An Algorithmic Contribution to a Sraffian Measurement of Technological Progress

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ABSTRACT

In this paper, we propose a measure of technological progress which is based on the information embedded in standard input-output tables. Well known duality properties enable one to establish a connection between the quantities necessary as inputs, the associated output and some auxiliary prices (like the *wage-profit curves*). Properly tailored *wage-profit frontiers* may provide a basis for the measurement of technological progress. However, the computation of these *wage-profit frontiers* is not trivial. A brute force algorithm for the computation of the wage-profit frontiers has high combinatorial complexity that would make its precise computation intractable. Thanks to an efficient algorithm (VFZ-*algorithm*) that we have been able to devise, we can now compute it. Here, we present and apply this algorithm. As a result, we can now use these wage-profit frontiers as benchmarks against which to measure technological progress. Two new indices have been defined: the VFZ–*index* and the VFZ–*ranking*. One of the two indices, which generates a ranking between a given set of economic regions, is independent of the chosen numeraire. This too is an important and robust feature. These new tools have been applied to the OECD input-output data 1970–2005.

Keywords: Technological Change, Convergence, Input–output analysis, Technological Frontier, Sraffian Schemes, Computational Techniques.
JEL classifications: C61, C63, C67, O47

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1 Introduction

In this article, we propose a measurement of the technological progress of an economic region based on the information about its interdependent and intricate production structure, which is recorded in standard input-output tables. We are proposing two new indexes of economic performance, which we hope will be useful tools in the evaluation of the state of economic regions.

We are aware of the difficulties associated with the search of a good index and have tried to avoid the imposition of unnecessary assumptions. The index number problem can be seen as that of finding an aggregator function that maps a vector of quantities into a scalar. As discussed in depth by various authors, this function should have some desirable properties (see Fisher (1922), Frisch (1930, 1936), Samuelson and Swamy (1974)). But, it is a well known result, a function which matches all these requirements is possible only for trivial and often irrelevant cases or, it does not exist. Hence, the search of an index is the search for a satisfactory one. We think that our two indexes are highly satisfactory.

Frisch (1936), mostly elaborating on Fisher (1922), did pose the problem of the definition of a proper index number very clearly:

> *"The problem of how to construct an index number is as much one of economic theory as of statistical technique. Indeed, all discussions about the "best" index formula, the "most correct" weights, etc., must be vague and indeterminate so long as the meaning of the index is not exactly defined. Such a definition cannot be given on empirical grounds only but requires theoretical considerations. . . . The index-number problem arises whenever we want a quantitative expression for a complex that is made up of individual measurements for which no common physical unit exists. The desire to unite such measurements and the fact that this cannot be done by using physical or technical principles of comparison only, constitute the essence of the index-number problem and all the difficulties center here"* (Frisch (1936), p. 1).

Inevitably, even in the case of production theory and the measurement of technological progress, the *"indices"* have been *"exactly defined"* at the cost of loss of generality and in some cases, at the cost of the consistency with the observations.

Samuelson (1962) has shown that corresponding to any neoclassical function there is a dual function: the *factor price frontier.* Assuming linearity and continuity, Samuelson's method allows for the mapping of heterogeneous, multiple product production to a neoclassical surrogate production function which has neoclassical properties - like those of the constant rate of returns aggregate Cobb-Douglas. But when the linearity assumption is relaxed, the mapping into a surrogate aggregated neoclassical production is not always verified (see Ferguson (1969) (pp. 254–270) and Samuelson (1966)).

Bruno (1969) has shown that there exists a *generalized factor price frontier* in a Linear Leontief model, but this *generalized factor price frontier* does not always exhibit neoclassical properties. Burmeister and Kuga (1970) have shown that a similar *generalized factor price frontier* exists in a multi-sector model with neoclassical production functions.

Furthermore Farrell (1957), a precursor of Data Envelopment Analysis, has indicated a way to measure productive efficiency by assuming the existence of a universally optimal (or efficient) production function which has isoquants that are consistent with neoclassical postulates [1].

The assumptions which appear to be essential, so as to guarantee the neoclassical properties of the aggregated surrogate production function, are those that impose a structure on the production technology (see, for example, the imposed definition as in Shephard (1970) (p.13-14), or the neoclassical conditions as in Sato (1974)).

The most important conditions are those of continuity, convexity and of positive, continuous and smooth marginal rates of substitution among the factors of production. The empirical observations are point observations and whether substitutions among the factors are possible depends on the specific technology that is available. And inferences on the particular shape of a *'generalized'* production function can be done only by an arbitrary choice of shape for the surrogate production function or of the properties of the production set. Whether or not it is reasonable to assume that the production functions are with flexible or fixed proportions (that is with flexible or fixed coefficients), has been a controversial matter at times, which led to the adoption of sets of

[1] As pointed out by Afriat (2003) (pp.119-20), Farrell's approach and the more recent Data Envelopement Analysis are substantially the same. While the Data Envelopment Analysis does not rely on the definition of a specific functional form, it is well known that its validity relies on the assumption that the underlying production function is neoclassical, by imposing convexity. On this see also Petersen (1990), Bogetoft (1996) and Bogetoft et al. (2000)

postulates that were consistent with theoretical approach preferred by the researcher.

For example, it is most likely that the raw data is almost always not convex, but a common practice (see for example Farrell (1957), or Bogetoft et al. (2000)), is to convexify the data so as to assure that the estimated production function has the desired properties. Whether it is reasonable to do so is a debatable[2].

In this paper, we do not *convexify* the technological set and propose a measure of technological progress which is based on the empirical observations that are present in the input-output tables: we compute a set of auxiliary prices and use these prices, and not the quantities, for the measurement of actual technological progress.

Due to well known duality properties, we establish a connection betwee the quantities necessary as inputs and the associated output and the *wage-profit curve.* Extending the work of Sraffa (1960), Samuelson (1962) and Bruno (1969), we define and interpret the outer envelope of all the possible *wage-profit curves* as the *wage-profit frontier.* We use this *wage-profit frontier* as a benchmark against which to measure technological progress.

By comparing the production prices associated to country specific input–output tables, we will be able to construct an index which allows us to pick an efficient set of discrete methods of production. This efficient set of methods is what we use for the construction of our indices, and for the measurement of technological progress.

In heterogeneous production, whether a new technique is superior with respect to previous one would also depend on the prices of the other inputs and therefore, on the methods available for their production as well. Given that we make very mild assumptions with respect to the properties of the aggregate production function and that we do not impose convexity, our method is, in our view, quite robust.

[2]Electrical energy may be produced with different methods. For example: nuclear, coal, wind, hydraulic and so on. Clearly, the set of the necessary inputs producing a unit of output would be not convex. While a unit of output can be produced by a linear combination of the different techniques, marginal substitutions of the different factors may be inefficient or simply impossible. Moreover, chemical reactions have to be made, almost always, with fixed proportions and the cases in which the methods of productions are with fixed proportions are countless. Samuelson, 1962, as well as others, was fully aware that methods of production are, at least in the short run, with fixed coefficients because methods are different. But, he introduced a method so as to work with a surrogate flexible coefficients aggregate production function. Later, Samuelson (1966), admitted that the surrogate production function would not always exhibit desired neoclassical properties.

In Section, 2 we will review the relation that exists between *wage-profit frontier* (the *factor-price frontier*) and the *optimal-transformation-frontier*, and claim their duality. In Section 3 we will define the *technological frontier* as the outer envelope that is computed from all the possible *wage-profit curves*. The mathematical notion of an envelope is conceptually straightforward, but the *brute force* algorithm associated with the computation of such an envelope, for every single point, is computationally infeasible. Thanks to an efficient algorithm that we have been able to devise which exploits a result by Bruno et al. (1966) and Bharadwaj (1970) , we can compute several versions of the *wage-profit frontier*. This algorithm is described in Section 4.

The new, properly tailored *wage-profit frontiers* are then used to compute two new indices of technological progress. These indices are defined in Section 5. The OECD data set is described in Section 6 and in the Appendix A. In Section 7 the results of the computations are presented. Section 8 summarizes the main findings, while Section 9 concludes the paper. Appendix B reports additional results.

2 Technological Progress and the Quantity–Price Dualities

Here we base our analysis on input-output models. different production methods (activities) that are available for the production of a single output. These methods are extracted from the set of available input-output tables. We will assume here that b_{ii} of commodity i can be produced with t_i different alternative methods.

$$\phi(z_i, :, i) : a_{i1}^{z_i}, a_{i2}^{z_i}, \ldots, a_{in}^{z_i}, \ell_i^{z_i} \mapsto b_{ii}^{z_i} \tag{2.1}$$

where: $i = 1, \ldots, n; j = 1, \ldots, n; \quad z_i = 1, ..., s_i$; and $a_{ij}^{z_i} \in \mathbb{Q}$. s_i is the number of available methods for the production of good i and n is the number of goods.

The set of methods for the production of good i can be represented in matrix notation as:

$$\Phi(1 : s_i, 1 : (n + 2), i) = \begin{bmatrix} a_{i1}^1 & a_{i2}^1 & \ldots & a_{in}^1 & \ell_i^1 & b_{ii}^1 \\ a_{i1}^2 & a_{i2}^2 & \ldots & a_{in}^2 & \ell_i^2 & b_{ii}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{i1}^{s_i} & a_{i2}^{s_i} & \ldots & a_{in}^{s_i} & \ell_i^{s_i} & b_{ii}^{s_i} \end{bmatrix} \tag{2.2}$$

Obviously, the cardinality t_i of the above set of methods can be very large and subsets of the above methods can have, in principle, a great variety of topological properties. For example, subsets of the above methods can be such that they fulfill standard neoclassical properties, or they may contradict them. After a moment of reflection, one can see that the above set of discrete methods can approximate a flexible coefficients production function as well as a fix coefficients production function. It all depends on the number and the structure of these alternative methods. As correctly pointed out by Bruno (1969) (p. 51), *"any neo-classical technology could be simulated by a 'very dense' spectrum of discrete techniques"*. Actually, Bruno's statement can be generalized to any technology and not just to *"any neo-classical technology"*.

Figure 2.1 reports an example on how this can be done. Let us assume that for the production of the same quantity of output, $\bar{b}_{ii}$, there are several observations that are associated, reporting the use of different proportions of two factors of production, the circled points $(a_{i1}^1,\, a_{i2}^1) \ldots (a_{i1}^6,\, a_{i2}^6)$. These discrete points may be linked so as to generate a *'dense'* isoquant. This can be achieved by adding virtual points that lie on the thick line. Clearly, if we assume that the observed points are all efficient, the thick line is not convex and hence it is not an isoquant of a *neo-classical* production function.

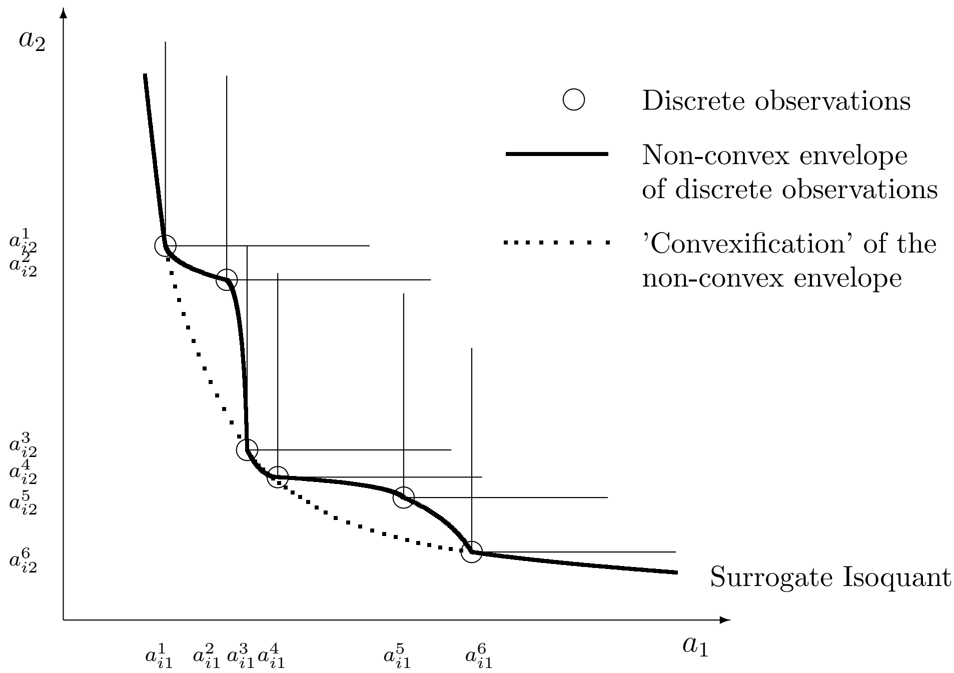
Farrell (1957), Samuelson (1962) and those following modern Data Envelopment Analysis (see Charnes et al. (1978)) would assume that the points (a_{i1}^2), a_{i2}^2 and $(a_{i1}^5,\, a_{i2}^5))$ are inefficient and that these points will not belong to the efficient isoquant. The production set is subsequently *convexified* by imposing that the *'dense'* hypothetical new points lie on the dotted line. With this procedure, the production technology is modified so as to be consistent with the Shepard's type of *neo-classical* properties.

In this paper, we will not construct any envelope of the data related to the quantities used – whether *neo-classical* or not will not matter – but simply use the quantities as they are.[3]

In essence, whether the overall production system approximates a neoclassical production function depends on the actual structure of Φ.

[3]As it will be also evident from below, this does not mean that the openly inefficient observations will play a role. Eventual observations that may lie to north-east of the circled observations will be excluded automatically by the method employed. Such case would be considered from the point of view of technological progress as moving backward.

Fig. 2.1: Artificial construction of a *dense* isoquant and eventual *convexification* of the production set



The set of all the available methods is given by the following set of activities $\mathbf{\Phi} = \{\Phi(:,:,1) \cup \Phi(:,:,2) \ldots \cup \Phi(:,:,t_i) \ldots, \Phi(:,:,n)\}$.[4]. Hence, a n-commodities output vector can be generated by using one combination of the methods which belongs to set $\mathbf{\Phi}$. There are a total $\boldsymbol{s} = \prod_{i=1}^{n} s_i$ of these combinations. Given one of these combinations, $\bar{\mathbf{z}} = [\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_n]'$, we have one production possibility which can be represented by the following

$$\mathbf{A}^{\bar{\mathbf{z}}} = \begin{bmatrix} a_{11}^{\bar{z}_1} & a_{12}^{\bar{z}_1} & \ldots & a_{1n}^{\bar{z}_1} \\ a_{21}^{\bar{z}_2} & a_{22}^{\bar{z}_2} & \ldots & a_{2n}^{\bar{z}_2} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1}^{\bar{z}_n} & a_{n2}^{\bar{z}_2} & \ldots & a_{nn}^{\bar{z}_n} \end{bmatrix}; \mathbf{L}^{\bar{\mathbf{z}}} = \begin{bmatrix} \ell_1^{\bar{z}_1} \\ \ell_2^{\bar{z}_2} \\ \vdots \\ \ell_n^{\bar{z}_n} \end{bmatrix}; \mathbf{B}^{\bar{\mathbf{z}}} = \begin{bmatrix} b_{11}^{\bar{z}_1} & 0 & \ldots & 0 \\ 0 & b_{22}^{\bar{z}_2} & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & b_{nn}^{\bar{z}_n} \end{bmatrix};$$

[4]Alternatively, one can see $\mathbf{\Phi}$ as a multi–dimensional array, whose maximum number of rows is given by $max\{s_1, s_2, \ldots, s_i, \ldots, s_n\}$, the number of columns is $n+2$ (the n inputs, labour and output) and the number of metrices that is equal to the number of goods. Each matrix $\Phi(:,:,i)$ contains information about all the possible discrete methods. The users of Matlab and/or Mathematica will be familiar with the notation and the structure presented here.

$$\mathbf{XA}^{\bar{\mathbf{z}}}, \mathbf{XL}^{\bar{\mathbf{z}}} \mapsto \mathbf{XB}^{\bar{\mathbf{z}}} \tag{2.3}$$

Where $\mathbf{X}$ is a semipositive diagonal matrix, which represents the intensity of the utilization of the methods used (the activity levels). The system is defined as being productive for all those cases in which $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{X}$ are such that $\mathbf{e}'(\mathbf{XB} - \mathbf{XA}) \geq 0$, where $\mathbf{e}$ is the unit or summation vector.

For a productive system and for a given endowment of the factors of production, we can construct the n–dimensional *production possibility frontier* for the entire economy.[5]

Any intricate productive system can be examined from the point of view of (a). the quantities that are used as factors of production or (b). the values that are necessary for that intricate productive system to reproduce itself. This duality is a principle which has been used since the birth of political economy, but it was introduced formally since the pioneering work of von Neumann (45 6).

The prices used for the dual problem are not market prices, but computed analytical prices, which are based on the actual quantities observed. They can be interpreted in many different ways. They can be seen as Adam Smith's *natural prices* or Ricardo-Marx's *production prices*, Seton's *eigenprices*, long term *competitive equilibrium prices*; Walrasian *market clearing prices*, *shadow prices* and so on. In order to not confine them to any particular interpretation in the sequel, we will refer to our computed prices as *auxiliary prices*.

Given the productive system 2.3, the prices that would assure the system to be productive also for the periods to come are precisely those which allow the following accounting relation to hold:

$$(\mathbf{I} + \mathbf{R})\mathbf{XA}^{\bar{\mathbf{z}}}\mathbf{p} + \mathbf{XL}^{\bar{\mathbf{z}}}w = \mathbf{XB}^{\bar{\mathbf{z}}}\mathbf{p} \tag{2.4}$$

where $\mathbf{I}$ is the nxn identity matrix and $\mathbf{R}$ is a diagonal nxn matrix where each diagonal element represents a profit rate, r_{ii}.

[5]Moreover, once a proper subset of $\mathbf{B}$ is defined as being consumption goods, one can construct the n-dimensional *optimal transformation frontier* which is the *'... dynamic analog of an economy's production possibility frontier, namely the locus of maximal combinations of the* per capita *consumption and the rates of growth of the various capital goods (Bruno (1969) (p.39)'.* The dynamic characterization of growth and/or of change is given by the sequence $\{\mathbf{X}_t, \mathbf{X}_{t+1}, \cdots, \mathbf{X}_T\}\}$ from where individual rates of growth, g_i, can be derived. For a recent use of the production possibility frontier in the context of *'more realistic'* input-output tables, see Samuelson (2001)

For a given matrix $\mathbf{R}$ and uniform wage rate w, there exists a price vector $\mathbf{p}$, which would allow the system to be productive also during the subsequent periods.

$$\mathbf{p}^{\bar{\mathbf{z}}} = [\mathbf{X}\mathbf{B}^{\bar{\mathbf{z}}} - (\mathbf{I} + \mathbf{R})\mathbf{X}\mathbf{A}^{\bar{\mathbf{z}}}]^{-1}\mathbf{X}\mathbf{L}^{\bar{\mathbf{z}}}w \tag{2.5}$$

An important result is that the re-proportion matrix $\mathbf{X}$ does not influence the determination of the price vector $\mathbf{p}$.

$$\mathbf{p}^{\bar{\mathbf{z}}} = [\mathbf{B}^{\bar{\mathbf{z}}} - (\mathbf{I} + \mathbf{R})\mathbf{A}^{\bar{\mathbf{z}}}]^{-1}\mathbf{L}^{\bar{\mathbf{z}}}w \tag{2.6}$$

Equations 2.5 2.6 encapsulate a very important result known as the *non-substitution-theorem*: relative prices for a given system ($\mathbf{X}\mathbf{A}^{\bar{\mathbf{z}}}$, $\mathbf{X}\mathbf{L}^{\bar{\mathbf{z}}}$, $\mathbf{X}\mathbf{B}^{\bar{\mathbf{z}}}$) are independent of the intensities of the different activities, $\mathbf{X}$ [6]

Once the choice of a *numéraire* $\eta'\mathbf{p} = 1$ is made, the wage rate is given by:

$$w^{\bar{\mathbf{z}}} = [\eta'[\mathbf{B}^{\bar{\mathbf{z}}} - (\mathbf{I} + \mathbf{R})\mathbf{A}^{\bar{\mathbf{z}}}]^{-1}\mathbf{L}^{\bar{\mathbf{z}}}]^{-1} \tag{2.7}$$

This is the *wage-profit curve* associated to system $\bar{\mathbf{z}}$. Substituting 2.7 into 2.6 we obtain the price vector

$$\mathbf{p}^{\bar{\mathbf{z}}} = [\mathbf{B}^{\bar{\mathbf{z}}} - (\mathbf{I} + \mathbf{R})\mathbf{A}^{\bar{\mathbf{z}}}]^{-1}\mathbf{L}^{\bar{\mathbf{z}}}[\eta'[\mathbf{B}^{\bar{\mathbf{z}}} - (\mathbf{I} + \mathbf{R})\mathbf{A}^{\bar{\mathbf{z}}}]^{-1}\mathbf{L}^{\bar{\mathbf{z}}}]^{-1} \tag{2.8}$$

The price vector $\mathbf{p}^{\bar{\mathbf{z}}}$ is a function of the particular set of methods $\bar{\mathbf{z}}$ and of the profit rates $r_{11}, r_{22}, \ldots, r_{nn}$. Obviously, these prices are not market prices, but are auxiliary prices. That is, prices that would allow

[6] On the origins of the *non-substitution-theorem*, see Arrow (1951), Koopmans (1951), Samuelson (1951). A more recent treatment is given in Mas-Colell et al. (1995), pp.159-60. See also Zambelli (2004).

On the importance of the independence of prices from conditions of demand and of the specific and supposed marginal productivity requirements the comments made by Champernowne (45 6) (p. 12) on von Neumann (45 6) are particularly illuminating:
'By reducing the role of the worker-consumer to that of a farm animal, he can focus attention on those parts of the mechanism determining prices and the rate of interest, which depend on supply conditions alone and not on the tastes of consumers. This emphasis is important because the orthodox analysis has distributed attention evenly between marginal utility and conditions of supply; since supply is often more elastic than demand, prices in the long run do over a wide field reflect contrasts in cost rather than conditions of consumers' demands: a price-theory focussing attention on costs can give a very clear and yet an approximately true account'.

the *accounting* equilibrium between buyers and sellers of the factors of productions such that the same production activity could take place during next production cycle.

3 The Technological Frontier

We can now attempt a measurement of technological progress, by comparing the prices associated with the employment of old methods with the prices associated with the employment of new ones. The system will exhibit a technological improvement when the auxiliary price is lower than the previous price or, when, for given profit rates $\mathbf{R}$, the associated wage rate, $w^{\bar{\mathbf{z}}}$, is higher or the highest.

Equation 2.7 is meaningful only when the n profit rates, the matrix $\mathbf{R}$, are given explicit numerical values.

A simplifying and meaningful special case would be the one in which the rates of profit are uniform, so that $r = r_{11} = r_{22} = \ldots = r_{nn}$. [7]. A special case of equation 2.7 is given by:

$$w^{\bar{\mathbf{z}}} = [\eta'[\mathbf{B}^{\bar{\mathbf{z}}} - \mathbf{A}^{\bar{\mathbf{z}}}(1+r)]^{-1}\mathbf{L}^{\bar{\mathbf{z}}}]^{-1} \tag{3.1}$$

where $r = \{r \in \mathbb{Q} : 0 \leq r \leq \mathcal{R}^{\bar{\mathbf{z}}}\}$. $\mathbb{Q}$ is the set of rational numbers and $\mathcal{R}^{\bar{\mathbf{z}}}$ is the maximum rate of profit of system $\bar{\mathbf{z}}$. This is the *wage-profit curve* associated to system $\bar{\mathbf{z}}$, for the case in which the profit rate is uniform for all industries.

Although 3.1 is a well known relation, its empirical importance may have been somewhat underestimated. For each combination of methods $\bar{\mathbf{z}}$, there is a corresponding *wage-profit curve.* The outer envelope of all possible *wage-profit curves* is the *wage-profit frontier.* Several characteristics of the *wage-profit curves* and of the *wage-profit frontier* are useful for the construction of our index.

1. The *wage profit curves* and *frontiers* are scale independent. This is a result of the *non-substitution* theorem. Hence, two different productive systems, say, one associated with a small country and the other with a big country, can be compared using the same framework.

[7] Clearly, there is a cloud of possible values that the individual profit rates could take and that would guarantee a set of values for which the reproduction of the system could take place. The choice of the *uniform rate of proifit* finds its principlal justification from the fact that it allows to work in a two dimensional space. In practice the graphical representation of of the *wage-profit rates frontier* collapses from a $n + 1$ hyperspace to a 2 dimensional space

2. Comparison between two *wage-profit curves* is independent of the cardinality of their productive systems. Two systems having different cardinality, say n and m, can still be compared as long as they have the same *numéraire*. The only requirement is that the *numéraire* is a transformation based on the subset of commodities, which are common to both systems.

3. The *wage-profits curve*, 2.7 or 3.1, is dual with respect to the *production possibilities frontier* 2.3. Clearly, for a given set of profit rates, if $w^{\bar{\bar{\mathbf{z}}}} > w^{\bar{\mathbf{z}}}$ this means that the $w^{\bar{\bar{\mathbf{z}}}}$ has, for the associated auxiliary prices, a higher purchasing power with respect to $w^{\bar{\mathbf{z}}}$. This is possible only if the *production possibilities frontier* associated with the methods $\bar{\bar{\mathbf{z}}}$ is superior with respect to the *production possibilities frontier* associated with the method $\bar{\mathbf{z}}$.

4. All the possible linear combinations of two methods $\bar{\bar{\mathbf{z}}}$ and $\bar{\mathbf{z}}$ will result in a set of *wage-profit curves* or *frontiers* which will be dominated either by $w^{\bar{\bar{\mathbf{z}}}}$ or by $w^{\bar{\mathbf{z}}}$. This exstends to any subset of the methods in $\mathbf{\Phi}$

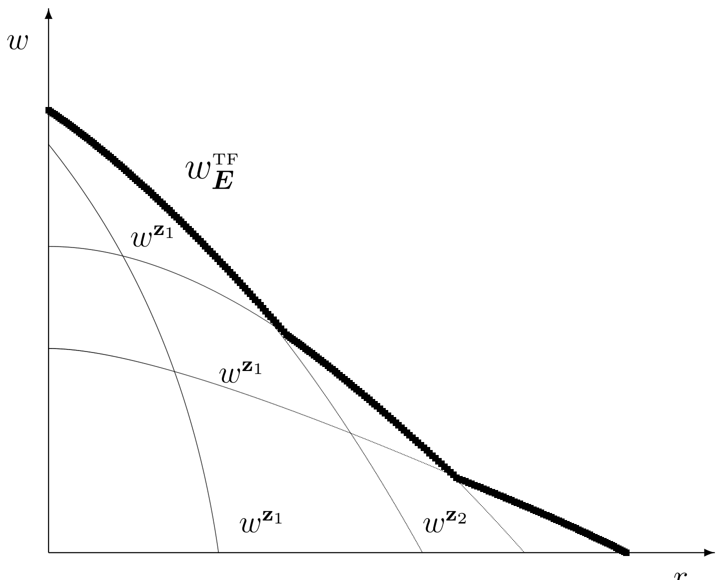


Fig. 3.1: *The technological frontier*

Given any subset $\boldsymbol{E} = \left\{\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_m\right\}$ of $\mathbf{\Phi}$, the

$$w^{\mathrm{TF}}_{\boldsymbol{E}} = \max\left\{w^{\mathbf{z}_1}, w^{\mathbf{z}_2}, ..., w^{\mathbf{z}_m}, \right\} \tag{3.2}$$

An example of the *technological frontier* is illustrated in Figure 3.1.

5. Clearly, not all the *wage-profit curves* associated to $\boldsymbol{E}$ contribute to the formation of the *technological frontier*, $w^{\mathrm{TF}}_{\boldsymbol{E}}$. The subset of methods of $\boldsymbol{E}$ which enters the frontier, $\boldsymbol{E}^{\mathrm{TF}}$, represents the most productive system of methods, which is a combination of different sets. This is, in our view, the composite benchmark commodity that needs to be used in order to measure increases of productivity due to technological progress [8].

6. Whether the *technological frontier*, w^{TF}, is consistent with the neo-classical framework or not will depend on the particular structure of the set of methods. Hence, this approach is general.

4 The Velupillai-Fredholm-Zambelli *technological frontier* - Algorithm

The computation of the *wage profit frontier* is not a simple matter. There exist a brute force algorithm which allows a precise computation of the w^{TF}_{Φ}. But the implementation of this algorithm (see below) becomes computationally intractable as the cardinality of the set of methods increases. However, we have been able to find a tractable algorithm that allows a drastic reduction of the computational effort. For example, with the cardinality of the data set that we use in this

[8] Velupillai and Zambelli (1993) is a first search in this direction. Velupillai (1993, p. 5-6) states:
Production structures carry with them natural prices corresponding to particular analytical assumptions about the economics of the production system. What is needed is a device to extract these prices from the observed data of a functioning economy. Thus the natural questions for the production approach relate to indices of productivity and the optimality of price systems supporting production structures from particular economic viewpoints. The measures encapsulate the price-mediated interaction between resource allocation and income distribution for efficient production. The conceptional tools include the optimal transformation frontier *... and the* factor price frontier. *The framework yields efficiency indices of different production systems and, by implication, real output comparisons between different economies. Constructing indices for these conceptual categories, as remarked earlier, is the main task of this paper.*
That is the 1993 paper. We think that this task, whose foundations were set in 1993, has been fully accomplished in 'this' paper

paper, see below, the computation of w_{Φ}^{TF} with a desktop computer using the brute force algorithm would take several decades, while the use of our algorithm allows its computation in just a few hours. Given that we think that this is an important contribution of the present paper we have taken the liberty of calling it the VFZ-*algorithm*, which leads to the computation of the VFZ-*technological frontier*, which allows the computation of two indices of technological progress: the VFZ-*index* and the VFZ-*rankings*.

4.1 A brute-force algorithm

1. input data, i.e. individual input–output tables and organize it into the multiple dimension array, $\mathbf{\Phi}$ (see equation 2.2)
2. enumerate all possible combinations of methods $\boldsymbol{E}_{\mathbf{\Phi}} = \{\mathbf{z}_{\mathrm{j}}\}$ with $j = 1, ..., \boldsymbol{s}$ with $\boldsymbol{s} = \prod_{i=1}^{n} s_i$
3. compute sequentially, for $j = 1$ to $\boldsymbol{s}$, the *wage profit frontier*, $w^{\mathbf{z}_j}$ 3.1 and retain the value for wages w which dominate the previously computed *wage profit frontiers*. That is, compute (see 3.2)

$$w_{\boldsymbol{E}_{\{j\}}}^{\mathrm{TF}} = \max\left\{w_{\boldsymbol{E}_{\{j-1\}}}^{\mathrm{TF}}, w^{\mathbf{z}_{\mathrm{j}}}\right\}$$

However, when using the above algorithm the computational complexity of the problem implies that it is practically impossible to compute the technological frontier for even small datasets, sinceall possible combinations of techniques must be evaluated for each rate of profit. Using the *Big-O notation*, the time-complexity is (at least) $O(\boldsymbol{s})$. This implies that no matter how powerful a computer that will be developed within, say the next century, it will always be possible to include additional available data, such that the algorithm will not halt within any reasonable time frame[9].

4.2 The VFZ-*algorithm* and the VFZ-*technological frontier*

The computational complexity can however be drastically reduced (in the order of $\boldsymbol{s}$ to $\boldsymbol{s}^{1/n}$) by exploiting an important result concerning

[9]With N input-output tables, n sectors, the total number of systems, *wage-profit frontiers* are N^n. The OECD input-output tables used below is formed of $N = 64$ tables with $n = 23$ sectors, which means $64^{23} \approx 3.5 \cdot 10^{41}$ unique systems. Running a whole year, the computer must evaluate $1.1 \cdot 10^{34}$ systems per second, each including several matrix operations. Nothing close to such a computer exists today or will exist within any reasonable time frame.

switch points. Bruno et al. (1966) and Bharadwaj (1970) have shown that:

i) *"At a switch point the adjacent production system differ in the method of production for* only one *of the commodities common to them (Bharadwaj (1970) (p.423), emphasis added)"*;
ii) *"The choice of the value unit* [the numéraire] *does not affect the maximum number of switching possibilities* [and their correspondence to the profit rate]*(Bharadwaj (1970) (p.424)")*

Using any point on any frontier, the following procedure climbs the individual wage-profit frontiers using the switch points as stepping stones, so to speak. This is how we reduce drastically the computational time.

1. import data and convert it into matrices of technical coefficients
2. choose an initial point on any frontier
3. from this point, while $r > 0$, lower the profit rate one increment[10] and compute the wage rate without changing the techniques, save this as $\overline{w}$
    (a) one by one, change the techniques (piecemeal), i.e., $n \cdot (N-1)$ times, and for each system
        i. if the profit rate is smaller than the maximum profit rate, compute the wage rates
        ii. if this wage rate is greater than $\overline{w}$, then we have passed a switch point. Fix the new set of techniques and the associated wage rate. Else, use $\overline{w}$
4. Now reverse the procedure, while $w > 0$, increase the profit rate one increment and compute the wage rate without changing the techniques, save this as $\overline{w}$
    (a) one by one, change the techniques and for each system:
        i. if the profit rate is smaller than the maximum profit rate, compute the wage rates.
        ii. if this wage rate is greater than $\overline{w}$, then we have passed a switch point. Fix the new set of techniques and the associated wage rate. Else, use $\overline{w}$
5. go to point # 3 as long as loop # 3 and # 4 do not produce identical results, else terminate and collect the results

[10]In the actual computation the step-size is fixed at $\frac{1}{1000}$. Between $\frac{1}{500}$ and $\frac{1}{1000}$ the number of switch points increased, which implies that the algorithm missed some switch points. No changes in the results were found when the step-size was narrowed to $\frac{1}{2000}$.

Both algorithms can be implemented with no serious demand on the available memory, but unlike the brute-force algorithm, the VFZ-*algorithm* cannot be run in parallel.

An easy way to verify the outcome from the Piecemeal algorithm is to apply the two algorithms on a tractable subset and check whether they yield identical results. This has been verified with positive results.[11]

The full set of results for the eight OECD countries and for eight time periods can be computed within a few hours, using the VFZ-*algorithm* and a standard desktop computer.

5 Two measures of technological progress based on the VFZ-*technological frontier*

The VFZ-*technological frontier* can be used to measure the technological progress and the relative economic performances of the different economic systems, countries. We have constructed two different indices of performance: the VFZ-*index* and the VFZ-*ranking*.

The VFZ-*index* measures the level of development as the ratio between the system specific *wage-profit curve* and the VFZ-*technological frontier*. The VFZ-*index* is dependent on the choice of the *numéraire*, but has the advantage of assessing the degree of economic backwardness or forwardness in terms of the globally efficient production frontier that is captured by the VFZ-*technological frontier*. In essence, it is an assessment of the actual development of the particular national system with respect to the benchmark represented by the VFZ-*technological frontier*.

The VFZ-*ranking* computes the relative performances based on the contribution of the economic systems to the formation of the efficient global VFZ-*technological frontier*. As Bharadwaj (1970) has shown, the switch points of the *wage-profit frontier* are independent of the *numéraire*, therefore, the contributions of the economic systems do not change with it. A ranking between the different systems can be made by exploiting this fact. Obviously, an economic system that con-

[11] There exist one potential problem: it is theoretical possible, by some fluke, that the envelope is not connected by intersections with the initially chosen frontier. However, the probability of this occurring tends to zero as the number of techniques tends to infinity.

tributes substantially, and more than others, to the formation of the VFZ-*technological frontier* can be considered as being forward in technological development with respect to those not contributing at all. This does not mean that we have to expect that the economic system necessarily performs better than others. Whether this technological forwardness is actually exploited so as to assure, for example, full employment level or high level of per-capita output or income is another matter which is not discussed in this paper [12].

It has to be stressed that the VFZ-*index* is an 'absolute' measurement of actual potential economic performance, while the VFZ-*ranking* is a 'relative' measure of the access to more advanced, and potentially more productive, industry-level production methods. The computations of these two indices require the computation of the VFZ-*technological frontier*. Hence, for the reason explained above, they have never been computed before.

5.1 The VFZ-*index*

Given a set of systems $\boldsymbol{E}$ (derived from combinations af the available methods, $\mathbf{\Phi}$), the VFZ-*index* provides a measure of the average efficiency relative to the VFZ-*technological frontier*, $w_{\boldsymbol{E}}^{\mathrm{VFZ}}$, associated with the subset.

For the jth country, at time t, the VFZ-*index* is computed as:

$$VFZ_{j,t}^{index} = \frac{1}{m}\sum_{i=1}^{m}\left[w_{j,t}(r_i)/w_{\boldsymbol{E}_t}^{\mathrm{VFZ}}(r_i)\right] \qquad (5.1)$$
$$j = 1, 2, ..., N, \quad t = 1, 2, ..., T$$

where: $r_i = \{0 \leq r_i \leq r_m = \mathcal{R}_{\boldsymbol{E}_t}^{\mathrm{VFZ}}, i = 1, \ldots, m\}$; $\mathcal{R}_{\boldsymbol{E}_t}^{\mathrm{VFZ}}$ is the maximun rate of profit of VFZ-*technological frontier*; m is the number of points of the rate of profits domain of the VFZ-*technological frontier* $w_{\boldsymbol{E}_t}^{\mathrm{VFZ}}$;

The closer the index is to unity, the more efficient is the technology used in the single country relative to the theoretical maximum computed from the entire set of production activities.

The advantages of the VFZ-*index* over conventional ones are:

[12] This becomes obvious when one considers the fact that to any *wage-profit curve* there is associated an infinity of re-proportioning or level of activities matrix $\mathbf{X}$. Clearly, a great variety of employment levels and per-capita incomes can be consistent with different activity levels, $\mathbf{X}$, it all depends on the actual structure and demand level of the regions involved

1. The method is non-parametric and non-stochastic.
2. Technology, value, and aggregation are fully integrated through the auxiliary prices, hence to some extent circumvents the standard index number and value problems.
3. The indices are time-invariant, i.e., they are fully determined within a single accounting period.[13]
4. The stability of the switch points greatly limits the sensitivity of changes to the *numéraire*.
5. The interdependence among industries is endogenously captured by changes in the prices of production.
6. The indices will not change as a consequence of simple changes in the scale of production in a single industry, but only if real technological innovations are observed in one or more industries.[14]
7. In the study of convergence, the benchmark/reference point is determined from the system as a whole and not simply a 'leading country'.

5.2 The VFZ-*ranking*

The VFZ-*technological frontier* is a piecemeal function formed with v intervals, where for each interval, a fixed combination of methods, $\bar{\mathbf{z}}$, holds. This is independent of any *numéraire*. We have exploited this fact to construct a *numéraire-free* index of performance. Our approach is to consider the level of forwardness and backwardness of economic regions, and relate them also with the contributions of the methods of these regions to the formation of the global VFZ-*technological frontier*. Here it is not, as in the case of the VFZ-*index*, the distance with respect to the VFZ-*technological frontier* that matters, instead, the actual number of methods belonging to a region that contribute to the computation of the VFZ-*technological frontier*.

In order to also take account of those methods that are not the most 'efficient', but are almost as efficient, we have generated a scheme in which the methods can be ordered as being first, second, third, ... and last (N^{th}). A method would be ranked second, provided it contributes the most to the new, lower, VFZ- technological frontier, once the first-ranked method is removed. method ranked first is removed from the

[13]However, updating the entire dataset with new data, say the 2010 OECD tables, will almost certainly change the intertemporal technological frontier, but the within-period ranking will remain unaffected.

[14]By real technological innovations, we mean changes in the matrix of technological coefficients and/or in the corresponding (normalised) vector of labour inputs.

set of methods and it is the one that would contribute to the new, and lower, VFZ-*technological frontier* [15] ; it would be ranked third when the methods ranked first and second are removed and its contribution to the new VFZ-*technological frontier* and so on.

Once these rankings have been generated, they are aggregated using the Borda Counts weights (see for example Saari (1985)). That is, the first would weighted with value 1, the second with value 1/2, the third with value 1/3 ... the N^{th} with value $1/N$. These values are used to determine the ranking of the different regions by summing all the values associated to the methods of the region. Clearly, if the methods employed in a region are all superior with respect to the others, the highest value would be equal to the number of commodities. Hence, it is appropriate to normalize this value with respect to the number of commodities, i.e industries or sectors. In this way, the highest possible performance value, as in the case of VFZ-*index* would be 1. But in this case a high perfomance of one region would imply a much lower performance of the other regions [16]. This is not so in the case of VFZ-*index*.

This index has been called VFZ-*ranking*. Being totally independent from the choice of the *numéraire*, we think that this is a very strong measure of economic performance.

6 Data and the Choice of *Numéraire*

We study three versions of the VFZ-*technological frontier*; the contemporary $w^{\text{VFZ}}_{\text{CTF}}(r_i, \boldsymbol{E}_t)$, the rolling $w^{\text{VFZ}}_{\text{RTF}}(r_i, \boldsymbol{E}_1 \cup \boldsymbol{E}_2 \cup ... \cup \boldsymbol{E}_t)$, and the intertemporal $w^{\text{VFZ}}_{\text{ITF}}(r_i, \boldsymbol{\Phi})$, where $\boldsymbol{E}_t$ denotes the set of techniques used at time t and $\boldsymbol{E}_{\boldsymbol{\Phi}}$ the total set of systems made from all the combinations of techniques available. An obvious analytical property of these three versions of the technological frontier is that[17]:

$$w^{\text{VFZ}}_{\text{CTF}}(r_i, \boldsymbol{E}_t) \leq w^{\text{VFZ}}_{\text{RTF}}(r_i, \boldsymbol{E}_1 \cup \boldsymbol{E}_2 \cup ... \cup \boldsymbol{E}_t) \leq w^{\text{VFZ}}_{\text{ITF}}(r_i, \boldsymbol{E}_{\boldsymbol{\Phi}}) \qquad (6.1)$$

[15]Here we would like to stress that the calculations of a new VFZ-*technological frontier* each time that a method is 'removed' would not be tractable if we had not found an efficient way to compute it.

[16]In the highly unlikely case in which a region dominates in all the industries the value would be 1, in the case in which it would alwasys perform seconfd best, the value would be 1/2, in the case in which all the industries would berform third best the value would be 1/3 and so on.

[17]Since $\boldsymbol{E}_t \subseteq \{\boldsymbol{E}_1 \cup \boldsymbol{E}_2 ... \cup \boldsymbol{E}_t\} \subseteq \{\boldsymbol{E}_{\boldsymbol{\Phi}}\} \; \forall \; t = 1, 2, ..., T$

For the actual computation of the technological frontiers, we have chosen the OECD 1970–2005 input–output tables for the US, Germany, the UK, France, Canada, Denmark, Japan, and Australia. All based on the the ISIC 2 or ISIC 3 classifications with 35 and 48 industries, repectively.[18] The tables contain both the domestic inter-industrial flow and industry-specific imports of capital goods.

Some problems of comparability do exist between the two methods of classification, but steps have been taken to minimize these problems. The initial 48 and 35 industries have been aggregated into 23 industries following standards of national accounting. The main reason for doing so is that there are differences practices followed by the in the specific input-output tables due to different national statistical bureaus. We think and hope that our aggregation furthers the comparability over time and across time [19].

Unfortunately, the tables are not available for all countries, for all time periods. To further increase comparability, we chose to substitute the missing tables with the most commensurable table, typically the table from the previous accounting period in the same country. For details, see Table A.1 in Appendix A.

We use data from the OECD on the industry-level 'compensation of employees' as labour inputs and use this to distribute the total employment in hours to the single industries. When available, we use detailed industry-level employment data from The Groningen Growth and Development Centre.[20]

There is a fundamental problem related to the units of accounting, since the tables are denominated in current values of the national currency. Macro-industry deflators have been computed as the differences between macro-industry GDP denomination in current and base period prices, respectively, and used to deflate the value denominated tables. This is probably the best available proxy for the physical flow among industries found in the OECD input–output tables. Appendix A contains additional information on the data used.

As a *numéraire*, we chose the vector of domestic net product of the USA from the base year 2000, normalised by the total hours worked.

[18] See www.OECD.org.

[19] We have investigated whether different aggregations would change the qualitative results presented here. For aggregations that are consistent with the ISIC 2 or ISIC 3, in the sense of aggragating 'neighbouring' industries, we found that the qualitative results, i.e. the relative positions, do not change significantly.

[20] See www.GGDC.net.

7 Efficiency, Technological Change, and Convergence

A general problem associated with the measurement of technological progress is due to the well known observation that different production activities use different sets of factors of production. For example, one can consider the production of energy; nuclear energy, wind mills, hydro-power, solar-energy, oil, coal, gas, etc (see footnote 2 page 4). Without a robust measure of efficiency, an assessment of the most efficient production process is almost always impossible. In the case of heterogeneous production whether a method is more efficient than another would depend also on the methods used on the production of the other factors of production which in turn would depend also on the methods used by the producers of the methods of production that they use.

There might be reasons which are different from technological superiority that may lead to the adoption of a specific production structure. We do not try to assess what these reasons might be, but simply register that once production has occurred certain quantities of inputs have been used and a certain amount of output has been produced: this is given by data. Given the dual relation existing between the *production possibility frontier* and the VFZ-*technological frontier* we can try to asses technological progress in terms of the artificially generated auxiliary prices. The VFZ-*technological frontier* is the outer frontier of all possible *wage-profit curves* which are associated with all possible combinations of the methods of production of all the economic systems involved. We think it to be a robust benchmark against which we measure the technological efficiency of the different individual regional or national systems.

7.1 The empirical technological frontiers

Figure 7.1 shows the complete collection of contemporary and rolling VFZ–*technological frontiers*. Analogous to the study of the *wage-profit curves* for the individual countries, an outward shift of the VFZ–*technological frontiers* imply, in the context of the approach presented here, an unambiguous technological progress. Only in the case in which two *wage-profit curves* or *frontiers* intersect, it cannot be unambiguously determined whether or not a higher level of productivity for the whole system has been reached. [21]

The contemporary VFZ-*technological frontiers*, $w^{\mathrm{VFZ}}_{\mathrm{CTF}}(r_i, \boldsymbol{E}_t)$, show a clockwise and steady shift outwards, while the rolling VFZ-*technological frontiers*, $w^{\mathrm{VFZ}}_{\mathrm{RTF}}(r_i, \boldsymbol{E}_1,\ \boldsymbol{E}_2, ..., \boldsymbol{E}_t)$, show a more parallel shift. This difference provides a first-hand insight into the nature of the global technological progress. Here we can interpret the clockwise shifts of the contemporary technological frontiers as a global labour-saving technological progress. Since the value of the circulating capital does not necessarily change monotonically with the profit rate, this interpretation is not fully unambiguous.

In the context of this paper, we assume, following standard practices, that the index numbers representing the inputs and the output do represent the same class of commodities. As time evolves, new combinations of inputs do allow the production of the outputs. The complexity of interactions is very high and therefore it is not at all clear that a particular combination would be efficient when inserted in a different context, where different methods are used for the production of other commodities. The production structure of a national system is the result of a complex set of events and hence a particular combination of inputs used in the past may not be realized at a later point in time. The rolling technological frontier $w^{\mathrm{VFZ}}_{\mathrm{RTF}}$ does capture technological progress. This is particularly so when the relation between inputs and output is considered as a method of production.

The problem of intersection(s) between frontiers does not exist for the rolling technological frontier, since these, by construction, will never intersect. Consequently, together with the other frontiers, this property makes the rolling technological frontier a strong analytical tool.

An observed difference between the contemporary and rolling frontiers implies that there exist some combinations of the old and new production techniques, which are more productive than all combinations of the techniques currently used.[22].

[21] It is not always the case that the adoption of a new method indicates that the method is superior, simply because its evaluation in terms of cost-benefits would depend on the adoption of the methods of the other industries. Furthermore, both the market prices and the auxiliary prices do depend on the methods used for the whole system (and other very important contingent factors).

[22] However, it could be argued that some old techniques of production should be discarded from the set of techniques forming the rolling (and inter-temporal) technological frontier. These could be techniques that are both (under some circumstances) superior to contemporary techniques, but practically obsolete. And hence *de facto* no longer exist in the *book of available blueprints*. But this type of analysis, although very relevant, goes beyond the scope of the present study

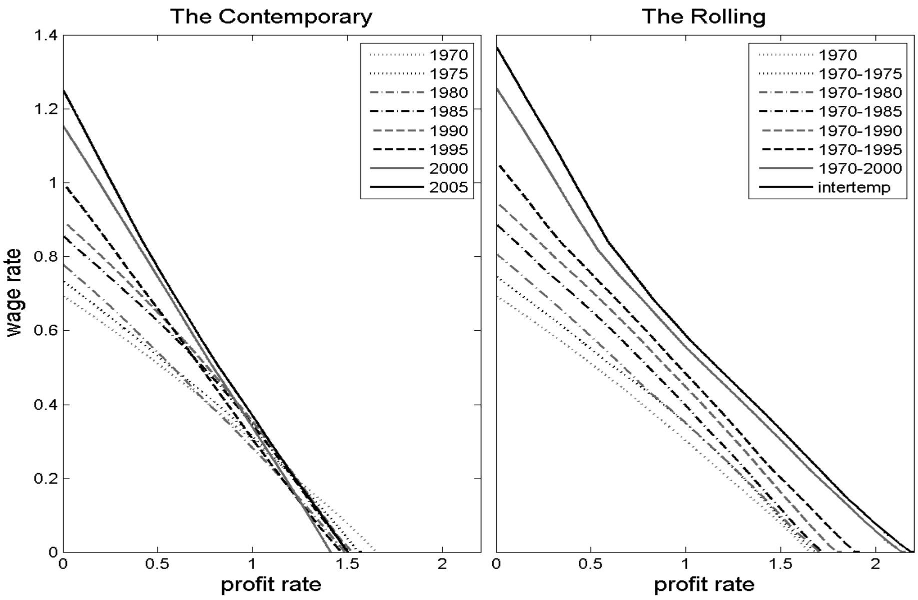


Fig. 7.1: *The contemporary,* w^{VFZ}_{CTF}*, and rolling* w^{VFZ}_{RTF}*, technological frontiers*

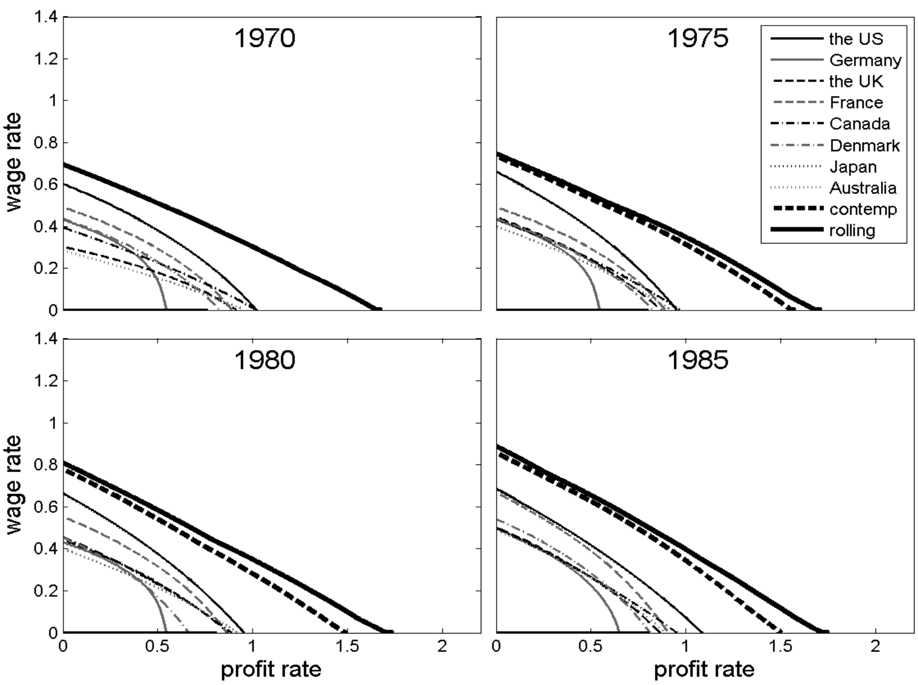


Fig. 7.2: *Contemporary, rolling and country specific wage-profits technological frontiers: 1970–1985*

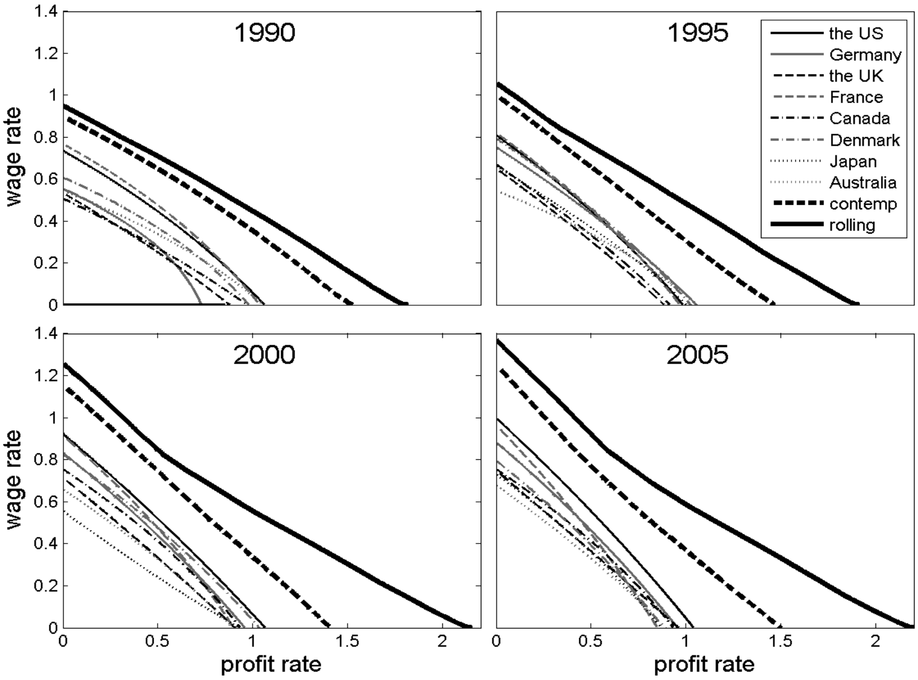


Fig. 7.3: *Contemporary, rolling and country specific wage-profits technological frontiers: 1990–2005*

Figure 7.2 and 7.3 show the wage-profit curves for the individual countries together with the contemporary and rolling technological frontiers. Figure 7.2 for the period 1970–1985 and Figure 7.3 for 1990–2005. As expected, the US is the leading country from the 1970s, but the US wage-profit frontiers do not shift as much as that of the other countries' frontiers in the 1970s, i.e., there is an evidence of a slowdown in the US and catching up by the other countries.[23] See also Figure B.2 and B.3 in the statistical companion, where the frontiers are presented country-by-country.

7.2 The VFZ-*index* applied to the OECD input-output data set

The values of the VFZ-*index* computed for eight countries and the economy as a whole are collected in Tables[24] 7.1, 7.2, 7.3, 7.4 . Table 7.1 reports the values of the VFZ-*index* when the contemporary

[23]See Degasperi and Fredholm (2010) for a discussion of the US productivity slowdown.

[24]The values inside brackets of these table are relative to missing input-output tables, they are computed anyway using the method described in Appendix A

wage-profit frontier, $w^{\text{VFZ}}_{\text{CTF}}(r_i, \boldsymbol{E}_t)$ is used as benchmark. Clearly, as shown in the figures of the previous section, the contemporary wage-profit frontier evolves through time. It is particularly interesting the stable values exhibited by the United States that, for the whole sample period, range between 0.66 and 0.69. Meanwhile, Germany, France and Denmark, which started at a much lower level, during the twenty years ranging from 1970 to 1990-95, have reached the United States levels. Germany has gone from 0.37 to 0.64; France from 0.52 to 0.70; Denmark from 0.43 to 0.67. Other countries, according to the results presented here, have moved slowly upward, but far from exhibiting a clear catching up. The other interesting observation is the fact that for the period going from 1995 to 2005, it is only the United States (and at a lower level Canada) that keeps the same distance with respect to the contemporary technological frontier: all the other countries fall. This does also indicate the technological development could have been driven, in the recent past and to a large extent, by the United States.

This observation is confirmed when we measure the country performances having as benchmark the inter-temporal technological frontier, $w^{\text{VFZ}}_{\text{ITF}}\ (r_i, \boldsymbol{E}_{\boldsymbol{\Phi}})$, see Table 7.3. The VFZ-*index* for the United States shows that the United States have shown no sign of technological development throughout the 80s, a minor improvement is observed from 1985 to 1995, but the real expansion is captured by the sharp increase exhibited from 1995 to 2005: jumping from 0.39 (1995) to 0.49 (2005).

The last row of Table 7.3 reports the measure of distance to the inter-temporal VFZ-*technological frontier* of the contemporary technological frontier. From this index, it is clear that the technological progress has taken place in two jumps: the first from 1980 to 1985 (going from 0.47 to 0.55) and the second from 1995 to 2000 (going from 0.56 to 0.64). While the technological progress observed from 1980 to 1985 was paralleled by all the countries with improvements of similar magnitudes, the jump observed from 1995 to 2000 is to be attributed principally to the United States (and Canada).

When we analyze the data reported in Table 7.3 we observe that the contemporary technological frontier has, for the period going from 1970 to 2005, increased from 0.45 to 0.66. This corresponds to a compounded growth rate of 1.1 percent per year, which is far less than the often reported 1.5–2 percent. Over a period of 35 years the difference between a growth rate of 1.1 and 2.0 percent corresponds to an increase by factor of 1.5 and 2, respectively.

For the single countries, the difference between the level of 1970 and

2005 corresponds to a compounded growth rate of: the US 1.0, Germany 1.2, the UK 2.0, France 1.2, Canada 1.5, Denmark 1.4 and Australia 2.1 percent per year, these growth rates are surprisingly small. Especially, the 1.0 percent annual growth for US. [25]

If these results — as here implicitly claimed — provide an alternative measure with respect to the usual indices of technological progress, then they are indeed interesting.

	1970	1975	1980	1985	1990	1995	2000	2005
the US	0.68	0.69	0.68	0.66	0.67	0.66	0.69	0.68
Germany	(0.37)	(0.36)	0.36	0.39	0.43	0.64	0.59	0.57
the UK	0.32	(0.44)	0.43	0.43	0.41	0.48	0.47	0.46
France	(0.52)	0.51	0.55	0.60	0.70	0.67	0.63	0.58
Canada	0.42	0.44	0.43	0.44	0.42	0.51	0.54	0.50
Denmark	(0.43)	0.43	0.38	0.46	0.54	0.67	0.62	0.49
Japan	...	...	...	...	...	0.54	0.35	0.45
Australia	0.29	0.40	0.39	0.42	0.50	0.48	0.44	0.42

Table 7.1: *The VFZ-index values: Contemporary,* $w^{VFZ}_{CTF}(r_i, \boldsymbol{E}_t)$

	1970	1975	1980	1985	1990	1995	2000	2005
the US	1	1	1	1	2	3	1	1
Germany	(5)	(7)	7	7	5	4	4	3
the UK	6	(4)	4	5	7	8	6	6
France	(2)	2	2	2	1	1	2	2
Canada	4	3	3	4	6	6	5	4
Denmark	(3)	5	6	3	3	2	3	5
Japan	...	...	...	...	...	5	8	7
Australia	7	6	5	6	4	7	7	8

Table 7.2: *The VFZ-index positions: Contemporary,* $w^{VFZ}_{CTF}(r_i, \boldsymbol{E}_t)$

[25] The OECD data bank does not report all the input-output tables. Appendix A contains additional information on the data used. With respect to Japan, we have found a great gap between the input-output tables available before 1995 and those available starting from 1995. We consider the values computed up to 1990 to be unreliable, and hence its value is here not reported. It must also be noted that between 1990 and 1995 the input–output tables change from the ISIC 2 to the present ISIC 3 standard of accounting. Whether or not this greatly influence our results is *pro tempore* unknown.

	1970	1975	1980	1985	1990	1995	2000	2005
the US	0.32	0.33	0.32	0.36	0.38	0.39	0.46	0.49
Germany	(0.16)	(0.16)	0.16	0.20	0.24	0.38	0.39	0.41
the UK	0.15	(0.20)	0.20	0.23	0.23	0.28	0.31	0.34
France	(0.24)	0.24	0.26	0.33	0.40	0.40	0.42	0.41
Canada	0.20	0.21	0.21	0.23	0.24	0.30	0.36	0.36
Denmark	(0.20)	0.20	0.18	0.24	0.31	0.40	0.41	0.35
Japan	...	...	...	...	...	0.32	0.23	0.32
Australia	0.13	0.19	0.19	0.23	0.29	0.29	0.29	0.30
Contemporary	0.45	0.48	0.47	0.55	0.57	0.56	0.64	0.66

Table 7.3: *The VFZ-index values: Benchmark, Inter-temporal VFZ*-technological frontier, $w_{ITF}^{VFZ}(r_i, \mathbf{\Phi})$

	1970	1975	1980	1985	1990	1995	2000	2005
the US	1	1	1	1	2	3	1	1
Germany	(5)	(7)	7	7	6	4	4	3
the UK	6	(4)	4	5	7	8	6	6
France	(2)	2	2	2	1	2	2	2
Canada	4	3	3	4	5	6	5	4
Denmark	(3)	5	6	3	3	1	3	5
Japan	...	...	...	...	...	5	8	7
Australia	7	6	5	6	4	7	7	8

Table 7.4: *The VFZ-index positions: Benchmark, Inter-temporal VFZ*-technological frontier, $w_{ITF}^{VFZ}(r_i, \mathbf{\Phi})$

An interesting point is that the US - as well as the other countries - do not seem to be facing a technology constraint. The distance between the actual economic structures and the VFZ-*technological frontier* is noticeable. Naturally, this depends on the availability of the foreign production techniques. Some techniques might be country-specific, i.e, cannot be transferred; a great deal will probably not be transferable, but internal to multinational corporations, which at least limits the transferability; and some (if not most) production techniques require a great deal of human capital, which in one way or another also must be transferred. In any case, we observe that the US - as well as the other countries - from the 1980s has been approaching the inter-temporal technological frontier, but we also observe that there is, still potentially, a long way to go [26].

The same goes for Germany and Denmark. While the UK and Australia remain behind.

7.3 The VFZ-*ranking* applied to the OECD input-output data set

The VFZ-*ranking* is a *numéraire* free measure of performance. As explained above, subsection 5.2, it is a measurement of the performance of the individual industries in terms of first, second and n-th best contributions to the formation of the VFZ-technological frontier. It is not a measure of the actual state of the implementation of the technological progress, which is captured by the VFZ-*index* as discussed in the previous section, but does measure the contribution of the economic region with respect to the formation of the most efficient global outer frontier represented by the VFZ-*technological frontier*, $w^{\text{VFZ}}_{\text{CTF}}(r_i, \boldsymbol{E}_t)$. It is also a concise measure of the relative position of the regions. A sure *'winning'* region is the one which contributes-dominates all the others because the methods of production for each sector dominates the methods of all the other regions. This case of total dominance would mean that the value of the VFZ-*ranking* would be 1.

[26]Particularly striking is the very low position of the actual wage-profit frontier of Japan. For the year 2000 it was the worst in terms of the wage-profit frontier and for the year 2005 only Australia had an actual wage-profit frontier that was lower than that of Japan. In terms of technological progress we have that the VFZ-*index* measures the actual technological progress, while the VFZ-*rankings* the relative position with respect the potential contribution to the worldwide potential frontier of production. As we will see below, for the year 2005, Japan performs in 2005 rather poorly, but second in the relative positions

Given this, the highest possible VFZ-*ranking* for the second region, where all the methods of this region are second best, would be 1/2. Given this, the highest possible value for the VFZ-*ranking* for the country performing third-best would be 1/3 and so on. Clearly, what has just been described is a case of perfect relative dominant ordering. But the situation is rarely clear cut and the VFZ-*ranking* values are weighted averages. A value between 1 and 1/2 means that, on the average, the methods are between being the first best and the second best, a value between 1/2 and 1/3 would imply that they are between second and third best and so on.

The VFZ-*index* is useful in order to assess the actual development of an economic region, but, as explained above, the VFZ-*ranking* does indicate the degree of technological forwardness or backwardness in terms of the importance of the individual industries.

From the values reported in Table 7.5 and the relative positions,Table 7.6, with respect to the VFZ-*rankings*, we can conclude that Canada has experienced a remarkable fall in the level of technological progress, going from 0.6 in 1970 and falling to 0.36 in 1995. Although Canada remains above average, the loss in position is noticeable. The US has experienced a fall in the technological performance from the 70s to the whole of 90s. However, it has kept an important relative position as the second best. It is only in the years 2000 and 2005 that the United States does express an unambiguous leadership. The VFZ-*ranking* relative to 2000 is back to the levels shown in 1970. Contrary to what one would expect, Germany does exhibit a modest performance in both the VFZ-*ranking* values (Table 7.5) and relative position (Table 7.6).

A striking result is the poor performance of the UK and Australia. Contrary to the UK and Australia, France which was performing well in terms of the VFZ-*index* in the case of the VFZ-*ranking*, is not performing as well. This can be interpreted as a well balanced domestic interdependent production structure. In particular, France has reached a leadership role in 1990, a role that was overtaken by Denmark in 1995. An inspection to the VFZ-rankings of 1990 and 1995 shows that all countries were performing rather poorly - or alternatively - that there was not a clear indication of leadership. When we compare the highest values for the years in the VZF-*ranking*, we see that the leader of 1990 (France, 0.48) and of 1995 (Denmark, 0.45) are those having the lowest VFZ-*ranking* [27].

[27] 1970 Canada 0.6, 1975 Canada 0.58, 1980 Canada 0.52, 1985 Canada 0.52, 1990 France 0.47, 1995 Denmark 0.45, 2000 United States 0.53, 2005 United States 0.51

	1970	1975	1980	1985	1990	1995	2000	2005
the US	**0.50**	**0.45**	**0.44**	**0.43**	**0.41**	**0.42**	**0.53**	**0.51**
Germany	(0.38)	(0.37)	**0.39**	**0.35**	0.30	0.32	0.31	0.31
the UK	0.18	(0.24)	0.23	0.25	0.23	0.20	0.29	0.28
France	(0.35)	**0.34**	**0.35**	**0.41**	**0.48**	0.31	0.29	0.32
Canada	**0.60**	**0.58**	**0.52**	**0.52**	**0.47**	**0.36**	**0.40**	**0.37**
Denmark	(0.32)	0.31	**0.35**	**0.38**	**0.39**	**0.45**	**0.37**	0.30
Japan	...	...	...	...	...	**0.35**	0.28	**0.39**
Australia	0.24	0.31	(0.31)	0.24	0.31	0.30	0.24	0.24

Table 7.5: *The VFZ-ranking values*

	1970	1975	1980	1985	1990	1995	2000	2005
the US	2	2	2	2	3	2	1	1
Germany	(3)	(3)	3	5	6	5	4	5
the UK	7	(7)	7	6	7	8	6	7
France	(4)	4	4	3	1	6	5	4
Canada	1	1	1	1	2	3	2	3
Denmark	(5)	6	5	4	4	1	3	6
Japan	...	...	...	...	...	4	7	2
Australia	6	5	6	7	5	7	8	8

Table 7.6: *The VFZ-ranking values. Relative positions*

8 A summing up: technological progress and catching up

The value-addition of this paper is the discovery of the VFZ-*algorithm*, see section 4, which allows the computation of actual technological frontiers from huge collections of production techniques. This is done without going through stochastic, *ad hoc*, and hence unsatisfactory, short cuts. This is an important breakthrough that allows the computation of powerful measures of systemic technological progress.

An important property of the wage-profit frontiers, and consequently, also of the the VFZ-*technological frontier* is that they are scale independent, see section 2, page 9. This means that the performance of small economic regions as well as of big ones can all be measured with the same method.

Based on this powerful tool, we have tried to develop synthetic indices of economic progress: the VFZ-*index* and the VFZ-*ranking*. These indices are useful to measure the intra-temporal economic progress and the inter-temporal technological development.

Here, we have applied these new tools to the OECD input-output data in order to measure the performance of 8 leading countries for the period starting from 1970 to 2005. We can conclude the following:

1. The global technological progress has been, for the period considered, around 1.1% per year.
2. This evolution can be divided into two periods (see Table 7.3, p.26):
    (a) **Catching-up:** Period 1970-1995, characterized by a low global overall technological progress, 0.88:%
        i. During this period the United States has maintained the high technological level reached at the beginning of the period, 1970, but with a very low growth rate, 0.79%;
        ii. Practically all the countries have converged towards the United States level;
            A. A set of countries have, by the year 1995, reached the United States levels: Germany (6% growth rate) ; France (2.6% growth rate); Denmark (2.8% growth rate);
            B. Another set of countries have exhibited catching-up, but never reached the United States levels: In 1995, Canada (2% growth rate) has reached a VFZ-*index*

value of 0.3 while the US had 0.39; For the same year, United Kingdom (2.5% growth rate) reached a VFZ-*index value* of 0.28 while the US had 0.39, Japan reached a VFZ-*index value* of 0.32 while the US had 0.39[28]; Australia (3.3% growth rate) reached a VFZ-*index value* of 0.29 while the US had 0.39 .

(b) **Restart:** Period 1995-2005, characterized with higher technological progress, 2% ;

i. During these years, after having kept its position gained 25 years before, the United States has exhibited a sustained growth, where the VFZ-*index* has gone from the 1995 level of 0.39 to 0.49 in 2005, implying a rate of growth of 2.3%;

ii. A clear indication of leadership by the United States is also shown by high value of the VFZ-ranking; the values for the years 2000 and 2005 are above the already high value of 1970 (see Table 7.5, p.29);

iii. Apart from Canada, which has continued in catching-up, all the other countries have remained at the same level in 2005 as they had in 1995 and in one case, Denmark, which has exhibited a definite and sharp fall.

3. An inspection of VFZ-*index* values of Table 7.1 p.25, where the contemporary VFZ-*technological frontier* is the benchmark, and of the values of Table 7.3, p.26, where the inter-temporal VFZ-*technological frontier* is the benchmark, indicate that although there is a clear leadership in technological progress, it is also the case that the *wage-profit curves* of the individual countries are substantially below the global *wage-profit frontiers*. Hence, it seems that there is scope for improvement.

9 Concluding Remarks

In this paper, we have presented a computationally efficient algorithm - the VFZ-*algorithm* - that allows the computation of the efficient *wage-profit frontier*, called the VFZ-*technological frontier*. As shown, the VFZ-*technological frontier* is a theoretical construction that, due to

[28] As mentioned above, the data for Japan antecedent the period 1995-2005 seems to be somewhat flawed and hence it is here not considered in terms of its rate of growth

duality properties, can be used to measure technological progress in terms of the changes of the auxiliary prices. Once the computation of VFZ-*technological frontier* was made tractable, it has been straightforward to define two important measures of technological performance, the VFZ-*index* and the VFZ-*ranking*.

We have applied the above tools to the OECD input-output data bank. This has led to important empirical results, which we have summarized in the previous section. The results appear interesting and somewhat illuminating and a review comparing results from other studies could be useful.

Another important direction for further investigation is the comparison of the productivities of the different sectors that can be conducted by extracting information with respect to the auxiliary prices associated to it. An interesting characteristic of the method used here is that the prices are all measured in terms of the per-capita Net National Product of the United States, year 2000, but their values vary as a function of the specific individual local structures. That is, the auxiliary prices are a function of the local production conditions. Hence, the virtual purchasing power (or virtual exchange values) of the commodities produced in one system might be very different from the values produced in another system. This information can be used further to measure the sectoral technological progress. Research on this direction would be necessary and, we think, may shed a new light on productivities and efficiency comparisons [29].

A related issue concerns the difference that exist between actual market prices and our virtual prices. As we have pointed out in Section 2, the assumption of a uniform rate of profit, although very standard, is a convenient assumption that allows to work with a simple two dimensional space - instead of an n-dimensional one.

Furthermore, the VFZ-*technological frontier* can be seen as a tool for the study of comparative and absolute advantages. This too would be a possible direction for further research.

A last word. The VFZ-*algorithm* does exploit an important result found in Bruno et al. (1966) and Bharadwaj (1970). Both expand on Sraffa (1960)'s fundamental work *Production of Commodities by means of Commodities (*in particular Ch.12). Those papers were written as a contribution to what is often considered an important, but maybe

[29] A similar attempt has been made, 25 years ago, by Wassily Leontief (1985) himself. Thanks to the algorithms available and constructed for this paper, we are confident that interesting results would emerge

useless, theoretical field: capital theory. To be able to use the result presented there for an empirical application is a further element in support of pure theory. Whether a theoretical result might or might not have a practical application cannot be determined *a priori*. We hope that this paper is a further example of a useful, but unexpected and hence not programmable, link between pure theory and applications.

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A Data Description

Table A.1 shows which OECD input–output tables that are available from the period 1970–2005. Tables are not necessarily available from

	1970	1975	1980	1985	1990	1995	2000	2005
the US	×	×	×	×	×	×	×	×
Germany			×	×	×	×	×	×
the UK	×		×	×	×	×	×	×
France		×	×	×	×	×	×	×
Canada	×	×	×	×	×	×	×	
Denmark		×	×	×	×	×	×	×
Japan	×	×	×	×	×	×	×	×
Australia	×	×		×	×		×	×

Table A.1: *Available input–output tables*

the exact five year intervals, e.g., the US tables here labelled 1970 and 1975 are actually the 1972 and 1977 tables, respectively.[30]

The list below shows how the original tables have been aggregated down to the 23 × 23 used in this study. The numbers in the brackets refer to their respective ISIC 2 and ISIC 3 classification, *viz.* {[ISIC 3],[ISIC 2]}.

1. Agriculture, hunting, forestry, and fishing {[1],[1]}
2. Mining and quarrying {[2–3],[2]}
3. Food products, beverages, and tobacco {[4],[3]}
4. Textiles, textile products, leather, and footwear {[5],[4]}
5. Wood and products of wood and cork {[6],[5]}
6. Pulp, paper, paper products, printing, and publishing {[7],[6]}
7. Coke, refined petroleum products, and nuclear fuel {[8],[9]}
8. Chemicals {[9–10],[7–8]}
9. Rubber and plastics products {[11],[10]}
10. Other non-metallic mineral products {[12],[11]}
11. Metals {[13–14],[12–13]}
12. Fabricated metal products, except machinery and equipment {[15],[14]}
13. Machinery and equipment, nec {[16],[15]}
14. Electrical machinery and apparatus {[17–20],[16–18]}
15. Transport equipment {[21–25],[19–22]}
16. Manufacturing nec; recycling (include furniture) {[25],[23–24]}
17. Production and distribution of electricity, gas, and water {[26–29],[25]}
18. Construction {[30],[26]}

[30]The full list of available tables are: the US {1972, 1977, 1982, 1985, 1990, 1995, 2000, 2005}, Germany {1978, 1986, 1990, 1995, 2000, 2005}, the UK {1968, 1979, 1984, 1990, 1995, 2000, 2003}, France {1977, 1980, 1985, 1990, 1995, 2000, 2005}, Canada {1971, 1976, 1981, 1986, 1990, 1995, 2000}, Denmark {1977, 1980, 1985, 1990, 1995, 2000, 2004}, Japan {1970, 1975, 1980, 1985, 1990, 1995, 2000, 2005}, and Australia {1968, 1974, 1986, 1989, 1999 ,2005}.

19. Wholesale and retail trade {[31],[27]}
20. Service activities (transport, hotels and restaurants) {[32–36],[28–29]}
21. Post and telecommunications {[37],[30]}
22. Business activities (finance, real estate, and R&D) {[38–43],[31–32]}
23. Public administration, education and health {[44–48],[33–35]}

Table A.2–A.9 show the macro-industry deflators used to convert the tables denominated in current prices (possible domestic currency) into tables denominated in fixed US 2000 prices. The transition to the EURO has been taken into account in the tables below. The deflators are computed as the ratio between GDP in constant prices and GDP in current prices and when necessary also divided by the dollar-domestic currency exchange rate (www.sourceoecd.org). The missing values marked with a '–' correspond with the unavailable OECD tables.

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	1.23	0.84	0.71	0.74	0.65	0.67	1.00	0.91
II Industry including energy	2.83	1.89	1.14	1.09	1.02	0.96	1.00	0.93
III Construction	4.97	3.28	2.03	1.79	1.43	1.26	1.00	0.71
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	2.13	1.53	1.12	1.05	0.97	0.87	1.00	0.98
V Financial intermediation; real estate, business activities	4.79	3.45	2.34	1.88	1.47	1.23	1.00	0.88
VI Other service activities	5.15	3.57	2.37	1.98	1.53	1.24	1.00	0.82

Table A.2: *Macro-industry deflators for the US 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	–	–	0.53	0.60	0.64	1.00	1.03	1.39
II Industry including energy	–	–	0.81	0.60	0.57	1.02	1.03	1.01
III Construction	–	–	1.15	0.84	0.71	1.02	1.03	0.98
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	–	–	0.78	0.63	0.60	1.01	1.03	1.03
V Financial intermediation; real estate, business activities	–	–	1.01	0.69	0.61	1.02	1.03	0.94
VI Other service activities	–	–	0.98	0.75	0.67	1.08	1.03	0.97

Table A.3: *Macro-industry deflators for Germany 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	5.97	–	2.26	2.43	1.40	1.06	1.57	1.37
II Industry including energy	11.3	–	3.65	2.21	1.86	1.64	1.57	1.57
III Construction	18.4	–	4.44	3.14	2.15	2.00	1.57	1.35
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	13.2	–	4.41	2.87	1.91	1.72	1.57	1.51
V Financial intermediation; real estate, business activities	14.8	–	4.50	2.87	2.01	1.71	1.57	1.34
VI Other service activities	21.5	–	5.81	3.84	2.33	1.94	1.57	1.38

Table A.4: *Macro-industry deflators for the UK 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	–	0.25	0.23	0.17	0.15	0.98	1.06	1.09
II Industry including energy	–	0.34	0.26	0.17	0.16	1.04	1.06	1.12
III Construction	–	0.56	0.38	0.26	0.21	1.19	1.06	0.86
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	–	0.39	0.30	0.20	0.17	1.05	1.06	0.95
V Financial intermediation; real estate, business activities	–	0.53	0.42	0.29	0.22	1.22	1.06	0.94
VI Other service activities	–	0.58	0.41	0.26	0.21	1.18	1.06	0.91

Table A.5: *Macro-industry deflators for France 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	3.53	1.63	1.00	0.96	0.95	0.78	0.81	–
II Industry including energy	3.95	2.37	1.38	1.18	1.04	0.94	0.81	–
III Construction	3.33	1.89	1.31	1.18	0.92	0.87	0.81	–
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	2.63	1.76	1.20	0.96	0.85	0.82	0.81	–
V Financial intermediation; real estate, business activities	4.26	2.50	1.58	1.17	0.94	0.87	0.81	–
VI Other service activities	4.40	2.58	1.71	1.29	1.02	0.90	0.81	–

Table A.6: *Macro-industry deflators for Canada 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	–	.104	.094	.074	.078	.101	.119	.143
II Industry including energy	–	.305	.246	.178	.149	.140	.119	.113
III Construction	–	.360	.290	.222	.173	.147	.119	.104
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	–	.244	.210	.134	.120	.117	.119	.109
V Financial intermediation; real estate, business activities	–	.329	.253	.179	.150	.128	.119	.109
VI Other service activities	–	.360	.285	.197	.151	.136	.119	.103

Table A.7: *Macro-industry deflators for Denmark 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	.0135	.0083	.0067	.0063	.0060	.0056	.0065	.0071
II Industry including energy	.0111	.0078	.0063	.0058	.0058	.0059	.0065	.0074
III Construction	.0313	.0152	.0104	.0087	.0073	.0066	.0065	.0066
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	.0123	.0082	.0067	.0062	.0062	.0061	.0065	.0068
V Financial intermediation; real estate, business activities	.0177	.0119	.0091	.0078	.0069	.0064	.0065	.0068
VI Other service activities	.0288	.0137	.0104	.0086	.0075	.0066	.0065	.0067

Table A.8: *Macro-industry deflators for Japan 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	3.32	2.09	–	0.98	0.73	–	0.88	0.75
II Industry including energy	3.78	2.76	–	0.94	0.88	–	0.80	0.55
III Construction	4.45	2.72	–	1.05	0.88	–	0.77	0.65
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	5.35	3.42	–	1.02	0.84	–	0.77	0.69
V Financial intermediation; real estate, business activities	7.30	4.98	–	1.34	0.99	–	0.80	0.65
VI Other service activities	5.04	2.96	–	1.20	1.01	–	0.79	0.61

Table A.9: *Macro-industry deflators for Australia 1970–2005*

B The Wage-profit and Intertemporal Technological Frontiers

Figure B.1 shows the wage-profit frontiers forming the intertemporal technological frontier, and Figure B.2 and B.3 show the wage-profit frontiers for the individual countries together with the intertemporal technological frontier.

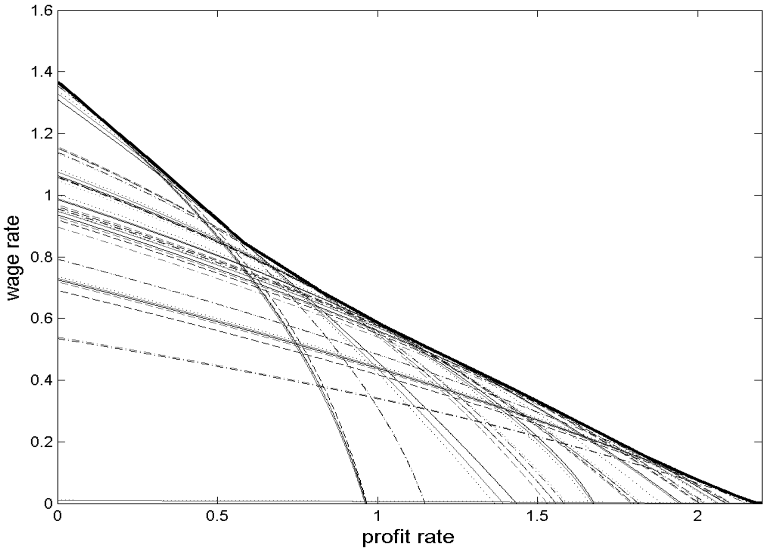

Fig. B.1: *The intertemporal technological frontier*

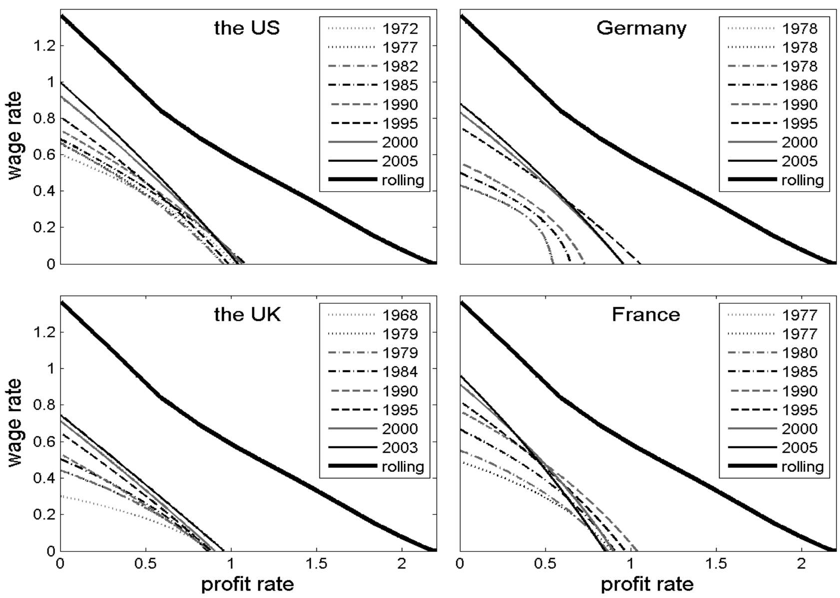

Fig. B.2: *Wage-profit frontiers and the intertemporal technological frontier: the US, Germany, the UK, and France*

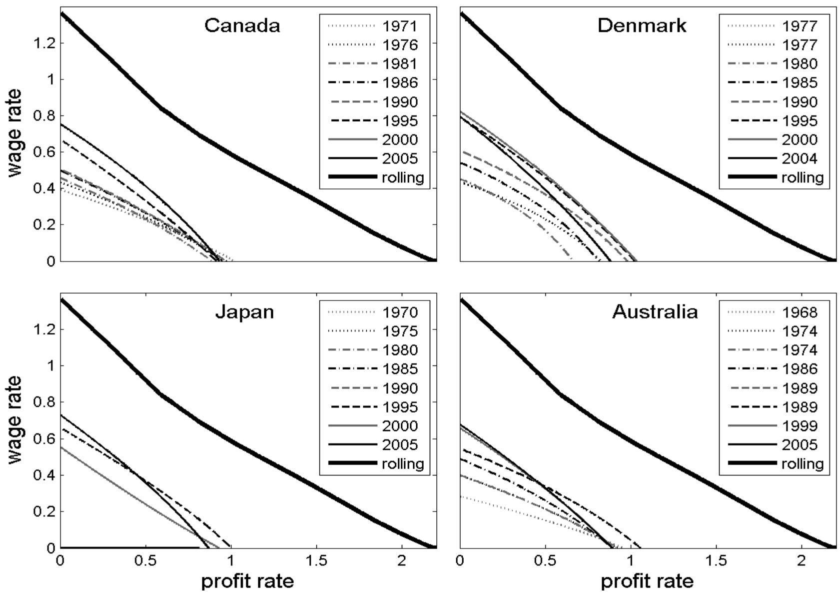

Fig. B.3: *Wage-profit frontiers and the intertemporal technological frontier: Canada, Denmark, Japan, and Australia*