# Dynamically Complex Balanced Economic Growth with Innovation

## -A Synthesis of the Theories of Ricardo, Keynes and Schumpeter -

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## 1. Introduction

In this paper, we try to synthesize Ricardo's labor theory, Keynes' theory of effective demand and Schumpeter's theory that the fundamental factor of economic development lies in the creative destruction, by game theory [1]. R&D which brings about innovation needs enormous funds and human capital. In particular, we should consider that human capital, if it succeeds in innovation, creates value and the value is involved in the productions. Therefore, labor theory concerning human capital is inevitable. In addition, this design must be described in the macroeconomic structure which involves Keynesian fundamental equation (i.e. savings is equal to investments), because we analyze the capitalism economy. On the other hand, from the Schumpeter's point of view, we must investigate whether there exists a mechanism in the economy which fosters entrepreneur's spirit which brings about innovation and achieves economic development. One purpose of this paper is to show the dynamically complex balanced economic growth which is defined precisely later in this paper is theoretically possible if we succeed in synthesizing three theories.

On the other hand, if economy achieves the balanced growth, economy would reach full employment. However, in the model presented in this paper, once economy achieves full employment, players of the game ,capitalists and workers in our model, would cease savings. In this case, no one could invest, money (or assets) market would vanish and capitalism would collapse. We call this phenomenon *the saturation collapse of the economy.* Can capitalism survive? The answer for this problem could be required in the Schumpeterian point of view, i.e. innovation. We will analyze how economy could avoid the saturation collapse and hold the level of unemployment low. Keyword is cyclical R&D. This cyclical R&D is expected to work to avoid these problems.

However, if economy does not involve a mechanism which could achieve this cyclical R&D, it becomes important to make political supports for R&D sectors. These policies are considered as the R&D version of the Keynesian policy. Capitalism economy always experiences tidal waves of innovation. Innovation is supported by R&D firms achieve. Although R&D reflects the strategy with which firms intend to acquire monopolistic

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[1] Our model is also Keynesian in the meaning that it involves Kaldor's fundamental equation.

surplus profits by innovation, it is essentially the strategy for the survival of the firms, as Schumpeter pointed out[2]. R&D and innovation construct the engine of economic development. During some decades in the past, the fiscal and monetary policies which are supported by the theory of Keynes have been achieved. However, considering from the historical point of view, innovation has been playing an essential role. This fact implies that economy needs Schumpeter's point of view and requires to construct a new theory. For this purpose, it is necessary to rearrange the multiple economic theories and synthesize them. Therefore, let us survey the theories on R&D and innovation which generate the design of this paper from both microeconomic and macroeconomic points of view. This study enables us to confirm the position of our model among economic theories.

Firstly, let us survey microeconomic theories. The theories on R&D have been promoted mainly by the development of game theory. The pioneering studies in this field are Loury(1979), Dasgupta and Stiglitz(1980), Lee and Wilde(1980) , Reinganum(1981) and Malueg,D.A., and S. O. Tsutsui(1997). These multiple analyses in game theory imply that simple Schumpeter's hypothesis that the larger the scale of firm, the more advantageous the R&D becomes cannot necessarily be correct. These fruitful studies imply that the theory, if it is related not only to R&D but also to innovation, needs to be described by game theory. However, it must be noted that theories of R&D and innovation described by game theory are constricted in the field of microeconomics. Therefore, we must pump these fruitful game theories into macroeconomics. In addition, if R&D theories are applied in the macroeconomic field, we presume that it must be constructed by the complex and evolutional processes. This means that the theory which is constructed only on the basis of the concept of equilibrium could only explain the restricted phenomena.

Secondly, let us survey theories of innovation from macroeconomics. This field also involves multiple studies. Now we classify these into two categories. One category is endogenous economic growth theory. The representative studies in this category are Romer,P.M.(1990), Aghion,P.,and P. Howitt(1992), Freeman,S.,Hong Dong-Pyo, and Dan Peled, (1998). These studies depend on the concept of equilibrium. However, as is mentioned above, when we consider R&D in the field of macroeconomics, the concept of equilibrium can only explain the restricted phenomena. Hence, we must concentrate our attentions on the game theoretic and evolutional phenomena including the concept of equilibrium such as Nash equilibrium and balanced economic growth.

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[2] Schumpeter (1954), p.83.

Another category is the genealogy of Ricardo and Keynes. This school involves Pasinetti,L.L. and R.M.,Goodwin. As Soviotti(2006) pointed out, the theories of both categories treat the complexity of innovation. The conflicts of these schools mean the multiplicity of innovation. This paper is on the basis of the genealogy of Ricardo. Therefore, next, we will consider the characteristic of the theories by Pasinetti and Goodwin.

Innovation involves two types, product innovation and process innovation. Considering these innovations from the macroeconomic point of view, they bring about the changes in the relationships between industries, destroy the old input-output structure of production process and generate new one. As is pointed out by Schumpeter[3], this process can be considered as the process of generating new combinations. Although the concept of new combination includes five cases, the structure of economy, in particular, the structure of production is destroyed in consequence of the new combinations. Expressing these phenomena by the technical term of input-output analysis, the input-output coefficient matrix is destroyed and newly reconstructed. The more drastic the innovation is, the more violent the destruction is. Hence, for analyzing the effect of innovation on the macro-economy, it becomes important to analyze the change of the matrix totally. This analysis is achieved by Pasinetti(1981), using the concept of vertically integrated sector .

By the way, multi-sector models involve the characteristic problems, such as dynamical stability, choice of technology and income distribution, etc. Among them, we concentrate our attentions on the study by Sraffa(1960). Sraffa's study plays an important role to analyze production theory, theory of labor value and income distribution. Of course, Sraffa didn't focus his attentions on innovation, but successors to Sraffa, Pasinetti and Goodwin, have developed the theories which involve innovation. Although their studies are based on the common foundation, i.e.Sraffa, there are many differences between their theories. For example, Pasinetti introduces the concept of vertically integrated sector and analyzes the changes in coefficients. However, for this sake, he had to abandon the analysis of economic fluctuations. On the other hand, Goodwin analyzes economic fluctuation, technical progress and income distribution. But, for this sake, he had to keep the technical coefficients constant. But, these differences make it possible for each theory to complement the other. Our model is based on vertically integrated sectors, but also consider about economic growth.

So far, we have surveyed theories on R&D and innovation from both microeconomic

[3] Schumpeter(1934),p.65-74.

and macroeconomic points of view. In this stage, we have to point out that there exists a deep gap between them. Hence, it is required to achieve a new theoretical approach to vanish this gap. The theories which have the possibility to vanish this gap are evolutional economic theory, game theory and complex dynamic theory. The studies on which we concentrate our attention are Iwai(1984),(2000) and (2001). Iwai abandoned the concept of equilibrium and presented the model which described the diffusion process of new product and new technology. These studies make a definite contrast with the studies such as Segerstrom, Anant and Dinopoulos(1990), which analyzed the product life cycle on the basis of equilibrium theory. But, the theories on which we focus are game theory and complex dynamic theory. Let us explain the relationship between these theories and our paper.

Firstly, let us start with game theory. We have surveyed microeconomic game theory above. These studies are also applied in the macroeconomic theory. The origin of the macroeconomic, game theoretic models is Lancaster(1971).Players in his model are capitalists and workers. The purpose of this study is to show the inefficiency of capitalism, using the method of differential game. This study is developed by Basar,T.,Haurie,A.,and G.Ricci(1985) , Pohjola,M.(1985), Kaitala,V., and M. Pohjola(1990) and Benabou,Roland and J. Tirole(2006). But, these studies are unrelated to R&D and innovation. The study which introduces R&D and innovation into maco-dynamic game is Akimoto(2006). Although this study is written in Japanese, it reconstructs the macroeconomic-R&D-game which has households and investors as players. We follow this game in this paper.

Secondly, let us consider about complex dynamic theory. In this category, the representative authors are Lorenz and Goodwin. Goodwin analyzes Schumpeterian dynamical aspects using nonlinear models and presents chaotic dynamics including some attractors. However, the study on which we focus in this paper is Feichtinger and Sorger(1988). They considered a scientist who has two main activities, basic R&D and the application of the knowledge obtained by basic R&D. They showed the existence of a periodic solution between basic R&D and its application by dynamic programming. The characteristic structure of this model is that the state variable is controlled not directly but indirectly. Although there is a mistake about the analysis of the steady state, this study is applicable to macroeconomics and implies the importance of the periodic R&D in the macro-economy. This study is also achieved in Akimoto(2006)(which is written in Japanese). The model in this paper shows by game theory that the cyclical R&D is inevitable for the balanced economic growth. Although a scientist makes the decision between basic R&D and its application in Feichtinger and Sorger(1988), we instead introduce the player who determines the strategy between process innovation and product innovation. These two types of innovation are essential. They may play a complementary role in the macro-economy. Hence, the problems which must be considered are set as follows. Whether does a mechanism exist which generates these types of innovation in the economy? If it does not exist, what kind of policy should be made? And what balance between two types of innovation is desired for the balanced economic growth?

Based on the consciousness mentioned above, this paper is constructed. For this sake, we try to synthesize some theories. This paper is organized as follows. In section 2, we construct the game theoretic model which is based on Ricardo, and show the existence of the balanced economic growth path and the possibility of the crisis of saturation collapse. In this section, we do not introduce R&D and innovation into the model.

In section 3, we make some preparations for applying multi-sector model to our model and introducing R&D and innovation. The theory considered in this section is Pasinetti's vertically integrated analysis. The vertically integrated sectors are constructed up to the sectors of the higher order, theoretically up to infinity. However, in his study (1981), he constructs simplified model which includes only the sectors of the first order to explain the essence of technical progress. We introduce the sectors of the higher order up to infinity. This introduction is necessary to reconstruct the Ricardian game theoretic model in section 4 where R&D is introduced. The analysis in this section is essentially the same with Pasinetti's analysis.

In section 4, we reconstruct the game theoretic model starting with the vertically integrated sectors. Firstly, we consider the unit on the basis of labor theory. Using this unit, we compose the intensive model from vertically integrated sectors.

In section 5, we introduce R&D to the model in section 4. Our purpose is to analyze the conditions that make it possible to hold the balanced economic growth which is measured by the unit defined in section 4. The balanced economic growth is expressed by the complex and evolutionary path with innovation.

## 2. BALANCED ECONOMIC GROWTH IN GAME THEORETIC RICARDIAN MODEL

### 2.1 Model

In this section, we construct a game theoretic model which has capitalists, workers and capital distributor as players of the game. Capital distributor is the personification of money (or asset) market in our game and decides the distribution of capital goods between production sectors.

### 2.1.1 Construction of model

The model contains production sectors, price system and the fundamental equation of Kaldor's model. Let us construct our model.

**Production sectors**

We assume that the economy has two production sectors, namely the sector producing consumption goods and the sector producing capital goods. We call the former sector 0 and the latter sector 1 and allocate suffixes 0 and 1 respectively. The technology of each sector is expressed by labor coefficient $n_i$ and capital-output ratio $b_i$ ( $i = 0,1$ ). Coefficients $n_i$ and $b_i$ are positive constants in this section. Furthermore, let $X_i$ and $N_i$ represent the output and the labor employed in sector $i$ respectively. The relationship of the output of sector $i$, $X_i$, and capital stocks in sector $i$, $K_i$, is represented by

$$X_i = \frac{1}{b_i} K_i .$$

We denote the amount of capital stocks in the economy by $K$ and denote the ratio of the distribution of capital stocks to sector 0 by $v$ and the ratio to sector 1 by $1-v$. Then, we obtain

$$K_0 = vK, \tag{2-1}$$

$$K_1 = (1-v)K, \tag{2-2}$$

$$X_0 = \frac{1}{b_0} vK, \tag{2-3}$$

$$X_1 = \frac{1}{b_1}(1-v)K. \tag{2-4}$$

In addition, we obtain

$$N_0 = n_0 X_0 \quad , \tag{2-5}$$

$$N_1 = n_1 X_1 , \tag{2-6}$$

$$N = N_0 + N_1 , \tag{2-7}$$

where $N$ is the total amount of labor employed. We put some assumptions about capital stocks.

**Assumption2.1** In the process of production, capital stocks are not worn out.

**Assumption2.2** Capital stocks can move freely between sector 0 and sector 1.

**Assumption2.3** Capital stocks are employed completely.

By these assumptions, we obtain

$$X_1 = \dot{K} (= \dot{K}_0 + \dot{K}_1) , \tag{2-8}$$

where dot ( $\cdot$ ) denotes $\cdot \equiv \frac{d}{dt}$ and $t$ denotes time. Although (2-8) defines the differential equation of capital stocks $K$, the determination of distribution of capital stocks to each sector needs the level of profit rate. Namely, profit rate affects the distribution of capital stocks between two sectors. Therefore, next, we construct the price system which contains the profit rate.

**Price system**

Let $p_i$ denote the price of the product of sector $i$ $(i = 0,1,)$. We assume that the prices are determined as the sum of the wages which are paid to the labor employed and the profits which are required for the use of the capital. Then, from assumption 2.1, we obtain

$$\begin{cases} p_0 = n_0 w + \pi b_0 p_1, \\ p_1 = n_1 w + \pi b_1 p_1, \end{cases} \tag{2-9}$$

where $w$ and $\pi$ denote wage rate and profit rate respectively. We can solve equation (2-9) immediately. We obtain

$$p_0 = \left( n_0 + \frac{\pi n_1 b_0}{1 - \pi b_1} \right) w, \tag{2-10}$$

$$p_1 = \frac{n_1}{1 - \pi b_1} w. \tag{2-11}$$

In equation (2-11), $\frac{n_1}{1 - \pi b_1}$ is the quantity of labor which is required in the whole economy to produce one unit of capital[4]. Since the prices must be positive, we assume

$$\pi < \frac{1}{b_1} \quad . \tag{2-12}$$

Equations (2-10) and (2-11) contain four unknowns: $p_0, p_1, \pi, w$. As is explained later, profit rate $\pi$ is determined in the process of the distribution of capital stocks between two sectors. Therefore, the price system has one degree of freedom. So we choose $p_0$ as the *nume'raire* to close the price system. That is,

$$p_0 = 1. \tag{2-13}$$

**Equilibrium condition : fundamental equation of Kaldor's model**

[4] This is explained by expanding the right-hand side of (2-11):

$$\frac{n_1}{1 - \pi b_1} = n_1 + (\pi b_1) n_1 + (\pi b_1)^2 n_1 + \cdots,$$

When $p_0$ and $p_1$ are determined under some profit rate $\pi$ and some wage rate $w$ and $X_0, X_1, N_0$ and $N_1$ are determined in the production sectors, GDP $Y$ which is equal to the sum of total profits $\Pi$ and total wages $W$ in our model should be determined in the economy at the same time. We will show this fact. From assumption2.1, we obtain

$$Y = p_0 X_0 + p_1 X_1 . \tag{2-14}$$

Substituting (2-1)-(2-4),(2-10) and (2-11) for(2-14) , we obtain

$$Y = \left\{ \frac{\pi n_1}{1-\pi b_1} w K_0 + \left( \frac{1}{1-\pi b_1} - 1 \right) \frac{n_1}{b_1} w K_1 \right\} + \left( \frac{n_0}{b_0} K_0 + \frac{n_1}{b_1} K_1 \right) w . \tag{2-15}$$

It is easy to verify that

$$\begin{aligned} W &= (n_0 X_0 + n_1 X_1) w \\ &= \left( \frac{n_0}{b_0} K_0 + \frac{n_1}{b_1} K_1 \right) w, \end{aligned} \tag{2-16}$$

$$\begin{aligned} \Pi &= \pi p_1 K_0 + (p_1 X_1 - N_1 w) \\ &= \frac{\pi n_1}{1-\pi b_1} w K_0 + \left( \frac{1}{1-\pi b_1} - 1 \right) \frac{n_1}{b_1} w K_1 . \end{aligned} \tag{2-17}$$

Therefore, from equation (2-14), we have

$$Y = \Pi + W . \tag{2-18}$$

(2-18) shows GDP $Y$ is distributed to capitalists and workers. Therefore, from equilibrium condition, i.e. investments are equal to savings, we obtain

$$p_1 X_1 = s_p \Pi + s_w W \tag{2-19}$$

where $s_p$ and $s_w$ denote saving rate of capitalists and workers respectively and $0 \le s_p \le 1$ and $0 \le s_w \le 1$. As is explained in the following section, $s_p$ and $s_w$ are the strategies of capitalists and workers.

**Players of the macroeconomic game**

Now, we construct a macroeconomic model with game theory. Players of the game are capitalists, workers and capital distributor. As is mentioned above, capital distributor is the personification of money (or asset) market. Capitalists' and workers' objects are to maximize the profit rate and the wage rate, respectively. Therefore, we can define these problems as

$$\max_{s_p} \pi, \qquad \text{s.t. } 0 \le s_p \le 1 \tag{2-20}$$

$$\max_{s_w} w, \qquad \text{s.t. } 0 \le s_w \le 1 \tag{2-21}$$

where $s_p$ and $s_w$ are the strategies of the players.

Next, let us see the equation (2-19). It represents the equilibrium condition which requires the distribution of capital stocks. Therefore, the economy must contain the process by which capital stocks are distributed between two production sectors. How is this distribution determined? In our game model, savings enter into sector 1 as investments. So, we set a player who decides the distribution of capital stocks. We call this player capital distributor. Her problem is to maximize profit. In other words, she acts like investors in money (or assets) markets. Capital distributor decides the distribution of capital stocks between two production sectors. Her strategy is the investment ratio $v$. We define her problem as follows:

$$\max_{v} \pi, \quad \text{s.t. } 0 \le v \le 1. \tag{2-22}$$

**To complete the model**

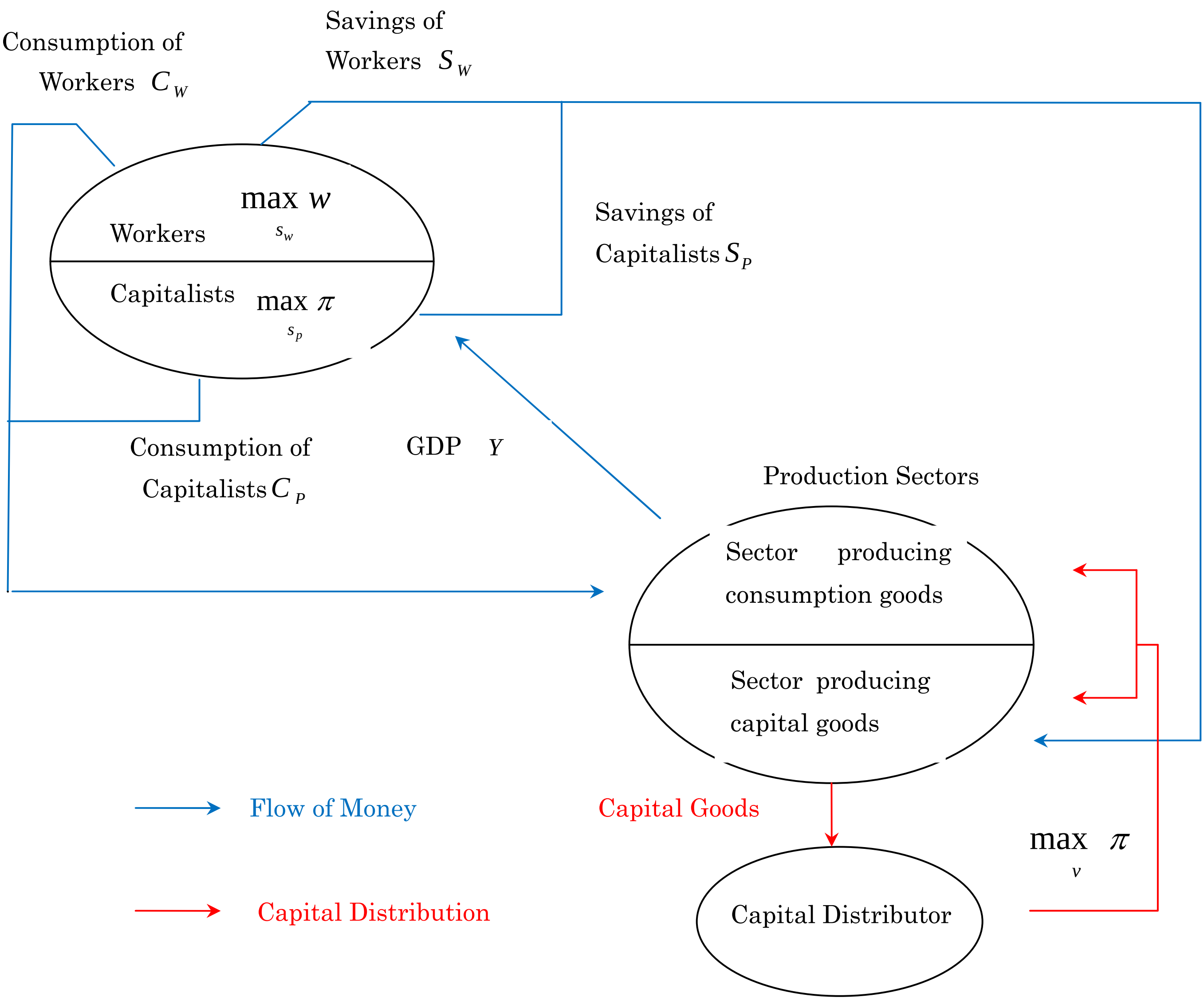
We have constructed the model. We have 15 unknowns: $X_1, X_2, p_1, p_2, K, K_1, K_2, N,$ $N_1,$ $N_2, w, \pi, s_p, s_w v.$ For these unknowns, we have 15 equations: (2-1)~(2-8), (2-10),(2-11),(2-13), (2-19)~(2~22). These conditions complete the equation system.

### 2.1.2 Structure of the model

Let us explain the macroeconomic structure of the model. Figure2-1 shows this structure. In production sectors, sector 0 and sector 1, GDP is produced. It is distributed between capitalists and workers. They consume some part of their income. The rest, savings, are invested into sector 1. Capital goods produced are handed to capital distributor whose problem is to decide the distribution rate $v$ to maximize the profit rate.

Since our model is constructed as a game, information structure is important. The information about the model described in Fig.2-1 is assumed to be the common knowledge between players. Namely, the game is constructed on the complete and perfect information. Decision making is achieved simultaneously. Hence, the solution of the game, if it would exist, is a Nash equilibrium.

**Figure2-1　Flows of Money and Capital Goods in the Game**




### 2.2 Searching for solution

Referring to the conclusion ahead, the game has a Nash equilibrium which generates the balanced economic growth. The proof of the existence of the Nash equilibrium will be showed in the following paragraph. In this paragraph, we will demonstrate the process of finding the equilibrium which helps us to understand how players search for the equilibrium.

2.2.1 Profit function

Let us focus our attentions on equation (2-19). Substitute (2-1) , (2-2) for (2-16) ,(2-17). Next, substitute these (2-16) , (2-17) and (2-4), (2-11) for (2-19) and rearrange them. Then, we obtain

$$\pi = \frac{\frac{n_1}{b_1}(1-s_w) - \left\{\frac{n_1}{b_1} - \left(\frac{n_1}{b_1} - \frac{n_0}{b_0}\right) s_w\right\} v}{(s_p - s_w)n_1 + b_1 s_w\left(\frac{n_1}{b_1} - \frac{n_0}{b_0}\right) v} = A + \frac{B}{C + Dv} \quad , \qquad (2\text{-}23)$$

where

$$A = -\frac{\frac{n_0}{b_0} s_w + (1-s_w)\frac{n_1}{b_1}}{b_1 s_w\left(\frac{n_1}{b_1} - \frac{n_0}{b_0}\right)} \quad , \quad B = \frac{n_1\left[\frac{n_1}{b_1} s_p(1-s_w) - \frac{n_0}{b_0} s_w(1-s_p)\right]}{b_1 s_w\left(\frac{n_1}{b_1} - \frac{n_0}{b_0}\right)} \quad ,$$

$$C = (s_p - s_w)n_1 \; , \qquad D = b_1 s_w\left(\frac{n_1}{b_1} - \frac{n_0}{b_0}\right).$$

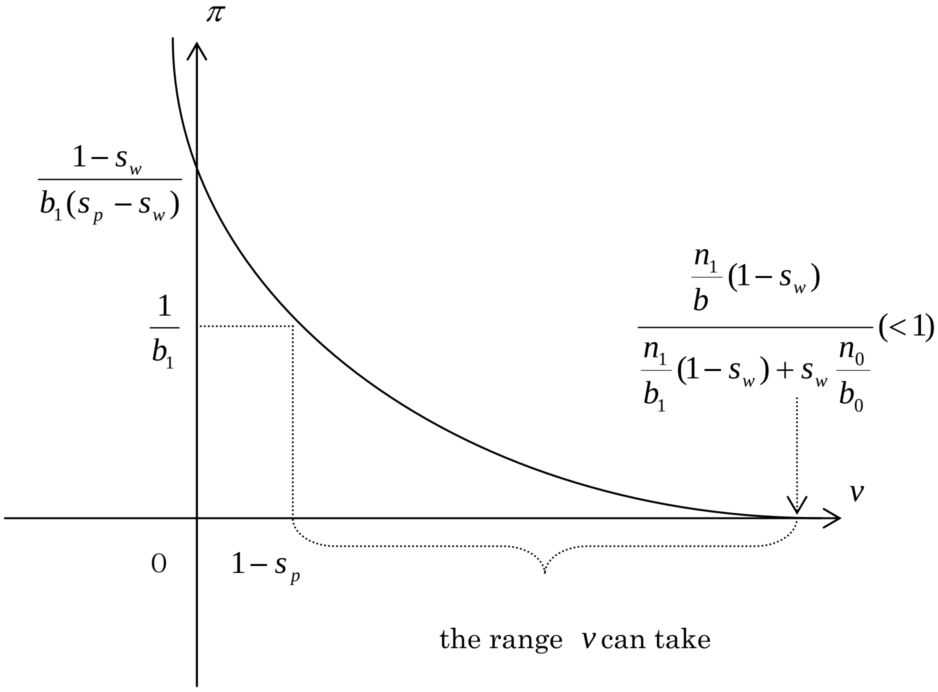
The equation (2-23) expresses a hyperbola of the variable $v$. Although $A$, $B$, $C$ and $D$ contain the strategies of players, $s_w$, $s_p$, we may depict the graph of (2-23) in Fig.2-2, considering $s_w$, $s_p$ .as parameters. The signs of 'plus or minus' of $A$, $B$, $C$ and $D$ depend on the conditions of the technical coefficients. However, as we point out later, the condition $\frac{n_0}{b_0} \geq \frac{n_1}{b_1}$ could not be allowed economically. Hence, we put the following assumption.

**Assumption2.4** It is assumed to be held that

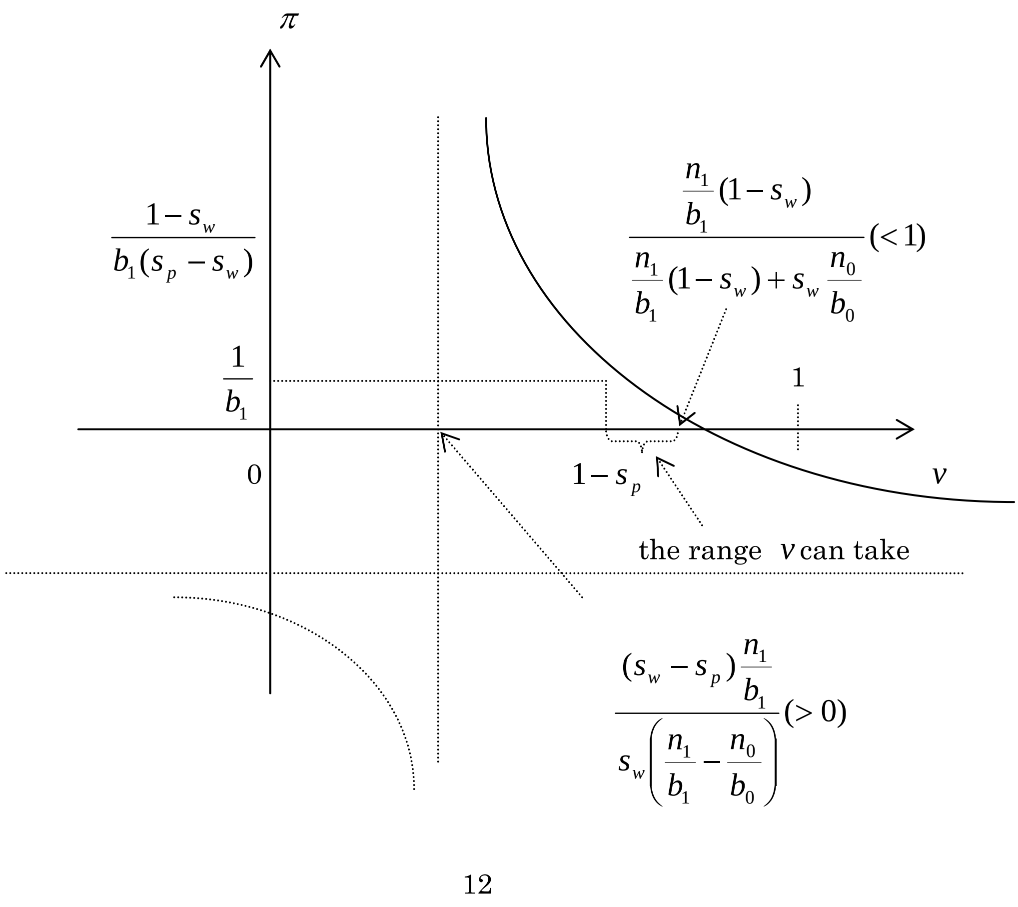
$$\frac{n_0}{b_0} < \frac{n_1}{b_1} \qquad (2\text{-}24)$$

Let us depict the graph of (2-23). Since $A < 0, D > 0$ by the assumption 2.4, the figure of the hyperbola depends on the signs of $B$ and $C$. The graphs are depicted in Fig.2-2(a)-(c).

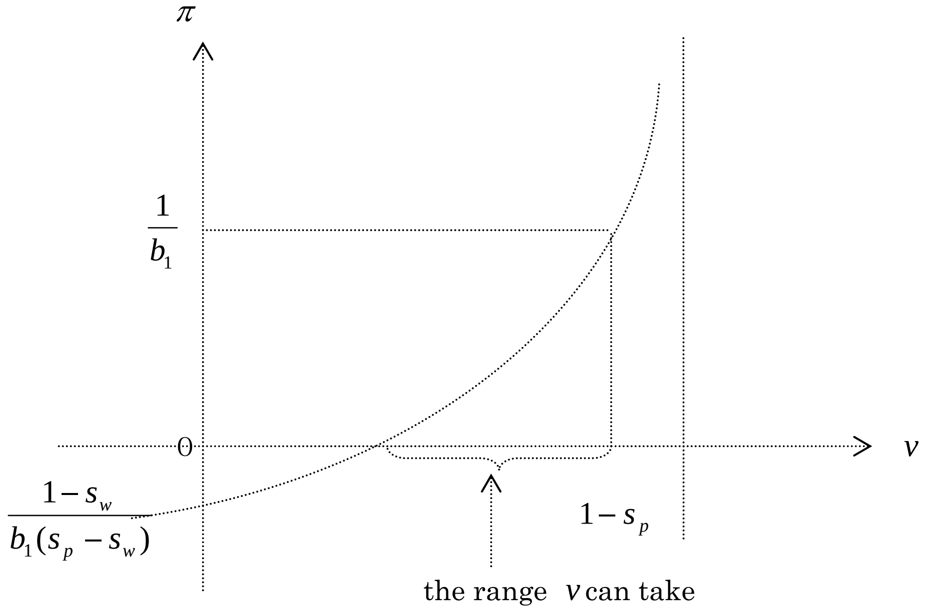
**Figure2-1** (a) the case of $B > 0$, $s_p > s_w$




(b) the case of $B > 0$, $s_p < s_w$



(c) the case of $B<0$, $s_p<s_w$




By assumption2.4, the case of $B<0$ and $s_p>s_w$ does not occur.

Next, let us analyze the process of decision making of the players.

**The problem of capital distributor**

As is shown in Fig.2-2, we have three cases. As is defined in (2-22),whatever case occurs, capital distributor intends to maximize the profit rate. Therefore, she necessarily chooses the strategy $v=1-s_p$ to get the profit rate $\pi=\frac{1}{b_1}$.

**The problem of workers**

On the other hand, we obtain the wage-profit curve

$$\pi=\frac{1-n_0 w}{b_1+(n_1 b_0-n_0 b_1)w} \tag{2-25}$$

from (2-10) and (2-13). If the capital distributor chooses the strategy $v=1-s_p$, the wage rate $w$ becomes $w=0$. Workers whose problem is defined by (2-21) intend to avoid this situation's happening. There exists only one strategy with which workers could avoid it. That is, workers have only to choose the strategy which accomplishes $B=0$ in (2-23). This condition is denoted by

$$\frac{b_1}{n_1}s_p(1-s_w)=\frac{b_0}{n_0}s_w(1-s_p) \tag{2-26}$$

Workers should choose the strategy $s_w (\in (0,1))$ which satisfies (2-26) for the capitalists' strategy $s_p (\in (0, \frac{1}{b_1}))$ .

**The problem of capitalists**

If workers choose the strategy which satisfies (2-26), the profit rate which capitalists intend to maximize becomes $\pi = A(< 0)$ (see assumption2.4). Therefore, capitalists have to avoid the situation of $\pi = A$. For this sake, the strategy remained to capitalists is the one which satisfies

$$C + Dv = 0. \qquad (2\text{-}27)$$

**Again, the problem of capital distributor**

If workers and capitalists choose the strategies which satisfy (2-26) and (2-27) , capital distributor could not determine the profit rate by (2-22) and (2-23). But, whatever situation occurs, her problem is to get the maximum profit rate which is available under any situation. To get the profit rate, capital distributor certainly considers that the two production sectors must be sustained forever, because one of the production sectors vanishes, the economy would be destroyed and the profit rate could not be obtained. Capital distributor must consider how much profit rate she could get, if she constructs the balanced economic growth. Therefore, firstly, she certainly makes the following calculations to construct the balanced economic growth.

From (2-2) and (2-8), we obtain

$$\dot{K} = X_1 = \frac{1}{b_1} K_1,$$

and

$$\frac{\dot{K}}{K_1} = \frac{1}{b_1}.$$

On the other hand, since $K_0 = vK$ , $K_1 = (1-v)K$ , we obtain

$$\dot{K}_0 = v\dot{K} = \frac{v}{b_1} K_1,$$

$$\dot{K}_1 = (1-v)\dot{K} = \frac{(1-v)}{b_1} K_1.$$

We can denote these equations by matrix. The equations can be denoted by

$$\begin{bmatrix} \dot{K}_0 \\ \dot{K}_1 \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{v}{b_1} \\ 0 & \dfrac{1-v}{b_1} \end{bmatrix} \begin{bmatrix} K_0 \\ K_1 \end{bmatrix}.$$

Caluculating the eganvalues and the eganvactors of the matrix:

$$\begin{bmatrix} 0 & \dfrac{v}{b_1} \\ 0 & \dfrac{1-v}{b_1} \end{bmatrix},$$

we obtain

$$\lambda_0 = 0, \quad \lambda_1 = \frac{1-v}{b_1}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad h_1 = \begin{bmatrix} \dfrac{v}{b_1} \\ \dfrac{1-v}{b_1} \end{bmatrix}$$

where the eganvector $h_i$ corresponds to the eganvalue $\lambda_i \ (i = 0,1)$. Therefore, the capital vector which denotes the balanced economic growth is

$$\begin{bmatrix} K_0 \\ K_1 \end{bmatrix} = \overline{K} \exp\left\{ \left( \frac{1-v}{b_1} \right) t \right\} \begin{bmatrix} \dfrac{v}{b_1} \\ \dfrac{1-v}{b_1} \end{bmatrix} \tag{2-28}$$

where $\overline{K}$ denotes the initial level of capital stocks. On the balanced economic growth path, it is easily verified that

$$\frac{\dot{K}_0}{K_0} = \frac{\dot{K}_1}{K_1} = \frac{1}{b_1}(1-v).$$

Furthermore, the growth rate of the capital stocks $\dfrac{\dot{K}}{K}$ is calculated as follows:

$$\frac{\dot{K}}{K}=\frac{\dot{K}_0+\dot{K}_1}{K_0+K_1}=\frac{\dfrac{\dot{K}}{K_1}}{1+\dfrac{K_0}{K_1}}=\frac{1}{b_1\left(1+\dfrac{K_0}{K_1}\right)}=\frac{1-v}{b_1}.$$

Next consider about the sustainability of two production sectors. All paths denoted by (2-28) are not necessarily sustainable, because sector 1 provides sector 2 with capital goods. In particular, since we are now analyzing the balanced economic growth path, the following condition is required;

$$\dot{X}_1=\dot{X}_2.$$

in addition to (2-28). Since this means

$$\dot{X}_i=\frac{1}{b_i}\dot{K}_i,$$

we obtain

$$\frac{\dot{K}_0}{\dot{K}_1}=\frac{b_0}{b_1}. \tag{2-29}$$

Since we can have the following equation from (2-28) and (2-29),

$$\frac{\dot{K}_0}{\dot{K}_1}=\frac{v}{1-v}=\frac{b_0}{b_1}$$

we obtain

$$v=\frac{b_0}{b_0+b_1}. \tag{2-30}$$

(2-30) determines the distribution of capital stocks corresponding to the balanced economic growth.

**Again, the problem of workers**

Workers know that capital distributor calculates (2-30). However, even if workers know this calculation, they could not change the strategy which satisfies (2-26). That is, if workers deviate from (2-26), capital distributor would turn back to her problem (2-22), select the strategy $v=1-s_p$ and achieve $w=0$ (see Fig. 2.2).

**Again, the problem of capital distributor**

Although so far we have analyzed the strategies of players, the profit rate is not determined yet. In this situation, capital distributor must certainly analyze what happens in the game. In this stage, capital distributor would concentrate her attentions on the money market where savings of workers are handed to capitalists. The fact that the profit rate cannot be determined by the strategies of players means that the determination of profit rate cannot but depend on the condition of the money market[5]. Figure 2-3 shows the relationship of the balance between demand and supply of money. Firstly, although capitalists have to invest their money from their income to the production process, they spend a part of their money for consumption. In Fig.2-3, capitalists' consumption is denoted by $C_p$ .This part is considered as the shortage of capital. Therefore, in our model, this shortage should be supplied in the money market. Suppliers of capital are workers. Thus, savings of workers which are denoted by $S_w$ in Fig.2-2 are supplied as capital in the money market. Therefore, we obtain

$$C_p = S_W . \qquad (2\text{-}31)$$

Since the following equations are formed by definition;

$$C_p = (1 - s_p)\Pi,$$

$$S_w = s_w W$$

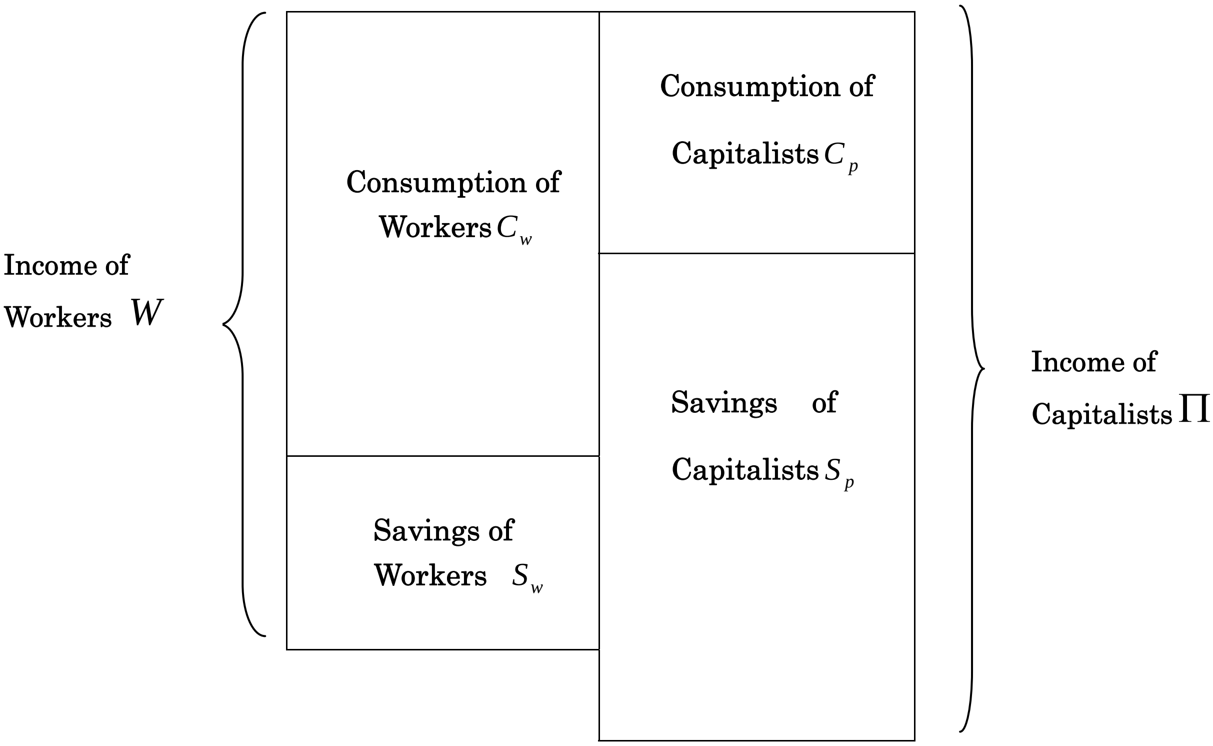
we obtain

$$\pi = \frac{1}{b_0 + b_1}, \qquad (2\text{-}32)$$

by using (2-16), (2-17) and the equilibrium of the game (2-33) which is proved later. Therefore, capitalists could get the profit rate (2-32) by adopting the strategy (2-30) and sustaining the balanced economic growth. As is shown later, the profit rate (2-32) is the best one which capitalists could acquire.

[5] Recall that players intend to avoid the situation where the profit rate is determined by (2-23). (2-23) is equivalent to (2-19). As is confirmed easily, these strategies of the players which are shown to become (2-33) later cannot determine the profit rate through (2-19)

Figure2-2. Consumptions and Savings of the Players




Considering the above process of constructing the strategies, we may expect that the candidate of Nash equilibrium is

$$\left(s_w^*,\ s_p^*,\ v^*\right)=\left(\frac{n_1}{n_0+n_1},\frac{b_1}{b_0+b_1},\frac{b_0}{b_0+b_1}\right). \tag{2-33}$$

Then, we obtain the following proposition.

**Proposition2.1 (Nash equilibrium, balanced economic growth and natural economy)**

(i) (2-33) is the Nash equilibrium and constitutes the balanced economic growth.

(ii) The profit rate and the wage rate under (2-33) are

$$\begin{cases} \pi^* = \dfrac{1}{b_0+b_1}, \\ w^* = \dfrac{1}{n_0+n_1}. \end{cases} \tag{2-34}$$

(iii) (2-34) achieved by (2-34) is the counterpart of the profit rate and the wage rate in the natural economic system defined by Pasinetti[6]. In the natural economic system,

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[6] Pasinetti(1981), pp.128,147,148.

total profits turn out to be the total amount of capital goods produced (i.e. total new investments) and total wages turn out to be equal to total consumption. Total profits are required as a share from the final national income prior to total wages. That is, total wages emerge as a kind of residual or surplus that remains over and above what has been charged for profit[7].

(Proof) (i) We shall show each player's strategy is the best response to the other players' strategies in (2-33).

(1) Suppose that the strategies $s_p^*$ and $v^*$ are given. Substituting $v^* = 1 - s_p^*$ for (2-23), we obtain $\pi = \frac{1}{b_1}$ (see Fig.2-2). Then, the wage rate is $w = 0$. Workers intend to avoid $w = 0$. For this purpose, workers have to choose the strategy which satisfies (2-26). Then, workers obtain the wage rate $w^*$ in (2-34). Therefore, the best response strategy of workers to the strategies $s_p^*$ and $v^*$ is $s_w^*$.

(2) Suppose that the strategies $s_w^*$ and $v^*$ are given. Then, the profit rate is $\pi = 0$, because the numerator of (2-23) becomes zero. Capitalists intend to avoid $\pi = 0$. For this purpose, capitalists have to choose the strategy which satisfies (2-27). The strategy is given as $s_p^*$, and the profit rate which corresponds to these strategies is given by $\pi^*$ in (2-34).

(3) Suppose that the strategies $s_w^*$ and $s_p^*$ are given. In this case, we obtain $B = 0$ and $\pi = A < 0$ in (2-23). Therefore, capital distributor intends to avoid the situation where the profit rate is determined by (2-23). For this purpose, capital distributor has only to choose the strategy $v$ which satisfies (2-27). The strategy is given by $v^*$ in (2-33).

From (1),(2) and (3), we conclude (2-33) is the Nash equilibrium.

(ii) Calculating (2-31) by using (2-16),(2-17) and (2-33), we obtain (2-32). Furthermore, substituting (2-32) for (2-25), we obtain $w = \frac{1}{n_0 + n_1}$.

(iii) From (2-31), i.e. $(1 - s_p)\Pi = s_w W$, we obtain

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7 Ibid.,pp.143-148.

$$p_0 X_0 = (1 - s_p)\Pi + (1 - s_w)W = W, \tag{2-35}$$

and from (2-14),(2-18) and (2-35), we obtain

$$p_1 X_1 = \Pi. \tag{2-36}$$

In the process of above calculation, the profit rate is determined prior to the wage rate. Hence, total wages are interpreted as a surplus that remains over and above what has been charged for profit.

In addition, since $b_i$ denotes the one unit of capital goods required to produce one unit of goods in sector $i$ $(i = 0,1)$, $\pi = \dfrac{1}{b_0 + b_1}$ gives the amount of the product which could be produced by one unit of capital. Therefore, it denotes the opportunity cost of using capital goods in the production process. It is the counterpart of natural rate of profit defined by Pasinetti(1981)[8]. (QED)

Proposition2.1 shows that the balanced economic growth and the natural economy are achieved at the same time in the macroeconomic model considered. As is pointed out, it is Pasinetti who analyzes the natural economy precisely. He points out as follows[9]..

To conclude, in the natural economic system, profits turn out to be equal to new investments; and wages turn to be equal to consumption; hence values of consumption goods turn out to be proportional to quantities of labor – not only in the economic system as a whole but also (and most importantly) in each single (vertically integrated) sector.

Pasinetti's model is a generalized multi-sector model which involves technical progress. Our model in this paper is interpreted as a special model of Pasinetti's model which has only one vertically integrated sector[10] corresponding to one consumption goods. Therefore, by using our model, we can calculate the balanced economic growth path corresponding to the natural economic system defined by Pasinetti. Let us try this calculation.

Firstly, let us calculate the condition that total wages are equal to total consumption;

$$p_0 X_0 = W. \qquad \text{(P-1)} \ .$$

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[8] Pasinetti(1981), pp.128-131.

[9] Ibd.,pp.147-148.

[10] We shall analyze a generalized multi-sector model which involves both process innovation and product innovation in section 4 and section 5.

By (2-3),(2-10) and (2-11), the left side of (P-1) is calculated as follows;

$$p_0 X_0 = \left( \frac{n_0}{b_0} + \frac{\pi n_1}{1 - \pi b_1} \right) w K_0 = \frac{n_0}{b_0} w K_0 + \pi p_1 K_0 = N_0 w + \pi p_1 K_0 .$$

Substituting this and (2-16) for (P-1) gives

$$\pi = \frac{X_1}{K_0 + K_1} = \frac{1}{\left( 1 + \frac{K_0}{K_1} \right) b_1} . \qquad \text{(P-2)}$$

In addition, from (2-28) and (2-29), we obtain

$$\frac{K_0}{K_1} = \frac{\dot{K}_0}{\dot{K}_1} = \frac{b_0}{b_1} .$$

Therefore, (P-2) becomes

$$\pi = \frac{1}{b_0 + b_1} . \qquad \text{(P-3)}$$

We could obtain (P-3) which is the same with $\pi^*$ in (2-34). Notice from the process of above calculation that (P-3) is not obtained by the adjustment process of economic power. In other words, although Pasinetti(1981) analyzes the existence of the natural profit rate, he does not show the economic process of how the natural profit rate is achieved. Therefore, there is no process which achieves the natural economic system in his analysis. However, our game theoretic model gives the process which is given by the economic adjustment. This means that the balanced economic growth and the natural economic system are accomplished within the game theoretic economy.

### 2.3 Balanced economic growth and the constraint of labor

Let us consider about the constraint of labor. If natural economy is achieved, total wages are equal to total consumption and total profits are equal to total investments to the production of capital goods. Namely, the market of consumption goods and the market of capital goods are both in equilibrium in natural economy. However, capital stocks would continue to grow and economy would encounter the constraint of labor under the equilibrium achieved by (2-33). In other word, the amount of labor employed in the balanced economic growth at time $t$ , $N_t$ , is given by

$$N = \frac{n_0 + n_1}{b_0 + b_1} K_0 \exp\left[\left(\frac{1}{b_0 + b_1}\right) t\right],$$

where $K_0$ denotes the initial level of capital stocks. It would reach at $\overline{N}$ ,which denotes the total labor existing in the economy, sooner or later. The economy would confront the full employment. Once the full employment is achieved, the economy could not continue to grow. How do players of the game act in this situation? We can present the following proposition.

**Proposition 2.2 ( Nash equilibrium under full employment)**

Under the full employment, the strategy pair

$$\left(s_w^{**}, s_p^{**}\ v^{**}\right) = (0,0,1) \qquad (2\text{-}37)$$

is the Nash equilibrium.

(Proof) We shall show each player's strategy is the best response to the other players' strategies in (2-37).

(1) Suppose that the strategies $s_w^{**}$ and $s_p^{**}$ are given. Since the economy has reached at full employment and could not grow, capital distributor could not distribute capital goods to sector 1. Therefore, she cannot but choose $v = 1$.

(2) Suppose that the strategies $s_p^{**}$ and $v^{**}$ are given. Substituting $s_p^{**}$ and $v^{**}$ for (2-23), we obtain

$$\pi = \frac{s_w}{s_w b_1}.$$

If workers choose the strategy $s_w \neq 0$, the profit rate would be $\pi = \frac{1}{b_1}(> 0)$. Then, the wage rate becomes $w = 0$. Workers intend to avoid $w = 0$ and choose the strategy $s_w = 0$.

(3) Suppose that the strategies $s_w^{**}$ and $v^{**}$ are given. Substituting $s_w^{**}$ and $v^{**}$ for (2-23), we obtain

$$\pi = \frac{0}{s_p n_1 b_0}.$$

If capitalists choose the strategy $s_p \neq 0$, the profit rate would be $\pi = 0$. Capitalists intend to avoid $\pi = 0$ and choose the strategy $s_p = 0$. (QED)

Under the Nash equilibrium (2-37), the following proposition is given.

**Proposition 2.3 (Saturation collapse of capitalism)**

If the full employment is achieved, money market would be destroyed and the capitalism would collapse because of the saturation.

(Proof) Under(2-37), savings of players become zero. Therefore, money market would vanish. Then, the process which establishes the route from savings to investments would be destroyed and capitalism would collapse. (QED)

What happens in the economy? From (2-14)~(2-18), we obtain

$$Y = p_0 X_0 = \left(\frac{n_0}{b_0}\right) wK_0 + \left(\frac{\pi n_1}{1-\pi b_1}\right) wK_1 = W + \Pi. \qquad (2\text{-}38)$$

Equation (2-38) shows economy produces only consumption goods, wages of workers and profits of capitalists are all spent on consumption. Capitalists would live as if they live on annuity. Economy would lose the mechanism which determines profit rate and wage rate and would not determine the income distribution between workers and capitalists. The destruction of money market demonstrates the collapse of capitalism.

As is shown, it is impossible to construct the balanced economic growth forever in our game theoretic model. What is the reason? Our model is simple Ricardian model. We propose the two reasons for this problem.

(i) Although our model has Kaldor's fundamental equation (2-19) and seems to be Keynesian model, the model also has the feature of classical model. That is, the model is constructed by considering supply side and does not involve demand side. Therefore, the model in this section does not have the adjustment process in goods markets.

(ii) The second reason is decisive. The model in this section has the constant technical coefficients,i.e. $b_i$, $n_i$ $(i = 0,1)$. Therefore, the model involves no innovation and cannot analyze the structural changes of the economy which are brought about by innovation.

When we consider about production structure and innovation, we have to analyze both qualitative and quantitative changes in the production sector. For instance, process innovation changes the structure of production. On the other hand, product innovation constructs the new sectors and brings about multiple reconstruction of the economy. Sector 0 and sector 1 in our model actually involve multiple sectors and evolve complexly. In these circumstances, structural change occurs and the technical coefficients $n_i$ , $b_i (i = 0,1)$ change complexly. Therefore, we must construct the multi-sector model at the first stage and then reconstruct a new Ricardian model by introducing innovation at the second one.

In section 3, we shall generalize Pasinetti's model which involves demand side and make preparations for reconstructing the new synthetic model. Under these reparations, we will try to conquer the problem of 'Saturation collapse of capitalism' in section 4 and in section5.

## 3. GENERALIED MODEL OF PASINETTI'S VERTICALLY INTEGRATED SECTOR

As is mentioned in section 1, the purpose of this paper is to construct the game theoretic model which involves innovation. This study needs to introduce multi-sector model, in particular the vertically integrated sectors constructed by Pasinetti. However, Pasinetti(1981) adopted the simplified version which involves the vertically integrated sectors of finitely higher order. This simplified model makes it possible to analyze the essential of technical progress clearly. However, our game model needs the complete model which involves the vertically integrated sectors of infinitely higher order. Therefore, in this section, we construct the generalized model of Pasinetti's vertically integrated sector which can analyze the effect of innovation on economy.

The model presented in this section plays a role of infusing R&D and innovation into the model in section2. This study is tried in section 5.

### 3.1 Structure of model and definition of notation

Suppose that the number of final commodity produced in the economy is $n$ .Therefore, we have $n$ sectors which produce these consumption goods. Economy also has some production sectors which produce capital goods. We now define the notations which describe the economy.

(i) We call $n$ commodities commodity $1,\cdots,$commodity $n$ and call the sector which produces commodity $j$ sector $j$ $(j = 1,\cdots,n)$ . Let $A$ be $(n \times n)$ matrix, the $j$ th column of which presents the physical stocks of capital goods(both circulating and fixed) required for the production of one physical unit of commodity $j$ . Moreover, let $A^*$ be $(n \times n)$ matrix which corresponds to matrix $A$ and represents the circulating capital goods and the worn-out part of capital goods.

(ii) By matrices $A$ and $A^*$, we may define vertically integrated sectors[11]. Furthermore, we may define the vertically integrated sectors of the $k$ th order infinitely $(k = 1, 2, \cdots)$. We call the vertically integrated sector of the first order corresponding to sector $j$ sector $k_j^1$. We also call the vertically integrated sector of the second order corresponding to sector $k_j^1$ sector $k_j^2$. With the same argument, we may define the sector $k_j^i$ sequentially and infinitely $(i = 1, 2, \cdots; j = 1, \cdots, n)$. Let $X_j^0$ denote the output of sector $j$ $(j = 1, \cdots, n)$ and put $X^0 = (X_1^0, \cdots, X_n^0)^t$ where vector $(\cdot)^t$ denotes the transposition of vector$(\cdot)$. In addition, let $X_j^i$ denote the output of sector $k_j^i$ and put $X^i = (X_1^i, \cdots, X_n^i)^t$ $(i = 1, 2, \cdots; j = 1, \cdots, n)$.

Furthermore, we define vertically integrated labor coefficients of sector $j$ and sector $k_j^i$ as follows.

$a_j^0$ ; the vertically integrated labor coefficient of sector $j$ $(j = 1, \cdots, n)$

$a^0 = \left(a_1^0, a_2^0, \cdots, a_n^0\right)$

$a_j^i$ ; the vertically integrated labor coefficient of sector $k_j^i$

$(i = 1, 2, \cdots; j = 1, \cdots, n)$

$a^i = \left(a_1^i, a_2^i, \cdots, a_n^i\right) \qquad (i = 1, 2, \cdots)$

On the other hand, we define the price of each sector as follows.

$p_j^0$ ; the price of commodity $j$ $(j = 1, \cdots, n)$

$p^0 = \left(p_1^0, \cdots, p_n^0\right)^t$

$p_j^i$ ; the price of capital goods produced by sector $k_j^i$ $(i = 1, 2, \cdots; j = 1, \cdots, n)$

$p^i = \left(p_1^i, \cdots, p_n^i\right)^t \qquad (i = 1, 2, \cdots)$

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11 See Pasinetti(1973),pp.5-7.

(iii) Sector $l$ is the household sector which supplies labor for the vertically integrated sectors and demands final goods which involve consumption goods (i.e., commodities) and capital goods. We define the notation of sector $l$ as follows.

$d_j^0$ ; the demand coefficient (the demand per capita)for commodity $j$

$(j = 1,\cdots\cdots,n)$

$d^0 = (d_1^0,\cdots\cdots,d_n^0)^t$

$d_j^i$ ; the demand coefficient(the demand per capita) for capital goods produced by

sector $k_j^i$ : the new investment for the capital goods

$(i = 1,2,\cdots\cdots; j = 1,\cdots\cdots,n)$

$d^i = (d_1^i,\cdots\cdots,d_n^i)^t \qquad (i = 1,2,\cdots\cdots)$

$X_L$ ; the total quantity of labor existing in sector $l$

**3.2 Matrix of capital stocks and matrix of its flows**

Let $\Lambda$ denote the coefficient matrix of capital stocks which is constituted by the vertically integrated sectors defined starting from each commodity infinitely sequentially. Matrix $\Lambda$ may be represented by the definition as follows.

$$
\begin{array}{cc}
\begin{array}{cccccc}
1\ 2\cdots\cdots n & k_1^1\ k_2^1\cdots\cdots k_n^1 & k_1^2\ k_2^2\cdots\cdots k_n^2 & & k_1^i\ k_2^i\cdots\cdots k_n^i & \cdots\cdots
\end{array} & \\
\Lambda = \left[\begin{array}{c:c:c:c:c:c}
0_n & 0_n & 0_n & \cdots\cdots & 0_n & \cdots\cdots \\ \hdashline
I_n & 0_n & 0_n & \cdots\cdots & 0_n & \cdots\cdots \\ \hdashline
0_n & I_n & 0_n & \cdots\cdots & 0_n & \cdots\cdots \\ \hdashline
\multicolumn{6}{c}{\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots} \\ \hdashline
0_n & 0_n & 0_n & \cdots\cdots & I_n & \cdots\cdots \\ \hdashline
\multicolumn{6}{c}{\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots}
\end{array}\right]
&
\begin{array}{l}
1,2,\cdots,n \\
k_1^1,k_2^1,\cdots,k_n^1 \\
k_1^2,k_2^2,\cdots,k_n^2 \\
\\
k_1^{i+1},k_2^{i+1},\cdots,k_n^{i+1} \\
\\
\end{array}
\end{array}
$$

(3-1)

where $0_n$ and $I_n$ denote null matrix and identity matrix of $n \times n$ respectively. Matrix $\Lambda$ can be explained as follows. Sector $j$ is accompanied by sector $k_j^i$ $(j = 1,\cdots,n)$. For sector $k_j^1$, sector $j$ requires one unit of capital stocks which is measured by *the unit of vertically integrated productive capacity* to produce one unit of commodity $j$ [12]. Therefore, we arrange '1' at the point of intersection corresponding to sector $j$ and sector $k_j^1$. Furthermore, we arrange '1' at the point of intersection corresponding to sector $k_j^i$ and sector $k_j^{i+1}$ with the same reason $(i = 1,2,\cdots\cdots; j = 1,\cdots,n)$. In (3-1), these arrangements of '1' are expressed by matrix $I_n$. For simplicity, we put the following assumption for these capital stocks.

**Assumpiotn3.1** We assume that the flow of depreciated capital stocks (i.e., worn out part of capital stocks) occurring by the production of one unit of commodity $j$ is denoted by $\sigma_j^0$ and the one occurring in sector $k_j^i$ by the production of one unit of capital stocks measured by *the vertically integrated productive capacity* is also denoted by $\sigma_j^i$ $(i = 1,2,\cdots; j = 1,\cdots\cdots,n)$.

Then, these flows are expressed by the matrix $\Lambda^*$:

$$\Lambda^* = \begin{bmatrix} 0_n & & & & & \\ \sigma^0 & 0_n & & & & \\ & \sigma^1 & 0_n & & & \\ & & \ddots & \ddots & & \\ & & & \sigma^i & 0_n & \\ & & & & \ddots & \ddots \end{bmatrix} \tag{3-2}$$

[12] Pasinetti(1973),p.6

where

$$\sigma^0 = \begin{bmatrix} \sigma_1^0 & & & & \\ & \sigma_2^0 & & & \\ & & \sigma_3^0 & & \\ & & & \ddots & \\ & & & & \sigma_n^0 \end{bmatrix}, \quad \sigma^i = \begin{bmatrix} \sigma_1^i & & & & \\ & \sigma_2^i & & & \\ & & \sigma_3^i & & \\ & & & \ddots & \\ & & & & \sigma_n^i \end{bmatrix} (i = 1, 2, \cdots) \quad . \tag{3-3}$$

### 3.3 Output system

#### 3.3.1 Model

Let us construct output system by (3.1) and (3.2). For simplicity, we consider the vertically integrated sectors of higher order up to the $m$-th order. Namely, we consider sector $k_j^i$ under $i = 1, \cdots\cdot, m; j = 1, \cdots\cdot, n$. If one wants to construct the model with infinitely defined vertically integrated sectors, he has only to put $m \to \infty$.

Firstly, since the demand coefficient vector for consumption goods is represented by $d^0$, the output vector $X^0$ is expressed by

$$X^0 = d^0 X_L. \tag{3-4}$$

Secondly, since the demand coefficient for sector $k_j^i$ involves the new investment and the replacement of worn-out capital, the output of sector $k_j^i$, $X_j^i$, is denoted by

$$X_j^i = \sigma_j^{i-1} X_j^{i-1} + d_j^i X_L. \quad (i = 1, \cdots\cdot, m; j = 1, \cdots\cdot, n) \tag{3-5}$$

We denote (3-5) by using vector $X^i$ as follows.

$$X^i = \sigma^{i-1} X^{i-1} + d^i X_L. \quad (i = 1, \cdots\cdot, m) \tag{3-6}$$

(3-4) and (3-6) constitute the output system. Since the production requires the input of labor, we have to satisfy the constraint of it. It is expressed by

$$\sum_{i=0}^{m} a^i X^i \leq X_L. \tag{3-7}$$

Since $a^i X^i$ in the right-hand side of (3-7) represents the labor employed in sector $i$ $(i = 1, \cdots\cdot, m)$, the right-hand side represents the total labor employed in the economy. If equality is held in (3-7), then the full employment is achieved.

#### 3.3.2 Condition for full employment

We may solve (3-4) and (3-6) immediately. The solutions $X^0, X^1, \cdots\cdot, X^m$ are given by

$$\begin{cases} X^0 = d^0 X_L, \\ X^i = \left[ \sum_{s=0}^{i-1} \left\{ \left( \prod_{k=s}^{i-1} \sigma^k \right) d^s \right\} + d^i \right] X_L . \qquad (i = 1, \cdots, m) \end{cases} \tag{3-8}$$

That is, we obtain the solution recursively starting from the solution $X^0$. In addition,(3-8) must satisfy the constraint (3-7). Substituting (3-8) for (3-7), we obtain

$$a^0 d^0 + \sum_{i=1}^{m} \left[ a^i \left[ \sum_{s=0}^{i-1} \left\{ \left( \prod_{k=s}^{i-1} \sigma^k \right) d^s + d^i \right\} \right] \right] \le 1. \tag{3-9}$$

In particular, the condition for full employment becomes

$$a^0 d^0 + \sum_{i=1}^{m} \left[ a^i \left[ \sum_{s=0}^{i-1} \left\{ \left( \prod_{k=s}^{i-1} \sigma^k \right) d^s + d^i \right\} \right] \right] = 1. \tag{3-10}$$

Using matrix and vector, the system (3-4) and (3-6) is represented by

$$\begin{bmatrix} -I_n & & & & & & d^0 \\ \sigma^0 & -I_n & & & & & d^1 \\ & \sigma^1 & -I_n & & & & d^2 \\ & & \ddots & \ddots & & & \vdots \\ & & & \sigma^{m-2} & -I_n & 0_n & d^{m-1} \\ & & & & \sigma^{m-1} & -I_n & d^m \\ a^0 & a^1 & a^2 \cdots\cdots & & a^{m-1} & a^m & -1 \end{bmatrix} \begin{bmatrix} X^0 \\ X^1 \\ X^2 \\ \vdots \\ \vdots \\ X^m \\ X_L \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \end{bmatrix} . \tag{3-11}$$

The equation (3-11) is linear and homogeneous. The condition for non-trivial solution is given by

$$\begin{vmatrix} -I_n & & & & & & d^0 \\ \sigma^0 & -I_n & & & & & d^1 \\ & \sigma^1 & -I_n & & & & d^2 \\ & & \ddots & \ddots & & & \vdots \\ & & & \sigma^{m-2} & -I_n & 0_n & d^{m-1} \\ & & & & \sigma^{m-1} & -I_n & d^m \\ a^0 & a^1 & a^2 \cdots\cdots & & a^{m-1} & a^m & -1 \end{vmatrix} = 0 \cdot \tag{3-12}$$

By developing this determinant, we obtain (3-10). See appendix for the proof of this development. The condition (3-12) is equivalent to (3-10). Therefore, if the (3-11) does not have non-trivial solutions, then there exists unemployment in the economy. Of course, the condition for unemployment is expressed by

$$a^0 d^0 + \sum_{i=1}^{m} \left[ a^i \left[ \sum_{s=0}^{i-1} \left\{ \left( \prod_{k=s}^{i-1} \sigma^k \right) d^s + d^i \right\} \right] \right] < 1. \qquad (3\text{-}13)$$

### 3.4 Price system

#### 3.4.1 Equations of price system

In price system, we also consider the vertically integrated sectors of higher order up to the $m$-th order. Since the vertically integrated sectors of the $m$-th order don't been accompanied by any sectors, they produce their products only by the input of labor[13]. Then, the price equation is given by

$$\begin{cases} p^0 = (\sigma^0 + \pi I_n) p^1 + (a^0)^t w, \\ p^1 = (\sigma^1 + \pi I_n) p^2 + (a^1)^t w, \\ p^2 = (\sigma^2 + \pi I_n) p^3 + (a^2)^t w, \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ p^{m-1} = (\sigma^{m-1} + \pi I_n) p^m + (a^{m-1})^t w, \\ p^m = (a^m)^t w. \end{cases} \qquad (3\text{-}14)$$

The first equation shows that the price is formed by the replacements of worn-out parts of capital stocks, $\sigma^0 p^1$, the profits required for the use of capital goods, $\pi p^1$, and the wages, $(a^0)^t w$. The prices $p^1, \cdots, p^{m-1}$ have the same structure. The last equation shows that the prices $p^m$ are constructed only by the wages $(a^m)^t w$, because the production requires only input of labor.

In this stage, let us consider the structure of expenditure. Since the average demand per capita for the product of each sector is denoted by $d^i$ $(i = 0,1,\cdots,m)$, the average expenditure per capita is denoted by $\sum_{i=0}^{m} (d^i)^t p^i$. On the other hand, the output per

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[13] The sectors of the $m$-th order do not require the input of capital stocks. Therefore, if $d^m$ appears, then put $d^m = 0$. This gives no effect to our analysis.

capita is also denoted by $\sum_{i=0}^{m}(d^i)^t p^i$ , because we consider the economy where supply is determined by demand (see (3-8)). Since wages and profits are paid from the value of this supply, we obtain

$$\sum_{i=0}^{m}(d^i)^t p^i = \pi\sum_{i=0}^{m-1}(X^i)^t p^{i+1} / X_L + w. \tag{3-15}$$

Since $X^i \ / \ X_L$ is determined by (3-8), substituting this for (3-15) gives

$$\sum_{i=0}^{m}(d^i)^t p^i = \pi(X^0)^t p^1 + \pi\sum_{i=1}^{m-1}\left[\sum_{s=0}^{i-1}\left\{\left(\prod_{k=s}^{i-1}\sigma^k\right)d^s + d^i\right\}\right]^t p^{i+1} + w. \tag{3-16}$$

**3.4.2 Condition for proper demand**

We may solve (3-14) directly. The solutions are given by

$$\begin{cases} p^i = \left[\sum_{s=i}^{m-1}\left\{\left(\prod_{k=i}^{s}\left(\sigma^k + \pi I_n\right)\right)a^{s+1}\right\} + a^i\right]w, (i = 0,1,\cdots\cdot, m-1) \\ p^m = a^m w. \end{cases} \tag{3-17}$$

The prices contain the labor input both directly and indirectly to produce the productions. The price system (3-14) and the constraint (3-16) are represented by matrix and vectors. We may express them as follows.

$$\begin{bmatrix} -I_n & (\sigma^0+\pi I_n) & & & & & (a^0)^t \\ & -I_n & (\sigma^1+\pi I_n) & & & & (a^1)^t \\ & & -I_n & (\sigma^2+\pi I_n) & & & (a^2)^t \\ & & & \ddots & \ddots & & \vdots \\ & & & & -I_n & (\sigma^{m-1}+\pi I_n) & (a^{m-1})^t \\ & & & & & -I_n & (a^m)^t \\ (d^0)^t & (d^1-\pi\theta^0)^t & (d^2-\pi\theta^1)^t & \cdots & \cdots & (d^m-\pi\theta^{m-1})^t & -1 \end{bmatrix} \begin{bmatrix} p^0 \\ p^1 \\ p^2 \\ \vdots \\ p^{m-1} \\ p^m \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3-18}$$

where $\theta^i$'s denote

$$\begin{cases} \theta^0 (= X^0 / X_L) = d^0, \\ \theta^i (= X^i / X_L) = \sum_{s=0}^{i-1}\left\{\left(\prod_{k=s}^{i-1}\sigma^k\right)d^s\right\} + d^i, (i = 1,\cdots\cdot, m), \end{cases} \tag{3-19}$$

(see (3-8)).

The equation (3-18) is linear and homogeneous. The condition for non-trivial solution is given by

$$\begin{vmatrix} -I_n & (\sigma^0+\pi I_n) & & & & & (a^0)^t \\ & -I_n & (\sigma^1+\pi I_n) & & & & (a^1)^t \\ & & -I_n & (\sigma^2+\pi I_n) & & & (a^2)^t \\ & & & \ddots & \ddots & & \vdots \\ & & & & -I_n & (\sigma^{m-1}+\pi I_n) & (a^{m-1})^t \\ & & & & & -I_n & (a^m)^t \\ (d^0)^t & (d^1-\pi\theta^0)^t & (d^2-\pi\theta^1)^t & \cdots & \cdots & (d^m-\pi\theta^{m-1})^t & -1 \end{vmatrix} = 0 \quad . \qquad (3\text{-}20)$$

In (3-18), the prices are determined as (3-17) independently of the last row of the matrix. Therefore, whether (3-18) has non-trivial solutions or not depends on the equation corresponding to the last line of the matrix. Moreover, this equation is equivalent to (3-16). This means that (3-16) for which we substitute (3-17) is equivalent to (3-18)'s having non-trivial solutions. On the other hand, we obtain (3-10) by developing (3-20). See appendix for the proof of this development. This also means that we may obtain (3-10) by substituting (3-17) for (3-16) and rearranging terms.

(3-10) is the condition for the full employment. Then, let us consider (3-16) which is equivalent to (3-10) to analyze (3-10) from the price system's point of view. (3-16) can be interpreted as showing the distribution of GDP per capita. However, from a different point of view, since the left-hand side of (3-16) shows the average expenditure per capita, we may say that (3-16) shows the level of expenditure per capita is at the proper one. In other words, if the following inequality occurs:

$$\sum_{i=0}^{m}(d^i)^t p^i - \pi\left(X^0\right)^t p^1 - \pi\sum_{i=1}^{m-1}\left[\sum_{s=0}^{i-1}\left\{\left(\prod_{k=s}^{i-1}\sigma^k\right)d^s + d^i\right\}\right]^t p^{i+1} < w, \qquad (3\text{-}21)$$

then the level of expenditure is low. This means that the unemployment occurs because of the low level of expenditure. Indeed, substituting (3-17) for (3-21) brings about (3-13).

### 3.5 Conclusion of this section

As is mentioned above, to construct the game theoretic model which involves innovation needs to introduce the multi-sector model. For this purpose, we have constructed the generalized model of Pasinetti's vertically integrated sector. The model presented in this section plays the role of inspiring R&D and innovation into the model in section2.

## 4. RECONSTRUCTION OF GAME-THEORETIC RICARDIAN MODEL BY VERTICALLY INTEGRATED SECTORS

In this section, we shall synthesize the game-theoretic model in section 2 and vertically integrated sectors in section 3. This synthesis is the preparation for constructing the generalized model which contains R&D and innovation. The synthesis will be tried in section 5.

The synthesis is completed by recomposing the game model in section 2 starting from vertically integrated sectors. In this work, we have to always confirm the problem of unit. This confirmation is very important, because each vertically integrated sector has a characteristic unit of goods and we have to measure the products with a common unit to reconstruct the Ricardian model which condenses the vertically integrated sectors. The common unit must be expressed by the fundamental factor of production, labor. And all structures must be expressed by the terms of labor.

Now, let us reconstruct the model. For this purpose, we need the following assumption, because we must match the situations of the model in section 3 to that in section 2.

**Assumption 4.1** Capital stocks are not worn out, i.e., $\sigma_j^i = 0$ $(i, j = 0,1,2,\cdots)$ in (3-3).

### 4.1 Definition of unit

We start with the same situation with that in section 2. That is, the structure of the macro-economy is expressed in Fig.2.1. The model has three players, workers, capitalists and capital distributor. Although the model in section 2 has sector 0 (the sector producing consumption goods) and sector 1(the sector producing capital goods), we will start from the economy which has $n$ commodities and $n$ vertically integrated sectors. We will try to condense these into the basic model in section 2. The symbols used denote the same meanings with those in section 2 and section 3.

First, remember that we adopted the consumption goods as numeraire,i.e. $p_0 = 1$,in section 2(see (2-13)). Therefore, we must define the unit of consumption goods, starting from $n$ commodities, to adopt it as a numeraire which is the counterpart of (2-13). The price system in vertically integrated analysis is given by (3-17); $p^i\,(i=0,1,\cdots\cdot,m-1)$. In addition, we denoted the average consumption vector $d^0=(d_1^0,\cdots\cdot,d_n^0)^t$. Hence, we must define one unit of the composite goods which are constituted by $n$ commodities and inherit the ratio of the components from the vector $d^0=(d_1^0,\cdots\cdot,d_n^0)^t$. We put the following definition.

**Definition 4.1** Let $d^{0*}$ be the vector which satisfies

$$p^0\ d^{0*}=1 \tag{4-1}$$

and the ratio of components of which is the same with that of the vector $d^0$. We call the vector $d^{0*}\left(=\left(d_1^{0*},\cdots\cdot,d_n^{0*}\right)^t\right)$ *the unit vector of consumption goods* and define it as one unit of composite goods which are constituted by $n$ commodities.

Apparently, the vector which satisfies the conditions of Def.4.1 does exist. Moreover, we may express the relationship of the two vectors, $d^0$ and $d^{0*}$ as follows:

$$d^0=\mu d^{0*} \tag{4-2}$$

where the constant $\mu(>0)$ denotes the average consumption per capita for the unit vector of consumption goods[14].

Since we have defined the unit of consumption goods by the unit vector of consumption goods $d^{0*}$, we must define the price of the vector $d^{0*}$. For this purpose, we must concentrate our attentions on the quantities of labor input into the vector. By assumption 4.1 and the price system (3-17), the quantities of labor input into the vector $d^{0*}$ are expressed by

$$\sum_{j=1}^{n}\frac{p_j^0}{w}d_j^{0*}=\sum_{j=1}^{n}\left[\sum_{s=0}^{\infty}\left(\pi^{s+1}a_j^{s+1}\right)+a_j^0\right]d_j^{0*}\quad.$$

Therefore, we define the price of the vector $d^{0*}$, $P^0$,by

$$P^0=\sum_{j=1}^{n}\left[\sum_{s=0}^{\infty}\left(\pi^{s+1}a_j^{s+1}\right)+a_j^0\right]d_j^{0*}w. \tag{4-3}$$

Next, let us consider about capital/output ratio. Since we have defined the unit of consumption goods by the vector $d^{0*}$, we must define the unit of capital stocks

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14 It is important to interpret the theoretical meanings of $\mu$. Regarding this point, we will consider in paragraph 5.4.

corresponding to the vector $d^{0*}$. Although we have one kind of capital goods in section 2, the vertically integrated analysis in section 3 has $n$ commodities and each commodity has the characteristic capital stocks defined by the composite goods denoted by $n \times 1$ vectors. As is shown in (3-1), vertically integrated sector $j$ must prepare one unit of capital stocks measured by vertically integrated productive capacity, i.e.the characteristic $n \times 1$ vector of capital goods, from sector $k_j^1$ for the production of one unit of commodity $j$ $(j = 1, \cdots, n)$. In other words, each consumption goods has the different unit of capital goods. Therefore, we need to get back to the foundation to reconstruct our model. Let us turn back briefly to the construction of vertically integrated sector according to Pasinetti(1973).

**Pasinetti 's formulation**

Let $A^C$ and $A^F$ be the square matrices of $n \times n$ which represent the stocks of circulating-capital goods and the stocks of fixed capital goods respectively. Each sector $i$ has to replace all circulating goods and a fraction $\sigma^i$ which denotes the worn-out part of fixed capital goods $(i = 1, \cdots, n)$. A diagonal matrix $\sigma$ has all the $\sigma^i$ s on the main diagonal[15]. And we define $A^* = A^C + A^F \sigma$. The row vector $l$ of $n \times 1$ denotes the labor-output coefficient vector. Then, the vertically integrated sectors of the first order are defined by the following matrix and vector;

$$H = A(I - A^*)^{-1},$$
$$a^0 = l(I - A^*)^{-1}.$$

In addition, we may define vertically integrated sectors of the higher order by the matrix $H^{i+1}$ and the vector $a^i = l(I - A^*)^{-1} H^i = a^0 H^i$ $(i = 1,2,.....)$.

By the above preparation, we shall define the unit of capital stocks which corresponds to the unit vector of consumption goods $d^{0*}$. Since the quantity of labor required to produce the vector $d^{0*}$ is expressed by

$$\frac{p^1}{w} d^{0*} = \sum_{j=1}^{n} \frac{p_j^1}{w} d_j^{0*} = \sum_{j=1}^{n} \left[ \sum_{s=1}^{\infty} \left( \pi^s a_j^{s+1} \right) + a_j^1 \right] d_j^{0*},$$

we may define the unit of capital stocks as follows.

[15] Note that $\sigma = 0$ in our model by assumption 4.1

**Definition 4.2** We define the vector of capital stocks, $Hd^{0*}$, which is required to produce the unit vector of consumption goods $d^{0*}$ as the unit of capital goods and call it *the unit vector of capital stocks.* In addition, we define the price of the unit *vector of* capital goods vector, $P^1$, by

$$P^1 = \sum_{j=1}^{n}\left[\sum_{s=1}^{m-1}\left(\pi^s a_j^{s+1}\right) + a_j^1\right] d_j^{0*} w. \tag{4-4}$$

Since $a^i = a^0 H^i$, $P^1$ is expressed as follows:

$$\begin{aligned} P^1 &= (a^1 + \pi a^2 + \pi^2 a^3 + \pi^3 a^4 + \cdots)d^{0*}w \\ &= (a^0 H + \pi a^0 H^2 + \pi^2 a^0 H^3 + \pi^3 a^0 H^4 + \cdots)d^{0*}w \\ &= a^0 (I + \pi H + \pi^2 H^2 + \pi^3 H^3 + \cdots)Hd^{0*}w \\ &= a^0 (I - \pi H)^{-1} Hd^{0*} w. \end{aligned} \tag{4-5}$$

where we put $m \to \infty$.

Now, we synthesize the vertically integrated sectors and our Ricardian model in the section 2, starting from the former. Our first purpose is to define the labor coefficient $\bar{n}_1$ and capital/output ratio $\bar{b}_1$ which satisfy the conditions of (2-11) and (4-5). The price of the capital goods shown by (2-11) is

$$p_1 = \frac{n_1}{1 - \pi b_1} w. \tag{2-11}$$

(4-5) and (2-11) have the same structure. Comparing the structures of two prices, it is natural to define the labor coefficient of sector 1, $\bar{n}_1$, as follows:

$$\bar{n}_1 = a^0 Hd^{0*} (= a^1 d^{0*}). \tag{4-6}$$

On the other hand, we may define the capital/output ratio of sector 1 as is shown in Fug.4-1. Namely, since the value of $a^0 (I - \pi H)^{-1} Hd^{0*}$ is determined in (4-5), we have only to define $\bar{b}_1$ corresponding to the value of $a^0 (I - \pi H)^{-1} Hd^{0*}$. We set the following definition.

**Definition 4.3** We define the $\bar{b}_1$ which satisfies (4-7) as the capital/output ratio of sector 1, i.e.,

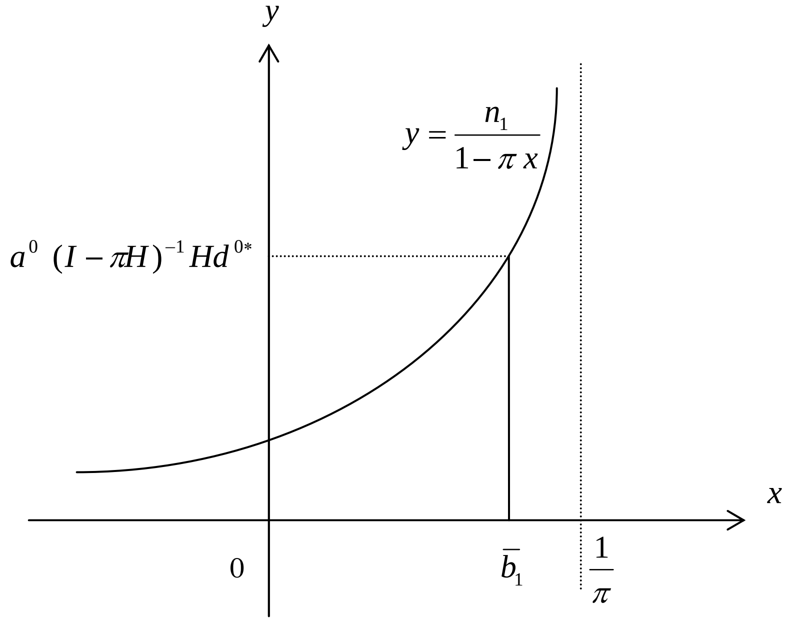
$$a^0 (I - \pi H)^{-1} Hd^{0*} = \frac{\bar{n}_1}{1 - \pi \bar{b}_1} \tag{4-7}$$

where $\bar{n}_1 = a^0 Hd^{0*} (= a^1 d^{0*})$.

Then, we may express the price of the capital goods $P^1$ as follows;

$$P^1 = \frac{\bar{n}_1}{1-\pi \bar{b}_1} w. \tag{4-8}$$

**Figure 4.1** the determination of $\bar{b}_1$ ($n_1 = a^0 H d^{0*}$)




The unit vector of capital stocks $Hd^{0*}$ is the stocks which are required to produce the unit vector of consumption goods $d^{0*}$. However, the unit vector of capital stocks $Hd^{0*}$ is also the produced products and requires the vector of capital stocks $H^2 d^{0*}$ to be produced. Notice that the vector $H^2 d^{0*}$ is different from the vector $Hd^{0*}$. Therefore, we cannot measure $H^2 d^{0*}$ by $Hd^{0*}$ which is defined as the unit vector of capital stocks. This means that we have to solve the problem of the unit of capital stocks if we intend to reconstruct the model which contains sector 0 and sector 1(see section 2) from vertically integrated sectors.

To find the clue of the solution of this problem, we have to go back to the fundamental factor of production, labor. The left-hand side of (4-7) represents the quantities of labor which are required to produce the unit vector of capital stocks $Hd^{0*}$ in the whole economy. On the other hand, the labor coefficient $\bar{n}_1$ in (4-8) is defined by (4-6). The term of $a^0\ Hd^{0*}$ stands for the quantity of labor which is required to produce one unit of capital stocks $Hd^{0*}$ in the vertically integrated sectors of the first order. In addition, the right side of (4-8) is developed as follows;

$$\frac{\bar{n}_1}{1-\pi\bar{b}_1}w = \bar{n}_1\{+\pi\bar{b}_1 + (\pi\bar{b}_1)^2 + (\pi\bar{b}_1)^3 + \cdots\cdots\}w \quad . \tag{4-9}$$

(4-9) has the analogous structure with (4-5). As is mentioned above, it is natural to adopt the definition (4-6). On the other hand, $\bar{b}_1$ is determined so as to equalize $\frac{\bar{n}_1}{1-\pi\bar{b}_1}$ to the quantity of labor which is required to produce one unit of the capital stocks $Hd^{0*}$ in the whole economy. Therefore, what we are doing is that we define the quantity of labor which could produce one unit of capital stock $Hd^{0*}$ and at the same time calculate the coefficient $\bar{b}_1$ which could equalize the left-hand side of (4-8) to the quantity of labor. At this stage, remember the definition that the capital/output ratio of sector 0 is equal to one, i.e. $\bar{b}_0 = 1$ which is explained again later in (4-12). The coefficient $\bar{b}_0(=1)$ is defined for the unit vector of capital stocks $Hd^{0*}$. Therefore, (4-7) implies that $\bar{b}_1$ is theoretically measured by the unit vector of capital stocks $Hd^{0*}$ in terms of labor. It is only 'labor' that plays the role of common measure in the complex economy. This fact is also emphasized in section 5 where we construct the evolutionary model with R&D and innovation.

On the other hand, the price of the unit vector of consumption goods $d^{0*}$ may be defined and computed as follows;

$$P^0 = \sum_{j=1}^{n}\left[\sum_{s=0}^{\infty}\left(\pi^{s+1}a_j^{s+1}\right) + a_j^0\right] d_j^{0*} w$$

$$\begin{aligned}
&= (a^0 + \pi a^1 + \pi^2 a^2 + \pi^3 a^3 + \cdots)d^{0*}w \\
&= \{a^0 + (\pi a^0\ H + \pi^2\ a^0\ H^2 + \pi^3 a^0\ H^3 + \cdots)\}d^{0*}w \\
&= \{a^0 + \pi\, a^0\ (I + \pi H + \pi^2 H^2 + \pi^3 H^3 + \cdots)H\}d^{0*}w \\
&= \{a^0 d^{0*} + \pi\, a^0\ (I - \pi H)^{-1} H d^{0*}\}\ w .
\end{aligned} \tag{4-10}$$

Let $\bar{n}_0$ and $\bar{b}_0$ be labor coefficient and capital/output ratio of sector 0 obtained from vertically integrated sectors respectively. Next we shall define $\bar{n}_0$ and $\bar{b}_0$. The price of consumption goods in section 2 is (2-10), i.e.,

$$p_0 = \left(n_0 + \frac{\pi n_1 b_0}{1-\pi b_1}\right)w . \tag{2-10}$$

(4-10) and (2-10) have the same structure. From this structure, it is natural to define the labor coefficient $\bar{n}_0$ as follows;

$$\bar{n}_0 = a^0 d^{0*}. \qquad (4\text{-}11)$$

On the other hand, the capital/output ratio $\bar{b}_0$ becomes

$$\bar{b}_0 = 1 \qquad (4\text{-}12)$$

by the definition of vertically integrated sector. Then, the price of the unit vector of consumption goods, $P^0$, becomes

$$P^0 = \left( \bar{n}_0 + \frac{\pi \bar{n}_1 \bar{b}_0}{1 - \pi \bar{b}_1} \right) w, \quad (\bar{b}_0 = 1) \qquad (4\text{-}13)$$

We may confirm that (4-6)~(4-13) have the coordination with (2-10) and (2-11). We have finished the study of constructing the price system from vertically integrated sectors.

### 4.2 Reconstruction of model

We may now reconstruct our game theoretic model in section 2 using (4-6)~(4-13). It is important to note that the two sectors, i.e. sector 0 and sector 1, are introduced starting from the usual input-output model, via vertically integrated sectors. They may be constructed theoretically from usual input-output analysis. Now let us start our study, paying our attentions to the problem of the unit.

**Production sectors**

The structure of the model is the same with that of section2. We adopt the labor coefficient $\bar{n}_i$ and capital/output ratio $\bar{b}_i, (i = 0, 1)$ defined in paragragh4.1. Let $X_i$ and $N_i$ denote the output and the labor employed in each sector respectively$(i = 0, 1)$. At this place, it is important to confirm the unit. Namely, $X_0$ and $X_1$ are measured by the unit vector of consumption goods $d^{0*}$ and the unit vector of capital stocks $Hd^{0*}$ respectively. And we define

$$X_i = \frac{1}{\bar{b}_i} K_i \quad (i = 0, 1.) \qquad (4\text{-}14)$$

where capital stock $K_i$ is also measured by the unit vector of capital stocks $Hd^{0*}$. By (4-12), $\bar{b}_0 = 1$. The capital/output ratio $\bar{b}_1$ is defined in Def.4.3. Let $K$ denote the amount of capital stocks which exist in the economy. Since the ratio of distribution of capital stocks to sector 0 is $v$ and the one to sector 1 is $1-v$, we obtain

$$K_0 = vK, \tag{4-15}$$

$$K_1 = (1-v)K, \tag{4-16}$$

$$X_0 = \frac{1}{\bar{b}_0} vK, \tag{4-17}$$

$$X_1 = \frac{1}{\bar{b}_1}(1-v)K. \tag{4-18}$$

In the model of this section, the variable $v$ is also the strategy of capital distributor. Therefore, we assume that capital distributor knows the calculation of the unit and makes her decisions. In addition, we obtain

$$N_0 = \bar{n}_0 X_0 \ , \tag{4-19}$$

$$N_1 = \bar{n}_1 X_1, \tag{4-20}$$

$$N = N_0 + N_1 \tag{4-21}$$

where $N$ denotes the total quantity of labor employed.

We also adopt the assumptions regarding capital stocks in section 2, i.e. assumption2.1, assumption2.2 and assumption2.3. Thus we obtain

$$\dot{K}(=\dot{K}_0 + \dot{K}_1) = X_1. \tag{4-22}$$

**Price system**

The prices are given by (4-8) and (4-13) where $\bar{b}_0 = 1$. Since the prices must be positive, we assume

$$\pi < \frac{1}{\bar{b}_1} \ . \tag{4-23}$$

From (4-1) and (4-3), we obtain

$$P^0 = 1. \tag{4-24}$$

which is the counterpart of (2-13) and means that we adopt the unit vector of consumption goods $d^{0*}$ as the *nume'raire*.

**Equilibrium condition : fundamental equation of Kaldor's model**

Finally, we also adopt the equilibrium condition. The way of introduction of it is the same with that in section 2. Hence, we show the results directly:

$$P^1 X_1 = s_p \Pi + s_w W \tag{4-25}$$

where

$$W = \left( \frac{\bar{n}_0}{\bar{b}_0} K_0 + \frac{\bar{n}_1}{\bar{b}_1} K_1 \right) w \ , \tag{4-26}$$

$$\Pi = \frac{\pi \bar{n}_1}{1-\pi \bar{b}_1} wK_0 + \left( \frac{1}{1-\pi \bar{b}_1} - 1 \right) \frac{\bar{n}_1}{\bar{b}_1} wK_1 \tag{4-27}$$

and $s_p$ and $s_w$ represent the saving rates of capitalists and that of workers respectively.

$s_p$ and $s_w$ are the strategies of capitalists and workers respectively, as is the same with sector 2.

**Players of the macroeconomic game**

The definition of the game is the same with that in section 2. Namely, players of the game are capitalists, workers and capital distributor and we define their problems as follows respectively;

$$\max_{s_p} \pi, \quad \text{s.t.} \ 0 \le s_p \le 1 \tag{4-28}$$

$$\max_{s_w} w, \quad \text{s.t.} \ 0 \le s_w \le 1 \tag{4-29}$$

$$\max_{v} \pi, \quad \text{s.t.} \ 0 \le v \le 1. \tag{4-30}$$

**To complete the model**

We have 15 unknowns, $X_0, X_1, P^0, P^1, K, K_0, K_1, N, N_0, N_1, s_p, s_w v$ , $w$, $\pi$ .For these unknowns, we propose 15 equations, (4-8),(4-13) (4-15)~(4-22),(4-24),(4-25),(4-28)~(4-30). These conditions complete the equation system.

**4.3 Analysis**

The way of analysis of the model and the results of it are completely the same with those in section 2.

## 5. Model with R&D and innovation

We shall construct a model with R&D and innovation by integrating the models in section 2 and section 4. Hence, the framework of the model is the same with that in section 2 and in section 4. We will infuse R&D and innovation into the model in section 4 and analyze the essential roles of R&D and innovation in the economy. This analysis constructs the main core of this paper. As is analyzed in section 2, once full employment is achieved on the balanced economic growth, money market would be destroyed and the capitalism would collapse because of the saturation. Our main theoretical interest is the problem that ‘Can capitalism survive ? ’. The key word for this problem is ‘*cyclical innovation*’.

Behind the process of production, there exist many technologies. These technologies are created by intellectual labor. Namely technologies which exist are the result of R&D efforts made by human beings. As Schumpeter pointed out[16], these efforts for innovation construct the fundamental element of economic development. The inputs of the efforts for innovation always exist behind the production of goods and generate new value, i.e. new knowledge and technology. Although human beings cannot predict precisely what happens regarding technologies, we may make the theories concerning what is required for the technologies to achieve the proper economic growth and sustain it.

Firstly, innovation contains process innovation and product innovation. The former is the type of technological change which gives improvements in the production of already existing goods, and the later is the type of innovation which provides new product which doesn’t exist in the economy. These types of innovation occur in the complex forms and bring about the complex structural changes in the economy. To analyze these problems, we will put our view points on vertically integrated analysis and infuse the two types of innovations into the model in section 4. For this purpose, we put the following assumption.

**Assumption 5.1** The R&D efforts for product innovation are made in sector 0 (i.e. the sector producing consumption goods) and those for process innovation are made in sector 1(i.e. the sector producing capital goods).

Therefore, sector 0 produces consumption goods and carries out R&D for product innovation, and sector 1 produces capital goods and carries out R&D for process innovation.

### 5.1 Redefinition of Unit

In section 4.1, we have defined the unit vector of consumption goods and the unit vector of capital stocks. Since we infuse R&D and innovation into our model, we have to also infuse structural change into it. Therefore, we have to define the unit vector of consumption goods and the unit vector of capital stocks corresponding to the structural changes.

Innovation may occur at any time and the change of technology occurs at the same

[16] Schumpeter(1931), II.

time. These new changes appear as time goes on. Of course, these technological changes construct new technical systems in the economy. Thus, we line up the new technical systems in order which appear in the economy as time goes on and give the suffix $\tau(=1,2,3,\cdots)$ to the variables in order which we have defined in section 4. We call the technical system which has the suffix $\tau$ the technical system $\tau$. For instance, the coefficients and variables such as $\bar{n}_i(\tau), \bar{b}_i(\tau), X_i(\tau)$ and $K_i(\tau)$ represent $\bar{n}_i, \bar{b}_i$ $X_i$ and $K_i$ which may be defined under the technical system $\tau$ respectively $(i=0,1)$.

Using this notation, the number of the commodies under the technical system $\tau$ is denoted by $n(\tau)$ and matrices $A^C$, $A^F$ and $A^*(=A^C+A^F\delta)$ are denoted by $A^C(\tau)$, $A^F(\tau)$ and $A^*(\tau)$ respectively. They are the matrices of $n(\tau)\times n(\tau)$. Therefore, the matrix $H$ and the vector $a^i$ which define the vertically integrated sector also depend on the technical system $\tau$ and are denoted by

$$H(\tau)=A(\tau)(I-A^*(\tau))^{-1},$$
$$a^i(\tau)=l(\tau)(I-A^*(\tau))^{-1}H^i(\tau)=a^0(\tau)H^i(\tau),\ (i=1,2,\ldots\ldots)$$

respectively.

In the same way, we may redefine the counterparts of (4-1),(4-2),(4-3),(4-4),(4-6),(4-7),(4-8),(4-11),(4-12) and (4-13). We redefine those briefly.

**Definition 5.1** ( counterpart of Def.4.1)

We call the vector $d^{0*}(\tau)=(d_1^{0*}(\tau)\cdots\cdots,d_{n(t)}^{0*}(\tau))'$ which satisfies

$$p^0(\tau)\,d^{0*}(\tau)=1 \qquad (5\text{-}1)$$

*the unit vector of consumption goods under the technical system* $\tau(=1,2,3,\cdots)$.

Note that $d^0(\tau)$ and $d^{0*}(\tau)$ are $n(\tau)\times 1$ vectors. The price $p^0(\tau)$ is the price vector which is defined in (3-17) and depends on the technical system $\tau$. We also put

$$d^0(\tau)=\mu(\tau)d^{0*}(\tau) \qquad (5\text{-}2)$$

which is the counterpart of (4-2). The price of the vector $d^{0*}(\tau)$, i.e. $P^0(\tau)$, is defined by

$$P^0(\tau)=\sum_{j=1}^{n(t)}\left[\sum_{s=0}^{\infty}\left(\pi^{s+1}(\tau)a_j^{s+1}(\tau)\right)+a_j^0(\tau)\right]d_j^{0*}(\tau)w(\tau) \qquad (5\text{-}3)$$

which is the counterpart of (4-3).

**Definition 5.2** (counterpart of Def.4.2)

We call the vector $H(\tau)d^{0*}(\tau)$ *the unit vector of capital stocks under the technical system* $\tau$ and define its price $P^1(\tau)$ by

$$P^1(\tau)=\sum_{j=1}^{n(\tau)}\left[\sum_{s=1}^{m-1}\left(\pi^s(\tau)a_j^{s+1}(\tau)\right)+a_j^1(\tau)\right]d_j^{0*}(\tau)w(\tau) \tag{5-4}$$

which is the counterpart of (4-4). Noting $a^i(\tau)=a^0(\tau)H^i(\tau)$, (5-4) becomes

$$P^1(\tau)=a^0(\tau)(I-\pi(\tau)H(\tau))^{-1}H(\tau)d^{0*}(\tau)w(\tau)\quad, \tag{5-5}$$

which is the counterpart of (4-5).

Finally, we define the technical coefficients $\overline{n}_i$ and $\overline{b}_i$ under the technical system $\tau$ by

$$\overline{n}_i=\overline{n}_i(\tau),\overline{b}_i=\overline{b}_i(\tau).\qquad (i=0,1) \tag{5-6}$$

Note that $\overline{b}_0(\tau)=1$ for all $\tau$ by definition.

It is very important to confirm that all units and all variables vary corresponding to the change of the technical system $\tau$. Although our model seems to be stationary, the change which occurs in the model is inevitable and complex. It contains *the creative destruction* defined by Schumpeter[17].

### 5.2 Construction of model

We shall construct the model with R&D and innovation which has the same structure with that in section 4, remembering assumption 5.1. For simplicity, we omit the suffix $\tau$ on which all coefficients and all variables except the strategies of players depend.

**Production sectors**

The outputs $X_0$ and $X_1$ are measured by the vectors $d^{0*}(\tau)$ and $H(\tau)d^{0*}(\tau)$ respectively. From the definition of strategy $v$, we obtain

$$K_0=vK, \tag{RD-1}$$

$$K_1=(1-v)K, \tag{RD-2}$$

and

$$X_0=\frac{1}{\overline{b}_0}vK,\quad (\overline{b}_0=1) \tag{RD-3}$$

[17] Schumpeter (1954), pp.81-86.

$$X_1 = \frac{1}{\overline{b}_1}(1-v)K. \tag{RD-4}$$

Next let us infuse R&D into the model. The labor coefficient $\overline{n}_i$ is defined in (5-6). Remembering assumption5.1, we assume that the quantity of labor $n_i^{RD}$ is input into R&D in sector $i$ at the same time when sector $i$ produces one unit of production $(i = 0,1)$. We represent the total coefficient of labor in sector $i$ by

$$\hat{n}_i = \overline{n}_i + n_i^{RD}. \tag{5-7}$$

As the counterpart of assumption 2.4, we put

$$\frac{\overline{n}_0}{\overline{b}_0} < \frac{\overline{n}_1}{\overline{b}_1}. \tag{5-8}$$

In addition, we obtain

$$N_0 = \hat{n}_0 X_0 \ , \tag{RD-5}$$

$$N_1 = \hat{n}_1 X_1, \tag{RD-6}$$

$$N = N_0 + N_1. \tag{RD-7}$$

We also follow the assumptions about capital stocks, i.e. assumption2.1, assumption2.2 and assumption2.3. Therefore, we obtain

$$\dot{K}(=\dot{K}_0 + \dot{K}_1) = X_1. \tag{RD-8}$$

**Price system**

The price of the product of sector 0 becomes

$$P^0 = \left(\hat{n}_0 + \frac{\pi \hat{n}_1 \overline{b}_0}{1-\pi \overline{b}_1}\right) w, \quad (\overline{b}_0 = 1)$$

$$= \left(\overline{n}_0 + \frac{\pi \overline{n}_1 \overline{b}_0}{1-\pi \overline{b}_1}\right) w + \left(n_0^{RD} + \frac{\pi\, n_1^{RD}\, \overline{b}_0}{1-\pi \overline{b}_1}\right) w. \tag{RD-9}$$

The price (RD-9) may be divided into the two parts:

$$\begin{cases} \overline{P}^0 = \left(\overline{n}_0 + \dfrac{\pi \overline{n}_1 \overline{b}_0}{1-\pi \overline{b}_1}\right) w, \\ \\ P^{0\,RD} = \left(n_0^{RD} + \dfrac{\pi\, n_1^{RD}\, \overline{b}_0}{1-\pi \overline{b}_1}\right) w, \end{cases} \tag{5-9}$$

where $\overline{P}^0$ contains the quantity of labor required to produce the unit consumption goods vector $d^{0*}(\tau)$ and $\overline{P}^{0RD}$ contains the quantity of labor input to R&D when one unit of $d^{0*}(\tau)$ is produced.

The price of the product of sector 1 also becomes

$$P^1 = \frac{\hat{n}_1}{1-\pi \bar{b}_1} w, \qquad \text{(RD-10)}$$

which may also be divided into the two parts;

$$\begin{cases} \bar{P}^1 = \dfrac{\bar{n}_1}{1-\pi \bar{b}_1} w, \\ \\ P^{1RD} = \dfrac{n_1^{RD}}{1-\pi \bar{b}_1} w. \end{cases} \qquad \text{(5-10)}$$

Since the prices must be positive, we assume

$$\pi < \frac{1}{\bar{b}_1} \ , \qquad \text{(5-11)}$$

which is the counterpart of (2-12).

To complete the price system, we adopt the unit vector of consumption goods $d^{0*}(\tau)$ as the *nume'raire* and put

$$P^0 = 1. \qquad \text{(RD-11)}$$

which is the counterpart of (4-24).

**Equilibrium condition : fundamental equation of Kaldor's model**

The way of introducing equilibrium condition is the same with that in section 2.Therefore, we show the results directly. The total wages $\hat{W}$ become

$$\hat{W} = \left( \frac{\hat{n}_0}{\bar{b}_0} K_0 + \frac{\hat{n}_1}{\bar{b}_1} K_1 \right) w \ = \left( \frac{\bar{n}_0}{\bar{b}_0} K_0 + \frac{\bar{n}_1}{\bar{b}_1} K_1 \right) w \ + \left( \frac{n_0^{RD}}{\bar{b}_0} K_0 + \frac{n_1^{RD}}{\bar{b}_1} K_1 \right) w. \qquad \text{(5-12)}$$

We denote the first term and second one of the right side of (5-12) by $\overline{W}$ and $W^{RD}$

respectively. Therefore, the total wages $\hat{W}$ become

$$\hat{W} = \overline{W} + W^{RD}. \qquad \text{(5-13)}$$

On the other hand, the total profits $\hat{\Pi}$ become

$$\begin{aligned} \Pi &= \frac{\pi \hat{n}_1}{1-\pi \bar{b}_1} wK_0 + \left( \frac{1}{1-\pi \bar{b}_1} - 1 \right) \frac{\hat{n}_1}{\bar{b}_1} wK_1 \\ &= \frac{\pi \bar{n}_1}{1-\pi \bar{b}_1} wK_0 + \left( \frac{1}{1-\pi \bar{b}_1} - 1 \right) \frac{\bar{n}_1}{\bar{b}_1} wK_1 + \frac{\pi n_1^{RD}}{1-\pi \bar{b}_1} wK_0 + \left( \frac{1}{1-\pi \bar{b}_1} - 1 \right) \frac{n_1^{RD}}{\bar{b}_1} wK_1. \end{aligned} \qquad \text{(5-14)}$$

We denote the first and second terms of the right side of (5-14) by $\overline{\Pi}$ and $\Pi^{RD}$

respectively. Therefore, the total profits $\hat{\Pi}$ become

$$\hat{\Pi} = \overline{\Pi} + \Pi^{RD}. \tag{5-15}$$

Thus, by the equilibrium condition, we obtain

$$(\overline{P}^1 + P^{1RD})X_1 + P^{0RD}X_0 = s_p\hat{\Pi} + s_w\hat{W},$$

which also becomes

$$\overline{P}^1 X_1 + (P^{1RD}X_1 + P^{0RD}X_0) = (s_p\overline{\Pi} + s_w\overline{W}) + (s_p\Pi^{RD} + s_w W^{RD}). \tag{RD-12}$$

(RD-12) shows the fundamental equation of Kaldor's model with R&D.

**Players of the macroeconomic game**

The definition of the game is the same with that in section 2. Namely, players of the game are capitalists, workers and capital distributor and we define their problems as follows respectively;

$$\max_{s_p} \pi, \quad \text{s.t. } 0 \le s_p \le 1 \tag{RD-13}$$

$$\max_{s_w} w, \quad \text{s.t. } 0 \le s_w \le 1 \tag{RD-14}$$

$$\max_{v} \pi, \quad \text{s.t. } 0 \le v \le 1. \tag{RD-15}$$

Finally we have to refer to $n_0^{RD}$ and $n_1^{RD}$. We assume capital distributor determines the value of $n_0^{RD}$ and $n_1^{RD}$. That is, $n_0^{RD}$ and $n_1^{RD}$ are her strategies. We also assume that the savings $s_p\Pi^{RD} + s_w W^{RD}$ are handled to capital distributor. Note that the savings $s_p\Pi^{RD} + s_w W^{RD}$ are determined when capital distributor determines her strategies $n_0^{RD}$ and $n_1^{RD}$. As is shown later, capital distributor cannot but choose a certain strategy. This strategy is important for the economy to realize the balanced economic growth. Thus, we have to investigate the conditions of $n_0^{RD}$ and $n_1^{RD}$ to guarantee the existence of Nash equilibrium and the balanced economic growth. T

**To complete the model**

We have 15 unknowns, $X_0, X_1, P^0, P^1, K, K_0, K_1, N, N_0, N_1, s_p, s_w v$ , $w$ , $\pi$ .For these unknowns, we have 15 equations, (RD-1)~(RD-15). These conditions complete the equation system.

Finally we have to refer to $n_0^{RD}$ and $n_1^{RD}$ . In (RD-12), the savings of players are handed to capital distributor. Therefore, capital distributor could determine $n_0^{RD}$ and $n_1^{RD}$ in addition to $v$. As is shown later, capital distributor may cannot but choose a certain strategy. This strategy is important for the economy to realize the balanced economic growth. Thus, we have to investigate the conditions of $n_0^{RD}$ and $n_1^{RD}$ to guarantee the existence of Nash equilibrium and the balanced economic growth. This problem constitutes the main purpose of this section.

### 5.3 Analysis

We have only to repeat the analysis made in section2. To avoid unnecessary reputation, we will show the results directly.

The profit rate function becomes

$$\pi = \frac{\frac{\hat{n}_1}{\bar{b}_1}(1-s_w) - \left\{ \frac{\hat{n}_1}{\bar{b}_1} - \frac{n_0^{RD}}{\bar{b}_0} - \left( \frac{\hat{n}_1}{\bar{b}_1} - \frac{\hat{n}_0}{\bar{b}_0} \right) s_w \right\} v}{(s_p - s_w)\hat{n}_1 + \left\{ s_w \bar{b}_1 \left( \frac{\hat{n}_1}{\bar{b}_1} - \frac{\hat{n}_0}{\bar{b}_1} \right) - \bar{b}_1 \left( \frac{n_1^{RD}}{\bar{b}_1} - \frac{n_0^{RD}}{\bar{b}_0} \right) \right\} v} = \hat{A} + \frac{\hat{B}}{\hat{C} + \hat{D}v} \qquad \text{(5-16)}$$

where

$$\hat{A} = - \frac{\frac{\hat{n}_0}{\bar{b}_0} s_w + (1-s_w)\frac{\hat{n}_1}{\bar{b}_1} - \frac{n_0^{RD}}{\bar{b}_0}}{s_w \bar{b}_1 \left( \frac{\hat{n}_1}{\bar{b}_1} - \frac{\hat{n}_0}{\bar{b}_0} \right) - \bar{b}_1 \left( \frac{n_1^{RD}}{\bar{b}_1} - \frac{n_0^{RD}}{\bar{b}_0} \right)} \ ,$$

$$\hat{B} = \frac{\hat{n}_1 \left[ \frac{\bar{n}_1}{\bar{b}_1} s_p (1-s_w) - \frac{\bar{n}_0}{\bar{b}_0} s_w (1-s_p) - (1-s_p)(1-s_w)\left( \frac{n_1^{RD}}{\bar{b}_1} - \frac{n_0^{RD}}{\bar{b}_0} \right) \right]}{s_w \bar{b}_1 \left( \frac{\hat{n}_1}{\bar{b}_1} - \frac{\hat{n}_0}{\bar{b}_0} \right) - \bar{b}_1 \left( \frac{n_1^{RD}}{\bar{b}_1} - \frac{n_0^{RD}}{\bar{b}_0} \right)} \ ,$$

$$\hat{C} = (s_p - s_w)\hat{n}_1 \ , \quad \hat{D} = s_w \bar{b}_1\left(\frac{\hat{n}_1}{\bar{b}_1} - \frac{\hat{n}_0}{\bar{b}_0}\right) - \bar{b}_1\left(\frac{n_1^{RD}}{\bar{b}_1} - \frac{n_0^{RD}}{\bar{b}_0}\right).$$

The function (5-16) is the counterpart of (2-23).

By the way, the formal difference between the model in section 2 and that in this section is whether model contains R&D or not. Thus, the variables $n_0^{RD}$ and $n_1^{RD}$ have the key to guarantee the existence of Nash equilibrium and the balanced economic growth. Then, we propose a rule about R&D.

**The equilibrium R&D rule;** We call the R&D variables $n_0^{RD}(\tau)$ , $n_1^{RD}(\tau)$ which satisfy

$$\frac{n_0^{RD}(\tau)}{\bar{b}_0(\tau)} = \frac{n_1^{RD}(\tau)}{\bar{b}_1(\tau)}, \qquad (\bar{b}_0(\tau) = 1) \tag{5-17}$$

*the equilibrium R&D rule under technical system* $\tau$ and denote them by $(n_0^{RD*}(\tau), n_1^{RD*}(\tau))$ . (For simplicity, we will omit the suffix $\tau$ in the following.)

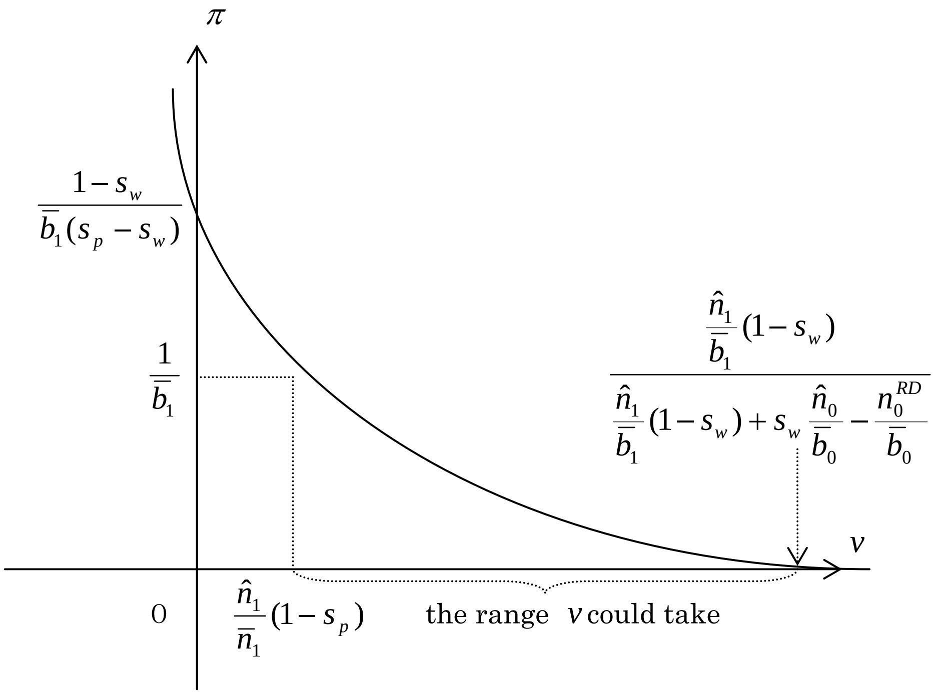
We obtain the following dilemma of capital distributor.

**Dilemma of capital distributor** Capital distributor cannot but keep the strategies which satisfy (5-17).

(Consideration) If Capital distributor abandons the equilibrium R&D rule (5-17), she could determine the profit rate by (5-16). In this case, under the problem of (RD-15), she cannot but choose the strategy $v = \frac{\hat{n}_1}{\bar{n}_1}(1 - s_p)$ which realizes the maximum profit rate $\pi = \frac{1}{\bar{b}_1}$ (see Fig.5-1). Then, the wage rate becomes $w = 0$ which destroys all in our game and capital distributor could not realize any profit rate.

We wished to propose the above dilemma as a proposition. Although we expect that no Nash equilibrium exists in the case of $v\left( > \frac{\hat{n}_1}{\bar{n}_1}(1 - s_p)\right)$, we could not exclude the possibility of the existence of Nash equilibrium in this case . Hence, we refrain ourselves from proposing the dilemma as proposition. Thus, we treat the equilibrium R&D rule as a condition.

**Figure 5-1** the case of $\hat{B} > 0$, $s_p > s_w$




**Proposition 5.1 (Nash equilibrium, balanced economic growth and natural economy)**
(i) *Under the equilibrium condition (5-17),* the combination of the strategies, $\left(\hat{s}_w^*,\ \hat{s}_p^*,\ \hat{v}^*\right)$, becomes the Nash equilibrium[18], i.e.,

$$\left(\hat{s}_w^*,\ \hat{s}_p^*,\ \hat{v}^*\right)=\left(\frac{\hat{n}_1^*\left\{\left(n_0^{RD*}+n_1^{RD*}\right)+\bar{b}_0\bar{b}_1\left(\frac{\bar{n}_1}{\bar{b}_1}-\frac{\bar{n}_0}{\bar{b}_0}\right)\right\}}{\bar{b}_0\bar{b}_1(\hat{n}_0^*+\hat{n}_1^*)\left(\frac{\bar{n}_1}{\bar{b}_1}-\frac{\bar{n}_0}{\bar{b}_0}\right)},\ \frac{(n_0^{RD*}+n_1^{RD*})+\bar{b}_0\bar{b}_1\left(\frac{\bar{n}_1}{\bar{b}_1}-\frac{\bar{n}_0}{\bar{b}_0}\right)}{\bar{b}_0(\bar{b}_0+\bar{b}_1)\left(\frac{\bar{n}_1}{\bar{b}_1}-\frac{\bar{n}_0}{\bar{b}_0}\right)},\ \frac{\bar{b}_0}{\bar{b}_0+\bar{b}_1}\right) \tag{5-18}$$

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18 We expect that (5-17) construct Nash equilibrium with (5-18). But, we cannot prove this completely, because we could not exclude the possibility of the existence of Nash equilibrium in the case of $v\left(>\frac{\hat{n}_1}{\bar{n}_1}(1-s_p)\right)$.

where $\hat{n}_i^* = \overline{n}_i + n_i^{RD*}\ (i = 1,2)$.

(ii) Under the equilibrium condition (5-17) and the strategies (5-18), the profit rate $\hat{\pi}^*$ and the wage rate $\hat{w}^*$ become

$$\begin{cases} \hat{\pi}^* = \dfrac{1}{\overline{b}_0 + \overline{b}_1}, \\ w^* = \dfrac{1}{\hat{n}_0 + \hat{n}_1}. \end{cases} \tag{5-19}$$

(Proof) (i) We may assume that (5-17) is hold. To introduce (5-18), we have only to use the same logic which is used to verify proposiiton2.1 with (2-23). Namely, (5-16) is formally the same with (2-23) under the R$D rule of (5-17). In section 2, we solved the equations (2-26), (2-27) and (2-30) to obtain (2-33). Hence, we have only to solve the following equations about $s_w$, $s_p$ and $v$ under (5-17);

$$\begin{cases} \dfrac{\overline{b}_1}{\overline{n}_1} s_p(1-s_w) = \dfrac{\overline{b}_0}{\overline{n}_0} s_w(1-s_p), \\ \hat{C} + \hat{D}v = 0, \\ v = \dfrac{\overline{b}_0}{\overline{b}_0 + \overline{b}_1}. \end{cases} \tag{5-20}$$

(ii) Adopt the same way used to obtain (2-34). Namely, we cannot determine the profit rate by (5-17) and (5-18). Hence, it depends on the condition of the money balance of the market shown in (2-31). We may determine the profit rate by solving (2-31) under

$C_p = (1-s_p)\hat{\Pi}$ and $S_w = s_w\hat{W}$. (QED)

As is shown in Prop.5.1, the Nash equilibrium exists and constructs the balanced economic growth and the natural economic system under the condition (5-17). However, in the balanced economic growth, we would confront the full employment and the saturation collapse of economy as is stated in section2. Can capitalism survive? We shall consider this problem.

### 5.4 Can capitalism survive?

Let us promote our analysis, remembering that we have infused product innovation and process innovation into our game.

What does technical progress mean? Technical progress brings complex changes in the economy. It is useful to return back to Pasinetti(1981) who investigates the problem of technical progress theoretically in the context of labor theory, applying the vertically integrated analysis. We have to pay our attentions to the treatments of wage rate and profit rate. Pasinetti chooses the wage rate as the *nume'raire*, puts $w=1$ and introduces the natural profit rate to verify that technical progress means the reduction of labor input[19]. On the other hand, in our model, the wage rate and the profit rate are given by (5-19) as the solutions of the game and show complex movements depending on the developments of the coefficients $n_i(\tau), b_i(\tau)(i=0,1)$. However, if the technical system $\tau+1$ occurs by innovation, the price system of (3-17) has to satisfy

$$p_j^0(\tau+1) \leq p_j^0(\tau) \ , \quad (i=1,\cdot\cdots\cdot, n(\tau)) \tag{5-21}$$

because, if not, the technical system $\tau+1$ could not take t
he place of the technical system $\tau$. Through this adjustment process, the technical system $\tau+1$ could be created.

Now we are ready to propose some lemmas.

**Lemma 5.1** (i) If R&D for product innovation is operated in sector 0 under the technical system $\tau$, i.e. $n_0^{RD}(\tau)>0$, we obtain $\overline{n}_0(\tau+1)>\overline{n}_0(\tau)$ in the equilibrium of the game[20].

On the other hand, $\overline{b}_0(\tau+1)=\overline{b}_0(\tau)(=1)$. This means that the unit of '1' under the technical system $\tau+1$ contains more quantities of labor than those under the technical system $\tau$. We cannot obtain any relationship between $\overline{b}_1(\tau+1)$ and $\overline{b}_1(\tau)$, because measures which are defined by *the unit vector of capital stocks* change between two technical systems(see Def.5.2).

(ii) If R&D for process innovation is operated in sector 1, i.e. $n_1^{RD}(\tau)>0$, we cannot obtain any relationship between $n_i(\tau+1)$ and $n_i(\tau)$ and cannot also get any

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19 Pasinetti(1981),pp.206-207.

20 In this case, we cannot obtain any information about wage rate, because $\hat{n}_i(\tau)$ contains the quantity of labor input into R&D (see(5-19)).

relationship between $b_1(\tau+1)$ and $b_1(\tau)$.

(Proof) (i) $d^0(\tau+1)$ is the vector which contains $d^0(\tau)$ and has the higher dimension than $d^0(\tau)$. Thus, we obtain

$$\overline{n}_0(\tau+1) = a^0(\tau+1)d^{0*}(\tau+1) > a^0(\tau)d^{0*}(\tau) = \overline{n}_0(\tau).$$

$\overline{b}_0(\tau+1) = \overline{b}_0(\tau) = 1$ by definition. Moreover, notice that the vector $H(\tau+1)d^{0*}(\tau+1)$ contains the vector $H(\tau)d^{0*}(\tau)$ in this case. Thus, the unit of '1' under the technical system $\tau+1$ contains more quantities of labor than those under the technical system $\tau$.

(ii) Since process innovation brings about complex structural changes in production process, it is impossible to analyze its effects. However, we have to point out the Pasinetti's analysis, 'Technical progress is ultimately revealed to be diminution of labor inputs. To put it in another way, all technical progress is, in the end, labor-saving'[21]. This analysis is important and essential. Hence, if we follow this analysis, we could conclude $\overline{n}_i(\tau+1) \le \overline{n}_i(\tau)$. However, as is mentioned above, the *nume'raire* in our model is different from Pasinetti's one, i.e. $w=1$. On the contrary, we adopt $P^0 = 1$ (RD-11). In addition, the vector $d^{0*}(\tau+1)$ is different from the vector $d^{0*}(\tau)$ (see (4-1) and (4-2)). Furthermore, the profit rate in (5-19) which is included in the price system (RD-9) and (RD-10) is different from 'natural profit rate' defined by Pasinetti[22]. We have to investigate the effect of the difference on the price system. Therefore, we cannot conclude $\overline{n}_i(\tau+1) \le \overline{n}_i(\tau)\,(i=1,2)$. (QED)

Thus, R&D brings about complex structural changes. However, under the equilibrium R&D rule, the change in $b_1(\tau)$ means the mutual change in $n_0^{RD}(\tau)$ and $n_1^{RD}(\tau)$. Namely, the intensity of R&D between $n_0^{RD}(\tau)$ and $n_1^{RD}(\tau)$ would mutually change. Therefore, we put this analysis as a lemma.

**Lemma5.2** Under the equilibrium rule (5-17), the mutual changes in $n_0^{RD}(\tau)$ and $n_1^{RD}(\tau)$ occurs necessarily, because of the change in $b_1(\tau)$ . The changes are irregular

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[21] Pasinetti(1981),pp.206-207.

[22] Ibid., pp.156-175.

but may seem to be cyclical.

(Proof) Suppose that it happens to be $b_1(\tau)=1$ under the technical system $\tau$. In this case, R&D becomes $n_1^{RD}(\tau) = n_0^{RD}(\tau)$ by (5-17). However, even if R&D satisfies $n_1^{RD}(\tau) = n_0^{RD}(\tau)$, it is impossible for $b_1(\tau+i)=1$ to be held for all $i$ $(=1,2,....)$ because innovation occurs. Thus, the technical coefficient $b_1$ becomes $b_1(\tau+i)\neq 1$ sooner or later for some $i$. Then, we can write the following scenario. Suppose that $b_1(\tau)>1$ for some $\tau$. Since $b_1(\tau)>1$, it follows that $n_1^{RD}(\tau) > n_0^{RD}(\tau)$ from (5-17). The condition of $n_1^{RD}(\tau) > n_0^{RD}(\tau)$ means that the intensity of process innovation is relatively strong and it brings about the probability that $b_1$ decreases and $b_1(\tau)<1$ is accomplished. But, once it becomes $b_1(\tau)<1$, it follows that $n_1^{RD}(\tau) < n_0^{RD}(\tau)$ from (5-17). The condition of $n_1^{RD}(\tau) < n_0^{RD}(\tau)$ means that the intensity of product innovation is relatively strong and it brings about the probability that $b_1$ increases and $b_1(\tau)>1$ is accomplished.

Although this scenario does not show the accurate paths of R&D and innovation, we can expect that the change is irregular but has a cyclical property. (QED)

**Lemma 5.3 (process innovation and $\mu$)** If R&D for process innovation is operated in sector 1, i.e. $n_1^{RD}(\tau)>0$, we obtain $\mu(\tau+1)\leq \mu(\tau)$.

(Proof) Since we are considering process innovation, the composition of consumption goods does not change. Hence, the average consumption vector $d^0$ is invariant. Then, we obtain $|d^{0*}(\tau+1)| \geq |d^{0*}(\tau)|$ by Def.5.1 and (5-21), where $|\quad|$ denotes the norm of vector. Since $d^0 = \mu(\tau)d^{0*}(\tau) = \mu(\tau+1)d^{0*}(\tau+1)$, we obtain $\mu(\tau+1)\leq \mu(\tau)$. (QED)

On the other hand, product innovation brings about more complex changes.

**Lemma 5.4 (product innovation and $\mu$)**

If R&D for product innovation is operated in sector 0, i.e. $n_0^{RD}(\tau)>0$, it essentially brings about $\mu(\tau+1) \geq \mu(\tau)$ except the special cases where the product innovation by $n_0^{RD}(\tau)$ makes the price of the existing consumption goods lower enough or the new product affects (i.e. reduces) the average demand for existing consumption goods.

(Proof) In the case that the new product does not affect the average demand for existing consumption goods, the vector $d^0(\tau+1)$ contains $d^0(\tau)$ as its component. That is, $d^0(\tau+1)$ is expressed as $d^0(\tau+1) = (d^0(\tau)^t, \cdots)^t$ where vector $(\cdot)^t$ denotes the transposition of vector $(\cdot)$. In this situation, we have to hold $p^0(\tau)d^{0*}(\tau) = p^0(\tau+1)d^{0*}(\tau+1) = 1$. In addition, $\mu(\tau)$ and $\mu(\tau+1)$ are determined by $d^0(\tau) = \mu(\tau)d^{0*}(\tau)$ and $d^0(\tau+1) = \mu(\tau+1)d^{0*}(\tau+1)$ respectively by (5-2). Therefore, as long as the prices of existing consumption goods, i.e. $p^0(\tau)$, do not decline rapidly by the product innovation by $n_0^{RD}(\tau)$, it must be held that $\mu(\tau+1) \geq \mu(\tau)$.

Intuitively, the probability of the occurrence of the special case where the product innovation makes the price of the existing consumption goods lower enough is very low, because the case is brought about by the decreases in wage rate (see (5-19)). Since new product appears and nothing has happened to the existing consumption goods except the case where the new product reduces the average demand for existing consumption goods rapidly, $\mu(\tau+1) \geq \mu(\tau)$ is held essentially. (QED)

Let us begin our analysis under above preparations. Firstly, if we have macroeconomic points of view, we have to analyze the labor constraint. This constraint is expressed by

$$(\bar{n}_0(\tau) + n_0^{RD}(\tau))(\mu(\tau)N) + (\bar{n}_1(\tau) + n_1^{RD}(\tau))((\mu(\tau)N) \leq N,$$

which becomes

$$\{(\bar{n}_0(\tau) + n_0^{RD}(\tau)) + (\bar{n}_1(\tau) + n_1^{RD}(\tau))\}\mu(\tau) \leq 1. \quad (5\text{-}22)$$

(5-22) is the game theoretic R&D version of (3-9) in vertically integrated analysis. To investigate what occurs in the economy, we put following definition.

**Dedinition5.2 (innovation and $\mu$)**

We call the innovation *employment-creative innovation* if $\mu(\tau+1) > \mu(\tau)$ and call the innovation *labor-saving innovation* if $\mu(\tau+1) < \mu(\tau)$.

The variable $\mu(\tau)$ is interpreted as standing for the price level of the unit vector of consumption goods $d^{0*}$. Since the price system in our model is based on labor theory, judging from the meaning of (5-22), we may state that the decrease in $\mu$ means the occurrence of labor-saving innovation and the increase in $\mu$ means that of employment-creative innovation.

Secondly, the constraint of consumption goods in the equilibrium becomes

$$\mu(\tau)N \leq X_0(\tau)\left(=\frac{1}{b_0(\tau)+b_1(\tau)}K_0(\tau)\exp\left\{\frac{1}{b_0(\tau)+b_1(\tau)}\left(t-t^0(\tau)\right)\right\}\right), \quad ( b_0(\tau)=1 )$$

(5-23)[23]

where $t$ denotes time and $t^0(\tau)$ represents the time when the technical system $\tau$ is adopted. (5-23) shows that the demand for consumption goods cannot exceed the supply of them. The right-hand side of (5-23) expresses the output of consumption goods in the equilibrium (i.e., in the balanced economic growth). The conditions (5-22) and (5-23) must be satisfied by controlling the total amount of R&D, $n_0^{RD}$ and $n_1^{RD}$. We call the conditions (5-22) and (5-23) *absolute constraints of R&D*. On the other hand, if the equilibrium R&D rule (5-17) is not satisfied, the Nash equilibrium which accomplishes the balanced economic growth does not exist. Hence, the equilibrium R&D rule (5-17) may be called *relative constraint of R&D*.

Now our final purpose is to search for the mechanism which could accomplish the balanced economic growth, solve the problem of unemployment and at the same time evade the saturation collapse of capitalism shown in Prop.2.3. For this problem, we propose the following hypothesis. Although the hypothesis is not proved mathematically rigidly, it constructs the core of this paper.

***Hypothesis* (Generation of stabilizer by innovation under R&D version of Keynesian policy)**

*The equilibrium R&D rule (5-17) generates a stabilizer which works to accomplish the balanced economic growth, holds unemployment in a certain acceptable range and at the same time evades the saturation collapse of capitalism under the R&D version of Keynesian policy. The stabilizer forms the waves of R&D where the change in intensity*

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23 See the calculation of balanced economic growth in paragraph 2.2.

*between product innovation and process innovation occurs mutually, R&D holding both absolute constraint and relative constraint. The stabilizer forms dynamically complex balanced economic growth.*

**Consideration about hypothesis**

Let us consider about the condition and the reason by which our hypothesis is proposed.

Firstly, let us analyze (5-22). Suppose that the inequality comes out in (5-22) and the difference between left hand side and right one is large enough. In this case a large unemployment exists in the economy. Namely, unemployment occurs because of the shortage of demand as is pointed by Keynes. On the other hand, if the equality comes out in (5-22), full employment is accomplished. However, in this case, the economy confronts the saturation collapse as is shown in Prop.2.3. Thus, the best condition for the economy is the one where the value of the left-hand side of (5-22) is in the neighborhood of 1. If this condition is accomplished, the unemployment becomes as small as possible. Thus, the problem we have to solve is whether the economy contains a mechanism which insures the condition or not.

Secondly, suppose that it happens to be $b_1(\tau)=1$ under the technical system $\tau$. But, as is mentioned in the proof of lemma 5.2, it is impossible for $b_1(\tau+i)=1$ to be held for all $i$ $(=1,2,....)$. Thus, the technical coefficient $b_1$ becomes $b_1(\tau+i)\neq 1$ sooner or later for some $i$. Hence, we will analyze the four cases which could happen in the economy.

(I) Suppose that the unemployment rate is high (that is, the left hand side of (5-22) is smaller than 1 considerably) and $b_1(\tau)<1$ under the technical system $\tau$.

Since $b_1(\tau)<1$, it follows that $n_1^{RD}(\tau)<n_0^{RD}(\tau)$ from (5-17). Then, it also follows that $\mu(\tau+1)\geq \mu(\tau)$ except the special cases from Lemma5.4. Thus, the unemployment rate declines by (5-22).

But, $n_1^{RD}(\tau)<n_0^{RD}(\tau)$ brings about the change in $b_1$ and it may happen that $b_1>1$ where R&D satisfies $n_1^{RD}(\tau)>n_0^{RD}(\tau)$ and labor-saving innovation may occur before the problem of unemployment is solved enough. In this case, the economy requires the aid of policy which brings about product innovation from outside of the economy.

Note that the condition $b_1(\tau)<1$ does not continue forever by lemma 5.2.

(II) Suppose that the unemployment rate is high (that is, the left hand side of (5-22) is smaller than 1 considerably) and $b_1(\tau)>1$ under the technical system $\tau$.

Since $b_1(\tau) > 1$, it follows that $n_1^{RD}(\tau) > n_0^{RD}(\tau)$ from(5-17). Then, it also follows that $\mu(\tau+1) < \mu(\tau)$ from Lemma5.3. Thus, the unemployment rate increases by (5-22). However, $n_1^{RD}(\tau) > n_0^{RD}(\tau)$ brings about the decrease in $b_1$ and accomplishes $b_1(\tau) < 1$ (see Lemma5.2).Thus, it will be held that $\mu(\tau+1) > \mu(\tau)$ and the unemployment rate will decrease.

Note that although the unemployment increases at the early stage of (II), this bad situation does not continue forever because of the equilibrium R&D rule. However, at this stage, the economy needs the aid of the policy which accelerates the product innovation and makes it possible to evade from the problem of unemployment.

(III) Suppose that the unemployment rate is low (that is, the left hand side of (5-22) is in the neighborhood in 1) and $b_1(\tau) < 1$ under the technical system $\tau$.

Since $b_1(\tau) < 1$, it follows that $n_1^{RD}(\tau) < n_0^{RD}(\tau)$ from (5-17). Then, it also follows that $\mu(\tau+1) > \mu(\tau)$ with a high probability from Lemma5.4. Thus, the unemployment rate decreases by (5-22) and the probability that the economy experiences the saturation collapse becomes high. However, $n_1^{RD}(\tau) < n_0^{RD}(\tau)$ brings about an increase in $b_1$ and accomplishes $b_1(\tau) > 1$(see Lemma5.2). Thus, it will be held that $\mu(\tau+1) < \mu(\tau)$ and the unemployment rate will increase. Thus, the crisis of saturation collapse will be avoided. Note that although the probability of saturation collapse of the economy is high at the early stage of (III), this bad situation does not continue forever because of the equilibrium R&D rule. However, at this stage, the economy needs the aid of the policy which accelerates the process innovation and makes it possible to evade from the saturation collapse of the economy.

(IV) Suppose that the unemployment rate is low (that is, the left-hand side of (5-22) is in the neighborhood in 1) and $b_1(\tau) > 1$ under the technical system $\tau$.

Since $b_1(\tau) > 1$, it follows that $n_1^{RD}(\tau) > n_0^{RD}(\tau)$ from (5-17). Then, it also follows that $\mu(\tau+1) < \mu(\tau)$ from Lemma5.3. Thus, the unemployment rate increases by (5-22). Thus, crisis of the saturated collapse will be avoided.

But, $n_1^{RD}(\tau) > n_0^{RD}(\tau)$ brings about a change in $b_1$ and it may happen that $b_1 < 1$.

Thus, R&D becomes $n_1^{RD}(\tau) < n_0^{RD}(\tau)$. Hence, there is a probability that the employment begins to increase before the saturation collapse of economy is solved enough. In this case, the economy requires the aid of policy which brings about process innovation from the outside of the economy. Note that the condition $b_1(\tau) < 1$ does not continue forever by lemma 5.2.

We have finished the consideration on our hypothesis. In this stage, we must emphasis two points. In all cases considered above, the economy needs the aid of policy from the outside of the economy. In this stage, we should emphasize that although the equilibrium R&D rule (5-17) generates the cyclical R&D which plays the role of constructing the stabilizer of the economy, it does not include the capacity of recognizing the crisis and solving it objectively. This implies that the economy needs the aid of policy. Particularly, the case (II) and the case (III) need the strong aid of policy to avoid the crisis, because the reflection by the equilibrium R&D rule to avoid crisis are slower than that in the case (I) and the case (IV).

The other point is important. The equilibrium R&D rule has the power for accomplishing the balanced economic growth. We have an image for the balanced economic growth that it is expressed by the linear path. But, in our model, the complex change in the unit occurs (see for example (5-1), (5-2) and (5-6) ) behind the balanced economic growth. The change in the unit implies the change in economic structure which could not represented even by non-linear system. The change is complex and evolutionary.

The equilibrium R&D rule (5-17) has the possibility that it plays the role of stabilizer of the economy. It may avoid deep unemployment and saturated collapse. If economy (thus capitalism) can survive, it depends on whether R&D is accomplished properly and economy has a mechanism which works for this proper R&D. In our model, it depends on whether the equilibrium R&D rule (5-17) is held and government helps to avoid the crisis by proper policy. In particular, we could not show the equilibrium R&D rule is brought into Nash equilibrium. This means the problem: who makes the decision. It is the synonym for the problem pointed out by Schumpeter; does entrepreneur exist?

We finally propose the following proposition.

***Proposition 5.2*** *(R&D version of Keynesian policy: the control of effective demand by R&D)*

*To hold the balanced economic growth with innovation, a mechanism of keeping (5-17) and the policy which sustains the equilibrium R&G rule are inevitable. The policy is cyclical and keeps the R&D variables which hold the effective demand in the proper* *range where unemployment is not serious and the saturation collapse is avoided. In addition, we should point out that what we have discussed in this section is the control of the effective demand by R&D (see (5-22)). This implies that the control is interpreted as the R&D version of Keynesian policy.*

## 6. Conclusion

In this paper, we tried to synthesize Ricardo's labor theory, Keynesian theory and Schumpeter's idea that the fundamental factor of economic developments lies in the creative destruction, by adopting game theory. For concluding this paper, we want to point out some problems concerning to R&D and innovation.

Firstly, as we mentioned in section1, viewing from microeconomic point, the theories on R&D have been promoted mainly by the development of the game theory. In this field, many significant studies are reported. However, not only in R&D theory but also in the whole economics, there exists a deep gap between microeconomics and macro-economics. Hence, it is necessary to propose the theory which could vanish this gap. Game theory would play an important role as is shown in this paper. The introduction of game theory means the importance of the infusion of many fruits of microeconomic game theory to macroeconomics.

In addition, as is mentioned in section 1, the theories which have the possibility to vanish this gap are evolutional economic theory, game theory and complex dynamic theory. Among them, we focus on Feichtinger and Sorger(1988). They considered a scientist who has two main activities, basic R&D and the application of the knowledge obtained by basic R&D. By using dynamic programming theory, they showed the existence of a periodic solution between basic R&D and its application. Although their theory is a microeconomic one, there is a room for applying it to macroeconomics. This study is tried in Akimoto(2006)(which is written in Japanese). If we succeed to vanish the gap between microeconomics and macroeconomics which exists in Feichtinger and Sorger(1988), we could construct the theory which supports our *R&D version of Keynesian policy*(see Prop.5.2).

Secondly we want to focus on Schumpeter's sense of crisis for capitalism; Can capitalism survive[24]? Schumpeter pointed out the importance of 'creative destruction' and analyzed the five 'new combinations'[25]. However, there is no theory which deepens and formulates these important concepts. If it is possible, it would be described by

[24] Schumpeter(1954), pp.81-110.

[25] Schumpeter(1934), fourth printing(1951)p.66.

evolutionary economics, complex dynamics and game theory totally. The synthesis of these theories would provide a new economics' field.

Now, final point is very important. Most important approach in this paper is labor theory. In this paper, all variables are connected with and measured by labor theory. Vertically integrated analysis developed by Pasinetti plays an important role. In our model in section 5, even R&D is measured not by capital but by labor. The labor input into R&D is intellectual one. It must be pointed out that the formulation and the definition of intellectual labor are theoretically fragile in this paper. It is expressed only by the input of labor in this paper. If they are concerned with production theory, all problems must be based on labor theory. The fundamental factor which constitutes and promotes economy is labor, even if it is physical or intellectual.

However, we cannot treat physical labor and intellectual labor in the same dimension. Their characters are different. Therefore, if we want to introduce the two different types of labor theoretically precisely in a model, we must construct a common measure among them. Is there such a common measure which includes labor theory? The rigid fact is that the energy of human beings are inspired both to the two different types of labor. Therefore, if a common measure exists, it is measured by the energy. This implies that we should construct a theory of energy. As mentioned above, there exists a deep gap between microeconomics and macroeconomics and it is needed to vanish this gap. Hence, firstly the energy of theory should be defined in the dimension of microeconomics and in the next stage we should construct the macroeconomic theory, starting from the microeconomic definition and keeping logical consistency and theoretical compatibility.

## References


Aghion,P.,and P. Howitt(1992), "A Model of Growth through Creative Destruction," *Econometrica*, Vol.60,No.2,pp.323-351.

Akimoto,K.,(2006), *Macroeconomic Analysis of R&D; an approach from the Analysis Economic Model,* Kyushu University Press, Japan, ( written in Japanese).

Basar,T.,Haurie,A.,and G.Ricci(1985), "On the Dominance of Capitalists Leadership in a 'Feedback-Stackelberg' Solution of a Differential Game Model of Capitalism," *Journal of Economic Dynamics and Control*, Vol.9. pp.101-125.

Benabou,Roland and J. Tirole(2006),"Belief in a Just World and Redistributive Politics," Quarterly Journal of Economics, Vol.121,pp.699-745.

Dasgupta,P., and J.Stiglitz[1980], "Uncertainty, Industrial Structure ,and the Speed of R&D," *Bell Journal Economics* Vol.11,pp.1-28

Freeman,S.,Hong Dong-Pyo, and Dan Peled, (1998)" Endogenous Cycles and Growth with Indivisible Technological Development," *Review of Economic Studies*, Vol.2,pp.403-432.

Feichtinger,G., and G. Sorger[1988], "Periodic Research and Development," in *Optimal Control Theory and Economic Analysis 3*, G.Feichtinger(ed.), North- Holland.

Iwai, K.(1984a), " Schumpeterian Dynamics ; An Evolutionary Model of Innovation and Imitation," *Journal of Economic Behavior and Organization*, Vol.5,pp.159-190.

Iwai ,K.,(2000), "A Contribution to Evolutionary Theory of Innovation, Imitation and Growth," *Journal of Economic Behavior and Organization*,Vol.43,pp.167-198.

Iwai, K.(2001), " Schumpeterian Dynamics ; A Disequilibrium Theory of Long Run Profit," in Punzo,L.F.(ed.),C*ycles Growth and Structural Change,* pp.169-200.

Kaldor,N.(1955-1956), "Alternative Theories of Distribution," Review of Economic Studies, Vol.23. pp.83-100.

Kaitala,V., and M. Pohjola,(1990),"Economic Development and Agreeable Redistribution in Capitalism: Efficient Game Equilibria in a Two-class Neoclassical Model," *International Economic Review*,Vol.31,No.2,pp.421-438.

Lancaster,K.(1973), "The Dynamic Inefficiency of Capitalism," Journal of Political Economy,Vol.81,pp.1092-1109.

Lee,T. and L.L. Wilde[1980], " Market Structure and Innovation; a reformulation," *Quarterly Journal of Economics*,Vol.94,pp.429-436.

Leontief,W.W.(1951), *The Structure of American Economy,1919-1939, An Empirical Application of Equilibrium Analysis*, Oxford University Press, Yew York.

Leontief,W.W.(1966), *Input-Output Economics*, Oxford University Press, New York.

Loury,G.C.[1979], "Market Structure and Innovation," *Quarterly Journal of Economics*,Vol.93,pp.395-410.

Malueg,D.A., and S. O. Tsutsui, (1997), "Dynamics R&D Competition with Learning," *Rand Journal of Economics*, Vol.2,No.4,pp.751-772.

Medio,A.(1980), "A Classical Model of Business Cycles," in E.J.Nell(ed.) *Growth, Profits, and Property*, Cambridge University Press.

Morishima,M.,(1989),*Ricardo's Economics :A General Equilibrium Theory of Distribution and Growth,* Cambridge University Press.

Pasinetti,L.L.(1973),"The Notion of Vertical Integration in Economic Analysis," *Metroeconomica*,Vol.25,pp.1-29.

Pasinetti,L.L.(1974), *Growth and Income Distribution, Essays in Economic Theory*, Cambridge University Press.

Pasinetti,L.L.(1977), *Lectures on the Theory of Production*, Columbia University Press.
Pasinetti.L.L.(1981), *Structural Change and Economic Growth; a theoretical essay on the dynamics of the wealth of Nation,* Cambridge University Press.
Pasinetti,L.L.(1990), "Normalized General Coordinates and Vertically Integrated Sectors in a Simple Case," in *Nonlinear and Multisectoral Macrodynamics*, K.Velupillai (ed.), The Macmillan Press.
Pohjola,M.(1983), "Nash and Stackelberg Solutions in a Differential Game Model of Capitalism," Journal of Economic Dynamics and Control, Vol.6,pp.173-186.
Pohjola,M.(1985), "Growth Distribution and Employment Modeled as a Differential Games," in *Optimal Control Theory and Economic Analysis 2*, Feichtinger,G.(ed.), North- Holland,1985.
Pohjola,M.(1986), "Applications of Dynamic Game Theory to Macroeconomics," in *Dynamic Games and Applications in Economics*, Bazar,T.(ed.),Springer-Verlag.
Reinganum,J.F.(1981), "Dynamic Games of Innovation," *Journal of Economic Theory*,Vol.25,pp.21-41
Reinganum, J.F.(1983), " Technology Adoption under Imperfect Information," *Bell Journal of Economics*, Vol.14,pp.57-69.
Reinganum,J.F.(1985), "Innovation and Industrial Evolution," Quarterly Journal of Economics,pp.81-99.
Ricci,G.(1985), "A Differential Game of Capitalism; a simulation approach," in Feichtinger,G.(ed.), Optimal Control Theory and Economic Analysis 2,North-Holland.
Ricci,G.(1988), "Application of Mathematics to the Goodwin Model," in Ricci,G. and K.Velupillai(eds.),*Growth Cycles and Multisectoral Economics: the Goodwin Tradition*,Springer-Verlag.
Romer ,P.M.(1990),"Endogenous Technological Change," *Journal of Political Economy*, Vol.98,pp.s71-s102.
Schumpeter,J.A.(1934), *The Theory of Economic Development* , Cambridge, Massachusetts: Harvard University Press.
Schumpeter,J.A.(1954),*Capitalism, Socialism and Democracy*, fourth edition, London , George Allen& Unwin :first edition New York , Harper &Row,1942.
Segerstrom,P.S., Anant,T.C.A. and E.Dinopoulos[1990], "A Schumpeterian Model of the Product Life Cycle," *The American Economic Review*, Vol.80,pp.1077-1091.
Soviotti(2006),"Trade Variety and the Economic Performance of Countries," :Paper to be presented at the 11th conference of the International Schumpeter Society , to be held in Sophia Antipolis on June,22-24,2006.

Sraffa,P.(1960), *Production of Commodities by Means of Commodities*, Cambridge University Press.

Yunfang Hu, Murray C.Kemp, and Koji Shimomura, "Endogenous Growth: Fragile Foundations?," *Review of Development Economics,* Vol.10(1),pp.113-115.

## Appendix (for section 3)

This appendix involves some lemmas which are needed to prove the development of the determinants in section 3.

**Lemma 3.1** The following equation holds:

$$\begin{vmatrix} -I_n & \sigma^0+\pi I_n & & & & & \\ & -I_n & \sigma^1+\pi I_n & & & & \\ & & \ddots & \ddots & & & \\ & & & -I_n & \sigma^{k-1}+\pi I_n & & \\ & & & & -I_n & \sigma^k+\pi I_n & \\ (d^0)^t & (d^1-\pi d^0)^t & (d^2-\pi(\sigma^0 d^0+d^1))^t & \cdots\cdots & & & \left(d^{k+1}-\pi\left[\sum_{s=0}^{k-1}\left\{\left(\prod_{i=s}^{k-1}\sigma^i\right)d^s\right\}+d^k\right]\right)^t \end{vmatrix}$$

$$=(-1)^{k+1}\left[\sum_{s=0}^{k}\left\{\left(\prod_{i=s}^{k}\sigma^i\right)d^s\right\}+d^{k+1}\right] \quad (k>1) \quad . \tag{A1}$$

**(Proof)** Let us analyze the structure of the determinant in (A1). The matrices $I_n$ and $\sigma^i\,(i=0,1,\cdots,k)$ are diagonal ones of $n\times n$. The elements which involve $(d^i)^t$ are row vectors of $1\times n$. Judging from the structure and the definition of determinant which requires the way to choose the elements from each row and from each column, we may develop the matrix in the case of $n>1$ with the same way with the case of $n=1$. Therefore, we prove the lemma in the case of $n=1$ and consider $I_n$ as 1, $0_n$ as 0, and $\sigma^i, d^i$ as real numbers respectively. Let us prove by mathematical induction.

(i) The case of $k=1$: we obtain

$$\begin{vmatrix} -I_n & \sigma^0+\pi I_n & 0_n \\ 0_n & -I_n & \sigma^1+\pi I_n \\ d^0 & d^1-\pi d^0 & d^2-\pi(\sigma^0 d^0+d^1) \end{vmatrix} = (-1)^2(\sigma^1\sigma^0 d^0+\sigma^1 d^1+d^2) \ .$$

Therefore, (A1) holds.

(ii) Let us investigate the case of $k=z+1$ , assuming that the case of $k=z(z\geq 1)$ is held. In the left-hand side of (A1), put $k=z+1$ and make the expansion of cofactors regarding the row which include $\sigma^{z+1}+\pi I_n$ .Then, we obtain

Left-hand side of (A1)=

$$(-1)(-1)^{2z+4}\begin{vmatrix} -I_n & \sigma^0+\pi I_n & & & \\ & \ddots & \ddots & & \\ & & & \sigma^{z-1}+\pi I_n & \\ & & & -I_n & \\ (d^0)^t & (d^1-\pi d^0)^t & \cdots\cdots & & \left(d^{z+2}-\pi\left[\sum_{s=0}^{z}\left\{\left(\prod_{i=s}^{z}\sigma^i\right)d^s\right\}+d^{z+1}\right]\right)^t \end{vmatrix}$$

$$+(-1)^{2z+5}(\sigma^{z+1}+\pi I_n)\begin{vmatrix} -I_n & \sigma^0+\pi I_n & & & \\ & -I_n & \sigma^1+\pi I_n & & \\ & & \ddots & \ddots & \\ & & & -I_n & \sigma^z+\pi I_n \\ (d^0)^t & (d^1-\pi d^0)^t & \cdots\cdots & & \left(d^{z+1}-\pi\left[\sum_{s=0}^{z-1}\left\{\left(\prod_{i=s}^{z-1}\sigma^i\right)d^s\right\}+d^z\right]\right)^t \end{vmatrix}$$

$$=(-1)^{z+2}\left[d^{z+2}-\pi\left[\sum_{s=0}^{z}\left\{\left(\prod_{i=s}^{z}\sigma^i\right)d^s\right\}+d^{z+1}\right]\right]$$

$$+(-1)^{z+2}(\sigma^{z+1}+\pi I_n)\left[\sum_{s=0}^{z}\left\{\left(\prod_{i=s}^{z}\sigma^i\right)d^s+d^{z+1}\right\}\right]$$

$$=(-1)^{z+2}\left[\sum_{s=0}^{z+1}\left\{\left(\prod_{i=s}^{z+1}\sigma^i\right)d^s\right\}+d^{z+2}\right].$$

Therefore, (A1) holds in the case of $k = z+1$. (QED)

The following matrix has the same structure with that in (A1). Therefore, we prove the following two lemmas in the case of $n=1$, assuming that $I_n$ is 1, $0_n$ is 0, and $\sigma^i, (a^i)^t, (d^i)^t$ are real numbers $(i = 0, 1, \cdots\cdots, k)$.

**Lemma3.2** The following equation holds:

$$\begin{vmatrix} -I_n & \sigma^0+\pi I_n & & & & & (a^0)^t \\ & -I_n & \sigma^1+\pi I_n & & & & (a^1)^t \\ & & \ddots & \ddots & & & \vdots \\ & & & -I_n & \sigma^{k-1}+\pi I_n & & \vdots \\ & & & & -I_n & & (a^k)^t \\ (d^0)^t & (d^1-\pi d^0)^t & (d^2-\pi(\sigma^0 d^0+d^1))^t & \cdots\cdots & \left(d^k-\pi\left[\sum_{s=0}^{k-2}\left\{\left(\prod_{i=s}^{k-2}\sigma^i\right)d^s\right\}+d^{k-1}\right]\right)^t & & -1 \end{vmatrix}$$

$$=(-1)^{k+1}\left\{-1+a^0d^0+a^1(\sigma^0d^0+d^1)+a^2(\sigma^1\sigma^0d^0+\sigma^1d^1+d^2)\right.$$

$$\left.+\cdots+a^k\left[\sum_{s=0}^{k-1}\left\{\left(\prod_{i=s}^{k-1}\sigma^i\right)d^s\right\}+d^k\right]\right\} \tag{A2}$$

(Proof) Let us prove by mathematical induction.

(i) The case of $k=1$

Left-hand side of (A2) $=\begin{vmatrix} -I_n & \sigma^0+\pi I_n & (a^0)^t \\ 0 & -I_n & (a^1)^t \\ (d^0)^t & (d^1-\pi d^0)^t & -1 \end{vmatrix} = -1+a^0d^0+a^1(\sigma^0d^0+d^1).$

Thus, (A2) holds.

(ii) Let us investigate the case of $k=z+1$, assuming that the case of $k=z (z\geq 1)$ is held. In the left-hand side of (A2), put $k=z+1$ and make the expansion of cofactors regarding the row which includes $(a^{z+1})^t$. By Lemma3.1 and the assumption of mathematical induction, we obtain

Left-hand side of (A2)=

$$
(-1)^{2z+4}(-1)\begin{vmatrix} -I_n & \sigma^0+\pi I_n & & & (a^0)^t \\ & \ddots & \ddots & & \vdots \\ & & & \sigma^{z-1}+\pi I_n & (a^{z-1})^t \\ & & & -I_n & (a^z)^t \\ (d^0)^t & (d^1-\pi d^0)^t \cdots\cdots & & \left(d^z-\pi\left[\sum_{s=0}^{z-2}\left\{\left(\prod_{i=s}^{z-2}\sigma^i\right)d^s\right\}+d^{z-1}\right]\right)^t & -1 \end{vmatrix}
$$

$$
+(-1)^{2z+5}(a^{k+1})^t\begin{vmatrix} -I_n & \sigma^0+\pi I_n & & \\ & \ddots & \ddots & \\ & & -I_n & \sigma^z+\pi I_n \\ (d^0)^t & (d^1-\pi d^0)^t \cdots\cdots & & \left(d^{z+1}-\pi\left[\sum_{s=0}^{z-1}\left\{\left(\prod_{i=s}^{z-1}\sigma^i\right)d^s\right\}+d^z\right]\right)^t \end{vmatrix}
$$

$$
=(-1)^{2z+5}(-1)^{z+1}\left\{-1+a^0d^0+a^1(\sigma^0d^0+d^1)+\cdots\cdots+a^z\left[\sum_{s=0}^{z-1}\left\{\left(\prod_{i=s}^{z-1}\sigma^i\right)d^s\right\}+d^z\right]\right\}
$$

$$
+(-1)^{2z+5}(a^{z+1})^t(-1)^{z+1}\left[\sum_{s=0}^{z}\left\{\left(\prod_{i=s}^{z}\sigma^i\right)d^s\right\}+d^{z+1}\right]
$$

$$
=(-1)^{z+2}\left\{-1+a^0d^0+a^1(\sigma^0d^0+d^1)+\cdots\cdots+a^{z+1}\left[\sum_{s=0}^{z}\left\{\left(\prod_{i=s}^{z}\sigma^i\right)d^s\right\}+d^{z+1}\right]\right\}.
$$

Therefore, (A2) also holds in the case of $k = z+1$. (QED)

**Lemma3.3** The following equation holds:

$$\begin{vmatrix} -I_n & & & & & d^0 \\ \sigma^0 & -I_n & & & & d^1 \\ & \sigma^1 & -I_n & & & d^2 \\ & & \ddots & \ddots & & \vdots \\ & & & -I_n & & d^{k-1} \\ & & & \sigma^{k-1} & -I_n & d^k \\ a^0 & a^1 & a^2 \cdots\cdots & a^{k-1} & a^k & -1 \end{vmatrix} \Bigg\} + d^i \Bigg]\Bigg]\Bigg). \qquad \text{(A3)}$$

$$_{i=1} \lfloor \quad \lfloor_{s=0} \lfloor ( \ _{j=s} \quad )$$

(Proof) In (A2), put $\pi = 0$. Then, (A3) is the transposed version of (A2). Transposition does not change the value of determinant. (QED)