Optimal abatement investment and environmental policies under pollution uncertainty

Enrico Saltari*and Giuseppe Travaglini†

(Preliminary draft. Please do not quote without permission)

Abstract

In this paper we present a continuous time model with reversible abatement capital in order to analyze the effects of environmental policies on the value of the firm and investment decisions. We show that the effects depend on what sort of future policy are implemented. We focus on investment effects of changes in corrective taxes to control the use of polluting inputs, and subsidies to promote abatement investment. We show that (1) while taxes have a depressive effect on capital accumulation, subsidies boost investment; (2) the impact of these policies on the value of the firm is ambiguous. This latter result has important empirical implications insofar as investment are based on the average value of the firm rather than the (unobservable) marginal value.

Key words: Pollution uncertainty; externality; capital reversibility; environmental policy.

JEL classification codes: E22, L51, H23, Q28.

*Dipartimento di Economia e Diritto, Facoltà di Economia, Università "La Sapienza" di Roma, Via del Castro Laurenziano 9, Roma, Italy. Correspondig author. Email: Enrico.Saltari@uniroma1.it.

†Dipartimento di Economia e Metodi Quantitativi, Facoltà di Economia, Università "Carlo Bo" di Urbino, Via Saffi 42, 61029, Urbino (PU), Italy

1 Introduction

This paper examines the effect of environmental policies on the abatement investment decisions of a competitive firm, facing pollution uncertainty. The relationship between economic decisions and pollution dynamics has been extensively studied over the last decade (Bretschger and Smulders, 2007; Egli and Steger 2007; Soretz, 2007). The standard problem concerns the optimal timing of a discrete policy that a society or a government should adopt to reduce emissions of environmental pollutant. Some common features characterize this class of models. Smulders and Gradus (1996), and Soretz (2007) model pollution as an externality which affects the productivity of inputs and/or utility of consumption. Fisher (2000) and Pindyck (2000) remark that irreversibility is a feature that frequently comes into play together with pollution, shaping the optimal abatement investment strategies. Pindyck (2000) emphasizes that abatement investment and pollution have to do with the effects of uncertainty, and that when irreversibility and uncertainty come together, delaying policy adoption may be optimal for an investor.

These models captured a growing attention and many theorists investigated the consequences of environmental policies on abatement investment strategies using the real option value approach (Fisher, 2000; Pindyck, 2002; Lin *et al.,* 2007; Ansar and Spark, 2009; Lin and Huang, 2010).

More recently, Lin and Huang (2011), and Saltari and Travaglini (2011) (hereafter LH and ST) extended the basic model of option value, shifting the attention from the net social benefits of a policy, to the net private benefits of a firm investing in clean capital goods. As they argue, an efficient energy-saving or abatement investment programme need to take account of the trajectory of costs and profits over time associated to the project, rather than just at a single point in time. Both these papers are related to the literature on optimal stopping time, but they start from different assumptions and reach different results.

Specifically, LH study the entry and exit strategy in energy industry, using an option value approach which allows to manage the flexibility of environmental real assets (Dixit and Pindyck, 1994). They combine the concept of Tobin's q - the viewpoint of firm value - and the concept of real option to determine the optimal stopping time of adopting a new energy-saving project. In their setup, the firm has the monopoly right to invest in a single, discrete project, and future discounted profits and costs are assumed to follow a geometric Brownian motion. As they show, the greater uncertainty makes waiting more valuable relative to investing at once, reducing the present value of the active "green" firm relative to the one of the idle.

However, ST argue that in many situations investments are made sequentially by the firm choosing the time path of its capital stock. Therefore, they sustain, it must be specified in more detail how abatement investment decisions are affected by adjustment costs of investment, and how alternative environmental policies impinge upon the optimal investment strategy chosen by the green firm. Following this idea, ST examine the effects of environmental policies aimed at stimulating private investment in pollution abatement capital. In their framework, firm operates in a competitive market, capital is irreversible, and pollution follows a geometric Brownian motion. Basically, aggregate pollution is seen as an externality which affects the investment decisions of any single firm. They show that, with irreversibility *and* uncertainty, environmental policies promoted to enforce abatement capital may generate the unexpected result of reducing the firm value and the abatement investment rate. Interestingly, ST underline that their results generalize LH's results because their model not only provides the optimal stopping time of the investment strategy, as it is in LH, but also how much to invest in a new abatement project at each instant of time.

The model developed herein is a version of the ST model because it studies the effects of environmental policies under the assumption of pollution uncertainty *but* reversible capital. The questions at the heart of the present paper are: is the blend between irreversibility and uncertainty a necessary condition to determine the counterintuitive effects of environmental policies on investment decisions of a firm? Are taxes and subsidies equivalent instruments to stimulate abatement investments?

To investigate these questions we scrutinize the properties of an investment model where aggregate pollution is an externality which affects negatively the productivity of inputs. We assume however that abatement investment is reversible. We further assume that the cost of investing is given by a quadratic cost of adjustment. In this setup, incentive-based instruments work by altering incentives for firm investment decisions. We focus on taxes on the level of particular inputs (such as oil), and subsidies on abatement capital. We get three main results. First, we show that corrective taxes on the polluting input will reduce investment while subsidies will stimulate them. Second, environmental policies can have a positive or negative impact on the value of the firm, depending on the magnitudes of parameters affecting the operating profit and the rents accruing to the firm. Finally, and for the same reasons, pollution uncertainty has an ambiguous impact on the value of the firm.

The organization of the paper is as follows. First we introduce the pollution process and the Bellman equation. We then relate the abatement investment to the form of the investment cost function, discussing the case of reversibility. Next we study the effects of environmental policies on both the value of the firm and the optimal abatement investment decision. The last section concludes.

2 The model

Let p_t be a variable that represents the stock of aggregate pollution, say the concentration of CO_2 in the atmosphere. The flow of pollution p_t evolves according to the geometric Brownian motion:

$$\frac{dp_t}{p_t} = (v - \varphi)dt + \sigma dz_t \tag{1}$$

where v is the instantaneous drift, σ is the instantaneous standard deviation, and dz is the increment to a Wiener process with mean of zero and standard deviation of $\sqrt{dt}$. Note that in equation (1) pollution growth is reduced by a factor φ which measures the environmental benefits associated to the aggregate abatement investments. In other words, the advantage in adding new units of abatement capital is given by the growth rate φ which quantifies the slowdown of pollution growth induced by the abatement activities of the whole economy. It is helpful to rewrite equation (1) as

$$\frac{dp_t}{p_t} = \mu dt + \sigma dz_t \tag{2}$$

where $\mu \equiv v - \varphi$ is the net instantaneous growth rate of pollution.

The firm undertakes gross abatement investment A_t and incurs depreciation at a constant rate $\delta \geqslant 0$. Thus, the change in the abatement capital stock M_t is:

$$dM_t = (A_t - \delta M_t)\, dt \tag{3}$$

and in steady state $A_t/M_t = \delta$.

Pollution enters as a negative externality in the production function of any single firm, decreasing the productivity of inputs (Smulders and Gradus, 1996; Bretschger and Smulders, 2007). Conceptually, we treat aggregate pollution as a negative technical change. The production function has the Cobb-Douglas form:

$$Y_t = U_t^{\alpha}\left(p_t^{-\beta} M_t\right)^{1-\alpha} \tag{4}$$

where α is the income share of the polluting input and satisfies the inequality $0 < \alpha < 1$. The Cobb-Douglas specification is, perhaps, restrictive (due to the unit elasticity of substitution), but it is adopted here for its analytical tractability (Bretschger and Smulders, 2007; Mulder and Degroot, 2007; Shadbegiana and Gray, 2005).

The firm produces at each instant of time an output Y_t using M_t units of abatement capital and U_t units of polluting input. Because of the pollution externality, the effective units of capital are reduced by a factor p_t^{β}, so that in efficiency units capital is M_t/p_t^{β}. In this formulation β is a measure of the strength of the effects of aggregate pollution on abatement capital: the higher is β, the lower is the capital in effective units (Smulders and Gradus, 1996). Equation (4) can be rewritten as

$$Y_t = p_t^{-\epsilon}\left(U_t^{\alpha} M_t^{1-\alpha}\right) \tag{5}$$

with $\epsilon = \beta(1-\alpha)$ to emphasize that pollution is a global externality affecting negatively the level of output.

The firm pays a fixed unit cost ω to use the polluting input U_t and an exogenously environmental tax rate τ to employ the input, so that total cost of U_t is $(\omega+\tau)\,U_t$. The tax τ internalizes the externality by inducing the firm to behave as if pollution costs enter its private cost function. In addition, tax τ alters the payoffs that firm faces because the operating profit at time t equals instantaneous revenue minus total cost,

$$\pi_t = p_t^{-\epsilon}\left(U_t^{\alpha} M_t^{1-\alpha}\right) - (\omega+\tau)\,U_t$$

The firm chooses U_t to maximize the instantaneous operating profit π_t. It is easily shown that

$$\max_{U_t} p_t^{-\epsilon}\left(U_t^{\alpha} M_t^{1-\alpha}\right) - (\omega+\tau)\,U_t = h p_t^{\theta} M_t \tag{6}$$

where $h = (1-\alpha)\left(\frac{\alpha}{\omega+\tau}\right)^{\frac{\alpha}{1-\alpha}} > 0$ and $\theta = -\beta < 0$. Note that $h p_t^{\theta}$ is the marginal revenue of abatement capital at time t, and the pollution effect on operating profit is negative.

2.1 The value of the firm

The value of the firm is

$$V(M_t, p_t) = \max_{A_s} E \int_t^{\infty} \left[h p_s^{\theta} M_s - c(A_s) + \eta A_s \right] e^{-rs} ds \tag{7}$$

where $r > 0$ is the (assumed) constant interest rate. In equation (7) the cost for increasing the stock of abatement capital by A_s is $c(A_s) - \eta A_s$, where $c(A_s)$ denotes the cost of investing at the rate A_s, and $0 \leq \eta \leq 1$ is a subsidy potentially given by the government to cover some of the expenses for expanding the abatement capital by A_s. Note that taxes and subsidies are different in their effects on profits. A firm gains an additional income from an abatement subsidy η only when it decides to invest; whereas, a tax τ results in a loss to the firms since it pays tax on all polluting inputs employed in production. As we will see, in detail, below, the asymmetric effect on profit of taxes and subsidies will shape the firm's investment abatement decisions, and the impact of a specific environmental policy.

The value of the firm satisfies the following Bellman equation

$$rV(M, p)\, dt = \max_{A} \left[\left(h p^{\theta} M - c(A) + \eta A \right) dt + E(dV) \right] \tag{8}$$

(we suppress time subscripts unless they are needed for clarity). Applying Ito's lemma to $E(dV)$ we get the expected change in the value of the firm over the time interval dt

$$E(dV) = \left[(A - \delta M) V_M + \mu V_p p + \frac{1}{2} \sigma^2 V_{pp} p^2 \right] dt \tag{9}$$

Since $V_M \equiv q$ is the marginal valuation of a unit of installed capital, substituting q for V_M in (9), and then substituting (9) in (8) yields

$$rV(M, p) = \max_{A} \left[h p^{\theta} M - c(A) + \eta A + (A - \delta M) q + \mu V_p p + \frac{1}{2} \sigma^2 V_{pp} p^2 \right] \tag{10}$$

This expression can be simplified by "maximizing out" the rate of investment to obtain:

$$rV(M, p) = h p^{\theta} M + \psi - \delta M q + \mu V_p p + \frac{1}{2} \sigma^2 V_{pp} p^2 \tag{11}$$

where

$$\psi \equiv \max_{A} \left[Aq + \eta A - c(A) \right] \tag{12}$$

In expression (12) ψ is the value of the *rents* obtained from undertaking investment at the rate A. When the green firm invests, it acquires Adt units of capital, whose value is $(qA + \eta A)\, dt$, but it pays $c(A)dt$ to increase the abatement capital. Hence, $Aq + \eta A - c\,(A)$ is the net value the firm gains per unit of time to invest at the rate A.

2.2 Investment, value and rents

Let us explore equation (12) a little further. We assume that abatement capital is reversible. We use a specialized version of the framework of Abel and Eberly (1997), under the assumption of reversible abatement capital stock. The case of irreversibility is studied in ST (2011).

We here assume that the cost of investing at time t is given by the following convex function:

$$c(A_t) = bA_t + \frac{\gamma}{2}A_t^2 \tag{13}$$

where bA_t is the fixed cost for purchasing abatement capital at the rate A_t; and $\frac{\gamma}{2}A_t^2$ is a quadratic cost of adjustment. Using (13), equation (12) becomes

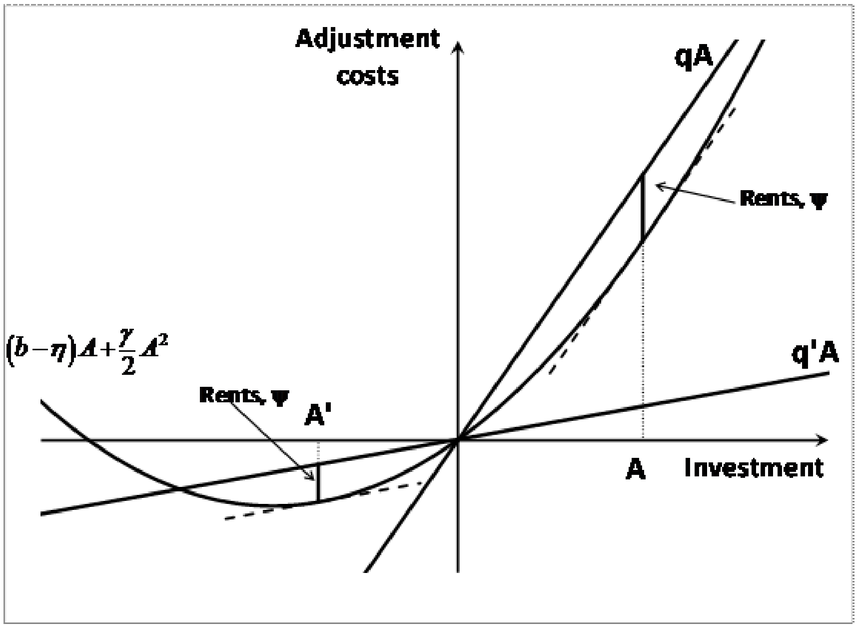
$$\psi \equiv \max_A \left[Aq - (b - \eta)\, A - \frac{\gamma}{2}A^2 \right] \tag{14}$$

Because the investment cost function is convex, the firm earns rents on inframarginal units of investment when investment is non zero, that is, when $q \neq b - \eta$ that is when $q + \eta \neq b$.

The rents ψ are illustrated in figure 2.2. The convex curve represents the net cost function $(b - \eta)\, A_t + \frac{\gamma}{2}A_t^2$. It is strictly convex, passes through the origin, and has a slope equal to $b - \eta$ at the origin. When $q > b - \eta$ the marginal value of capital is greater than the net cost of capital, so the straight line representing qA is steeper than $(b - \eta)\, A_t + \frac{\gamma}{2}A_t^2$ at the origin. In this case, q exceeds the net cost for some positive values of $A > 0$, and the optimal value of investment A is the value that maximizes $Aq - (b - \eta)\, A - \frac{\gamma}{2}A^2$. In figure 1 the rents ψ, for positive value of the investment, are shown as the vertical distance between the straight line and the curve at A.

Note, however, that when $q^{'} < b - \eta$ the straight line representing $q^{'}A$ is less steep than $(b - \eta)\, A_t + \frac{\gamma}{2}A_t^2$, at the origin. In this case, $q'A$ exceeds $(b - \eta)\, A_t + \frac{\gamma}{2}A_t^2$ for some negative values of investment. In this scenario, the optimal value of investment A' is negative, and the value of the rents ψ

and Settings/Administrator/Local Settings/Temporary Internet Files/Content.Outlook/X4MWN35I/adjustment.wmf



is again shown as the vertical distance between $q^{'}A$ and $(b-\eta)\,A_t+\frac{\gamma}{2}A_t^2$ at $A^{'}$. Since the optimal gross investment can be negative or positive when the cost function is given by equation (13), we consider this case as one in which abatement investment is *reversible.*

To sum up, the optimal investment solution takes the form

$$A^{*}=\frac{1}{\gamma}\left[q-(b-\eta)\right] \tag{15}$$

and the optimal rents are given by the expression

$$\psi^{*}=\left[q-(b-\eta)\right]^{2}\frac{1}{2\gamma} \tag{16}$$

Equations (11) and (16) together generate a non-linear second order partial differential equation. But, we imposed enough structure on our intertemporal problem to obtain an explicit solution. Appendix A.1 shows that the solutions below satisfy equations (11) and (16)

$$V(M,p)=q(p)M+G\left(p\right) \tag{17}$$

where

$$q(p) = \frac{hp^{\theta}}{r + \delta - \mu\theta - \frac{\theta}{2}(\theta - 1)\sigma^2} \equiv Bp^{\theta} \tag{18}$$

and

$$G(p) = \frac{1}{2\gamma}\left[\frac{\left(Bp^{\theta}\right)^2}{r - 2\mu\theta - \theta(2\theta - 1)\sigma^2} - 2\frac{Bp^{\theta}(b - \eta)}{r - \mu\theta - \frac{\theta}{2}(\theta - 1)\sigma^2} + \frac{(b - \eta)^2}{r}\right] \tag{19}$$

where $B = \frac{h}{r+\delta-\mu\theta-\frac{\theta}{2}(\theta-1)\sigma^2}$. Further, recall that $G(p) = 0$ when $q = b - n$, since at this point the slope of the adjustment cost function is equal to the marginal q, and the rents are equal to zero by definition (see figure 2.2).

To complete the analysis of equation (19), notice that the denominators in the formula are the growth-adjusted discount rates, assumed positive to assure convergence. In detail, since $b - \eta$ is a constant, there is no adjustment for the discount rate r. The adjustment factor is instead needed for the middle term in parenthesis since $q = Bp^{\theta}$. In fact, p^{θ} has the lognormal distribution whose expected growth rate is $\theta\mu + \frac{\theta}{2}(\theta - 1)\sigma^2$, so that r is adjusted by this factor. Finally, since $q^2 = \left(Bp^{\theta}\right)^2$, the corresponding expected growth rate is $2\theta\mu + \theta(2\theta - 1)\sigma^2$, and the discount rate must be adjusted in consequence.

Several results follow immediately from these conditions. First, we observe that the value of the firm is a linear function of the abatement capital stock M, since the slope of q is independent of M. For a competitive firm with constant returns to scale of production, the marginal operating profit depends only on the direct cost $(\omega + \tau)$ of the polluting input, and, thus, it is independent on the capital stock M. Second, the rents $G(p)$ and the value $V(M, p)$ depend on both q and η, given the interest rate r and the other parameters. Finally, investment A is an increasing function of q. But q depends on pollution p and taxes τ, whereas it is not affected by subsidies η.

Obviously, these features affect the value of the firm when the environmental policy changes. This focal matter is examined in the next section.

3 Environmental policy and its implications

In this section we address two policy issues: first, we investigate if, in the model described above, taxes and subsidies are equivalent instruments to stimulate abatement investments; second, we wonder, what are the effects of those policies on the value of the firm?

3.1 Taxes τ

To see what are the effects of an increase in tax on investment, recall that investment depends positively on q. Thus, it is straightforward to realize that an increase in τ reduces investment. In fact, taxes directly affects the cost of the polluting input U. We know that the marginal revenue of capital is hp^{θ} with $h = (1-\alpha)\left(\frac{\alpha}{w+\tau}\right)^{\frac{\alpha}{1-\alpha}} > 0$. Hence:

$$\frac{dh}{d\tau} < 0 \tag{20}$$

Expression (20) says that as τ increases h decreases, and, therefore, so does hp^{θ}. Hence, an increase of τ reduces the marginal q value of the firm and thus investment. Notice that this is a distorsive effect. The aim was to reduce pollution but the result is a reduction in abatement investment.

Let us now scrutinize the effect of a tax τ increase on the value of the firm. While the effects on q are clear, the same cannot be said for the rents. The change in τ also alters the present value of rents but in an indefinite direction since

$$\frac{\partial G(p)}{\partial h}\frac{\partial h}{\partial \tau} \lesseqgtr 0 \tag{21}$$

This is because the the sign of the first derivative is indeterminate:

$$\frac{\partial G(p)}{\partial h} = \frac{p^{\theta}}{\gamma}\frac{1}{r+\delta-\mu\theta-\frac{\theta}{2}(\theta-1)\sigma^2}\left[\frac{q}{r-2\mu\theta-\theta(2\theta-1)\sigma^2}-\frac{b-\eta}{r-\mu\theta-\frac{\theta}{2}(\theta-1)\sigma^2}\right] \tag{22}$$

The above derivative (22) is just an algebraic illustration of the relationship depicted in figure 2.2. Looking at this picture, it is evident that rents ψ are positive both when $q < b-\eta$ and when $q > b-\eta$, and equal to zero when $q = b-\eta$. Recall, however, that the present value of rents $G(p)$ depends directly on tax τ. Hence, ψ has a non linear pattern: it is positive but decreasing, until $q = b-\eta$ where it becomes equal to zero, and it is positive and rising afterwards. This nonlinearity shapes the function $G(p)$ and its relation with τ. This in turn means that a tax increase may reduce or increase the value of the firm.

3.2 Subsidies η

Let us now focus on the subsidy η. An increase in subsidy directly impacts on investment by decreasing the cost of purchasing capital goods. Differently from a tax increase on the polluting input, a subsidy increase stimulates investment in abatement capital by directly reducing its purchasing cost.

Let us now focus on the value of the firm. The incentive deriving from a subsidy increase directly affects the present value of rents $G(p)$, without affecting the q value. Looking at the effect of a change in subsidy, we get the expression

$$\frac{\partial G\left(p\right)}{\partial \eta}=\frac{1}{\gamma}\left\{\frac{q}{r-\theta\mu-\frac{\theta}{2}\left(\theta-1\right)\sigma^{2}}-\frac{b-\eta}{r}\right\}\gtreqless 0 \tag{23}$$

But, as in the case of a tax increase, the effect of an increase in subsidy on the value of the firm is indeterminate. At first sight, this result may appear counterintuitive. The ambiguity of the relationship depends on two features: firstly, subsidies affect directly the cost of the investment rate; secondly, the adjustment cost function is convex, and this non linearity affects the optimal investment decision.

The above equation (23) can also be clearly seen by looking again at figure 2.2. Begin in the region where the marginal cost of investment $(b-\eta+\gamma A)$ is grater than the marginal q value of the firm so that $A<0$. If we now increase the subsidy, disinvestment will decrease, thus reducing the rents accruing to the firm. This result implies that in this region $\partial G\left(p\right)/\partial\eta<0$. However, once past the value of $q=b-\eta$, where $A=0$, investment becomes positive, $A>0$. In this region an increase in subsidy will increase investment and thus the rents, implying that $\partial G\left(p\right)/\partial\eta>0$. To sum up, in this case too the effect of environmental policy on the value of the firm is ambiguous.

4 Conclusion

Lin and Huang (2011) demonstrated that the theory of Tobin's q and real options can be usefully employed to evaluate the feasibility of investing in energy-saving equipment. Saltari and Travaglini (2011) emphasized that, with irreversible capital and pollution uncertainty, the value of a firm captures the option value of the costly technology for disinvesting. Both these papers provide a new flexible thinking for decision making criteria in environmental issues. Nonetheless, these papers do not provide an analysis of the effect of environmental policies when abatement capital is reversible.

Here, we presented a continuous time model with reversible abatement capital in which the effect of environmental policies on the value of the firm and investments depends on what sort of policies are implemented, focusing in particular on investment effects of changes in corrective taxes, to control the use of polluting inputs, and subsidies, to promote abatement investment. We have shown that taxes and subsidies have opposite effects on investment and that the impact of these policies on the value of the firm is ambiguous.

In our framework, pollution is an externality which affects negatively the productivity of the firm. Abatement investment is affected by pollution and quadratic adjustment costs. Both, pollution and adjustment costs influence the impact of policies aimed at promoting abatement investment. Specifically, an increase in tax rate reduces the marginal revenue of capital, inducing a lower investment rate for any given pollution level. Therefore, if corrective taxes have the indirect effect of reducing the after-tax return of private capital, the firm can find optimal to lower investment rather than raise it. Conversely, subsidies will stimulate abatement investment.

Nonetheless, the effect of these policies on the value of the firm is indeterminate. The main reason is that when taxes or subsidies change the (present value of) rents change in a non-monotonic way because of the form of adjustment cost function. Thus, environmental policies have an ambiguous effect on the value of the firm, depending on the magnitudes of parameters affecting the operating profit and the rents. For the same reasons, even an increase of pollution uncertainty has ambiguous effects on the value of the firm. This ambiguity has important empirical implications since it implies the average $\hat{q}$ value of the firm changes in an unpredictable way.

This has important implications for the relationship among marginal q and average $\hat{q}$ value of the firm and the rate of investment. At each instant of time the firm chooses the rate of investment. In theory, as we saw above, investment depends on the marginal value of the firm, q. But the average value of the firm $\hat{q}$, namely the ratio of the market value of the existing capital to its replacement cost

$$\hat{q} = \frac{V(p)}{(b-\eta)M} = \frac{qM + G(p)}{(b-\eta)M} \tag{24}$$

is the only operational index that the firm can observe and use during investment process. Comparing average $\hat{q}$ with marginal q, we notice that the change in $\hat{q}$, in response of a change in either taxes or subsidy, is ambiguous, while that of marginal q is determinate. Thus, since environmental policies have ambiguous effects on $G(p)$, they will have indefinite effects on $\hat{q}$, as well. Thus, as far as investment decisions are based on the average value of the firm rather than its marginal value, the effects of changes in taxes or subsidies on investment is unpredictable.

Finally, in this paper we studied the evolution of both the value of firm and abatement investment under the assumption of ecological uncertainty. But, the analysis of this paper is easily extended to allow uncertainty in price and/or technology. Specifically, increased variance in the price or technological progress leads to an increase in the operating profit, leading to an increase in the optimal rate of investment. However, in this extended framework, the environmental policies are again ambiguous because of the quadratic adjustment costs. Therefore, the present framework does deliver predictions that appear to be robust when environmental and economic consideration suggest significant nonlinearities in behavior.

References

Abel, A.B. and J. C. Eberly, (1997). An exact solution for the investment and value of a firm facing uncertainty, adjustment costs, and irreversibility. *Journal of Economic Dynamics and Control* 21, 831–852.

Ansar, J. and R. Spark (2009). "The experience curve, option value and the energy paradox", *Energy Policy*, 37, 1012-1020.

Bretschger, L. and S. Smulders (2007). "Introduction to sustainable resource use and economic dynamics", in L. Bretschger and S. Smulders (eds), *Sustainable Resource Use and Economic Dynamics*, Berlin, Springer, 1-16.

Dixit, A. and R. S. Pindyck (1994). *Investment under Uncertainty.* Princeton University Press, Princeton New Jersey.

Egli, H. and T. M. Steger (2007). "A dynamic model of the environmental Kuznets curve: Turning point and public policy", in L. Bretschger and S. Smulders (eds), *Sustainable Resource Use and Economic Dynamics*, Berlin, Springer, 17-34.

Fisher, A. (2000). "Investment under uncertainty and option value on environmental economics", *Resource and Energy Economics,* 22, 197-204.

Lin, T. T., C. Ko and H. Yeh (2007). "Applying real options in investment decisions relating to environmental pollution", *Energy Policy,* 35, 2426-2432.

Lin, T. T. and S. Huang (2010). "An entry and exit model on the energy-saving investment strategy with real options", *Energy Policy,* 38, 794-802.

Lin, T. T. and S. Huang (2011). "Application of the modified Tobin's q to an uncertain energy-saving project with the real options concept", *Energy Policy,* 39, 408-420.

Mulder P. and H. L. F. De Groot (2007). "Sectoral energy and labor productivity Convergence", in L. Bretschger and S. Smulders (eds), *Sustainable Resource Use and Economic Dynamics*, Berlin, Springer, 165-190.

Pindyck, R. S. (2000). "Irreversibilities and the timing of environmental policy", *Resource and Energy Economics,* 22, 233-259.

Pindyck, R. S. (2002). "Optimal timing problems in environmental economics", *Journal of Economic Dynamics & Control,* 26, 1677-1697.

Saltari, E. and G. Travaglini (2011). "The effects of environmental policies on the abatement investment decisions of a green firm", forthcoming *Resource and Energy Economics,* doi:10.1016/j.reseneeco.2011.02.001.

Shadbegiana, R. J. and W. B. Gray (2005). "Pollution abatement expenditures and plant-level productivity: A production function approach", *Ecological Economics,* 54, 196-208.

Smulders, S. and R. Gradus (1996). “Pollution abatement and long-term growth”, *European Journal of Political Economy,* 12, 505-532.

Soretz, S. (2007). “Efficient dynamic pollution taxation in an Uncertain Environment”, in L. Bretschger and S. Smulders (eds), *Sustainable Resource Use and Economic Dynamics*, Berlin, Springer, 101-125.

Appendix

In this appendix we determine the value of the firm as a function of the aggregate pollution p. We assume that $V(M,p)$ is a linear function of M so that

$$V(M,p) = q(p)M + G\left(p\right) \tag{A.1}$$

To determine the functions $q(p)$ and $G\left(p\right)$, substitute equation (A.1) in (11) and use the expression for ψ in (16) to obtain:

$$\begin{aligned} rq(p)M + rG\left(p\right) = {} & hp^{\theta}M + \left[q-(b-\eta)\right]^2 \frac{1}{2\gamma} - \delta Mq + \\ & + \mu pq_p M + \mu pG_p + \frac{1}{2}\sigma^2 p^2 q_{pp} M + \frac{1}{2}\sigma^2 p^2 G_{pp} \end{aligned} \tag{A.2}$$

This differential equation must hold for all M. Therefore, the term multiplying M on the left-hand side must equal the sum of the terms multiplying M on the right-hand side. In addition, the term not involving M on the left-hand side must equal the sum of the terms not involving M on the right-hand side. This feature implies that

$$rq(p) = hp^{\theta} - \delta q + \mu pq_p + \frac{1}{2}\sigma^2 p^2 q_{pp} \tag{A.3}$$

and

$$rG\left(p\right) = \left[q-(b-\eta)\right]^2 \frac{1}{2\gamma} + \mu pG_p + \frac{1}{2}\sigma^2 p^2 G_{pp} \tag{A.4}$$

These equations have a recursive structure. The differential equation for $q(p)$ in (A.3) does not depend on $G\left(p\right)$, but the differential equation for $G\left(p\right)$ in (A.4) depends on $q(p)$. Hence, we employ a two steps procedure: we will solve equation (A.3) for $q(p)$, and then proceed to solve equation (A.4) for $G\left(p\right)$.

Equation (A.3) provides the marginal q value of installed abatement capital. It can be rewritten as

$$\frac{1}{2}\sigma^2 p^2 q_{pp}^{(i)} + \mu pq_p^{(i)} - (r+\delta)\, q^{(i)}(p) + hp^{\theta} = 0 \tag{A.5}$$

Simple substitution shows that the homogeneous part of the equation has solution of the form $q(p) = Cp^{\lambda}$, whereas the particular solution has the form

$$\frac{hp^{\theta}}{r+\delta-\mu\theta-\frac{1}{2}\sigma^2\theta\left(\theta-1\right)} \tag{A.6}$$

Intuitively, the particular solution is the present value of the marginal revenue hp^{θ}, and the denominator is the growth-adjusted discount rate, assumed that strictly positive to assure that the firm has a finite value. Thus, the general solution of equation (A.5) is:

$$q(p) = C_1 p^{\lambda_1} + C_2 p^{\lambda_2} + \frac{hp^{\theta}}{r + \delta - \mu\theta - \frac{\sigma^2}{2}\theta(\theta - 1)} \tag{A.7}$$

The two terms in equation (A.7) $C_1 p^{\lambda_1}$ and $C_2 p^{\lambda_2}$ are the speculative components of value. We can rule them out by invoking economic considerations. Since $\lambda_2 < 0$, that power of p goes to infinity as pollution p goes to zero. To prevent the value from diverging, we must set the corresponding coefficient $C_2 = 0$. The other root is $\lambda_1 > 1$ and this implies that the marginal value of the green firm rises as the value of aggregate pollution increases. Restricting our attention to the fundamental value of $q(p)$ implies that $C_1 = C_2 = 0$. Hence:

$$q(p) = Bp^{\theta}, \text{ where } B \equiv \frac{h}{r + \delta - \mu\theta - \frac{\sigma^2}{2}\theta(\theta - 1)} \tag{A.8}$$

We will employ this solution to compute the value of the intercept term $G(p)$ which represents the present value of rents associated to the adjustment cost function. It is determined by the differential equation (A.4). Since the rents are $\frac{1}{2\gamma}[q - (b - \eta)]^2$, the second order differential equation is:

$$rG(p) = \frac{1}{2\gamma}[q - (b - \eta)]^2 + \mu p G_p + \frac{1}{2}\sigma^2 p^2 G_{pp} \tag{A.9}$$

The solution of the homogenous part is:

$$G(p) = D_1 p^{z_1} + D_2 p^{z_2} \tag{A.10}$$

where $z_1 > 1$ and $z_2 < 0$, and D_1 and D_2 are constants to be determined. Again, the solution of the homogenous part represents bubbles unrelated to the fundamentals and can thus can be eliminated. Hence, the solution of (A.9) is given by the particular solution:

$$G(p) = \frac{1}{2\gamma}\left[\frac{q^2}{r - 2\theta\mu - \theta(2\theta - 1)\sigma^2} - \frac{2q(b - \eta)}{r - \theta\mu - \frac{\theta}{2}(\theta - 1)\sigma^2} + \frac{(b - \eta)^2}{r}\right] \tag{A.11}$$

where $G(p)$ indicates the present value of the rents accruing to the firm because of the presence of adjustment costs. It can be verified, by direct substitution, that this expression satisfies equation (A.9). Further $G(p) = 0$ when $q = b - n$, because at this point the rents are equal to zero.

Adding the two components of the value of the firm, we get the solution:

$$V(M, p) = q(p)M + G(p) \tag{A.12}$$

which is the sum of two parts: the present value of profits associated to the assets in place, $q(p)M$, and the present value $G(p)$ of the rents accruing to the firm because of the presence of adjustment costs.