beneficiaries and consumers simply find themselves consuming a sub-optimal brand mix. With respect to such results, we find that when goods are initially not independent but imperfect substitutes, the creation of a cocktail might prompt firms to sell their components at a premium rather than at a discount and such practice might not necessarily be welfare-reducing. Similarly to our setting, Armstrong (2011) extends the standard model of bundling to allow for substitutable products and finds that firms have an incentive to introduce bundling discounts when the demand for a bundle is elastic relative to the demand for stand-alone products. In particular, he finds that separate firms often have a unilateral incentive to offer inter-firm bundle discounts when products are substitutes, although this depends on the detailed form of substitutability. Bundle discounts mitigate the innate substitutability of products, which can relax competition between .firms. Our model focuses less on market elasticity and directly relates the incentives to discount or to sell at a premium to the degree of substitutability. It also generalizes Armstrong's results to market structures in which products might be vertically differentiated and present different (possibly asymmetric) weights in the complementary relationship implied by the cocktail. Moreover, differently from Armstrong, we also focus on cases in which markets can be characterized by firms producing goods that do not belong to cocktails or by multiproduct duopolists.

Lucarelli et al. (2010), empirically analyze the welfare effects of cross-firm bundling in the pharmaceutical industry[3] In their data-set firms cannot price discriminate because each drug is produced by a different firm and it is a physician that creates the bundle in her office from the component drugs. However, they are able to analyze the economic effects of cocktails by performing a series of counterfactual exercises on the estimated demand and profit-maximizing condition.Their conterfactual analysis is consistent with the main results of our model. First, they find that cocktail regimens increase profits for all firms involved in the cocktail (and also for entrants producing new drugs) but harm consumers. This occurs because cocktail regimens result in higher drug prices; and the effect of internalizing pricing externalities dominates the business stealing effect of substitute products. Second, they find that the incremental profits from creating a

[3] The title of their paper clearly inspired ours, as the sale of cocktails at a discount or at a premium is indeed equivalent to the sale of bundles of substitute goods supplied by independent competitors.

bundle are sometimes as large as the incremental profits from a merger of the same two firms. Finally, in the third conterfactual, they allow a firm to set two separate drug prices, one for its stand-alone regimen and the other for its component drug in a cocktail regimen. This is equivalent to the case that we analyze in Section 5, where a firm has two separate drugs, one used by itself and the other in a cocktail regimen, and applies very well to the Kaletra/Norvir case illustrated above. Although Lucarelli at al.do not observe a similar situation in their colorectal drug market, they implement this exercise as an "out-of-sample" validation test for their static Nash pricing assumption. Consistently with our theoretical model and with the Abbott case, they find that firms set the price of the cocktail component higher than the stand-alone drug when this flexibility is allowed.

The paper is organized as follows. Section 2 derives demand functions when a cocktail of imperfectly substitute goods is introduced in an oligopolistic market and Section 3 determines the effect of such nenwly available product on the Bertrand-Nash equilibrium prices, quantities and profits. In Section 4, we find first that a multiproduct monopolist would have no incentive to engage in price discrimination when its two products can be used in a cocktail by consumers and then we observe how such conclusions changen when oligopolistc competition among subtitutes is introduced and firms can coordinate on a bilateral discount (or premium, if feasible) when the two goods are purchased to compose a cocktail. Particularly, we determine the optimal profit-sharing rule and find cases under which "bundling among rivals" might benefit consumers. Section 5 analyzes the impact of cocktails on the introduction of new products in the market, studying separately the case of the entry of new firms from the case in which it's one of the two incumbents that introduces a new product or "innovation". In this part, we focus on the impact of innovations on equilibrium prices, profits, consumer surplus and on the ability to continue coordinating on a discount (or premium) when a cocktail composed of the exisiting product is purchased. In Section 6 we recognize that coordination on price discriminatory practices might not be feasible but that firms might be able to indipendently and simultanenously price their products differently when consumers purchase them to form a cocktail. In such case the resulting Bertrand-Nash equilibrium is similar to the outcome of a Prisoner's Dilemma, where both firms would find selling at a premium a dominant strategy, but where profits are lower than under uniform pricing. Section 7 extends some of the results to the case of vertically differentiated products and to different dosages of the components Section 8 concludes. Proofs of Proposition

2 The Model

Consider first a standard model in which two substitute goods are sold (e.g., two drugs to cure HIV/AIDS). The market demands for the two goods are derived from the following social welfare function (Dixit, 1979)

$$U(q_1, q_2, M) = M + (\alpha_1 q_1 + \alpha_2 q_2) - \frac{\beta}{2}\left(q_1^2 + q_2^2 - \gamma q_1 q_2\right) \tag{1}$$

where M is the total expenditure on other goods different from 1 and 2 and γ measures the degree of substitutability between the two goods ($\gamma \in [0, 1]$). Parameters α_1 and α_2 represent the quality of goods 1 and 2 respectively. In that follows, we assume $\alpha_1 = \alpha_2 = \alpha$ because we want to isolate the impact on prices of the introduction of a cocktail from that due to an asymmetric quality distribution.

Following Shubik and Levitan (1980), we set

$$\beta = n - (n-1)\gamma = 2 - \gamma \tag{2}$$

where n is the number of products available (here $n = 2$), to prevent changes in n and γ to affect total market demand. Using such normalization, the demands for the two products are

$$q_1 = \frac{2\alpha(1-\gamma) - p_1(2-\gamma) + \gamma p_2}{4(1-\gamma)} \tag{3}$$

$$q_2 = \frac{2\alpha(1-\gamma) - p_2(2-\gamma) + \gamma p_1}{4(1-\gamma)} \tag{4}$$

Consider now this same market when some consumers, as before, use only one drug, whereas some others are prescribed a cocktail of the two substitute goods by their physician. For the latter type of consumers, then, the two drugs works as complements. In fact, it is sometimes the case that in the US oncology market a pharmaceutical cocktail composed (in a fixed proportion) of two or more already approved drugs is also approved by the Food and Drug Administration because it demonstrates superior efficacy, fewer side-effects or greater convenience. In the market for colorectal cancer chemotherapy drugs, cocktails of two or more substitute drugs are often used to treat patients who suffer strong side - effects when treated with one single drug in the amount prescribed by the organizations such as The National Comprehensive Cancer Network (NCCN).

Such organizations also recommend the amount of each drug that doctors should use in each cocktail/regimen, based on the dosages used in clinical trials or in actual practice (Lucarelli et al., 2010). Physicians prescribe either one of the two single drugs or the cocktail, according to the characteristics of the patient. Consumption of the three available regimes amount then to q_1, q_2 and q_3 respectively. The social welfare function then becomes

$$U(q_1, q_2, q_3, M) = M + \alpha(q_1 + q_2) + \alpha_3 q_3 - \frac{1}{2}\left(q_1^2 + q_2^2 + q_3^2 + \gamma \sum_{i=3} \sum_{j \neq i} q_i q_j\right) \tag{5}$$

As a first approximation, we assume that $\alpha_3 = \alpha$, i.e., combining the two goods does not lead to a product of different quality. However, considering that the introduction of cocktails is often motivated by their superior quality (as in the US pharmaceutical markets), we will later present an extension with $\alpha_3 \neq \alpha$.

The demands for the three products are

$$q_1 = \frac{3\alpha(3-\gamma) - p_1(3-\gamma) + \gamma(p_2 + p_3)}{9(1-\gamma)} \tag{6}$$

$$q_2 = \frac{3\alpha(3-\gamma) - p_2(3-\gamma) + \gamma(p_1 + p_3)}{9(1-\gamma)} \tag{7}$$

$$q_3 = \frac{3\alpha(3-\gamma) - p_3(3-\gamma) + \gamma(p_1 + p_2)}{9(1-\gamma)} \tag{8}$$

Assume that the newly developed and approved cocktail combines goods 1 and 2 in proportions r_1 and r_2, r_1, $r_2 > 0$ and $r_1 + r_2 = 1$. One unit of the cocktail then costs

$$p_3 = r_1 p_1 + r_2 p_2. \tag{9}$$

Substituting p_3 from (9) into demands (6) - (8), we obtain demands as a function of p_1 and p_2 only:

$$q_1^c = \frac{3\alpha(1-\gamma) - p_1(3-\gamma(1+r_1)) + \gamma p_2(2-r_1)}{9(1-\gamma)} \tag{10}$$

$$q_2^c = \frac{3\alpha(1-\gamma) - p_2(3-\gamma(2-r_1)) + \gamma p_1(1+r_1)}{9(1-\gamma)} \tag{11}$$

$$q_3^c = \frac{3\alpha(1-\gamma) - p_1((3-\gamma)r_1 - \gamma) - p_2((3-\gamma)(1-r_1) - \gamma)}{9(1-\gamma)} \tag{12}$$

3 The Effects of the Cocktail

In order to clarify that the incentives to offer cocktails at a discount or a premium typically arise in oligopolistic settings only, consider briefly as a benchmark a multiproduct monopolist selling both drugs 1 and 2 with no fixed costs. Its marginal costs are constant, common to both products and normalized to zero. In such setting, the profit function would be $\Pi = p_1\, q_1 + p_2\, q_2$ and the profit- maximizing price and output leveles would simply amount to $p_1^M = p_2^M = \frac{a}{2}$ and $q_1^M = q_2^M = \frac{\alpha}{4}$. If the cocktail becomes available and total demand remains unchanged through 2, then profit-maximizing prices and outputs remain unchanged. Unsurprisingly, the introduction of a cocktail does not affect the monopolist's behavior.

Results differ if the two separate drugs are offered by two independent firms. If the cocktail is not available, the Bertrand-Nash equilibrium prices are $p_1^{nc} = p_2^{nc} = \frac{2\alpha(1-\gamma)}{4-3\gamma}$, which, not surprisingly, are lower that p_M^i, $i = 1, 2$. Quantities are $q_1^{nc} = q_2^{nc} = \frac{\alpha(2-\gamma)}{2(4-3\gamma)}$ and profits $\Pi_i^{nc} = \frac{\alpha^2(2-\gamma)(1-\gamma)}{(4-3\gamma)^2}$, $i = 1, 2$.

When the new regimen becomes available, each firm's demand becomes $D_i = q_i + r_i q_3$, $i = 1, 2$ and, substituting the definitions of q_i $(i = 1, 2)$ and q_3 from (6) - (8), profits are

$$\Pi_1 = \frac{p_1\left[3\alpha(1+r_1)(1-\gamma) - p_1((1+r_1^2)(3-\gamma) - 2r_1\gamma) - p_2\left(r_1(3-\gamma)(1-r_1) - 2\gamma\right)\right]}{9(1-\gamma)} \quad (13)$$

$$\Pi_2 = \frac{p_2\left[3\alpha(2-r_1)(1-\gamma) - p_1\left(r_1(3-\gamma)(1-r_1) - 2\gamma\right) - p_2(2(3-2\gamma)(1-r_1) + r_1^2(3-\gamma))\right]}{9(1-\gamma)} \quad (14)$$

In order to separate the impact of the creation of a cocktail from the possible asymmetry in the dosage of the two drugs (i.e., $r_1 \neq r_2$), we now assume $r_1 = \frac{1}{2}$, leaving the asymmetric case for later.

Profit functions become

$$\Pi_1 = \frac{p_1\left(6\alpha(1-\gamma) - p_1(5-3\gamma) - p_2(1-3\gamma)\right)}{12(1-\gamma)} \quad (15)$$

$$\Pi_2 = \frac{p_1\left(6\alpha(1-\gamma) - p_1(5-3\gamma) - p_2(1-3\gamma)\right)}{12(1-\gamma)} \quad (16)$$

From the first order conditions , each firm's reaction function results:

$$p_i = \frac{6\alpha(1-\gamma) - (1-3\gamma)p_j}{2(5-3\gamma)}, i = 1, 2 \quad (17)$$

Notice that the slope of such function changes its sign as γ increases In particular, $\frac{dp_i}{dp_j} \lesseqgtr 0$ iff $\gamma \gtreqless \frac{1}{3}$.In other words the degree of substitutability across products influences the nature itself of the competition. In other words, the creation of the cocktail renders the two substitute goods complementary at some degree, as well, and a particular version of the Cournot problem (also known as "complementary monopoly") might emerge. In fact, when the two goods become complements, each separate firm does not fully recognize the negative impact that a price increase has on its rival's demand, generating a negative externality that tend to increase prices. When $\gamma > \frac{1}{3}$, however, the goods are close substitutes and this latter characteristics prevails and reaction functions remain positively sloped. On the other hand, when $\gamma < \frac{1}{3}$, the Cournot problem becomes dominant and firms react to their rivals' price reduction with a price increase As we will see, this switch will have a clear impact on the firms' incentives to offer the cocktail at a discount. At the moment, we simply verify the that Bertrand-Nash equilibrium prices are $p_1^c = p_2^c = \frac{6\alpha(1-\gamma)}{11-9\gamma}$, while the quantities sold of each regimen (i.e., demands of the two separate drugs, q_1^c, q_2^c and of the cocktail q_3^c) are $q_i^c = \frac{\alpha(5-3\gamma)}{33-27\gamma}$, $i = 1, 2, 3$. Given that q_3 consists of r_1 units of q_1 and r_2 units of q_2, then the equilibrium quantities are $q_1^* = q_2^* = \frac{3\alpha(5-3\gamma)}{2(33-27\gamma)}$. Finally, equilibrium profits are $\Pi_1^c = \Pi_2^c = \frac{3\alpha^2(1-\gamma)(5-3\gamma)}{(11-9\gamma)^2}$.

We can prove the following results.

Proposition 1 *When a cocktail is available,* $p_i^c > p_i^{nc}$ *and* $q_i^* < q_i^{nc}$. $\Pi_i^c > \Pi_i^{nc}$ *(*$i = 1, 2$*) iff* $\gamma > 0.175$, *so that the introduction of a cocktail can reduce each firm's profits.*

Thus, the introduction of the cocktail increases prices. for any $\gamma \in (0, 1)$ *Ceteris paribus*, the creation of a complementary relationship between the two goods softens competion,even if this effect does not necessarily favors firms. In fact, when γ is particularly low ($\gamma < 0.175$), the negative externality characterizing the Cournot problem is particularly severe (the low degree of substituability infact is not enough to push prices sufficiently down through competition), so that the demands for the two goods drop significantly, lowering each firm's profit. In conclusion, there might be cases in which at the same time the introduction of cocktails softens price competition, at consumers' expenses, and penalizes rather than favors firms. Notice that this is a particularly relevant conclusion for the market of drugs. In fact, if the quality of a cocktail is not significantly higher from that of the existing therapies, its presence might harm both sides of the market. This justifies the attention with which the Food and Drug Administration (FDA) approves new pharmaceutical cocktails in the US, as they need to exhibit superior efficacy, fewer side effects or greater convenience relative to existing drugs. The "convenience" argument might however work ambiguously, because firms can use discounts to price discriminate against those consumers using the original therapies, as correctly pointed out by Gans and King (2006) and Armstrong (2011). This is why, in the next Section we investigate whether such incentives to price discriminate are present for any degree of substitutability and whether they always generate discounts rather than premiums, favouring those consumers purchasing the cocktail.

4 Discounting Cocktails

Consider first the multiproduct monopolist's incentive to price discriminate. Particularly, consider a case in which the monopolist offers the cocktail at a price $p_3 = r_1 p_1 + r_2 p_2 - \delta$, where δ is a real number representing the discount (or premium, if $\delta < 0$) that a consumer receives when she buys one unit of the cocktail. The monopolist's profits are $\Pi_M = p_1\, q_1 + p_2\, q_2 + (r_1\, p_1 + r_2\, p_2 - \delta)\, q_3$ and, substituting the expressions for demands q_1 - q_3 from (6) - (8), equilibrium prices are $p_{1M}^{\delta} = p_{2M}^{\delta} = \frac{3\alpha+2\delta}{6}$ and quantities are $q_{1M}^{\delta} = q_{2M}^{\delta} = q_{3M}^{\delta} = \frac{3\alpha(1-\gamma)-2\delta}{18(1-\gamma)}$. Then, monopolist's profits as a function of the discount δ become $\Pi_M^{\delta}(\delta) = \frac{9\alpha^2(1-\gamma)-8\delta^2}{36(1-\gamma)}$, which is monotonically decreasing in $|\delta|$. Then, the monopolist would find it optimal to set $\delta = 0$ and no price discrimination occurs.[4]

Proposition 2 *When the cocktail is available, price discrimination is not optimal for a multiproduct monopolist.*

Then, when quality differentiation is absent and demand size remains constant as the number of products available grows, a monopolist does not have any incentive to offer the cockail at a discount (premium).

Conclusions might differ when competition is introduced. Particularly, we now consider the incentive to set a bilateral discount (or premium) by two independent firms producing the two separate drugs. Again, the cocktail will be offered at a unit price

[4] As we will see later, the incentive to price discriminate, selling the cocktail either at a discount or at a premium, may arise when the cocktail's quality differs from the quality of the initial components.

$p_3 = r_1 p_1 + r_2 p_2 - \delta$, where firm 1 bears a portion k of the discount (or enjoys a portion k of the premium when $\delta < 0$), whereas firm 2 contributes for the remaining portion $1 - k$. The effective revenues per-unit of the cocktail become then $r_1 p_1 - k\,\delta$ and $r_2 p_2 - (1-k)\,\delta$ for firms 1 and 2, respectively. Profits therefore are $\Pi_1 = p_1 q_1 + (r_1 p_1 - k\,\delta)\, q_3$ and $\Pi_2 = p_2 q_2 + (r_2 p_2 - (1-k)\,\delta)\, q_3$. Equilibrium prices and quantites are ($p_1^\delta = \frac{2(1-\gamma)(9\alpha(3-\gamma)+\delta(k(11-9\gamma)+8))}{3(33-38\gamma+9\gamma^2)}$, $p_2^\delta = \frac{2(1-\gamma)(9\alpha(3-\gamma)+\delta(19-9\gamma-k(11-9\gamma)))}{3(33-38\gamma+9\gamma^2)}$) and ($q_1^\delta = \frac{\alpha(45-87\gamma+51\gamma^2-9\gamma^3)-2\delta(8-\gamma(5+\gamma)+k(9\gamma^2-20\gamma+11))}{9(1-\gamma)(9\gamma^2-38\gamma+33)}$, $q_2^\delta = \frac{\alpha(45-87\gamma+51\gamma^2-9\gamma^3)-2\delta(19-\gamma(25-8\gamma)-k(9\gamma^2-20\gamma+11))}{9(1-\gamma)(9\gamma^2-38\gamma+33)}$, $q_3^\delta = \frac{3\alpha\left(3\gamma^2-8\gamma+5\right)+4\delta(6-5\gamma)}{9(1-\gamma)(11-9\gamma)}$), respectively. Following Gans and King (2006), firms 1 and 2 coordinate, choosing δ and then k to maximize joint profits $\Pi_1^\delta + \Pi_2^\delta$. Proceeding by backward induction they choose k for every level of δ and then δ given the optimal sharing rule k, $1-k$. We find that, for any δ, the optimal sharing rule is $k^* = \frac{1}{2}$[5]. Under such rule, we can now prove the following

Proposition 3 *Under the optimal sharing rule $k = \frac{1}{2}$, firms coordinate on a discount when $0 < \gamma < \frac{1}{3}$, while they coordinate on a premium when $\frac{1}{3} < \gamma < 1$.*

Proof. When $k = \frac{1}{2}$, joint profits become $\Pi_1^\delta + \Pi_2^\delta = \frac{2[27\alpha^2(1-\gamma)^2(5-3\gamma)+3\alpha(1-4\gamma+3\gamma^2)\delta-(123-200\gamma+81\gamma^2)\delta^2]}{9(11-9\gamma)^2(1-\gamma)}$ Maximizing such function with respect with δ,we obtain $\delta^* = \frac{3\alpha(1-4\gamma+3\gamma^2)}{(246-400\gamma+162\gamma^2)}$. Notice that $\delta^* > 0$ if and only if $0 \leq \gamma < \frac{1}{3}$, so that a discount would be profitable if and only if the degree of substitutability is sufficiently low. ■

Then, as noticed in 17, when $\gamma < \frac{1}{3}$ complementarity prevails and results are consistent with those obtained by Gans and King (2006), establishing that discounts can be used as a device to soften price competition on the separate prices. In fact, in such case, $p_i^{\delta *} = \frac{3a(1-\gamma)(45-37\gamma)}{246-400\gamma+162\gamma^2}$, $i = 1, 2$ and $p_i^{\delta *} > p_i^c$ iff $\gamma < \frac{1}{3}$. However, when $\gamma > \frac{1}{3}$,the strength of substitutability is stronger and the firms would find profitable to sell the cocktail at a premium (if this is feasible) rather than at a discount, making competition on single products fiercer. As a result, consumer surplus can increase through price discrimination, and the following proposition establishes that this happens indeed for intermediate degrees of substitutability

Proposition 4 $CS^\delta > CS^c$ *iff* $\frac{1}{3} < \gamma < 0.78$. *Then, when cocktails are sold at a premium, consumers might benefit from price discrimination.*

[5] See Appendix

This result appears interesting when working on price competition without quality differentation. Even when the three regimen are perceived as being of the same quality (so that, as established earlier, the introduction of the cocktail always damages consumers if firms are constrained to price both their stand-alone products and their components of a cocktail the same), there is a rationale for allowing firms to price discriminate when the benefit of a fiercer competition on stand alone products more than counterbalances the cost of a cocktail sold at a premium. Notice that consumer surplus remains lower than under uniform pricing at the extremes of the distribution of γ When γ is particularly low, this happens because of the Cournot problem. When γ is very high, the reason is to be found on the particulary large premium imposed for the cocktail dominating over the very low prices of separate goods.

5 The impact of cocktails on the introduction of new therapies in the market

In the early 2000s Abbot launched Kaletra, a drug fro trating HIV/AIDS. At the time Abbott was already selling Norvir, which was used in a cocktail regimen to help boost the performance of its competitor's drug. Shortly after the launch of Kaletra, Abbot increased Norvir's price four-fold while pricing Kaletra more competitively, presumably to drive customers from the cocktail regimen to its new stand-a.lone regimen. Our model is consistent with such behavior and in order to demonstrate that we proceed in two steps. We first analyze the case in which a third firm enters this particular drug market with a new product, and we show that entry is more difficult when a cocktail of the two existing products is available, and this independently of the quality level of the newly-available therapy. In other words, we show that, ceteris paribus, the introduction of a cocktail requires a new product to be of higher quality than without it in order to be profitably produced. Second, consistently with the case described above, we explicitly recognize that new therapies are very often introduced by the incumbents themselves. Thus, we analyze how the incentives of a duopolistic firm to introduce a third therapy change when the two exisiting ones are also combined into a cocktail.

5.1 The effect of cocktails on the entry of new firms.

We first assume the presence of three firms, each producing an imperfectly substitutable drug such that, without the cocktail, the social welfare function and the demand for the three products are those listed in (5)-(8). We also assume for the moment that products are not vertically differentiated (so that $\alpha_i = \alpha, i = 1, 2, 3$). In this setting, the (symmetric) Bertrand-Nash equilibrium involves each firm pricing its product at $p_i^{nc} = \frac{3\alpha(1-\gamma)}{2(3-2\gamma)}$ and obtaining a profit of $\Pi_i^{nc} = \frac{\alpha^2(3-\gamma)(1-\gamma)}{4(3-2\gamma)^2}$, $i = 1, 2, 3$.

When the new cocktail is introduced, it becomes the fourth available (same-quality) therapy. Consumption of the four products amount then to q_1, q_2, q_3 and q_4 respectively. The social welfare function becomes

$$U(q_1, q_2, q_3, q_4, M) = M + \alpha \sum_{i=4} q_i - \frac{\beta}{2}\left(\sum_{i=4} q_i^2 + \gamma \sum_{i=4}\sum_{j\neq i} q_i q_j\right) \tag{18}$$

and using again the normalization for β in 2, the demand for each of the four products is

$$q_i^{c3} = \frac{4\alpha(1-\gamma) + \gamma \sum\limits_{j\neq i} p_j - p_i(4-\gamma)}{16(1-\gamma)}, \qquad i = 1, .., 4 \tag{19}$$

Firms'demands become $D_i = q_i + r_i q_4$, $i = 1, 2$ and $D_3 = q_3$, respectively. If we again assume $r_1 = \frac{1}{2}$ and consider that $p_4 = \frac{p_1+p_2}{2}$, substituting these demands into the three firms' profit functions we obtain

$$\Pi_1 = \frac{p_1\left[24\alpha(1-\gamma) + p_2(9\gamma-4) - 6p_3\gamma - p_1(20-9\gamma)\right]}{64(1-\gamma)} \tag{20}$$

$$\Pi_2 = \frac{p_2\left[24\alpha(1-\gamma) + p_1(9\gamma-4) - 6p_3\gamma - p_2(20-9\gamma)\right]}{64(1-\gamma)} \tag{21}$$

$$\Pi_3 = \frac{p_3\left[8\alpha(1-\gamma) + 3\gamma(p_1+p_2) - 2p_3(4-\gamma)\right]}{32(1-\gamma)} \tag{22}$$

From the system of the three first order conditions, we derive the following Bertrand-Nash equilibrium prices: $p_1^{c3} = p_2^{c3} = \frac{6\alpha(8-\gamma)(1-\gamma)}{88-\gamma(76-9\gamma)}$, $p_3^{c3} = \frac{\alpha(44-9\gamma)(1-\gamma)}{88-\gamma(76-9\gamma)}$ The quantities sold of each regimen are $q_i^{c3} = \frac{\alpha(160-92\gamma+9\gamma^2)}{16(88-76\gamma 9\gamma^2)}$, $i = 1, 2, 4$ and $q_3^{c3} = \frac{\alpha(176-80\gamma+9\gamma^2)}{16(88-76\gamma 9\gamma^2)}$. Given that q_4 consists of r_1 units of q_1 and r_2 units of q_2, then the equilibrium quantities for firms 1 and 2 are $q_1^{*3} = q_2^{*3} = \frac{3\alpha(160-92\gamma+9\gamma^2)}{32(88-76\gamma 9\gamma^2)}$. Finally, equilibrium profits are

$\Pi_1^{c3} = \Pi_2^{c3} = \frac{9\alpha^2(1-\gamma)(20-9\gamma)(8-\gamma)^2}{16(88-76\gamma+9\gamma^2)^2}$and $\Pi_3^{c3} = \frac{\alpha^2(1-\gamma)(4-\gamma)(44-9\gamma)^2}{16(88-76\gamma+9\gamma^2)^2}$. Using these results, it is easy to conclude that $\Pi_3^{c3} < \Pi_i^{nc}$ for every $\gamma \in (0,1)$ so that the following Proposition holds

Proposition 5 *The presence of a cocktail reduces the profits of Firm 3. Then, ceteris paribus, a cocktail makes entry less profitable.*[6]

It could be interesting to combine this conclusion to our previous results, considering how the incentives to set a bilateral discount (or premium) change when two independent firms producing the two drugs combined in a cocktail face the competition of a stand-alone rival. Again, the cocktail will be offered at a unit price $p_4 = r_1p_1 + r_2p_2 - \delta$, and the effective revenues per-unit of the cocktail are $r_1\, p_1 - k\,\delta$ and $r_2\, p_2 - (1-k)\,\delta$ for firms 1 and 2, respectively. Profits therefore are $\Pi_1 = p_1\, q_1 + (r_1\, p_1 - k\,\delta)\, q_4$, $\Pi_2 = p_2\, q_2 + (r_2\, p_2 - (1-k)\,\delta)\, q_4$ and $\Pi_3 = p_3 q_3$ When $r_1 = \frac{1}{2}$, we find again that, for any δ, the sharing rule that maximizes firm 1 and 2' s joint profits is $k = \frac{1}{2}$ Using this result, we obtain first the optimal amount of δ, $\delta' = \frac{2\alpha(-16+58\gamma-47\gamma^2+5\gamma^3)}{2624-4512\gamma+2468\gamma^2-457\gamma^3+27\gamma^4}$, which is effectively a discount (premium) if and only if $0 \leq \gamma < \frac{2}{5}$($\frac{2}{5} < \gamma < 1$). At such value, the Bertrand-Nash equilibrium prices and quantities are $p_1^{\delta3} = p_2^{\delta3} = \frac{6\alpha(240-478\gamma+288\gamma^2-53\gamma^3+3\gamma^4)}{2624-4512\gamma+2468\gamma^2-457\gamma^3+27\gamma^4}$, $p_3^{\delta3} = \frac{2\alpha(1312-2704\gamma+1753\gamma^2-388\gamma^3+27\gamma^4)}{2624-4512\gamma+2468\gamma^2-457\gamma^3+27\gamma^4}$and $q_1^{\delta3} = q_2^{\delta3} = \frac{\alpha(14336-20544\gamma+9280\gamma^2-1515\gamma^3+81\gamma^4)}{32(2624-4512\gamma+2468\gamma^2-457\gamma^3+27\gamma^4)}$, $q_3^{\delta3} = \frac{\alpha(4-\gamma)(1312-1392\gamma+361\gamma^2-27\gamma^3)}{16(2624-4512\gamma+2468\gamma^2-457\gamma^3+27\gamma^4)}$[7]. Equilibrium profits amount to $\Pi_1^{\delta3} = \Pi_2^{\delta3} = \frac{\alpha^2(1-\gamma)(8-\gamma)^2(61-27\gamma)}{16(2624-4512\gamma+2468\gamma^2-457\gamma^3+27\gamma^4)}$, $\Pi_3^{\delta3} = \frac{\alpha^2(4-\gamma)(1-\gamma)(1312-1392\gamma+361\gamma^2-27\gamma^3)^2}{16(2624-4512\gamma+2468\gamma^2-457\gamma^3+27\gamma^4)^2}$. The values $p_i^{c3}$and $p_i^{\delta3}$, $i = 1,2$ can be easily compared with p_i^c and p_i^δ, noticing that the higher number of competitors in the market for a given demand size has pro-competitive effect, that is $p_i^{c3} < p_i^c$ and $p_i^{\delta3} < p_i^\delta$, $i = 1,2$. All such results are summarized in the following proposition:

Proposition 6 *The presence of a third competitor, ceteris paribus, lowers prices and enlarges the range of γ such that it is optimal to set a discount.*

[6]This result can be generalized to new products exhibiting better perfomances. In fact, when a cocktail produced by the incumbents is available, new products will be profitable, net of quality costs, only if they perform better compared to what they'd need to without the cocktail.

[7]See the Appendix for a detailed demonstration of such results.

5.2 The effect of cocktails on the introduction of new products by the incumbents.

Consider now, consistently with the Abbot case, the introduction of third product by one of the two incumbents. In particular, we assume that firm 1 produces two products, the existing drug 1 and the newly introduced drug 3, while firm 2 continues producing drug 2. If the cocktail between 1 and 2 were not available, the social welfare and the demand functions would be the same as in (5) -(8). The profit functions of the two firms would be $\Pi_1 = p_1 q_1 + p_3 q_3$ and $\Pi_2 = p_2 q_2$, and solving the system of the three first order conditions, we get thfollowing Bertrand equilibrium prices: $p_1^* = \frac{-\alpha_3\gamma^2+\alpha(36-42\gamma+7\gamma^2)}{12(6-6\gamma+\gamma^2)}$, $p_2^* = \frac{\alpha(18-21\gamma+4\gamma^2)-\alpha_3\gamma(3-2\gamma)}{6(6-6\gamma+\gamma^2)}$, $p_3^* = \frac{(6-\gamma)(\alpha_3(6-5\gamma)-\alpha\gamma)}{12(6-6\gamma+\gamma^2)}$, where α_3 is the quality of the new product. In this section we allow for $\alpha_3 \neq \alpha$, since vertical differentiaition provides very interesting implications on the effect of higher-quality innovations on social welfare.[8].

Quantities are $q_1^* = \frac{\alpha\left(-8\gamma^3+69\gamma^2-162\gamma+108\right)-\alpha_3\gamma\left(4\gamma^2-33\gamma+36\right)}{108(1-\gamma)(\gamma^2-6\gamma+6)}$, $q_2^* = \frac{(3-\gamma\left(\alpha\left(4\gamma^2-21\gamma+18\right)+\alpha_3\gamma(2\gamma-3)\right)}{54(1-\gamma)(\gamma^2-6\gamma+6)}$ and $q_3^* = \frac{\alpha_3\left(-4\gamma^3+51\gamma^2-144\gamma+108\right)-\alpha\gamma\left(8\gamma^2-51\gamma+54\right)}{108(1-\gamma)(\gamma^2-6\gamma+6)}$.

Comparing these results with the case in which there are two products and they are not combined in a cocktail we can state the following results. When the new good has the same quality of the existing ones ($\alpha_3 = \alpha$) the following Lemma applies, ralize to ($\alpha_3 \neq \alpha$).

Lemma 7 *The introduction of a new good with $\alpha_3 = \alpha$ by firm 1 increases p_1 and decreases p_2. It increases Π_1^* and decreases Π_2^*, so that firm 1 will have the incentive to introduce it. Consumer surplus decreases.*

This result can be now generalized to the case ($\alpha_3 \neq \alpha$) with the following Proposition:

Proposition 8 *If $\alpha_3 \neq \alpha$, the introduction of a new good by firm 1 increases p_1 iff $\alpha_3 > \frac{3\gamma^3-14\gamma^2+12\gamma}{4\gamma^2-3\gamma^3}$. Profits increase iff either $R < R_1(\gamma)$ or $R > R_2(\gamma)$. Consumer surplus increases iff either $R < R_1^{CS}(\gamma)$ or $R > R_2^{CS}(\gamma)$. Given that $R_1^{CS}(\gamma) < R_1(\gamma)$ and $R_2^{CS}(\gamma) < R_2(\gamma)$, there exists values or R such that the introduction of the new good is profitable for firm 1 but harms consumers.*

[8]Notice that when α_3 is particularly high, product 2 might be forced out of the market, and only products 1 and 2 would then be produced by a multiproduct monopolist. Symmentrically, when α_3 is particularly low, it might not be priced above or equal to the marginal cost (here zero) and would then be unprofitable. To avoid this latter case we assume from now on that $\frac{\alpha_3}{\alpha} > \frac{\gamma}{6-5\gamma}$.

So far we have excluded the possibility that consumers bought a cocktail of goods. We now introduce this possibility, assuming that the cocktail is formed by good 1, already produced by firm 1 and the good produced by firm 2. The two goods are again combined in fixed proportions r_1 and r_2, with $r_1 = r_2 = \frac{1}{2}$.

The Bertand-Nash equilibrium prices in this case are $p_1^{3c} = \frac{3\alpha\left(13\gamma^2-60\gamma+48\right)+\alpha_3(2-5\gamma)\gamma}{3(21\gamma^2-98\gamma+88)}$, $p_2^{3c} = \frac{2\left(3\alpha\left(7\gamma^2-30\gamma+24\right)+\alpha_3\gamma(7\gamma-10)\right)}{3(21\gamma^2-98\gamma+88)}$ and $p_3^{3c} = \frac{(\gamma-4)(3\alpha\gamma+\alpha_3(19\gamma-22))}{2(21\gamma^2-98\gamma+88)}$.

Quantities are $q_1^{3c} = \frac{\alpha\left(63\gamma^3-420\gamma^2+816\gamma-480\right)+7\alpha_3\gamma\left(3\gamma^2-20\gamma+20\right)}{48(\gamma-1)(21\gamma^2-98\gamma+88)}$, $q_2^{3c} = \frac{\alpha\left(63\gamma^3-408\gamma^2+816\gamma-480\right)+\alpha_3\gamma\left(21\gamma^2-64\gamma+52\right)}{48(\gamma-1)(21\gamma^2-98\gamma+88)}$, $q_3^{3c} = \frac{3\alpha\gamma\left(7\gamma^2-34\gamma+32\right)+\alpha_3\left(7\gamma^3-86\gamma^2+240\gamma-176\right)}{16(\gamma-1)(21\gamma^2-98\gamma+88)}$ and $q_4^{3c} = \frac{\alpha\left(21\gamma^3-138\gamma^2+272\gamma-160\right)+\alpha_3\gamma\left(7\gamma^2-34\gamma+32\right)}{16(\gamma-1)(21\gamma^2-98\gamma+88)}$, where q_4^{3c} is the quantity of the cocktail. Profits for the two firms now become:

$$\begin{aligned}\Pi_1^{3c} &= \frac{9\alpha^2\left(441\gamma^5-4886\gamma^4+20112\gamma^3-38208\gamma^2+33984\gamma-11520\right)}{144(\gamma-1)\left(21\gamma^2-98\gamma+88\right)^2}+ \\ &+ \frac{6\alpha\alpha_3\gamma\left(441\gamma^4-4886\gamma^3+18044\gamma^2-25560\gamma+12192\right)}{144(\gamma-1)\left(21\gamma^2-98\gamma+88\right)^2}+ \\ &+ \frac{\alpha_3^2\left(441\gamma^5-9422\gamma^4+59896\gamma^3-154568\gamma^2+172656\gamma-69696\right)}{144(\gamma-1)\left(21\gamma^2-98\gamma+88\right)^2}\end{aligned} \tag{23}$$

$$\Pi_2^{3c} = \frac{(9\gamma-20)\left(3\alpha\left(7\gamma^2-30\gamma+24\right)+\alpha_3\gamma(7\gamma-10)\right)^2}{144(\gamma-1)\left(21\gamma^2-98\gamma+88\right)^2} \tag{24}$$

Consumer surplus is

$$\begin{aligned}CS^{3c} &= \frac{6\alpha\alpha_3\gamma(8352-18072\gamma+13724\gamma^2-4214\gamma^3+441\gamma^4)}{288(-1+\gamma)(88-98\gamma+21\gamma^2)^2}+ \\ &+ \frac{\alpha_3^2(-69696+172656\gamma-156488\gamma^2+63160\gamma^3-10766\gamma^4+441\gamma^5)}{288(-1+\gamma)(88-98\gamma+21\gamma^2)^2}+ \\ &+ \frac{9\alpha^2(-19200+50880\gamma-50592\gamma^2+23616\gamma^3-5222\gamma^4+441\gamma^5)}{288(-1+\gamma)(88-98\gamma+21\gamma^2)^2}\end{aligned} \tag{25}$$

By comparing prices, profits and consumers surplus in this case with those in the case in which firm 1 produces one good only, we reach conclusions similar to those obtained above, where a cocktail was not available.

Particularly, there exists thresholds $R_1^{3c}(\gamma)$, $R_1^{3c}(\gamma)$, such that the introduction of the product 3is profitable for firm 1 iff $R < R_1^{3c}(\gamma)$ and $R > R_2^{3c}$ Similarly, there exist thresholds $R_1^{CS3c}(\gamma)$, $R_1^{CS3c}(\gamma)$, such that the introduction of product 3 enhances consumer surplus iff $R < R_1^{CS3c}(\gamma)$ and $R > R_2^{CD3c}$. Again, $R_1^{CS3c}(\gamma) < R_1^{3c}(\gamma)$ and $R_2^{CS3c}(\gamma)$ lies above $R_2^{3c}(\gamma)$. Then, there exists an area, such that the introduction of product 3 is profitable for firm 1 but detrimental for consumers.

Differently from the previous case, however, the degree of differentiation (difference between α and α_3) required to render the introduction of product 3 profitable and also welfare enhancing for consumers is higher (namely, the thresholds $R_1^{3c}(\gamma)$ and $R_1^{CS3c}(\gamma)$ are smaller than before, whereas $R_2^{3c}(\gamma)$ and $R_2^{CS3c}(\gamma)$ are larger.

5.3 Discounting Cocktails in the Presence of New Products

Consider now the incentives to set bilateral discounts (or premia) by firms 1 and 2 when they sell a cocktail q_4 consisting of $r_1 = 1/2$ units of good 1 and $r_2 = 1/2$ units of good 2, given that firm 1 also produces a second good, q_3. Using the same methodology applied in the previous sectiton, we obtain the following Nash equilibrium prices:
$p_{1d}^{3c} = \frac{6\alpha\left(17\gamma^2-89\gamma+72\right)+\delta\left(9\gamma^2-128\gamma+\left(99\gamma^2-284\gamma+176\right)k+128\right)}{9(21\gamma^2-98\gamma+88)}$,
$p_{2d}^{3c} = \frac{2\left(12\alpha\left(7\gamma^2-25\gamma+18\right)+\delta\left(63\gamma^2-206\gamma+\left(-63\gamma^2+142\gamma-88\right)k+152\right)\right)}{9(21\gamma^2-98\gamma+88)}$ and $p_{3d}^{3c} = \frac{66\alpha\left(\gamma^2-5\gamma+4\right)+\gamma\delta(-9\gamma+(27\gamma-44)k+4)}{6(21\gamma^2-98\gamma+88)}$.

Quantities then are $q_{1d}^{3c} = \frac{6\alpha\left(21\gamma^3-140\gamma^2+239\gamma-120\right)+\delta\left(-21\gamma^2-184\gamma+\left(231\gamma^2-568\gamma+352\right)k+256\right)}{72(\gamma-1)(21\gamma^2-98\gamma+88)}$,
$q_{2d}^{3c} = \frac{6\alpha\left(21\gamma^3-118\gamma^2+217\gamma-120\right)+\delta\left(213\gamma^2-752\gamma+\left(-219\gamma^2+568\gamma-352\right)k+608\right)}{72(\gamma-1)(21\gamma^2-98\gamma+88)}$,
$q_{3d}^{3c} = \frac{3\alpha\left(7\gamma^3-47\gamma^2+84\gamma-44\right)+\gamma\delta(-11\gamma+(19\gamma-22)k+14)}{12(\gamma-1)(21\gamma^2-98\gamma+88)}$ and $q_{4d}^{3c} = \frac{3\alpha\left(7\gamma^3-43\gamma^2+76\gamma-40\right)+\delta\left(216\gamma+\gamma^2(k-47)-192\right)}{12(\gamma-1)(21\gamma^2-98\gamma+88)}$.

Firms 1 and 2 offer the cocktail at a discount (or a premium), setting a price $p_4 = (p_1 + p_2)/2 - \delta$. As before, we assume that they choose the pricing policy for the cocktail cooperatively, i.e., maximizing joint profits $\Pi_1^{3c} + \Pi_2^{3c}$, where $\Pi_1^{3c} = p_1 q_1 + (\frac{1}{2}p_1 - k\delta)$. In such a setting we obtain the following result:

Proposition 9 *1. When $\gamma \leq 0.029$, the two firms sell their products at a discount $\delta^{3c} > 0$, of which firm 1 bears a share k^{*3c} and firm 2 a share $1 - k^{*3c}$. The share k^{*3c} is decreasing in γ and $k^{*3c} = \frac{1}{2}$ for $\gamma = 0$.*

2. When $\gamma > 0.029$, only firm 2 price discriminates in the market, applying a discount $\delta_0^{3c} > 0$ if $\gamma \leq 0.44$ and a premium $\delta_0^{3c} < 0$ if $\gamma > 0.44$. Hence $k = 0$.

We now compare the two cases in Proposition 9 with the no-discount case, starting with the interval $0 < \gamma \leq 0.029$:

Proposition 10 *By practicing a coordinated discount, both firm 1 and 2 can charge a higher price on their standalone products* ($p_{1d}^{3c} > p_1^{3c}$, $p_{2d}^{3c} > p_2^{3c}$, $p_{3d}^{3c} > p_3^{3c}$)*. Firm 1 gains from the coordinated discount, whereas firm 2 gains for* $0.000619 < \gamma \leq 0.029$*. Consumer surplus is always reduced by coordinated discount practices.*

Proof. See Appendix. ■

The case with $\gamma > 0.029$ is discussed in the following Proposition.

Proposition 11 *If* $\gamma > 0.029$*, given* $k = 0$*, firm 1 charges a higher price on product 1* ($p_1^0 > p_1^{3c}$) *if and only if* $\gamma \in (0.029, 0.44]$*, i.e., when firm 2 practices a discount on its product when sold in the cocktail. Firm 1 always charges a higher price on its newly introduced product 3* ($p_3^0 > p_3^{3c}$)*, while firm 2 always charges a higher price on its standalone product 2* ($p_2^0 > p_2^{3c}$)*. Firm 1 always gains from firm 2's price discrimination, whereas firm 2 is always losing. Consumer surplus is always reduced by price-discriminatory practices.*

From Proposition 11 it can be immediately inferred that firm 2 will never agree to coordinated price discriminatory policy to which firm 1 does not actively participate ($k = 0$). Therefore, the following corollary ensues:

Corollary 12 *If* $\gamma > 0.029$*, a coordinated policy of price discrimination in the sale of the cocktail* q_4 *is not feasible.*

Therefore, to analyze firm 1's incentives to introduce the new product 3, we need to compare profits in the two-product case (in which both firm 1 and 2sell a unique good, which can be combined in a cocktail) with those in the three-product case with price discriminatory practices. Therefore, the analysis will be limited to the case in which $\gamma \leq 0.029$, in which $k = k^{*3c} > 0$ and only a discount is applied. The results of this comparison are reported in the following Proposition.

Proposition 13 *If* $\gamma \leq 0.029$*, firm 1 always gains from the introduction of a new product, whereas firm 2 is harmed. Finally, consumer surplus decreases.*

Proof. See Appendix. ■

6 Uncoordinated Cocktail Pricing

7 Extensions

7.1 Different Cocktail Quality

In this extension we assume that the cocktail's quality differs from the quality of its single components. Particularly, the cocktail may represent an improvement on the existing products. In case of drugs, it may cure the same symptoms with less side effects. This should be the most relevant case when drug are concerned, since the Food and Drug Administration concedes marketability of a drug cocktail when it has superior efficacy, fewer side-effects or greater convenience. The cocktail may also be of lesser quality. In case of drugs, this may mean that, although it has fewer side effects, and it is more tolerable for some patients, a cocktail requires taking more pills during the day or implies a longer cure. Using a different example, a cocktail of lesser quality is perfectly plausible when the goods provided are visits to a museum. In that case, seeing two museums in a day may reduce the time spent in each visit, reducing the quality of the visit itself. Still, some consumers may enjoy the possibility to visit two museums rather than one.

The analysis in this Section confirms and somehow generalizes the results in Sections 3 and 4. Particularly, the introduction of a cocktail increases the prices of the standalone products if the cocktail's quality is not lower than the quality of its components. If the cocktail is of lesser quality, prices increase if and only if γ is high enough.

In order to analyze this case, we rewrite the utility function of the representative consumer in terms of relative qualities. Particularly, we normalize the quality of the two standalone components to 1 ($\alpha = 1$) and set the relative quality of the cocktail as Q, where $Q > 1$ ($Q < 1$) implies higher (lower) cocktail quality.

$$U(q_1, q_2, q_3, M) = M + (q_1 + q_2) + Qq_3 - \frac{1}{2}\left(q_1^2 + q_2^2 + q_3^2 + \gamma \sum_{i=3} \sum_{j \neq i} q_i q_j\right) \tag{26}$$

7.2 Asymmetric Cocktails

7.3 Endogenous r_1

So far we have assumed that the cocktail is exogenously given and that each firm participates with $r_1 = r_2 = \frac{1}{2}$. We now remove this assumption. In this subsection, both firms choose whether to supply a cocktail to the market and then they choose r_1 to maximize joint profits. The timing of the game then becomes:

1. Firms decide whether to supply a cocktail;
2. They decide whether to engage in price discrimination or not;
3. They choose r_1;
4. If they price discriminate, they choose δ;
5. If δ is chosen cooperatively, firms then choose k;
6. Firms compete à la Bertrand, setting prices.

We consider first the case in which firms decide to supply the cocktail but not to engage in price discrimination. We establish the following result.

Proposition 14 **a)** *If* $\gamma \geq \frac{1}{3}$, *the unique value of* r_1 *that maximizes joint profits* $\Pi_1^c + \Pi_2^c$ *is* $r_1 = \frac{1}{2}$.

b) *If* $0.175 \leq \gamma < \frac{1}{3}$, *there exist multiple equilibria, in which different cocktails can be formed. If firms have equal bargaining power and are symmetric, however,* $r_1 = \frac{1}{2}$.

c) *If* $\gamma < 0.175$ *no cocktail is formed.*

Proof. The proof is rather straightforward, although algebraically tedious. By differentiating the expression $\Pi_1^c + \Pi_2^c$ with respect to r_1 we obtain a polynomial of 9th degree.

Solving it with respect to r_1, we find that $r_1 = \frac{1}{2}$ is always a solution. We then find six roots that are complex $\forall \gamma \in [0, 1]$. Finally, we have two roots, $\tilde{r}_1 = \frac{3-\gamma-\sqrt{9\gamma^2-30\gamma+9}}{2(3-\gamma)}$ and $\tilde{\tilde{r}}_1 = \frac{3-\gamma+\sqrt{9\gamma^2-30\gamma+9}}{2(3-\gamma)}$ that are real for $\gamma \geq \frac{1}{3}$ and complex for $\gamma < \frac{1}{3}$. Moreover, $0 < \tilde{r}_1 < \tilde{\tilde{r}}_1 < 1 \; \forall \gamma \in [0, \frac{1}{3}]$.

a) Let us consider first the case $\gamma \geq \frac{1}{3}$. In that case there is only one real solution in the interval $[0,1]$, i.e., $r_1 = \frac{1}{2}$. At $r_1 = 0$, $\frac{\partial}{\partial r_1}[\Pi_1^c + \Pi_2^c] < 0$, whereas, at $r_1 = 1$, $\frac{\partial}{\partial r_1}[\Pi_1^c + \Pi_2^c] > 0$. Hence $r_1 = \frac{1}{2}$ is a maximum of $\Pi_1^c + \Pi_2^c$.

b) When $0.175 \leq \gamma < \frac{1}{3}$, at $r_1 = 0$, $\frac{\partial(\Pi_1^c+\Pi_2^c)}{\partial r_1} > 0$, whereas, at $r_1 = 1$, $\frac{\partial(\Pi_1^c+\Pi_2^c)}{\partial r_1} < 0$. Given that there are three real roots in the interval $r_1 \in [0,1]$, $\frac{\partial(\Pi_1^c+\Pi_2^c)}{\partial r_1}$ changes sign three times. It is positive for $0 \leq r_1 < \tilde{r}_1$, then negative for $\tilde{r}_1 < r_1 < \frac{1}{2}$. This means that $\tilde{r}_1$ is a maximum. Again, $\frac{\partial(\Pi_1^c+\Pi_2^c)}{\partial r_1} > 0$ for $\frac{1}{2} < r_1 < \tilde{\tilde{r}}_1$ and negative for $\tilde{\tilde{r}}_1 < r_1 < 1$. Then, $\tilde{\tilde{r}}_1$ too is a maximum. Clearly, $r_1 = \frac{1}{2}$ is a minimum.

The combined profits $\Pi_1^c + \Pi_2^c$ take the same value at $r_1 = \tilde{r}_1$ and at $r_1 = \tilde{\tilde{r}}_1$. In fact $(\Pi_1^c + \Pi_2^c)|_{r_1=\tilde{r}_1} = \frac{a^2}{4} = (\Pi_1^c + \Pi_2^c)|_{r_1=\tilde{\tilde{r}}_1}$. Then $\frac{a^2}{4}$ is the maximum value that joint profits reach in the interval $[0,1]$. Hence, the two firms should pick r_1 equal to either $\tilde{r}_1$ or $\tilde{\tilde{r}}_1$ and would enjoy higher profits than without a cocktail.[9]

Numerical simulations show that, $\forall \gamma \in [0, \frac{1}{3}]$, Π_1^c is monotonically increasing in r_1, with $\Pi_1^c|_{r_1=0} < \Pi_1^{nc}$ and $\Pi_1^c|_{r_1=1} > \Pi_1^{nc}$, so that there exists a unique value $0 < r_{11} < 1$ such that $\Pi_1^c \lesseqgtr \Pi_1^{nc}$ iff $r_1 \lesseqgtr r_{11}$.

Similarly, there exists a unique value $0 < r_{12} < 1$, with $r_{11} < r_{12}$, such that $\Pi_2^c \gtreqless \Pi_2^{nc}$ iff $r_1 \lesseqgtr r_{12}$.[10]

Then, r_{11} represents the minimum r_1 firm 1 is prepared to accept, whereas r_{12} is the maximum r_1 firm 2 is willing to accept. Then, in the equilibrium, $r_1 \in [r_{11}, r_{12}]$. The final value will depend on the relative bargaining power of the two firms, with r_1 closer to r_{12} the stronger firm 1's bargaining power.

At $r_1 = \frac{1}{2}$, $\Pi_1^c = \Pi_2^c$, hence firms with equal bargaining power are most likely to set $r_1 = \frac{1}{2}$, if $\Pi_i^c|_{r_1=\frac{1}{2}} > \Pi_i^{nc}$.

c) We know from Proposition 1 that, with $r_1 = \frac{1}{2}$, $\Pi_i^c < \Pi_i^{nc}$ when $\gamma < 0.175$.

It is possible to prove that r_{11}, the minimum share that firm 1 is prepared to accept to agree to form the cocktail, is greater than r_{12}, the maximum firm 2 is willing to accept, for all $\gamma < 0.175$. To prove it, substitute $r_1 = \frac{1}{2}$ into Π_i^c $(i = 1,2)$ and obtain $\Pi_i^c - \Pi_i^{nc} = \left\{ \frac{\alpha^2(1-\gamma)(13\gamma-9\gamma^2-2)}{(11-9\gamma)^2(4-3\gamma)^2} \right\}$. Solving it with respect to γ, there are

[9] When $\gamma = 0$ we have the maxima at $r_1 = 0$ and $r_1 = 1$.

[10] An analytical proof for firm 1 is available upon request from the authors. The proof for firm 2 is analogous. Such proofs are very long and tedious, while adding little to the discussion. Therefore they have been omitted.

three roots: $\gamma_1 = 0.175$, $\gamma_2 = 1.27$ and $\gamma_3 = 1$. The only admissible value is γ_1, so that $\Pi_i^c \geq \Pi_i^{nc}$ iff $\gamma \geq \gamma_1$. Then, setting $\gamma < \gamma_1$ into Π_i^c and solving $\Pi_i^c - \Pi_i^{nc} = 0$ numerically for r_1, we find $r_{11} < r_{12}$ (whereas, setting a value $\gamma > \gamma_1$, and solving for r_1, we find $r_{11} < r_{12}$, which allows a cooperative solution and the formation of a cocktail as seen above).

■

8 Conclusions

We have analyzed the effects of the introduction of a cocktail composed of a fixed proportion of two existing stand-alone products in a Bertrand duopoly with imperfectly substitutable goods. We have shown that a cocktail rises the Bertrand equilibrium prices as it introduces a certain degree of complementarity. Moreover, it creates an incentive to sell the separate components at a discount or at premium depending on the degree of substitutability. While discounts are always detrimental to consumers, premiums might increase consumer surplus. We have then analyzed the entry of new substitutes that are not part of a cocktail. A cocktail reduces the opportunity of a single-product entrant to earn positive profits, but an incumbent can profitably introduce an innovation whenever goods are close substitutes or if they are highly differentiated. Interestingly, a higher quality incumbent's innovation can be at the same time profitable and welfare reducing, so that its introduction might harm consumers. On the other hand, it is never the case that an innovation that increases consumer surplus is not profitable.

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A Appendix

B Proof of Proposition 1

Note first that $p_i^c - p_i^{nc} = \frac{2\alpha(1-\gamma)}{(11--9\gamma)(4-3\gamma)} > 0$ and $q_i^c - q_1^{nc} = -\frac{\alpha(1-\gamma)}{(11--9\gamma)(4-3\gamma)} < 0$, $i = 1, 2$ for any $\gamma \in [0, 1)$. As for profits, $\Pi_i^c - \Pi_i^{nc} = -\frac{\alpha^2(1-\gamma)(9\gamma^2-13\gamma+2)}{(11--9\gamma)^2(4-3\gamma)^2} > 0$, $i = 1, 2$ iff $.175 < \gamma < 1$.

Results differ if the two separate drugs are offered by two independent firms. If the cocktail is not available, the Bertrand-Nash equilibrium prices are $p_1^{nc} = p_2^{nc} = \frac{2\alpha(1-\gamma)}{4-3\gamma}$, which, not surprisingly, are lower that p_M^i, $i = 1, 2$. Quantities are $q_1^{nc} = q_2^{nc} = \frac{\alpha(2-\gamma)}{2(4-3\gamma)}$ and profits $\Pi_i^{nc} = \frac{\alpha^2(2-\gamma)(1-\gamma)}{(4-3\gamma)^2}$, $i = 1, 2$.

At the moment, we simply verify the that Bertrand-Nash equilibrium prices are $p_1^c = p_2^c = \frac{6\alpha(1-\gamma)}{11-9\gamma}$, while the quantities sold of each regimen (i.e., demands of the two separate drugs, q_1^c, q_2^c and of the cocktail q_3^c) are $q_i^c = \frac{\alpha(5-3\gamma)}{33-27\gamma}$, $i = 1, 2, 3$. Given that q_3 consists of r_1 units of q_1 and r_2 units of q_2, then the equilibrium quantities are $q_1^c = q_2^c = \frac{3\alpha(5-3\gamma)}{2(33-27\gamma)}$. Finally, equilibrium profits are $\Pi_1^c = \Pi_2^c = \frac{3\alpha^2(1-\gamma)(5-3\gamma)}{(11-9\gamma)^2}$.

C The Optimal Sharing Rule when the Cocktail is offered at a discount.

When the the two firms coordinate on a discount (or premium), their goal is to maximize joint profits. When $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ and $r_1 = r_2$, substituting the equilibrium prices and quantites p_1^δ, p_2^δ, q_1^δ, $q_2^\delta$and q_3^δ into the profit function we obtain:

$$\begin{aligned}
\Pi_1^\delta + \Pi_2^\delta &= \frac{81\alpha^2(5-3\gamma)(3-4\gamma+\gamma^2)^2}{27(11-9\gamma)^2(1-\gamma)(3-\gamma)^2} + \\
&+ \frac{18\alpha\delta(3-4\gamma+\gamma^2)(8(5-8\gamma+3\gamma^2)-k(77-118\gamma+45\gamma^2))}{27(11-9\gamma)^2(1-\gamma)(3-\gamma)^2} + \\
&+ \frac{4\delta^2 A}{27(11-9\gamma)^2(1-\gamma)(3-\gamma)^2}
\end{aligned} \tag{27}$$

where $A = (16(1-\gamma)^2(5-3\gamma) - k(1760 - 4003\gamma + 3131\gamma^2 - 945\gamma^3 + 81\gamma^4) - k^2(11 - 20\gamma + 9\gamma^2)^2)$ and

$$\begin{aligned}
\Pi_2^\delta &= \frac{81\alpha^2(5-3\gamma)(3-4\gamma+\gamma^2)^2}{27(11-9\gamma)^2(1-\gamma)(3-\gamma)^2} + \\
&+ \frac{18a\delta(3-4\gamma+\gamma^2)(37-54\gamma-21\gamma^2-k(77-118\gamma+45\gamma^2))}{27(11-9\gamma)^2(1-\gamma)(3-\gamma)^2} + \\
&- \frac{4\delta^2 B}{27(11-9\gamma)^2(1-\gamma)(3-\gamma)^2}
\end{aligned} \tag{28}$$

where $B = (1801 - 4235\gamma + 3553\gamma^2 - 1257\gamma^3 + 162\gamma^4 - k(2002 - 4883\gamma + 4327\gamma^2 + 1665\gamma^3 - 243\gamma^4) + k^2(11 - 20\gamma + 9\gamma^2)^2)$.

Both expressions depend on the discount δ and on k, and the same applies to their sum. In particular, $\Pi_1^\delta + \Pi_2^\delta$ depends on k in a quadratic form and, for a given δ, the first order condition for profit maximization w.r.t. k is very simple. In fact, $\frac{\partial(\Pi_1^\delta+\Pi_2^\delta)}{\partial k} = \frac{8(1-2k)(1-\gamma)\delta^2}{27(3-\gamma)}$ and $\frac{\partial(\Pi_1^\delta+\Pi_2^\delta)}{\partial k} = 0$ when $k = \frac{1}{2}$.

D Proof of Proposition 4

Substituting $k^* = \frac{1}{2}$ and $\delta^* = \frac{3\alpha(1-4\gamma+3\gamma^2)}{(246-400\gamma+162\gamma^2)}$ on the Bertrand-Nash equilibrium prices, quantites and profits we obtain $p_i^{\delta*} = \frac{3a(1-\gamma)(45-37\gamma)}{246-400\gamma+162\gamma^2}, i = 1, 2$; $q_i^{\delta*} = \frac{\alpha(111-155\gamma+54\gamma^2)}{6(123-200\gamma+81\gamma^2)}, i = 1, 2$; $q_3^{\delta*} = \frac{\alpha(57-82\gamma+27\gamma^2)}{369-600\gamma+243\gamma^2}$ and $\Pi_i^{\delta*} = \frac{\alpha^2(1-\gamma)(61-36\gamma)}{492-800\gamma+324\gamma^2}, i = 1, 2$, respectively. In such equilibrium, consumer surplus is defined as $CS^{\delta*} = U(q_1^{\delta*}, q_2^{\delta*}, q_3^{\delta*}, M) - \left\{ \sum_{i=1}^{2} p_i^{\delta*} q_i^{\delta*} + \frac{q_3^{\delta*}(p_1^{\delta*}+p_2^{\delta*}-\delta^*)}{2} + M \right\} = \frac{\alpha^2(6216-17393\gamma+18019\gamma^2-8187\gamma^3+1377\gamma^4)}{4(123-200\gamma+81\gamma^2)^2}$. Without discount, consumer surplus was $CS^c = U(q_1^c, q_2^c, q_3^c, M) - \left\{ \sum_{i=1}^{2} p_i^c q_i^c + \frac{q_3^c(p_1^c+p_2^c)}{2} + M \right\} = \frac{\alpha^2(5-\gamma)^2}{2(11-9\gamma)^2}$ and $CS^{\delta*} - CS^c > 0$ iff $\frac{1}{3} < \gamma < .78$. In such a range for the degree of substitutability, firms would profitably sell the cocktail at a premium, so that in such case price discrimination is welfare enhancing.

E Proof of Lemma 7

Given the Bertrand-Nash equilibrium prices and quantities p_i^*, q_i^*, $i = 1, 2, 3$, equilibrium profits are

$$\begin{aligned}\Pi_1^* &= \frac{\alpha^2\left(32\gamma^5-459\gamma^4+2340\gamma^3-5184\gamma^2+5184\gamma-1944\right)}{648(\gamma-1)\left(\gamma^2-6\gamma+6\right)^2}+ \\ &+ \frac{2\alpha\alpha_3\gamma\left(16\gamma^4-237\gamma^3+1116\gamma^2-1836\gamma+972\right)}{648(\gamma-1)\left(\gamma^2-6\gamma+6\right)^2}+ \\ &+ \frac{\alpha_3^2\left(8\gamma^5-183\gamma^4+1332\gamma^3-3780\gamma^2+4536\gamma-1944\right)}{648(\gamma-1)\left(\gamma^2-6\gamma+6\right)^2}\end{aligned} \tag{29}$$

and $\Pi_2^* = \frac{(\gamma-3)\left(\alpha\left(4\gamma^2-21\gamma+18\right)+\alpha_3\gamma(2\gamma-3)\right)^2}{324(\gamma-1)(\gamma^2-6\gamma+6)^2}$.
Consumer surplus in this case is

$$\begin{aligned}CS^* &= \frac{\alpha^2\left(32\gamma^5-495\gamma^4+2862\gamma^3-7614\gamma^2+9072\gamma-3888\right)}{1296(\gamma-1)\left(\gamma^2-6\gamma+6\right)^2}+ \\ &+ \frac{2\alpha\alpha_3\gamma\left(16\gamma^4-201\gamma^3+810\gamma^2-1242\gamma+648\right)}{1296(\gamma-1)\left(\gamma^2-6\gamma+6\right)^2} \\ &+ \frac{\alpha_3\left(8\gamma^5-219\gamma^4+1422\gamma^3-3834\gamma^2+4536\gamma-1944\right)}{1296(\gamma-1)\left(\gamma^2-6\gamma+6\right)^2}\end{aligned} \tag{30}$$

When $\alpha_3 = \alpha$, equilibrium prices are $p_1^* = p_2^* = \frac{\alpha(6-7\gamma+\gamma^2)}{2(6-6\gamma+\gamma^2)}, p_3^* = \frac{\alpha(3-\gamma)(1-\gamma)}{(6-6\gamma+\gamma^2)}$. Particularly, p_1^* and p_3^* (the price of the good produced by firm 2) can be easily compared with $p_i^{nc} = \frac{2\alpha(1-\gamma)}{4-3\gamma}$, $i = 1, 2$, finding that $p_1^* > p_i^{nc} > p_3^*$ for every $\gamma \in [0, 1]$. Similarly, when $\alpha_3 = \alpha$, $\Pi_1^* = \frac{\alpha^2(6-\gamma)^2(1-\gamma)(3-2\gamma)}{18(\gamma^2-6\gamma+6)^2}$ and $\Pi_2^* = \frac{\alpha^2(3-\gamma)^3(1-\gamma)}{9(\gamma^2-6\gamma+6)^2}$ and it is easy to verify that $\Pi_2^* < \Pi_i^{nc} < \Pi_1^*$, $i : 1, 2$.Finally, consumer surplus when $\alpha_3 = \alpha$ equals $CS^* = \frac{\alpha^2\left(162-252\gamma+135\gamma^2-29\gamma^3+2\gamma^4\right)}{36(\gamma^2-6\gamma+6)^2}$ and can be compared to $CS^{nc} = U(q_1^{nc}, q_2^{nc}, M) - \left\{\sum_{i=1}^{2} p_i^{nc} q_i^{nc} + M\right\} = \frac{\alpha^2(2-\gamma)^2}{2(4-3\gamma)^2}$. Particularly $CS^* < CS^{nc}$ for every $\gamma \in (0, 1)$.

F Proof of Proposition 8

When $\alpha_3 \neq \alpha$,and using the profits functions introduced in the Proof of Lemma 7, we first notice that $\Pi_1^* \geq \Pi_1^{nc}$ whenever $\gamma \geq 0.836$. If $\gamma < 0.836$, then $\Pi_1^* > \Pi_1^{nc}$ if either $R < R_1(\gamma)$ or $R > R_2(\gamma)$, where $R = \frac{\alpha_3}{\alpha}$ and $R_1(\gamma)$, $R_2(\gamma)$ are the solutions to $\Pi_1^* - \Pi_1^{nc} = 0$. $R_1(\gamma)$ and $R_2(\gamma)$ are plotted in Figure 1, panel (a). It can be seen that $R_1(\gamma) > 0$ for $\gamma > 0.55$ and that it takes its maximum value at $\gamma = 0.836$, where $R_1(0.836) = 0.56$. Thus we can have that $\Pi_1^* > \Pi_1^{nc}$ for α_3 much lower than α. It can also be seen that $R_2(\gamma)$ is always positive and increasing, with $R_2(0) = 0.7$ and $R_2(0.836) = 0.76$. Then $\Pi_1^* > \Pi_1^{nc}$ also when $\alpha_3 > \alpha$ but for the introduction of the new product to be profitable we don't need that α_3 is much higher than α. From Figure 1(a), profitability requires R to lie outside the area delimited by $R_1(\gamma)$ and $R_2(\gamma)$. Notice that the higher γ (i.e., the closer substitutes the goods), the more differentiated the goods have to be in order for the production of good 3 to be profitable for firm 1.

Similar conclusions can be obtained for consumer surplus. Again, $CS^* > CS^{nc}$ iff either $R < R_1^{CS}(\gamma)$ or $R > R_2^{CS}(\gamma)$. However, Figure 1, panel (b) shows that $R_1^{CS}(\gamma)$ lies below $R_1(\gamma)$, whereas $R_2^{CS}(\gamma)$ lies above $R_2(\gamma)$. In order for the introduction of the new good to be welfare enhancing for consumers, α and α_3 have to differ quite substantially. Thus, the degree of product differentiation that ensures profitability for the introduction of product 3 is much lower than the degree of differentiation that guarantees higher consumer surplus. In any case, the required degree of differentiation increases with γ.

In conclusion, from Figure 1(b) it can be seen that the introduction of the new product by firm 1 may be profitable but not good for consumers. This occurs when R lies between the (smaller) area delimited by $R_1(\gamma)$, $R_2(\gamma)$ and the area delimited by $R_1^{CS}(\gamma)$, $R_1^{CS}(\gamma)$.

G Proof of Proposition 9

Substituting equilibrium prices and quantities, profits as a function of δ and k are

$$\Pi_{1d}^{3c} = \frac{\alpha^2(\gamma-1)\left(882\gamma^3-8575\gamma^2+25614\gamma-21672\right)\delta}{18(21\gamma^2-98\gamma+88)^2}+$$
$$+\frac{\alpha\delta\left(-3843\gamma^3+25294\gamma^2-44392\gamma+\left(6237\gamma^3-42506\gamma^2+78344\gamma-44352\right)k+23040\right)}{54(21\gamma^2-98\gamma+88)^2}+$$
$$+\frac{\delta^2\left(-567\gamma^4+23382\gamma^3-82504\gamma^2+100352\gamma+2\left(17577\gamma^4-213282\gamma^3+745832\gamma^2-993856\gamma+450560\right)k-40960\right)}{648(\gamma-1)(21\gamma^2-98\gamma+88)^2}+$$
$$+\frac{\left(26649\gamma^4-130266\gamma^3+241304\gamma^2-199936\gamma+61952\right)\delta^2k^2}{648(\gamma-1)(21\gamma^2-98\gamma+88)^2} \text{ and}$$

$$\Pi_{2d}^{3c} = \frac{\left(14175\gamma^4-69102\gamma^3+124216\gamma^2-99968\gamma+30976\right)\delta^2 k^2}{324(\gamma-1)(21\gamma^2-98\gamma+88)^2}+$$
$$+\frac{\delta\left(36\alpha^2(9\gamma-20)\left(7\gamma^2-25\gamma+18\right)^2-6\alpha(\gamma-1)\left(-4473\gamma^3+24098\gamma^2-39848\gamma+\left(8505\gamma^3-47458\gamma^2+82216\gamma-44352\right)k+21312\right)\right)}{324(\gamma-1)(21\gamma^2-98\gamma+88)^2}+$$
$$+\frac{\delta^2\left(31185\gamma^4-268218\gamma^3+826456\gamma^2-1043648\gamma-8\left(5670\gamma^4-43317\gamma^3+123698\gamma^2-149224\gamma+64064\right)k+461056\right)}{324(\gamma-1)(21\gamma^2-98\gamma+88)^2}, \text{ for}$$

firms 1 and 2, respectively.

Firms 1 and 2 maximize joints profits choosing δ. Then, they choose how to share the burden of the discount (if $\delta > 0$) or the gains from the price premium (if $\delta < 0$) by fixing k, again cooperatively. Proceeding by backward induction, maximizing total profits with respect to k for every possible value of $\delta \in \left[-\inf, \frac{1}{2}q_1 + \frac{1}{2}q_2\right]$,

$$k^{3c} = \frac{24\alpha\gamma\left(567\gamma^3 - 1805\gamma^2 + 2206\gamma - 968\right) + \left(27783\gamma^4 - 133254\gamma^3 + 243752\gamma^2 - 199936\gamma + 61952\right)\delta}{\left(54999\gamma^4 - 268470\gamma^3 + 489736\gamma^2 - 399872\gamma + 123904\right)\delta} \tag{31}$$

It is immediate to see from (31) that, if $\delta < 0$ (i.e., the good is sold at a premium), k^{3c} is negative, meaning that the firm producing two goods would be wiling to transfer money to the other competitor. To simplify the analysis, we assume this is ruled out. So, if $\delta < 0$, $k^{3c} = 0$.

We then need to check when this is the case. Assume first that $\delta > 0$ and check whether such assumption is consistent with $0 \leq k \leq 1$. In order to do that, we substitute $k = k^{3c}$ into the expression for total profits $\Pi_1^{3c} + \Pi_2^{3c}$, obtaining

$$\begin{aligned}\Pi^{Tot} &= \frac{\alpha^2(\gamma-1)^2\left(24444\gamma^3 - 101313\gamma^2 + 135392\gamma - 61568\right) - 2\alpha\left(1191\gamma^4 - 4203\gamma^3 + 5444\gamma^2 - 2944\gamma\right.}{2(\gamma-1)\left(54999\gamma^4 - 268470\gamma^3 + 489736\gamma^2 - 399872\gamma + 123904\right)} \\ &+ \frac{\left(18387\gamma^4 - 90176\gamma^3 + 165072\gamma^2 - 135168\gamma + 41984\right)\delta^2}{2(\gamma-1)\left(54999\gamma^4 - 268470\gamma^3 + 489736\gamma^2 - 399872\gamma + 123904\right)}\end{aligned}$$

We then maximize (32) with respect to δ, obtaining

$$\delta^{3c} = \frac{\alpha\left(1191\gamma^4 - 4203\gamma^3 + 5444\gamma^2 - 2944\gamma + 512\right)}{18387\gamma^4 - 90176\gamma^3 + 165072\gamma^2 - 135168\gamma + 41984} \tag{33}$$

and substituting δ^{3c} from (33) into (31):

$$k^{*3c} = \frac{-5151\gamma^3 + 11508\gamma^2 - 9088\gamma + 256}{-1191\gamma^3 + 3012\gamma^2 - 2432\gamma + 512} \tag{34}$$

It is immediate to check that $0 \leq k^{*3c} \leq 1$ if and only if $\gamma \leq 0.029$, and that k^{*3c} is decreasing in γ, with $k^{*3c} = \frac{1}{2}$ when $\gamma = 0$. Then, for $\gamma \leq 0.029$, $\delta > 0$: firms apply a discount on the cocktail and $k^{*3c} < 1$, decreasing from $\frac{1}{2}$ when $\gamma = 0$ to 0 when $\gamma = 0.029$.

When $\gamma > 0.029$, k^{*3c} hits the lower boundary and becomes zero. We then substitute $k = 0$ into the expression for total profits and maximize with respect to δ. We obtain

$$\delta_0^{3c} = \frac{12\alpha\left(-315\gamma^4 - 283\gamma^3 + 2870\gamma^2 - 3136\gamma + 864\right)}{61803\gamma^4 - 513054\gamma^3 + 1570408\gamma^2 - 1986944\gamma + 881152} \tag{35}$$

which is positive (implying discount) for $\gamma \leq 0.44$ and negative (premium) for $\gamma \geq 0.44$. This implies that, for $\gamma \geq 0.029$, the firm producing two goods charges the same price p_1, no matter whether good 1 is sold on its own or combined with good 2 in a cocktail. Firm 2, however, will either sell at a discount (that it bears entirely) or at a premium (which it fully enjoys).